PDE/probability interactions :
kinetic equations, large time and propagation of chaos
CIRM, april 18-22, 2017

Tuesday 18

9h-10h30 : Thierry Bodineau (CNRS, École Polytechnique)
From hard-sphere dynamics to the Boltzmann equation, I

We will first review Lanford's strategy to prove the convergence of the density of the hard-sphere dynamics to the solution of the Boltzmann equation. We will focus on the propagation of chaos and on the emergence of the irreversibility. The second part of the lecture will be devoted to the evolution of a small perturbation of a dilute gas of hard spheres at equilibrium. We will show that, in the Boltzmann Grad limit, the linear and linearized Boltzmann equations can be recovered up to any large time (joint works with I. Gallagher, L. Saint-Raymond). Both equations are related to stochastic processes, namely the brownian motion and the fluctuating hydrodynamics.

11h-11h45 : Francis Fibet (Univ. Toulouse)
Asymptotically stable particle in cell methods for the Vlasov-Poisson system with a strong external magnetic field

I will speak about the numerical resolution of the Vlasov-Poisson system with a strong external magnetic field by particle methods. In this regime, classical particle methods are subject to stability constraints on the time and space steps related to the small Larmor radius and plasma frequency. Here, we propose an asymptotic-preserving particle scheme which is not subjected to these limitations. Our approach is based on first and higher order semi-implicit numerical schemes already validated on dissipative systems. Additionally, when the magnitude of the external magnetic field becomes large, this method provides a consistent particle discretization of the guiding-center equation, that is, incompressible Euler equation in vorticity form. We propose several numerical experiments which provide a solid validation of the method and its underlying concepts.

11h45-12h30 : Joaquin Fontbona (Univ. Chile)
Quantitative uniform propagation of chaos for Maxwell molecules

We prove propagation of chaos at explicit, mild polynomial rates in Wasserstein distance W_2, for Kac's N-particle system associated with the spatially homogeneous Boltzmann equation for Maxwell molecules (with and without cutoff). Our approach is based on novel, optimal transport-based probabilistic coupling techniques, developed in order to deal with (genuine) binary-jump interactions, and on a recent stabilization result for the particle system (obtained by M.Rousset). In particular, under suitable moments assumptions on the initial distribution, we establish a uniform-in-time estimate of order almost $N^{-1/3}$ for W_2^2. Joint work with Roberto Cortez.

14h-14h45 : Pierre DelMoral (INRIA - Bordeaux)
On the stability and the applications of interacting particle systems

We present a brief survey of a class of mean field type particle models arising in statistical physics, molecular chemistry, as well as in signal processing and data assimilation. These models include Feynman-Kac type interacting jumps processes, McKean-Vlasov type diffusions as well as Ensemble Kalman-Bucy filters. We connect the stability properties of the limiting nonlinear processes with the long time behavior of their mean field particle interpretations. We illustrate these results with a series of uniform propagation of chaos w.r.t. the time horizon.

This talk is based on a series of joint work with A.N. Bishop, A. Guionnet, A. Kurtzmann, M. Ledoux, L. Miclo, and J. Tugaut.

14h45-15h15 : Samir Salem (Univ. Aix-Marseille)

Propagation of chaos for aggregation equations with no-flux boundary conditions and sharp sensing zones

We consider a N-particle interacting particle system with the vision geometrical constraints and reflected noises, proposed as a model for collective behavior of individuals. We rigorously derive a continuity-type of mean-field equation with discontinuous kernels and the normal reflecting boundary conditions from that stochastic particle system as the number of particles N goes to infinity. More precisely, we provide a quantitative estimate of the convergence in law of the empirical measure associated to the particle system to a probability measure which possesses a density which is a weak solution to the continuity equation. This extends previous results on an interacting particle system with bounded and Lipschitz continuous drift terms and normal reflecting boundary conditions by Sznitman [J. Funct. Anal., 56, (1984), 311–336] to that one with discontinuous kernels. Joint work with Young-Pil Choi.

15h45-16h15 : Isabelle Tristani (CNRS, École normale supérieure)

On the Boltzmann equation without cutoff

In this talk, we are interested in the Boltzmann equation without cutoff for hard potentials. More precisely, we deal with the problem of convergence to equilibrium in the homogeneous case ; we also study the Cauchy problem for the inhomogeneous equation in a perturbative framework. This is a joint work with Frédéric Hérau and Daniela Tonon.

16h15-17h : Andreas Eberle (Univ. Bonn)

A coupling approach to the kinetic Langevin equation

The (kinetic) Langevin equation is an SDE with degenerate noise that describes the motion of a particle in a force field subject to damping and random collisions. It is also closely related to Hamiltonian Monte Carlo methods. An important question is, why in certain cases kinetic Langevin diffusions seem to approach equilibrium faster than overdamped Langevin diffusions.

