CONSTANT SCALAR CURVATURE METRICS IN KÄHLER AND SASAKI GEOMETRY

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Abstracts

Claudio Arezzo

Kähler constant scalar curvature metrics on blow ups and resolutions of singularities

Abstract: After recalling the gluing construction for Kähler constant scalar curvature and extremal (à la Calabi) metrics starting from a compact or ALE orbifolds with isolated singularities, I will show how to compute the Futaki invariant of the adiabatic classes in this setting, extending previous work by Stoppa and Odaka.

Besides giving new existence and non-existence results, the connection with the Tian-Yau-Donaldson Conjecture and the K-stability of the resolved manifold will be discussed. Joint work with A. Della Vedova, R. Lena and L. Mazzieri.

Sébastien Boucksom

Variational and non-Archimedean aspects of the Yau-Tian-Donaldson conjecture

Abstract: I will discuss some recent developments in the direction of the Yau-Tian-Donaldson conjecture, which relates the existence of constant scalar curvature Kähler metrics to the algebro-geometric notion of K-stability. The emphasis will be put on the use of pluripotential theory and the interpretation of K-stability in terms of non-Archimedean geometry.

Ruadhaí Dervan

Analytic and algebraic aspects of stable maps

Abstract: Kontsevich's version of Gromov-Witten theory rests on the notion of a ″stable map″ from an algebraic curve to a variety. I will discuss a notion of stability for maps between arbitrary varieties, which generalises Kontsevich's definition when the domain is a curve and Tian-Donaldson's definition of K-stability when the target is a point (joint with Julius Ross). The analogue of the Yau-Tian-Donaldson conjecture then predicts stability of a map is equivalent to the existence of certain canonical Kähler metrics on maps; one direction of this conjecture has been proven. I will also discuss some fundamental results linking the existence of these canonical metrics to the geometry of maps (joint with Lars Sektnan).

Eleonora Di Nezza
Complex Monge-Ampère equations with prescribed singularities

Abstract: Since the proof of the Calabi conjecture given by Yau, complex Monge-Ampère equations on compact Kähler manifolds have been intensively studied.

In this talk we consider complex Monge-Ampère equations with prescribed singularities. More precisely, we fix a potential and we show existence and uniqueness of solutions of complex Monge-Ampère equations which have the same singularity type of the model potential we chose. This result can be interpreted as a generalisation of Yau's theorem (in this case the model potential is smooth).

As a corollary we obtain the existence of singular Kähler-Einstein metrics with prescribed singularities on general type and Calabi-Yau manifolds.
This is a joint work with Tamas Darvas and Chinh Lu.

Joel Fine
From hypersymplectic to hyperkähler via G_2

Abstract: A triple of symplectic forms on a 4-manifold is called "hypersymplectic" if any non-zero linear combination of them is again symplectic. The prototypical example is the triple of Kähler forms associated to a hyperKähler metric. Donaldson has conjectured that when the 4-manifold is compact any hypersymplectic triple can be deformed via a path of cohomologous hypersymplectic triples to arrive at a hyperkähler triple. This conjecture is a special case of a "folklore" conjecture in symplectic geometry: a compact symplectic 4-manifold with $c_1 = 0$ and $b_+ = 3$ is actually Kähler.

I will describe an approach to Donaldson's conjecture which goes via G_2 geometry. A hypersymplectic structure on a 4-manifold X defines a G_2 structure on the 7-manifold $T^3 \times X$. There is a geometric flow, called the G_2-Laplacian flow, which attempts to deform a given G_2 structure into one with vanishing torsion. This is the G_2 analogue of Ricci flow. Applying this flow to $T^3 \times X$ we get a flow of hypersymplectic triples on X which attempt to deform the triple to a hyperkähler one. The main result of the talk - joint work with Chengjian Yao - is that if the scalar curvature of $T^3 \times X$ is bounded along the flow then a singularity cannot form. It is interesting to note that this result is seemingly stronger than what is currently known for Ricci flow with bounded scalar curvature.

