ALGEBRA, ARITHMETIC AND COMBINATORICS OF DIFFERENTIAL AND DIFFERENCE EQUATIONS

CIRM, LUMINY, 28 MAY - 1 JUNE, 2018

Abstracts

Frits Beukers

A supercongruence and hypergeometric motives

Abstract: In this lecture I discuss joint work with Eric Delaygue on supercongruences for certain truncated hypergeometric functions. There will also be a discussion of the hypergeometric motives that underlie these congruences.

Alin Bostan

Automated guess-and-prove for combinatorial functional equations

Abstract: Generating functions arising in combinatorics satisfy various types of functional equations. A fundamental question is to study the Diophantine nature of these generating functions. Through concrete examples, I will illustrate a computer algebra paradigm called "automated guess-and-prove", which is based on the algorithmic computation of Hermite-Padé approximants, and which allows to prove algebraicity and transcendence results in this combinatorial framework.

Mireille Bousquet-Mélou

Counting quadrant walks with large steps

Abstract: The enumeration of quadrant walks with small steps (that is, steps taken in some of the 8 directions N, S, E, W, NE, NW, SE, SW) is now well understood. In particular, their generating function is D-finite (i.e., solution of a linear differential equation) if and only if a certain group of rational transformation, associated with the set of allowed steps, is finite. It is far from obvious to extend the methods that led to this classification to quadrant walks with arbitrary steps. Fayolle and Raschel have already discussed the difficulties that one expects using the complex analysis approach that was very powerful in the small step case. In this talk, I will describe some progresses in the formal series approach, whose main ambition is to yield results when the group is finite.
Joint work with Alin Bostan and Steve Melczer.

Xavier Caruso

Un théorème de factorisation par les pentes des polynômes de Ore

Abstract: Il est bien connu que les polynômes classiques jouent un rôle important en algèbre linéaire (e.g. polynômes d'endomorphismes, polynôme caractéristique) et, notamment, que leurs propriétés de factorisation permettent de diagonaliser ou de trigonaliser (éventuellement par blocs) les applications linéaires. En algèbre semi-linéaire et en théorie des équations différentielles linéaires, une telle philosophie demeure à condition de remplacer les polynômes usuels par une variante non commutative de ceux-ci que l'on appelle les polynômes de Ore. Dans cet exposé, je présenterai un théorème de factorisation – par les pentes – des polynômes de Ore sur les corps ultramétriques complets et donnerai un algorithme itératif très simple permettant de calculer cette factorisation. Je discuterai également des applications de ce théorème à l'étude des représentations galoisiennes p-adiques, des équations différentielles p-adiques et des équations de Mahler.

Thomas Dreyfus

Algebraic independence among solutions of different functional equations

Abstract: Difference Galois theory aims at understanding what are the algebraic and differential relations satisfied by the solutions of one given functional equation. In this talk we will see how the latter may be used to answer to a more general problem. Given f and g solution of two different functional equations, can we give a tractable sufficient condition to ensures that f and g are algebraically independent?

Gwladys Fernandes

Algebraic and linear relations between values of Mahler functions in positive characteristic

Abstract: The aim of this talk is to discuss recent results concerning Mahler's method in positive characteristic. Let $\mathbb{K}$ be a function field. We will see that every algebraic relation over $\overline{\mathbb{K}}$ between the values, at a regular algebraic point, of functions $f_1(z), \dots, f_n(z)$ satisfying a linear Mahler system arises as the specialization of an algebraic relation over $\overline{\mathbb{K}}(z)$ between the functions themselves, providing that the extension $\overline{\mathbb{K}}(z)(f_1(z), \dots, f_n(z))$ is regular over $\overline{\mathbb{K}}(z)$. When $\mathbb{K}$ is a number field, this result was recently proved by P. Philippon, but the regularity condition is always satisfied. As we will see, this is no longer true in our setting.

