Introduction to Quantum Optics [Theory]

quantum control perspective

Peter Zoller

University of Innsbruck and IQOQI, Austrian Academy of Sciences

Lectures 3 + 4

Outline

uibk

Quantum Optical Systems & Control

Lectures 1+2: *Isolated / Driven Hamiltonian* quantum optical systems

- Basic systems & concepts of quantum optics - an overview
- Example / Application: Ion Trap Quantum Computer

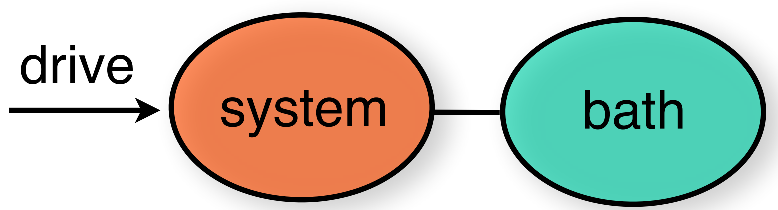
Lectures 3+4: ***Open*** quantum optical systems [a modern perspective]

- Continuous measurement theory, Quantum Stochastic Schrödinger Equation, master equation & quantum trajectories
- Example 1: *Chiral / Cascaded* Quantum Optical Systems & *Quantum Many-Body Systems*
- [Example 2: *Entanglement by Dissipation*]

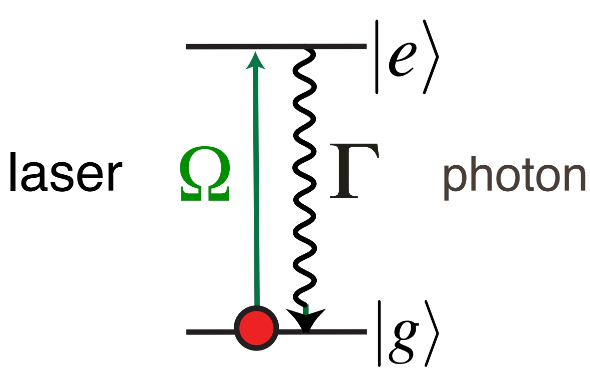
Theory of Quantum Noise: Quantum Optical Systems

The Quantum Stochastic Schrödinger Equation (QSSE)

- **quantum operations, Kraus operators**
  - formal quantum information theory
- **QSSE, master equations etc.**
  - quantum Markov processes
  - quantum optics

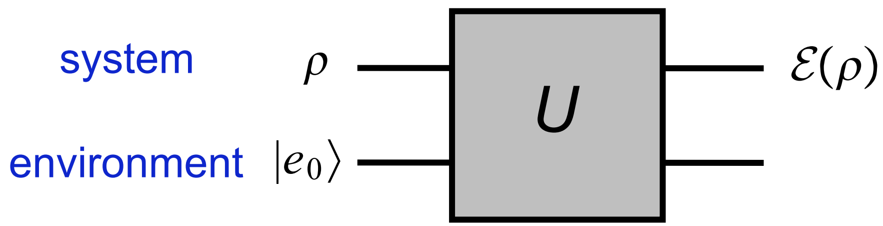


$$\frac{d\rho}{dt} = -i\,[H,\rho] + \mathcal{L}\rho$$ master equation



Quantum Operations

Evolution of a quantum system coupled to an environment:
open quantum system



$$\rho \longrightarrow \mathcal{E}(\rho) = \text{tr}_{\text{env}}[U(\rho \otimes \rho_B)U^\dagger]$$

quantum operation

ρ_B ↑ $|e_0\rangle\langle e_0|$

Operator sum representation:

$$\begin{aligned}\rho \longrightarrow \mathcal{E}(\rho) &= \text{tr}_{\text{env}}[U(\rho \otimes |e_0\rangle\langle e_0|)U^\dagger] \\ &= \sum_k \langle e_k|U(\rho \otimes |e_0\rangle\langle e_0|)U^\dagger|e_k\rangle \\ &= \sum_k E_k \rho E_k^\dagger \quad \text{with } E_k = \langle e_k|U|e_0\rangle\end{aligned}$$

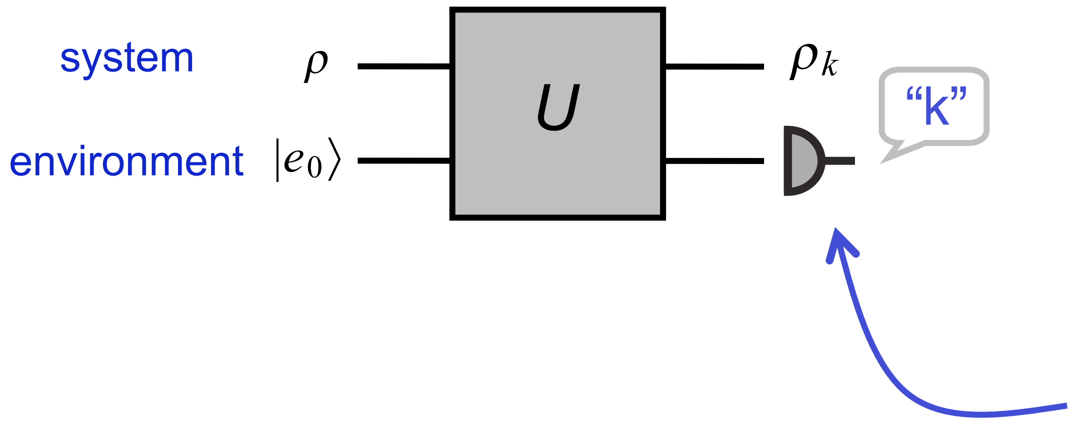
operation elements,
Kraus operator

Properties: $\sum_k E_k^\dagger E_k = 1$

- Ref.: Nielsen & Chuang, Quantum Information and Quantum Computations

Quantum Operations

Measurement of the environment: $P_k \equiv |e_k\rangle\langle e_k|$



state

$$\rho_k \sim \mathrm{tr}_{\mathrm{env}}(|e_k\rangle\langle e_k| U(\rho \otimes |e_0\rangle\langle e_0|) U^\dagger)$$

$$= E_k \rho E_k^\dagger$$

$$\rho_k = E_k \rho E_k^\dagger \,/\, \mathrm{tr}_{\mathrm{sys}}(E_k \rho E_k^\dagger) \qquad \text{(normalized)}$$

probability

$$p_k = \mathrm{tr}_{\mathrm{sys+env}}(|e_k\rangle\langle e_k| U(\rho \otimes |e_0\rangle\langle e_0|) U^\dagger)$$

$$= \mathrm{tr}_{\mathrm{sys}}(E_k \rho E_k^\dagger)$$

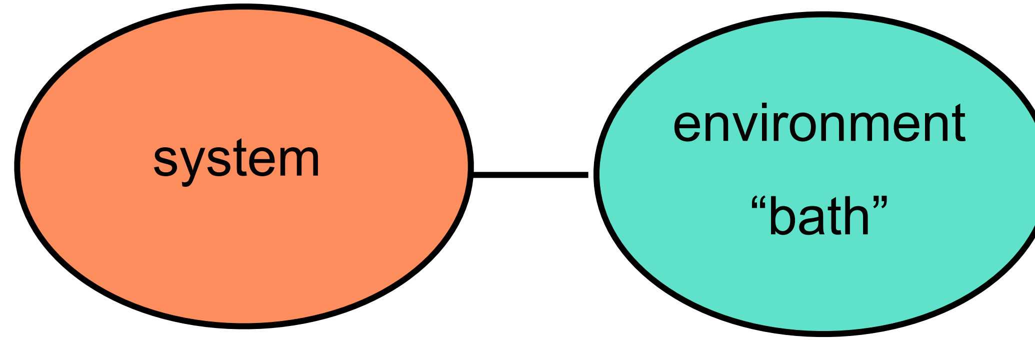
Remark: if we do not read out the measurement

$$\rho \longrightarrow \mathcal{E}(\rho) = \sum p_k \rho_k$$

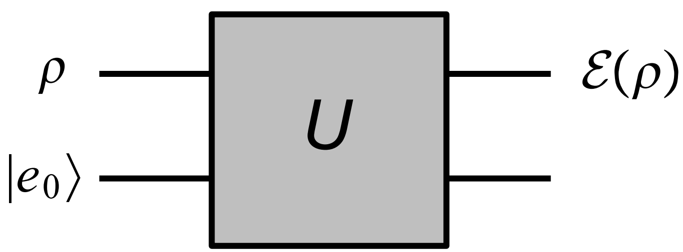
$$= \sum_k E_k \rho E_k^\dagger$$

Quantum noise & quantum optical systems

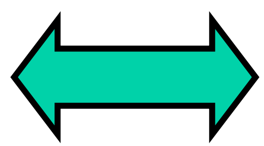
- decoherence
- state preparation
- read out



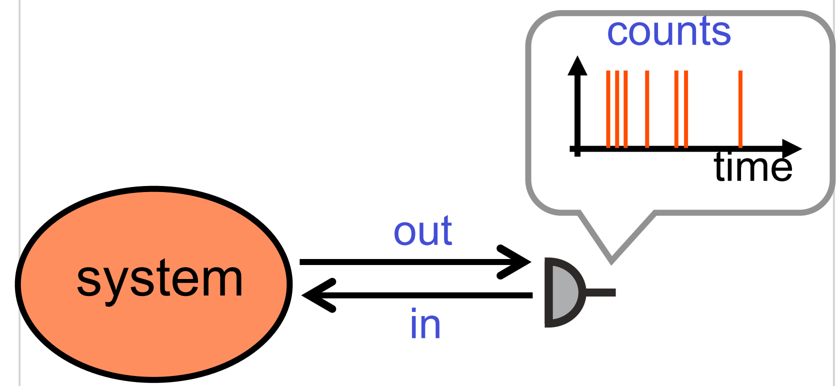
quantum operations



$$\rho \rightarrow \mathcal{E}(\rho) = \sum_k E_k \rho E_k^\dagger$$



quantum optics



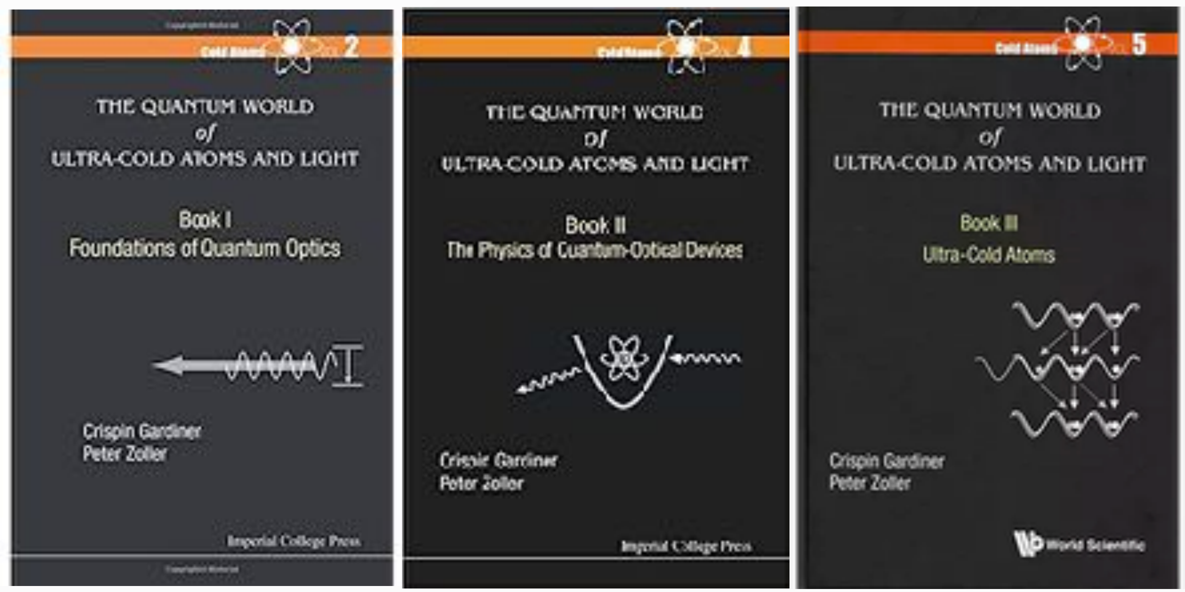
- ✓ master equation
- ✓ effect of observation on system: “preparation by quantum jumps”

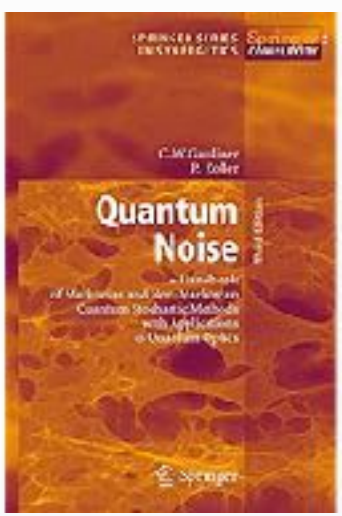
Literature

The Quantum World of Ultra-Cold Atoms and Light:

Book I: Foundations of Quantum Optics
Book II: The Physics of Quantum-Optical Devices
Book III: Ultra-cold Atoms

by Crispin W Gardiner and Peter Zoller





Quantum Noise

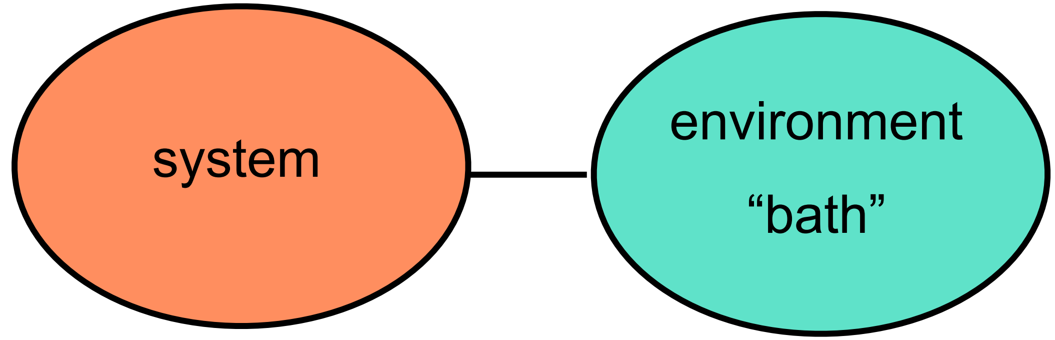
A Handbook of Markovian and Non-Markovian Quantum Stochastic Methods with Applications to Quantum Optics

by Crispin W Gardiner and Peter Zoller

`Standard Model' of Quantum Optics: System + Environment

Hamiltonian

$$H_{\text{tot}} = H_{\text{sys}} + H_B + H_{\text{int}}$$



H_{sys} unspecified

$$H_B = \int_B d\omega\, \omega\, b(\omega)^\dagger b(\omega) \quad \text{with } [b(\omega), b(\omega')^\dagger] = \delta(\omega - \omega')$$

harmonic oscillators

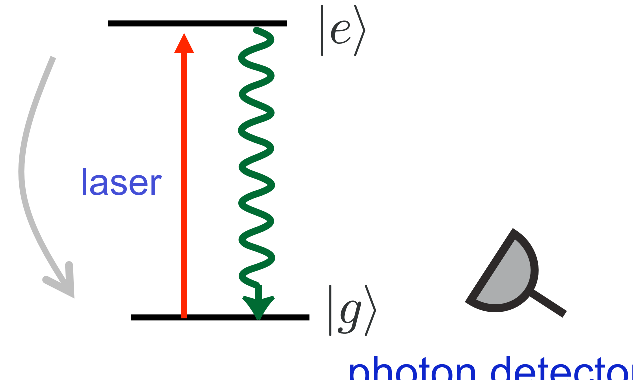
$$H_{\text{int}} = i \int_B d\omega\, \kappa(\omega) [c b^\dagger(\omega) - c^\dagger b(\omega)]$$

system operator

- driven two-level system + spontaneous emission

$$c \equiv |g\rangle\langle e|$$

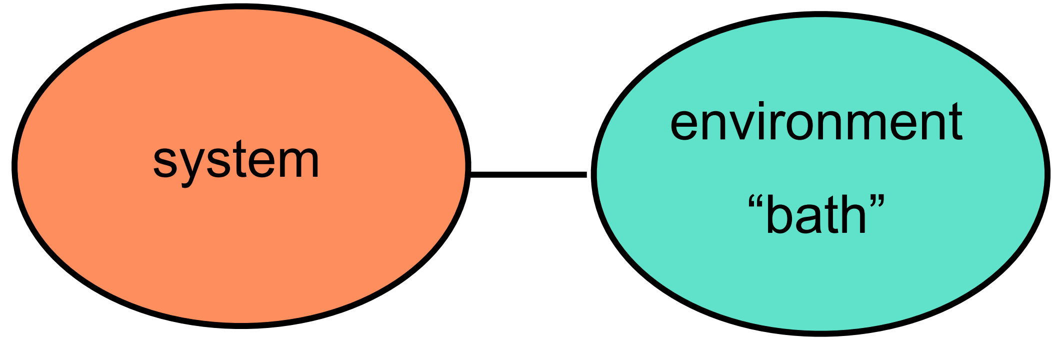
quantum jump operator



`Standard Model' of Quantum Optics: System + Environment

Hamiltonian

$$H_{\text{tot}} = H_{\text{sys}} + H_B + H_{\text{int}}$$



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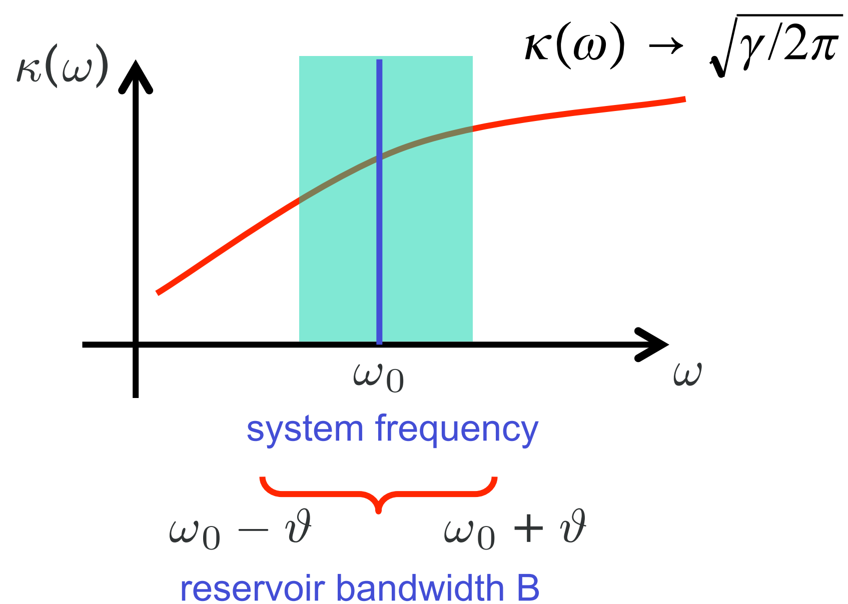
$$H_B = \int_B d\omega\, \omega\, b(\omega)^\dagger b(\omega) \quad \text{with } [b(\omega), b(\omega')^\dagger] = \delta(\omega - \omega')$$ harmonic oscillators

$$H_{\text{int}} = i\int_B d\omega\, \kappa(\omega)[c b^\dagger(\omega) - c^\dagger b(\omega)]$$

