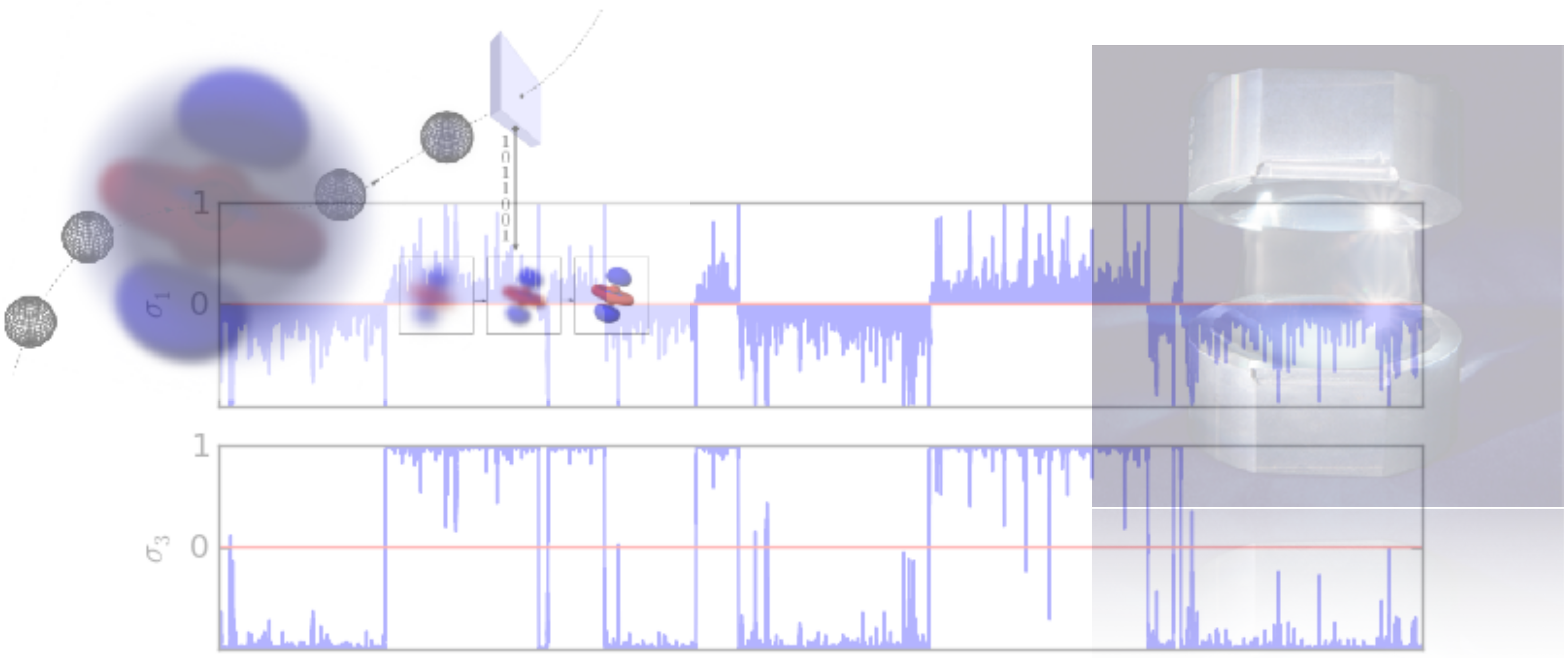
Statistical Aspects of Quantum State Monitoring for (and by) Amateurs

D. Bernard

« CIRM - April 2018 »



Four lectures:

1- Quantum non-demolition (QND) measurements

- What kind of experiments?
- What are indirect (repeated) measurements?
- Repeated POVMs & quantum trajectories
- Non-demolition measurements

