DIOPHANTINE GEOMETRY

CIRM, LUMINY, 21 - 25 MAY, 2018

Abstracts

Francesco Amoroso
Théorèmes de spécialisations et équations aux normes

Abstract : [travail en cours avec D. Masser et U. Zannier] We develop a new approach to treat families of norm form equations. We do not use Baker's method, applying instead the recent specialisation theorem of [1]. This allows us to prove that, under suitable assumptions, all solutions of a norm form diophantine equation over an algebraic function field come from specialisation of functional equations. For instance for Thomas cubic equation we get :

All diophantine solutions $(t, x, y) \in \mathbb{Z}^3$ *of Thomas cubic equation*

$$X(X - A_1(T)Y)(X - A_2(T)Y) + Y^3 = 1$$

(with $A_1, A_2 \in \mathbb{Z}[T],\ 0 < \deg(A_1) < \deg(A_2)$*) for* $t \in \mathbb{N}$ *(effectively) large enough, are specialisations of a functional solution* (T, X, Y).

Références

[1] F. Amoroso, D. Masser and U. Zannier, "Bounded Height Problems and Silverman Specialization Theorem" , "Bounded Height Problems and Silverman Specialization Theorem". *Duke Math J.*, **166**, no. 13 (2017), 2599–2642. Preprint. https ://hal.archives-ouvertes.fr/hal-01200626.

Fabrizio Barroero
Unlikely Intersections in families of abelian varieties and applications

Abstract : The Zilber-Pink Conjecture for families of abelian varieties describes the intersections of a subvariety of an abelian scheme with subgroup schemes of appropriate codimension. We will consider the case of a curve in an abelian scheme (everything defined over the algebraic numbers). In joint work with Laura Capuano we proved that the intersection of the curve with the union of all flat subgroup schemes of codimension at least 2 is finite, unless the curve is contained in a proper subgroup scheme. To have the full conjecture one would have to consider subgroup schemes that are not contained in flat ones and these arise in fibers with endomorphism ring larger than the generic one.

It turns out that one can deal with this problem when the abelian scheme is a fibered power of an elliptic scheme.

Finally, we will see applications of this kind of results to Diophantine equations in polynomials.

Daniel Bertrand

Unlikely intersections in a family of semi-abelian surfaces

Abstract : Let E_0 be an elliptic curve with CM, and let G be a one-dimensional family of extensions of E_0 by the multiplicative group. Then, G contains a dense set C of special curves, known as Ribet curves. In this joint work with H. Schmidt (see ArXiV :1803.04835v1), we consider a non-fibral irreducible curve W in G, and show that W meets C Zariski densely if and only if it is a translate of a Ribet curve by a multiplicative section. The proof involves the Pila-Masser-Zannier strategy, a new type of bounded height, and the Mordell-Lang theorem.

Nicolas Billerey

On the modularity of reducible Galois representations

Abstract : In this talk we extend the usual definition of modularity to reducible mod ℓ Galois representations. By analogy with the irreducible case, we address questions such as Serre's modularity conjecture (in various forms) and the level-raising problem. We present partial results on these questions that illustrate the differences with the classical situation. This is joint work with Ricardo Menares.

Laura Capuano

Linear relations in families of elliptic curves

Abstract : Let E_λ denote the Legendre elliptic curve of equation $Y^2 = X(X-1)(X-\lambda)$. Motivated by the work of Masser and Zannier on simultaneous torsion, we proved in a joint work with F. Barroero that given n linearly independent points $P_1(\lambda), \ldots, P_n(\lambda)$ on E_λ with coordinates in $\overline{\mathbb{Q}(\lambda)}$, there exist at most finitely many $\lambda_0 \in \mathbb{C}$ such that the specialized points $P_1(\lambda_0), \ldots, P_n(\lambda_0)$ satisfy two independent relations with integer coefficients on E_{λ_0}. This is a special case of conjectures about unlikely intersections in families of abelian varieties stated by different authors. In my talk, I will give an overview on these problems and present an outline of the proof, which uses the Pila-Zannier strategy combining o-minimality with other Diophantine ingredients.

Antoine Chambert-Loir

Géométrie diophantienne et espaces de Berkovich

Abstract : Les espaces analytiques au sens de Berkovich sont une des variantes des diverses théories géométriques d'espaces non archimédiens. Leur utilisation est assez pertinente en géométrie diophantienne, en ce qu'ils permettent d'étudier divers aspects non archimédiens, en particulier les théorèmes d'équidistribution de points de petite hauteur en géométrie d'Arakelov et leur application à la conjecture de Bogomolov sur les corps de fonctions (travaux de Yuan, Gubler et Yamaki). J'évoquerai aussi une notion de formes différentielles réelles et courants sur ces espaces, développée avec Antoine Ducros, qui permet d'envisager une théorie d'Arakelov non archimédienne proche de la situation complexe.