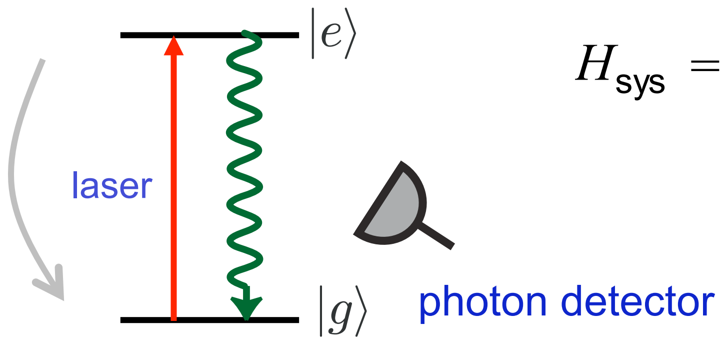
system operator



Examples

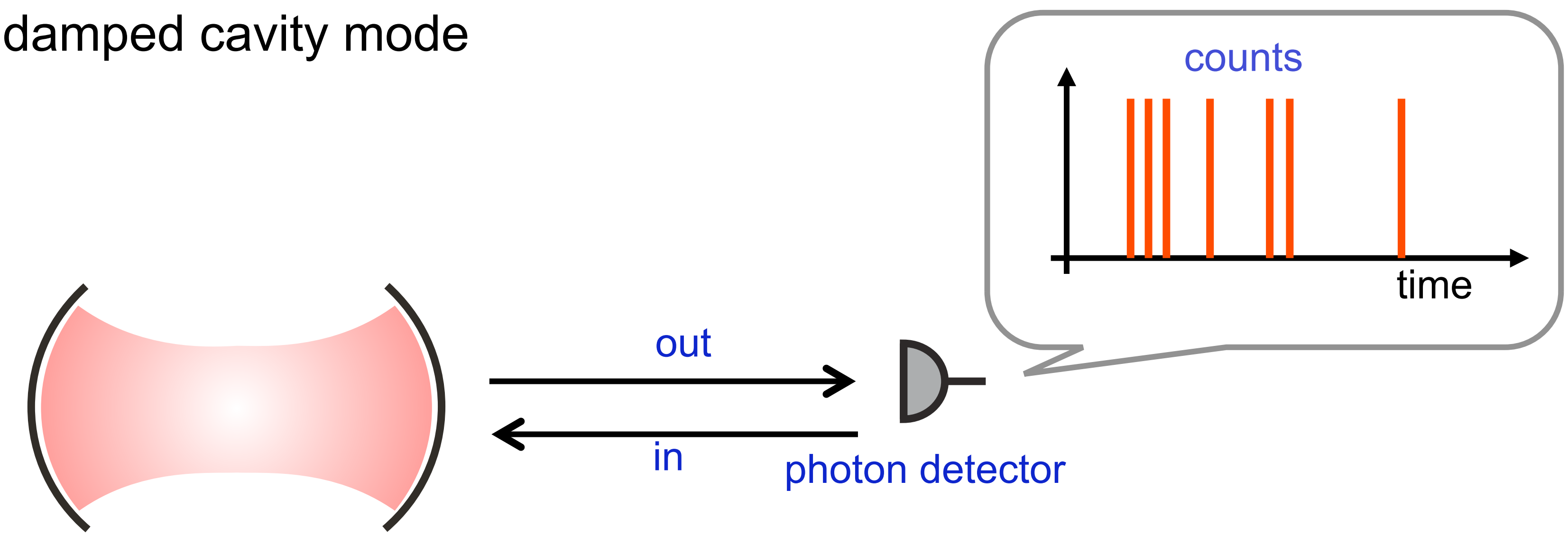
- driven two-level system undergoing spontaneous emission

$$c \equiv |g\rangle\langle e|$$



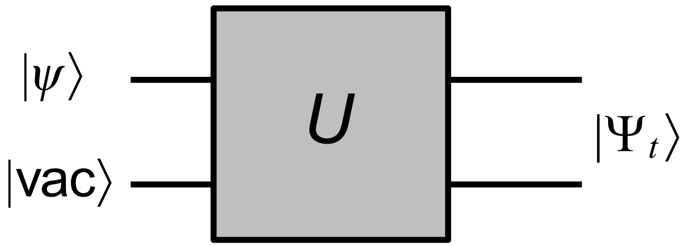
$$H_{\text{sys}} = \omega_{eg}\sigma_{ee} - \left(\tfrac{1}{2}\Omega e^{-i\omega_L t}\sigma_+ + \text{h.c.}\right)$$

- damped cavity mode



$$H_{\text{sys}} = \hbar\omega_0 a^\dagger a$$

Time evolution of the system + environment



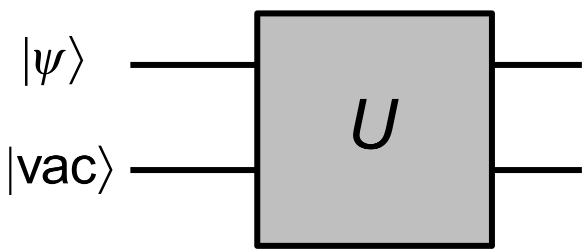
$$|\Psi_0\rangle \longrightarrow |\Psi_t\rangle = e^{-iH_{\text{tot}}t}|\Psi_0\rangle$$

Schrödinger equation:
system + environment

Time evolution of the system + environment

Questions:

- **We do not observe the environment:** reduced density operator



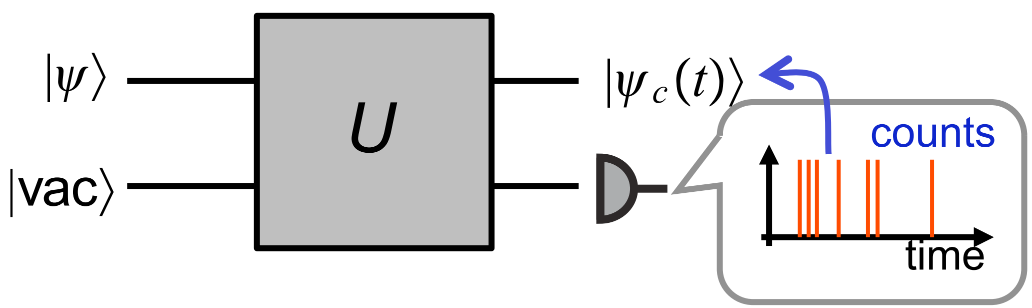
$$\rho_t = \mathrm{tr}_B |\Psi_t\rangle\langle\Psi_t|$$

master equation:

✓ decoherence

✓ preparation of the system
(e.g. laser cooling, optical pumping)

- **We measure the environment:** continuous measurement



conditional wave function:

✓ counting statistics

✓ effect of observation on system evolution (e.g. preparation of the (single quantum) system)

Integration of the Schrödinger Equation

- technical step: interaction picture with respect to bath:

$$|\Psi_t\rangle \to e^{-iH_B t}|\Psi_t\rangle$$ interaction picture

Schrödinger equation

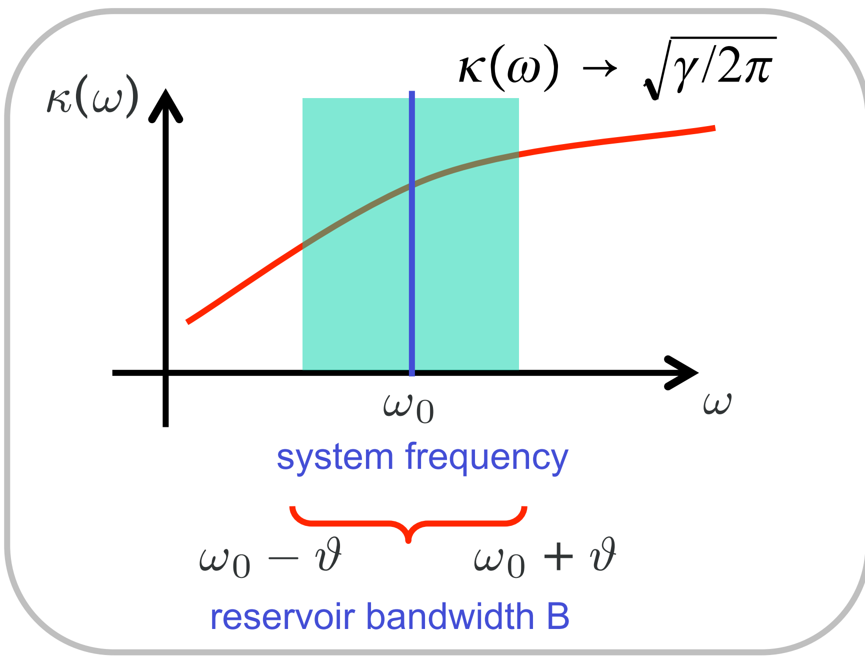
$$\frac{d}{dt}|\Psi_t\rangle = \left[-i\widetilde{H}_{\text{sys}} + \sqrt{\gamma}\, b(t)^\dagger c - \sqrt{\gamma}\, c^\dagger b(t)\right]|\Psi_t\rangle$$

$$b(t) := \frac{1}{\sqrt{2\pi}} \int_{\omega_0-\vartheta}^{\omega_0+\vartheta} b(\omega) e^{-i(\omega-\omega_0)t} d\omega$$

noise operators

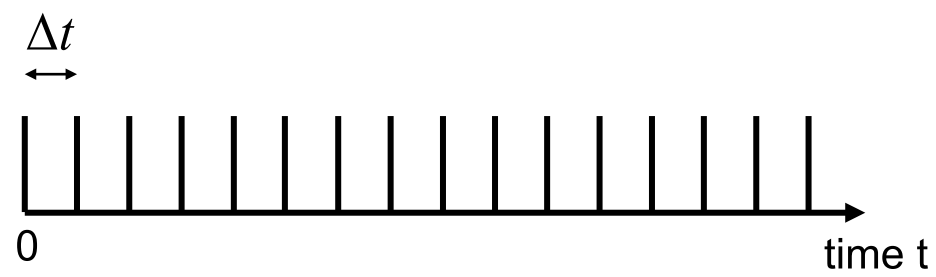
$$\left[b(t), b^\dagger(s)\right] = \delta_s(t-s) e^{-i\omega_0(t-s)}$$

"white noise on scale 1/B"



rotating frame

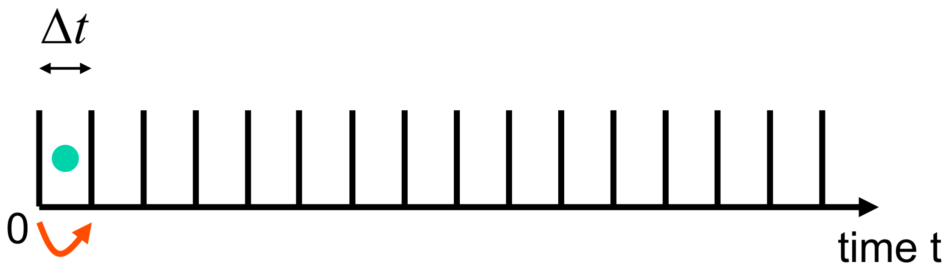
- We integrate the Schrödinger equation in small time steps



$$|\Psi(t = t_f)\rangle \;\; = U(\Delta t_f)\ldots U(\Delta t_1)U(\Delta t_0)|\Psi(0)\rangle$$

Q.: size of time step? hierarchy of time scales (see below: "coarse graining")

1st time step



- **First time step:** we start with the first interval and expand *U(Δt)* to second order in *Δt*

$$U(\Delta t)|\Psi(0)\rangle = \Big\{ \hat{1} - iH_{\text{sys}}\Delta t + \sqrt{\gamma}\, c \int_0^{\Delta t} b^\dagger(t)\, dt - \sqrt{\gamma}\, c^\dagger \int_0^{\Delta t} b(t)\, dt$$

$$+(-i)^2 \gamma c^\dagger c \int_0^{\Delta t} dt \int_0^{t_2} dt'\, b(t) b^\dagger(t') + \ldots \Big\} |\Psi(0)\rangle$$

$|\psi\rangle \otimes |\text{vac}\rangle$

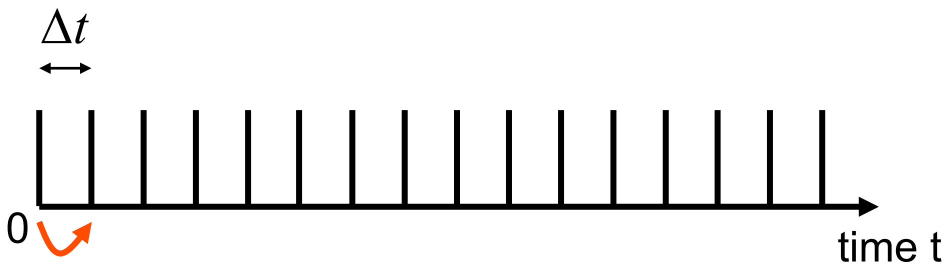
$$\int_0^t dt \int_0^{t_2} dt'\, b(t)\, b^\dagger(t')|\text{vac}\rangle = \int_0^t dt \int_0^{t_2} dt' \, [b(t), b^\dagger(t')]|\text{vac}\rangle$$

$$= \int_0^t dt \int_0^{t_2} dt'\, \delta(t-t')|\text{vac}\rangle$$

$$= \frac{1}{2}\Delta t |\text{vac}\rangle$$

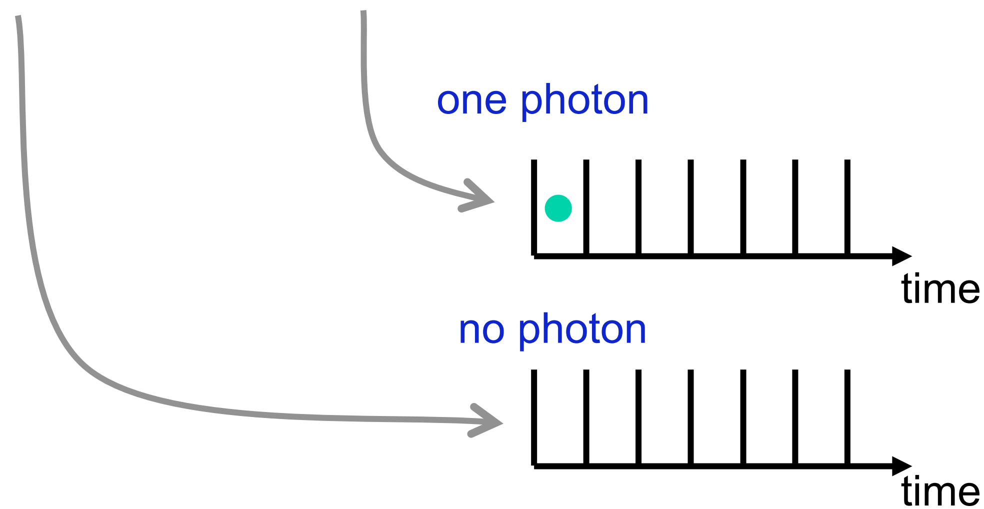
first order in Δt

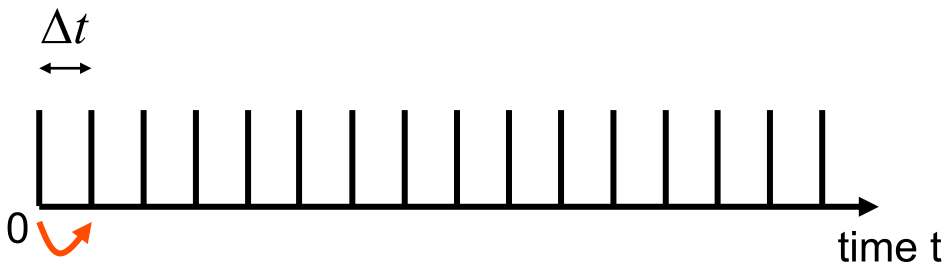
1st time step



- **First time step:** to first order in *Δt*

$$
\begin{aligned}
|\Psi(\Delta t)\rangle &= \hat{U}(\Delta t)|\Psi(0)\rangle \\
&= \left\{ \hat{1} - iH_{\text{eff}}\,\Delta t + \sqrt{\gamma}\, c \int_0^{\Delta t} b^{\dagger}(t)\, dt \right\} |\Psi(0)\rangle
\end{aligned}
$$





- **First time step:** to first order in *Δt*

$$
\begin{aligned}
|\Psi(\Delta t)\rangle &= \hat{U}(\Delta t)|\Psi(0)\rangle \\
&= \left\{\hat{1} - iH_{\text{eff}}\,\Delta t + \sqrt{\gamma}\; c\Delta B(t)^{\dagger}\right\}|\Psi(0)\rangle
\end{aligned}
$$

We define:

- effective (non-hermitian) system Hamiltonian

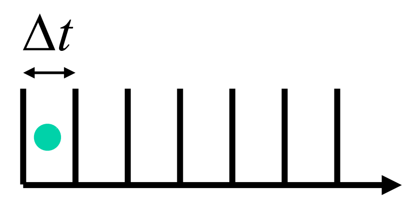
$$H_{\text{eff}} := H_{\text{sys}} - \frac{i}{2}\gamma c^{\dagger} c$$

- annihilation / creation operator for a photon in the time slot Δt :

$$\Delta B(t) := \int_{t}^{t+\Delta t} b(s)\, ds$$

annihilation / creation operator for a photon in the time slot Δt :

$$\Delta B(t) := \int_t^{t+\Delta t} b(s)\, ds$$



Remarks and properties:

- commutation relations:

$$[\Delta B(t), \Delta B^\dagger(t')] = \begin{cases} \Delta t & t = t' \quad \text{overlapping intervals} \\ 0 & t \neq t' \quad \text{nonoverlapping intervals} \end{cases}$$

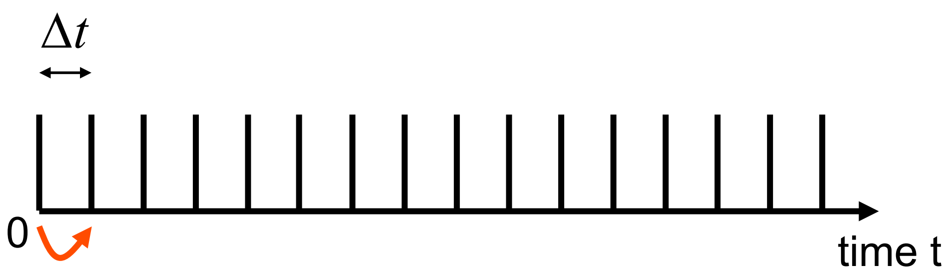
- one-photon wave packet in time slot Δt

$$\frac{\Delta B^\dagger(t)}{\sqrt{\Delta t}} |\text{vac}\rangle \equiv |1\rangle_t \quad \text{(normalized)}$$

- number operator of photon in time slot t:

$$N(t) = \frac{\Delta B^\dagger(t)}{\sqrt{\Delta t}} \frac{\Delta B(t)}{\sqrt{\Delta t}}$$

- $N(t)$ as set up commuting operators, $[N(t), N(t')] = 0$, which can be measured "simultaneously"



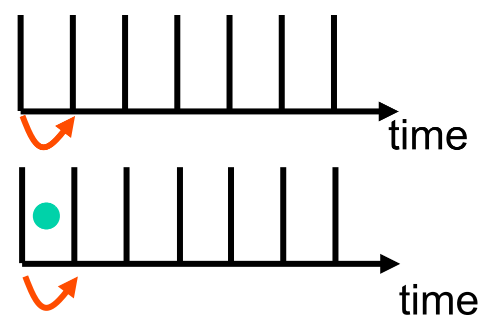
- **Summary of first time step:** to first order in *Δt*

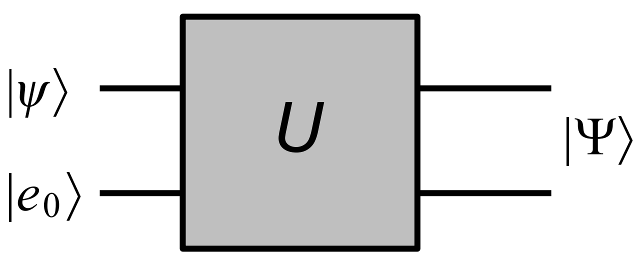
$$
\begin{aligned}
|\Psi(\Delta t)\rangle &= \left[1 - iH_{\text{eff}}\Delta t + \sqrt{\gamma}\, c\, \Delta B^{\dagger}(0)\right]|\Psi(0)\rangle \\
&= |\text{vac}\rangle \otimes \left(1 - iH_{\text{eff}}\Delta t\right)|\psi(0)\rangle \; + |1\rangle_t \otimes (\sqrt{\gamma\Delta t}\, c|\psi(0)\rangle) \\
&\equiv |\text{vac}\rangle \otimes E_0|\psi(0)\rangle \; + |1\rangle_t \otimes E_1|\psi(0)\rangle
\end{aligned}
$$

operation elements

where we read off the operation elements

$$
\begin{aligned}
E_0 &= 1 - iH_{\text{eff}}\Delta t \quad \text{(no photon)} \\
E_1 &= \sqrt{\gamma\Delta t}\, c \quad \text{(1 photon)}
\end{aligned}
$$

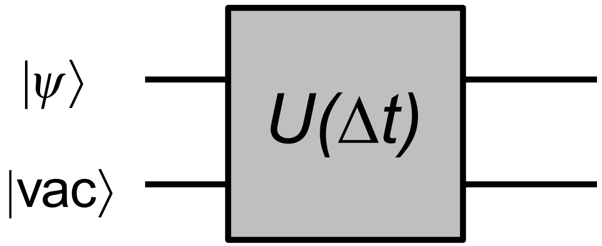




$$
\begin{aligned}
|\psi\rangle|e_0\rangle \to |\Psi\rangle &= U|\psi\rangle|e_0\rangle \\
&= \sum_k |e_k\rangle\langle e_k|U|e_0\rangle|\psi\rangle \equiv \sum |e_k\rangle E_k|\psi\rangle
\end{aligned}
$$

Discussion:

- **We do not read the detector:** reduced density operator

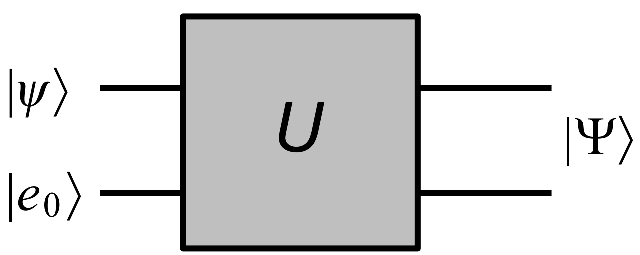


$$\begin{aligned}\rho(\Delta t) &= \mathrm{tr}_B|\Psi(\Delta t)\rangle\langle\Psi(\Delta t)| \\ &= E_0\rho(0)E_0^\dagger + E_1\rho(0)E_1^\dagger \\ &= \left(1 - iH_{\text{eff}}\Delta t\right)\rho(0)\left(1 - iH_{\text{eff}}\Delta t\right)^\dagger + \gamma c\rho(0)c^\dagger\Delta t\end{aligned}$$

no photon ... one photon

master equation:

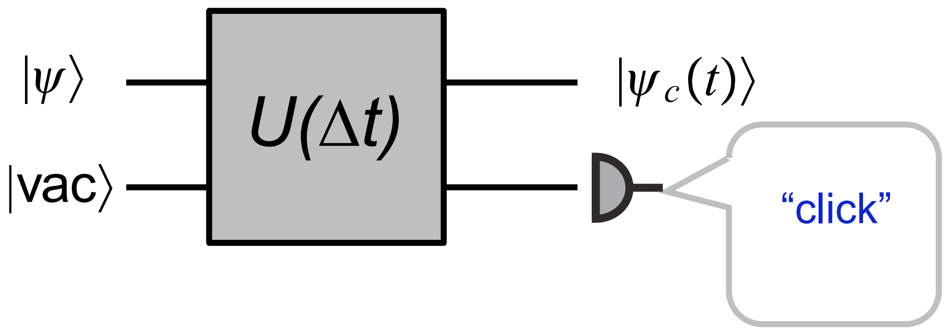
$$\begin{aligned}\rho(\Delta t) - \rho(0) &= -i\left(H_{\text{eff}}\rho(0) - \rho(0)H_{\text{eff}}^\dagger\right)\Delta t + \gamma c\rho(0)c^\dagger\Delta t \\ &\equiv -i\left[H_{\text{sys}}, \rho(0)\right]\Delta t + \frac{1}{2}\gamma(2c\rho(0)c^\dagger - c^\dagger c\rho(0) - \rho(0)c^\dagger c)\Delta t\end{aligned}$$



$$\begin{aligned}\rho \rightarrow \mathcal{E}(\rho) &= \mathrm{tr}_{\text{env}}[U(\rho\otimes|e_0\rangle\langle e_0|)U^\dagger] \\ &= \sum_k E_k\rho E_k^\dagger\end{aligned}$$

Discussion:

- **We read the detector:**



- **Click:** resulting state

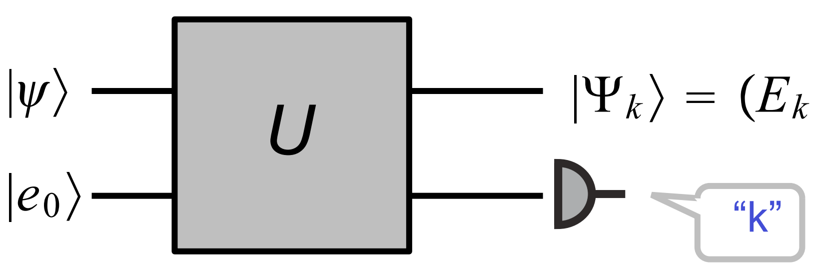
quantum jump operator

$$E_1|\psi(0)\rangle \equiv |\psi^{\text{click}}(\Delta t)\rangle = \sqrt{\gamma\Delta t}\, c|\psi(0)\rangle \quad \text{(quantum jump)}$$

with probability

$$p^{\text{click}} = \text{tr}_{\text{sys}}(E_1\rho(0)E_1) = \gamma\Delta t\, \|c\psi(0)\|^2$$

Rem.: density matrix $\rho_1(0) = E_1\rho(0)E_1/\text{tr}(\dots)$

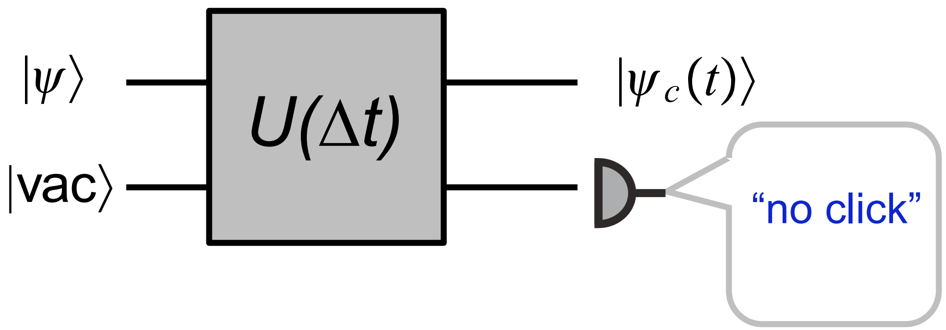


$$|\Psi_k\rangle = (E_k|\psi\rangle)|e_k\rangle/\|\dots\|$$

$$p_k = \|E_k\psi\|^2$$

Discussion:

- **We read the detector:**



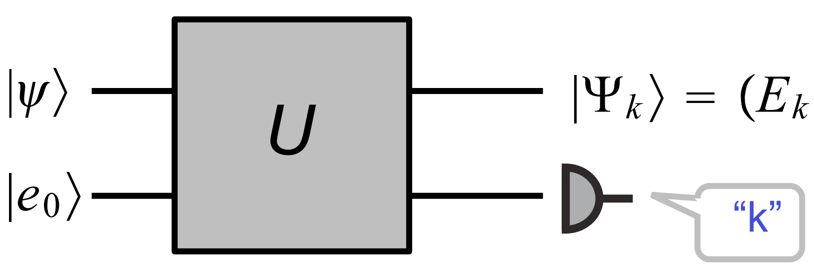
- **No click:** resulting state

decaying norm

$$E_0|\psi(0)\rangle \equiv |\psi^{\text{no click}}(\Delta t)\rangle = \left(1 - iH_{\text{eff}}\Delta t\right)|\psi(0)\rangle \approx e^{-iH_{\text{eff}}\Delta t}|\psi(0)\rangle$$

with probability

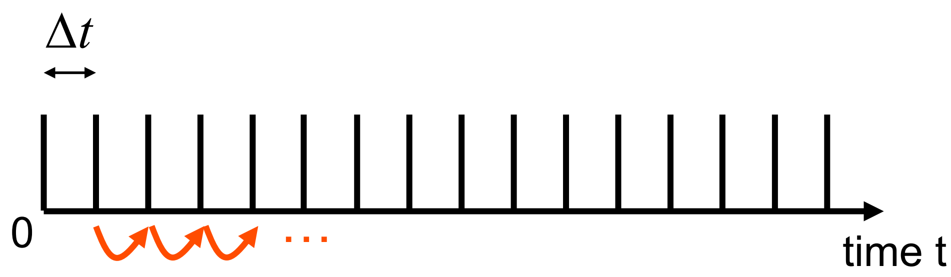
$$p^{\text{no click}} = \text{tr}_{\text{sys}}(E_0\rho(0)E_0) = \left\| e^{-iH_{\text{eff}}\Delta t}\psi(0) \right\|^2$$



$$|\Psi_k\rangle = (E_k|\psi\rangle)|e_k\rangle/||\ldots||$$

$$p_k = \|E_k\psi\|^2$$

2nd time step etc.



- **Second and more time steps:**

$$|\Psi(n\Delta t)\rangle = \left[1 - iH_{\text{eff}}\Delta t + \sqrt{\gamma}\, c\, \Delta B^{\dagger}((n-1)\Delta t)\right]|\Psi((n-1)\Delta t)\rangle$$

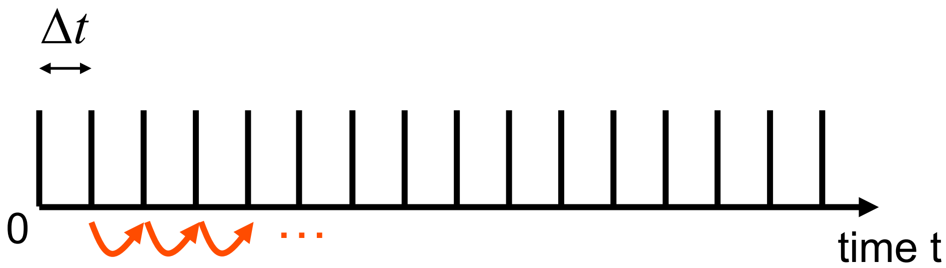
stroboscopic integration

$$\equiv \left[1 - iH_{\text{eff}}\Delta t + \sqrt{\gamma}\, c\, \Delta B^{\dagger}((n-1)\Delta t)\right] \times$$
$$\ldots \times \left[1 - iH_{\text{eff}}\Delta t + \sqrt{\gamma}\, c\, \Delta B^{\dagger}(0)\right]|\Psi(0)\rangle$$

✓ Note: remember … commute in different time slots

$$[\Delta B(t), \Delta B^{\dagger}(t')] = \begin{cases} \Delta t & t = t' \quad \text{overlapping intervals} \\ 0 & t \neq t' \quad \text{nonoverlapping intervals} \end{cases}$$

2nd time step etc.



- **Second and more time steps:**

$$|\Psi(n\Delta t)\rangle = \left[1 - iH_{\text{eff}}\Delta t + \sqrt{\gamma}\, c\, \Delta B^\dagger((n-1)\Delta t)\right]|\Psi((n-1)\Delta t)\rangle$$

stroboscopic integration

- **Ito Quantum Stochastic Schrödinger Equation**

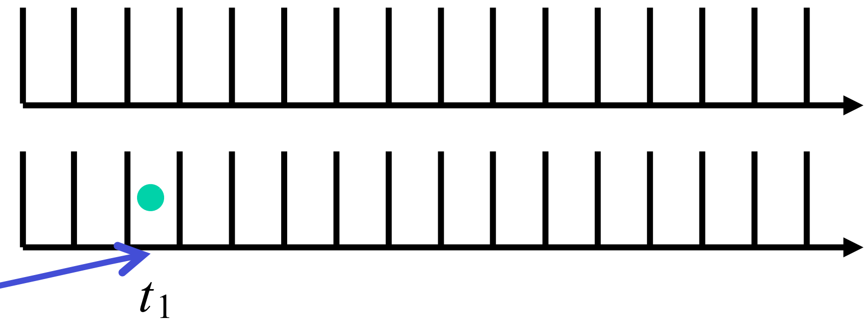
(I) $$dt\,|\Psi(t)\rangle = \left\{-iH_{\text{sys}}dt + \sqrt{\gamma}dB^\dagger(t)c\right\}|\Psi(t)\rangle \qquad (|\Psi(0)\rangle = |\psi_{\text{sys}}\rangle \otimes |\text{vac}\rangle)$$

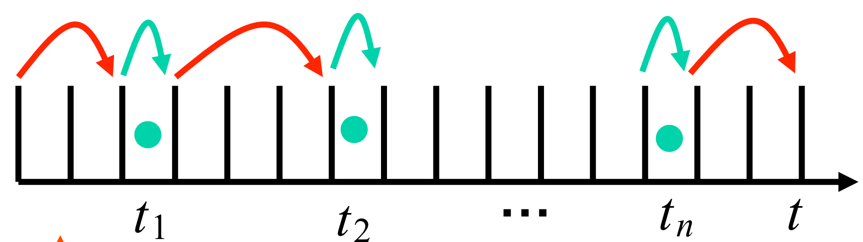
with Ito rules

$$\Delta B(t)\Delta B^\dagger(t)\,|\text{vac}\rangle = \Delta t\,|\text{vac}\rangle \quad \longrightarrow \quad dB(t)dB^\dagger(t) = dt$$

- **Wave function of the system + environment: entangled state**

$$
\begin{aligned}
|\Psi(t)\rangle \; &= |\text{vac}\rangle \otimes e^{-iH_{\text{eff}}t}|\psi(0)\rangle \\
&+ (\gamma\Delta t)^{1/2} \sum_{t_1} |1_{t_1}\rangle \otimes e^{-iH_{\text{eff}}(t-t_1)} c e^{-iH_{\text{eff}}t_1}|\psi(0)\rangle \\
&+ \ldots \\
&+ (\gamma\Delta t)^{n/2} \sum_{t_n > \ldots > t_1} |1_{t_1} 1_{t_2} \ldots 1_{t_n}\rangle \otimes e^{-iH_{\text{eff}}(t-t_n)} c \ldots c e^{-iH_{\text{eff}}t_1}|\psi(0)\rangle \\
&\ldots
\end{aligned}
$$





1. system time evolution $|\psi(t|t_1 t_2 \ldots t_n)\rangle$ for a specfic count sequence

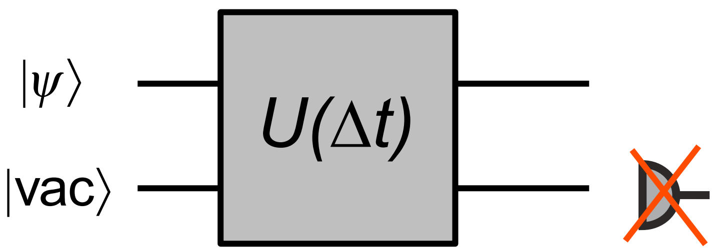
2. photon count statistics: probability densities

$$p_{(0,t]}(t_1, t_2, \ldots, t_n) = \| \psi(t|t_1 t_2 \ldots t_n) \|^2$$

click: $|\psi_t\rangle \rightarrow \sqrt{\gamma}\, c|\psi_{t+\Delta t}\rangle$

no click: $|\psi_0\rangle \rightarrow |\psi_t\rangle = e^{-iH_{\text{eff}}t}|\psi_0\rangle$

- **Tracing over the environment we obtain the master equation**



$$\frac{d}{dt}\rho(t) = -i\left[H_{\mathrm{sys}}, \rho(t)\right] + \frac{1}{2}\gamma\left(2c\rho(t)c^{\dagger} - c^{\dagger}c\rho(t) - \rho(t)c^{\dagger}c\right)$$

master equation

✓ Lindblad form

✓ coarse grained time derivative

Proof: $$\frac{d}{dt}\rho(t) = \mathrm{Tr}_B\left\{\underbrace{\ldots dB(t)\,|\Psi\rangle\langle\Psi|}_{=0} + \underbrace{\ldots|\Psi\rangle\langle\Psi|\,dB^{\dagger}(t)}_{=0} + dB^{\dagger}(t)\,|\Psi\rangle\langle\Psi|\,dB(t)\right\}$$

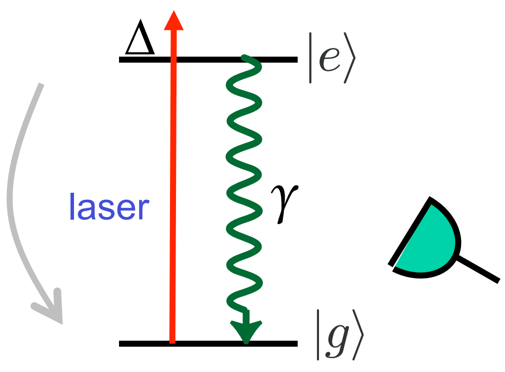
$$dB(t)dB^{\dagger}(t) = dt$$

Some Simple Examples

Example 1: Two-level atom + spontaneous emission
Master Equation

- **two-level system**

$$c \longrightarrow \sigma_+ = |e\rangle\langle g|$$



Hamiltonian

$$H_{\text{sys}} = \omega_{eg}\sigma_{ee} - \left(\tfrac{1}{2}\Omega(t)e^{-i\omega_L t}\sigma_+ + \text{h.c.}\right)$$

and in the rotating frame

$$H_{\text{sys}} = -\Delta\sigma_{ee} - \left(\tfrac{1}{2}\Omega\sigma_+ + \text{h.c.}\right)$$

jump operator

$$c \longrightarrow \sigma_+ = |e\rangle\langle g|$$

- **a quantum jump (detection of an emission)** *prepares the atom in the ground state*

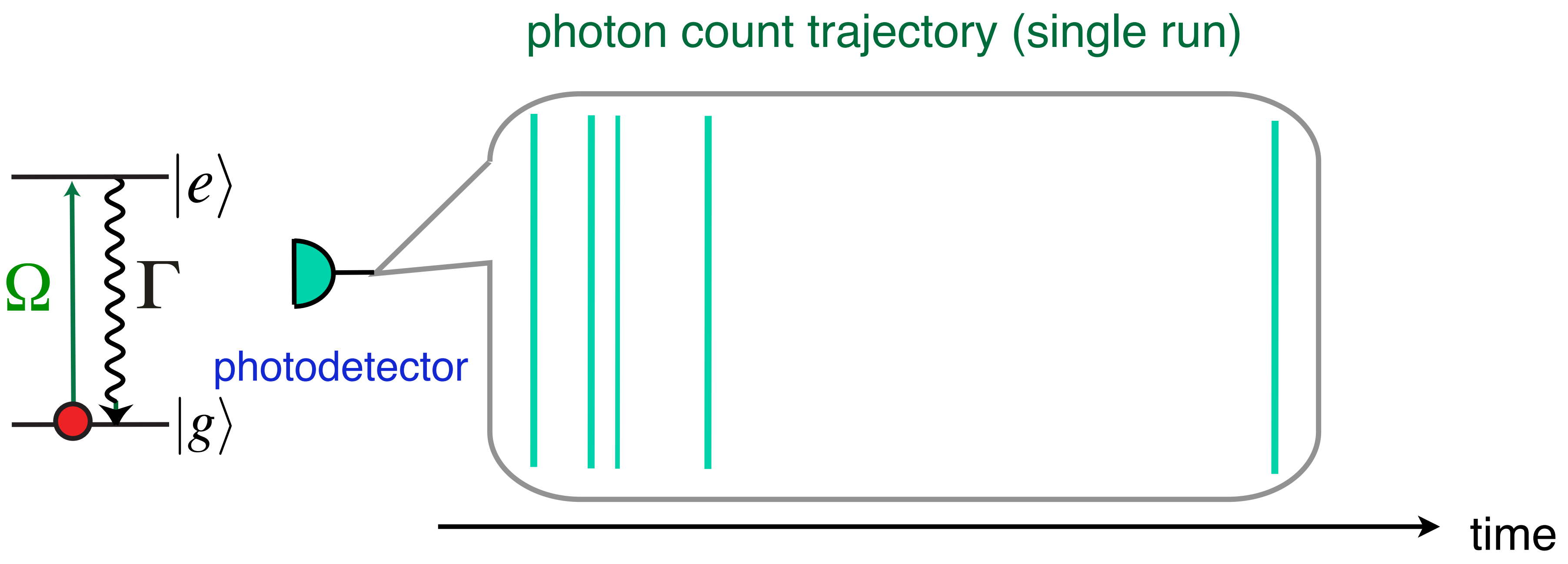
$$|\psi(t)\rangle \longrightarrow |\psi(t+dt)\rangle \sim \sigma_-|\psi(t)\rangle \equiv |g\rangle$$

probability for click in time interval *(t,t+dt]* $p_{(t,t+dt]} = \gamma|\langle e|\psi(t)\rangle|^2 dt$

- **master equation (Optical Bloch Equations)**

$$\frac{d}{dt}\rho = -i\left[H_{\text{sys}},\rho\right] + \tfrac{1}{2}\gamma(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-)$$

Example 2: Two-level atom
Evolution conditional to observation

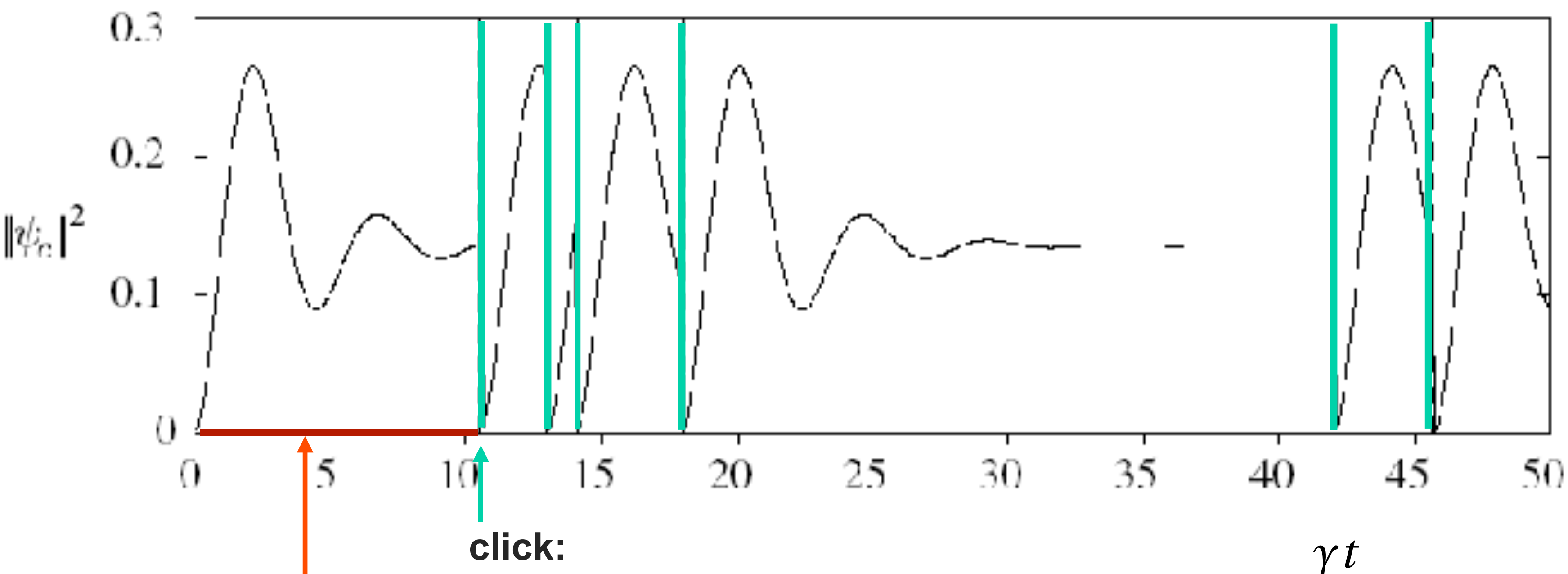


Evolution of the atom, *given* this counting trajectory?

conditional time evolution / wave function

Example 2: Two-level atom
Evolution conditional to observation

Fig.: typical quantum trajectory (upper state population)



click:

“quantum jump” = effect of detecting a photon on system

$$|\psi_{\rm sys}(t)\rangle \to \sqrt{\gamma}\sigma^- \, |\psi_{\rm sys}(t)\rangle$$

no click:

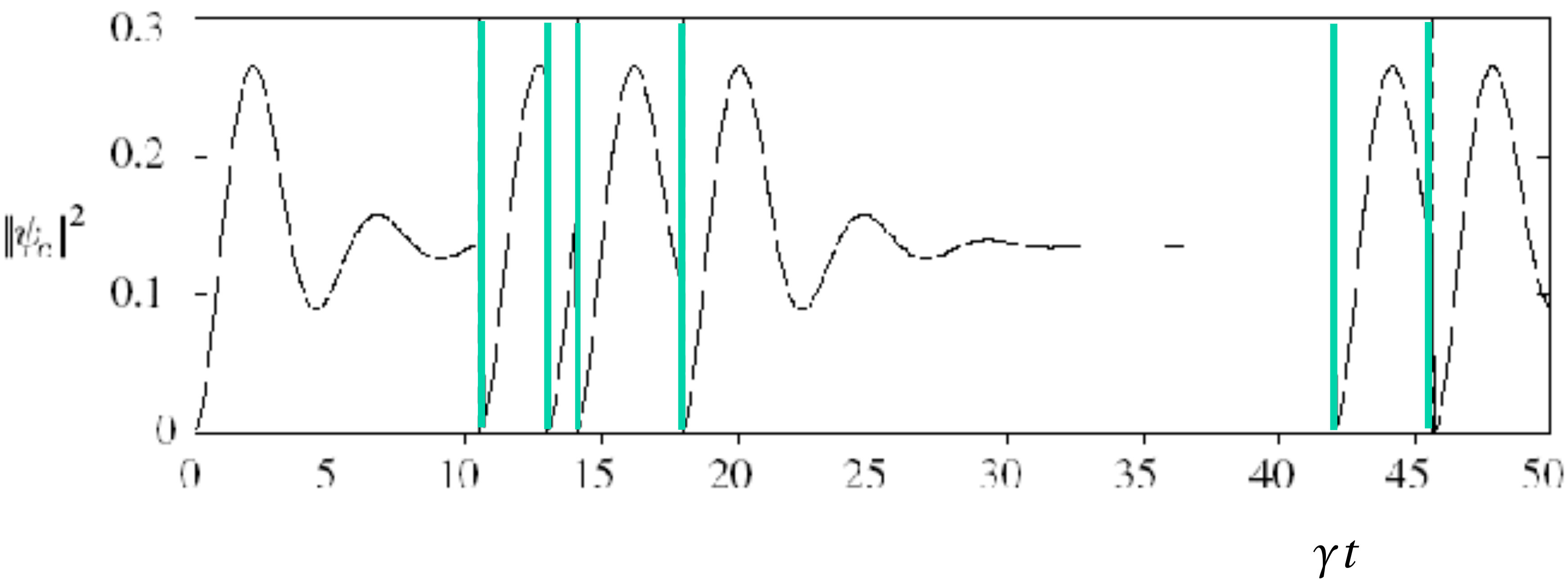
$$|\psi_{\rm sys}(t)\rangle = e^{-iH_{\rm eff}t} \, |\psi_{\rm sys}(0)\rangle$$

with Wigner -Weisskopf Hamiltonian

$$H_{\rm eff} = \left(\omega_{eg} - i\frac{1}{2}\gamma\right)\sigma_{ee} + \ldots$$

Example 2: Two-level atom
Evolution conditional to observation

Fig.: typical quantum trajectory (upper state population)



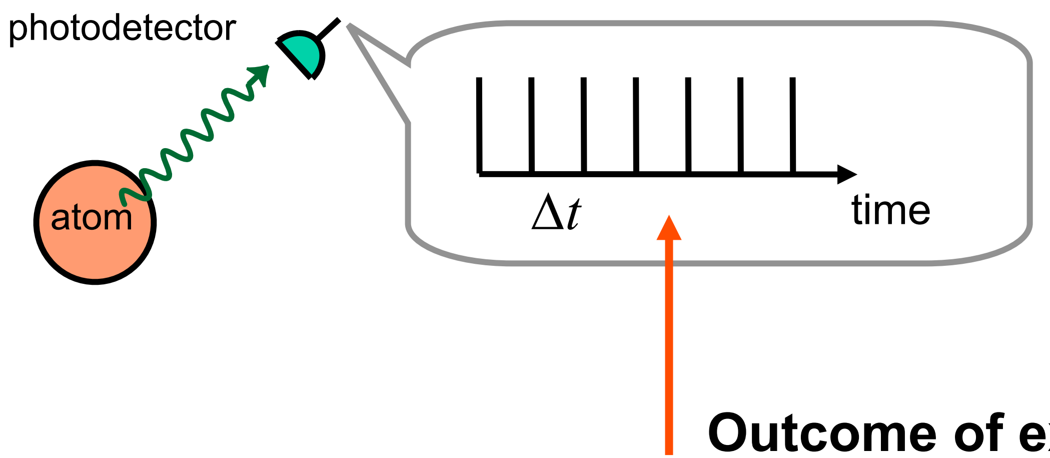
- Monte Carlo wave function simulation

stochastic wavefunction $|\psi(t)\rangle_{\text{sys}}$ (dim d)

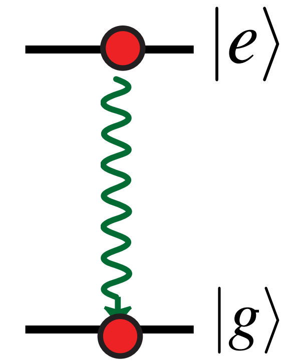
reduced density matrix $\rho(t) = \langle\, |\psi_{\text{sys}}(t)\rangle \langle \psi_{\text{sys}}(t)|\, \rangle_{\text{st}}$

DMRG + wave function simulation $\longleftrightarrow$ density matrix $\rho_{\text{sys}}(t)$ (dim $d \times d$)

Example 3: Two-level atom
Evolution conditional to observation



initial state:



$$|\psi_c(0)\rangle = c_g|g\rangle + c_e|e\rangle$$

Outcome of experiment:
We observe NO photon up to time t

Question: what is the state of the atom conditional to this observation after time t?

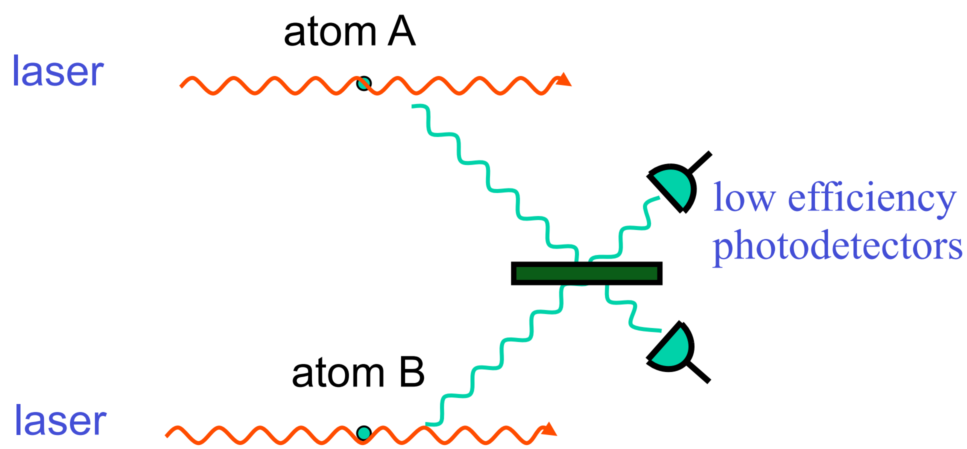
Answer: $$|\psi_c(t)\rangle = \frac{e^{-iH_{\text{eff}}t/\hbar}|\psi_c(0)\rangle}{\|\ldots\|} = \frac{c_g|g\rangle + c_e e^{-\gamma t/2}|e\rangle}{\|\ldots\|}$$

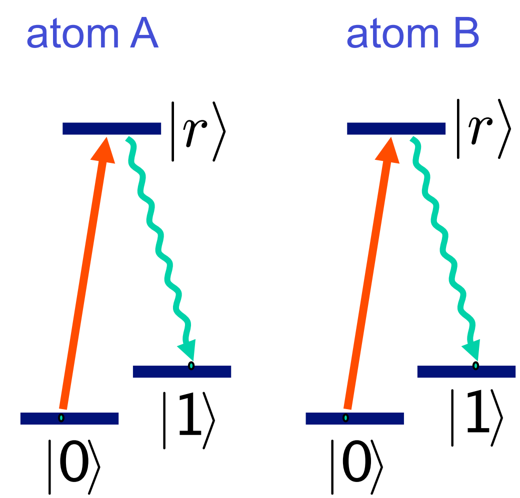
$\longrightarrow |g\rangle$ for $t \to \infty$

We *learn* that the system is in the ground state

Preparation of 2 atoms in a Bell state via measurement

- **System:** two atoms with ground states $|0\rangle$, $|1\rangle$ and excited state $|r\rangle$





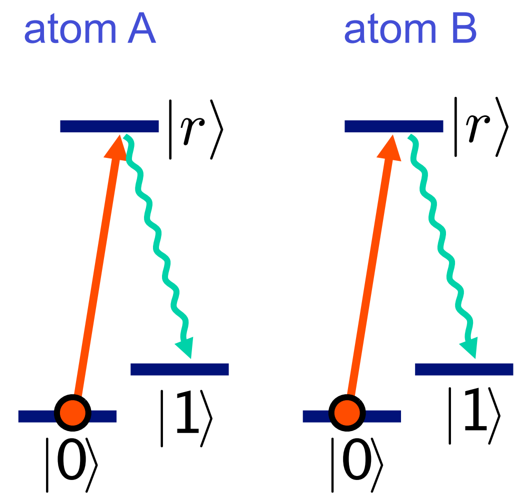
- Weak (short) laser pulse, so that the excitation probability is small.

- If no detection, pump back and start again.

- If detection, an entangled state is created.

$$\sim |0,1\rangle + |1,0\rangle$$

Process:



- preparation (by optical pumping)

$$|\Psi(t=0)\rangle = |\text{vac}\rangle|0\rangle_1|0\rangle_2$$

- excitation by a weak short laser pulse

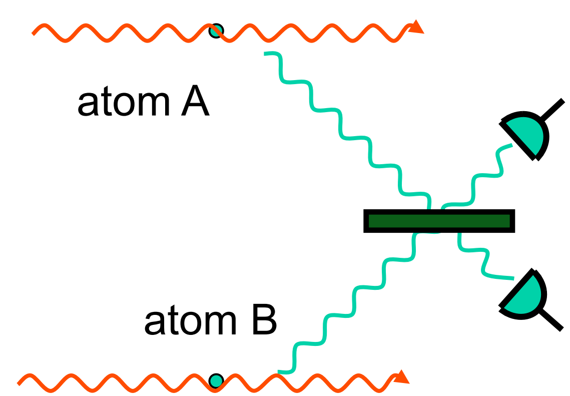
$$|\Psi(t=0^+)\rangle = |\text{vac}\rangle\,(|0\rangle_2 + \epsilon|r\rangle_2)(|0\rangle_2 + \epsilon|r\rangle_2)$$
$$= |\text{vac}\rangle\,[|0\rangle_1|0\rangle_2 + \epsilon(|r\rangle_1|0\rangle_2 + |0\rangle_1|r\rangle_2) + O(\epsilon^2)]$$

- spontaneous emission

$$|\Psi(t>0^+)\rangle = [|0\rangle_1|0\rangle_2 + \epsilon e^{-\gamma t/2}(|r\rangle_1|0\rangle_2 + |0\rangle_1|r\rangle_2)] \otimes |\text{vac}\rangle$$
$$+ \sum_{t_1} \Delta B_1^\dagger(t_1)|\text{vac}\rangle \otimes \epsilon\sqrt{\gamma}\, e^{-\gamma t_1/2}|1\rangle_1|0\rangle_2$$
$$+ \Delta B_2^\dagger(t_1)|\text{vac}\rangle \otimes \epsilon\sqrt{\gamma}\, e^{-\gamma t_1/2}|0\rangle_1|1\rangle_2 + O(\epsilon^2)$$

- We observe the fluorescence through a beam splitter

$$\Delta B_{1,2}^\dagger \rightarrow \frac{1}{\sqrt{2}}(\Delta B_1^\dagger \pm \Delta B_2^\dagger)$$



- Observation of a click prepares Bell state

$$|1\rangle_1|0\rangle_2 + |0\rangle_1|1\rangle_2$$

Open Quantum *Many-Body* Systems

Example 1:

→ Chiral Quantum Optics

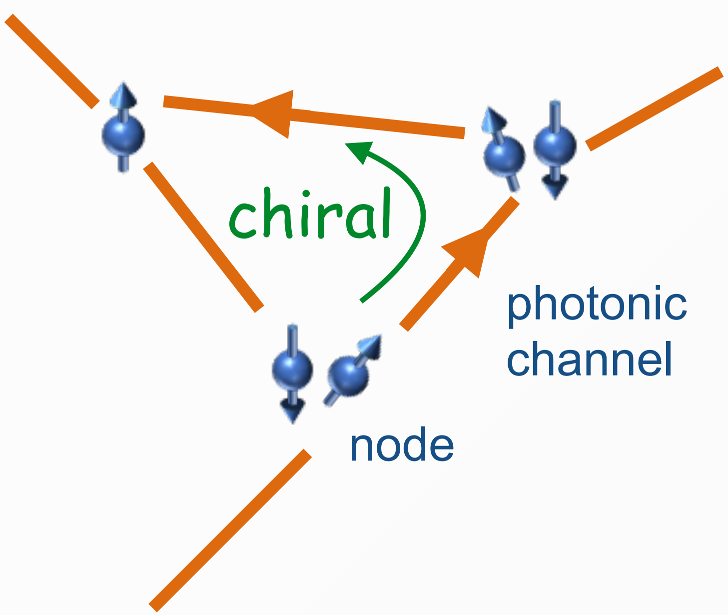
Example 2:

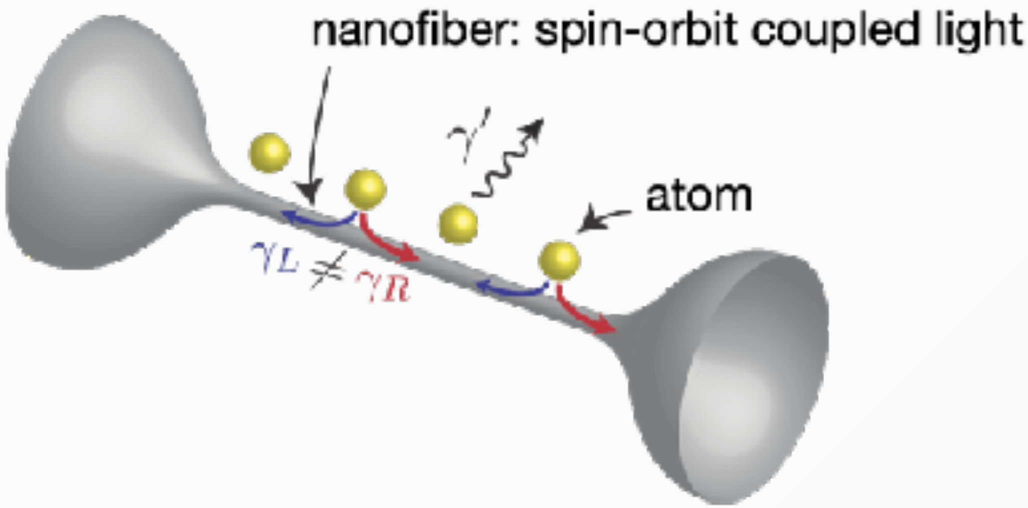
Entanglement by Dissipation [& Ion Experiment]

Example 1:

Chiral Quantum Optics

Theory: Cascaded Quantum Systems



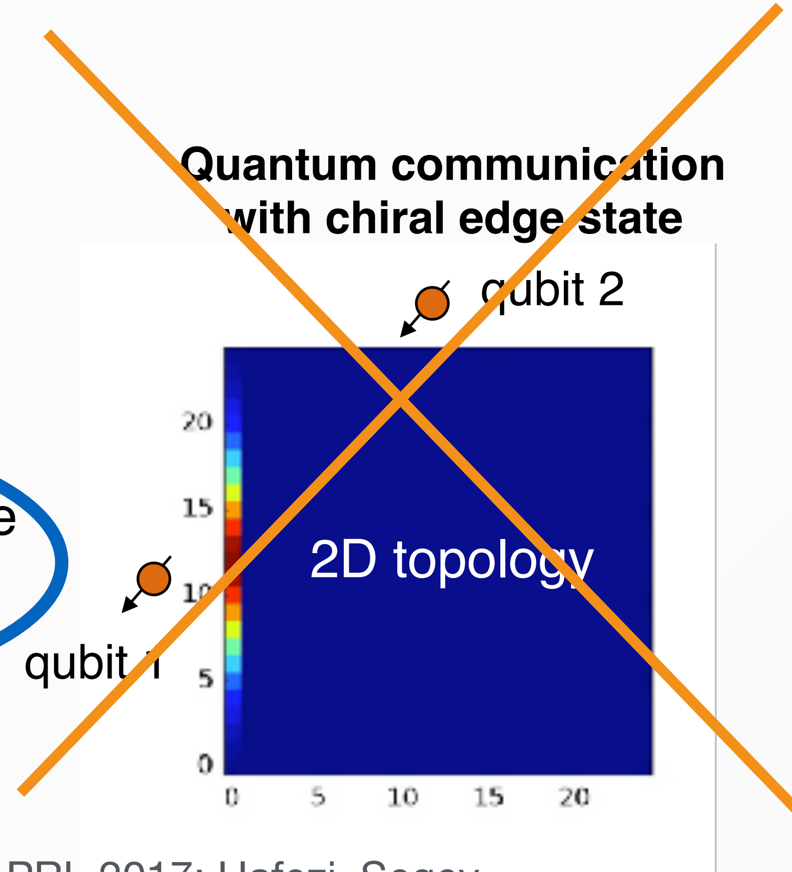


- *unidirectional* couplings appear naturally in *nanophotonic* devices

'Chiral' Quantum Optics & Nanophotonics

What is Chiral Quantum Optics?

- ✓ photonic nanostructure
- ✓ atoms, spin, …



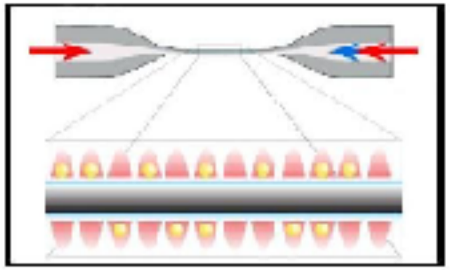
Ref.: topological quantum optics, Perczel, Lukin et al., PRL 2017; Hafezi, Segev, …

'Chiral' Quantum Optics & Nanophotonics

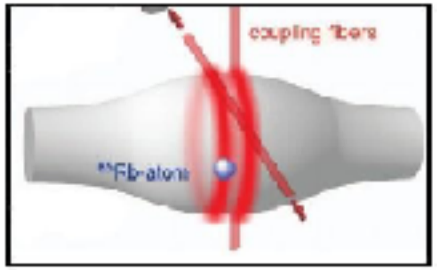
chiral coupling between light and quantum emitters

protected by *symmetry*,
not *topology*

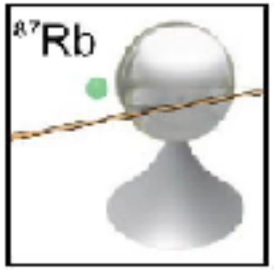
Nanophotonic devices: chirality appears naturally …

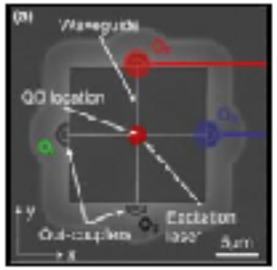


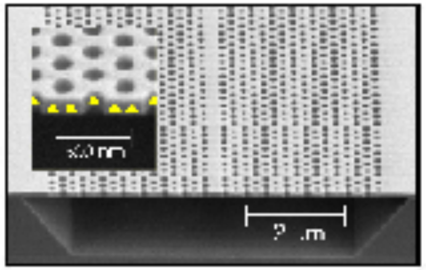
atoms & nanofibers



atoms & CQED







quantum dots &
photonic nanostructures

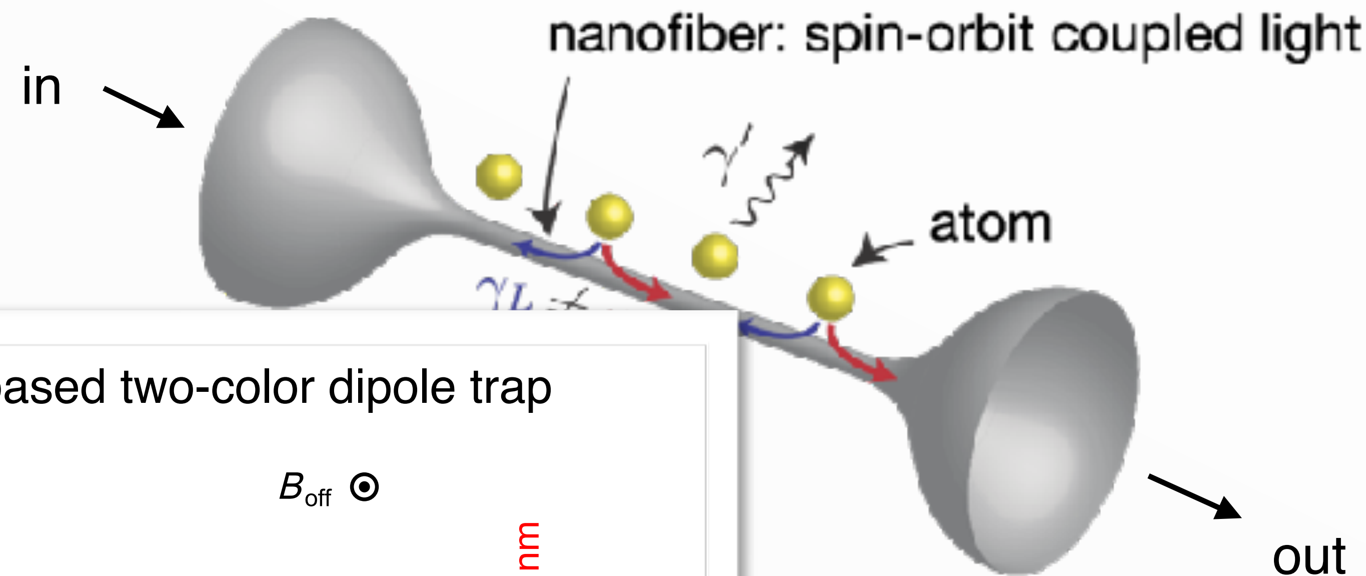
P. Lodahl, A. Rauschenbeutel, PZ et al., Nature Review 2017