2- Discrete quantum trajectories and open quantum walks

3- Continuous monitoring and quantum trajectories

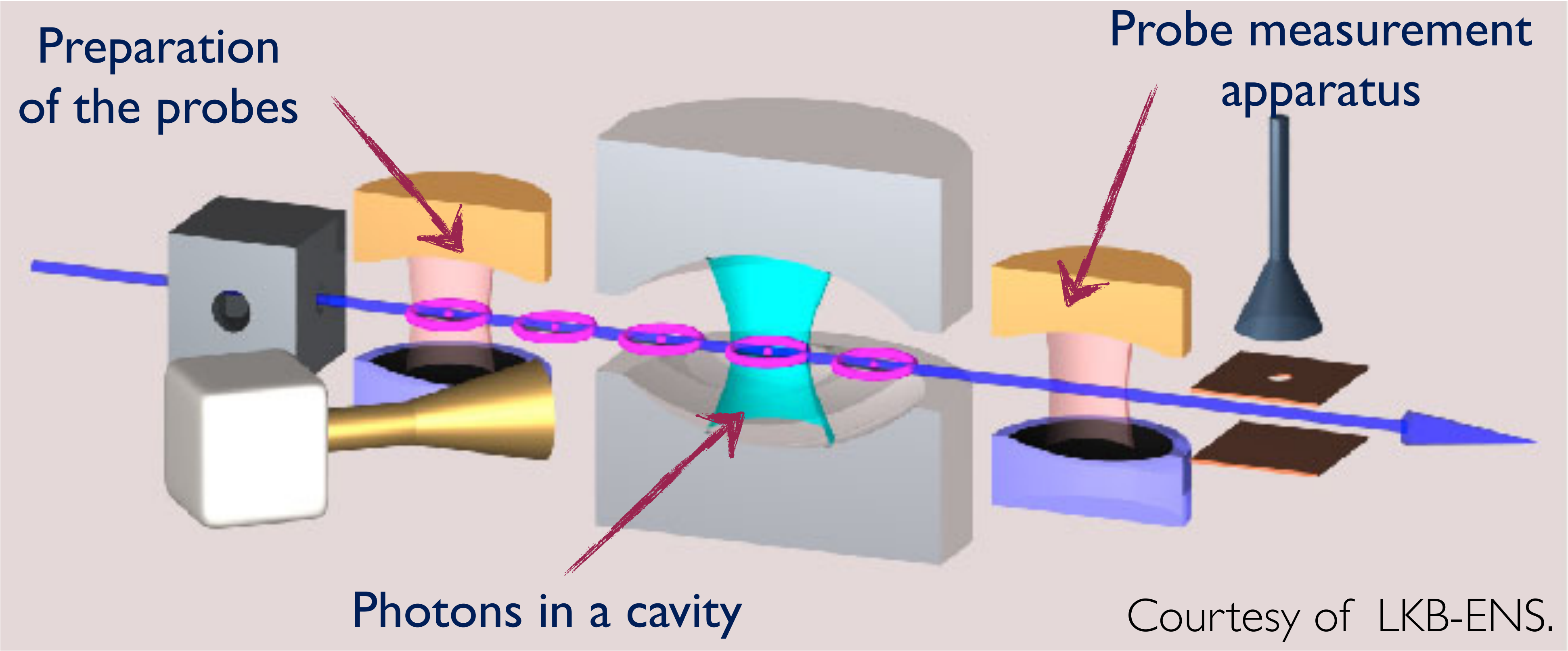
4- Strong monitoring limit

Lecture Notes: https://www.phys.ens.fr/~dbernard/

What kind of experiments?

— **Cavity QED:** (cf. I. Dotsenk's lectures)
Testing light/photon (the quantum system) with matter (the quantum probes).

System (S)= photons in a cavity.
Probes (P)= Rydberg atoms (two state systems)



— **Others: Circuit QED, etc...** (cf. B. Huard's lectures)

Progressive field-state collapse and quantum non-demolition photon counting

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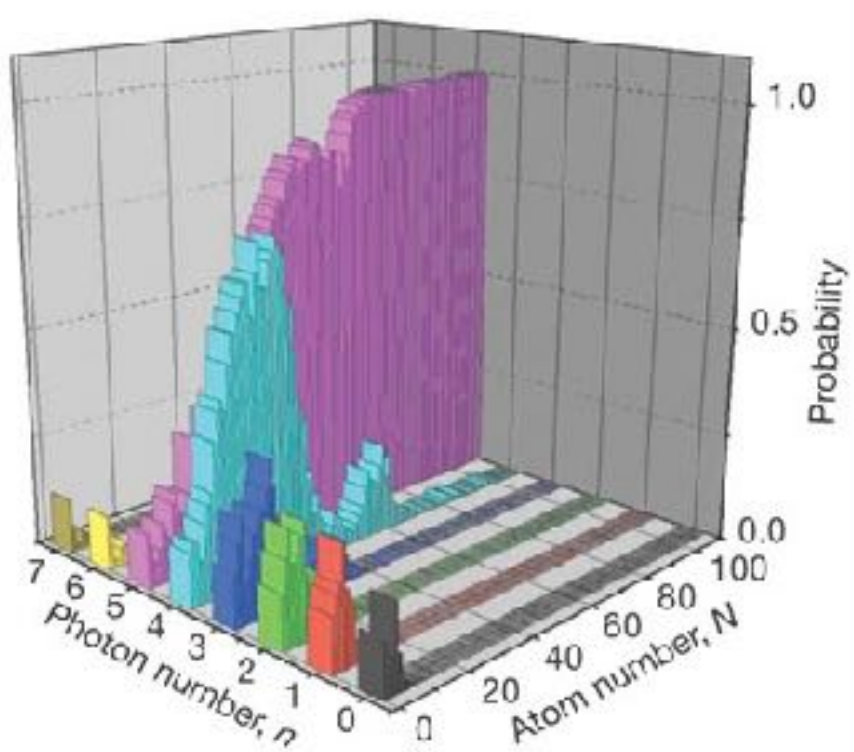
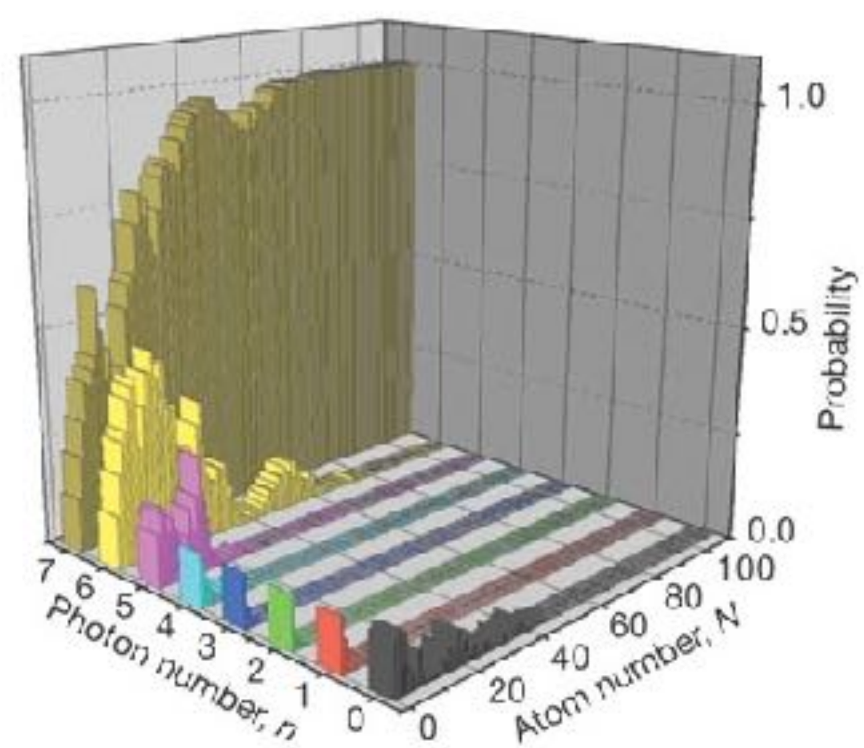
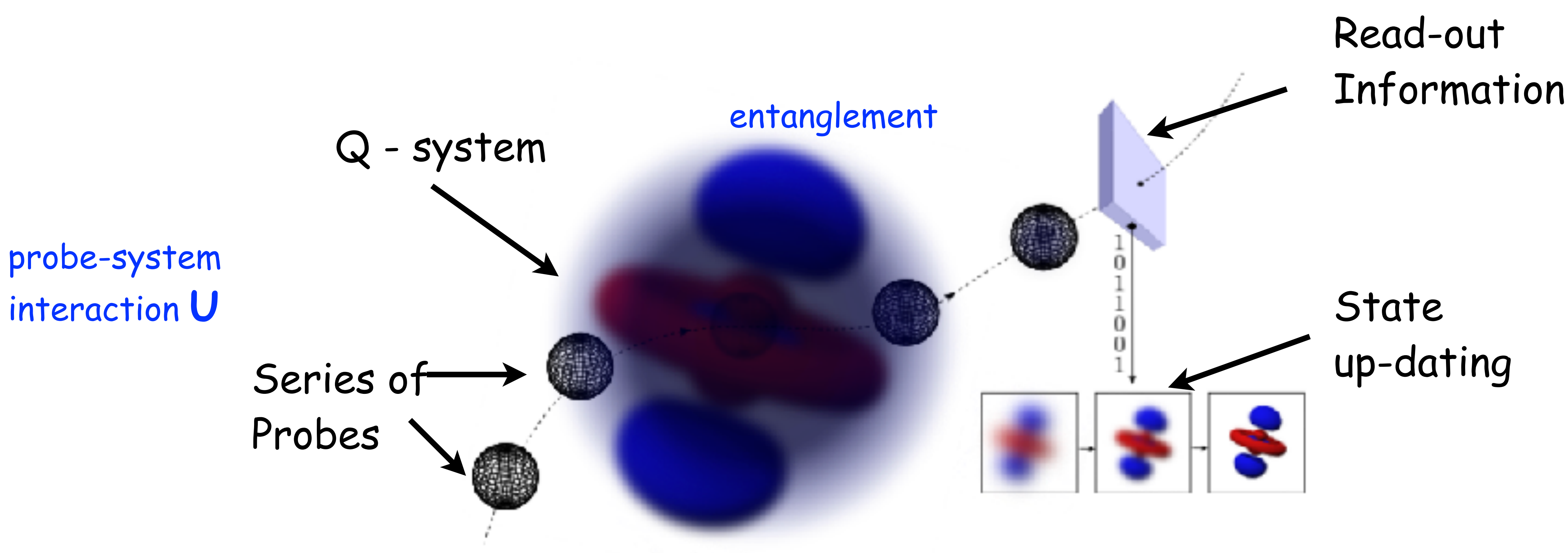
Figure 2 | Progressive collapse of field into photon number state. unity). **c**, Photon number probabilities plotted versus photon and atom numbers n and N. The histograms evolve, as N increases from 0 to 110, from a flat distribution into $n = 5$ and $n = 7$ peaks.





