REPRESENTATION THEORY, MATHEMATICAL PHYSICS AND INTEGRABLE SYSTEMS

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Abstracts

Alberto Cattaneo

Geometrical construction of reduced phase spaces

Abstract: The reduced phase space of a field theory is the space of its possible initial conditions endowed with a natural symplectic structure. An alternative to Dirac's method, relying on natural geometric aspects of variational problems, was introduced by Kijowski and Tulczijev. This method also has the advantage of having a natural generalization in the BV context. In this talk, I will explain the method and describe some examples, focusing in particular on the tetradic version of general relativity.

Kai-Chieh Chen

Kashaev-Reshetikhin Invariant and Some Conjectures

Abstract: I will introduce the Kashaev-Reshetikhin Invariant associated to $\mathfrak{sl}_2$ and give computation results for the trefoil knot. Based on this, I will talk about some conjectures about this invariant.

Qingtao Chen

Asymptotics of quantum 6j symbols evaluated at a (non-conventional) root of unity

Abstract: Kirillov-Reshetikhin's quantum-6j symbols play an important role in mathematical physics due to their relation to quantum gravity. Previously, Woodward formulated a conjecture on quantum 6j symbols evaluated at the usual root of unity $e^{2\pi i/r}$, which predicted polynomial growth in the variable r. Inspired by the Volume Conjecture for Reshetikhin-Turaev invariants and (modified) Turaev-Viro invariants evaluated at non-conventional roots of unity made by T. Yang and the speaker, it is possible to consider the asymptotics of quantum 6j symbol associated to a single tetrahedron (because the building blocks of Turaev-Viro invariants are quantum 6j-symbols). It turns out that the growth is exponential in r and we have conjectured that this growth rate is proportional to the hyperbolic volume of a single tetrahedron. We have proved the majority of the cases of this conjecture and also obtained secondary terms for the asymptotics of quantum 6j symbols. The very mysterious symmetric conjecture of quantum 6j symbols will also be discussed.
This is a joint work with Jun Murakami.

Philippe Di Francesco

Paths and arctic curves: the tangent method at work

Abstract: We present various applications of the so-called tangent method for determining the arctic curve which separates liquid from frozen regions in large random tilings of planar domains with specified boundaries.

Jørgen Ellegaard Andersen

Geometric Recursion

Abstract: Geometric Recursion is a very general machinery for constructing mapping class group invariants objects associated to two dimensional surfaces. After presenting the general abstract definition we shall see how a number of constructions in low dimensional geometry and topology fits into this setting. These will include the Mirzakhani-McShane identies, mapping class group invariant closed forms on TeichmÃ?ller space (including the Weil-Petterson symplectic form) and the Goldman symplectic form.

Laszlo Feher

Action-angle duality for a Poisson-Lie deformation of the BC(n) Sutherland system

Abstract: A group theoretic derivation of 3-parametric limiting cases of the 5-parametric deformations of the classical trigonometric BC(n) Sutherland system due to van Diejen will be presented. These integrable systems will be obtained by applying symplectic reduction to 'natural free systems' defined on the Heisenberg double of the Poisson-Lie group SU(2n). It will be explained how a single reduction leads to a pair of Liouville integrable Hamiltonian systems that enjoy action-angle duality. The construction generalizes the derivation of the Sutherland system and its dual by reduction of the cotangent bundle of SU(2n). A similar picture for the 5-parametric case is, apparently, still not known.
Based on joint works with T.F. Gorbe and with I. Marshall.

Nathan Geer

Holonomy braidings, biquandles and quantum invariants of links with SL(2,C) flat connectionss

Abstract: Rinat Kashaev and Nicolai Reshetikhin introduced the notion of "holonomy braiding" extending Vladimir Turaev's homotopy braiding to describe the behavior of cyclic representations of the unrestricted quantum group $\mathcal{U}_q\mathfrak{sl}_2$ at root of unity. In this talk, we describe a kind of Reshetikhin-Turaev functor for links and tangles in this situation. Our construction uses modified traces and (bi)quandles.
his is a joint work with Christian Blanchet, Bertrand Patureau-Mirand and Nicolai Reshetikhin.