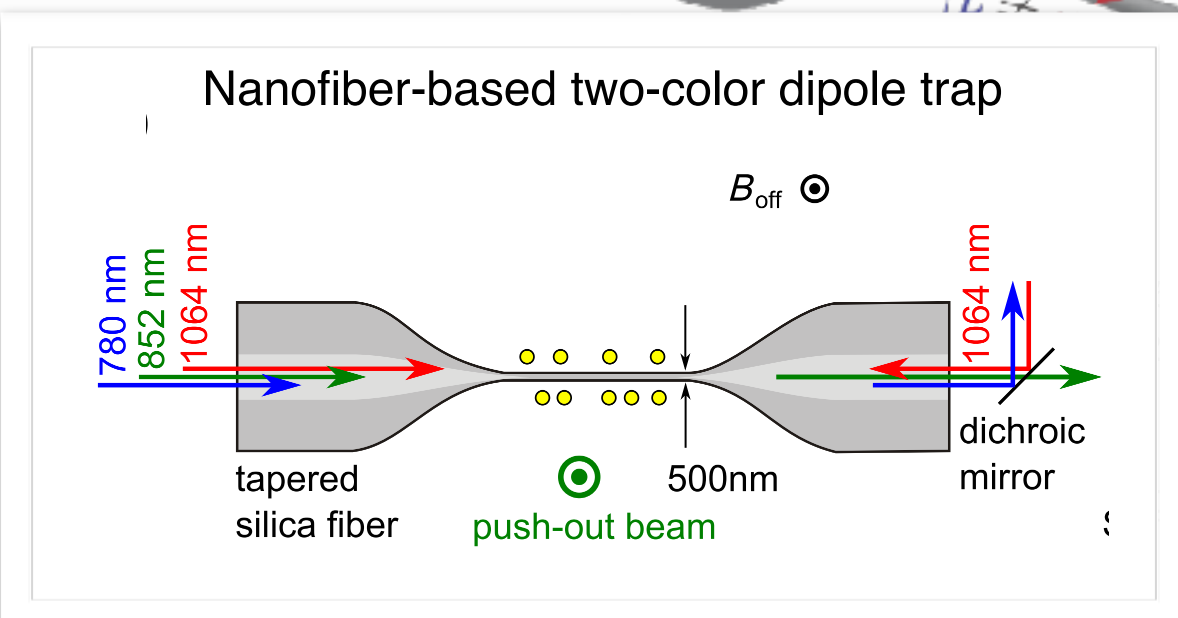
SCIENCE sciencemag.org 3 OCTOBER 2014 • VOL 346 ISSUE 6205

NANOPHOTONICS

Chiral nanophotonic waveguide interface based on spin-orbit interaction of light

Jan Petersen, Jürgen Volz,* Arno Rauschenbeutel*





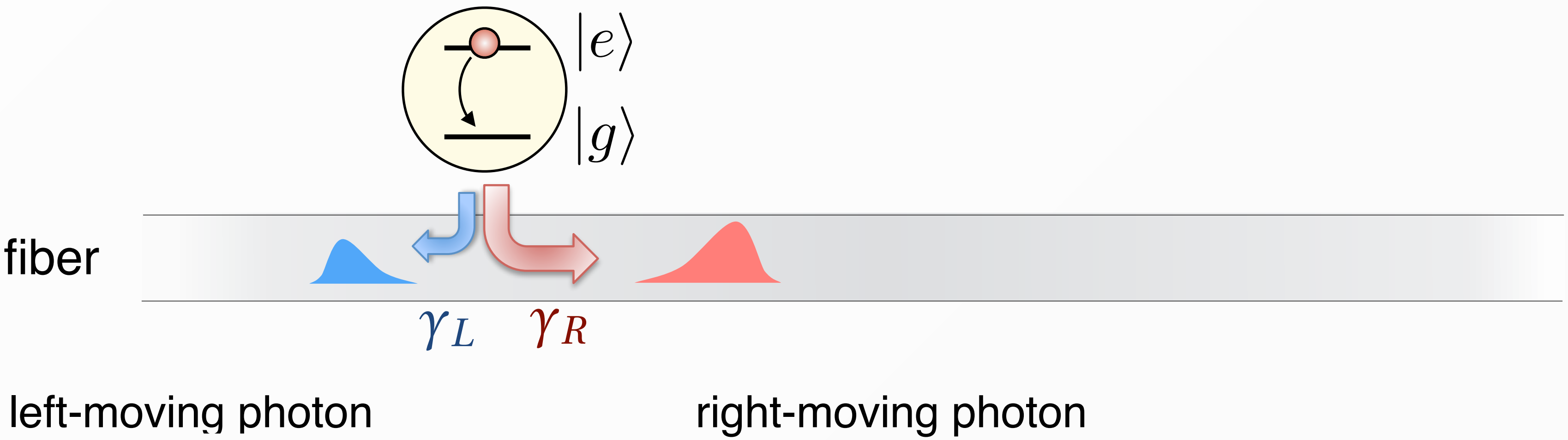
15)

Lukin-Vuletic-Park, Orozco, … + solid state

hotonic nanostructures

Chiral Quantum Optics

uibk



✓ 'chiral' atom-light interface:

broken left-right symmetry

$$\gamma_L \neq \gamma_R$$

Chiral Quantum Optics

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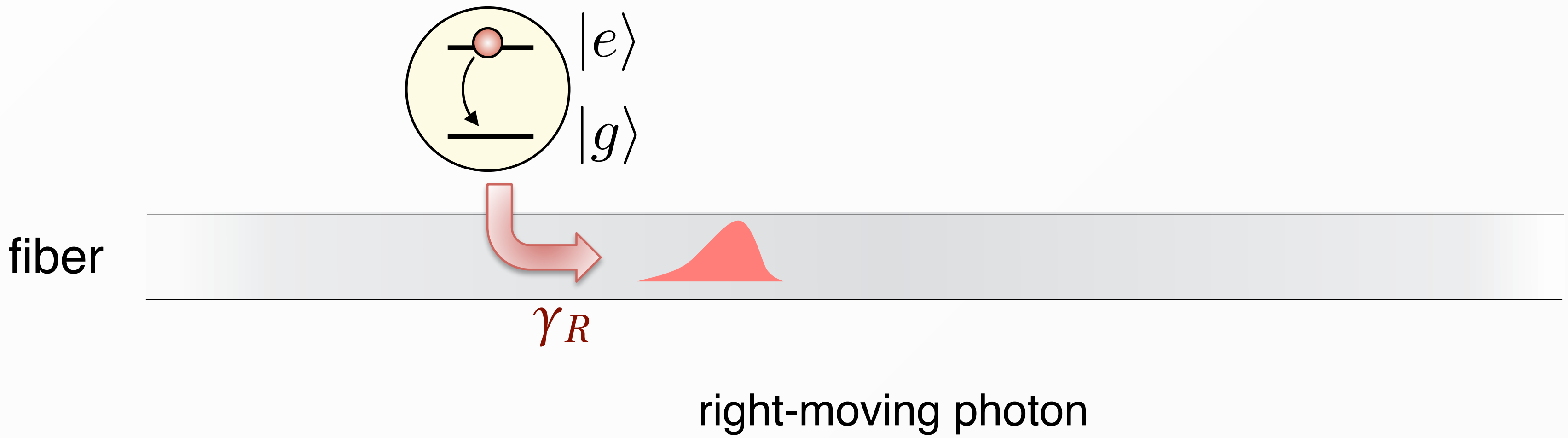

✓ 'chiral' atom-light interface:

broken left-right symmetry

$$\gamma_L = 0; \gamma_R$$

'chirality' ~ open quantum system

Chiral Photon-Mediated Interactions

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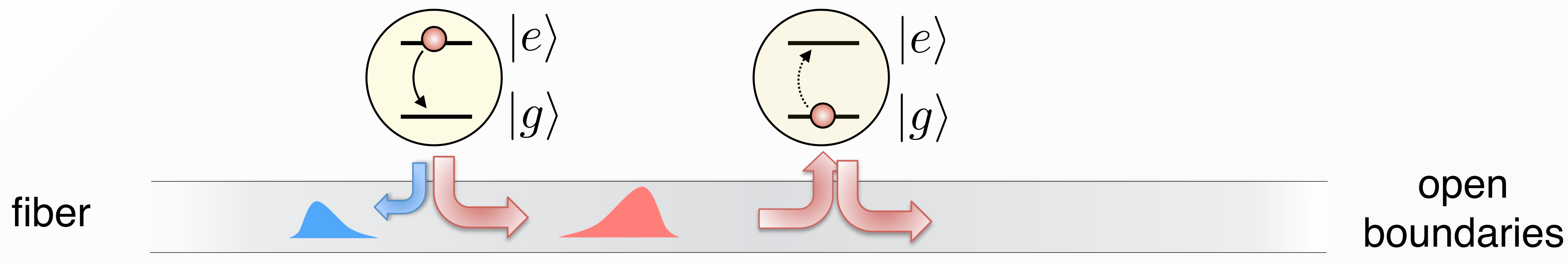


✓ 'chiral' interactions

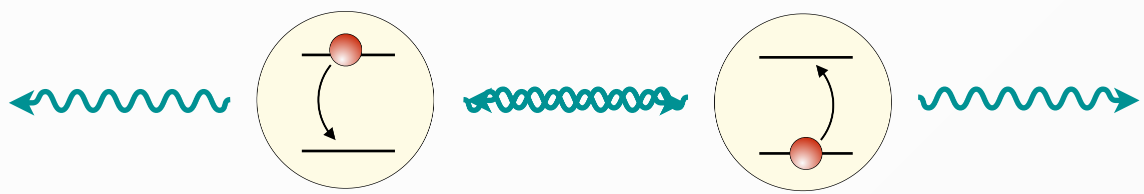
broken left-right symmetry

atoms only talk to atoms on the right

‘Chiral' Interactions … How to Model?

- **interactions mediated by photons**

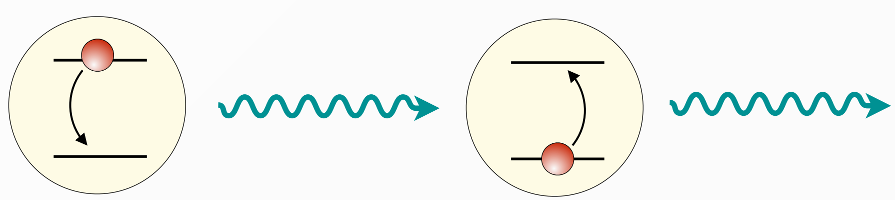
  - quantum optics we know



left - right symmetric

✓ dipole-dipole interaction $H \sim \sigma_1^- \sigma_2^+ + \sigma_1^+ \sigma_2^-$ by integrating out photons

  - chiral quantum optics

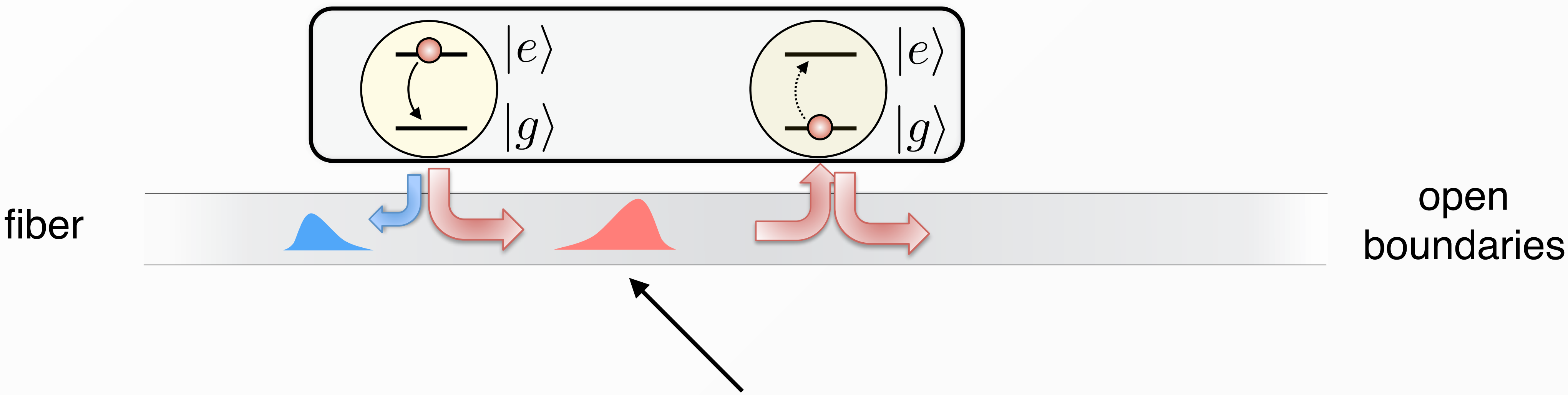


broken left - right symmetry

✓ unidirectional interaction $H \sim \sigma_1^- \sigma_2^+$?

Theory: ‘Cascaded Master equation’ = open quantum system

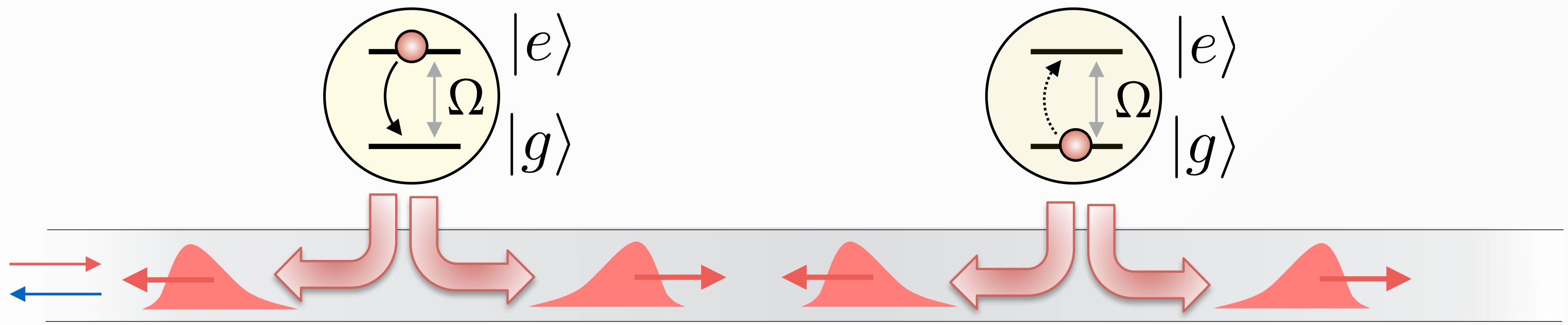
Theory - Master Equation



- We integrate the photons out as ‘quantum reservoir’ in Born-Markov approximation
- Master equation for reduced dynamics: density operator of atoms

$$\dot{\rho} = -\frac{i}{\hbar}\left[H_{\text{sys}}, \rho\right] + \mathscr{L}\rho$$

1. *Bidirectional* Master Equation



open boundaries

- **Master equation: symmetric**

driven atoms

1D dipole-dipole

$$\dot{\rho} \;=\; -i\,[H_{\mathrm{sys}} + \gamma \sin(k|x_1 - x_2|)(\sigma_1^+\sigma_2^- + \sigma_2^+\sigma_1^-), \rho]$$
$$+2\gamma \sum_{i,j=1,2} \cos(k|x_i - x_j|)(\sigma_i^-\rho\sigma_j^+ - \frac{1}{2}\{\sigma_i^+\sigma_j^-, \rho\}).$$

collective spontaneous emission

“Dicke" master equation for 1D: D E Chang et al 2012 New J. Phys. 14 063003

2. *Cascaded* Master Equation

uibk

open boundaries

- **Master equation: unidirectional**

$$\dot{\rho} = \mathscr{L}\rho \equiv -i(H_{\rm eff}\rho - \rho H_{\rm eff}^{\dagger}) + \sigma\rho\sigma^{\dagger}$$

Lindblad form

- non-Hermitian effective Hamiltonian

$$H_{\rm eff} = H_1 + H_2 - i\frac{\gamma}{2}\left(\sigma_1^+\sigma_1^- + \sigma_2^+\sigma_2^- + 2\sigma_2^+\sigma_1^-\right)$$

- quantum jump operator: collective

$$\sigma = \sigma_1^- + \sigma_2^-$$

C.W. Gardiner, PRL 1993;
H. Carmichael, PRL 1993

- **general case: N atoms, chiral**

positions of the atoms does not matter

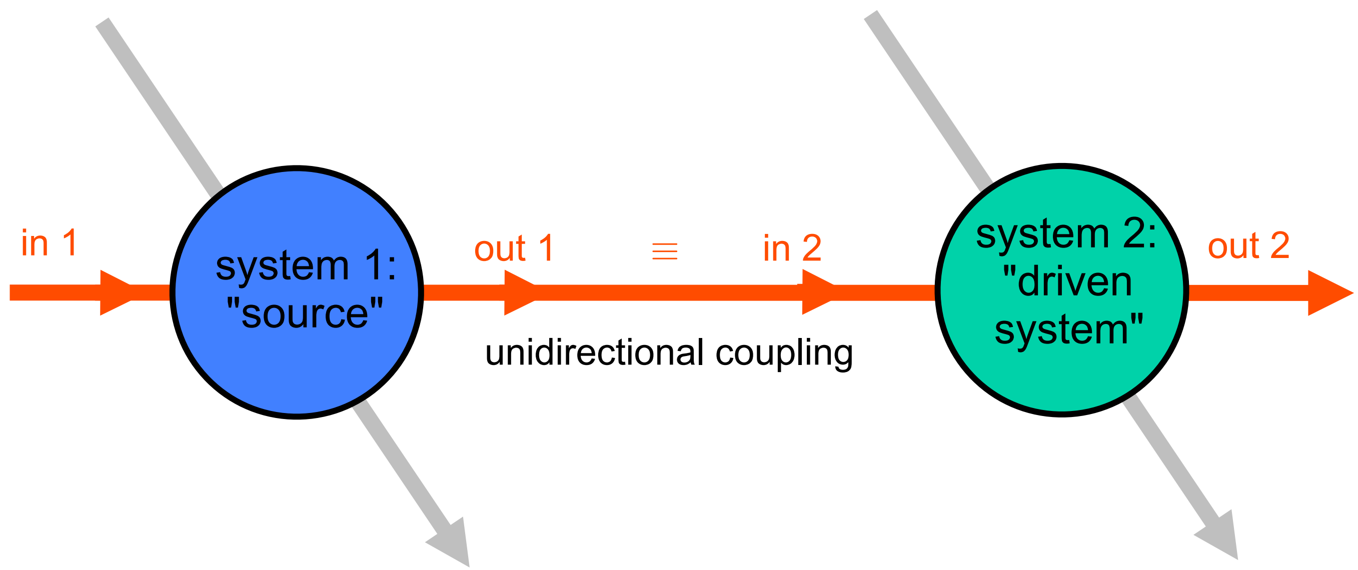
H. Pichler et al., PRA 2015

Theory Appendix

- QSSE for Cascaded / Chiral Systems
- Cascaded Master Equation

Cascaded Quantum Systems

Cascaded quantum systems: first system drives in a unidirectional coupling a second quantum system