Charlotte Hardouin

Galois theory and walks in the quarter plane

Abstract: In the recent years, the nature of the generating series of walks in the quarter plane has attracted the attention of many authors in combinatorics and probability. The main questions are: are they algebraic, holonomic (solutions of linear differential equations) or at least hyperalgebraic (solutions of algebraic differential equations)? In this talk, we will show how the nature of the generating function can be approached via the study of a discrete functional equation over a curve E, of genus zero or one. In the first case, the functional equation corresponds to a so called q-difference equation and all the related generating series are differentially transcendental. For the genus one case, the dynamic of the functional equation corresponds to the addition by a given point P of the elliptic curve E. In that situation, one can relate the nature of the generating series to the fact that the point P is of torsion or not.

Frédéric Jouhet

Congruences modulo cyclotomic polynomials and algebraic independence for q-series

Abstract: Many G-functions have coefficients satisfying conguences modulo prime numbers. These congruences are reminiscent to the ones discovered by Lucas for the binomial coefficients, which can be generalized to congruences modulo cyclotomic polynomials for the q-binomial coefficients. I will give a general congruence for multidimensional q-factorial ratios which, together with a specialization process, extends many known results of this kind. In terms of generating series, such congruences connect various classical power series to their q-analogs. By focusing on series with coefficients in Z[q], I will finally describe how to derive a propagation phenomenon: when these generating series are algebraically independent for q=1, this is also the case for their q-analogs.
This is joint work with Boris Adamczewski, Jason Bell, and Eric Delaygue. motives.

Kiran Kedlaya

Frobenius structures on hypergeometric equations

Abstract: We report on an ongoing effort to explicate the p-adic Frobenius structure on a hypergeometric differential equation, and to use this as an effective algorithm to compute the zeta functions of hypergeometric motives.

Pierre Lairez

Computing periods : applications to binomial sums, volumes and K3 surfaces

Abstract: The computation of periods is effective: given a multiple integral of a rational function with a parameter, one can compute a linear differential equation that it satifies, the Picard-Fuchs equation. This talk will demonstrate periods at work, featuring Maple, Magma and Sage. Using periods, we will compute binomial sums, volume of semi-algebraic sets and Picard rank of K3 surfaces.
Based on joint works with Alin Bostan, Mohab Safey El Din, Bruno Salvy and Emre Sertöz.

Marni Mishna

The Classification of Excursions

Abstract: Excursions are walks which start and end at prescribed locations. In this talk we consider the counting sequences of excursions, more precisely, the functional equations their generating functions satisfy. We focus on two sources of excursion problems: walks defined by their allowable steps, taken on integer lattices restricted to cones; and walks on Cayley graphs with a given set of generators. The latter is related to the cogrowth problems of groups. In both cases we are interested in relating the nature of the generating function (i.e. rational, algebraic, D-finite, etc.) and combinatorial properties of the models. We are also interested in the relation between the excursions, and less restricted families of walks.

Rahim Moosa

Around Jouanolou-type Theorems

Abstract: In the mid-90's, generalising a theorem of Jouanolou, Hrushovski proved that if a D-variety over the constant field C has no non-constant D-rational functions to C, then it has only finitely many D-subvarieties of codimension one. This theorem has analogues in other geometric contexts where model theory plays a role: in complex analytic geometry where it is an old theorem of Krasnov, in algebraic dynamics where it is a theorem of Bell-Rogalski-Sierra, and in meromorphic dynamics where it is a theorem of Cantat. I will report on work-in-progress with Jason Bell and Adam Topaz toward generalising and unifying these statements.