So far, convergence to equilibrium for kinetic Langevin diffusions has almost exclusively been studied by analytic techniques. In this talk, I present a new probabilistic approach that is based on a specific combination of reflection and synchronuous coupling of two solutions of the Langevin equation. The approach yields rather precise bounds for convergence to equilibrium at the borderline between the overdamped and the underdamped regime, and it may help to shed some light on the open question mentioned above.

17h30-18h : Pierre Monmarché (École Nationale des Ponts et Chaussées)

Uniform in time propagation of chaos for the Vlasov-Fokker-Planck equation in the convex case

Consider a system of N kinetic particles in a convex external potential interacting through a mean-field interaction. Hypocoercive methods yield a rate of relaxation to equilibrium in large time which is independent from N, from which a uniform in time propagation of chaos toward the Vlasov-Fokker-Planck equation may be established, together with an exponentially fast convergence in large time for the non-linear equation itself. The novelty from existing results is that the interaction is not supposed to be small.

18h-18h45 Stéphane Mischler (Univ. Paris-Dauphine)

Villani's program on constructive rate of convergence to the equilibrium : Part I - Coercivity estimates

In this talk I will first recall Villani's program on constructive rate of convergence to the equilibrium for the Boltzmann and Landau equations. I will next present new and comprehensive proofs for establishing optimal coercivity estimate for the Boltzmann (with cut-off) and the Landau operators, recovering some estimates established by Bobylev, Degond, Lemou, Mouhot, Baranger, Strain and many others.

Wednesday 19

9h-10h30 : Laurent Desvillettes (Univ. Paris 7)
Entropic structure of the Landau equation with Coulomb interaction, I

The Landau equation enables to understand the effect of binary collisions in a plasma. Like for the Boltzmann equation, the properties of the solutions of this equation are strongly related to its entropic structure (H theorem). We investigate both the smoothness properties and the properties related to the large time behavior of these solutions. We concentrate on the most interesting case from the physical point of view, that is the case of the Coulomb interaction. We also briefly explain how the methods developed for the Landau equation can help to understand the entropy structure of more complex equations related to collision kernels (coming out for example from the theory of weak turbulence).

11h-11h45 : Mario Pulvirenti (Univ. Roma 1)
On the Propagation of Chaos in Kinetic Theory

In this talk I discuss the concept of propagation of chaos for the various scaling limits (Mean-Field, Low-density and weak-coupling) occurring in Kinetic Theory. I also present recent results in this direction.

11h45-12h30 : Benjamin Jourdain (École Nationale des Ponts et Chaussées)
On a stochastic particle approximation of the Keller-Segel equation

The Keller-Segel partial differential equation is a two-dimensional model for chemotaxis. It is known to exhibit blow-up in finite time when the sensitivity χ of bacteria to the chemo-attractant is larger than 8π. We investigate its approximation by systems of N two-dimensional Brownian particles interacting through a singular attractive kernel in the drift term. In the very subcritical case $\chi < 2\pi$, the diffusion somehow dominates this singular drift and we obtain existence for the particle system and prove that the flow of empirical measures converge as $N \to \infty$ to a weak solution of the Keller-Segel equation.

For any solution of the system with $N \geq 2$ particles, pairs of particles do collide with positive probability so that the singularity of the drift term is visited. Nevertheless, it is possible to control the drift and obtain existence of the particle system until the first time when at least three particles collide. We check that this time is infinite iff $\chi < 8\pi(N-2)/(N-1)$. Last, we remark that in the system with $N = 2$ particles, the difference between the two positions provides a natural two-dimensional generalization of Bessel processes and study in details this system. Joint work with Nicolas Fournier.

14h-14h45 : Pietro Caputo (Univ. Roma 3)
Entropy production in nonlinear recombination models

We study the convergence to equilibrium of a class of nonlinear discrete recombination models. In contrast with previous analysis of these models, the convergence is measured in terms of relative entropy. The main result is a tight quantitative estimate for the entropy production functional. Along the way we establish some new inequalities generalizing Shearer's and related entropy inequalities. The problem is formulated within a general framework that we refer to as Reversible Quadratic Systems, which includes several nonlinear analogues of familiar Markov chains such as random walks and stochastic Ising models. This is based on joint work with Alistair Sinclair.

14h45-15h15 : Tom Holding (Warwick Univ.)
Propagation of chaos for Hölder continuous interaction kernels via Glivenko-Cantelli

We develop a new technique for establishing quantitative propagation of chaos for systems of interacting particles. Using this technique we prove propagation of chaos for diffusing particles whose interaction kernel is merely Hölder continuous, even at long ranges. Moreover, we do not require specially prepared initial data. On the way, we establish a law of large numbers for SDEs that holds over a class of vector fields simultaneously. The proofs bring together ideas from empirical process theory and stochastic flows.