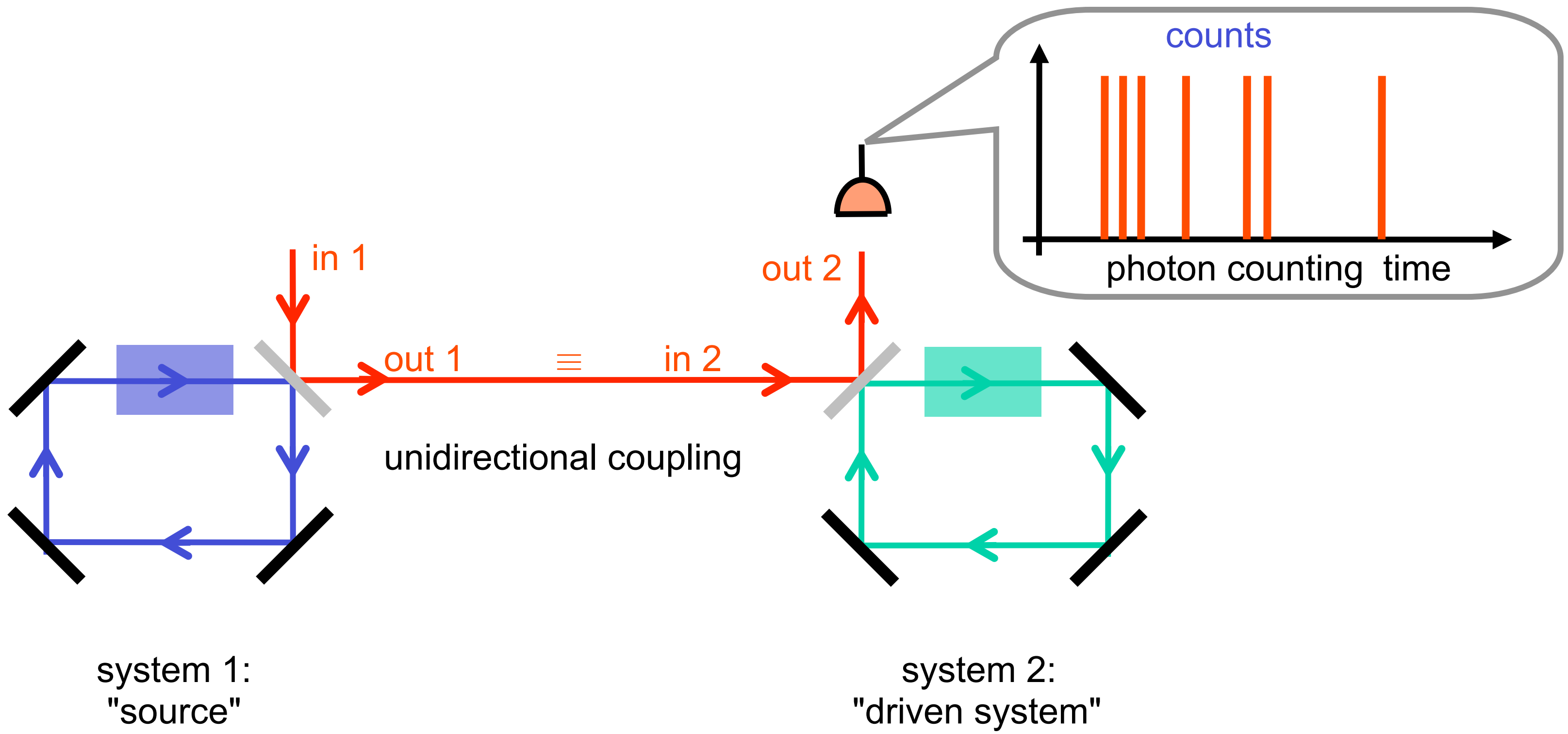


- Quantum Stochastic Schrödinger Equation *here*
- Master Equation

N=2 Cascaded Theory: C.W. Gardiner, PRL 1993; H. Carmichael, PRL 1993

Cascaded Quantum Systems

Example:



The Model

Hamiltonian

$$H = H_{\text{sys}}(1) + H_{\text{sys}}(2) + H_{\text{B}} + H_{\text{int}}$$

$$H_B = \int_{\omega_0-\vartheta}^{\omega_0+\vartheta} d\omega\, \omega\, b^\dagger(\omega) b(\omega)$$

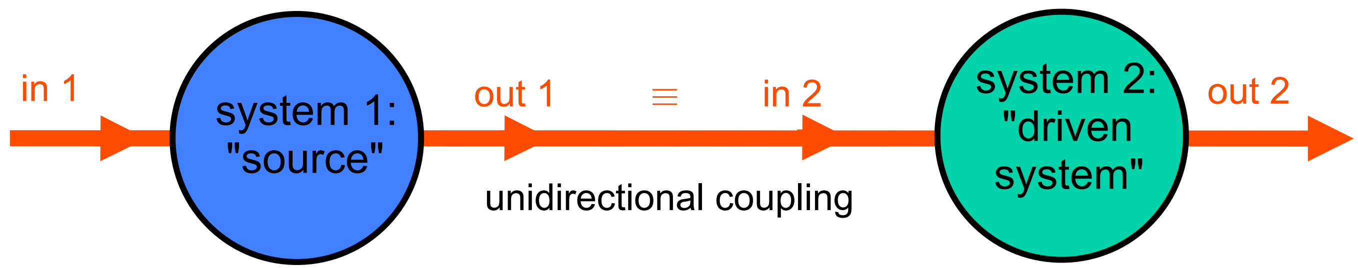
Interaction part

only right running modes

$$\begin{aligned} H_{\text{int}} = \quad & i\int d\omega \kappa_1(\omega) \left[b^\dagger(\omega) \underline{e^{-i\omega/c x_1}} c_1 - c_1^\dagger b(\omega) e^{+i\omega/c x_1} \right] \\ & + i\int d\omega \kappa_2(\omega) \left[b^\dagger(\omega) \underline{e^{-i\omega/c x_2}} c_2 - c_2^\dagger b(\omega) e^{+i\omega/c x_2} \right] \qquad (x_2 > x_1) \end{aligned}$$

unidirectional coupling

The Model



Interaction picture

time delay

$$H_{\text{int}}(t) = i\sqrt{\gamma_1}\left[b^\dagger(t)c_1 - b(t)c_1^\dagger\right] + i\sqrt{\gamma_2}\left[b^\dagger(t-\tau)c_2 - b(t-\tau)c_2^\dagger\right] \qquad (\tau \to 0^+)$$

where time ordering / delays reflects causality, and

$$b(t) = \frac{1}{\sqrt{2\pi}}\int_{-\vartheta}^{+\vartheta} d\omega\, b(\omega)\, e^{-\imath(\omega-\omega_0)t}$$

white noise operator

The Model

***Stratonovich* Quantum Stochastic Schrödinger Equation with time delays**

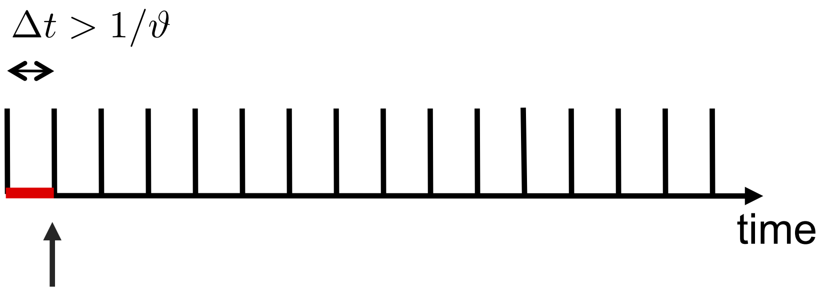
(S) $$\frac{d}{dt}|\Psi(t)\rangle = \left\{-i\left(H_{\rm sys}(1)+H_{\rm sys}(2)\right) + \sqrt{\gamma_1}\left[b^\dagger(t)c_1 - b(t)c_1^\dagger\right] + \sqrt{\gamma_2}\left[b^\dagger(t-\tau)c_2 - b(t-\tau)c_2^\dagger\right]\right\}|\Psi(t)\rangle \quad (\tau \to 0^+)$$

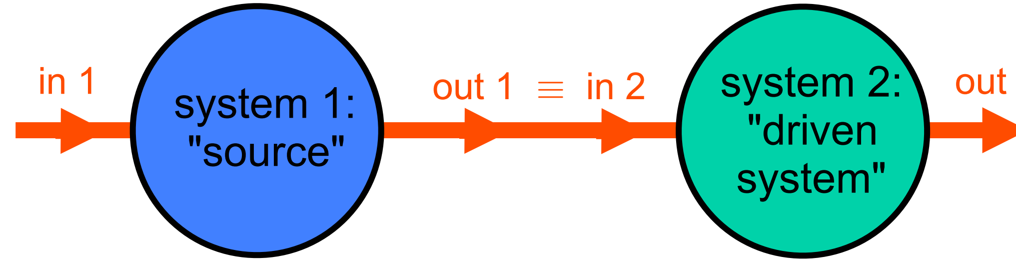
time delay

where time ordering / delays reflects causality

Scaling: $\sqrt{\gamma_i}\,c_i \to c_i$

Integrating the Schrödinger Eq.





First time step: (for time delay $\tau \to 0^+$)

$$|\Psi(\Delta t)\rangle = \Big\{ \hat{1} - iH_{\mathrm{sys}}(1)\Delta t + c_1 \int_0^{\Delta t} b^\dagger(t)\,dt - c_1^\dagger \int_0^{\Delta t} b(t)\,dt +$$

first system

$$- iH_{\mathrm{sys}}(2)\Delta t + c_2 \int_0^{\Delta t} b^\dagger(t^-)\,dt - c_2^\dagger \int_0^{\Delta t} b(t^-)\,dt$$

second system

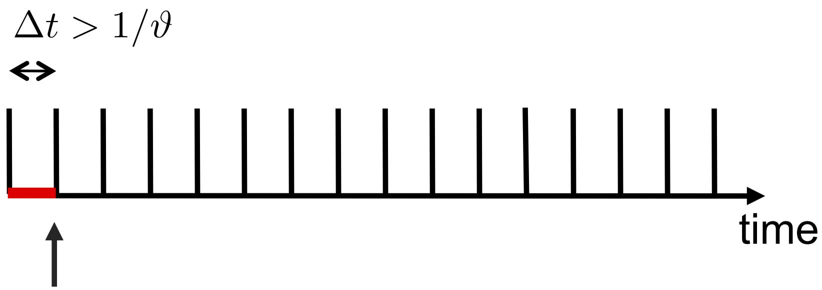
$$+ (-i)^2 \int_0^{\Delta t} dt_1 \int_0^{t_2} dt_2 \Big[\Big(-b(t_1)c_1^\dagger - b(t_1^-)c_2^\dagger\Big)\Big(b^\dagger(t_2)c_1 + b^\dagger(t_2^-)c_2\Big) + \dots\Big]\Big\}|\Psi(0)\rangle$$

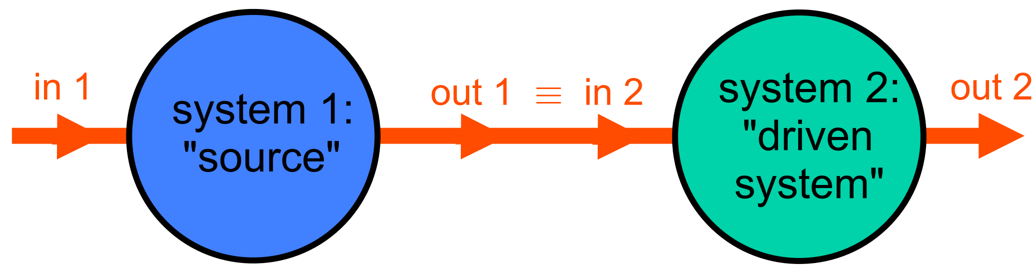
$$\left(-\frac{1}{2}c_1^\dagger c_1 + 0 - c_2^\dagger c_1 - \frac{1}{2}c_2^\dagger c_2\right)|\mathrm{vac}\rangle\Delta t$$

first system emits, second absorbs

causality & interaction

Integrating the Schrödinger Equation





First time step: (for time delay $\tau \to 0^+$)

quantum jump

$c = c_1 + c_2$

$$|\Psi(\Delta t)\rangle = \left\{ \hat{1} - i H_{\rm eff} \Delta t + \sqrt{\gamma}\, c\, \Delta B^\dagger(0) \right\} |\Psi(0)\rangle$$

- **effective (non-Hermitian) system Hamiltonian**

$$H_{\rm eff} = H_{\rm sys}(1) + H_{\rm sys}(2) - i\frac{1}{2} c_1^\dagger c_1 - i\frac{1}{2} c_2^\dagger c_2 - i c_2^\dagger c_1$$

coherent interaction: asymmetric

$$= \left\{ H_{\rm sys}(1) + H_{\rm sys}(2) + i\frac{1}{2}\left(c_1^\dagger c_2 - c_2^\dagger c_1\right) \right\} - i\frac{1}{2}(c_1^\dagger + c_2^\dagger)(c_1 + c_2)$$

quantum jump

… and more steps (as before in Lecture 2)

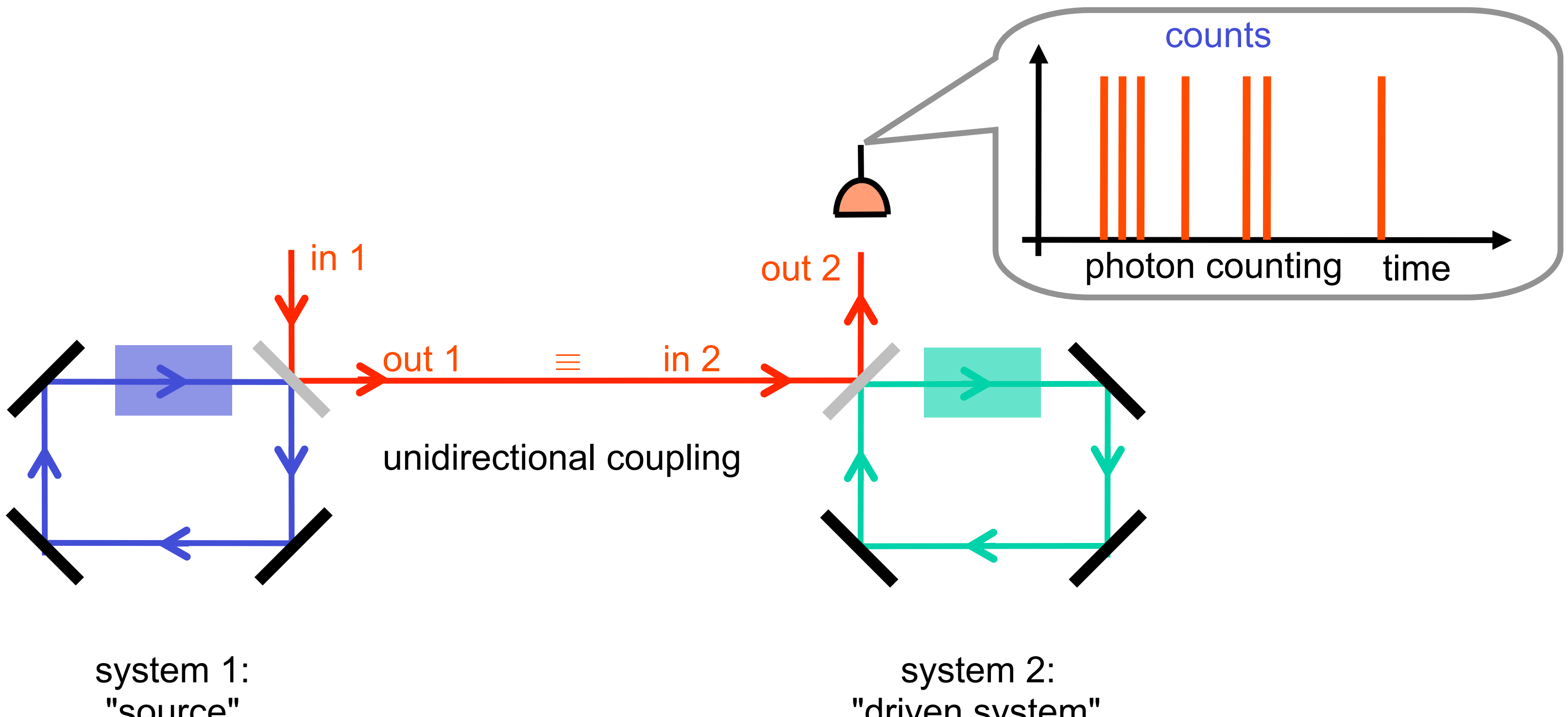
Cascaded Systems

Master Equation for Cascaded Quantum Systems

Version 1:

$$\frac{d}{dt}\rho = -i[H_{\text{sys}},\rho] + \frac{1}{2}\sum_{i=1}^{2}\left\{2c_i\rho c_i^\dagger - \rho c_i^\dagger c_i - c_i^\dagger c_i\rho\right\}$$
$$-\left\{[c_2^\dagger, c_1\rho] + [\rho c_1^\dagger, c_2]\right\}$$

asymmetric in 1 and 2



Cascaded Systems

Master Equation for Cascaded Quantum Systems

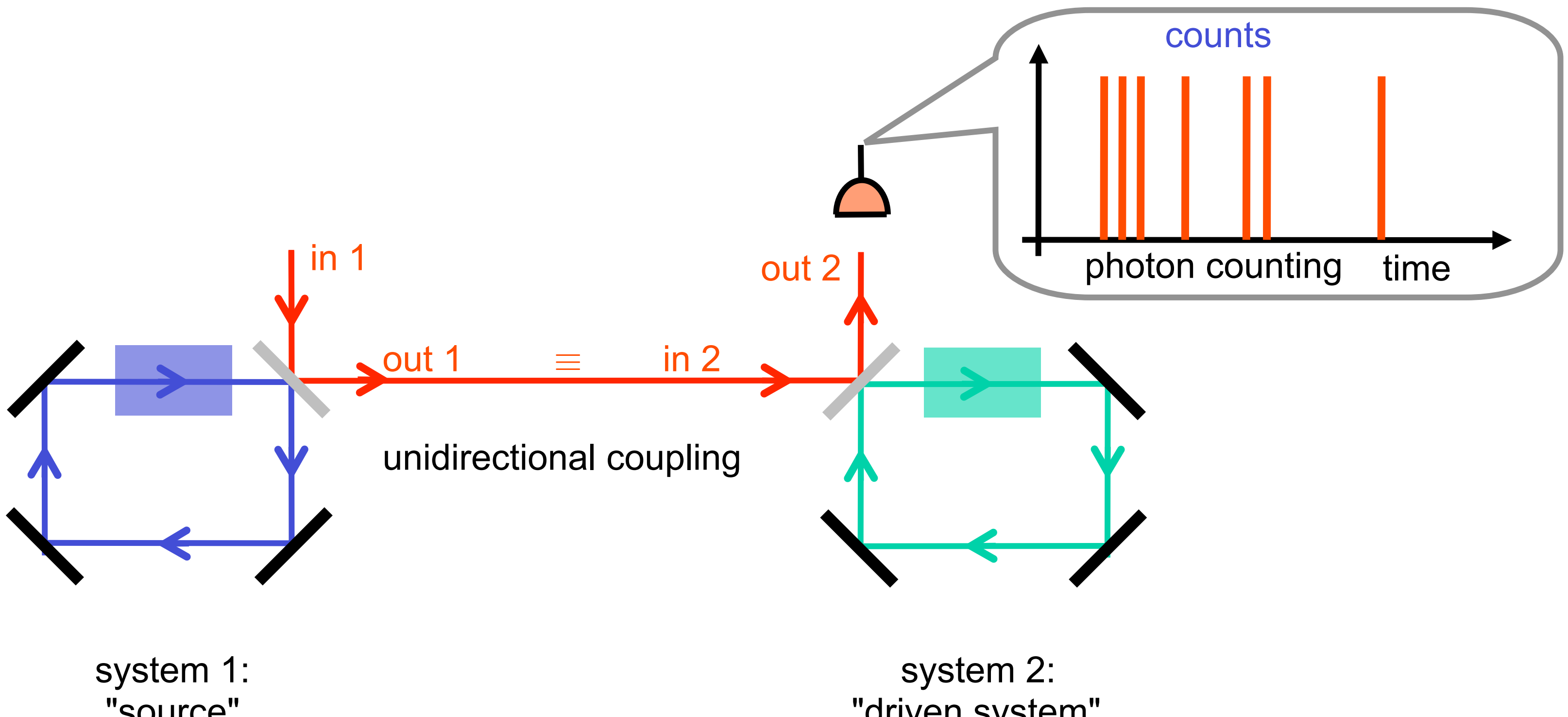
Version 2: Lindblad form

$$\frac{d}{dt}\rho = -i\left(H_{\text{eff}}\rho - \rho H_{\text{eff}}^{\dagger}\right) + \;\; c\rho c^{\dagger}$$

with jump operator $c \equiv c_1 + c_2$ and

coherent interaction

$$H_{\text{eff}} = H_{\text{sys}} + i\frac{1}{2}\left(c_1^{\dagger}c_2 - c_2^{\dagger}c_1\right) - i\frac{1}{2}c^{\dagger}c$$



End of Theory Appendix

Applications of *Chiral Atom-Light Interfaces*

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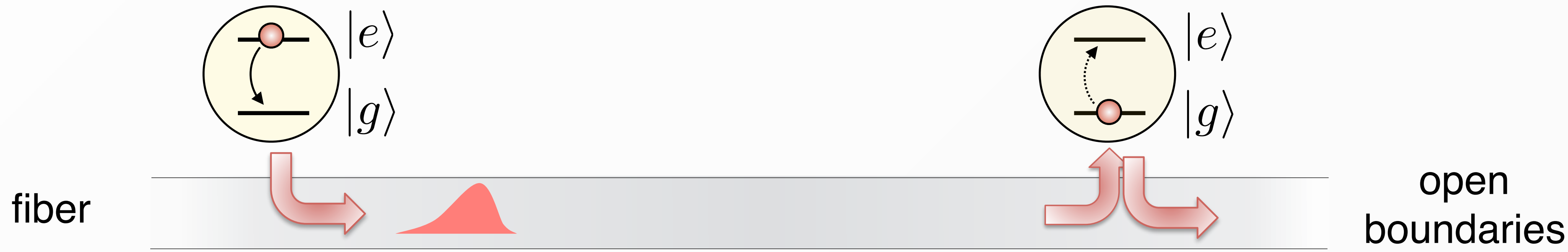
1. Quantum Information: *Chiral* Quantum Networks