What are (weak) indirect (repeated) measurements?

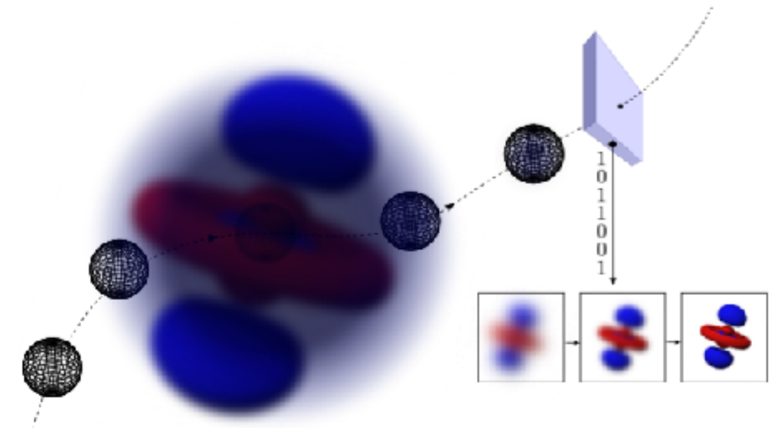
— **A probe**, prepared in a given (reference) state, interacts with **the system**, and (projectively) measured after interaction.



⟶ Probe measurements | information: output signal....
state up-dating....
Random by Q.M. rules

— How to model this process?

What are (weak) indirect measurements (I) ?



— **Interaction** [system-probe] :

system state → $\rho \otimes |\varphi\rangle\langle\varphi| \to U\,(\rho \otimes |\varphi\rangle\langle\varphi|)\,U^{\dagger}.$ ← unitary interaction operator

probe state

— **Probe measurement**: [measurement of a non degenerate probe observable with eigenstate |s> -> projection on $\mathbb{I} \otimes |s\rangle\langle s|$.]

$$\rho \to \frac{F_s\,\rho\,F_s^{\dagger}}{\pi(s)}, \quad \text{with probability } \pi(s) = \mathrm{Tr}(F_s\,\rho\,F_s^{\dagger}), \quad \text{with } F_s := \langle s|U|\varphi\rangle$$

The transformation of the system state is random (depending on the output signal)

CP-maps and Quantum Channels

— **POVM** [Positive Operator Valued

Set of operators Fs such that: $\sum_s F_s^\dagger F_s = \mathbb{I}.$

It ensures that: $\sum_s \pi(s) = 1 \text{ because } \sum_s \mathrm{Tr}(F_s \rho F_s^\dagger) = \mathrm{Tr}(\rho) = 1.$

OK. with countable (or continuous) labelling too (see next lectures).

— **Mean behaviour & CP-maps**:

$$\bar\rho \to \sum_s F_s \,\bar\rho\, F_s^\dagger$$

Let $\Phi(\bar\rho) := \sum_s F_s \,\bar\rho\, F_s^\dagger$ such transformations are called Completely Positive (CP) maps

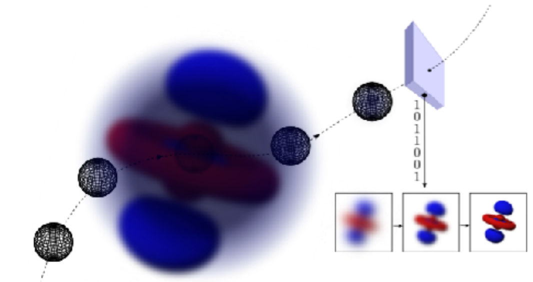
Theorem (Stinespring's theorem) "Auxiliary Reservoir":

Given a completely positive map Φ there exist a Hilbert space $\mathcal{K}$, a state ω on $\mathcal{K}$ and a unitary operator V on the tensor product $\mathcal{H}\otimes\mathcal{K}$ such that

$$\Phi(\rho) = \mathrm{Tr}_{\mathcal{K}}(V\,\rho\otimes\omega\,V^\dagger).$$

for any state ρ on $\mathcal{H}$.

