NON LINEAR FUNCTIONAL ANALYSIS

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CENTRE INTERNATIONAL DE RENCONTRES MATHEMATIQUES

BOOK OF ABSTRACTS

Florent Baudier (Texas A&M University)

Asymptotic structure and nonlinear geometry of Banach spaces

We will discuss the utilization of asymptotic structure and of Banach space-valued trees with branches of certain types, in connection to nonlinear Banach space geometry. We will study coarse rigidity properties of reflexive Banach spaces that are "very" smooth. An interesting application is a coarse version of a celebrated theorem of Tsirelson which states that there exists a Banach space that does not contain any ℓ_p-space nor c_0. In particular, there is a Banach space (Tsirelson's original space) that does not coarsely contain Hilbert space.
This is a joint work with G. Lancien and Th. Schlumprecht.

Bruno de Mendoça Braga (York University, Toronto, Canada)

On the rigidity of uniform Roe algebras of coarse spaces

Given a coarse space $(X, \mathcal{E})$, one can define a C^*-algebra $C^*_u(X)$ called the uniform Roe algebra of $(X, \mathcal{E})$. It has been proved by J. Špakula and R. Willet that if the uniform Roe algebras of two uniformly locally finite metric spaces with property A are isomorphic, then the metric spaces are coarsely equivalent to each other. In this talk, we look at the problem of generalizing this result for general coarse spaces and on weakening the hypothesis of the spaces having property A.

Marek Cúth (Charles University, Czech Republic)

Complexity of distances between metric and Banach spaces

I will talk about our recent work with M. Doucha and O. Kurka, where we investigate the complexity and reducibility between analytic pseudometrics coming from functional analysis and metric geometry, such as Gromov-Hausdorff, Kadets, and Banach-Mazur distances. This leads us to introduce the notion of Borel reducibility between pseudometrics which generalizes and is strictly stronger than the standard Borel reducibility between definable equivalence relations. Among our applications is that there exists a ball in the Gromov-Hausdorff pseudometric which is not Borel. This solves in negative a problem by I. Ben Yaacov, M. Doucha, A. Nies and T. Tsankov. Finally, we demonstrate that there are many many other distances that we leave untouched, which suggests there is enough further possible developement in the area.

Stephen Dilworth (University of South Carolina, USA)

Property (β) of Rolewicz and related asymptotic properties

We present some recent results concerning Banach spaces with the isomorphic (β) property, i.e., spaces admitting an equivalent norm with property (β) of Rolewicz. We also discuss related asymptotic properties, including the 'asymptotic midpoint uniformly convex' property and its isomorphic counterpart. We relate these properties to bilipschitz embeddings of infinitely branching binary trees and diamond graphs.

Michael Dymond (Universität Innsbruck - Austria)

Mapping n grid points onto a square forces an arbitrarily large Lipschitz constant.

We prove that the regular $n \times n$ square grid of points in the integer lattice $\mathbb{Z}^2$ cannot be recovered from an arbitrary n^2-element subset of $\mathbb{Z}^2$ via a mapping with prescribed Lipschitz constant (independent of n). This answers negatively a question of Feige. Our resolution of Feige's question takes place largely in a continuous setting and is based on some new results for Lipschitz mappings falling into two broad areas of interest, which we study independently. Firstly we discuss *Lipschitz regular mappings* on Euclidean spaces, with emphasis on their bilipschitz decomposability in a sense comparable to that of the well known result of Jones. Secondly, we build on work of Burago and Kleiner and McMullen on *non-realisable densities.* We verify the existence, and further prevalence, of strongly non-realisable densities inside spaces of continuous functions. This is joint work with Vojtěch Kaluža and Eva Kopecká.

Vladimir Fonf (Ben-Gurion University - Israel)

Norming subspaces of Banach spaces

Petr Hájek (Czech Technical University in Prague, Czech Republic)

Some remarks on separated sets in nonseparable Banach spaces

We will discuss some recent results in progress on the existence of uncountable families of vectors from the unit ball which are pairwise of distance greater than one.