Theo Johnson-Freyd

T-duality for finite groups

Abstract: I will describe a version of "T-duality" in which circles are replaced by finite cyclic groups. This T-duality appears naturally in fusion category theory and in the construction of "twisted orbifolds" of conformal field theories. As an application, I will compute the 't Hooft anomaly of monstrous moonshine.

Victor Kac

Integrable Hamiltonian partial differential and difference equations and related algebraic structures

Abstract: The related algebraic structures are additive and multiplicative Poisson vertex algebras. This turned out to be an adequate tool for the construction and study of integrable Hamiltonian partial differential and difference equations. The most famous of the former is the KdV equation, describing water waves in narrow channel. The most famous of the latter is the Volterra lattice, describing predator-pray interactions. Some knowledge of Lie algebras, but no knowledge of integrable systems, will be assumed.

Rinat Kashaev

The spectral problem of the modular oscillator in the strongly coupled regimes

Abstract: Motivated by applications for non-perturbative topological strings in toric Calabi-Yau manifolds, I will talk about the spectral problem for a pair of commuting modular conjugate (in the sense of Faddeev) Harper type operators with complex values of Planck's constant. The eigenvectors are expressed in terms of a special entire function on the complex plane with the Taylor expansion coefficients given in terms of specific q-orthogonal polynomials, while the eigenvalues are solutions of transcendental Bethe type equations.
This is a joint work with Sergey Sergeev.

David Keating

Random Tilings on the GPU

Abstract: We present a GPU implementation of Markov chain simulations of tilings, dimers, and the six vertex model.

Rinat Kedem

From completeness of Bethe ansatz to quantum Q systems

Abstract: A historical overview, starting with the conjectures of Kirillov and Reshetikhin about the completeness problem for Bethe ansatz in generalized Heisenberg spin chains, related insights about Fermonic formulas from CFT, and subsequent works leading up to quantum cluster algebras and quantum toroidal algebras.

Jiang-Hua Lu

Total positivity and Poisson cluster structures on shifted big cells in flag varieties

Abstract: We introduce a cluster structure on every shifted big cell in the flag variety of a complex semisimple Lie group, and we discuss the compatibility between this cluster structure and Lusztig's total positivity as well as the standard Poisson structure on the flag variety.

Olya Mandelshtam

Combinatorics of the ASEP on a ring and Macdonald polynomials

Abstract: The $\mathfrak{m}$-species asymmetric simple exclusion process ($\mathfrak{m}$-ASEP) on a ring is a Markov chain on a ring with n sites with each site either vacant or occupied by one of m classes of particles, and whose dynamics are dictated by parameter t with respect to their priority class: particles can hop right at rate 1 or left at rate t. At $t = 0$, the stationary probabilities of the states of the $\mathfrak{m}$-ASEP can be described by multiline queues of Ferarri and Martin. For the 2-ASEP, we show a new combinatorial description of these probabilities in terms of certain cylindric tableaux which are in bijection with the multiline queues. We then extend this result for general t, which furthermore gives a combinatorial formula for some classes of Macdonald polynomials.
This talk is based on ongoing work with Sylvie Corteel (Université Paris-Diderot) and Lauren Williams (Berkeley).

Sevak Mkrtchyan

The point processes at turning points of large lozenge tilings

Abstract: In the thermodynamic limit of the lozenge tiling model the frozen boundary develops special points where the liquid region meets with two different frozen regions. These are called turning points. It was conjectured by Okounkov and Reshetikhin that in the scaling limit of the model the local point process near turning points should converge to the GUE corners process. We will discuss various results showing that the point process at a turning point is the GUE corner process and that the GUE corner process is there in some form even when at the turning point the liquid region meets two frozen regions of arbitrary (non-lattice) rational slope. The last regime arises when weights in the model are periodic in one direction with arbitrary fixed finite period.

Gus Schrader

Cluster theory of Whittaker functions and the open Toda chain

Abstract: I will describe a cluster algebraic perspective on the theory of the quantum relativistic open Toda chain and its eigenfunctions, the q-Whittaker functions.
Joint work with A. Shapiro.

Vera Serganova

Spherical superreperesentations, Capelli eigenvalue problem and supersymmetric polynomials

Abstract: We study invariant differential operators on representations of supergroups associated with simple Jordan superalgebras, in the classical case this problem goes back to Kostant. Eigenvalues of Capelli differential operators give interesting families of polynomials such as super Jack polynomials of Sergeev and Veselov and factorial Schur polynomials of Okounkov and Ivanov. We also discuss connection with deformed Calogero-Moser systems in the super case.

