Introduction to Quantum Optics [Theory]

quantum control perspective

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Lectures 1 + 2

Outline

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Quantum Optical Systems & Control

Lectures 1+2: *Isolated / Driven Hamiltonian* quantum optical systems

- Basic systems & concepts of quantum optics - an overview
- Example / Application: Ion Trap Quantum Computer

Lectures 3+4: *Open* quantum optical systems [a modern perspective]

- Continuous measurement theory, Quantum Stochastic Schrödinger Equation, master equation & quantum trajectories
- Illustrations / Applications [in quantum information]

Literature

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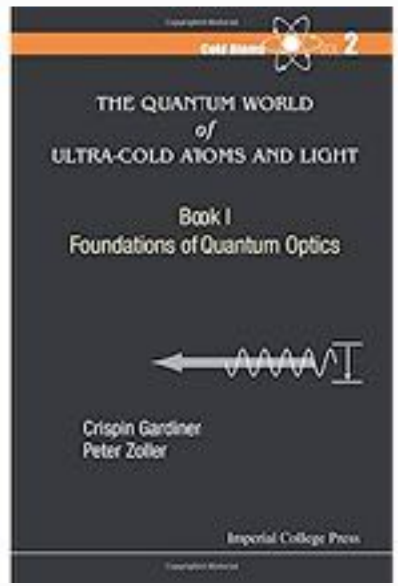
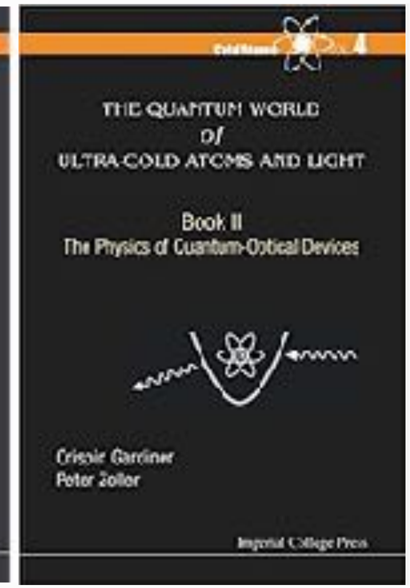
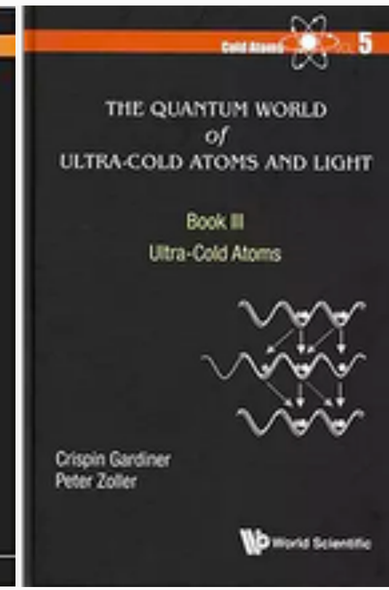
The Quantum World of Ultra-Cold Atoms and Light:

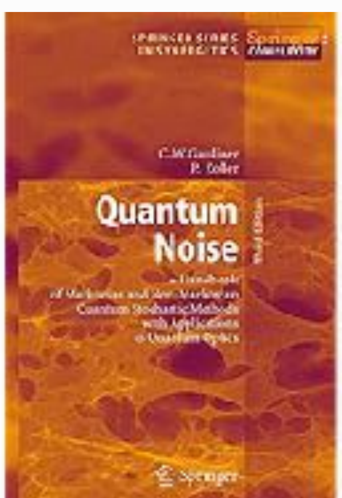
Book I: Foundations of Quantum Optics
Book II: The Physics of Quantum-Optical Devices
Book III: Ultra-cold Atoms

by Crispin W Gardiner and Peter Zoller









Quantum Noise

A Handbook of Markovian and Non-Markovian Quantum Stochastic Methods with Applications to Quantum Optics

by Crispin W Gardiner and Peter Zoller

A Short Preview

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What is Quantum Optics?

... a Short Tour & Overview

[Experiment + Theory]

- Quantum Properties of Light
- ⇨ [Quantum] Interaction of Light & Matter

control

... a Few Examples

Ex 1: Quantum Computing with Trapped Ions

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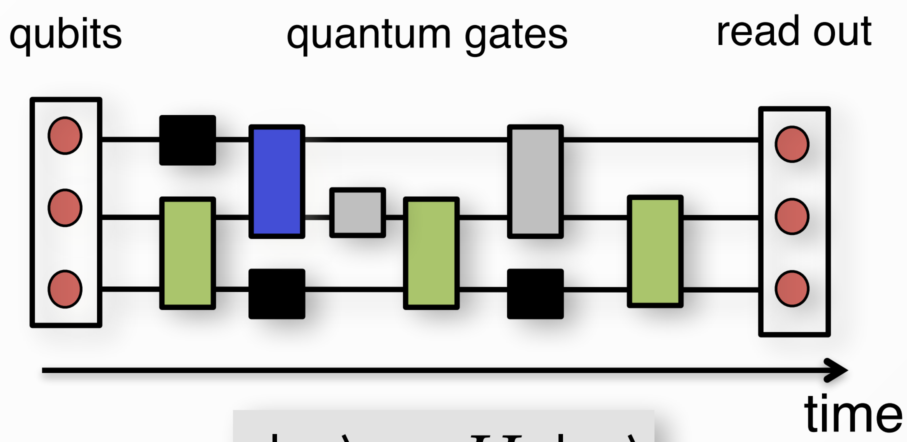
- **general purpose quantum computing**

quantum logic network model

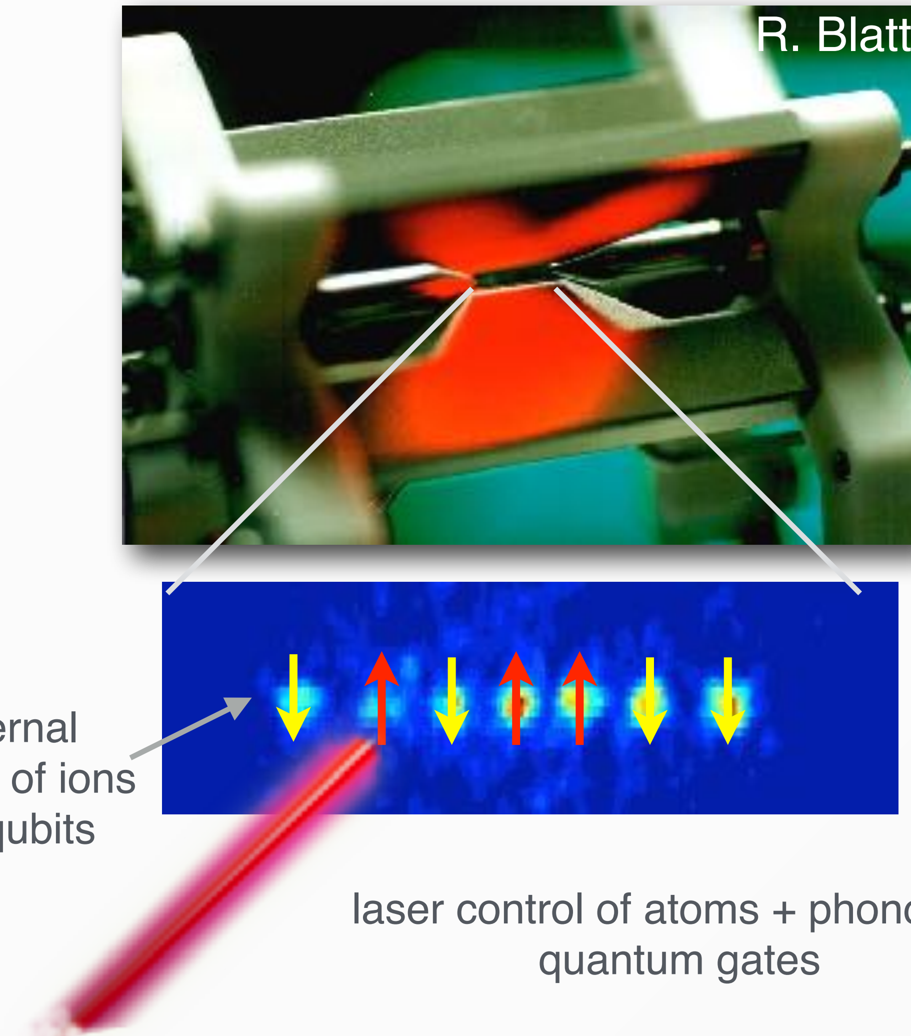


$$|\psi\rangle \rightarrow U_t\,|\psi\rangle$$

coherent Hamiltonian evolution

- quantum gates
- deterministic

- **atomic physics: trapped ions**



internal states of ions as qubits

laser control of atoms + phonons = quantum gates

Exp.: Innsbruck, NIST, JQI, MIT, Mainz, MPQ ...

