HARMONIC ANALYSIS OF ELLIPTIC AND PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS

CIRM, LUMINY, 23 - 27 APRIL 2018

Abstracts

Pascal Auscher

L^2 *boundary value problems for parabolic systems*

Abstract: The goal is to study these problems for parabolic systems in divergence form with measurable, bounded, transversally independent coefficients on a half-space. Ealier results are when coefficients are independent or smooth with respect to time. I will present the strategy based on a reduction to an evolution equation in the transverse variable involving half order time derivative and first order spatial derivatives and the Hardy space splitting resulting from the functional calculus of a bisectorial operator (which will be described in M. Egert's talk).
This talk is based on joint work with M. Egert and K. Nystrom.

Andrea Carbonaro

Maximal parabolic regularity for divergence-form operators with Neumann boundary conditions in rough domains

Abstract: Let $\Omega \subseteq \mathbb{R}^n$ be open and A be a complex uniformly accretive matrix function on Ω. Consider the divergence-form operator $L^A = -\mathrm{div}(A\nabla)$ with Neumann boundary conditions in Ω. We show that the associated parabolic problem $u'(t) + L^A u(t) = f(t)$, $u(0) = 0$ has maximal regularity in $L^p(\Omega)$, for all $p \in (1, +\infty)$ such that A satisfies an algebraic condition called p-ellipticity. The given range of exponents is optimal for this class of operators.
The talk is based on a work in progress with Oliver Dragičević

Li Chen

Gundy-Varopoulos martingale transforms and their projection operators on manifolds and vector bundles

Abstract: We study a class of operators which are projections of martingale transforms à la Gundy-Varopoulos for quite general diffusions on manifolds and vector bundles. As applications, we obtain dimension free L^p boundedness for generalized first order Riesz transforms on Lie groups of compact type, the Heisenberg group, SU(2), and for Riesz transforms on forms and spinors. This is a joint work with Rodrigo Bañuelos and Fabrice Baudoin.

Raphaël Danchin

Recent approaches based on harmonic analysis for the study of non regular solutions to the Navier-Stokes equations with variable density

Abstract: The inhomogeneous incompressible Navier-Stokes equations that govern the evolution of viscous incompressible flows with non-constant density have received a lot of attention lately. In this talk, we shall mainly focus on the singular situation where the density is discontinuous, which is in particular relevant for describing the evolution of a mixture of two incompressible and non reacting fluids with constant density, or of a drop of liquid in vacuum. We shall highlight the places where tools in harmonic analysis play a key role, and present a few open problems.

Moritz Egert

The Kato problem for parabolic systems in divergence form

Abstract: In my talk I will consider parabolic systems $L = \partial_t - \nabla_x \cdot A\nabla_x$ acting on the whole parabolic space $\mathbb{R}^{n+1}$. The coefficients A are bounded and allowed to depend measurably on time and all spatial variables. Surprisingly at first sight, L can be defined as a maximal accretive operator in $L^2(\mathbb{R}^{n+1})$ via a sesquilinear form on a natural energy space involving half-order time derivatives and first-order derivatives in space. Hence, there is a Kato square root type problem asking whether the domain of $\sqrt{L}$ coincides with the energy space. In a joint work with P. Auscher and K. Nyström we answered this in the affirmative. More generally, we established the bounded holomorphic functional calculus for an associated perturbed parabolic Dirac operator. I will discuss key ideas of the proof and – if time allows – try to explain the main difficulties in passing to an L^p-theory of these operators.

Dorothee Frey

Paradifferential and paracontrolled calculus in rough settings

Abstract: The paradifferential calculus plays an important roles in PDEs, in particular in the treatment of nonlinearities in Sobolev or Besov spaces. In recent years, it has found major applications in the rough path theory. Gubinelli, Imkeller and Perkowski have established a so-called paracontrolled calculus as an alternative approach to HairerÕs regularity structures in the context of singular stochastic PDEs.

We will discuss the basic principles of a paracontrolled calculus for singular stochastic PDEs, and show how it can be adapted to non-smooth settings where no Fourier transform is available.