Ronnie Nagloo

Functional Transcendence and some classical ODEs

Abstract: In this talk I will explain how one can show that any three distinct solutions of a Riccati equation are algebraic independent over $\mathbb{C}(t)$, provided that there are no solutions in the algebraic closure of $\mathbb{C}(t)$. This answers a very natural question in the theory. I will also explain how one can use this result to get a ?uniform? proof of the algebraic independence conjecture for all the generic Painlevé equations, namely: if $y_1,?,y_n$ are distinct solutions, then $y_1,y_1',?,y_n,y_n'$ are algebraically independent over $\mathbb{C}(t)$. If time permits, I will also explain how the Riccati equation can be use to answer transcendence questions about the Schwarz triangle functions.

Alexey Ovchinikov

Differential elimination, randomization, and applications

Abstract: We will discuss new results in differential elimination. Among other advantages, our new approach allows for randomizedcomputation to significantly increase the efficiency and tackle problems that could not be tackled before. This approach is based on uniform upper bounds for numbers of differentiations.
This is joint work with Gleb Pogudin and Thieu Vo.

Tanguy Rivoal

Exceptional algebraic values of E-functions at algebraic points

Abstract: E-functions are power series with algebraic coefficients (satisfying certain growth conditions) and solutions of linear differential equation with polynomial coefficients. They have been defined by Siegel in 1929 in order to generalize the diophantine properties of the simplest E-function, namely the exponential function, which takes transcendental values at every non-zero algebraic point. The situtation is more complicated in general, as an E-function may sometimes return an algebraic value when evaluated at a non-zero algebraic point. However, for any given E-function, they are only a finite number of such exceptional algebraic points.

In this talk, I will first explain the classical Diophantine results for E-functions, due to Siegel-Shidlovskii, Nesterenko-Shidlovkii and Beukers. I will then present an algorithm which, given an E-function $f(z)$ as input, outputs the finite list of algebraic numbers α such that $f(\alpha)$ is algebraic. This is a joint work with Boris Adamczewski.

Pierre Tarrago

Harmonic functions for alcove random walks of type A

Abstract: In this talk, I consider the problem of describing the set of harmonic functions for random walks on alcoves coming from affine Lie algebras of type A. These random walks are related to the combinatorics of the affine Coxeter group of type A, and the corresponding harmonic functions are solutions of a difference equation involving a special kind of symmetric functions. I will first briefly explain the geometric description of the set of harmonic functions due to Rietsch, and then I will present a recent joint work with Cédric Levouvey which gives a combinatoric approach to this description. The latter approach yields an explicit parametrization of the set of harmonic functions, and establishes a connection between the algebraic setting and the probabilistic behavior of the random walks."

Masha Vlasenko

Gamma functions, monodromy and Apéry constants

Abstract: In 1978 Roger Apéry proved irrationality of zeta(3) approximating it by ratios of terms of two sequences of rational numbers both satisfying the same recurrence relation. His study of the growth of denominators in these sequences involved complicated explicit formulas for both via sums of binomial coefficients. Subsequently, Frits Beukers gave a more enlightening proof of their properties, in which zeta(3) can be seen as an entry in a monodromy matrix for a differential equation arising from a one-parametric family of K3 surfaces. In the talk I will define Apéry constants for Fuchsian differential operators and explain the generalized Frobenius method due to Golyshev and Zagier which produces an infinite sequence of Apéry constants starting from a single differential equation. I will then show a surprising property of their generating function and conclude that the Apéry constants for a geometric differential operator are periods.
This is work in progress, joint with Spencer Bloch.

Jacques-Arthur Weil

Computing the Galois-Lie algebra of linear differential systems

Abstract: Given a (reductive) linear differential system, we show how to transform it into a "reduced form", from which its Galois-Lie algebra can be read. The construction uses decompositions of differential modules and Lie algebra conjugacy methods. We will show examples of a Maple implementation.
This is joint work with M. Barkatou, T. Cluzeau and L. Di Vizio.

Michael Wibmer

Free differential Galois groups

Abstract: In this talk we will discuss recent progress towards understanding the absolute differential Galois group of the field of rational functions.
This is joint work with Annette Bachmayr, Julia Hartmann and David Harbater.
