New a priori bounds for non-linear SPDEs

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Stochastic Partial Differential Equations

CIRM

Joint with A. Moinat and F. Otto

Aim of talk: Bounds for non-linear SPDEs

Quasilinear

$$\partial_t u - \nabla \cdot A(\nabla u) = \xi. \tag{Q}$$

- Control α-Hölder norm of ∇u by solution of linear problem $A = \mathrm{Id}$.
- Joint with F. Otto.

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Reaction diffusion equations

$$(\partial_t - \Delta)u = -|u|^{m-1}u + \xi \qquad m > 1. \tag{RD}$$

- Control u over compact set by distributional norm of ξ over larger set.
- Joint with A. Moinat.

Aim of talk: Bounds for non-linear SPDEs

Common theme: Ignore probability theory

- Think of $\xi =$ "*dW*" or multiplicative $\xi =$ "σ*dW*",

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Common theme: Ignore probability theory

- Think of $\xi =$ "*dW*" or multiplicative $\xi =$ "σ*dW*", but only use information on regularity of distribution ξ.

Quasilinear equation: Setup

$$\partial_t u - \nabla \cdot A(\nabla u) = \xi \qquad \text{(Q)}$$

The non-linearity

- $A\colon \mathbb{R}^d \to \mathbb{R}^d$ uniformly elliptic

$$\zeta \cdot DA(q)\zeta \geq \lambda|\zeta|^2 \quad \text{and} \quad |DA(q)\zeta| \leq |\zeta|.$$

- DA globally Lipschitz

$$|DA(q') - DA(q)| \leq \Lambda|q' - q|.$$

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The noise

Control in terms of solution to linear equation

$$\partial_t v - \Delta v = \xi.$$

Assume that ∇v is α Hölder (w.r.t. parabolic metric).

Example

$$dv = \Delta v \, dt + dW(t) \qquad x \in \mathbb{T}^d.$$

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- The assumption that $[\nabla v]_\alpha < \infty$ corresponds to Q (slightly better than) trace class.
- Assume that noise vanishes for $t \notin [0, 1]$.
- Well-known theory of variational solutions for non-linear equation. Solutions satisfy

$$\sup_{0 \le t \le \infty} \|u(t)\|_{L^2}^2 + \int_0^\infty \|\nabla u(t)\|_{L^2}^2 dt < \infty \qquad \text{a.s.}$$

Results

$$\partial_t u - \nabla \cdot A(\nabla u) = \partial_t v - \Delta v \qquad \text{over } \mathbb{R}_t \times \mathbb{R}_x^d.$$

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Lemma 1 - small α

There exists $\alpha_1 \in (0,1)$ such that for $\alpha_0 \in (0, \alpha_1)$

$$[\nabla u]_{\alpha_0} \leq C(d, \lambda, \alpha_0)[\nabla v]_{\alpha_0}.$$

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Lemma 2 - arbitrary α

Let α_0 be as in Lemma 1. Let L satisfy

$$[\nabla u]_{\alpha_0, P_{2L}} \leq L^{-\alpha_0}.$$

Then for $\alpha \in [\alpha_0, 1)$

$$[\nabla u]_{\alpha, P_L} \leq C(d, \lambda, \Lambda, \alpha_0, \alpha)\big(L^{-\alpha} + [\nabla v]_{\alpha, P_{2L}} + [j]_{\alpha, P_{2L}}\big).$$

Results

Corollary

Let α_0 be as in Lemma 1. Then for $\alpha \in (0,1)$

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Corollary: Stretched exponential bounds

If v is random and both $[\nabla v]_{\alpha_0}$ and $[\nabla v]_\alpha$ have Gaussian tails, then for $C \gg 1$

$$\mathbb{E}\Big(\exp\Big(\frac{1}{C}[\nabla u]^{2\min\{1,\frac{\alpha_0}{\alpha}\}}\Big)\Big) < \infty.$$

Related works:

Debussche, De Moor, and Hofmanová

$$\partial_t u = \nabla \cdot (A(u)\nabla u) + H(u)\dot{W}.$$

- Subtract v solution of $(\partial_t - \Delta)v = H(u)\dot{W}$.
- Remainder $w = u - v$ solves

$$\partial_t w - \nabla \cdot (A(u)\nabla w) = \nabla \cdot (A(u)\nabla v).$$

  Apply De Giorgi-Nash Theorem.
- Schauder theory to gives higher regularity.

Related works:

Otto, W. 2015

One dimensional equation, driven by space-time white noise

$$\frac{1}{T}u + \partial_t u - \partial_x^2 \pi(u) = \xi, \tag{1}$$

Derive bounds on $[u]_\alpha$.

- Argument: L^2 bounds using energy arguments. Upgrade to Hölder scale using Gaussian concentration inequalities.
- Our deterministic result (essentially) contains (1) by differentiating (Q) in space once.

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$$\partial_t \delta_y w - \nabla \cdot a_y \nabla \delta_y w = \nabla \cdot (a_y \nabla \delta_y v + \delta_y j)$$

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- "Chain rule"

$$a_y(t,x) = \int_0^1 DA(\theta \nabla u(t, x+y) + (1-\theta)\nabla u(t,x)) d\theta$$

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- We do not go to zero in y difference (quotient).

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- In order to bound differences of ∇w at scale ℓ we choose y at scale $r = \varepsilon \ell$.