Example of CP-maps (random unitaries).



Repeated POVMs & Quantum Trajectories

— Series of probes interact recursively with the system and are recursively measured.

The system evolution: $\rho_n \to \rho_{n+1} = \dfrac{F_s\,\rho_n\,F_s^\dagger}{\pi_n(s)}$, with probability $\pi_n(s) = \mathrm{Tr}(F_s\,\rho_n\,F_s^\dagger)$.

The output signal: series of measurement outputs $(s_1, \cdots, s_n, \cdots)$.

— This specifies a Markov chain (on system states) called « **Quantum Trajectories** »

The probability space is that of the sequences of output, equipped with a natural filtration Fn and with an induced probability measure P.

— **Iterated CP-maps**:

If the output signal is not recorded, the mean system state evolves via:

$$\bar\rho_n \to \bar\rho_{n+1} = \Phi(\bar\rho_n) = \Phi^{n+1}(\rho_0). \qquad \text{with} \qquad \Phi:\ \bar\rho \to \Phi(\bar\rho) := \sum_s F_s\,\bar\rho\,F_s^\dagger.$$

Quantum non-demolition measurements (I)

— If one want the series of repeated POVMs to be close to what a **von Neumann measurement** would be, we have to impose that a preferred state basis is preserved.

that is $|k\rangle\langle k| \to |k\rangle\langle k|$ with probability one.

Let « **Pointer states** » = states |k>

The non-demolition condition:

— This imposes that the **interaction U preserves the pointer states.**

$$U = \sum_k |k\rangle\langle k| \otimes U_k,$$ and hence $$F_s = \sum_k |k\rangle \langle s|U_k|\varphi\rangle \langle k|.$$

unitary on probes ↗ ↖ diagonal on pointer states

—> Check that then indirect measurement preserve the pointer basis.

— Conditioned probabilities:

For k fixed, the numbers $p(s|k) := |\langle s|U_k|\varphi\rangle|^2$ specified a probability measure on the probe outputs, $\sum_s p(s|k) = 1$. These are the distributions of the outputs conditioned on the system to be in the pointer state $|k\rangle$. There is one such distribution for each pointer state.

Quantum non-demolition measurements (II)

POVM diagonal on pointer states : $$F_s = \sum_k |k\rangle \langle s|U_k|\varphi\rangle \langle k|.$$

— The system evolution under repeated QND POVMs:

$$\rho_n \to \rho_{n+1} = \frac{F_s\,\rho_n\,F_s^\dagger}{\pi_n(s)}, \quad \text{with probability } \pi_n(s) = \mathrm{Tr}(F_s\,\rho_n\,F_s^\dagger).$$

Look at the **evolution of the diagonal matrix elements** of the system density matrix (in the pointer basis):

$$Q_n(k) := \rho_n(k,k).$$

[a probability measure]

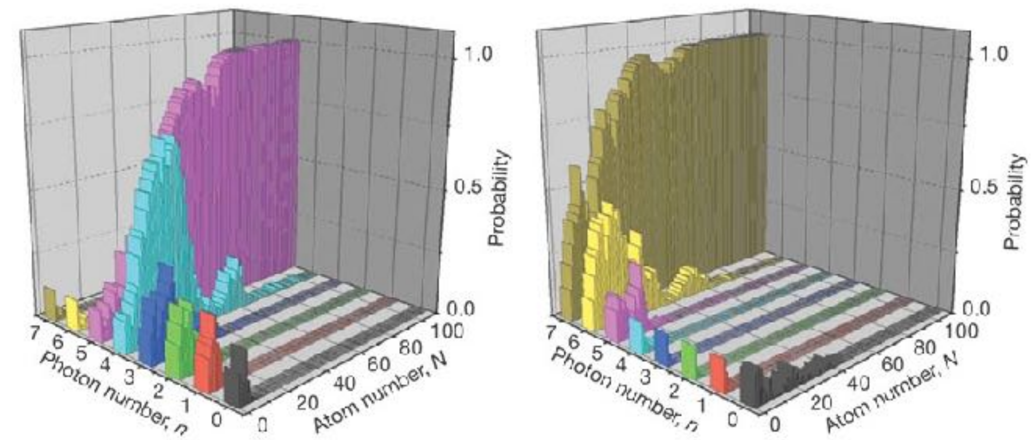
$$Q_n(k) \to Q_{n+1}(k) = \frac{p(s|k)\,Q_n(k)}{\pi_n(s)},$$

with probability $$\pi_n(s) = \mathrm{Tr}(F_s \rho_n F_s^\dagger) = \sum_k p(s|k)\,Q_n(k).$$

Quantum non-demolition measurements (III)

— Repeated non-demolition POVM and convergence

A precise description
of **progressive collapses**
as in cavity QED experiments.



— « Convergence/Progressive collapse »:

– *The sequences* $n \to Q_n(k)$ *converge a.s. and in* $\mathbb{L}^1$ *for any* k.

– *The limit distribution is peaked:* $Q_\infty(k) := \lim_{n\to\infty} Q_n(k) = \delta_{k;k_\infty}$ *for some random target pointer* k_∞.

– *The random target* k_∞ *is distributed according to the initial distribution:* $\mathbb{P}[k_\infty = k] = Q_0(k)$.

– *The convergence to the target is exponential fast with*

$$Q_n(k)/Q_n(k_\infty) \simeq \exp[-nS(k_\infty|k)],$$

with $S(k_\infty|k)$ *the relative entropy* $S(k_\infty|k) = -\sum_s p(s|k_\infty) \log\left[\frac{p(s|k)}{p(s|k_\infty)}\right]$.