Ex 1: Quantum Computing with Trapped Ions

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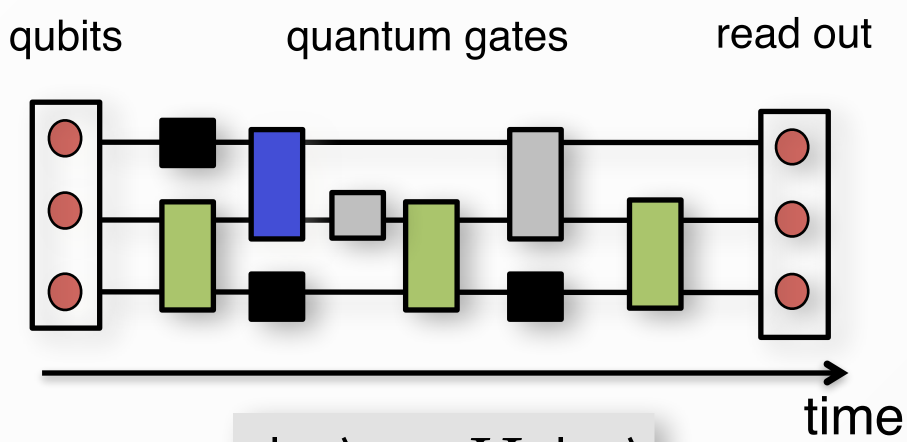
- **general purpose quantum computing**

quantum logic network model



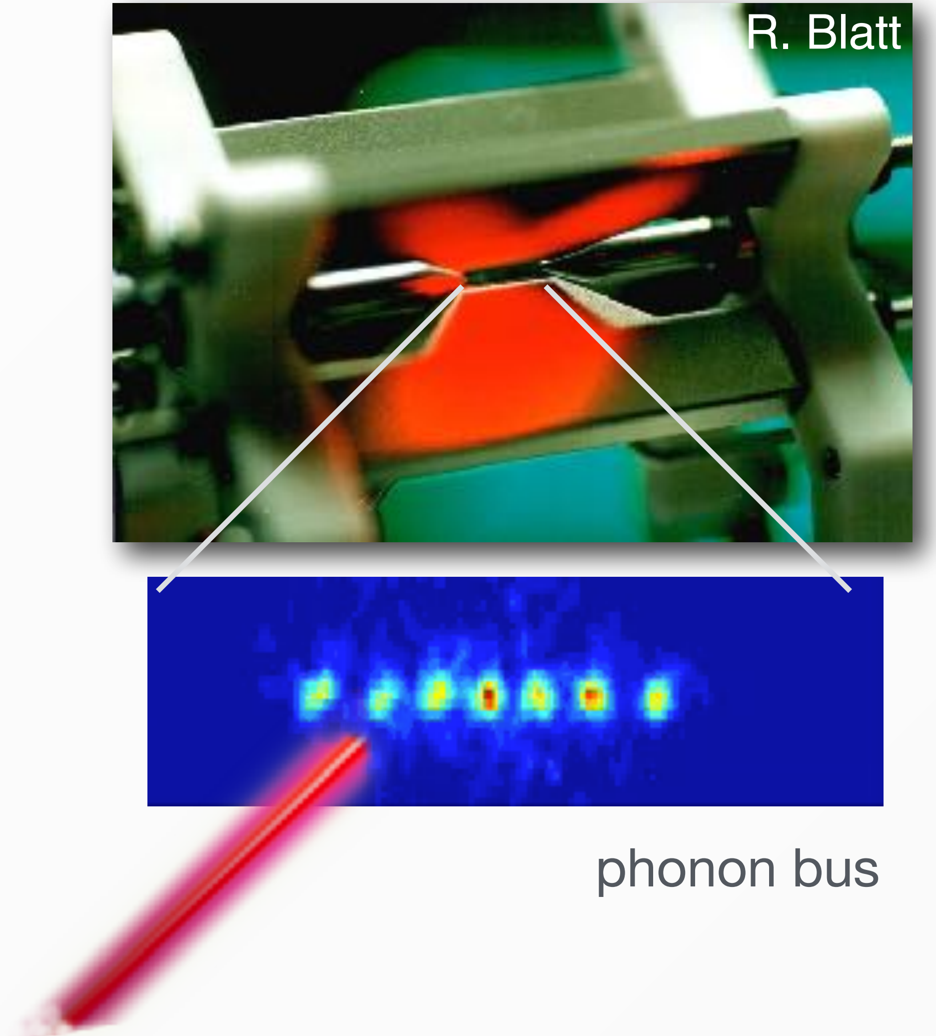
$$|\psi\rangle \rightarrow U_t\,|\psi\rangle$$

coherent Hamiltonian evolution

- quantum gates
- deterministic

coherent control

- **atomic physics: trapped ions**



phonon bus

… in Lecture 1+2 we will go in detail over the underlying theory.

Ex 2: Atomic Hubbard Models - QSimulators

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Quantum Many-Body

- **Hubbard models etc.**

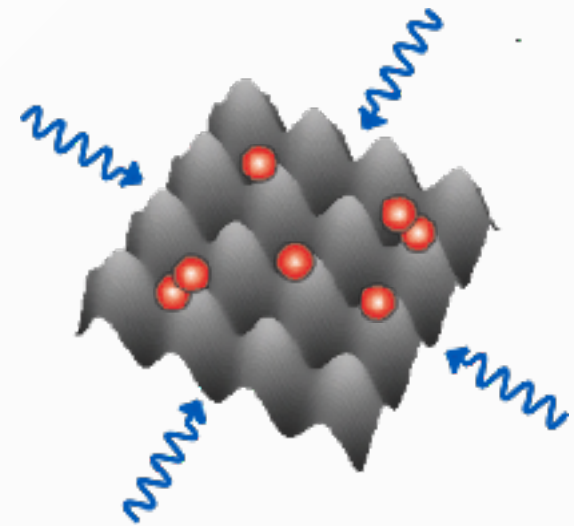
$$\hat{H} = -\sum_{\alpha\neq\beta} J_{\alpha\beta}\hat{a}^{\dagger}_{\alpha}a_{\beta} + \frac{1}{2}U\sum_{\alpha}\hat{a}^{\dagger}_{\alpha}\hat{a}^{\dagger}_{\alpha}\hat{a}_{\alpha}\hat{a}_{\alpha}$$