Proof of Lemma 1

Apply De Giori-Nash

$$[\delta_y w]_{\alpha_1, P_\ell(z)} \lesssim \ell^{-\alpha_1} \inf_k \|\delta_y w - k\|_{P_{2\ell}(z)} + \ell^{1-\alpha_1} \|a_y \nabla \delta_y v + \delta_y j\|_{P_{2\ell}(z)}.$$

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Taking supremum over $|y| \le r$

Elementary manipulation of LHS and RHS lead to

$$\frac{1}{r^{1+\alpha_0}} \sup_{|y|\le r} \inf_k \|\delta_y w - k\|_{P_r(z)}$$
$$\lesssim \left(\frac{r}{\ell}\right)^{\alpha_1 - \alpha_0} [\nabla w]_{\alpha_0} + \left(\frac{\ell}{r}\right)^{1-\alpha_1} ([\nabla v]_{\alpha_0} + [j]_{\alpha_0}).$$

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Norm equivalence

$$[\nabla w]_{\alpha_0} \lesssim \sup_{z,r} \frac{1}{r^{1+\alpha_0}} \sup_{|y|\leq r} \inf_k \|\delta_y w - k\|_{P_r(z)}.$$

Reaction diffusion equations

$$(\partial_t - \Delta)u = -|u|^{m-1}u + \xi \qquad m > 1. \tag{RD}$$

- ξ distribution of (negative) regularity $\alpha - 2$.
- Assume that (RD) holds on $P_0 := [0,1] \times \{x \colon |x| < 1\}$.

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Theorem: Space-time "Coming down from ∞"

$$\|u\|_{P_R} \leq C(\varepsilon, \alpha, d, m) \max \left\{ R^{-\frac{2}{m-1} - \varepsilon}, [\zeta]_{\alpha-2, P_0}^{\frac{2}{2+(m-1)\alpha}} \right\}.$$

- $\|u\|_{P_R}$ = supremum norm over smaller cylinder

$$[R^2, 1] \times \{x \colon |x| < 1 - R\}$$

.

Discussion

$$\|u\|_{P_R} \lesssim \max\left\{ R^{-\frac{2}{m-1}-\varepsilon}, [\zeta]_{\alpha-2,P_0}^{\frac{2}{2+(m-1)\alpha}} \right\}.$$

- Bound does not depend on boundary conditions.

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- If ξ random with $[\zeta]_{\alpha-2,P_0}$ Gaussian moments, then for $C \gg 1$

$$\mathbb{E}\Big(\exp\Big(\frac{1}{C} \|u\|_{P_R}^{2+(m-1)\alpha} \Big) \Big) < \infty.$$

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  - Better than Gaussian integrability.
  - Interplay between regularity and integrability.
- ε should not be there.

Optimal integrability

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Special case: $\xi =$ space-time white noise (say over $\mathbb{T}^1$).
(RD) describes reversible Markov process w.r.t. measure

$$\mu(du) \propto \exp\Big(-\frac{1}{2}\int_{\mathbb{T}^1} u^4 dx\Big)\nu(du)$$

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Under μ

$$\mathbb{E} \exp\Big(\frac{1}{C} \|u\|_{L^4}^4 \Big) < \infty \qquad \text{and} \qquad \mathbb{E} \exp\Big(\frac{1}{C} [u]_\alpha^2 \Big) < \infty \quad \alpha < \frac{1}{2}.$$

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Interpolating $\|u\|_{L^4}$ and $[u]_\alpha$ leads to (2).

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Lemma

For all $p < \infty$

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- Proof by iterated testing against localised version of $|u|^{p-1}$.
- Relies heavily on damping non-linearity. Regularity improvement of heat operator not (really) used.

Idea of proof 2: Reducing to the smooth case

Regularise (RD)

$(\cdot)_T =$ smoothing at scale T.

$$(\partial_t - \Delta) u_T = -|u_T|^{m-1} u_T + \xi_T + \mathrm{Error}(T),$$

where $\mathrm{Error}(T) = [|u_T|^{m-1} u_T - (|u|^{m-1} u)_T]$ commutator.

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Apply smooth case

$$\|(u)_T\|_{p,P_R} \lesssim \max\Big\{ \Big(\frac{C_p^2}{(R-R')^2} \Big)^{\frac{1}{m-1}}, \big(\|(\zeta)_T\|_{\infty,P_{R'}}\big)^{\frac{1}{m}}, \big(\|(u)_T^m - (u^m)_T\|_{\infty,P_{R'}}\big)^{\frac{1}{m}} \Big\}. \tag{3}$$

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Commutator

$\|(u)_T^m - (u^m)_T\|_{\infty,P_R}$

- Bound in terms of $\|u\|$ and $[u]_\alpha$.
- which in turn can be bounded, using the (unregularized) equation once more and local Schauder estimates.

Discussion

- Argument is heavily inspired by Hairer's notion of sub-criticality:
  - Use heat operator on small scales.
  - Use non-linearity on large scales.
- Motivation: Find a method to derive a priori bounds within the framework of regularity structures. Then local Schauder estimate should be replaced by version of Hairer's Integration Theorem.
- For now ϕ_2^{2m} also looks promising, higher dimensions...?

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- Treat SPDE with deterministic techniques.

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Common theme:

- Treat SPDE with deterministic techniques.
- Careful use of regularisation at various scales.