William B. Johnson (Texas A&M University - USA)

Ideals in $L(L_p)$

I'll discuss the Banach algebra structure of the spaces of bounded linear operators on ℓ_p and $L_p := L_p(0,1)$. The main new results are

1. The only non trivial closed ideal in $L(L_p)$, $1 \le p < \infty$, that has a left approximate identity is the ideal of compact operators (joint with N. C. Phillips and G. Schechtman).
2. There are infinitely many; in fact, a continuum; of closed ideals in $L(L_1)$ (joint with G. Pisier and G. Schechtman).

The second result answers a question from the 1978 book of A. Pietsch, "Operator ideals".

Benoît Kloeckner (Université Paris-Sud, France)

Regularity is a counter-curse to dimensionality

An "empirical measure" is a random measure giving a mass $1/n$ to each of n points of a random process (e.g. independent identically distributed random points). A way to define the distance to a limit measure is to use duality against a class of functions (e.g. Lipschitz function, giving birth to the L^1 Wassesrtein metric). One motivation for this lies in Monte-Carlo methods, where the integral of a function with respect to the limit law is approximated by the integral of the same function with respect to the empirical measure. When estimating the integral of a single function, the error usually decreases as the square root of n; but to estimate the integrals of a large number of functions, one is lead to estimate the worse error in the considered class. For an IID sample in large dimension d, and in duality to Lipschitz functions, the convergence is very slow: the distance decreases as the $1/d$ power of n. The goal of the talk will be to present a simple method to bound above the duality distances between the empirical measure and the limit measure. The method is based on classical functional decompositions (Fourier series, wavelets) and in particular shows that if we consider the class of C^k functions with $k > d/2$, we recover the optimal convergence rate proportional to the square root of n.

Ana Khukhro (Université de Neuchâtel, Switzerland)

Finite quotients in coarse geometry

The study of groups often sheds light on problems in various areas of mathematics. Whether playing the role of certain invariants in topology, or encoding symmetries in geometry, groups help us understand many mathematical objects in greater depth. In coarse geometry, one can use groups to construct examples or counterexamples with interesting or surprising properties. In this talk, we will introduce one such metric object arising from finite quotients of finitely generated groups, and survey some of its useful properties and associated constructions.

Manor Mendel (The Open University of Israel)

On gentle partitions of unity and Lipschitz extension of finite metrics

Let T be an arbitrary N-point metric space, Y an arbitrary Banach space, $X \supset T$ arbitrary super space, and $f : T \to Y$ arbitrary 1-Lipschitz map. What bounds on the best Lipschitz extension $F : X \to Y$ can be given as a function of N?
This question was first raised by Johnson, Lindenstrauss and Schechtman in the 80's. They proved that there always exists an extension F with $\|F\|_{\mathrm{Lip}} = O(\log N)$.
On the other hand, Johnson and Lindenstrauss previously proved that even for the special case when Y is Hilbert space there is an example where every extension must have Lipschitz constant at least $\Omega(\sqrt{\log N/\log\log N})$. The upper bound was improved to $O(\log N/\log\log N)$ by Lee and Naor and the lower bound was improved to $\Omega(\sqrt{\log N})$ very recently by Naor and Rabani.
This gap between the upper and lower bound remains an interesting open problem and our main goal in this talk is to draw attention to it. We observe that all previous work on devising extension for mappings into arbitrary Banach space uses a technique called *gentle partition of unity* [GPU], first made explicit by Lee and Naor. We present a simple extension formula that achieves the Lee-Naor bound of $O(\log N/\log\log N)$, and then show a matching lower bound of $\Omega(\log N/\log\log N)$ on the GPU bound when T is the discrete torus of N points.
Based on a joint work with Assaf Naor and Yuval Rabani.

