STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS

CIRM, LUMINY, 14 - 18 MAY 2018

Abstracts

Dirk Blömker
Random initial conditions for semi-linear PDEs

Abstract: We analyze the effect of random initial conditions on the local well-posedness of semi-linear PDEs, like for example Burgers, KPZ, Kuramoto–Sivashinky, or many other models. The main aim is to investigate to what extent recent ideas on singular stochastic PDEs can prove to be useful in this framework.

We discuss several aspects where random initial conditions improve the results, but also discuss the limitations of it. For example, in some cases one is able to prove the local existence and uniqueness of solutions for random (Gaussian) initial conditions with a regularity that would not allow for a proof in the case of deterministic initial conditions. But it seems to be impossible to break the barrier formed by the theory of critical spaces.
Joint work with Giuseppe Cannizzaro and Marco Romito.

Zdzislaw Brzezniak
Weak martingale solutions for the stochastic nonlinear Schrödinger equation driven by pure jump noise

Abstract: I will speak about the existence of weak martingale solutions for the stochastic nonlinear Schrödinger equation driven by pure jump noise. In order that the L^2-norm of the solution remains constant in time we use the so called Marcus canonical form. The interactions of it with the nonlinearity cause some unexpected difficulties which require using Paley–Littlewood theory. The talk is based on a joint work Fabian Hornung and Utpal Manna.

Carsten Chong
Path properties of the solution to the stochastic heat equation with Lévy noise

Abstract: We consider sample path properties of the solution to the stochastic heat equation driven by a Lévy space-time white noise. When viewed as a stochastic process in time with values in an infinite-dimensional space, the solution is shown to have a càdlàg modification in fractional Sobolev spaces of index less than $-d/2$. Concerning the partial regularity of the solution in time or space when the other variable is fixed, we determine critical values for the Blumenthal–Getoor index of the Lévy noise such that noises with a smaller index entail continuous sample paths, while Lévy noises with a larger index entail sample paths that are unbounded on any non-empty open subset.

Robert C. Dalang

Global solutions to reaction-diffusion equations with super-linear drift and multiplicative noise

Abstract: We consider a stochastic heat equation with additive non-linearity b and multiplicative non-linearity σ, in the case where the drift b is super-linear: $|b(z)| \geqslant |z|(\log|z|)^{1+\varepsilon}$, for some $\varepsilon > 0$. When $\sigma \equiv 0$, it is well known that such PDEs frequently have non-trivial stationary solutions. By contrast, Bonder and Groisman (2009) have shown that when σ is constant and $\sigma \neq 0$, there is finite-time blowup. We prove that the Bonder–Groisman condition is unimproveable by showing that the reaction-diffusion equation with noise is "typically" well posed when $|b(z)| = O(|z| \log_+ |z|)$ as $|z| \to \infty$.
This is joint work with Davar Khoshnevisan (University of Utah) and Tusheng Zhang (University of Manchester).

Anne De Bouard

Stochastic homogenization of the Landau–Lifshitz equation

Abstract: The theory of micromagnetism, which describes the magnetization of ferromagnetic materials at the mesoscopic scale has been the subject of extensive studies since its construction in the 1940s by W. F. Brown and Landau–Lifshitz. Nowadays, there is a strong demand from of a large community of physicists and engineers for more complex models involving possibly stochastic (spatial and temporal) perturbations. The use of random structures is in particular natural for modern magnets, obtained as alloys of several materials with different magnetic properties. We will study the homogenization of these materials, described by the Landau–Lifshitz equation with random coefficients.
This is a joint work with F. Alouges, B. Merlet and L. Nicolas.

Henri Elad Altman

Integration by parts formulae for the laws of Bessel bridges, and Bessel-like SPDEs

Abstract: In the early 2000s, Zambotti introduced a family of Bessel-like SPDEs, parametrized by a real number d larger or equal to 3. The solutions to these equations define Markov processes on the set of nonnegative functions, and exhibit a rich behavior reminiscent of Bessel processes. In fact, their unique invariant measure corresponds to the law of the d-dimensional Bessel bridge. A long-standing open problem is to extend such results to d less than 3. A partial answer to the question was provided recently, with the derivation of integration by parts formulae for bridges of dimension less than 3. In my talk, I will explain these formulae in detail. The particular case where d equals 1, which corresponds to the law of the reflecting Brownian bridge, will be mentioned.

