

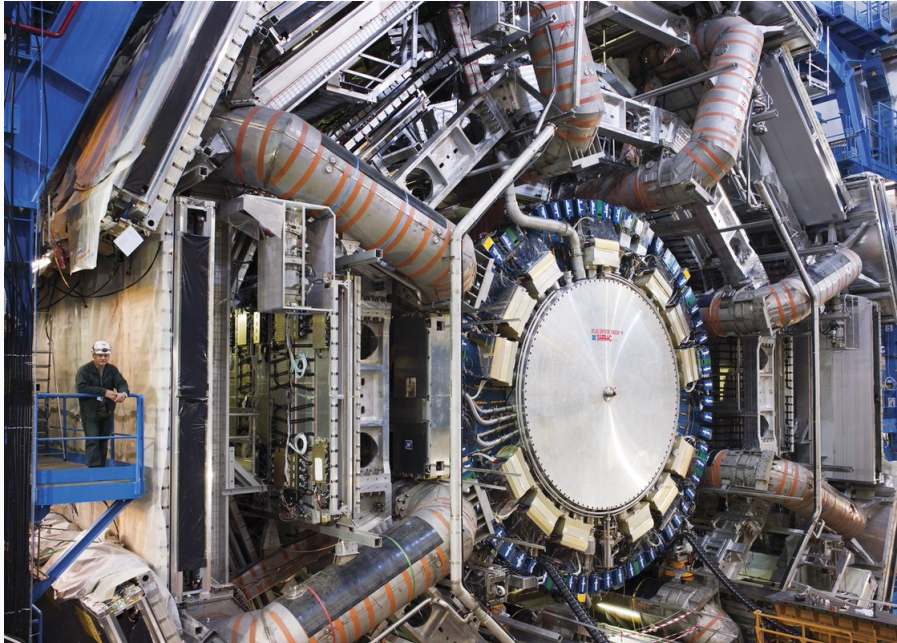
Yaël Frégier

Artois University

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Quantum field theory

Particle physics : 2 ~ 5 billions €/years



N-point functions

$$\langle \Phi_1(x) \dots \Phi_n(x) \rangle$$

N-point functions

$$\langle \Phi_1(x) \dots \Phi_n(x) \rangle =$$

N-point functions

$$\langle \Phi_1(x) \dots \Phi_n(x) \rangle = \int \Phi_1(x) \dots \Phi_n(x) d\mu$$

N-point functions

$$\begin{aligned}\langle \Phi_1(x) \dots \Phi_n(x) \rangle &= \int \Phi_1(x) \dots \Phi_n(x) d\mu \\ &= \int \Phi_1(x) \dots \Phi_n(x) e^{-S(x)} dx\end{aligned}$$

N-point functions in maths

Gromov-Witten

Gromov-Witten, Jones Polynomial

Gromov-Witten, Jones Polynomial, Mirror symmetry

Gromov-Witten, Jones Polynomial, Mirror symmetry, Kontsevich knot invariant

N-point functions in maths

Gromov-Witten, Jones Polynomial, Mirror symmetry, Kontsevich knot invariant, Kontsevich formality map

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$$\langle \Phi_1(x) \dots \Phi_n(x) \rangle$$

Problem with definition

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\mathcal{M}

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\mathcal{M} = *space of fields*

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\mathcal{M} = *space of fields*

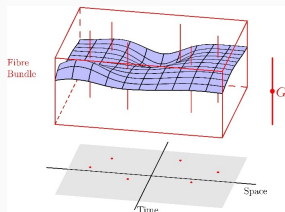
$\Gamma(M, P)$

Problem with definition

$$\langle \Phi_1(x) \dots \Phi_n(x) \rangle = \int_{\mathcal{M}} \Phi_1(x) \dots \Phi_n(x) d\mu$$

\mathcal{M} = *space of fields*

$\Gamma(M, P)$



$$S(x)$$

$$S(x) =$$

$$S(x) = C$$

$$S(x) = C +$$

$$S(x) = C + a_1 x$$

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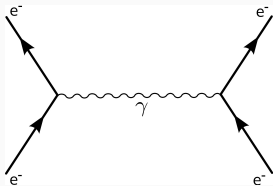
$$S(x) = C + a_1x + a_2x^2$$

$$S(x) = C + a_1x + a_2x^2 +$$

$$S(x) = C + a_1x + a_2x^2 + \textit{higher orders}$$

reduction to the kinetic term

$$S(x) = C + a_1x + a_2x^2 + \textit{higher orders}$$



$$S(x) = C + a_1x + a_2x^2$$

$$S(x) = a_1x + a_2x^2$$

$$S(x) = a_2 x^2$$

$$\langle \Phi_1(x) \dots \Phi_n(x) \rangle$$

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$$\begin{aligned} \langle \Phi_1(x) \dots \Phi_n(x) \rangle &:= \int \Phi_1(x) \dots \Phi_n(x) d\mu \\ &:= \end{aligned}$$