Akiro Futaki
Hessian formula for moment maps and the structure of automorphism group on conformally Kähler, Einstein-Maxwell metrics

Abstract: The square norm of the moment map appears in many subjects and plays important roles. In this talk, I will focus on the case of Calabi energy on the space of Kähler metrics and the structure theorem of the group of automorphisms obtained from the Hessian formula. We apply the same idea to other relevant problems such as conformally Kähler, Einstein-Maxwell metrics.

Yoshinori Hashimoto
Relative balanced metrics and relative stability

Abstract: Suppose that a polarised Kähler manifold admits a constant scalar curvature Kähler metric. If the automorphism group is discrete, a foundational theorem of Donaldson states that it admits a sequence of balanced metrics, whose existence is known to be equivalent to asymptotic Chow stability of the manifold. When the automorphism group is non-discrete, the situation is more complicated since naive generalisation of Donaldson's theorem does not hold. This leads to the notion of balanced metrics and Chow stability that are "relative" to the automorphisms, as proposed by various works. There are several, a priori non-equivalent, definitions of these objects, and their relationship is subtle. This talk aims to clarify the similarities and differences among these notions, and survey some recent results in this area, including the ones of the speaker.

An-Min Li
Extremal Metrics on Toric Manifolds and Affine Techniques

Abstract: In a sequence of papers, Donaldson initiated a program to study the extremal metrics on toric manifolds and solved the problem for cscK metrics on toric surfaces. For toric manifolds, the equation of extremal metrics can be reduced to a real 4th-order partial differential equation on the Delzant polytope, called the Abreu equation. The affine techniques play important role. In joint papers with Bohui Chen and Li Sheng we apply the affine techniques to extend the existence result in dimension 2 to extremal metrics. We have also some results for higher dimensions. In this talk we explain our main idea and methods.

Andrea Loi
Kähler immersions of homogeneous Kähler manifolds into complex space forms

Abstract: The study of Kähler (holomorphic and isometric) immersions of a given Kähler manifold (M, g) into finite or infinite dimensional complex space forms started with the pioneering work of Eugenio Calabi (Annals of Mathematics 1953). In his paper, Calabi provides a classification of such immersions when (M, g) is itself a (finite dimensional) complex space form. This talk is an overview of the results obtained by the author and his collaborators in the last ten years when (M, g) is homogeneous. In particular a complete classification of the homogeneous Kähler manifolds which can be Kähler immersed into a given finite of infinite dimensional complex space form is given. The talk ends with the discussion of some open problems in the Kähler -Einstein case.

Heather Macbeth
Kâhler-Ricci solitons on crepant resolutions

Abstract: By a gluing construction, we produce steady Kahler-Ricci solitons on equivariant crepant resolutions of $\mathbb{C}^n/G$, where G is a finite subgroup of $SU(n)$, generalizing Cao's construction of such a soliton on a resolution of $\mathbb{C}^n/\mathbb{Z}_n$.
This is joint work with Olivier Biquard.

Gideon Bahir-Maschler

Kähler metrics associated with Lorentzian metric in dimension four

Abstract: We give a construction associating a family of Kähler metrics to any semi-Riemannian metric g on a four-manifold M which is equipped with two distinguished vector fields satisfying certain properties. In most of our examples the domain of such a Kähler metric g_K coincides with M. Under certain conditions the metrics g_K and g share various first order properties, like a joint Killing field.
Our examples are Lorentzian, including de Sitter spacetime, gravitational plane waves and Petrov type D metrics such as the Kerr metric. For SKR-type Kähler metrics, which include the classical extremal metric conformal to Page's Einstein metric, we provide an ansatz which inverts the construction: it produces Lorentzian metrics to which the SKR metric is associated.
This is joint work with Amir Aazami.

Sean Paul

Discriminants, Resultants, and a theorem of Ding and Tian

Abstract: I will offer a new proof (valid for any Hodge class) of the main result of Ding and Tian's seminal 1992 Inventions paper on the generalized Futaki invariant. The proof is based on a reformulation of Tian's so-called CM polarization. Our reformulation consists in the construction of a pair of globally generated line bundles. The associated morphisms into projective space are used to deduce Ding and Tian's theorem.