Perla El Kettani

A stochastic mass conserved reaction-diffusion equation with nonlinear diffusion

Abstract: In this talk, we study a stochastic mass conserved reaction-diffusion equation with a linear or nonlinear diffusion term and an additive noise corresponding to a Q-Brownian motion. We prove the existence and the uniqueness of the weak solution. The proof is based upon the monotonicity method.
This is joint work with D. Hilhorst and K. Lee.

Benjamin Fehrman

Well-posedness of stochastic porous media equations with nonlinear, conservative noise

Abstract: In this talk, which is based on joint work with Benjamin Gess, I will describe a pathwise well-posedness theory for stochastic porous media equations driven by nonlinear, conservative noise. Such equations arise in the theory of mean field games, as an approximation to the Dean–Kawasaki equation in fluctuating hydrodynamics, to describe the fluctuating hydrodynamics of a zero range process, and as a model for the evolution of a thin film in the regime of negligible surface tension. Our methods are based on the theory of stochastic viscosity solutions, where the noise is removed by considering a class of test functions transported along underlying stochastic characteristics. We apply these ideas after passing to the equation's kinetic formulation, for which the noise enters linearly and can be inverted using the theory of rough paths.

Franco Flandoli

Stochastic solutions of 2D fluids

Abstract: We revise recent contributions to 2D Euler and Navier–Stokes equations with and without noise, but always in the case of stochastic solutions. The role of white noise initial conditions will be stressed and related to some questions about turbulence.

Paul Gassiat

On the speed of propagation for stochastic Hamilton-Jacobi equations

Abstract: We study the speed of propagation of initial data for Hamilton–Jacobi equations with multiplicative rough (typically stochastic) time dependence. We first show that, in contrast with the classical (deterministic) case, in general this speed may be infinite as soon as the driving noise has unbounded variation. In the case where the Hamiltonian is convex in the gradient, we show that the range of dependence is bounded by a multiple of the length of a piecewise linear path obtained by connecting the successive extrema of the original path. When the driving path is a Brownian motion, this implies finite speed of propagation.
Based on a joint work with B. Gess, P. Souganidis and P.L. Lions.

Benjamin Gess

Path-by-path regularization by noise for scalar conservation laws

Abstract: In this talk we will revisit regularizing effects of noise for nonlinear SPDE. In this regard we are interested in phenomena where the inclusion of stochastic perturbations leads to increased regularity of solutions as compared to the unperturbed, deterministic case. Closely related, we study effects of production of uniqueness of solutions by noise, i.e. instances of SPDE having a unique solution, while non-uniqueness holds for the deterministic counterparts. The talk will concentrate on a path-by-path regularization by noise result in the case of nonlinear scalar conservation laws. In particular, this proves regularizing properties for scalar conservation laws driven by fractional Brownian motion and generalizes the respective results obtained in [G., Souganidis; Comm. Pure Appl. Math. (2017)]. We show that (ρ, γ)-irregularity is a sufficient path-by-path condition implying improved regularity. In addition, we introduce a new path-by-path scaling property which is also shown to be sufficient to imply regularizing effects.

Martin Hairer
Quasilinear singular SPDEs

Abstract: tba

Erika Hausenblas
Controllability, irreducibility and support theorems of SPDEs

Abstract: In the talk we first introduce some controllability properties for deterministic systems, like approximate controllability, exact controllability and solid controllability. As next, we introduce stochastic systems described by SPDEs. From these controllability properties one can derive properties of the Markovian semigroup of a stochastic system. Here we give two examples, once irreducibility of the Markovian semigroup of the stochastic wave equation and existence of a density of the 2D Navier–Stokes equation on finite dimensional subspaces. Here, we assume that only a finite numbers of modes are perturbed by some Lévy noise.

Martina Hofmanova
Global solutions to elliptic and parabolic Φ^4 models in Euclidean space

Abstract: I will present some recent results on global solutions to singular SPDEs on $\mathbb{R}^d$ with cubic nonlinearities and additive white noise perturbation, both in the elliptic setting in dimensions $d = 4, 5$ and in the parabolic setting for $d = 2, 3$. A motivation for considering these equations is the construction of scalar interacting Euclidean quantum field theories. The parabolic equations are related to the Φ^4_d Euclidean quantum field theory via Parisi–Wu stochastic quantization, while the elliptic equations are linked to the Φ^4_{d-2} Euclidean quantum field theory via the Parisi–Sourlas dimensional reduction mechanism. We prove existence for the elliptic equations and existence, uniqueness and coming down from infinity for the parabolic equations. Joint work with Massimiliano Gubinelli.