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$\langle x \rangle$

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$$\frac{\partial}{\partial B_i} e^{(B, X)} = x_i e^{(B, X)}$$

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$$Z(B) = \int e^{-\frac{1}{2}(X,AX) + (B,X)} dx$$

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$$\frac{\partial}{\partial B_i} Z(B) = \int x_i e^{-\frac{1}{2}(X, AX) + (B, X)} dx$$

$$\frac{\partial}{\partial B_i} Z(B)|_{B=0} = \int x_i e^{-\frac{1}{2}(X, AX) + (B, X)} dx$$

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$$\frac{\partial}{\partial B_i} Z(B)|_{B=0} = \langle x_i \rangle$$

$$\frac{\partial}{\partial B_{i_1}} Z(B) =$$

$$\frac{\partial}{\partial B_{i_1}} Z(B) = \int x_{i_1} e^{-\frac{1}{2}(X, AX) + (B, X)} dx$$

$$\frac{\partial}{\partial B_{i_1}} \cdots \frac{\partial}{\partial B_{i_k}} Z(B) = \int x_{i_1} e^{-\frac{1}{2}(X, AX) + (B, X)} dx$$

$$\frac{\partial}{\partial B_{i_1}} \cdots \frac{\partial}{\partial B_{i_k}} Z(B) = \int x_{i_1} \cdots x_{i_k} e^{-\frac{1}{2}(X, AX) + (B, X)} dx$$

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$$\Phi_1\left(\frac{\partial}{\partial B}\right) \cdots \Phi_n\left(\frac{\partial}{\partial B}\right) Z(B)|_{B=0} = \int x_{i_1} \cdots x_{i_k} e^{-\frac{1}{2}(X, AX)} dx$$

$$\Phi_1\left(\frac{\partial}{\partial B}\right)\dots\Phi_n\left(\frac{\partial}{\partial B}\right)Z(B)|_{B=0} = \int \Phi_1(x)\dots\Phi_n(x)e^{-\frac{1}{2}(X,AX)}dx$$

A 50 billion € trick

$$\langle \Phi_1(x) \dots \Phi_n(x) \rangle$$

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$$\langle \Phi_1(x) \dots \Phi_n(x) \rangle = \Phi_1\left(\frac{\partial}{\partial B}\right) \dots \Phi_n\left(\frac{\partial}{\partial B}\right) Z(B)|_{B=0}$$

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$$Z(B)$$

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$$Z(B) =$$

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$$Z(B) = \int e^{-\frac{1}{2}(X, AX) + (B, X)} dx$$

$$\langle \Phi_1(x) \dots \Phi_n(x) \rangle = \Phi_1\left(\frac{\partial}{\partial B}\right) \dots \Phi_n\left(\frac{\partial}{\partial B}\right) Z(B)|_{B=0}$$

$$Z(B) = e^{\frac{1}{2}(B, A^{-1}B)} \int e^{-\frac{1}{2}(x, Ax)} dx$$

$$\langle \Phi_1(x) \dots \Phi_n(x) \rangle = \Phi_1\left(\frac{\partial}{\partial B}\right) \dots \Phi_n\left(\frac{\partial}{\partial B}\right) Z(B)|_{B=0}$$

$$Z(B) = e^{\frac{1}{2}(B, A^{-1}B)} Z(0)$$

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$$Z(B)/Z(0) = e^{\frac{1}{2}(B, A^{-1}B)} Z(0)$$

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$$Z(B)/Z(0) = e^{\frac{1}{2}(B, A^{-1}B)}$$