Christopher Daw

Unlikely intersections in Shimura varieties

Abstract : In this talk, we will give our perspective on the Zilber-Pink conjecture for Shimura varieties, following our recent attempts to obtain conditional results in that direction. In particular, we will discuss the recent result of Mok, Pila, and Tsimerman, which we refer to as the hyperbolic Ax-Schanuel conjecture, as well as analogues of the structure theorems of Bombieri, Masser, and Zannier, and arithmetic problems arising from attempts to adapt the usual Pila-Zannier method. This is based on joint work with Jinbo Ren.

Gabriel Dill

Unlikely intersections between isogeny orbits and curves

Abstract : In the spirit of the Mordell-Lang conjecture, we consider the intersection of a curve in a family of abelian varieties with the images of a finite-rank subgroup of a fixed abelian variety A_0 under all isogenies between A_0 and some member of the family. After excluding certain degenerate cases, we can prove that this intersection is finite if everything is defined over the field of algebraic numbers. This proves a slightly modified version of the so-called André-Pink-Zannier conjecture over the algebraic numbers in the case of curves. We can even allow translates of the finite-rank subgroup by abelian subvarieties of controlled dimension if we strengthen the degeneracy hypotheses suitably. In my talk, I give some historical background and motivation for this problem and present an outline of the proof, which follows the Pila-Zannier strategy.

Vesselin Dimitrov

On some new results in the Mordell and Bogomolov problems

Abstract :

I will report on two recent developments around these classical topics, and discuss their connection.

(1) The algebraic sets of a theoretically minimum Arakelov height in an arithmetic variety are stable under taking an intersection. This result turns out to yield a new solution of the Bogomolov conjecture, by means of a reduction to the torsion case (Manin-Mumford).

(2) A generalization of Bombieri and De Diego's quantitative estimates on the number of K-rational points with large canonical height on a curve C/K of a fixed genus $g > 1$ and a given Mordell-Weil rank over K of its Jacobian. The point of our generalization is to introduce a new parameter that quantifies the meaning of "large height," so as to make possible the application of a new height lower bound of Gao and Habegger to the old problem of uniformly estimating $\#C(K)$. I will discuss this result and its relation to the previous, as well as to the work of David and Philippon (from their paper *Minorations des hauteurs normalisées des sous-variétés des puissances des courbes elliptiques*, IMRS, 2007).

This is a joint work with Ziyang Gao and Philipp Habegger.

Ziyang Gao

Around a height inequality

Abstract : With Philipp Habegger we have recently proved a height inequality for 1-parameter families of abelian varieties. It has several applications, for example it can prove the Geometric Bogomolov Conjecture over characteristic 0. In this talk, I will start by explaining its application towards counting the rational points of curves. Then I'll explain some generalization of this method to an arbitrary family of curves. I'll focus on how the mixed Ax-Schanuel for universal abelian varieties, extension of a recent result of Mok-Pila-Tsimerman, applies to this problem. This is work in progress, joint with Vesselin Dimitrov and Philipp Habegger.

Gareth Jones

Counting for functions on the disc

Abstract : I will discuss a counting result for algebraic points of bounded height and bounded degree on the graph of a function on the disc. Under the assumption of exponential decay along the interval, and assuming that our function is bounded, we give a power-log-bound. Our bound is also polynomial in the degree of the points. We exploit this to give a dynamical application. This is all joint work with Gareth Boxall and Harry Schmidt.

Samuel Le Fourn
Quadratic Chabauty and* L*-functions

Abstract : The Chabauty method has long stood as the most efficient tool to effectively determine rational points on algebraic curves. In the context of Serre's uniformity problem, its realization for modular curves has been decisive to prove there are no nontrivial rational points on families of modular curves whose jacobian admits a "rank zero quotient". Thanks to Kolyvagin-Logachev theorems, this can be obtained through nonvanishing of L-functions of modular forms. In this talk, I will present a work in progress with Dogra and Siksek, proving (again with L-functions) the existence of a "rank=dimension" quotient of jacobians of modular curves from two families, and how it allows to apply the new "quadratic Chabauty" method devised by Balakrishnan, Besser, Dogra and Müller.

Davide Lombardo
Endomorphism rings of Jacobians : from theory to practice and back again

Abstract : Over the last 200 years mathematicians have developed a very rich theory of abelian varieties ; however, certain kinds of explicit calculations have remained out of reach of our computational power until quite recently, when theoretical and technological advances have led to a renewed interest in the computational side of the theory. In this talk I will discuss one of the fundamental algorithmic problems one would like to solve, namely that of determining the endomorphism ring of the Jacobian of an explicitly given curve over a number field ; in particular, I will describe a method to provably compute the endomorphism ring of a genus 2 Jacobian, and discuss its applicability in practical situations. If time permits, I will also describe the problems one faces when trying to extend the result to Jacobians of curves of higher genus ; this leads back to interesting theoretical questions, which take the form of a peculiar kind of local-global principle for endomorphisms.