[hypothesis: all conditioned probability p(.|k) are distincts]

Proof of progressive collapse

— Qn(k) is a (bounded) **martingale**.

[Naively: it is preserved in mean]

$$\mathbb{E}[Q_{n+1}(k)\,|\mathcal{F}_n] = \sum_s \frac{p(s|k)\,Q_n(k)}{\pi_n(s)}\,\pi_n(s) = \sum_s p(s|k)\,Q_n(k) = Q_n(k)$$

— The « martingale convergence theorem » then says that **Qn(k) converges a.s.** and in L1. Looking at the fixed point condition implies that Qn(k) is peaked:

That is: there is a certain pointer state $|k_\infty\rangle$ such that $Q_\infty(k) = \delta_{k;k_\infty}$.

— Because it converges in L1, we get:

$$\mathbb{P}[k_\infty = k] = \mathbb{E}[\delta_{k_\infty = k}] = \mathbb{E}[Q_\infty(k)] = Q_0(k),$$

which are **von Neumann rules**.

— The exponential decay follow from an explicit formula for Qn(k):

$$\frac{Q_n(k)}{Q_n(k_\infty)} = \prod_s \left(\frac{p(s|k)}{p(s|k_\infty)}\right)^{N_n(s)} \simeq \prod_s \left(\frac{p(s|k)}{p(s|k_\infty)}\right)^{np(s|k_\infty)},$$

with $N_n(s)$ the number of the value s occurs in the n first output measurement asymptotically at large n, $N_n(s) \simeq np(s|k_\infty) + \cdots$

How to read the pointer state?

— By looking at the **histogram** in the output signal (s1,s2,….,sn,…) for a given series of iterated indirect measurement:

Because asymptotically at large n, $N_n(s) \simeq np(s|k_\infty) + \cdots$

— Comparing the **histogram** with the (given) conditioned probability p(s|k) **allow to identify** k_\infty if all the p(.|k) are different.

What about if we don't know the initial Q(k)?

— This is actually the real situation [as no need to do a measure of Q(k)]

start with some a priori trial distribution, say $\widehat{Q}_0(k)$, update it recursively using Bayes' rules:

$$\widehat{Q}_n(k) \to \widehat{Q}_{n+1}(k) := \frac{p(s_{n+1}|k)\,\widehat{Q}_n(k)}{\widehat{Z}_n},$$

But the output signals (s1,s2,…) are distributed with the 'true' distribution Q(k). And the 'trial' and the 'true' distribution have the same limit (asymptotically)

Mixing, decoherence or not?

— If initially pure, the system state remains pure: **no mixing**.

— But if we don't record the output signals, or don't measure the output probes, then the series of probes form a reservoir and induce decoherence.

The off-diagonal matrix element are updated as follows:

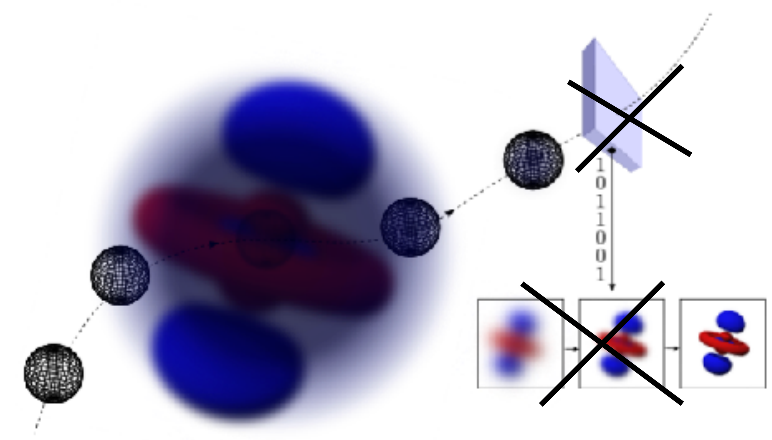
$$\rho_n(j,k) \to \rho_{n+1}(j,k) = \frac{u_s(j)u_s(k)^*}{\pi_n(s)}\,\rho_n(j,k), \quad \text{with proba } \pi_n(s) = \sum_k p(s|k)Q_n(k).$$

Hence, in mean:

$$\bar{\rho}_n(j,k) = [\langle\varphi|U_j^* U_k|\varphi\rangle]^n\,\rho_0(j,k) \to 0, \quad \text{exponentially}$$

That is: **exponential decoherence in the pointer basis** (as usual…)

« Macroscopic » pointer states



— After n probe-system interaction (without measurement)

$$\sum_k C_k\,|k\rangle \to \sum_k C_k|k\rangle \otimes |\Phi_k\rangle^{(n)}$$

with « macroscopic states » $$|\Phi_k\rangle^{(n)} = \sum_{s_1,\cdots,s_n} \big(\prod_{j=1}^{n} \langle s_j|U_k|\varphi\rangle\big)\,|s_1\rangle \otimes \cdots \otimes |s_n\rangle$$

These macroscopic states are asymptotically/exponentially orthogonal

$$^{(n)}\langle\Phi_k|\Phi_l\rangle^{(n)} = \sum_{[s]} Z_n([s]|k)^*\,Z_n([s]|l) \simeq e^{-nD(k|l)}$$

where the sum is over partition [s] of given frequencies of occurrences of output s: $N_s(n) = nq_s$

Four lectures:

1- Quantum non-demolition (QND) measurements

2- Discrete quantum trajectories and open quantum walks

Repeated indirect measurement as quantum walks
Open quantum walks & examples
Basics: ergodicity, detailed balance, etc.
Dilation and geometrisation

3- Continuous monitoring and quantum trajectories

4- Strong monitoring limit

Repeated indirect measurement as quantum walks

— **Repeated POVM as Open Quantum Walks (OQW)**

- From repeated POVMs, we get output signals (s1,s2,...).
- At each step we can decide to extract some data X(n) from these outputs,

[i.e. the Xn is a fonction of the signal (s1,...sn) up to step n].

[i.e. they are randomly distributed in some set].

- At each step, the updating of the outputs yields a updating of the data X(n).

—> We can view this **updating as a « walker » moving** on the set of possible data values and whose **position is X(n)** at step n.