Raphael Kruse
On a randomized Milstein method for S(P)DEs

Abstract: The Milstein scheme is a well-known numerical method for the temporal approximation of stochastic ordinary differential equations. The standard derivation is based on an iterated application of Itô's lemma to the coefficient functions and therefore requires some smoothness. In recent years the Milstein method has also been generalized to infinite dimensional stochastic evolution equations. However, the smoothness conditions then usually turn out to be too restrictive for most applications. In this talk we propose a new drift-randomized version of the Milstein method that relaxes the smoothness assumptions on the drift coefficient function considerably in the case of SODEs and for semilinear SPDEs. In particular, it is no longer necessary to impose a differentiability condition on the semilinearity. We also show that the randomization technique reduces the smoothness assumption for time-dependent coefficients considerably.
Joint work with Yue Wu (TU Berlin).

Guanglian Li

On the decay rate of the singular values of bivariate functions

Abstract:

In this work, we establish a new truncation error estimate of the singular value decomposition (SVD) for a class of Sobolev smooth bivariate functions $\kappa \in L^2(\Omega, H^s(D))$, $s \geqslant 0$, and $\kappa \in L^2(\Omega, \dot{H}^s(D))$ with $D \subset \mathbb{R}^d$, where $H^s(D) = W^{s,2}(D)$ and $\dot{H}^s(D) = \{v \in L^2(D) \colon (-\Delta)^{s/2} v \in L^2(D)\}$ with $-\Delta$ being the negative Laplacian on D coupled with specific boundary conditions. To be precise, we show the order $\mathcal{O}(M^{-s/d})$ for the truncation error of the SVD series expansion after the Mth term. This is achieved by deriving the sharp decay rate $\mathcal{O}(n^{-1-\frac{2s}{d}})$ for the square of the nth largest singular value of the associated integral operator, which improves on known results in the literature. We then use this error estimate to analyze an algorithm for solving a class of elliptic PDEs with random coefficient in the multi-query context, which employs the Karhunen–Loève approximation of the stochastic diffusion coefficient to truncate the model.

Reference:

M. Griebel and G. Li, *SIAM J. Numer. Anal.*, 56(2), 974–993, 2018.

Jonathan Mattingly

Approximate/exact controllability and ergodicity for (additive noise) SPDEs/SODEs

Abstract:

tba

Romeo Mensah

Scale interactions in stochastic fluid models

Abstract:

In this talk, we take a look at the compressible Navier-Stokes system driven by random forces. We consider weak martingale solutions to the system that dissipates energy and state the known existence results. We then explore how a generalized version of the system — when additional deterministic forces are explicitly considered — relates with the stochastic incompressible Navier–Stokes system using singular limit analysis.

Jean-Christophe Mourrat

Quantitative stochastic homogenization, theory and practice

Abstract:

Divergence-form operators with random coefficients homogenize over large scales. Recently, major progress has allowed to make this convergence quantitative. I will outline some of the main results in this direction, and then focus on some related insights concerning the computational aspects of homogenization.
Joint work with Armstrong, Hannukainen and Kuusi.

Alexandra Neamtu
Pathwise mild solutions for quasilinear stochastic partial differential equations

Abstract:
Stochastic partial differential equations (SPDEs) have become a key modelling tool in applications. Yet, there are many classes of SPDEs, where the existence and regularity theory for solutions is not completely developed. Here we contribute to this aspect and provide mild solutions for a broad class of quasilinear Cauchy problems, including — among others — cross-diffusion systems as a key application. Our solutions are local-in-time and are derived via a fixed point argument in suitable function spaces. The main idea is to combine the classical theory of deterministic quasilinear parabolic partial differential equations with recent theory of evolution semigroups. We show how these techniques can be applied to the Shigesada–Kawasaki–Teramoto model.
This is joint work with Christian Kuehn (TU Munich).