Pavlos Motakis (Texas A&M University, USA)

Joint spreading models and uniform approximation of bounded operators

We introduce the notion of ℓ-joint spreading models, $\ell \in \mathbb{N}$, which concerns the simultaneous asymptotic behavior of ℓ sequences in a Banach space X. We also introduce a property of Banach spaces called the uniform approximation on large subspaces (UALS) which concerns bounded linear operators on X. A Banach space X has the UALS property if there exists $C > 0$ such that whenever $A \in \mathcal{L}(X)$ and $W \subset \mathcal{L}(X)$, with W convex and compact, are such that for some $\varepsilon > 0$ and for every $x \in X$ there exists $B \in W$ with $\|A(x) - B(x)\| \leq \varepsilon\|x\|$ then there exists a subspace Y of X of finite codimension and a $B \in W$ with $\|(A - B)|_Y\|_{\mathcal{L}(Y,X)} \leq C\varepsilon$.
We connect these two notions by proving that whenever a (reasonably nice) Banach space X or its dual X^* has a unique ℓ-joint spreading model then X satisfies the UALS property. We deduce that ℓ_p, for $1 \leq p < \infty$, Asymptotic ℓ_p spaces, for $1 \leq p \leq \infty$, and $C(K)$ spaces, for countable compacta K, all satisfy the UALS property. We also prove that certain spaces, such as $L_p[0,1]$, $1 \leq p \neq 2 \leq \infty$, and $C[0,1]$, fail the UALS.
This is joint work with S. A. Argyros, A. Georgiou, and A.-R. Lagos.

Assaf Naor (Princeton University - USA)

Coarse dimension reduction

In the spirit of the classical Johnson-Lindenstrauss dimension reduction lemma, we discuss the following type of questions: can any n-points metric space be embedded into some normed space of dimension $o(logn)$ with Lipschitz distorsion $O(logn)$? We improve on a result of Jiri Matousek by showing the following theorem, which states the impossibility of coarse dimension reduction as follows: if ω and Ω are two functions from the positive real line to itself which increase to infinity, there is a strictly positive real number β (which can be explicitly computed from the two functions) such that for all n, there is a finite metric space of cardinality n which cannot coarsely embed into a finite dimensional normed space X with a control given by ω and Ω unless $dim(X) \geq n^\beta$.

Mikhail Ostrovskii (St. John's University, New York - USA)

Accurate pasting of embeddings of locally finite metric spaces from embeddings of their finite pieces

It is known that if finite subsets of a locally finite metric space M admit bilipschitz embeddings into a Banach space X with uniformly bounded distortions, say all distortions are $\leq C$, then M admits a bilipschitz embedding into X with distortion $\leq D \cdot C$, where D is an absolute constant. One of the main goals of the talk is to show that for many Banach spaces, for example for such spaces as ℓ_p ($p \neq 2, \infty$) the constant D is equal to 1^+ in the following sense: The statement above does not hold for $D = 1$, but holds for any $D = 1 + \varepsilon$ with $\varepsilon > 0$ (joint work with Sofiya Ostrovska).

Eva Pernecká (Czech Technical University in Prague - Czech Republic)

Pełczyński's property (V^*) of certain Lipschitz-free spaces

For a metric space M equipped with a distinguished element 0, the Lipschitz-free space $\mathcal{F}(M)$ is a predual of the space of all real-valued Lipschitz functions on M vanishing at 0. Adapting the proof of a classical Bourgain's result, we show that for every compact subset M of a superreflexive Banach space, the Lipschitz-free space $\mathcal{F}(M)$ has the Pełczyński's property (V^*). As a consequence, $\mathcal{F}(M)$ is weakly sequentially complete and hence does not contain c_0. This result extends an earlier work due to Cúth, Doucha and Wojtaszczyk and relates to a question posed by Godefroy and Lerner concerning the property (X) for $\mathcal{F}([0,1]^n)$. The talk is based on a joint work with T. Kochanek.