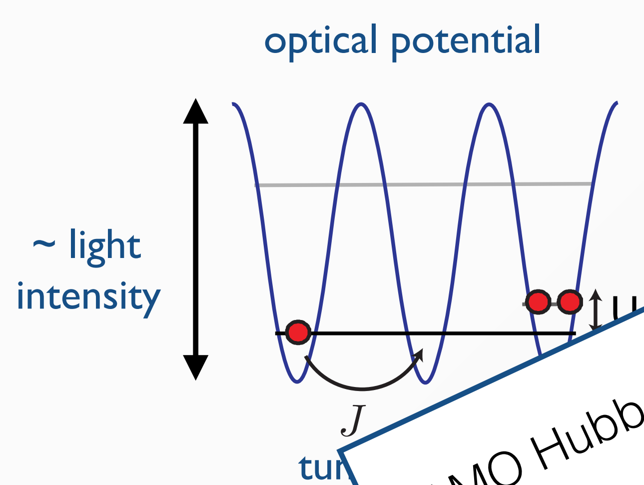
Hubbard Hamiltonian

Bosons, Fermions
- strongly correlated system
- quantum phase transitions

Atomic Physics

- **atoms in optical lattices**





Analog quantum simulation: “always on”

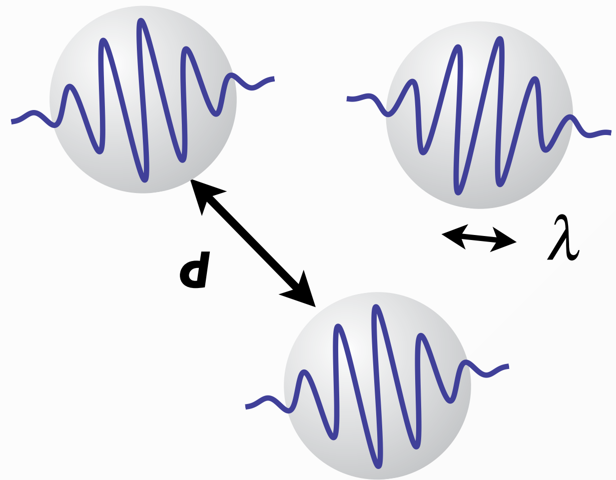
- We “build” a quantum system with desired Hamiltonian & *controllable parameters*, e.g. Hubbard models of atoms in optical lattices

AMO Hubbard toolbox

From Artificial Quantum Matter to Real Materials

Quantum Regime

$$\lambda/d \gtrsim 1$$

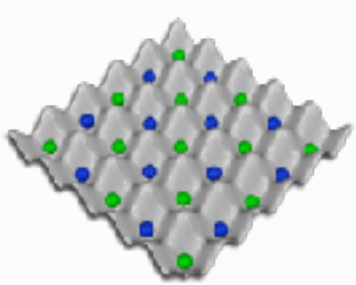


de Broglie Wavepackets

Universality of Quantum Mechanics!

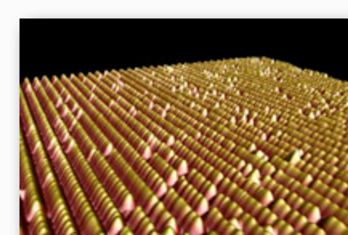
Ultracold Quantum Matter

- ***Densities:*** *$10^{14}/cm^3$*
- ***Temperatures:*** *few nK*



Real Materials

- ***Densities:*** *10^{24}-$10^{25}/cm^3$*
- ***Temperatures:*** *mK – several hundred K*

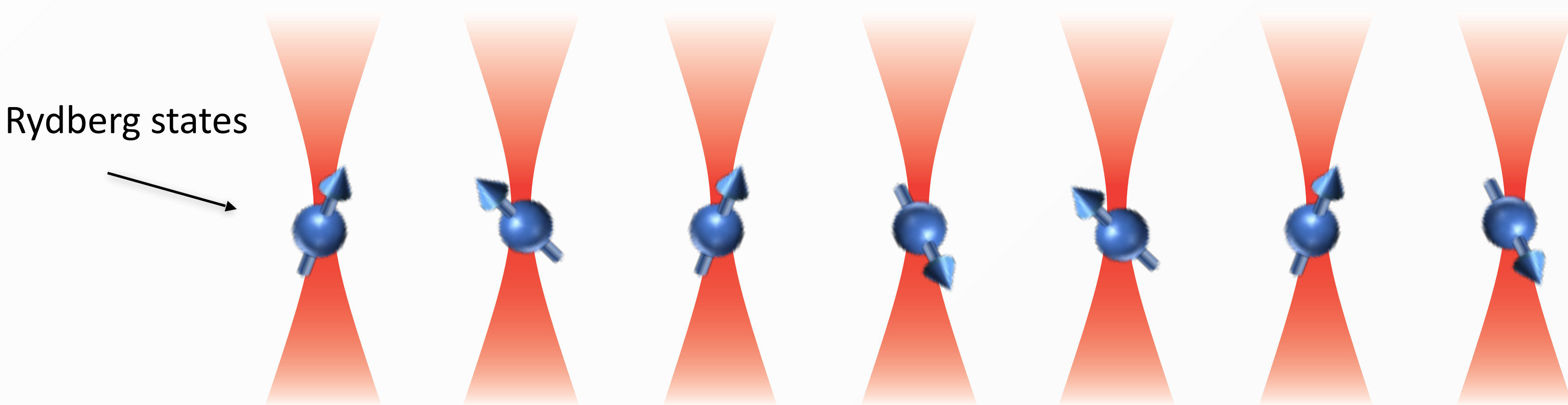


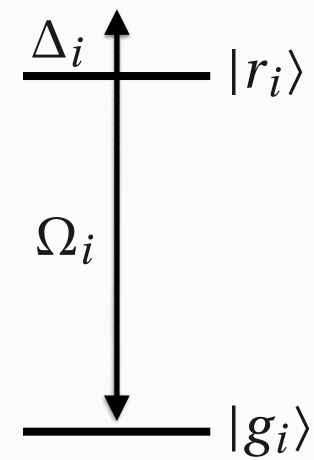
(Neuchatel)

Same λ/d!

Rydberg Spin-Models [as Quantum Simulator]

Hamiltonian Engineering





Hamiltonian

$$H = \sum_i \frac{1}{2}\Omega_i \sigma_x^i - \sum_i \Delta_i n_i + \sum_{i<j} V_{ij} n_i n_j$$

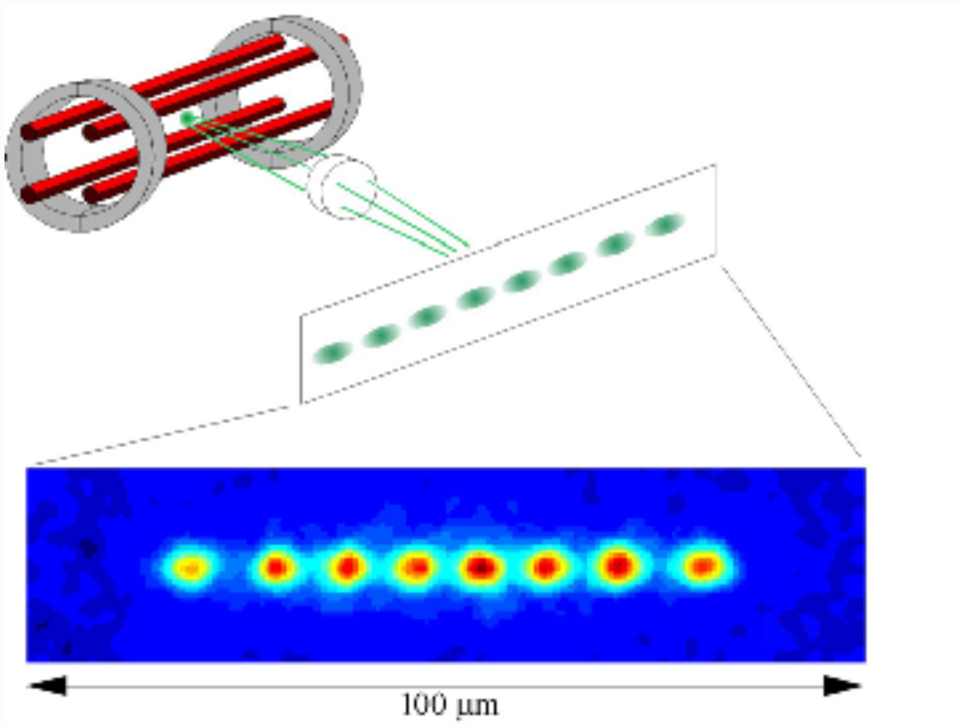
$$\sigma_x^i = |g_i\rangle\langle r_i| + |r_i\rangle\langle g_i|$$