15h45-16h15 : Laure Pédèches (Univ. Toulouse)
Flocking for stochastic variations of the Cucker-Smale model

We look into various stochastic alterations of Cucker and Smale's mean-field, kinetic model for a population of autonomous agents without central direction. We are interested in their asymptotic behaviour, and, more precisely, their "flocking" and "swarming" properties. They significantly differ depending on the structure of the Brownian noise added to the original deterministic dynamics, modelling for instance a random environment or the free will of each individual.

16h15-17h : Maria Gualdani (George Washington Univ.)
New estimates for the Landau equation with soft potentials

One of the most interesting physical cases in gas and plasma physics is collision of particles under the influence of Coulomb potential. For this potential the Boltzmann equation is not a valid model anymore. The reason is that the momentum exchanged among particles during a collision is divergent. The physical explanation of such mathematical divergence is that grazing collisions cannot be neglected when the potential is of Coulomb type. This problem was known by Landau, who in 1936 derived from the Boltzmann equation a kinetic equation that describes collisions of particles in plasmas where grazing collisions are predominant. Many properties of this equation are known ; however the issue of existence and uniqueness of smooth solution is a long standing open problem. In particular it is not yet clear if solutions become unbounded in a finite time. In this talk we present two new results on smoothness and regularity : any weak radial solution to the Landau equation with Coulomb potential that belongs to $L^{3/2}$ it is automatically bounded and smooth. The second result concerns local bounds which do not deteriorate as time grows for any solution to the Landau equation with moderately soft-potentials. This is a joint work with N. Guillen.

17h30-18h : Julien Reygner (École Nationale des Ponts et Chaussées)
Convergence rates for a particle approximation of conservation laws

Sticky particle dynamics are introduced in order to approximate the solution to either scalar or diagonal systems of conservation laws in one space dimension, with large but monotonic data. This talk will focus on deriving explicit convergence rates as well as designing an efficient numerical scheme based on this approximation procedure. Joint work with Benjamin Jourdain.

Thursday 20

9h-10h30 : Thierry Bodineau (CNRS, École Polytechnique)
From hard-sphere dynamics to the Boltzmann equation, II

11h-11h45 : Pierre Degond (Imperial College, London)
Coarse-graining of collective dynamics models

In this talk, we will report on some new individual-based models of collective dynamics and their coarse-graining into continuum models. The applications span from collective cell dynamics (such as social bacteria or sperm) to flocking of birds or fish. Models of social behavior are best set up at the individual scale where behavioral rules can be easily introduced and tested. However, the complexity of individual-based models increases rapidly with the number of individuals and their calibration or control can hardly be implemented at this level. To overcome this limitation, one often uses continuum model that describe the system through average quantities such as densities or mean orientation. But the downside of most models in the literature is that the link between the rules at the individual behavior and the coefficients in the macroscopic model are not known exactly and are at best extrapolated from heuristic consideration. Here, we propose a systematic and mathematical rigorous way to derive continuum models from collective dynamics models. It relies on the introduction of a new concept, the "generalized collision invariants", which permit to overcome the lack of physical invariance in most systems undergoing collective dynamics. In this talk, we will review some recent developments of these concepts and how they can be used to model systems of practical scientific importance.

11h45-12h30 : Maxime Hauray (Univ. Aix-Marseille)
Propagation of chaos for the 3D homogenous Landau equation with moderalty soft potential

Abstract : I will present results obtained in collaboration with Nicolas Fournier, on the propagation of chaos for the Landau equation. The difficulty here is the presence of a singularity in the interaction kernel that appears in equation. For mild singularities, we obtain quantitative results of convergence using a weak-strong stability result for the Landau equation, and a perturbation of it, that allows to apply it also to empirical measures associated to particles system approximating the Landau equation. For stronger singularity, we obtain a qualitative result of convergence, relying on the techniques introduced previously with Stéphane Mischler for the case of vortex, but with several improvement in order to control the possibly degenerate Landau diffusion.

Friday 21

9h-10h30 : Laurent Desvillettes (Univ. Paris 7)
Entropic structure of the Landau equation with Coulomb interaction, II

11h-11h45 : Irene Gamba (Univ. Texas, Austin)
The Cauchy problem for the quantum Boltzmann equation for bosons at very low temperature

This model approximates the evolution of quasiparticles in a dilute gas of bosons at very low temperature by a Boltzmann problem with a cubic kinetic transition probability kernel. The solution to this equation couples to the quantum density evolution of the condensate, modeled by a coupled system of Gross-Pitaevskii and quantum Boltzmann equation for bosons. At this first stage, we prove existence and uniqueness for the quantum Boltzmann model after deriving a priori qualitative properties including propagation and creation of polynomial moments, by means of ODE methods in Banach spaces by characterizing an invariant bounded, convex, closed solutions subset of integrable solutions with bounded mass differentiable in time. We also show the propagation and creation of Mittag-Leffler moments that characterize the exponential order of the tails decay. This is a work in collaboration with Ricardo J. Alonso and Minh Binh Tran.

