COHOMOLOGY OF ALGEBRAIC VARIETIES

CIRM, LUMINY, 15 - 19 OCTOBER, 2018

Abstracts

Joseph Ayoub

New realisations for motives over positive characteristic fields

Abstract: Let K be an algebraically closed field of characteristic zero endowed with a rank one valuation having residue field k of characteristic $p > 0$. We construct a realisation functor

$$R : DM(k) \to D(A)$$

from the category of Voevodsky motives to the derived category of A-modules having the following properties. (1) The ring A is a K-algebra which has an explicit description reminiscent to the description of algebras of abstract periods. (2) The functor R is monoidal and takes a geometric motive to a perfect complex of A-modules. (3) There are comparison maps relating R with the classical realisations. In particular, there is a comparison isomorphism to rigid cohomology which on the algebra A is given by an explicit integration map.

If time permits, we also state some conjectures about the ring A and the realisation functor R.

Patrick Brosnan

Fourier transform and sheaves associated to families of Hessenberg varieties

Abstract: Hessenberg varieties are certain closed subvarieties of flag varieties defined by Lie theoretic conditions and depending on a Lie algebra element. By varying the Lie algebra element we get a projective family of varieties which is flat over the locus of regular elements of the Lie algebra. In particular, we get a monodromy action on the cohomology of the smooth fibers. When the Lie algebra is $\mathfrak{gl}_n$, this monodromy action has a combinatorial interpretation. In fact, there is a combinatorial formula for it. Moreover, there are long-standing conjectures due to Stanley and Stembridge concerning a certain type of positivity in the formula. I will explain joint work with Tim Chow which explains how to use Fourier transform to compute the monodromy action (in all types), and also proves that the total direct image of the constant sheaf is a sum of IC sheaves of local systems supported in codimension 0.

Hélène Esnault
Cohomological dimension in pro-ℓ-towers

Abstract: We give a proof without use of perfectoid geometry of Scholze's vanishing theorem for étale cohomology with $\mathbb{F}_\ell$-coefficients beyond the dimension of projective varieties in a specific pro-ℓ tower in characteristic not equal to ℓ.

Javier Fresán
Symmetric power moments of Kloosterman sums, I

Abstract: We construct motives over the rational numbers associated with symmetric power moments of Kloosterman sums, and prove that their L-functions extend meromorphically to the complex plane and satisfy a functional equation conjectured by Broadhurst and Roberts. Although the motives in question turn ou to be classical, the strategy consists in first realizing them as exponential motives and computing their Hodge numbers by means of the irregular Hodge filtration. We show that all Hodge numbers are either zero or one, which implies potential automorphicity thanks to recent results of Patrikis and Taylor. The first talk will be concerned with the arithmetic aspects and in the second one we will present the Hodge theoretic computations. Joint work with Claude Sabbah and Jeng-Daw Yu.

Ofer Gabber
Remarks on vanishing cycles and comparison of oriented products and rigid toposes

Abstract: to be announced

Annette Huber-Klawitter
Periods of* 1*-motives

Abstract: (joint work with G. Wüstholz) Roughly, 1-dimensional periods are the complex numbers obtained by integrating a differential form on an algebraic curve over $\bar{\mathbf{Q}}$ over a suitable domain of integration. One of the alternative characterisations is as periods of Deligne 1-motives.

We clear up the linear relations between these numbers, proving Kontsevich's version of the period conjecture for 1-dimensional periods. In particular, a 1-dimensional period is shown to be algebraic if and only if it is of the form $\int_\gamma(\phi + df)$ with $\int_\gamma \phi = 0$. We also get formulas for the spaces of periods of a given 1-motive, generalising Baker's theorem on logarithms of algebraic numbers.

The proof is based on a version of Wüstholz's analytic subgroup theorem for 1-motives.

June Huh

Kazhdan-Lusztig theory for matroids

Abstract: There is a remarkable parallel between the theory of Coxeter groups (think of the symmetric group or the dihedral group) and matroids (think of your favorite graph or point configuration), based on a combinatorial cohomology theory. I will give an overview of the similarity, and report on my joint work with Tom Braden, Jacob Matherne, Nick Proudfoot, and Botong Wang on a cohomological approach to classical conjectures in combinatorics.

Peter Jossen

Cohomology of varieties with potential

Abstract: There are several interesting cohomology theories attached to pairs (X, f), consisting of an algebraic variety X and a regular function f (the potential) on X. I will present one of these cohomology theories, which takes values in a tannakian category Perv_0, whose objects are certain perverse sheaves on the affine line. My plan is then to explain how to compute tannakian Galois groups in this category Perv_0, and to explain how these groups are related to motivic Galois groups of motives attached to pairs (X, f).Joint work with J. Fresán.

Emanuele Macrì

Derived categories of cubic fourfolds and non-commutative K3 surfaces

Abstract: The derived category of coherent sheaves on a cubic fourfold has a subcategory which can be thought as the derived category of a non-commutative K3 surface. This subcategory was studied recently in the work of Kuznetsov and Addington-Thomas, among others. In this talk, I will present joint work in progress with Bayer, Lahoz, Nuer, Perry, Stellari, on how to construct Bridgeland stability conditions on this subcategory. This proves a conjecture by Huybrechts, and it allows to start developing the moduli theory of semistable objects in these categories, in an analogue way as for the classical Mukai theory for (commutative) K3 surfaces. I will also discuss a few applications of these results.