Eulalia Nualart
Asymptotics for some non-linear stochastic heat equations

Abstract:
Consider the following stochastic heat equation,

$$\frac{\partial u_t(x)}{\partial t} = -\nu(-\Delta)^{\alpha/2} u_t(x) + \sigma(u_t(x))\dot{F}(t, x), \quad t > 0,\ x \in \mathbb{R}^d.$$

Here $-\nu(-\Delta)^{\alpha/2}$ is the fractional Laplacian with $\nu > 0$ and $\alpha \in (0, 2]$, $\sigma : \mathbb{R} \to \mathbb{R}$ is a globally Lipschitz function, and $\dot{F}(t, x)$ is a Gaussian noise which is white in time and colored in space. Under some suitable conditions, we will explore the effect of the initial data on the spatial asymptotic properties of the solution. We also prove a strong comparison principle thus filling an important gap in the literature.
Joint work with Mohammud Foondun (University of Strathclyde).

Felix Otto
Singular SPDE with rough coefficients

Abstract:
We are interested in parabolic differential equations $(\partial_t - a\partial_x^2)u = f$ with a very irregular forcing f and only mildly regular coefficients a. This is motivated by stochastic differential equations, where f is random, and quasilinear equations, where a is a (nonlinear) function of u.

Below a certain threshold for the regularity of f and a (on the Hölder scale), giving even a sense to this equation requires a renormalization. In the framework of the above setting, we present recent ideas from the area of stochastic differential equations (Lyons' rough path, Gubinelli's controlled rough paths, Hairer's regularity structures) that allow to build a solution theory. We make a connection with Safonov's approach to Schauder theory.
This is based on joint work with H. Weber, J. Sauer, and S. Smith.

Stanislav Shaposhnikov

Nonlinear diffusion processes and Fokker–Planck–Kolmogorov equations

Abstract:

At last decade there is an extremely higher interest to finite and infinite dimensional nonlinear diffusion processes with coefficients depending on distributions of processes. Such problems arise in different physical, biological and economic models. Main questions concern with the existence, uniqueness and convergence to stationary distribution. Moreover, different estimates of distances between transition probabilities play the crucial role. We discuss all of that problems, in particular, we present several new results concerning of the convergence to stationary distribution in the case when there are several invariant measures.

Jonas Tölle

Gradient flows for the stochastic Amari neural field model

Abstract:

We shall discuss aspects of a nonlocal SPDE with applications to the so-called Amari-type neural field model, which are mean-field models for neural activity in the cortex. In particular, under suitable assumptions on the coupling kernel, we show that via an infinite dimensional change of coordinates the equation admits a gradient flow structure, which has consequences for regularity, long-time behavior and uniqueness of invariant distributions.
Joint work with Christian Kuehn, TU Munich.

Mark Veraar

Pathwise mild solutions and optimal regularity estimates for parabolic SPDEs with adapted coefficients

Abstract:

In this talk I will present recent results on optimal regularity estimates for a large class of parabolic SPDEs with multiplicative noise. The results are an unexpected extension of previous results. In particular, we are able to cover systems equations with adapted and measurable coefficients in time; a case which was completely open problem. One of the new tools is the so-called "pathwise mild solution", introduced by Pronk and the speaker in 2014. This is a new type of mild solution formula for the solution of an SPDE. Other tools to obtain the required regularity estimate are from stochastic calculus and harmonic analysis.
The talk is based on joint work with Pierre Portal.

Hendrik Weber

New a priori bounds for non-linear SPDEs

Abstract: In this talk I will present two results on a priori bounds for non-linear stochastic partial differential equations (SPDEs). The first bound concerns solutions to uniformly parabolic quasilinear equation with a noise term and the second bound concerns stochastic reaction diffusion equations. In both cases the noise is assumed to be additive and not too irregular so that solutions can be constructed by classical methods.

I will show how detailed bounds on these solutions can be obtained by purely deterministic methods, viewing the SPDE as a PDE with irregular right hand side. For the quasilinear case I will present an optimal bound on the parabolic C^α norm. In the case of the stochastic reaction diffusion equation I will show a non-linear localisation effect where solutions can be bounded on a compact set in terms of the noise on a slightly larger bounded set uniformly over all possible choices of boundary conditions.

In both bounds we obtain good integrability properties when specialising to the case of Gaussian noise - stretched exponential bounds on the C^α norm in the quasi-linear equation and even better than Gaussian bounds in the reaction diffusion equation.
This is based on joint work with F. Otto and A. Moinat.