11h45-12h15 : Max Fathi (CNRS, Toulouse)
Ricci curvature and functional inequalities for interacting particle systems

I will present a few results on entropic Ricci curvature bounds, with applications to interacting particle systems. The notion was introduced by M. Erbar and J. Maas and independently by A. Mielke. These curvature bounds can be used to prove functional inequalities, such as spectral gap bounds and modified logarithmic Sobolev inequalities, which measure the rate of convergence to equilibrium for the underlying dynamic. Based on joint works with J. Maas and M. Erbar.

14h-14h45 : Florent Malrieu (Univ. Tours)
Long time behaviour of some McKean-Vlasov equation with non-convex confinement potential

The propagation of chaos and long time behaviour of McKean-Vlasov equations with convex confinement potential and smooth attraction is well understood. Nevertheless, if the confinement is no longer convex, the McKean-Vlasov equation may have several invariant measures and then the propagation of chaos cannot be uniform in time. One of the interesting issues in this context is the description of the basin of attraction of each invariant measure. I will present several results, mainly on the Kuramoto model, and open questions.

14h45-15h30 : Juan J. L. Velázquez (Univ. Bonn)
Long time asymptotics of homoenergetic solutions for the Boltzmann equation

Homoenergetic solutions are a particular class of solutions of the Boltzmann equation which were introduced in the 1950's by Galkin and Truesdell. They are useful to describe the dynamics of Boltzmann gases under shear, expansion or compression in nonequilibrium situations. Homoenergetic solutions are much simpler than the general solutions of the Boltzmann equation. Their well posedness theory, which has many similarities with the theory of homogeneous solutions of the Boltzmann equation was studied by Cercignani in the 1980's. However, the corresponding long time asymptotics theory differs much of the analogous theory for homogeneous solutions. Actually, in the case of homoenergetic solutions, the long time asymptotics cannot always be described using Maxwellian distributions. For several collision kernels the long time behaviour of homoenergetic solutions is given by particle distributions which do not satisfy the detailed balance condition. In this talk I will describe different possible long time asymptotics of homoenergetic solutions of the Boltzmann equation as well as some open problems in this direction. (Joint work with R.D. James, S. Müller and A. Nota).

Saturday 22

9h30-10h15 Tony Lelièvre (École Nationale des Ponts et Chaussées)
Title : Metastability : a journey from stochastic processes to semiclassical analysis

A stochastic process is metastable if it stays for a very long period of time in a region of the phase space (called a metastable region) before going to another metastable region, where it again remains trapped. Such processes naturally appear in many applications, metastability being related to a two time scale mechanism : the small time scale corresponds to the vibration period within the metastable regions and the large time scale is associated with the transitions between metastable states. For example, in molecular dynamics, the metastable regions are typically associated with the atomic conformations of a molecule (or an ensemble of molecules), and one is actually interested in simulating and studying the transitions between these conformations.

In this talk, I will explain how the exit events from a metastable state can be studied using an eigenvalue problem for a differential operator. This point of view is useful to build very efficient algorithms to simulate metastable stochastic processes (using in particular parallel architectures). It also gives a new way to prove the Eyring-Kramers laws and to justify the parametrization of an underlying Markov chain (Markov state model), using techniques from semiclassical analysis.

10h45-11h15 Kleber Carrapatoso (Univ. Montpellier)
Long-time behaviour of the Landau equation

I will present recent results, obtained in collaboration with S. Mischler, on the long-time behaviour of solutions to the inhomogeneous Landau equation with Coulomb potential.

11h15-12h Laurent Miclo (CNRS, Toulouse)
On Markov intertwinings

As part of the audience of this meeting belongs to the PDE community, we will begin by giving a feeling of Markov processes through the simple example of top-to-random card shuffle and by showing how the notion of strong stationary times enabled Aldous and Diaconis (1986) to study its quantitative convergence to equilibrium. The goal of the talk is to extend the underlying principle to elliptic diffusions on manifolds, via Markov intertwinings, which correspond to a weak similar relation between semigroups (preserving non-negativity and the function 1). Instead of only looking at the motion of one diffusive particle, we will couple it with dynamical domains containing the particle and whose boundary evolution is a stochastic modification of the mean curvature flows. The (remote) hope is a new probabilistic approach to Hörmander's theorem.