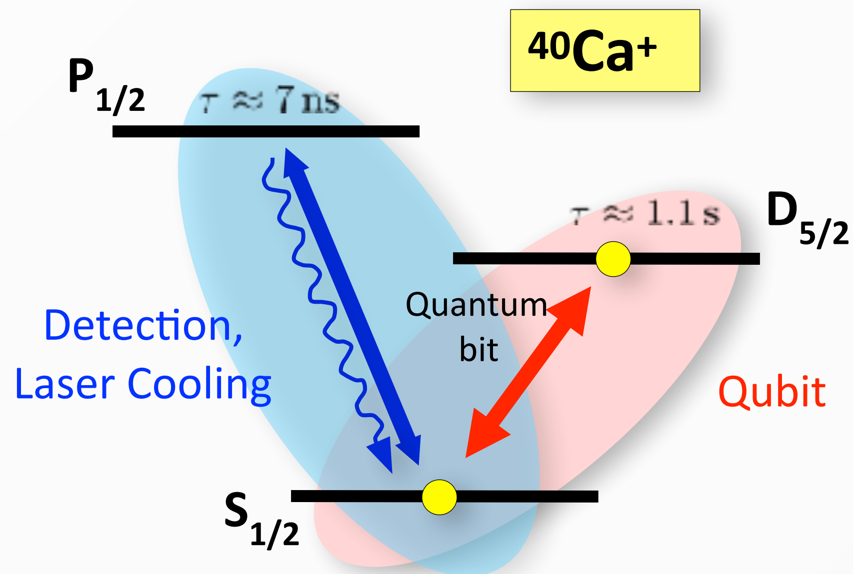
$$V_{ij} = C_6 / r_{ij}^6$$

$$n_i = |r_i\rangle\langle r_i| \equiv \frac{1}{2}(1+\sigma_z^i)$$

Exp: Lukin-Greiner-Vuletic groups (Harvard - MIT), Browaeys (Palaiseau), Saffman (Wisconsin), Biedermann (Sandia)

Example: Trapped Ion Quantum Computer





Theory 1. Single Trapped Laser-Driven Ion

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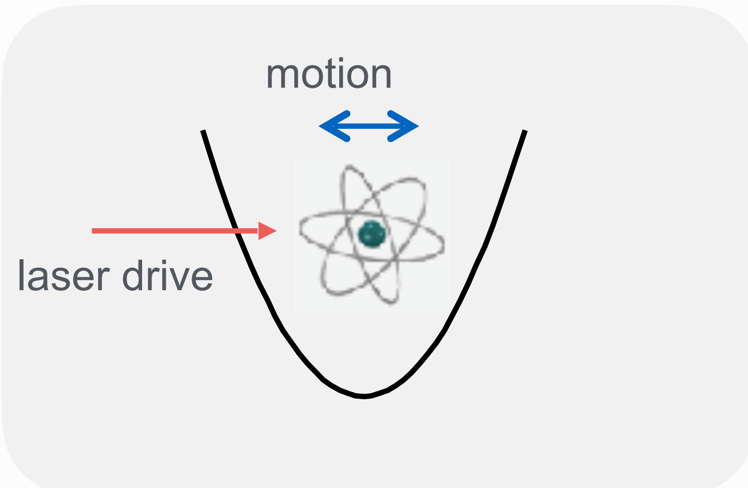
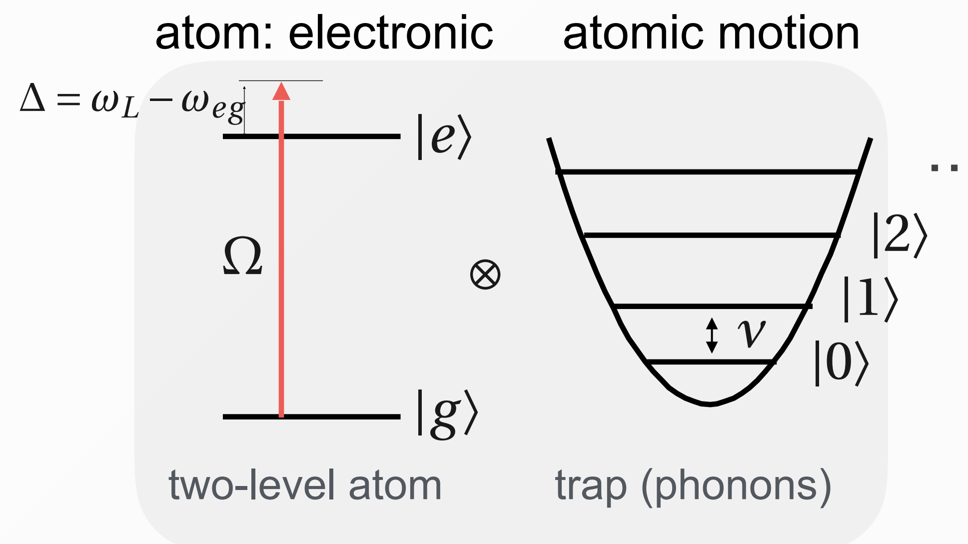
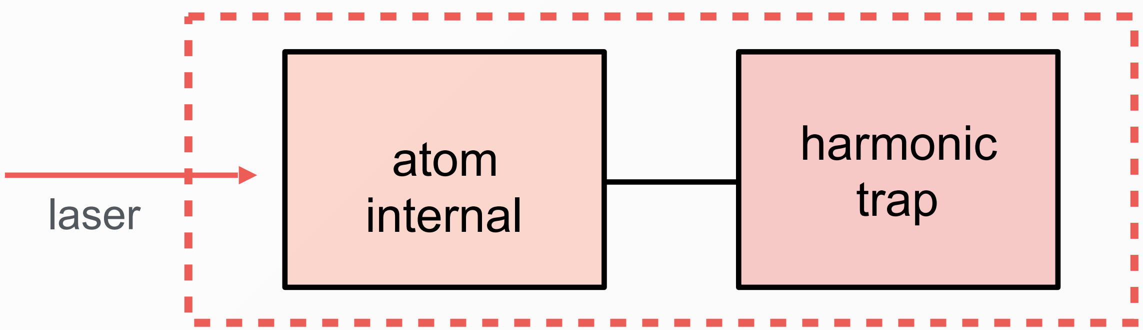
- **driven two-level atom in a harmonic oscillator trap**





Lectures 1+2:



system of interest

✓ coherent control [quantum gates etc.]

isolated driven system

System Hamiltonian

$$H = \frac{\hat{p}^2}{2M} + \frac{1}{2} M \nu^2 \hat{x}^2 + \hbar\omega_{eg}|e\rangle\langle e| - \hbar\left(\frac{1}{2}\Omega e^{ik\hat{x}-i\omega t}|e\rangle\langle g| + \text{h.c.}\right)$$

Engineer interesting quantum states?