Example:

- Choose to only keep the last digit as data: i.e. X(n):= s_n
- The walker is equipped with the quantum system : an internal « quantum gyroscope ».
- It moves on the complete graph [with vertices the possible value of the output s].

$$X_{(n-1)} \to X_{(n)} = s_n \text{ and } \rho_{n-1} \to \rho_n = F_{s_n}\rho_{n-1}F_{s_n}^\dagger / \mathrm{Tr}(F_{s_n}\rho_{n-1}F_{s_n}^\dagger)$$

$$\text{with probability } \pi_{s_n} = \mathrm{Tr}(F_{s_n}\rho_{n-1}F_{s_n}^\dagger)$$

- Other choices: see below.

Open quantum walks

Players: « A walker and its internal quantum gyroscope »:
The walker moves on oriented graph Lambda, with position « x »
A quantum system is attached to the walker with Hilbert space H and state « rho ».

Data: Transition matrices on edges B_{xy} *such that* $\sum_y B^*_{xy} B_{xy} = \mathbb{I}$ *for all* x

Definition [Attal et al]

Let Λ *be an oriented graph. Let* $\mathcal{H}$ *be a Hilbert space. Let* B_{xy} *be (bounded) operators* B_{xy} *on* $\mathcal{H}$ *associated to any edge* $x \to y$ *of* Λ *such that* $\sum_y B^*_{xy} B_{xy} = \mathbb{I}$ *for all* x*. Let* (x, ρ) *be the position* $x \in \Lambda$ *and the internal state* ρ *of the walker. The open quantum walk (OQW) with transition matrices* B_{xy} *is the Markov chain defined by the moves*

$$(x, \rho) \to (y, \frac{B_{xy}\, \rho\, B^*_{xy}}{\pi_{xy}}), \quad \text{with probability } \pi_{xy} = \mathrm{Tr}(B_{xy} \rho B^*_{xy}).$$

Example: Homogeneous OQW on the line

The two transition matrices
s.t. $B^*_+ B_+ + B^*_- B_- = \mathbb{I}.$

$$(x_n; \rho_n) \to \Big(x_{n+1} = x_n + \epsilon_{n+1}, \rho_{n+1} = \frac{B_{\epsilon_{n+1}} \rho_n B^*_{\epsilon_{n+1}}}{\pi_n(\epsilon_{n+1})}\Big).$$

$$\epsilon_{n+1} = \pm \quad \text{with probability } \pi_n(\pm) = \mathrm{Tr}(B_+ \rho_n B^*_{\perp}).$$

Open quantum walks (II):

— **A CP-map is associated to OQWs**:

Acting on (extended) density matrix of the form $\sum_x \rho_x \otimes |x\rangle\langle x|$. in $\mathcal{H} \otimes \mathbb{L}^2(\Lambda)$.

$$\mathfrak{P}\Big(\sum_x \rho_x \otimes |x\rangle\langle x|\Big) = \sum_{x;y} (B_{xy} \rho_x B_{xy}^*) \otimes |y\rangle\langle y|.$$

or

$$\mathfrak{P}(\rho)_x = \sum_y B_{yx} \rho_y B_{yx}^*.$$

— There is **a dual (more geometrical) point of view**:
Instead of repeating POVM on the internal state,
OQW may be viewed as coming from **evolution + position measurement**.

(i) evolve the extended density matrix $\rho \otimes |x\rangle\langle x|$, with this CP-map.
(ii) perform a von Neumann measurement of the position.

— Relation with control & feedback (because the POVMs $(B_{xy})_y$ depend on the information X on the system).....

Examples:

— The simplest example is of course for homogeneous quantum walk on the line, and for spin 1/2 internal space.
The choice is then in the transition matrices s.t. $B_+^\dagger B_+ + B_-^\dagger B_- = \mathbb{I}$

— Non-demolition measurement as OQW:

In this example $\mathcal{H} = \mathbb{C}^2$, $\Lambda = \mathbb{Z}$ and the transition matrices $B_\pm := B_{x;x\pm1}$ are diagonal – this is the non-demolition hypothesis. Let us parametrize them as $B_\pm = \begin{pmatrix} \sqrt{p_\pm} & 0 \\ 0 & \sqrt{q_\pm} \end{pmatrix}$ with $p_+ + p_- = 1$ and $q_+ + q_- = 1$.

For an internal density matrix with diagonal matrix element Qn and (1-Qn), the moves are:

$$(x_n, Q_n) \to \big(x_{n+1} = x_n \pm 1, Q_{n+1} = p_\pm Q_n/\pi_n(\pm)\big),$$

$$\text{with probability } \pi_n(\pm) = p_\pm Q_n + q_\pm(1 - Q_n)$$

Detailed analysis can be done (as before.. equivalence)...

Example: a crossover from diffusive to ballistic behaviors

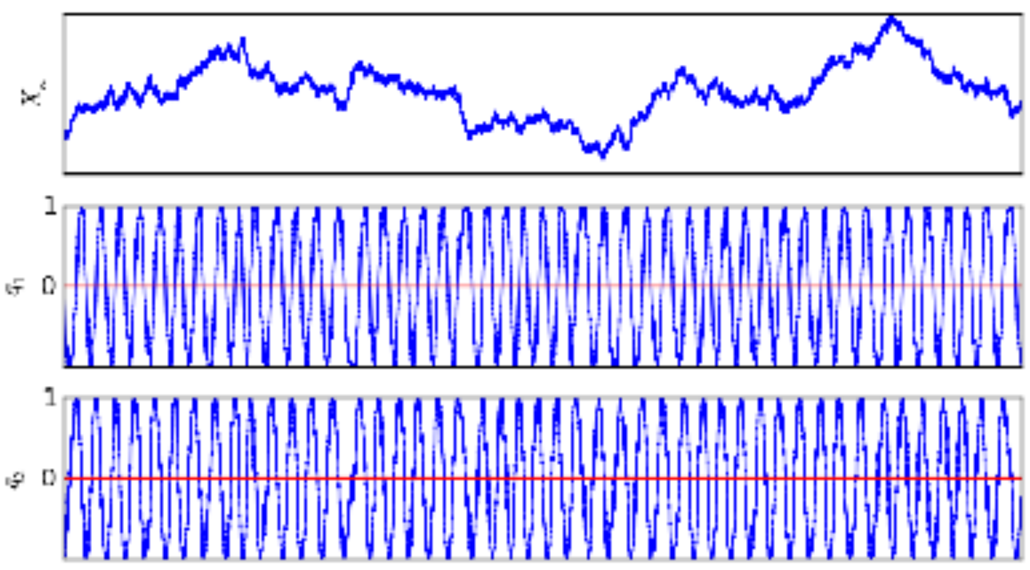
— A choice for the simulations:

$$B_+^\dagger B_+ + B_-^\dagger B_- = \mathbb{I} \quad \text{with} \quad \delta = \sqrt{u^2+v^2+r^2+s^2}$$

$$B_+ = \delta^{-1}\begin{pmatrix} u & r \\ s & v \end{pmatrix} \quad \text{and} \quad B_- = \delta^{-1}\begin{pmatrix} -v & s \\ r & -u \end{pmatrix}$$

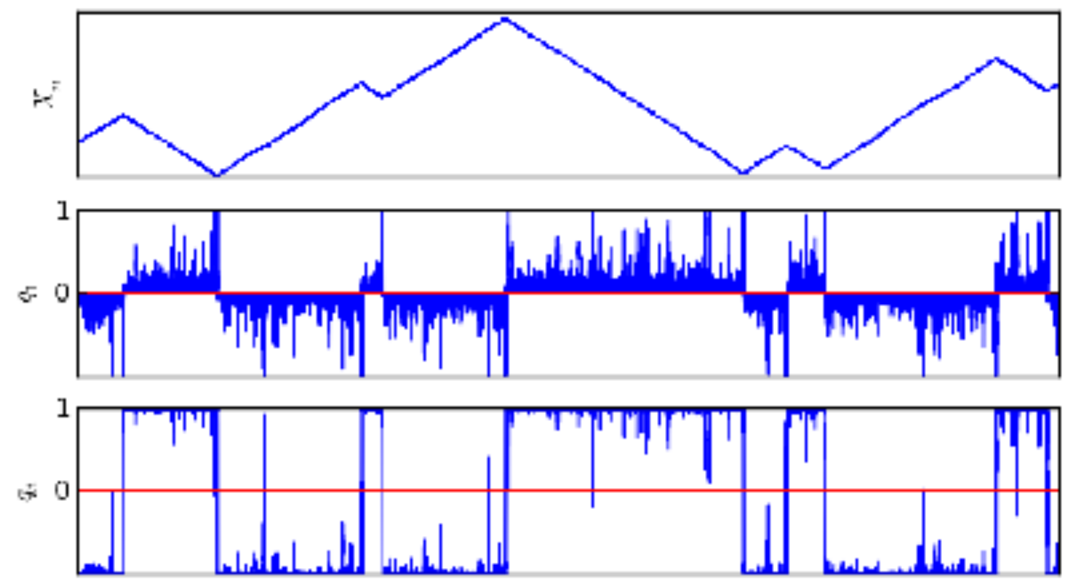
A Brownian like regime.

$u = 1.005,\ v = 1.00$ and $r = -s = 0.00015$



A ballistic like (but diffusive) regime.

$u = 1.1,\ v = 1.00$ and $r = -s = 0.00015$



— Trajectories are ballistic, with seesaw profiles induced by gyroscope flips, and large mean free paths. But at very large time the position is Gaussian, with large effective diffusion constant.

Basics: ergodicity, detailed balance, etc.

— Ergodicity (via Kummerer-Maassen theorem)

> *Theorem* (Kummerer-Maassen) "Ergodicity":
> *Let (x_n, ρ_n) an OQW. Then, we have the almost sure convergence of its time average:*
>
> $$\frac{1}{n}\sum_{k=0}^{n} \rho_k \otimes |x_k\rangle\langle x_k| \to \sum_x \rho_x^{\text{inv}} \otimes |x\rangle\langle x|, \quad \text{a.s.},$$
>
> *when $n \to \infty$, with $\mathfrak{P}(\rho^{\text{inv}}) = 0$, or equivalent $\sum_x B_{xy}\rho_x^{\text{inv}} B_{xy}^* = \rho_y^{\text{inv}}$.*

In particular:

$$\lim_{n\to\infty} \frac{N_n(y)}{n} = \text{Tr}(\rho_y^{\text{inv}}),$$

$$\lim_{n\to\infty} \frac{1}{N_n(y)} \sum_{k=0}^{n} \rho_k \, \mathbb{I}_{x_k = y} = \frac{\rho_y^{\text{inv}}}{\text{Tr}(\rho_y^{\text{inv}})}.$$

with Nn(y) the number of time the site y has been visited.

Proof: $\hat{\rho}_k = \rho_k \otimes |x_k\rangle\langle x_k|$. Consider the sum of its iteration $\sum_{k=0}^{n} \hat{\rho}_k$. Since $\mathbb{E}[\hat{\rho}_n|\mathcal{F}_{n-1}] = \mathfrak{P}(\hat{\rho}_{n-1})$, its Doob decomposition is

$$\sum_{k=0}^{n} \hat{\rho}_k = M_n + \sum_{k=0}^{n-1} \mathfrak{P}(\hat{\rho}_k),$$

with M_n a martingale, $\mathbb{E}[M_n|\mathcal{F}_{n-1}] = M_{n-1}$. From the law of large for martingales and boundness arguments it follows that $\frac{1}{n}M_n \to 0$ and hence that $\frac{1}{n}\sum_{k=0}^{n}\hat{\rho}_k - \frac{1}{n}\sum_{k=0}^{n-1}\mathfrak{P}(\hat{\rho}_k)$ converges to 0. By iteration, this implies that $\frac{1}{n}\sum_{k=0}^{n}\hat{\rho}_k - \frac{1}{n}\sum_{k=0}^{n-1}\mathfrak{P}^m(\hat{\rho}_k)$ converges to 0 for any m. By summation this then implies that $\frac{1}{n}\sum_{k=0}^{n}\hat{\rho}_k - \frac{1}{n}\sum_{k=0}^{n-1}\left(\frac{1}{M}\sum_{m=0}^{M-1}\mathfrak{P}^m\right)(\hat{\rho}_k)$ also converges to 0. Now, the operation $\frac{1}{M}\sum_{m=0}^{M}\mathfrak{P}^m$ project on invariant states. Hence, $\frac{1}{n}\sum_{k=0}^{n}\hat{\rho}_k$ converges to an invariant state. □

Basics: ergodicity, detailed balance, etc.