Álvaro Lozano-Robledo
Recent progress in the classification of torsion subgroups of elliptic curves

Abstract : This talk will be a survey of recent results and methods used in the classification of torsion subgroups of elliptic curves over finite and infinite extensions of the rationals, and over function fields.

David Masser
Avoiding Jacobians

Abstract : It is classical that, for example, there is a simple abelian variety of dimension 4 which is not the jacobian of any curve of genus 4, and it is not hard to see that there is one defined over the field of all algebraic numbers $\overline{\mathbf{Q}}$. In 2012 Chai and Oort asked if there is a simple abelian fourfold, defined over $\overline{\mathbf{Q}}$, which is not even isogenous to any jacobian. In the same year Tsimerman answered "yes". Recently Zannier and I have done this over the rationals $\mathbf{Q}$, and with "yes, almost all". In my talk I will explain "almost all" the concepts involved.

Ricardo Menares

p-adic equidistribution of CM points and integral properties of singular moduli

Abstract : We establish equidistribution properties of CM points on the moduli space of p-adic elliptic curves. We obtain a description of the equidistribution properties of p-adic Hecke orbits as well. A key ingredient is a p-adic version of Linnik's theorem on the asymptotic distribution of integer points on spheres. A theorem of Habegger states that there are at most finitely many singular moduli that are units. As an application of our results on p-adic equidistribution, we show that for every finite set of primes S, there are at most finitely many singular moduli that are S-units.
This is a joint work with Sebastian Herrero and Juan Rivera-Letelier.

Lucia Mocz

A new Northcott property for Faltings height

Abstract : The Faltings height is a useful invariant for addressing questions in arithmetic geometry. In his celebrated proof of the Mordell and Shafarevich conjectures, Faltings shows the Faltings height satisfies a certain Northcott property, which allows him to deduce his finiteness statements. In this work we prove a new Northcott property for the Faltings height. Namely we show, assuming the Colmez Conjecture and the Artin Conjecture, that there are finitely many CM abelian varieties of a fixed dimension which have bounded Faltings height. The technique developed uses new tools from integral p-adic Hodge theory to study the variation of Faltings height within an isogeny class of CM abelian varieties. In special cases, we are able to use these techniques to moreover develop new Colmez-type formulas for the Faltings height.

Martin Orr

Unlikely intersections with Hecke correspondences

Abstract : I will talk about a theorem on unlikely intersections between a curve and Hecke correspondences in $A_g \times A_g$ (where A_g is the moduli space of principally polarised abelian varieties of dimension g). This is inspired by work of Habegger and Pila for the case $g = 1$.

The proof has two arithmetic steps which I will discuss : (1) a height bound, giving an effective version of the Siegel property from the reduction theory of arithmetic groups ; (2) a Galois bound, relying on the Masser-Wustholz isogeny theorem.

Pierre Parent

Stable models for modular curves in prime level

Abstract : We describe stable models for modular curves associated with all maximal subgroups in prime level, including in particular the new case of non-split Cartan curves.
Joint work with Bas Edixhoven.

Héctor H. Pastén Vásquez

Shimura curves and bounds for the abc conjecture

Abstract : I will explain some new connections between the abc conjecture and modular forms. In particular, I will outline a proof of a new unconditional estimate for the abc conjecture, which lies beyond the existing techniques in this context. The proof involves a number of tools such as Shimura curves, CM points, analytic number theory, and Arakelov geometry. It also requires some intermediate results of independent interest, such as bounds for the Manin constant beyond the semi-stable case. If time permits, I will also explain some results towards Szpiro's conjecture over totally real number fields which are compatible with the discriminant term appearing in Vojta's conjecture for algebraic points of bounded degree.

Jérôme Poineau

Local and global Berkovich spaces

Abstract : We will first give an introduction to p-adic analytic geometry in the sense of Berkovich. We will explain that it provides a convenient setting to study p-adic analogues of classical questions such as the inverse Galois problem over $\mathbb{Q}_p(T)$ (after Harbater and Serre-Liu) or the existence of zeroes of quadratic forms over $\mathbb{Q}_p(T)$ (after Harbater-Hartmann-Krashen and Mehmeti).

In the second part, we will use a more general definition of Berkovich spaces that allows arbitrary Banach rings as base rings, *e.g.* $\mathbb{Z}$ endowed with the usual absolute value. Over the latter, Berkovich spaces look like fibrations that contain complex analytic spaces as well as p-adic analytic spaces for every prime number p. We will shortly describe those spaces as well as some typical rings of functions that appear in this setting (Harbater's convergent arithmetic power series). We will finally explain how Berkovich spaces over $\mathbb{Z}$ can be used to parametrize certain natural families such as elliptic curves over arbitrary local fields, archimedean or not (in which case, one recovers only those with bad reduction, *i.e.* Tate curves).