Alexander Shapiro

Modular functor and positive representations

Abstract: It was conjectured by Frenkel and Ip that positive representations of quantum groups are closed under tensor products. This conjecture happens to be closely related to the modular functor conjecture by Fock and Goncharov. I will speak about joint works with Gus Schrader where we prove the above conjectures.

Fedor Smirnov
Quantum group invariance in integrable QFT

Abstract: I shall talk about an old, but not always correctly understood, paper which we wrote with N. Reshetikhin.

Noah Snyder
Exceptional Fusion Categories

Abstract: Fusion categories are tensor categories that look much like the category of complex representations of a finite group: they have duals, are semisimple, and have finitely many simple objects. In addition to finite groups, the main source of examples are the semisimplified quantum groups at roots of unity which played a key role in Kolya's work. Moore and Seiberg asked whether quantum group categories might explain all fusion categories. The goal of this talk is to survey the current state of knowledge about "exceptional" fusion categories which don't seem to come from groups or quantum groups. In a sense this talk will be more like an experimental physics talk, in that one is searching for "new particles" in various regimes (e.g. "low index subfactors") and seeing what you can find. The punchline is that we know one new large family of fusion categories (the Izumi quadratic categories) and four isolated examples (the Extended Haagerup Subfactors). This will include some of my own work joint with Bigelow, Grossman, Izumi, Morrison, Penneys, Peters, and others, but also summarize the work of many other people (especially Asaeda, Bisch, Ocneanu, Haagerup, Izumi, Jones, and Popa working in Subfactor theory).

Jasper Stokman
Vector-valued Harish-Chandra series and KZB equations

Abstract: I will introduce vector-valued Harish-Chandra series associated to split symmetric pairs and show that they produce common eigenfunctions for the quantum Hamiltonians of a new class of quantum integrable spin Calogero-Moser systems. For a natural subclass of vector-valued Harish-Chandra series I will show that they are also common eigenfunctions of a consistent system of first order differential equations, called KZB equations.

Catharina Stroppel
Quantum cohomology of Grassmannians via R-matrices and integrable systems

Abstract: IThe purpose of this talk is to connect in an easy example, namely the equivariant (quantum) cohomology of Grassmannians, classical Schubert calculus with integrable systems and quantum groups. We illustrate how the idea of Maulik and Okounkov of stable manifolds is connected with Schubert varieties and construct an action of a degenerate R-matrix on the equivariant cohomology and connect it with current algebras. Finally we illustrate how the quantum deformation arises naturally in this setup.

Peter Tingley

Crystal combinatorics from Lusztig's PBW bases

Abstract: In the early 1990s both Lusztig and Kashiwara developed a theory of canonical bases for certain Kac-Moody algebras (called canonical bases by Lusztig and global crystal bases by Kashiwara). The approaches are pretty different, and the work went in different directions: Lusztig used geometry and ultimately categorification, while Kashiwara went in a more combinatorial direction, developing the theory of crystals. In this talk we will discuss how Lusztig's theory of PBW bases can be used to develop crystal combinatorics directly (in finite type). This gives new insights into such classical combinatorial objects as Young tableaux, and also leads to some more exotic combinatorics.
Much of this is joint work with Jackson Criswell, Ben Salisbury and Adam Schultze, although in essence it is mostly due to Lusztig.

Harold Williams

The coherent Satake category

Abstract: We analyze the structure of the category of perverse coherent sheaves on the affine Grassmannian and its induced basis in equivariant K-theory. In joint work with Sabin Cautis we show that this category is rigid and possesses renormalized r-matrices formally similar to those appearing in the representation theory of affine quantum groups, and use these results to prove that simple perverse coherent sheaves assemble into a monoidal cluster structure in type A. This in turn makes visible a range of nontrivial connections between the coherent Satake category and other parts of mathematics and physics. In this talk we will sketch one: the relations among the symmetric monoidal subcategories comprising the monoidal cluster structure record symplecto-geometric relations among collections of Lagrangian 3-spheres in a Calabi Yau 3-fold associated to the periodic Toda system.
