CATEGORIES IN HOMOTOPY THEORY AND REWRITING

CIRM, LUMINY, SEPTEMBER 25 - 29, 2017

Speakers:

Steve Awodey: *Type theories and polynomial monads*
Frédéric Chapoton: *Operads in combinatorics and representation theory*
Pierre-Louis Curien: *A syntactic approach to polynomial functors and opertopes*
Yaël Frégier: *Physical origin of the BV formalism and shifted symplectic geometry*
Stéphane Gaussent: *Linear rewriting and quiver Hecke algebras*
Simon Henry: *On the homotopy hypothesis and new algebraic models for weak* ∞*-groupoids*
Éric Hoffbeck: *Shuffles of trees*
Martin Hyland: *Dialogues between enrichment and homotopy*
André Joyal: *On tribes and federations*
Joachim Kock: ∞*-operads as polynomial monads*
Victoria Lebed: *Yang-Baxter equation up to homotopy*
Maxime Lucas: *Normalisation strategies revisited*
Peter LeFanu Lumsdaine: *CwA's of equivalences / equivalences of CwA's*
Philippe Malbos: *Squier's theory for monoids and algebras*
Francois Métayer: *Homotopy theory of strict* ω*-categories and its connections with homology of monoids*
Viktoriya Ozornova: *2-Segal sets*
Simona Paoli: *Segal-type models of higher categories*
Dominic Verity: *A complicial compendium*
Marek Zawadowski: *Positive opetopes with contractions form a test category*

Abstracts

Steve Awodey
Type theories and polynomial monads

Abstract: A system of dependent type theory T gives rise to a natural transformation $\mathsf{p} : \mathsf{Terms} \to \mathsf{Types}$ of presheaves on the category Ctx of contexts, termed a "natural model of T". This map p in turn determines a polynomial endofunctor $\mathsf{P} : \widehat{\mathsf{Ctx}} \to \widehat{\mathsf{Ctx}}$ on the category of all presheaves. It can be seen that P has the structure of a monad just if T has Σ-types and a terminal type, and that p is itself a P-algebra just if T has Π-types. I will explain this rather unexpected connection between type theories and polynomial monads, and will welcome any insights from the other participants regarding it.

Frédéric Chapoton
Operads in combinatorics and representation theory

Abstract: It is now quite common to use Hopf algebras to help organise combinatorial objects or to gather representation theories of families of algebras. For instance, there are many Hopf algebras based on graphs or trees and various kinds of generalized symmetric functions. Operads can play a similar role, and we will illustrate this by several examples, some in the combinatorial realm and some related to representations of quivers, in particular some cyclic operads.

Pierre-Louis Curien
A syntactic approach to polynomial functors and opertopes

Abstract: We take the connection between Joyal's polynomial functors and Abbott-Altenkirch-Ghani's containers seriously, and provide a syntactic (and "type-theoretic") notation to describe polynomial functors and polynomial monads, and in particular the two polynomial monads involved in the construction of opetopes (Kock-Joyal-Batanin-Mascari) ("star" and "plus"). My current efforts culminate in a compact description of opetopes as formal languages over a layered alphabet, whose merits must be tested!

Yaël Frégier
Physical origin of the BV formalism and shifted symplectic geometry

Abstract: The aim of this talk is to give a mathematically friendly introduction to the ideas of perturbative expansion of path integrals, in order to motivate and explain the need for the BV formalism and shifted symplectic geometry.

Stéphane Gaussent
Linear rewriting and quiver Hecke algebras

Abstract: In this talk, we will introduce higher linear rewriting in terms of polygraphs. Then we will apply these techniques to study a diagrammatic presentation of quiver Hecke algebras, also known as KLR algebras, for Khovanov, Lauda and Rouquier. We will then explain how to recover the PBW properties of these algebras.

Simon Henry

On the homotopy hypothesis and new algebraic models for weak ∞-groupoids

Abstract: In his manuscript "Pursuing stacks" Grothendieck gave a definition of weak ∞-groupoids and conjectured that the homotopy category of his ∞-groupoids is equivalent to the homotopy category of spaces. This conjecture (the homotopy hypothesis) is still an open problem, and in fact there is lot of expected basic results concerning his definition of ∞-groupoids that are open problems. For these reasons, one prefer nowadays to use less problematic definitions of ∞-groupoids, typically involving simplicial sets, as a starting point for (weak) higher category theory. But Grothendieck's definition also has a lot of good properties not shared by the simplicial approaches: it is considerably closer to the intuitive notion of ∞-category, it has a more general universal property, it is simpler to extend to ∞-categories, but more importantly it can be defined within the framework of the Homotopy type theory program, while the definition of simplicial objects in this framework is one of the important open problem of this program.

In this talk I will discuss a new sort of definitions of weak ∞-groupoids that are inspired from Grothendieck's definition but that do not share any of its problems while retaining most of its advantages. If time permit, I also mention a precise and simple looking technical conjecture which implies that Gorthendieck definition is a special case of our framework, and hence also implies Grothendieck's homotopy hypothesis and most of the conjectures related to Grothendieck's definition.