Alexander Merkurjev

Versal torsors and retracts

Abstract: Let G be an algebraic group over a field and p a prime integer. We introduce the notion of a p-retract rational variety and prove that if $Y \to X$ is a p-versal G-torsor, then the classifying space BG is a stable p-retract of X. It follows that BG is p-retract rational if and only if there is a p-versal G-torsor $Y \to X$ with X a rational variety, that is all G-torsors over infinite fields are rationally parameterized. In particular, for such groups G the unramified Galois cohomology group is trivial.

Matthew Morrow

Relative integral p-adic Hodge theory

Abstract: Given a smooth scheme X over the ring of integers of a p-adic field, we introduce the notion of a relative Breuil-Kisin-Fargues module M on X. Each such M simultaneously encodes the data of a lisse étale sheaf, a module with flat connection, and a crystal, whose cohomologies are then intertwined by a relative form of the A_{inf} cohomology introduced in "Integral p-adic Hodge theory" by Bhatt-M-Scholze. They are moreover closely related to other work in relative p-adic Hodge theory, notably Faltings small generalised representations and his relative Fontaine Lafaille theory.
Joint with Takeshi Tsuji.

Andrei Negut

Hilbert schemes of K3 surfaces

Abstract: We give a geometric representation theory proof of a mild version of the Beauville-Voisin Conjecture for Hilbert schemes of K3 surfaces, namely the injectivity of the cycle map restricted to the subring of Chow generated by tautological classes. Although other geometric proofs of this result are known, our approach involves lifting formulas of Lehn and Li-Qin-Wang from cohomology to Chow, and using them to quickly solve the problem by invoking the irreducibility criteria of Virasoro algebra modules, due to Feigin-Fuchs.
Joint work with Davesh Maulik.

Wiesława Nizioł

Comparison theorems for rigid analytic varieties

Abstract: I will discuss p-adic comparison theorems for rigid analytic varieties.
This is joint work with Pierre Colmez.

Stefan Patrikis

Deformations of Galois representations: examples and applications

Abstract: The deformation theory of Galois representations, and in particular the question of how we can lift mod p representations of the Galois group of a number field to p-adic representations, is one of the main tools for understanding the conjectured modularity and motivic origin of p-adic representations. I will survey some illustrative examples in this subject and describe some applications to the construction of motives with exceptional Galois groups.
Some of this is joint work with N. Fakhruddin and C. Khare, and some is joint with G. Boxer, F. Calegari, M. Emerton, B. Levin, and K. Madapusi Pera.

Claude Sabbah

Symmetric power moments of Kloosterman sums, II

Abstract: We construct motives over the rational numbers associated with symmetric power moments of Kloosterman sums, and prove that their L-functions extend meromorphically to the complex plane and satisfy a functional equation conjectured by Broadhurst and Roberts. Although the motives in question turn out to be classical, the strategy consists in first realizing them as exponential motives and computing their Hodge numbers by means of the irregular Hodge filtration. We show that all Hodge numbers are either zero or one, which implies potential automorphicity thanks to recent results of Patrikis and Taylor. The first talk will be concerned with the arithmetic aspects and in the second one we will present the Hodge theoretic computations.
Joint work with Javier Fresan and Jeng-Daw Yu.

Takeshi Saito

Characteristic cycle of constructible sheaves and restriction to curves

Abstract: The characteristic cycle is defined for a constructible sheaf on a smooth scheme over a perfect field as a cycle on the cotangent bundle. After briefly recalling its basic properties, I explain that it is determined by its rank and the conductor of the restrictions to curves.

Christian Schnell

Extension theorems for holomorphic forms on complex spaces

Abstract: Suppose we have an algebraic (or holomorphic) differential form, defined on the smooth locus of an algebraic variety (or analytic space). Under what conditions does it extend to an algebraic (or holomorphic) differential form on a resolution of singularities? In 2011, Greb, Kebekus, Kovacs, and Peternell proved that such an extension always exists on algebraic varieties with klt singularities. I will explain how to generalize their result to a much larger class of singular spaces, with the help of Hodge modules and the decomposition theorem.
This is joint work with Kebekus.

Qizheng Yin

Special subvarieties in holomorphic symplectic varieties

Abstract: The classical Bogomolov-Mumford theorem states that every algebraic K3 surface admits a rational curve. In my talk I shall discuss generalizations of this theorem to holomorphic symplectic varieties, namely the search of special subvarieties swept out by rational/constant-cycle subvarieties. Various results and open questions will be presented, and the relation to the Beauville-Voisin conjectures will be mentioned.
Based on joint work with G. Oberdieck, J. Shen, and X. Zhao.

Weizhe Zheng

Compatible systems along the boundary

Abstract: A theorem of Deligne says that compatible systems of ℓ-adic sheaves on a smooth curve over a finite field are compatible along the boundary. I will present an extension of Deligne's theorem to schemes of finite type over the ring of integers of a local field. This has applications to the equicharacteristic case of some conjectures on ℓ-independence. I will also discuss the relationship with compatible wild ramification.
This is joint work with Qing Lu.