Theory 2. Single Trapped Laser-Driven Ion

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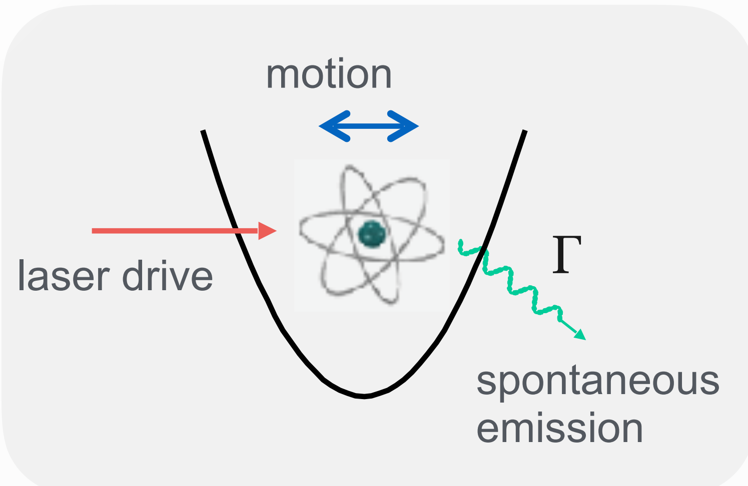
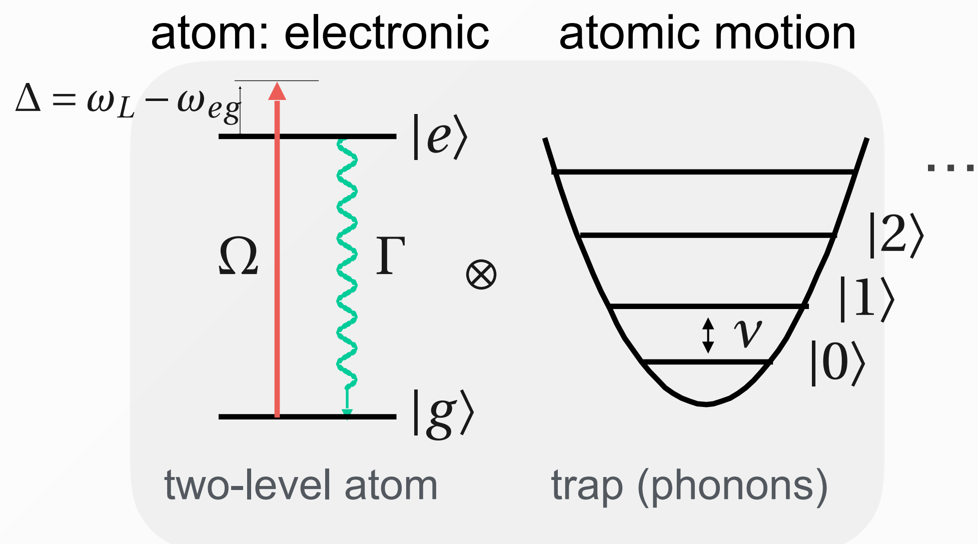
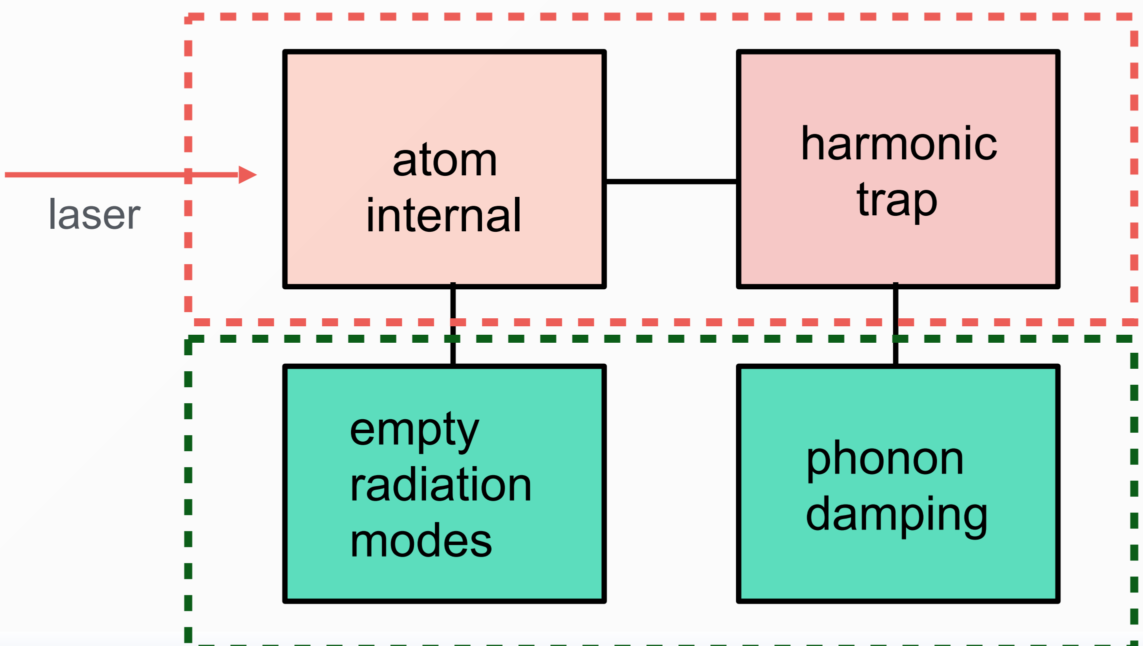
- driven two-level atom in a harmonic oscillator trap





Lectures 1+2:

Lectures 3+4:



system of interest

✓ coherent control [quantum gates etc.]

reservoirs

✓ decoherence :-(
✓ laser cooling :-)

open system

Ion Trap Quantum Computer

… as a Quantum Optical Problem

- single ion
  - Hamiltonians, examples for quantum state engineering
- many ions
  - Hamiltonians, entangling gate

Appendices:

- quantum information: qubits, quantum gates etc.
- From real atoms to two-level atoms & Rabi oscillations

Side-Remarks

From Real Atoms to Two-Level Systems

Rabi Oscillations

Two-Level Atom & Rabi Oscillations

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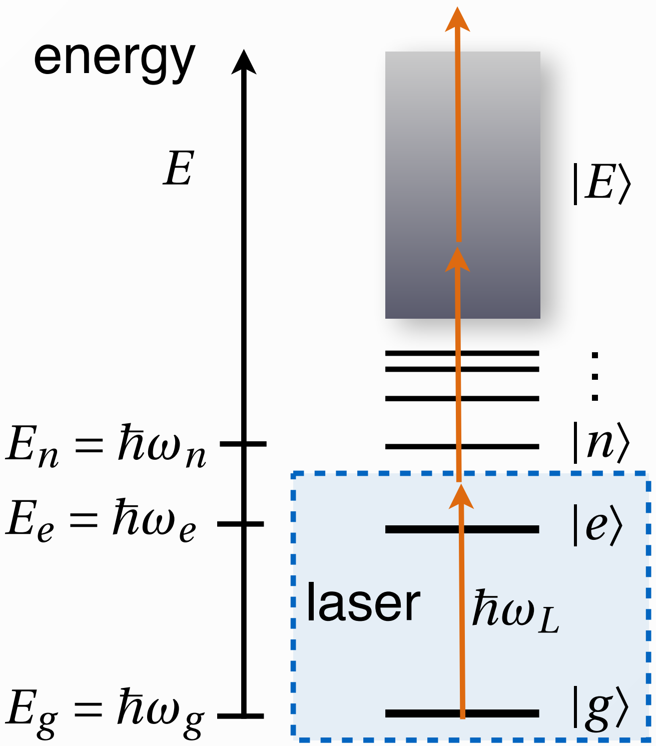
Real Atom



Electric field of laser

$$\vec{E}_{\rm cl}(\vec{x}=0,t) = \mathscr{E}\vec{\epsilon}e^{-i\omega t} + \mathscr{E}^*\vec{\epsilon}^*e^{+i\omega t}$$

$$\equiv \vec{E}_{\rm cl}^{(+)}(0,t) + \vec{E}_{\rm cl}^{(-)}(0,t)$$

Schrödinger Equation

$$i\hbar\frac{d}{dt}|\psi(t)\rangle = \left(H_{0A} - \vec{\mu}\cdot\vec{E}_{\rm cl}(\vec{x}=0,t)\right)|\psi(t)\rangle$$