Éric Hoffbeck

Shuffles of trees

Abstract: We study a notion of shuffle for trees which extends the usual notion of a shuffle for two natural numbers. Our notion of shuffle is motivated by the theory of operads and occurs in the theory of dendroidal sets. We give several equivalent descriptions of the shuffles, and prove some algebraic and combinatorial properties. In addition, we characterize shuffles in terms of open sets in a topological space associated to a pair of trees. This is a joint work with Ieke Moerdijk.

Martin Hyland

Dialogues between enrichment and homotopy

Abstract: One classical world of homotopy is that of chain complexes: in it the notion of homotopy takes a simple algebraic form. Both technically and conceptually a crucial aspect of the theory is the fact that the homotopy and so the derived category are triangulated. The relevant structure manifests itself in terms of weighted limits and colimits of chain complexes. A small amount of higher dimensional universality is used then needed to establish the triangulated structure.

Ideas from homotopy appear in seemingly quite different context in concurrency theory. For example distinct events may be essentially the same when they use indistinguishable names in the same fashion. Phenomena of this kind can be treated using groups of symmetries but in many cases it seems best to think in terms of bisimulation, a notion which is itself homotopy theoretic in character. The outcome is then some theory of structures-with-symmetry which can be seen as a kind of enriched category theory.

I shall sketch both these situations and try to make clear the value of basic enriched category theory in each.

André Joyal
On tribes and federations

Abstract: The notion of tribe is a categorical version of dependant type theory that can model Martin-Löf type theory. We develop the homotopy theory of tribes, including simplicial tribes. A federation of tribes is a Grothendieck fibrations whose fibers are tribes satisfying certain axioms; it can model a type system with two levels of contexts and deduction rules. The cubical type theory recently introduced by Cohen-Coquand-Huber-Mortberg is a federation of tribes.

Joachim Kock
∞-operads as polynomial monads

Abstract: Classical operads can fruitfully be regarded as monoids in the monoidal category of species/analytic functors under the substitution product. We establish an infinity version of this interpretation by developing the theory of polynomial functors over infinity categories. In the infinity world, analytic functors enjoy a representability feature not shared by classical analytic functors and operads: they are polynomial. We give a description of the free monad on an analytic endofunctor in terms of trees, and prove a nerve theorem implying that the infinity category of analytic monads is equivalent to the infinity category of dendroidal Segal spaces of Cisinski and Moerdijk, one of the known equivalent models for infinity operads. A byproduct of the development is a Joyal theorem for homotopical species. This is joint work with David Gepner and Rune Haugseng.

Victoria Lebed
Yang-Baxter equation up to homotopy

Abstract: In this talk we will discuss possible approaches to developing a homotopy theory for solutions to the Yang-Baxter equation. The role of tetrahedra, binary trees, Tamari lattices and associahedra from homotopy theory of associative structures will be played here by cubes, braids, higher Bruhat orders, and a series of polytopes starting from the octagon. Relations with cubical (co)homology will be discussed.

Maxime Lucas
Normalisation strategies revisited

Abstract: Starting from a convergent presentation of a monoid M, Squier showed how to extend it into a coherent presentation of M. This construction was then extended by Guiraud and Malbos in order to construct a polygraphic resolution of the monoid M. A central tool for the construction of this resolution is the notion of normalisation strategy. In the lowest dimension, a normalisation strategy (already used in usual rewriting) consists in the choice, for any word u, of a distinguished rewriting path from u to its normal form. In general a normalisation strategy consists in a function S which associates an $(n+1)$-cell to any n-cell, satisfying some conditions making S a natural transformation.

In this talk, we show how some modifications to Guiraud and Malbos's approach (and in particular the use of the Gray tensor product) lead to a great simplification of the construction of the normalisation strategy, suggesting a possible extension to other structures than just monoids. Finally, we show how a result from Berger and Moerdijk seems to validate our use of the Gray tensor product, up to some conjectures about the model structure of ω-categories.

Peter LeFanu Lumsdaine

CwA's of equivalences / equivalences of CwA's

Abstract:

I will present a technical tool in dependent type theory, and two different applications for it: one with very traditional logical motivations, one higher-categorical/homotopy-theoretic.

The technical tool is *CwA's of Reedy diagrams on an inverse category*, which we use in particular to construct the *Reedy span-equivalences* CwA and various related auxiliary CwA's. Roughly speaking, this gives for any model C of a suitable type theory T another model (C) of T whose objects are *equivalences* in C, in such away that "source" and "target" are T-logical functors (C) $\to$ T.

The higher-categorical application is developing the "homotopy theory of type theories". We give a notion of *equivalence* between CwA's modelling at least -types, along with classes of fibrations, cofibrations, etc, and show that for suitable type theories T, models of T with these classes of maps form a *left semi-model category* — i.e. not quite a Quillen model category, but close enough to serve for many purposes. The CwA's (C) provide path-objects in these categories, and as such are a key tool of the proof of the left semi-model structure.

The logical application, meanwhile, is in giving *conservativity/equivalence* results between dependent type theories. Such conservativity questions are a major natural source of interesting and difficult problems in type theory. I will survey some that can be solved using the tools set out above, and others that remain open problems.