Antonin Procházka (Université Bourgogne Franche-Comté - France)

Some isometric properties of Lipschitz free spaces

The Lipschitz free space $\mathcal{F}(M)$ is a Banach space constructed "around" a given metric space M in such a way that the Lipschitz maps defined on M become linear maps on $\mathcal{F}(M)$. The spectacular results of Godefroy and Kalton in this direction have underlined the usefullness of this concept in the non-linear geometry of Banach spaces. At the same time the norm on $\mathcal{F}(M)$ is closely related to the Wasserstein distance on the probability measures on M known in optimal transport. For these reasons the geometric properties of free spaces became a field of study in itself. In this talk we are going to survey several recent results on the interplay of the geometry of M and the geometry of $\mathcal{F}(M)$.
Joint work with L. C. García Lirola, C. Petitjean and A. Rueda Zoca.

Matias Raja (Universidad de Murcia, Spain)

Dentability index in $L^2(X)$ and super weak compactness

We will discuss some relations between the indices of Szlenk and dentability for certain convex subsets of the Bochner-Lebesgue spaces $L^p(X)$ with $1 < p < \infty$. As an application, we will give a somehow explicit estimation of the optimal modulus of convexity of super-reflexive spaces. Another related application is concerned with *super weak compactness*, a localisation of super-reflexivity. At the end we will present some nonlinear characterisations of super weakly compact convex sets.

Beata Randrianantoanina (Miami University - USA)

On a difference between two methods of low-distortion embeddings of finite metric spaces into non-superreflexive Banach spaces

In a recent paper, the speaker and M.I. Ostrovskii developed a new metric embedding method based on the theory of equal-signs-additive (ESA) sequences developed by Brunel and Sucheston in 1970's. This method was used to construct bilipschitz embeddings of diamond and Laakso graphs with an arbitrary finite number of branches into any non-superreflexive Banach space with a uniform bound on distortions that is independent of the number of branches.
In this talk we will outline a proof that the above mentioned embeddability results cannot be obtained using the embedding method which was used for trees by Bourgain (1986) and for binary branching diamonds and Laakso graphs by Johnson and Schechtman (2009), and which is based on a classical James' characterization of superreflexivity (the factorization between the summing basis and the unit vector basis of ℓ_1). Our proof uses a "self-improvement" argument and the Ramsey theorem.
Joint work with M.I. Ostrovskii.

Mikael de la Salle (CNRS - ENS Lyon, France)

L^2 spectral gap and group actions on Banach spaces

Exploring the relations between algebraic and geometric properties of a group and the geometry of the Banach spaces on which it can act is a fascinating program, still widely mysterious, and which is tightly connected to coarse embeddability of graphs into Banach spaces. I will present a recent contribution, joint with Tim de Laat, where we give a spectral (hilbertian) criterion for fixed point properties on uniformly curved Banach spaces.

Bünyamin Sarı (University of North Texas)

RIBE'S THEOREM AND KRIVINE STABILIZATION

Ribe's theorem is a local theorem; it states that uniformly homeomorphic Banach spaces have the 'same' finite dimensional subspaces. A priori its role in the nonlinear classification of infinite dimensional Banach spaces is not clear (except the special case of a Hilbert space). We will present an example where it does play a role when coupled with Krivine stabilization and the classical midpoint argument, and give a different proof of a theorem of Kalton and Randrianarivony.

Gideon Schechtman (Weizmann Institute - Israel)

OBSTRUCTIONS TO EMBEDDING SUBSETS OF SCHATTEN CLASSES IN L_p SPACES

I'll present a few inequalities on metric spaces holding for L_p and other natural spaces. Some of these inequalities can serve as the metric analogue of (Pisier's) property α and used as an obstruction to the Lipschitz (and uniform) embeddability of (some discrete subsets of) Schatten classes into L_p spaces.
Joint work with Assaf Naor

Thomas Schlumprecht (Texas A&M University, USA)

