On Grothendieck's homotopy hypothesis

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- Defining a path space for an ∞-groupoid (an ∞-groupoid of arrows).
- Constructing a "folk" Quillen model category of ∞-groupoids.
- Proving that ∞-groupoids "up to equivalence" are equivalent to homotopy types. (The homotopy hypothesis)

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However, it has been observed (Brunerie, Altenkirch, Rypáček) that Grothendieck's definition of ∞-groupoids can be formalized within the framework of homotopy type theory, which is not the case of any other definition of ∞-groupoid that we know of.

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Note: Grothendieck's definition has been extended by G.Maltsioniotis to a definition of weak ∞-categories, and S.Mimram and E.Finster recently gave a syntactic presentation of this definition that should also be interpretable in homotopy type theory.

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Note: Grothendieck's definition has been extended by G.Maltsioniotis to a definition of weak ∞-categories, and S.Mimram and E.Finster recently gave a syntactic presentation of this definition that should also be interpretable in homotopy type theory. But in this talk, we will focus on the case of ∞-groupoids.

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- We will give an "algebraic" and "globular" definition of weak ∞-groupoids, for which the homotopy hypothesis can be proved and such that this definition can be formalized within type theory.
- Under a simple looking technical conjecture regarding Grothendieck's definition, one can prove the original version of the homotopy hypothesis.

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- Type theory is "globular". But the formalism of type theory allows to do a lot more than Grothendieck's definitions: Any type has the structure of a ∞-groupoid (Lumsdaine, Van den berg, Garner, Bourke), but it also have a path space (its total identity type) which also has the structure of an ∞-groupoids.
- (Ara, Grothendieck) If X is a cofibrant object in a Quillen model category $\mathcal{C}$ where every objects is fibrant, one can construct an adjunction:

$$L : \infty - Gpd \leftrightarrows \mathcal{C} : \pi_\infty([X, \_])$$

such that $L(*) = X$. This should be thought of as a universal property of the category of weak ∞-groupoids.

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- Every trivial fibration admit a section.

Theorem (Brown, Moerdijk, Van den Berg)

The homotopy category of a path category $\mathcal{C}$ can be described as the category $\mathcal{C}$ with homotopy class of maps between the objects, where two maps f, g are homotopic if the map $(f, g) : X \to Y \times Y$ can be extended as:

$$X \to PY \twoheadrightarrow Y \times Y$$

The weak equivalences are exactly the maps that are invertible in the homotopy category.

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"Weak identity types" = Identity types where the J-computation rules is replaced by a weaker "introduction rules" which provide a higher identity term instead of the usual definitional equality.

Conversely, he also showed that one can give an interpretation of weak identity types in any path category. Essentially, path categories are the categorical models of type theory with weak identity types.

Definition

A Cylinder category is a category endowed with a class of maps called cofibrations and a class of maps called weak equivalences such that:

- Cofibrations and weak equivalences are stable under compositions and contains all isomorphisms.
- Weak equivalences satisfies the 2-out-of-6 property: if $f \circ g$ and $g \circ h$ are weak equivalences then f, g and h are weak equivalences.
- The category has an initial object 0 and for all other object X the map $0 \to X$ is a cofibration.
- Pushout of cofibrations exists and are cofibrations.
- Pushout of trivial cofibrations are trivial cofibrations.
- For every object X there is factorization of the co-diagonal map:

$$X \coprod X \hookrightarrow IX \overset{\sim}{\to} X$$

- Every trivial cofibration admit a retraction.

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We will see that (up to size problems/change of universe) those are essentially the only examples:

Any small cylinder category admit a fully faithful universal embedding into a Quillen (semi)model category, which preserves cofibrations, weak equivalences, cylinder objects , pushout along cofibrations, initial objects etc...

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Theorem (H.)

If $\mathcal{C}$ is a small cylinder categories, $\widetilde{\mathcal{C}}$ is a combinatorial semi-model category in which every object is fibrant.

Take the “free”(syntactical) type theory on one type $*$ and with weak identity types. The opposite of its context category $\mathcal{C}_{\mathbb{T}}$ is a cylinder category. One define the category of “type theoretic ∞-groupoids” as $\widetilde{(\mathcal{C}_{\mathbb{T}})^{op}}$.

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A Type theoretic ∞-groupoid is a globular set where all the operations definable on a type in type theory with weak identity type are defined.

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- Trivial fibrations are exactly the arrows that are fibrations and weak equivalences.
- For arrows with a cofibrant domain, being a trivial cofibration is the same as being a cofibration and a weak equivalence.

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For any semi-model category $\mathcal{D}$ whose objects are all fibrants there is an equivalence of categories between morphisms $\mathcal{C} \to Cof(\mathcal{D})$ and left Quillen functor $\widetilde{\mathcal{C}} \to \mathcal{D}$.

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In some sense, $\widetilde{\mathcal{C}}$ is the free semi-model category generated by $\mathcal{C}$ (among category whose objects are all fibrant), So if one wants to construct "free" semi-model category one just need to construct "free cylinder categories".

Definition

A pre-cylinder categories is a category with two classes of maps: weak equivalences and cofibrations such that:

- Cofibrations and weak equivalences are stable under compositions and contains all isomorphisms.
- Weak equivalences satisfies the 2-out-of-6 property: if $f \circ g$ and $g \circ h$ are weak equivalences then f, g and h are weak equivalences.
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- $\mathcal{C}$ satisfies the gluing lemma/cube lemma/Waldhausen axiom.

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If $f : \mathcal{C} \to \mathcal{D}$ is a morphism between (fibrant) cylinder categories which induces an equivalence between the homotopy categories then the Quillen functor $\widetilde{\mathcal{C}} \to \widetilde{\mathcal{D}}$ induced by f is a Quillen equivalence.

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Corollary: each cylinder coherator produces a combinatorial semi-model category of ∞-groupoids, any two such categories are equivalent.

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- The opposite of the syntactic category of the type theory with propositional identity type is a cylinder coherator. It defines the "type theoretic ∞-groupoids" mentioned before.
- The category of Grothendieck ∞-groupoid for a Grothendieck coherator can be describe as the completion of a pre-cylinder category. If one can proves that this pre-cylinder category is a cylinder category then we know that it is a cylinder coherator and hence this would implies Grothendieck Homotopy hypothesis.

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Let X be a a weak Grothendieck ∞-groupoid, which is finitely generated (in a polygraphic sens). Let a be an n-arrow of X, and let X^+ be the weak ∞-groupoid obtained from X by freely adding one n-arrow a' parallel to a and one $n+1$ arrow between a and a'.

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Then the natural map $X \to X^+$ is an equivalence (a bijection on the π_n for all n).

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