Constant Scalar Curvature Metrics in Kähler and Sasaki Geometry, CIRM January 2018

- SHORT TALKS SESSION -

THIBAUT DELCROIX

Mabuchi's K-energy functional on horosymmetric varieties

Spherical varieties form a very large class of almost-homogeneous varieties and as such a promising class on which one can study existence of canonical metrics. However it is very difficult to do Khler geometry in a uniform way on all spherical varieties. I will introduce a subclass containing homogeneous toric bundles and group compactifications on which I am able to adapt toric geometry techniques to obtain a combinatorial sufficient criterion of properness of Mabuchi's K-energy functional. I will also present applications to existence of cscK and log-Khler-Einstein metrics.

TAKAYUKI KOIKE

Non-projective K3 surfaces containing Levi-flat hypersurfaces

We show the existence of a non-projective K3 surface X which is not a Kummer surface and has a one-parameter family of Levi-flat hypersurfaces in which all the leaves are dense. We construct such X by patching two open complex surfaces obtained as the complements of tubular neighborhoods of elliptic curves embedded in blow-ups of the projective planes at general nine points.

HOMARE TADANO

Myers-Type Theorems, Diameter Bounds, and Gap Theorems for Sasaki Manifolds

After reviewing basic facts about Sasaki geometry, we shall give some Myers-type theorems for complete Sasaki manifolds. Moreover, we shall give upper and lower diameter bounds for compact gradient Sasaki-Ricci solitons in terms of the transverse scalar curvature. Finally, we shall give some gap theorems for compact gradient Sasaki-Ricci solitons by showing some necessary and sufficient conditions for the solitons to be Sasaki-Einstein.

JONATÁN TORRES OROZCO ROMÁN

Invariant solutions to the Yamabe equation on the Koiso-Cao soliton

We use the Hopf fibration to construct the non-trivial Ricci soliton on $\mathbb{CP}^2 \# \overline{\mathbb{CP}^2}$, from a different point of view. In particular, we discuss its Ricci and scalar curvatures, as well its volume and Yamabe constant. We will present it as a manifold where uniqueness to the Yamabe problem occurs.

Antonio Trusiani

Canonical Kähler-Einstein metrics on T_pX associated to Kähler classes

Given a compact Kähler manifold X and a Kähler class $\alpha \in H^2(X, \mathbb{R})$, the degenerations of all Kähler metrics in the Mabuchi space $\mathcal{H}(X, \alpha)$ corresponding to the deformation to the tangent space in $p \in X$ can be thought of as an infinite dimensional space contained in the "boundary" of $\mathcal{H}(X, \alpha)$. Thanks to a previous work of Witt Nystrm it can be described as a class of special plurisubharmonic functions on T_pX , called canonical growth. I will introduce the problem of finding Kähler-Einstein metrics belonging to such infinite dimensional space, i.e a Kähler-Einstein potentials on T_pX with canonical growth. This is a joint work with Witt Nyström.

MARKUS UPMEIER

Almost Kähler 4-manifolds of Constant Holomorphic Sectional Curvature are Kähler

We show that a closed almost Kähler 4-manifold of globally constant holomorphic sectional curvature $k \leq 0$ with respect to the canonical Hermitian connection is automatically Kähler. The same result holds for k < 0 if we require in addition that the Ricci curvature is J-invariant. The proofs are based on the observation that such manifolds are self-dual, so that Chern-Weil theory implies useful integral formulas, which are then combined with results from Seiberg-Witten theory.

CAROLINE VERNIER

Gluing methods in almost-Kähler geometry

Thanks to gluing methods, new examples of canonical metrics have been obtained on certain types of Kähler manifolds. For instance, canonical metrics on the blowup of a constant scalar curvature Kähler (cscK) manifold at a point or along a submanifold, or on the resolution of the singularities of a csck orbifold, have been constructed this way. The aim of my work is to use similar methods in the more general cases of almost-Kähler manifolds; that is, symplectic manifolds endowed with an almost complex structure that is compatible, but not necessarily integrable. In this framework, we build metrics with constant hermitian scalar curvature, which naturally generalise the Kähler constant scalar curvature metrics.

JUANYONG WANG

Iitaka's $C_{n,m}$ conjecture for Kähler fibrations over complex tori

In this talk, I will talk about a proof of Iitaka's $C_{n,m}$ conjecture for Kähler fibrations over complex tori. The method is essentially the same as the earlier article of Junyan Cao & Mihail Paun, except that we need a Kähler version of the positivity result of the direct image of the pluricanonical bundle as well as a Kähler version of the pluricanonical cohomology jump loci.