Most of the work presented is joint with Chris Kapulkin, and is available in *The homotopy theory of type theories*, https://arxiv.org/abs/1610.00037.

Philippe Malbos

Squier's theory for monoids and algebras

Abstract: Craig Squier proved that, if a monoid can be presented by a finite convergent string rewriting system, then it satisfies the homological finiteness condition left-$_3$ and the homotopical finiteness condition finite derivation type. Using these finiteness conditions, he constructed finitely presentable monoids with a decidable word problem, but that cannot be presented by finite convergent rewriting systems.

In the first part of this lecture, we will review notions of higher-dimensional rewriting theory and we will present Squier's results in the language of polygraphs and some applications of these results to coherence problems. In a second part, we will present the structure of linear polygraph as higher dimensional linear rewriting system and we will give an application to the construction of free resolutions for algebras. In particular, we will show how to construct polygraphic resolutions for algebras, starting from a convergent presentation, and how to relate these resolutions with the Koszul property.

François Métayer

Homotopy theory of strict ω-categories and its connections with homology of monoids

Abstract: In the first part, we describe the canonical model structure on the category of strict ω-categories and how it transfers to related subcategories. We then characterize the cofibrant objects as ω-categories freely generated by polygraphs and introduce the key notion of polygraphic resolution. Finally, by considering a monoid as a particular ω-category, this polygraphic point of view will lead us to an alternative definition of monoid homology, which happens to coincide with the usual one.

Viktoriya Ozornova
2-Segal sets

Abstract: Segal spaces is one possible concept for treating "categories up to homotopy". In a recent work by Dyckerhoff-Kapranov and Gálvez-Kock-Tonks, it was pointed out that many natural constructions which fail to have Segal spaces as output still have a certain kind of composition, which is formalized in the notion of a 2-Segal space by the former and decomposition space by the latter. In a joint work with J.Bergner, A.Osorno, M.Rovelli, and C.Scheimbauer, we gave a description of 2-Segal sets in terms of double categories. In an ongoing work, we promote this statement to homotopical level.

Simona Paoli
Segal-type models of higher categories

Abstract: Higher categorical structures find applications to diverse areas, such as homotopy theory, mathematical physics, logic and computer science, algebraic geometry. I will start this talk with an introduction to higher categories and to some of their connections with homotopy theory. I will then discuss a class of higher categories, called Segal-type models of weak n-categories, which are based on the combinatorics of multi-simplicial sets. This comprise the Tamsamani-Simpson model, as well as two new models which I have introduced, based on a new paradigm to weaken higher categorical structures.

Dominic Verity
A complicial compendium

Abstract: There comes a time to dust off an old idea and give it another try! In this case, I propose to revive some old work on (weak) *complicial sets*, a putative model of (∞,∞)-categories.

A complicial set is simply a simplicial set equipped with a specified *marking* of its simplices, of all dimensions, and satisfying a Kan-like horn filler condition. The strict variant of these gadgets, in which horn fillers are unique, finds its genesis in the work of Roberts as structures in which to value non-abelian cohomology theories. A decade later Street provided an explicit functor from the category of strict ω-categories to that of strict complicial sets. He conjectured that this functor would provide an equivalence of categories, a result which was ultimately settled by Verity.

Street also suggested that complicial sets, of the non-strict variety, should be regarded as being an efficient model for structures we would now call (∞,∞)-categories. That is to say, these are weak categorical structures with non-invertible cells at all positive dimensions, a point of view that we champion in this talk. To that end, we review the basic theory of complicial sets, develop a little of their higher category theory, build a few (∞,∞)-categories whose cells are bordisms, discuss some handy new complicial constructions, and suggest some future directions.

What better time and place could there be to revisit Roberts' complicial creation? This is, after all, the 40^{th} anniversary of its inaugural presentation to the *Colloquium on Operator Algebras and their Application to Mathematical Physics* here in Marseille.

Marek Zawadowski

Positive opetopes with contractions form a test category

Abstract: The category of positive opetopic sets **pOpeSet** can be defined as a full subcategory of the category of polygraphs **Poly**. An object of **pOpeSet** has generators whose codomains are again generators and whose domains are non-identity cells (i.e. non-empty composition of generators). The category **pOpeSet** is a presheaf category with the exponent being called the category of positive opetopes **pOpe**. The category **pOpe** has several descriptions. Its objects are called positive opetopes and morphisms are face maps only. Since **Poly** has a full-on-isomorphisms embedding into the category of ω-categories **oCat**, we can think of morphisms in **pOpe** as ω-functors that send generators to generators. The category of positive opetopes with contractions $\mathbf{pOpe}_{\iota}$ has the same objects and face maps as **pOpe**, but in addition it has some degeneracy maps. A morphisms in **pOpe**´ is an ω-functor that sends generators to either generators or to identities on generators. I will show that the category **pOpe**´ is a test category. The proof is combinatorial and it uses my description of the category $\mathbf{pOpe}_{\iota}$, as well as the other combinatorial description of **pOpe** due to T. Palm.