— Detailed balance (as usual, by relating reversed paths)

Definition-Proposition "Detailed balance":
(i) Detailed balance is said to be fulfilled if there exists a family of operators μ_x, $x \in \Lambda$, acting on $\mathcal{H}$ such that $B_{yx}\mu_y = \mu_x B_{xy}^*$.
(ii) In such case, $\rho_x^{\mathrm{inv}} := \mu_x \mu_x^*$ is $\mathfrak{P}$-invariant: $\mathfrak{P}(\rho^{\mathrm{inv}}) = \rho^{\mathrm{inv}}$.

Alternatively, an intertwining relation between the CP map and its dual.

Probabilities of reversed path/trajectories are then linked

$$\mathbb{P}[\Omega_{x_0\to x_\ell}\big|x_0, \rho_0 - \mu_{x_0}^2/\mathrm{Tr}(\mu_{x_0}^2)] \times \mathrm{Tr}(\mu_{x_0}^2) - \mathbb{P}[\overline{\Omega}_{x_\ell\to x_0}\big|x_\ell, \rho_\ell - \mu_{x_\ell}^2/\mathrm{Tr}(\mu_{x_\ell}^2)] \times \mathrm{Tr}(\mu_{x_\ell}^2).$$

The ratio of the probabilities to visit a path and its time reversed is proportional to the ratio of the asymptotic frequencies of visits of the final and initial points (as in classical)

— Irreducibility, decomposition (as, or close to, classical Markov chain)

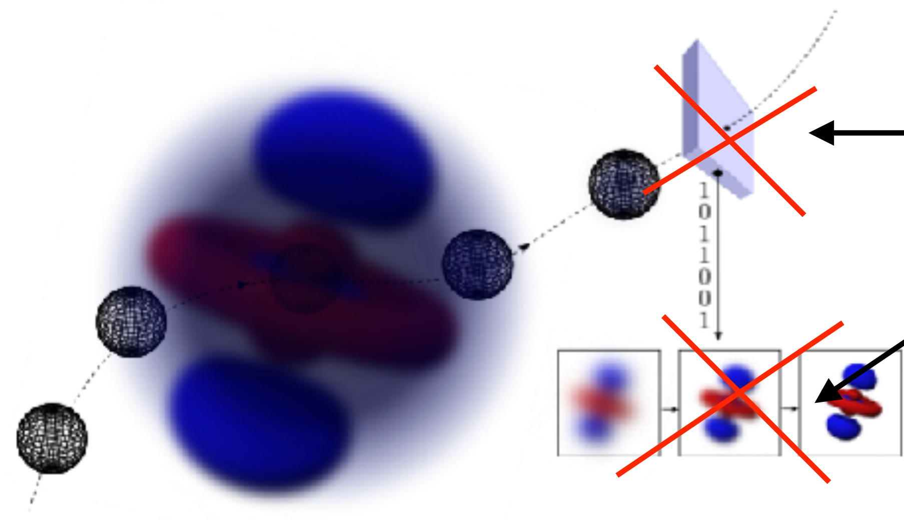
— Recurrence, transient, & harmonic measure,...

— etc....

Dilation and geometrisation

Recall what a (discrete) random process is.

What a **quantum stochastic process** could/should be?



No measurement…
No classical information…
No bayesian up-dating..

—> **Quantum Filtration….**

- The initial state (« vacuum state ») alias a measure:

$$\rho^0_{\rm sys}\otimes\rho_{\rm p}\otimes\cdots\otimes\rho_{\rm p}\otimes\cdots=\rho^0_{\rm sys}\otimes\rho_{\rm p}^{\otimes\infty}=:\Omega,$$

- The recursive evolution:

$$\Omega\to U_{0;1}\,\Omega\,U^\dagger_{0;1}\to U_{0;2}U_{0;1}\,\Omega\,U^\dagger_{0;1}U^\dagger_{0;2}\to\cdots.$$

- It entangles the system and the n-first probes, leaving the rest unchanged

$$\rho_n^{\rm tot}\otimes\rho_{\rm p}\otimes\rho_{\rm p}\otimes\cdots,$$

$$\rho^{\rm tot}_{n+1}=U_{0;n+1}\left(\rho_n^{\rm tot}\otimes\rho_{\rm p}\right)U^\dagger_{0;n+1},$$

- Filtration of algebras of observables

$$\mathcal{A}_n:=\mathcal{B}(H)_{\rm sys}\otimes\mathcal{B}(H)_{\rm probe}\otimes\mathcal{B}(H)_{\rm probe}\otimes\cdots$$

Example: A quantum process with classical walks:

If there is no internal space, OQW=RW: What is then the quantum process?

— Probes are spin 1/2 and the system Hilbert space is $L^2(\mathbb{Z})$ with basis $|x\rangle$, $x \in \mathbb{Z}$.
and the probe-system interaction is such that:

$$U|x\rangle \otimes |\varphi_p\rangle = \frac{1}{\sqrt{2}}[|x+1\rangle \otimes |+\rangle + |x-1\rangle \otimes |-\rangle].$$

— After n iteration, the system-probe (the n-first) state is

$$|\psi_n\rangle = \frac{1}{2^{n/2}} \sum_{\omega_n} |X(\omega_n)\rangle \otimes |\omega_n\rangle \qquad \text{with} \qquad |\omega_n\rangle := |\pm\rangle \otimes ... \otimes |\pm\rangle$$

—> kind of « Quantum parallelism ».

— If we measure |+/-> we are back to classical random walks (RW).
What happens if we measure the probe spin in a different direction?

That's it for today:

Thank you!

Next Time:

3- Continuous monitoring and quantum trajectories

4- Strong indirect quantum monitoring