Two-Level System (TLS)

$$|\psi(t)\rangle = a_g(t)|g\rangle + a_e(t)|e\rangle$$

keep resonant levels

Rotating Wave Approximation

$$\vec{\mu}\vec{E}_{\rm cl}(0,t) \longrightarrow \vec{\mu}_{eg}\vec{\epsilon}\vec{E}_{\rm cl}^{(+)}(0,t)|e\rangle\langle g| + \vec{\mu}_{ge}\vec{\epsilon}\vec{E}_{\rm cl}^{(-)}(0,t)|g\rangle\langle e|$$

absorption emission

Problem: We drive an atom near-resonant with a laser (no motion)

Two-Level Atom & Rabi Oscillations

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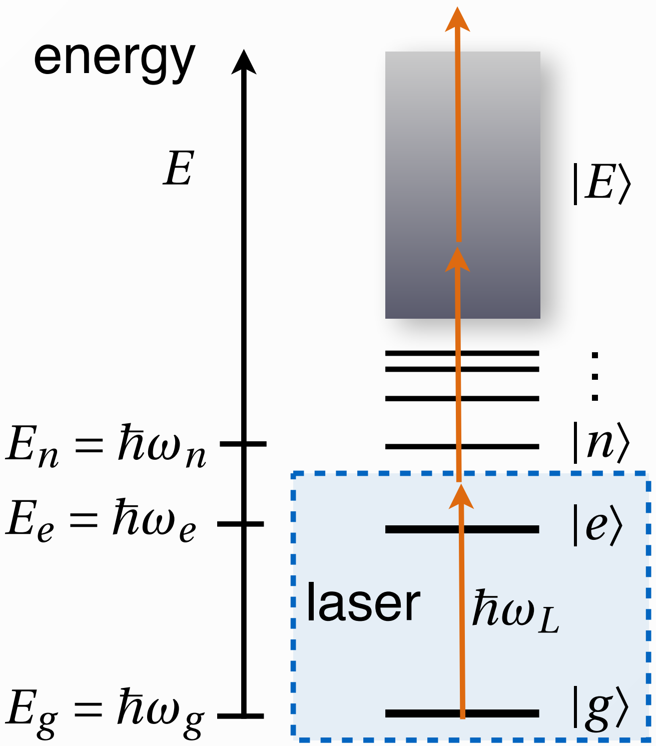
Real Atom



Hamiltonian TLS + RWA

$$H = \hbar\omega_{eg}|e\rangle\langle e| - \vec{\mu}_{eg}\vec{\epsilon}\mathscr{E}\, e^{-i\omega t}|e\rangle\langle g| - \vec{\mu}_{ge}\vec{\epsilon}\mathscr{E}^*\, e^{i\omega t}|g\rangle\langle e|$$

absorption (second term) — emission (third term)

Transformation to "rotating frame": $a_e(t) = \tilde{a}_e(t)e^{-i\omega t}$

$$i\frac{d}{dt}\begin{pmatrix} \tilde{a}_e \\ a_g \end{pmatrix} = \begin{pmatrix} -\Delta & -\vec{\mu}_{eg}\vec{\epsilon}\mathscr{E} \\ -\vec{\mu}_{ge}\vec{\epsilon}^*\mathscr{E}^* & 0 \end{pmatrix}\begin{pmatrix} \tilde{a}_e \\ a_g \end{pmatrix}$$

Parameters: Rabi frequency $\Omega_c \equiv 2\vec{\mu}_{eg}\vec{\epsilon}\mathscr{E}/\hbar \equiv \Omega e^{-i\varphi}$ and detuning $\Delta = \omega - \omega_{eg}$

Hamiltonian in "rotating frame"

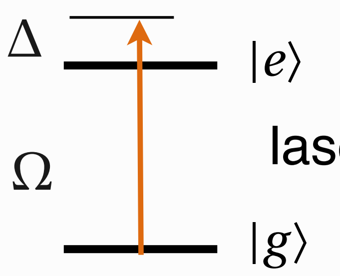
$$\tilde{H} = -\hbar\Delta|e\rangle\langle e| - \frac{1}{2}\hbar\Omega e^{-i\varphi}|e\rangle\langle g| - \frac{1}{2}\hbar\Omega e^{+i\varphi}|g\rangle\langle e|$$

Validity $\Omega, |\Delta| \ll \omega \approx \omega_{eg}$

Problem: We drive an atom near-resonant with a laser (no motion)

Two-Level Atom & Rabi Oscillations

Two-Level Atom



Hamiltonian in "rotating frame"

$$\tilde{H} = -\hbar\Delta|e\rangle\langle e| - \frac{1}{2}\hbar\Omega e^{-i\varphi}|e\rangle\langle g| - \frac{1}{2}\hbar\Omega e^{+i\varphi}|g\rangle\langle e|$$

Discussion: on-resonance Rabi oscillations $(\Delta = 0)$

$$U_t = e^{-i\tilde{H}t/\hbar} = \begin{pmatrix} \cos\frac{1}{2}\Omega t & -ie^{-i\varphi}\sin\frac{1}{2}\Omega t \\ -ie^{+i\varphi}\sin\frac{1}{2}\Omega t & \cos\frac{1}{2}\Omega t \end{pmatrix} \begin{matrix} \text{e} \\ \text{g} \end{matrix}$$

Examples:

- transition probability g → e

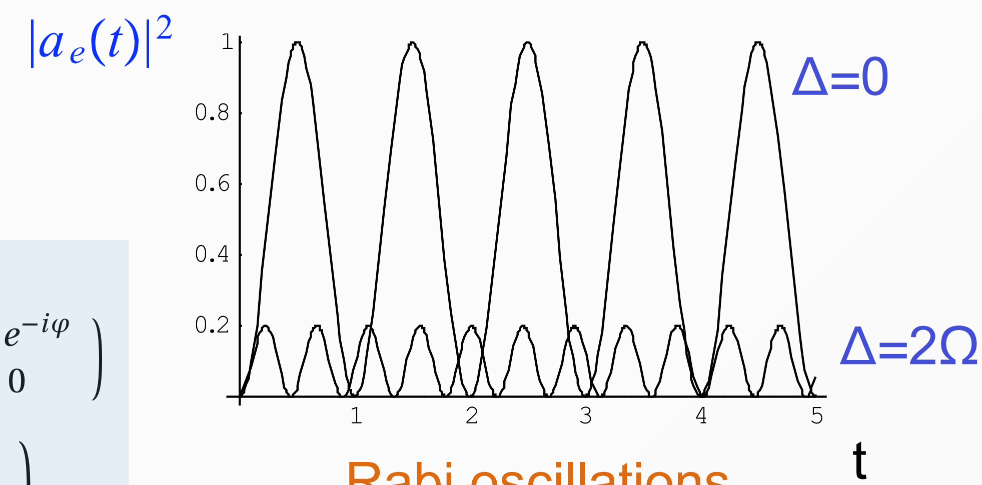
$$P_{e\leftarrow g}(t) = \frac{1}{2}[1-\cos\Omega t]$$

π-pulse $\Omega t = \pi$ inverts the TLS

$$U_{t=\pi/\Omega} = \begin{pmatrix} 0 & -ie^{-i\varphi} \\ -ie^{+i\varphi} & 0 \end{pmatrix}$$