$$S(x + \epsilon V) =$$

$$S(x + \epsilon V) = S(x)$$

$$S(x + \epsilon V) - S(V) =$$

$$S(x + \epsilon V) - S(V) = 0$$

$$dS(V) = 0$$

$$AV = 0$$

$$AV = 0$$

$$A^{-1}?$$

fields

fields

words

fields

gauge symmetry

words

fields

gauge symmetry

words

rewriting

fields

gauge symmetry

gauge fixing

words

rewriting

fields

gauge symmetry

gauge fixing

words

rewriting

normal form

fields

gauge symmetry

gauge fixing

BRST/BV

words

rewriting

normal form

fields

gauge symmetry

gauge fixing

BRST/BV

words

rewriting

normal form

Squier resolution

If

If

$$\mathcal{M} = G \times B$$

If

$$\mathcal{M} = G \times B$$

f

If

$$\mathcal{M} = G \times B$$

f, dx

If

$$\mathcal{M} = G \times B$$

f, dx, G – invariants

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f, dx, G – invariants

$$I =$$

If

$$\mathcal{M} = G \times B$$

f, dx, G – invariants

$$I = \int_{\mathcal{M}} f(x) dx$$

If

$$\mathcal{M} = G \times B$$

f , dx , G – invariants

$$I = \int_{\mathcal{M}} f(x) dx = \int_{\overline{\mathcal{M}}} \bar{f}(\bar{x}) \overline{dx}$$

$$\int_{\mathcal{M}} f(x) dx$$

$$\int_{\mathcal{M}} f(x) \delta_0(F(x)) dx$$

$$\int_{\mathcal{M}} f(x) \delta_0(F(x)) \det(dF(x)) dx$$

$$\int_{\mathcal{M}} f(x) e^{i\langle \lambda, F(x) \rangle} \det(dF(x)) dx$$

$$\int_{\mathcal{M} \times \Gamma^*} f(x) e^{i\langle \lambda, F(x) \rangle} \det(dF(x)) dx$$

$$\int_{\mathcal{M} \times \Gamma^*} f(x) e^{i\langle \lambda, F(x) \rangle} \det(\Lambda) \, dx$$

$$\int_{\mathcal{M} \times \Gamma^*} f(x) e^{i\langle \lambda, F(x) \rangle} e^{\langle \bar{c}, \Lambda c \rangle} dx$$

$$\int_{\mathcal{M} \times \Gamma^* \times \prod \Gamma} f(x) e^{i\langle \lambda, F(x) \rangle} e^{\langle \bar{c}, \Lambda c \rangle} dx$$

$$\int_{\mathcal{M} \times \Gamma^* \times \prod \Gamma \times \prod \Gamma^*} f(x) e^{i\langle \lambda, F(x) \rangle} e^{\langle \bar{c}, \Lambda c \rangle} dx$$

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$\mathcal{F}un(\overline{\mathcal{M}})$

$$\int_{\mathcal{M} \times \Gamma^* \times \prod \Gamma \times \prod \Gamma^*} f(x) e^{i\langle \lambda, F(x) \rangle} e^{\langle \bar{c}, \Lambda c \rangle} dx$$
$$\mathcal{F}un(\mathcal{M} \times \Gamma^* \times \prod \Gamma \times \prod \Gamma^*) \quad \mathcal{F}un(\overline{\mathcal{M}})$$

$$\int_{\mathcal{M} \times \Gamma^* \times \Pi\Gamma \times \Pi\Gamma^*} f(x) e^{i\langle \lambda, F(x) \rangle} e^{\langle \bar{c}, \Lambda c \rangle} dx$$

$$\mathcal{F}un(\mathcal{M} \times \Gamma^* \times \Pi\Gamma \times \Pi\Gamma^*) \quad S \in \mathcal{F}un(\overline{\mathcal{M}})$$

$$\int_{\mathcal{M} \times \Gamma^* \times \Pi\Gamma \times \Pi\Gamma^*} f(x) e^{i\langle \lambda, F(x) \rangle} e^{\langle \bar{c}, \Lambda c \rangle} dx$$

$$S_F \in \mathcal{F}un(\mathcal{M} \times \Gamma^* \times \Pi\Gamma \times \Pi\Gamma^*) \quad S \in \mathcal{F}un(\overline{\mathcal{M}})$$

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$$\mathcal{F}un(\mathcal{M} \times \Gamma^* \times \prod \Gamma \times \prod \Gamma^*) \quad \mathcal{F}un(\overline{\mathcal{M}})$$

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$$\mathcal{F}un(\mathcal{M} \times \Gamma^* \times \Pi\Gamma \times \Pi\Gamma^*) \rightarrow \mathcal{F}un(\overline{\mathcal{M}})$$

$$\int_{\mathcal{M} \times \Gamma^* \times \Pi\Gamma \times \Pi\Gamma^*} f(x) e^{i\langle \lambda, F(x) \rangle} e^{\langle \bar{c}, \Lambda c \rangle} dx$$
$$\mathcal{F}un(\mathcal{M} \times \Gamma^* \times \Pi\Gamma \times \Pi\Gamma^*) \xrightarrow{\sim} \mathcal{F}un(\overline{\mathcal{M}})$$

For example, if $S = -\frac{1}{2}(X, AX)$

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For example, if $S = -\frac{1}{2}(X, AX)$

$$S_F = -\frac{1}{2}(X, AX) + i \langle \lambda, F(x) \rangle + \langle \bar{c}, \Lambda c \rangle$$

which is non-degenerate!!!!!!!!!!