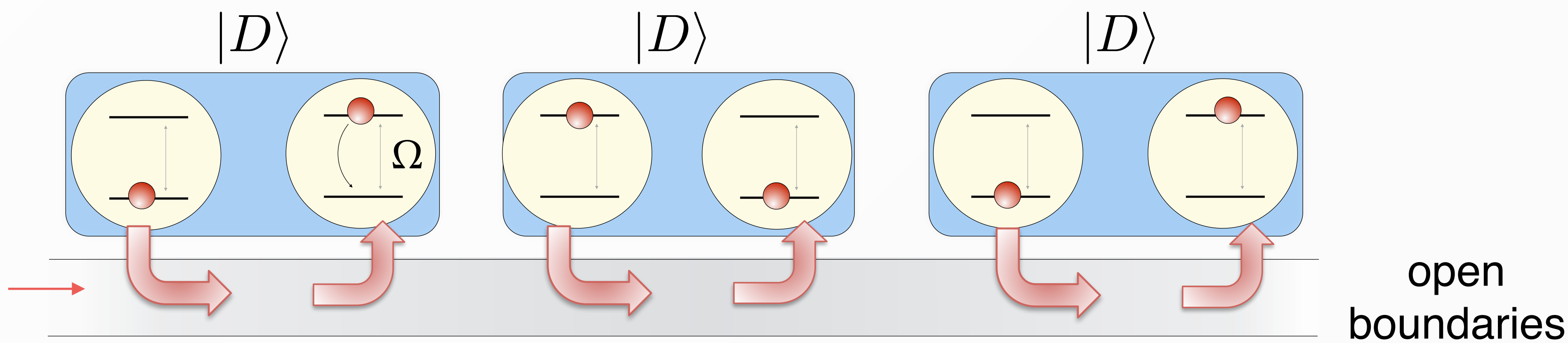
quantum state transfer protocol

$$\underbrace{\left(\alpha|g\rangle_1+\beta|e\rangle_1\right)}_{\text{qubit 1}}|0\rangle_p|g\rangle_1 \rightarrow |g\rangle_1\underbrace{\left(\alpha|0\rangle_p+\beta|1\rangle_p\right)}_{\text{wavepacket in waveguide}}|g\rangle_1 \rightarrow |g\rangle_1|0\rangle_p\underbrace{\left(\alpha|g\rangle_2+\beta|e\rangle_2\right)}_{\text{qubit 2}}$$

… with chiral coupling in principle perfect state transfer

theory: Cirac et al. 1997; Rabl et al. PRX 2017, Vermersch et al., PRL 2017, Gorshkov et al., PRA 2017
exp: Ritter, Rempe et al, Nature 2012; Schoelkopf et al, 2017; Wallraff et al. 2018

2. Driven-Dissipative Many-Body Quantum Systems



- **Unique, pure steady state:**

$$\rho(t) \xrightarrow{t\to\infty} |\Psi\rangle\langle\Psi|.$$

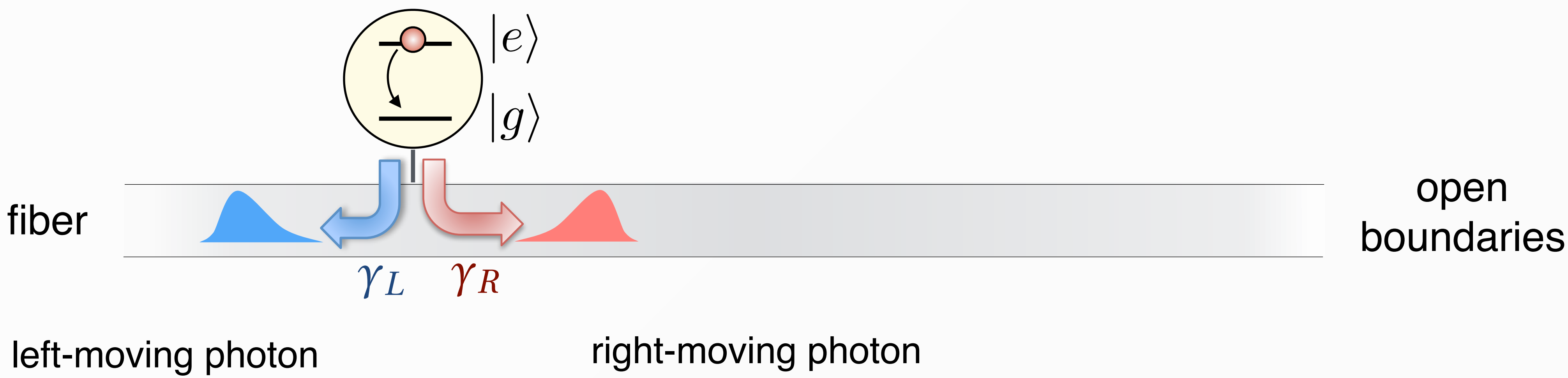
Entanglement by dissipation / non-equilibrium quantum phases

product of pure quantum spin-dimers/EPR $|\Psi\rangle = \bigotimes_{i=1}^{N} |D\rangle_{2i-1,2i}$

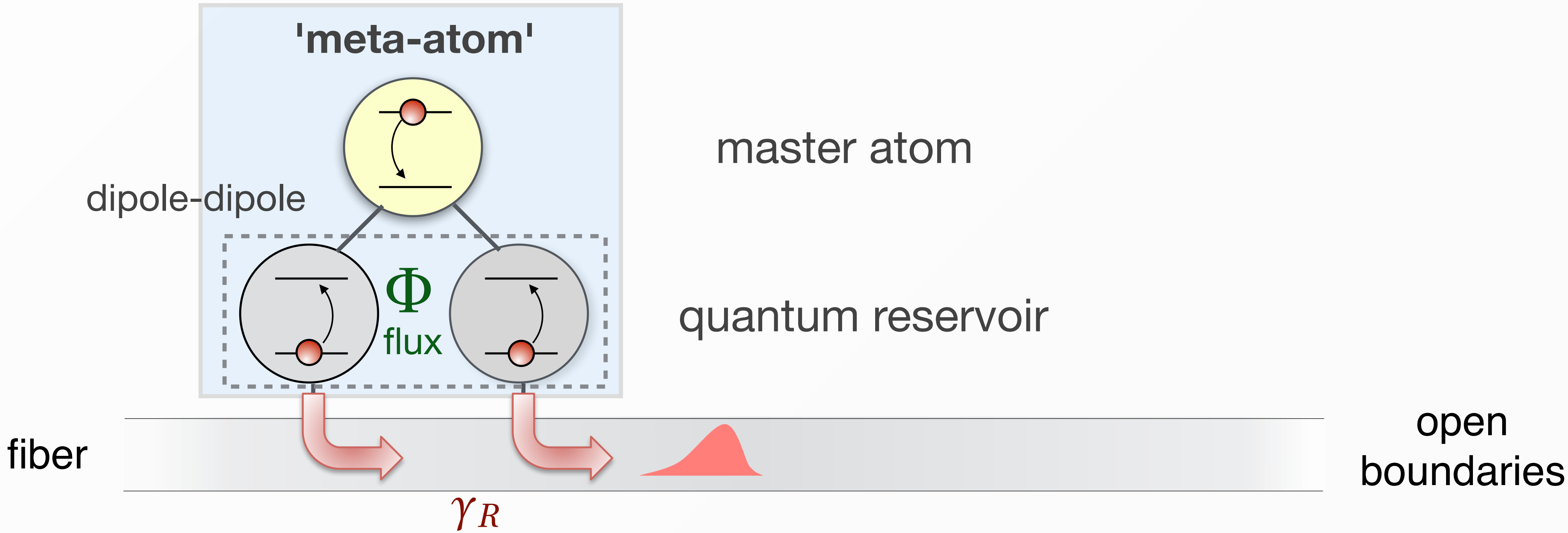
T. Ramos, H. Pichler, A.J. Daley, B Vermersch, P. Hauke, P.O. Guimond, K. Stannigel, P. Rabl, PZ, PRL 2014, PRA 2015, PRA 2017, PRL 2017 - see also: A Gorkov, D Chang, …

Engineering Chiral Coupling (1): nano-photonics

spin-orbit coupling in nano-photonics



Engineering Chiral Coupling (2): synthetic gauge field



two emitters: constructive / destructive interference

… could be implemented ‘as is’ with superconducting qubits / microwave

Can we do this in ‘free-space’ / no waveguide?

✓ 1D

✓ chiral

Simulating open system dynamics with spin waves

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two system spins with smooth absorbing boundary

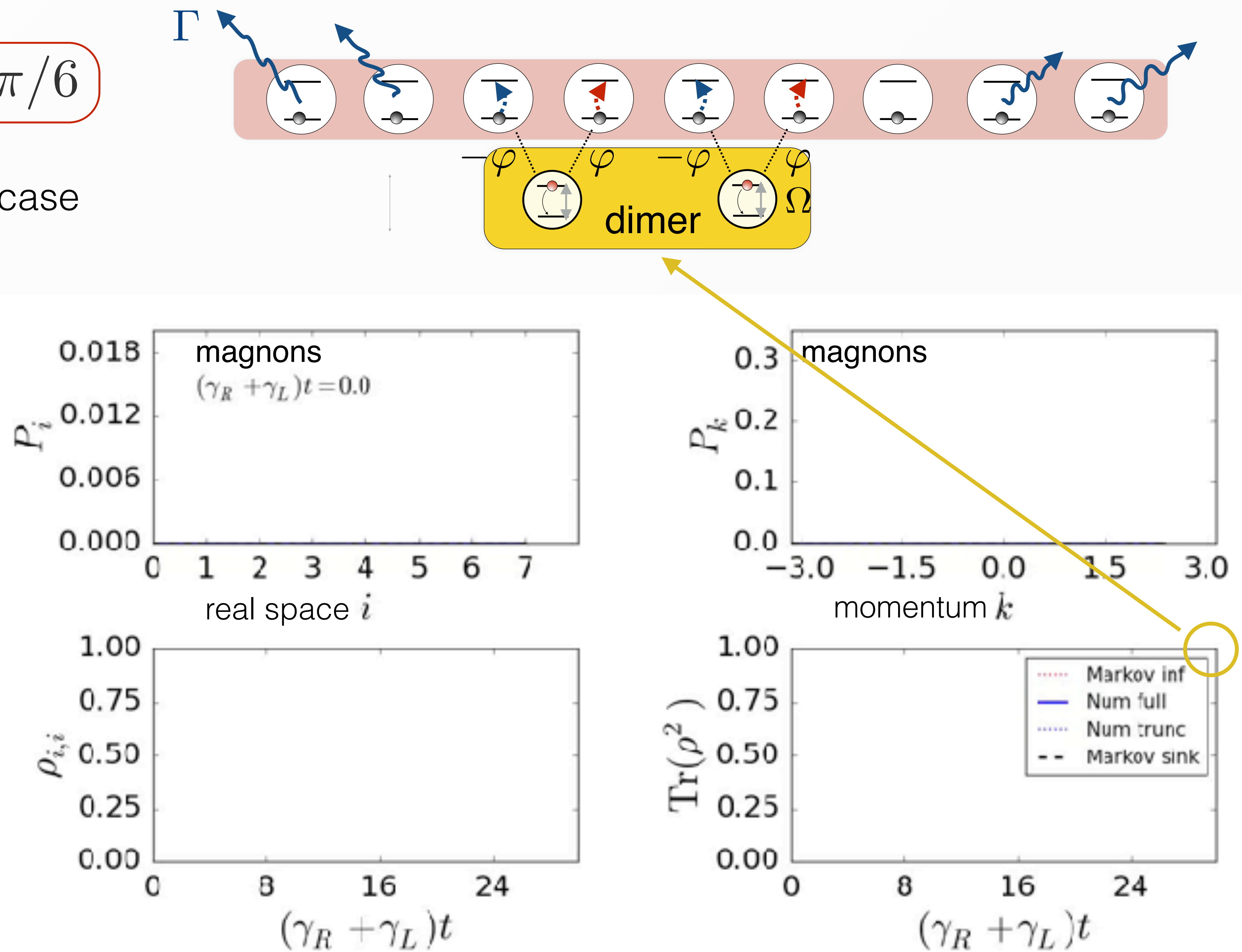
$\varphi = \pi/6$

↓

chiral case

reservoir spins:

system: spins



Open Quantum *Many-Body* Systems

Example 1:

Chiral Quantum Optics

→ **Example 2:**

Entanglement by Dissipation [& Ion Experiment]

Entanglement by [Engineered] Dissipation

theory:

Reviews

- M. Müller, S. Diehl, G. Pupillo, and P. Zoller, *Engineered Open Systems and Quantum Simulations with Atoms and Ions*, Advances in Atomic, Molecular, and Optical Physics (2012)

- C.-E. Bardyn, M. A. Baranov, C. V. Kraus, E. Rico, A. Imamoglu, P. Zoller, S. Diehl, *Topology by dissipation*, arXiv:1302.5135

experiments:

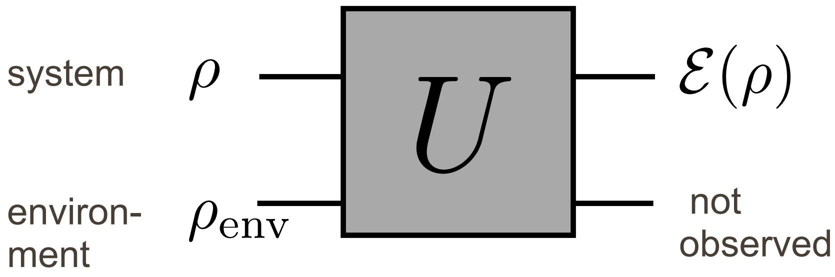
J. Barreiro, M. Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller & R. Blatt Nature 470, 486 (2011)

P. Schindler, M. Müller, D. Nigg, J. T. Barreiro, E. A. Martinez, M. Hennrich, T. Monz, S. Diehl, P. Zoller, and R. Blatt, Nat. Phys. 9, 1 (2013).

Krauter et al., Polzik & Cirac, PRL 2011 -- atomic ensembles

Open System Dynamics [& Decoherence ☹]

- **open system dynamics**



completely positive maps:

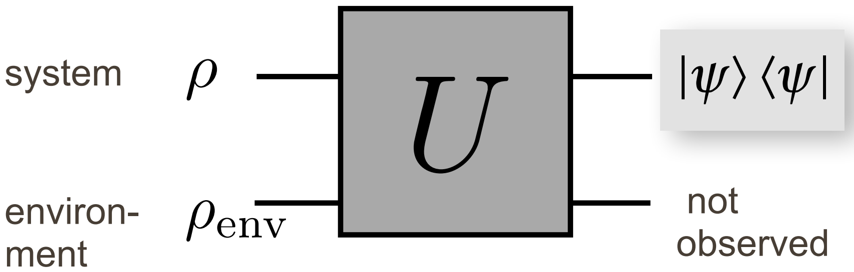
$$\rho \to \mathscr{E}(\rho) = \sum_k E_k \rho E_k^\dagger$$

Kraus operator

quantum control theory: open-loop [vs. closed loop = measurement + feedback]

Entanglement from (Engineered) Dissipation

- **open system dynamics**



engineering Kraus operators:

$$\rho \rightarrow \mathcal{E}(\rho) = \sum_k E_k \rho E_k^\dagger$$

$$\stackrel{!}{=} |\psi\rangle\langle\psi|$$

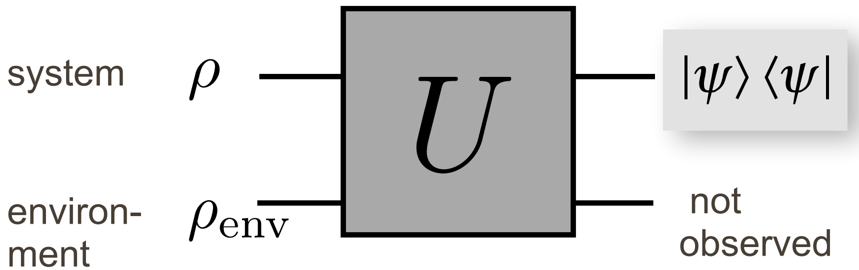
desired (pure)
quantum state

"cooling" into a pure state

- non-unitary
- deterministic

Entanglement from (Engineered) Dissipation

- **open system dynamics**



engineering Kraus operators:

$$\rho \rightarrow \mathcal{E}(\rho) = \sum_k E_k \rho E_k^\dagger$$

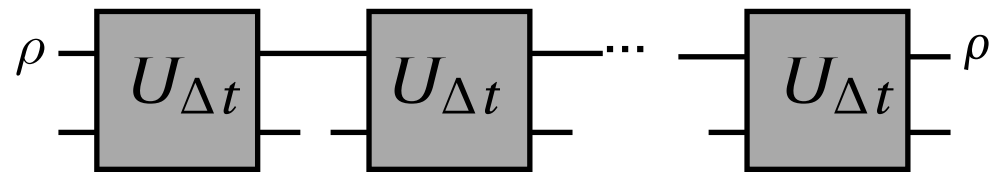
$$\stackrel{!}{=} |\psi\rangle\langle\psi|$$

desired (pure) quantum state

"cooling" into a pure state

- non-unitary
- deterministic

- **Markovian**



master equation:

$$\begin{aligned}\dot{\rho} &= -i[H,\rho] \\ &+ \sum_\alpha \gamma_\alpha \left(c_\alpha \rho c_\alpha^\dagger - \frac{1}{2} c_\alpha^\dagger c_\alpha \rho - \rho \frac{1}{2} c_\alpha^\dagger c_\alpha \right)\end{aligned}$$

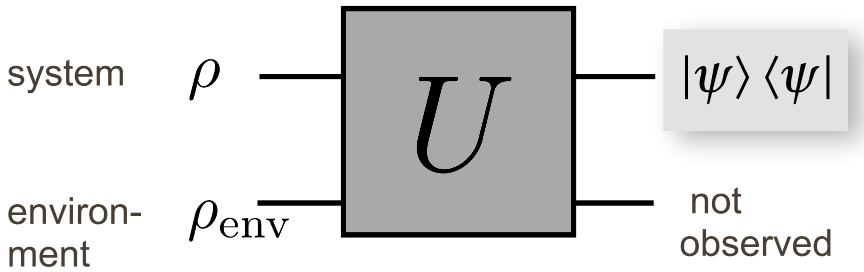
quantum jump operators

$$\rho(t) \xrightarrow{t\rightarrow\infty} |\psi\rangle\langle\psi|$$

pumping into a pure "dark state"

Entanglement from (Engineered) Dissipation

- **open system dynamics**



engineering Kraus operators:

$$\rho \rightarrow \mathcal{E}(\rho) = \sum_k E_k \rho E_k^\dagger \stackrel{!}{=} |\psi\rangle\langle\psi|$$

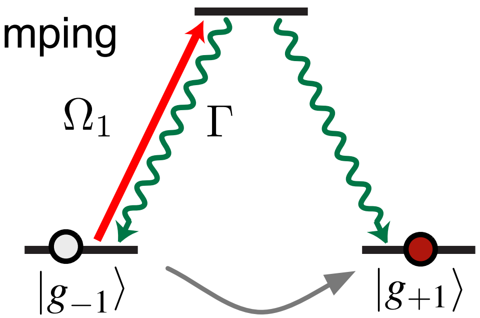
desired (pure)
quantum state

"cooling" into a pure state

- non-unitary
- deterministic

- **atomic physics: single particle**

optical pumping



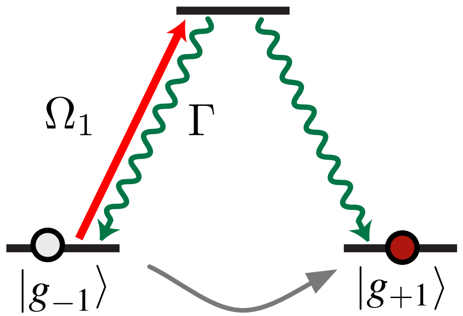
$$\rho(t) \xrightarrow{t\to\infty} |D\rangle\langle D|$$

pumping into a pure "dark state"

Q.: generalize to entangled states?