Tuomas Hytönen

Of commutators and Jacobians

Abstract: The boundedness (on L^p spaces) of commutators $[b, T] = bT - Tb$ of pointwise multiplication b and singular integral operators T has been well studied for a long time. Curiously, the necessary conditions for this boundedness to happen are generally less understood than the sufficient conditions, for instance what comes to the assumptions on the operator T. I will discuss some new results in this direction, and show how this circle of ideas relates to the mapping properties of the Jacobian (the determinant of the derivative matrix) on first order Sobolev spaces. This is work in progress at the time of submitting the abstract, so I will hopefully be able to present some fairly fresh material.

Peer Kunstmann

H^∞ *calculus for the Stokes operator on bounded Lipschitz domains*

Abstract: We show that the Stokes operator A on the Helmholtz space $L^p(\Omega)^d$ for a bounded Lipschitz domain $\Omega \subset \mathbb{R}^d$, $d \geqslant 3$, has a bounded H^∞-calculus for p close enough to 2. Our proof uses a new comparison theorem for A and the Dirichlet Laplace $-\Delta$ on $L^p(\Omega)^d$, which is based on "off-diagonal" estimates of the Littlewood-Paley decompositions of A and $-\Delta$, combined with results on the Stokes operator in bounded Lipschitz domains due to Shen.
Joint work with Lutz Weis.

David Rule

Bilinear Fourier integral operators with non-separable phase functions

Abstract: Very few boundedness results for bilinear Fourier integral operators

$$T_\sigma^\Phi(f,g)(x) = \iint \sigma(x,\xi,\eta)\widehat{f}(\xi)\widehat{g}(\eta)e^{i\Phi(x,\xi,\eta)}d\xi d\eta$$

seem to be known without simplifying assumptions on the phase function $\Phi(x,\xi,\eta)$, such as separability in the frequency variables. Motivated by PDE theory, we attempt to prove boundedness results for non-separable phases of the form $\Phi(x,\xi,\eta) = \phi_1(\xi) + \phi_2(\eta) + \phi_3(\xi+\eta)$.

Amongst other things, this in turn leads us to explore various ways to generalise the classical linear result of Seeger-Sogge-Stein. We prove the boundedness of linear Fourier integral operators on local Hardy spaces h^p without the necessity for the compact support of the amplitude. This can be done with the help of recent work of Ruzhanksy-Sugimoto and a variant of the Seeger-Sogge-Stein decomposition of frequency space, but, in contrast to the compactly supported case, only for $p > n/(n+1)$.

Xavier Tolsa

The weak-A_∞ *condition for harmonic measure*

Abstract: The weak-A_∞ condition is a variant of the usual A_∞ condition which does not require any doubling assumption on the weights. A few years ago Hofmann and Le showed that, for an open set $\Omega \subset \mathbb{R}^{n+1}$ with n-AD-regular boundary, the BMO-solvability of the Dirichlet problem for the Laplace equation is equivalent to the fact that the harmonic measure satisfies the weak-A_∞ condition. Aiming for a geometric description of the open sets whose associated harmonic measure satisfies the weak-A_∞ condition, Hofmann and Martell showed in 2017 that if $\partial\Omega$ is uniformly n-rectifiable and a suitable connectivity condition holds (the so-called weak local John condition), then the harmonic measure satisfies the weak-A_∞ condition, and they conjectured that the converse implication also holds. In this talk I will discuss a recent work by Azzam, Mourgoglou and myself which completes the proof of the Hofman-Martell conjecture, by showing that the weak-A_∞ condition for harmonic measure implies the weak local John condition.

Mark Veraar

H^∞*-calculus and the heat equation with rough boundary conditions*

Abstract: In this talk we consider the Laplace operator with Dirichlet boundary conditions on a smooth domain. We prove that it has a bounded H^∞-calculus on weighted L^p-spaces for power weights which fall outside the classical class of A_p-weights. Furthermore, we characterize the domain of the operator and derive several consequences on elliptic and parabolic regularity. In particular, we obtain a new maximal regularity result for the heat equation with very rough inhomogeneous boundary data.
The talk is based on joint work with Nick Lindemulder