Lars Sektnan

Extremal Poincaré type metrics and stability of pairs on Hirzebruch surfaces

Abstract: In this talk I will discuss the existence of complete extremal metrics on the complement of simple normal crossings divisors in compact Kähler manifolds, and stability of pairs, in the toric case. Using constructions of Legendre and Apostolov-Calderbank-Gauduchon, we completely characterize when this holds for Hirzebruch surfaces. In particular, our results show that relative stability of a pair and the existence of extremal Poincaré type/cusp metrics do not coincide. However, stability is equivalent to the existence of a complete extremal metric on the complement of the divisor in our examples. It is the Poincaré type condition on the asymptotics of the extremal metric that fails in general.
This is joint work with Vestislav Apostolov and Hugues Auvray.

Zakarias Sjöström Dyrefelt

On K-polystability of cscK manifolds with transcendental cohomology class

Abstract: This talk is concerned with the study of K-polystability of constant scalar curvature Kähler (cscK) manifolds admitting holomorphic vector fields. We introduce a natural notion of "geodesic K-polystability" that extends Donaldson's algebraic K-polystability for polarized manifolds to the general Kähler setting. The approach builds on exploiting and developing the relationship between test configurations and geodesic rays.

As a main result we show that cscK manifolds are geodesically K-polystable. In case the variety is polarized, or the automorphism group is discrete, we in particular recover the "easy" direction of the Yau-Tian-Donaldson conjecture this way. As a further application it follows that existence of cscK (or extremal) Kähler metrics implies equivariant K-polystability (resp. relative K-stability), thus building on recent results of R. Dervan.

Cristiano Spotti

Log Calabi-Yau surfaces

Abstract: I will discuss Ricci-flat metrics on klt Calabi-Yau log pairs (X, D) where X is a smooth surface and D a weighted singular curve, focusing on the asymptotic description of the metric near the singular points of D.
The talk is based on joint work with Martin de Borbon.

Ioana Suvaina

Asymptotically Locally Euclidean Kähler Manifolds

Abstract: The study of asymptotically locally Euclidean Kähler manifolds had a tremendous development in the last few years. This talk presents the main results and the open problems in this area. When the manifolds support an ALE Ricci flat Kähler metric the complex surfaces and their metric structures are well understood. The remaining case to be studied is that of ALE scalar flat Kähler manifolds. In this direction, the underlying complex manifolds are characterized.

Craig Van Coevering

A new obstruction for Sasaki-extremal metrics

Abstract: A Sasaki-extremal metric is a generalization of constant scalar curvature metrics on a Sasakian manifold, where the Futaki invariant is not assumed to be zero. Extremal metrics are unique up to the Hamiltonian holomorphic group action, so can be considered a "canonical" metric. On a Sasakian manifold one can consider the cone of Reeb vector fields $\mathcal{C}$, where each $\xi \in \mathcal{C}$ is a polarization. One can define the 'extremal cone' to be the open subset $E \subset \mathcal{C}$ of Reeb vector fields which admit a compatible Sasaki-extremal metric.
In joint work with Charles P. Boyer we give a generalization of the Lichnerowicz obstruction of Sasaki-Einstein metrics, due to Gauntlett, Martelli, Sparks and Yau, to an obstruction of Sasaki-extremal metrics. Using this obstruction, we will give many examples of Sasakian manifolds for which the extremal cone is empty. In particular, many weighted hypersurface examples are easily shown to have an empty extremal cone.

David Witt Nyström

Coupled Kähler-Einstein metrics

Abstract: In this talk I will explain the notion of coupled Kähler-Einstein (cKE) metrics, introduced by Jakob Hultgren and myself. These are k-tuples of Kähler metrics that satisfy certain coupled Kähler-Einstein equations, and they provide an alternative way of finding canonical metrics in a given Kähler class. Recently Hultgren found a necessary and sufficient condition for existence in the toric Fano case in terms of barycenters of polytopes, thus also proving the coupled version of the Yau-Tian-Donaldson conjecture in that case.