Dark States: Single Particle

- **optical pumping**



$$\rho(t) \xrightarrow{t\to\infty} |g_{+1}\rangle\langle g_{+1}|$$

pumping into a *pure* "dark state"

- **Optical Bloch Equations**

$$\dot{\rho} = -i[H,\rho] + \sum_\alpha \gamma_\alpha \left(c_\alpha \rho c_\alpha^\dagger - \frac{1}{2} c_\alpha^\dagger c_\alpha \rho - \rho \frac{1}{2} c_\alpha^\dagger c_\alpha \right)$$

quantum jump operator (nonhermitian)

- **steady state as a pure "dark state"**

$$H|D\rangle = E|D\rangle$$
$$\forall\alpha \quad c_\alpha|D\rangle = 0$$

conditions

$$\rho(t) \xrightarrow{t\to\infty} |D\rangle\langle D|$$

pumping into a pure state

Example: Bell state or stabilizer pumping

- concepts
- … and an ion trap experiment

Bell State Pumping

two spins / qubits

- **Bell States**

		$Z_1 Z_2$: $+1$	$Z_1 Z_2$: -1
$X_1 X_2$	$+1$	Φ^+	Ψ^+
	-1	Φ^-	Ψ^-

Bell states as eigenstates of (commuting) **stabilizer** operators $X_1 X_2$ and $Z_1 Z_2$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Bell State Pumping

two spins / qubits

- **Bell States**

		$Z_1 Z_2$ $+1$	$Z_1 Z_2$ -1
$X_1 X_2$	$+1$	Φ^+	Ψ^+
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$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

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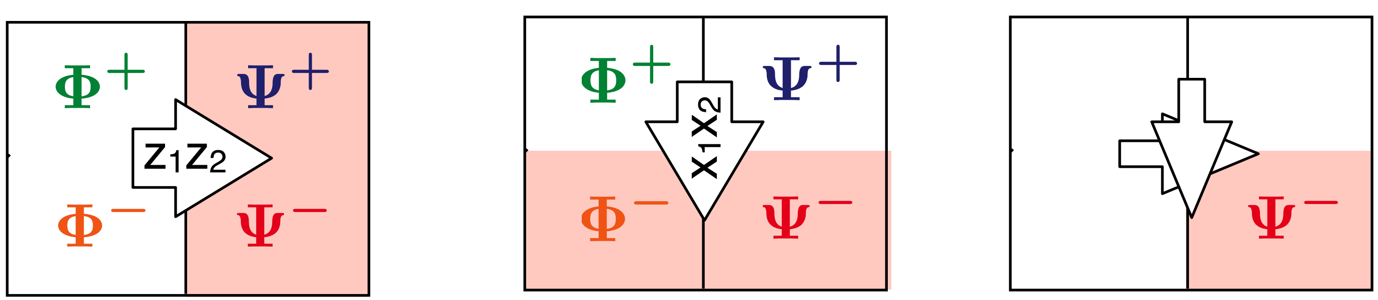
$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Bell states as eigenstates of (commuting) **stabilizer** operators $X_1 X_2$ and $Z_1 Z_2$

- **Goal: Bell state pumping** $\rho(t) \longrightarrow |\Psi^-\rangle\langle\Psi^-|$



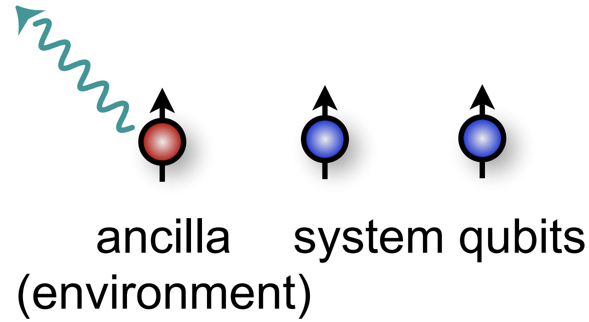
quantum jump operators

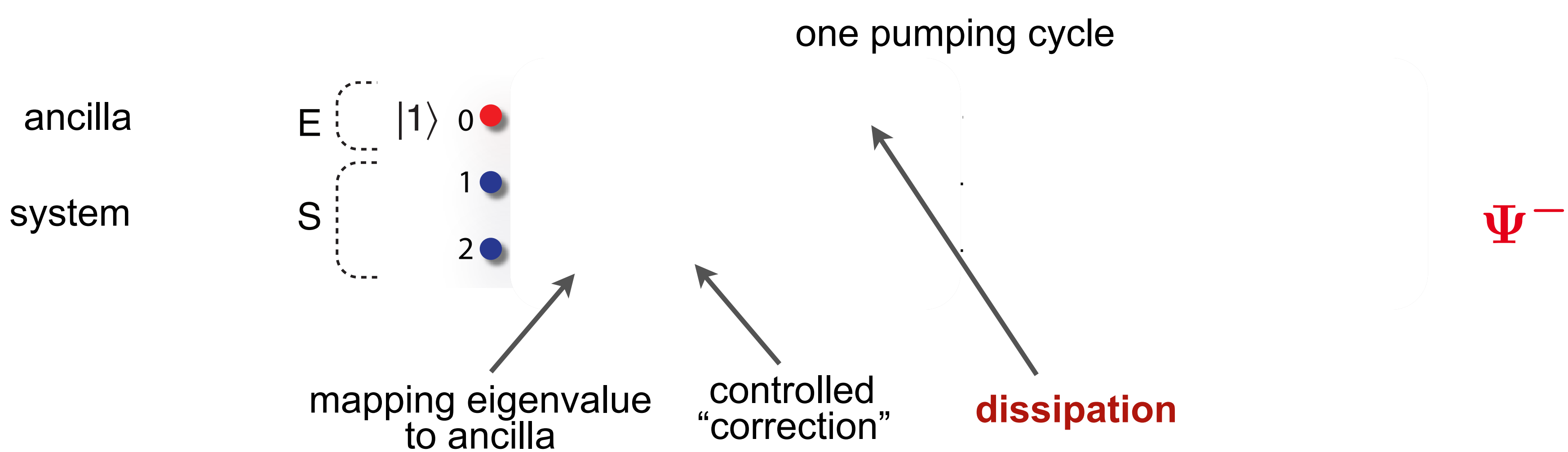
$c_1 = X_1(1 + Z_1 Z_2)$

$c_2 = Z_1(1 + X_1 X_2)$ ⟵ 2-particle operators ☹

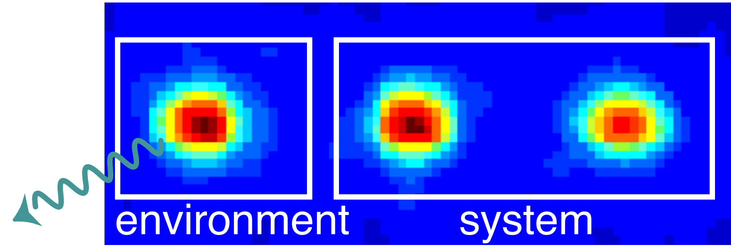
Bell State Pumping

- **quantum circuit**

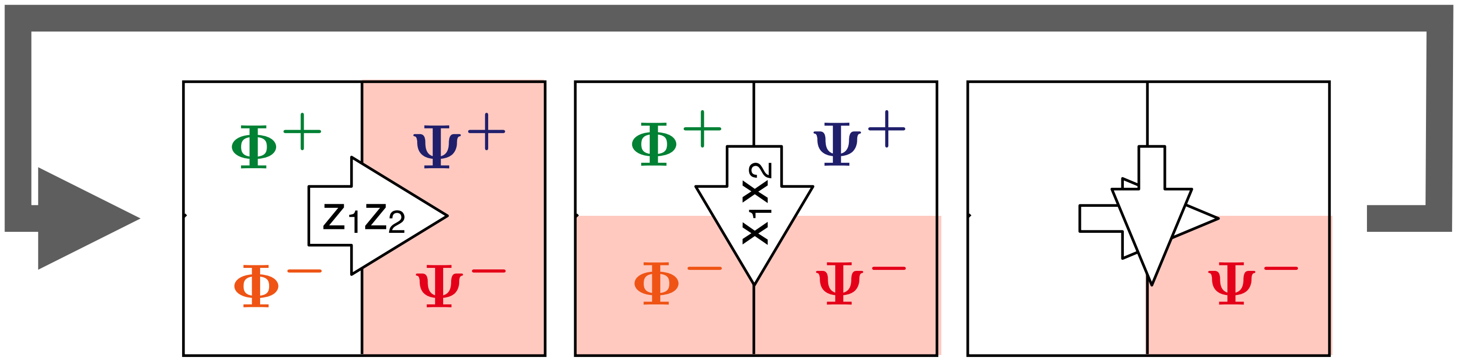


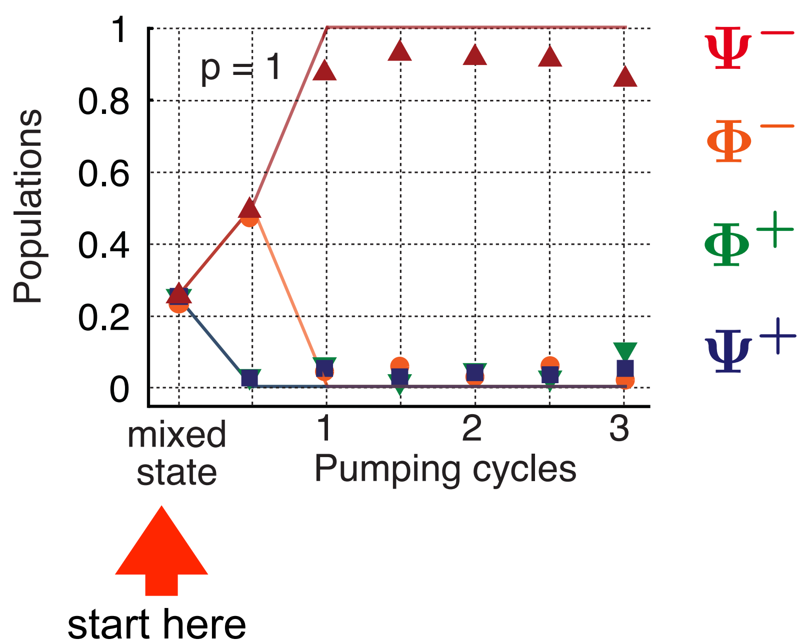


Bell State Pumping: Ion Experiment



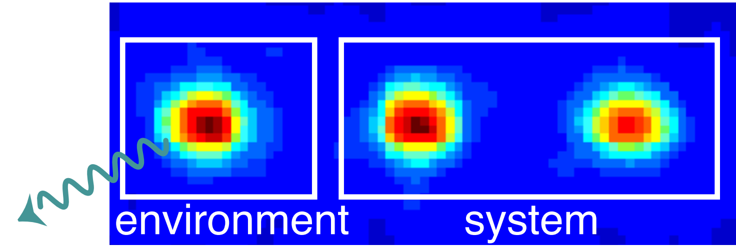
- **two step deterministic pumping**



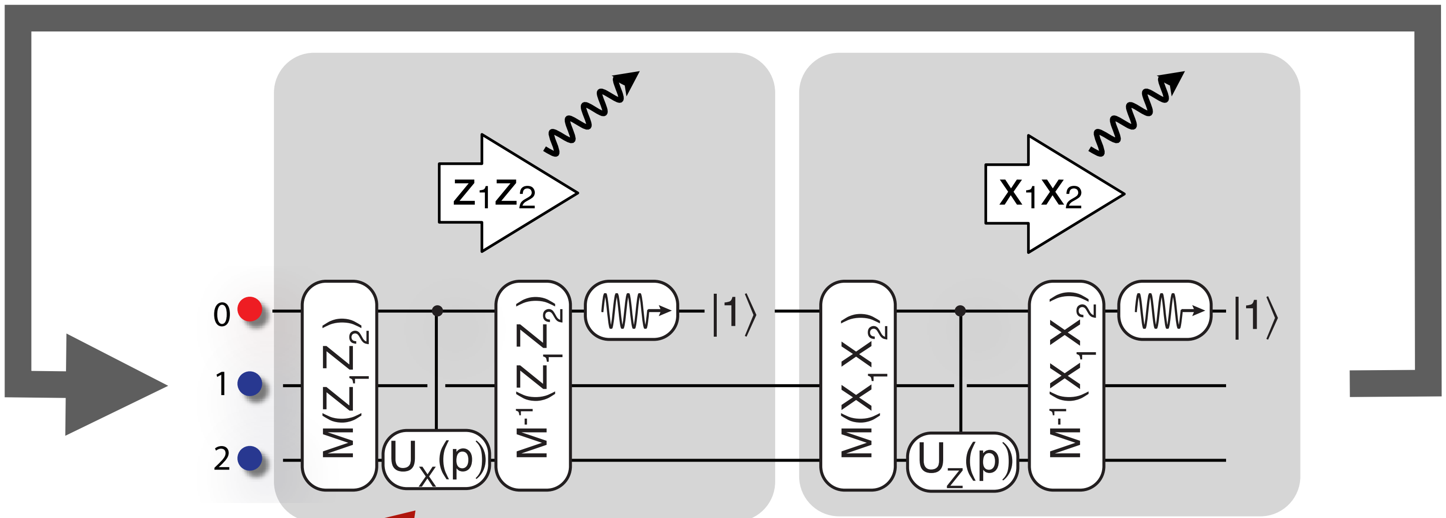


J. Barreiro, M.Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller and R. Blatt, Nature 2011

Bell State Pumping: Ion Experiment

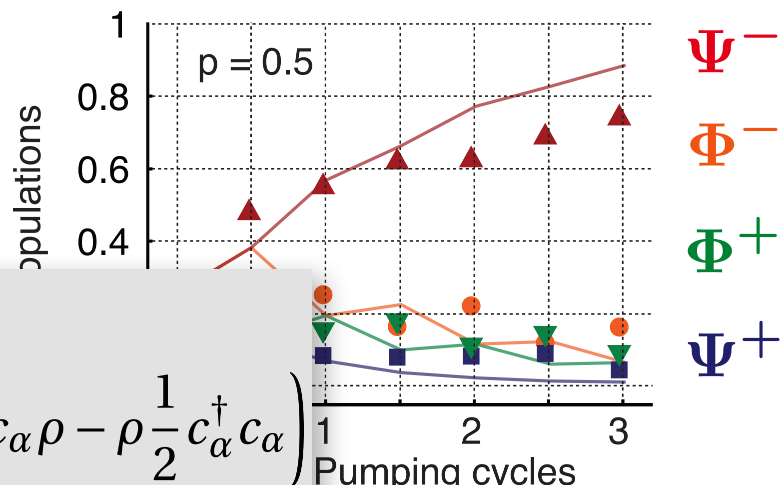


- **master equation limit:** probabilistic pumping



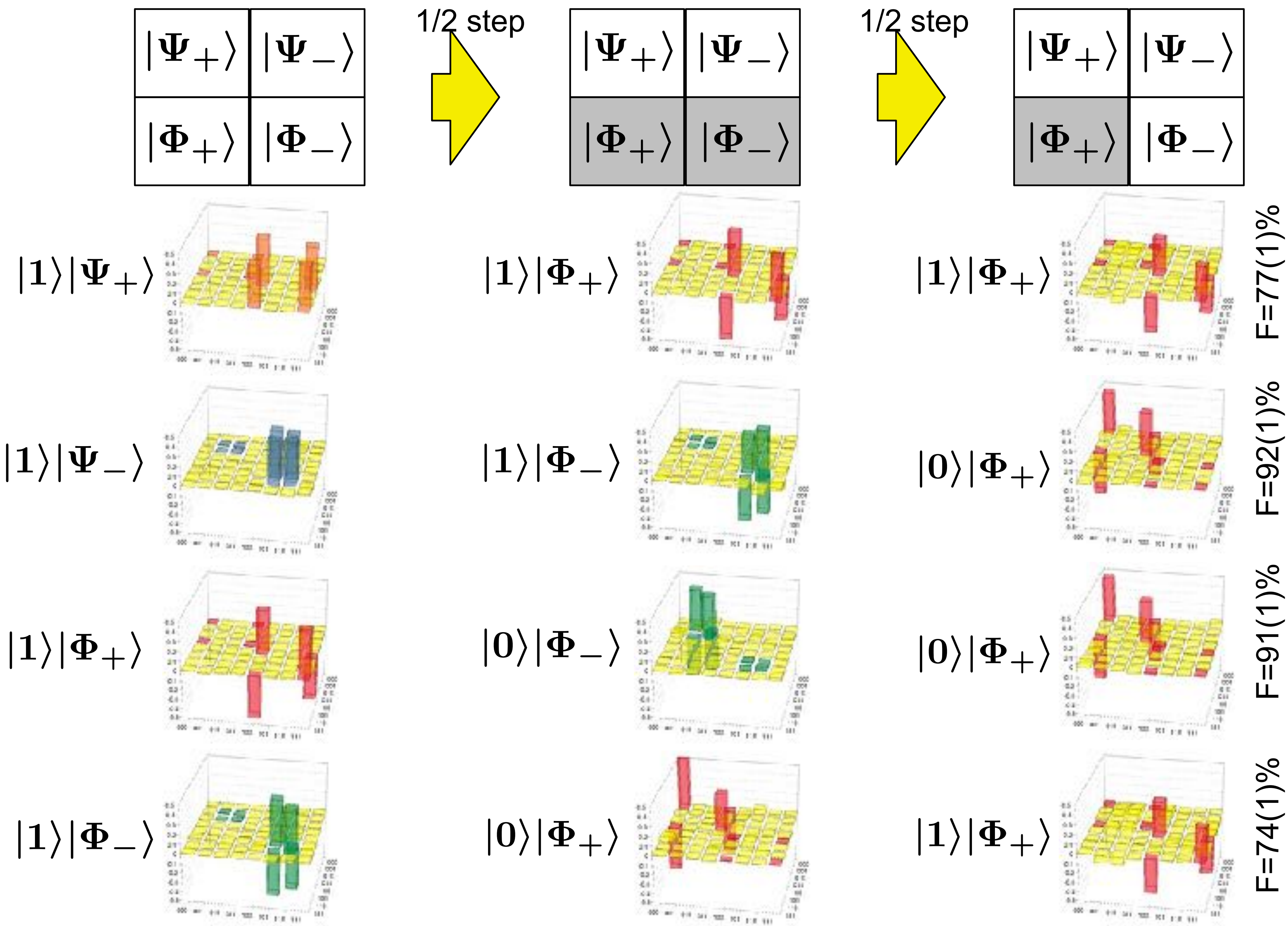
one pumping cycle

pumping probability p

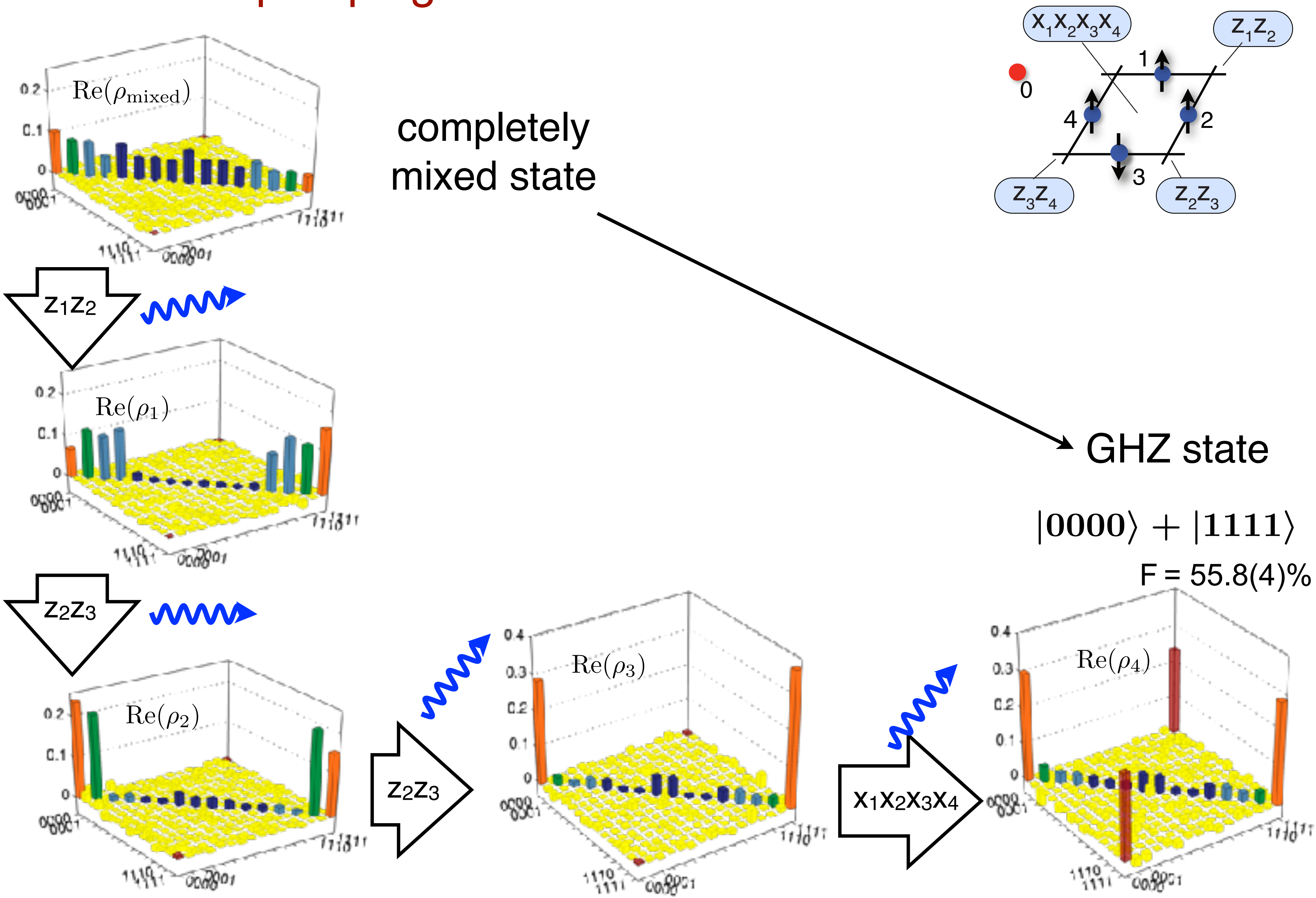


$$\dot{\rho} = -i[\cancel{H},\rho] + \sum_{\alpha=1}^{2} \gamma_\alpha \left(c_\alpha \rho c_\alpha^\dagger - \frac{1}{2} c_\alpha^\dagger c_\alpha \rho - \rho \frac{1}{2} c_\alpha^\dagger c_\alpha \right)$$

Bell State Pumping: Ion Experiment



Stabilizer pumping: 1+4 ions



J. Barreiro, M. Müller et al., Nature 470, 486 (2011)

Summary

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Quantum Optical Systems & Control

Lectures 1+2: *Isolated / Driven Hamiltonian* quantum optical systems

- Basic systems & concepts of quantum optics - an overview
- Example / Application: Ion Trap Quantum Computer

Lectures 3+4: ***Open*** quantum optical systems [a modern perspective]

- Continuous measurement theory, Quantum Stochastic Schrödinger Equation, master equation & quantum trajectories
- Example 1: *Chiral / Cascaded* Quantum Optical Systems & *Quantum Many-Body Systems*
- [Example 2: *Entanglement by Dissipation*]