ON COARSE EMBEDDINGS INTO $c_0(\Gamma)$

A classical result of Aharoni states that every separable metric space (in particular every separable Banach space) can be bi-Lipschitzly embedded into c_0. In this talk we discuss possible coarse, uniform, and lipschitz embeddings into $c_0(\Gamma)$, for uncountable sets Γ, and show that the non separable case differs largely from the separable one:
Let λ be a large enough cardinal number (assuming GCH it suffices to let $\lambda = \aleph_\omega$). If X is a Banach space with $\mathrm{dens}(X) \geq \lambda$, which admits a coarse (or uniform) embedding into any $c_0(\Gamma)$, then X fails to have nontrivial cotype, i.e. X contains ℓ_∞^n C-uniformly for every $C > 1$. In the special case when X has a symmetric basis, we may even conclude that it is linearly isomorphic with $c_0(\mathrm{dens}X)$.
Secondly we prove that for certain compacta K, with $\mathrm{dens(K)} \geq \lambda$, $C(K)$ does not coarsely embed into $c_0(\Gamma)$ for any set Γ.

Richard Smith (University College Dublin, Ireland)

TOPOLOGY AND THE APPROXIMATION OF NORMS

Topological methods have become established in the construction of equivalent norms (chiefly in the context of locally uniformly convex norms or strictly convex norms), and have been used to solve a number of problems that apparently could not be settled by the application of linear or geometric methods alone. A few years ago, the class of so-called 'w^*-locally relatively compact sets' was introduced by Fonf, Pallares, Troyanski and the speaker, as a topological device to aid in the construction of isomorphically C^∞-smooth or isomorphically polyhedral Banach spaces.
In this talk, we show how this device can be used to approximate norms. Let X be a Banach space and let **P** be a property of norms. We say that a norm $\|\cdot\|$ on X (equivalent to the original norm) can be *approximated by norms having* **P** if, given $\varepsilon > 0$, there exists another norm $|||\cdot|||$ on X, such that $\|x\| \leq |||\cdot||| \leq (1+\varepsilon)\|x\|$ for all $x \in X$. For a number of classes of Banach spaces X, including $c_0(\Gamma)$ (where Γ is an arbitrary set), certain Orlicz spaces and Lorentz predual spaces, and a class of $C(K)$ spaces (where K comes from a class of compact spaces having unbounded scattered height), we show that all equivalent norms on X can be approximated by C^∞-smooth norms or polyhedral norms.
Most of this work was undertaken jointly with V. Bible and S. Troyanski.

Romain Tessera (Université Paris-Sud, France)

Relative expansion and coarse embeddings into Banach spaces

In a joint work with Goulnara Arzhantseva, we construct a finitely generated group that does not coarsely embed into a Hilbert space, and yet does not contain expanders. To do so, we introduce and study sequences of finite regular graphs called "relative expanders". The existence of coarse embeddings into L^p will be discussed as well.

Alain Valette (Université de Neuchâtel, Switzerland)

Groups for which every isometric action is bounded or proper.

Fix $p > 1$. Say that a locally compact group G property BP_{L^p} (resp. PL) if every isometric action of G on L^p (resp. on any metric space) is either proper or has all orbits bounded. Shalom proved in 1999 that $SO(n, 1)$ and $SU(n, 1)$ have BP_{L^2}, and Cornulier showed in 2009 that every simple Lie group with finite center has PL. In joint work with R. Tessera, we show that BP_{L^p} and PL are equivalent for abelian groups and for amenable almost connected lie groups.

Michal Wojciechowski (IMPAN Warsaw - Poland)

On the Pełczyński conjecture on the number of Auerbach bases.

We give the estimate on the number of different Auerbach bases in an n-dimensional Banach space. When Plichko remarked that in every Banach space there are at least 2 different Auerbach Bases, Pełczyński conjectured that in an n-dimensional space it should be always at least n of them. We confirm this conjecture showing that there is at least $\frac{n(n-1)}{2} + 1$ of them. The estimate follows from the calculation of the Lusternik–Schnirelmann category of the flag variety. A better estimate is obtained for generic smooth Banach spaces using Morse theory. In this case the number of Auerbach bases is greater than the exponential function of a dimension.
Joint work with A. Weber.