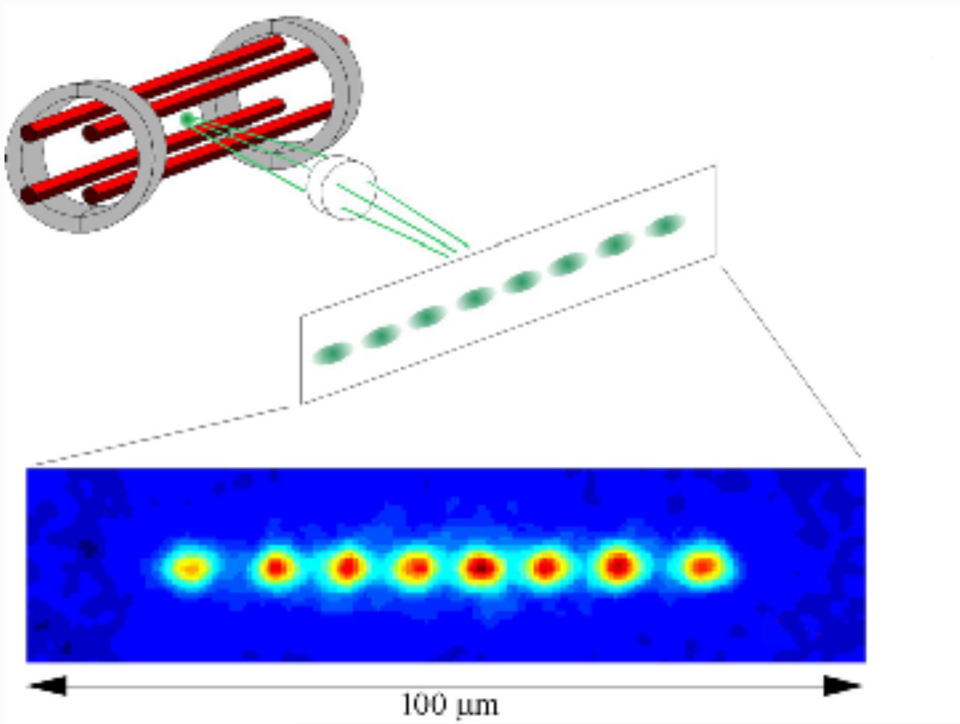
2π pulse

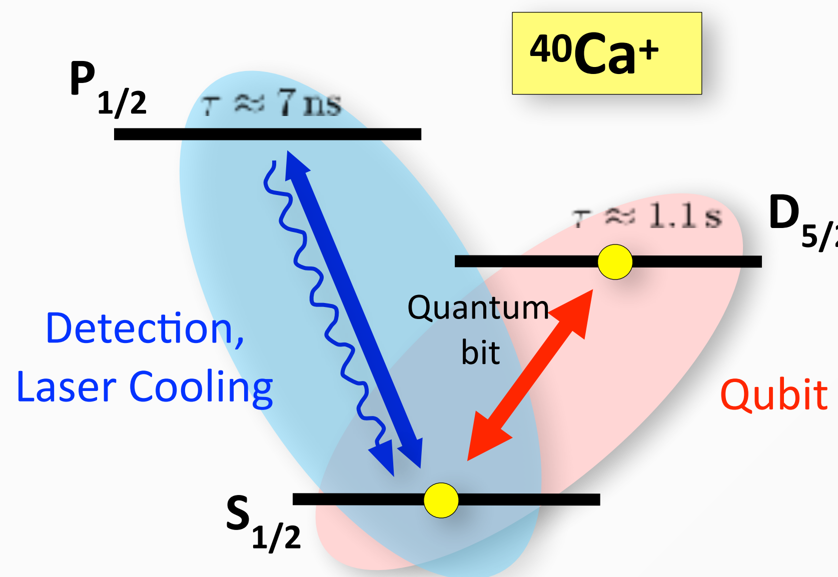
$$U_{t=2\pi/\Omega} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$



Rabi oscillations

Quantum State Engineering & Quantum Computing with Trapped Ions





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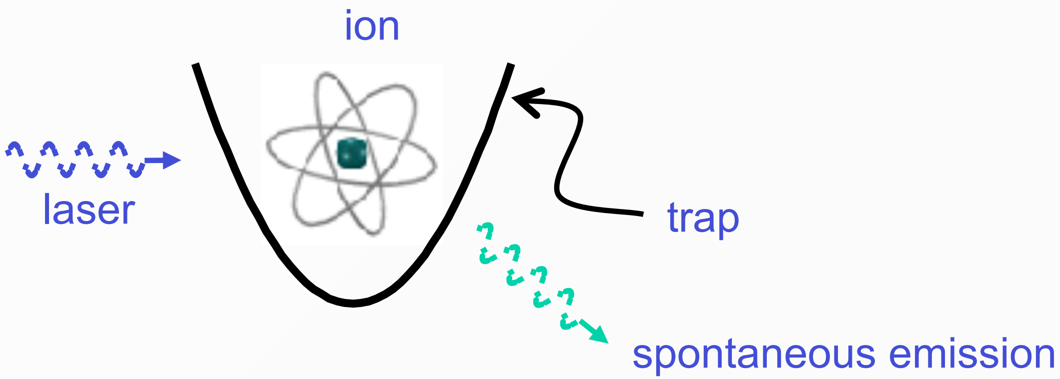
Model

Problem: We consider an ion moving in a trapping potential and interacting with laser light. The ion has internal degrees of freedom, the electronic excitation of the ion, and external degrees of freedom corresponding to the center of mass motion of the ion in the trap.



Single Laser-Driven Trapped Ion

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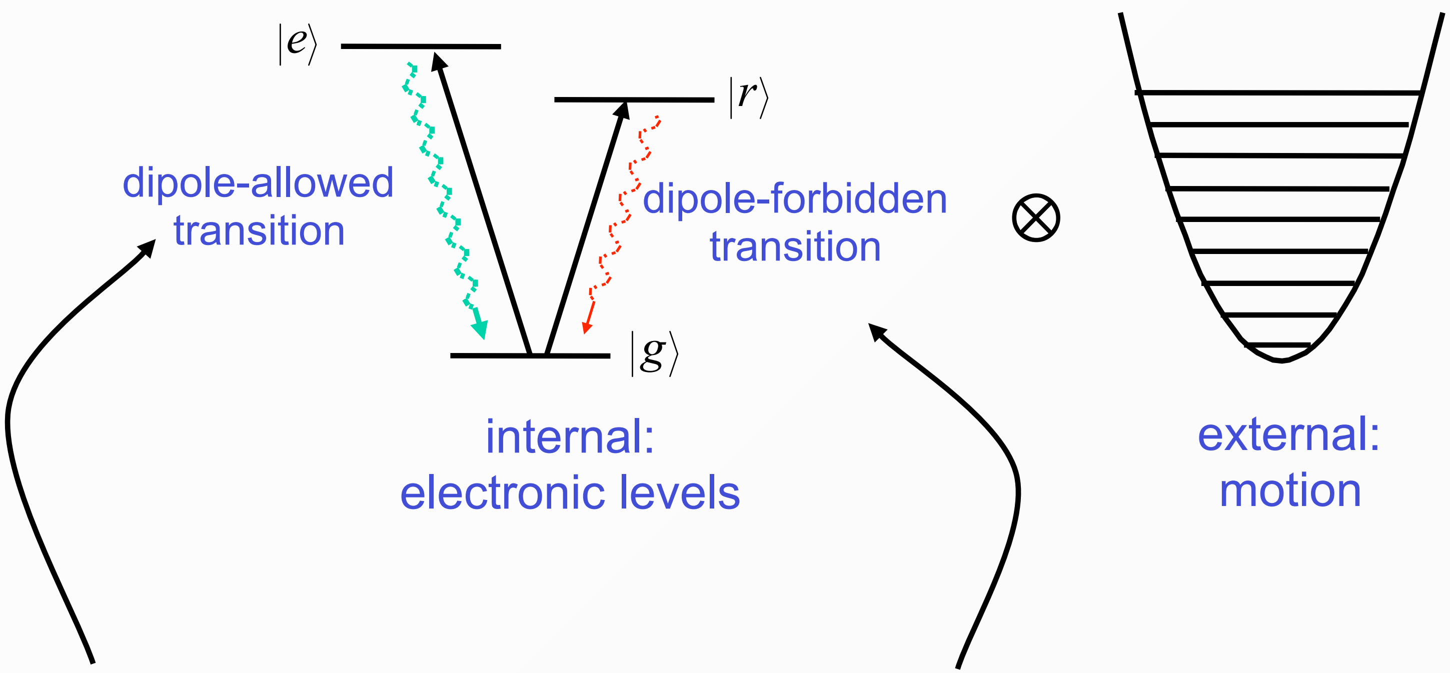
The System

- system = internal + external degrees of freedom



- strong dissipation
  - ✓ laser cooling / state preparation
  - ✓ state measurement

- small dissipation
  - ✓ Hamiltonian: quantum state engineering

Single Laser-Driven Trapped Ion

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Center of mass motion

We consider a single ion confined in a harmonic trap and interacting with one or several laser beams. We will assume that the lasers are directed along one of the principal axes of the harmonic potential. This assumption will simplify the problem, since it enables us to consider the ion motion in only one dimension.