Lorenzo Zambotti

Bessel-like SPDEs

Abstract: I will discuss integration by parts formulae on the law of the Bessel bridge of dimension less than 3 and show how this allows to conjecture the form of an associated SPDE. The most relevant case is the dimension equal to 1, which is expected to be the scaling limit of critical wetting models.

Rongchan Zhu

Conservative stochastic 2-dimensional Cahn–Hilliard equation

Abstract: We consider the stochastic 2-dimensional Cahn–Hilliard equation driven by the spatial derivative of space-time white noise. We use two different approaches to study this equation. First we prove that there exists a unique solution to the shifted equation by SPDE approach. Moreover, we use Dirichlet form approach to construct the probabilistically weak solution to the the original equation. By clarifying the precise relation between the solutions obtained by the Dirichlet forms approach and the shifted equation, we obtain that the Φ^4_2 field is invariant measure to the shifted equation. Furthermore, uniqueness of invariant measure has also been obtained.

Xiangchan Zhu

Stochastic heat equation taking values in a manifold

Abstract: In this talk we give the existence of martingale solutions to the stochastic heat equations with values in a manifold, which admits Wiener measure as an invariant measure by using Dirichlet form. Moreover, in finite volume case, exponential ergodicity has been obtained under the condition that the Ricci curvature is bounded from below. In infinite volume case, we obtain the exponential ergodicity of the solution if the Ricci curvature is strictly positive and the non-ergodicity of the process if the sectional curvature is negative.

Posters

Aurélien Deya
A non-linear wave equation with fractional perturbation

Abstract: tba

Mohammed Lakhdar Hadji and H. Rezgui
A stochastic model in image restoration

Abstract: In this work we propose a new algorithm for image processing using a stochastic model. Our contribution is to add a random term in the Perono–Malik model in order to control the restoration of missing parts in the image. Numerical results are presented to show quantitative, performance and competitiveness through texture based image retrieval.

References

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[2] G. Harmer, B. Davis, and D. Abott, "A review of stochastic resonance: Circuits and measurement", IEEE Transactions on Instrumentation and Measurement, vol. 51, pp. 299–309, 2002.
[3] Q. Ye, H. Huang, and C. Zhang, "Image enhancement using stochastic resonance [sonar image processing applications", in Proceedings of ICIP, vol. 1, 2004, pp. 263–266.
[4] "Noise-enhanced anisotropic diffusion for image scalar restoration", in Proceedings of the fifth PSIP congress (Physics in Signal and Image Processing, 2007).
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Claudine Leonhard
Investigating the impact of melt water on the ocean

Abstract: The accelerated melting of Greenland and Antarctica is perhaps one of the most alarming consequences of global warming. The fresh, cold melt water not only raises the sea-level, but also changes ocean circulation patterns since variations of temperature and salinity are two main drivers of circulation.

We investigate how the melting of ice sheets impacts ocean temperature and salinity in a regional model. Since large uncertainties remain in estimates of melt water influx from ice sheets [1], we employ a stochastic version of the primitive equations to incorporate the incomplete knowledge on melt water properties, and to handle model uncertainty. In order to obtain accurate results with reasonable computational effort, an associated goal is to develop an appropriate numerical scheme for the stochastic primitive equations.
This is joint work with Josefin Ahlkrona.

References

[1] J. Church, P. Clark, A. Cazenave, J. Gregory, S. Jevrejeva, A. Levermann, M. Merrifield, G. Milne, R. Nerem, P. Nunn, A. Payne, W. Pfeffer, D. Stammer, and A. Unnikrishnan. *Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*, book section 13, pages 1137–1216. Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA, 2013.

Tommaso Cornelis Rosati

The KPZ equation on the real line

Abstract: We use paracontrolled analysis to establish the existence of solutions to the Kardar–Parisi–Zhang equation on non-compact domains. The main tool at our disposal is a comparison principle, which allows to control the Cole–Hopf transform. Eventually we can prove that the the logarithm of the solution to the stochastic heat equation is a solution, in a paracontrolled sense, to the KPZ equation. We then study the link between the KPZ equation and the random polymer measure on the whole space exploiting some results regarding solutions to SDEs with singular drift.