Where are we?

Where are we?

If S non-degenerate:

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If S non-degenerate:

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S_F non degenerate on $\mathcal{M}_{BRST} := \mathcal{M} \times \Gamma^* \times \Pi\Gamma \times \Pi\Gamma^*$.

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$\exists?$

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$\exists \delta$

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$\exists \delta$ on $\mathcal{F}un(\mathcal{M}_{BRST})$

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$$H(\mathcal{F}un(\mathcal{M}_{BRST}))$$

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$\exists ? \delta$ on $\mathcal{F}un(\mathcal{M}_{BRST})$

$$H(\mathcal{F}un(\mathcal{M}_{BRST})) \simeq \mathcal{F}un(\overline{\mathcal{M}})?$$

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$\exists \delta$ on $\mathcal{F}un(\mathcal{M}_{BRST})$

$$H(\mathcal{F}un(\mathcal{M}_{BRST})) \simeq \mathcal{F}un(\overline{\mathcal{M}})?$$

Dependence on F ?

Differential on $\mathcal{F}un(\mathcal{M} \times \Gamma^* \times \Pi\Gamma \times \Pi\Gamma^*)$?

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$$\delta_{\Gamma} c^i := f_{jk}^i c^j c^k$$

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$$\delta_{\Gamma}^2 = 0$$

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$$\delta_{\Gamma} c^i := f_{jk}^i c^j c^k$$

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$$\delta_{\mathcal{M}}$$

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$$(\delta_{\mathcal{M}} + \delta_{\Gamma})$$

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$$(\delta_{\mathcal{M}} + \delta_\Gamma)^2 = 0 \iff \text{Jacobi} + \Gamma - \text{action}$$

Differential on $\mathcal{F}un(\mathcal{M} \times \Gamma^* \times \Pi\Gamma \times \Pi\Gamma^*)$?

$$\delta_\Gamma c^i := f_{jk}^i c^j c^k$$

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$$\delta_\Gamma^2 = 0 \iff \text{Jacobi}$$

$$(\delta_{\mathcal{M}} + \delta_\Gamma)^2 = 0 \iff \text{Jacobi} + \Gamma - \text{action}$$

$$\delta_{\mathcal{M}+\Gamma} f = 0 \iff f \text{ invariant}$$

$\delta_\lambda?$

$$\delta_\lambda \delta_{\bar{c}}?$$

$\delta_\lambda \delta_{\bar{c}}?$ **Proposition**

$$\delta_\lambda? \delta_{\bar{c}}?$$

Proposition

$$\int e^{-S_F}$$

$\delta_\lambda? \delta_{\bar{c}}?$

Proposition

$\int e^{-S_F}$ does not depend on F .

$\delta_\lambda? \delta_{\bar{c}}?$

Proposition

$\int g e^{-S_F}$ does not depend on F .

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Proposition

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S_F

$\delta_\lambda? \delta_{\bar{c}}?$

Proposition

$\int g e^{-S_F}$ does not depend on F .

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$$\frac{d}{dt} \int e^{-S_{F_t}}$$

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$\int g e^{-S_F}$ does not depend on F .

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$\int g e^{-S_F}$ does not depend on F .

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$$\frac{d}{dt} \int e^{-S_{F_t}} = \int \frac{d}{dt} e^{-S - \delta\Psi_{F_t}}$$

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$\int g e^{-S_F}$ does not depend on F .

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$$\frac{d}{dt} \int e^{-S_{F_t}} = \int \delta\left(\frac{d}{dt}\Psi_{F_t}\right) e^{-S - \delta\Psi_{F_t}}$$

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$\int g e^{-S_F}$ does not depend on F .

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$$\frac{d}{dt} \int e^{-S_{F_t}} = \int \operatorname{div} \delta \frac{d}{dt} \Psi_{F_t} e^{-S_{F_t}}$$

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Proposition

$\int g e^{-S_F}$ does not depend on F . ..if $\text{div}(\delta) = 0$.

$$S_F = S + \delta \Psi_F$$

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Proposition

$\int g e^{-S_F}$ does not depend on F . ..if $\text{div}(\delta) = 0$, g and S invariants.

$$S_F = S + \delta\Psi_F$$

$$\frac{d}{dt} \int e^{-S_{F_t}} = 0$$