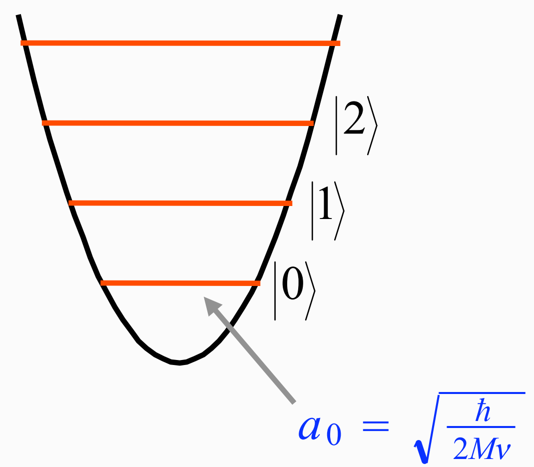
Hamiltonian describing the free motion of the ion in the trap

$$H_T = \frac{\hat{p}^2}{2M} + \frac{1}{2} M \nu^2 \hat{x}^2$$

$$\hat{x} = \sqrt{\frac{\hbar}{2M\nu}}\,(a + a^\dagger), \qquad \hat{p} = i\sqrt{\frac{\hbar M \nu}{2}}\,(a^\dagger - a)$$

$$H_T = \hbar \nu a^\dagger a$$

$$H_T|n\rangle = \hbar \nu n |n\rangle \quad (n = 0, 1, 2, \dots)$$



Remarks on the experimental situation:

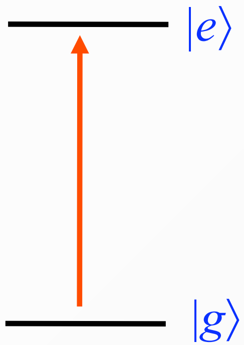
- Experiment: typical trapping frequencies are 1 MHz to 10 MHz
- Using laser cooling one can cool to the vibrational ground state $|0\rangle$ - see later.

Electronic excitation

internal structure of the ion has the form of a two–level system $\{|g\rangle, |e\rangle\}$

Hamiltonian

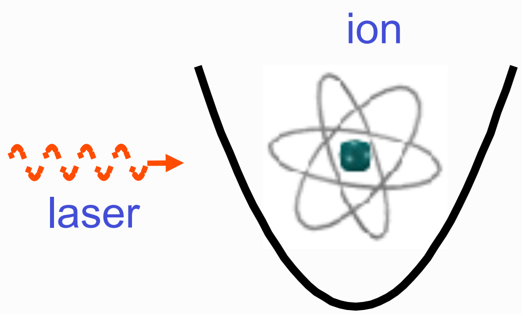
$$H_A = \hbar\omega_{eg}|e\rangle\langle e|$$

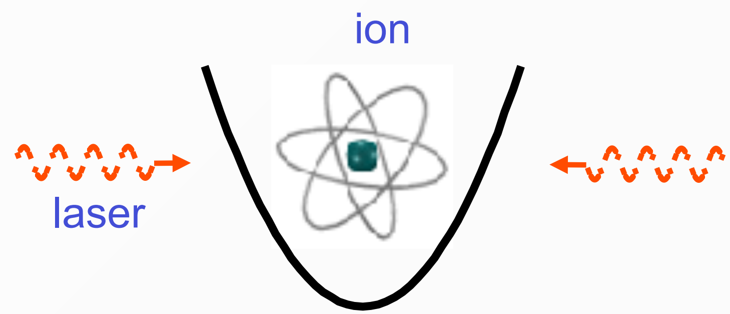


Single Laser-Driven Trapped Ion

Interaction Hamiltonian

The interaction Hamiltonian describing the excitation of the electron by the laser field of the form -$\vec{\mu} \cdot \vec{E}$ with $\vec{E}$ the classical driving field evaluated at the center of mass position x of the ion (which becomes an operator $\hat{x}$).





traveling wave or a *running wave* along the x-direction,

$$\vec{E}(x,t) = \mathcal{E}\vec{e}\, e^{ikx - i\omega t} + \text{c. c.},$$

$$\vec{E}(x,t) = \mathcal{E}\vec{e}\sin(kx + \phi)\, e^{-i\omega t} + \text{c. c.}$$

with wave vector $k = 2\pi/\lambda$. The parameter ϕ indicates the position of the ion in the standing wave, e.g. $\phi = 0$ at the node, $\phi = \pi/2$ at the antinode.

For a two-level atom and in the RWA this interaction Hamiltonian is of the form

$$H_1 = \begin{cases} -\hbar\left(\frac{1}{2}\Omega e^{ik\cdot\hat{x} - i\omega t}|e\rangle\langle g| + \text{h. c.}\right) & \text{running wave} \\ -\hbar\left(\frac{1}{2}\Omega \sin(k\hat{x} + \phi)e^{-i\omega t}|e\rangle\langle g| + \text{h. c.}\right) & \text{standing wave} \end{cases},$$

Single Laser-Driven Trapped Ion

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Remarks:

- Excitation by the laser transfers the electron to the excited state. This transition is associated with a momentum transfer to the center of mass degrees of freedom. (Note: for a traveling wave this is a momentum kick

$$|g\rangle|p\rangle \stackrel{H_1}{\rightarrow} |e\rangle e^{ik\hat{x}}|p\rangle \equiv |e\rangle|p+\hbar k\rangle$$

  with $|p\rangle$ a momentum eigenstate). *This opens the possibility of manipulating the center of mass motion of the ion via the laser.*

Single Laser-Driven Trapped Ion

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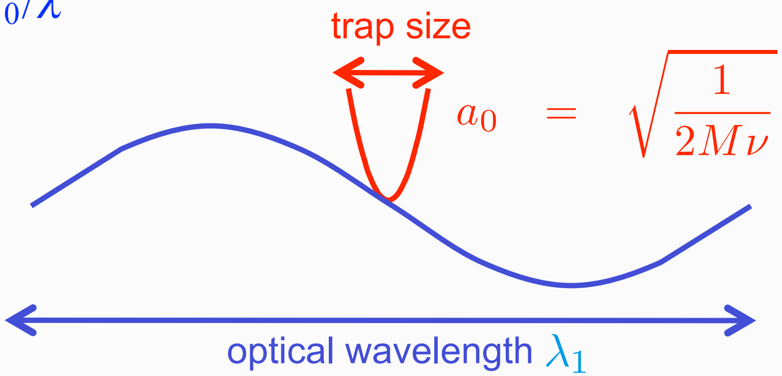
- *Lamb-Dicke parameter*: we have two length scales, the size of the ground state a_0 and the wavelength of the light λ. The ratio

$$\eta = 2\pi a_0/\lambda$$

is called Lamb-Dicke parameter.



The case $\eta \ll 1$ corresponds to a tight trap. We call this the Lamb-Dicke regime.

Another interpretation of the Lamb-Dicke parameter

recoil energy $\epsilon_R := \hbar k^2/2M$

a few to a few tens of kHz

$$\eta = \sqrt{\frac{\epsilon_R}{\hbar\nu}}$$

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Total Hamiltonian

- Traveling wave

$$H = \hbar \nu a^\dagger a - \hbar \Delta |e\rangle\langle e| - \left(\hbar \frac{1}{2} \Omega e^{i\eta(a+a^\dagger)} |e\rangle\langle g| + h.c. \right)$$

- Standing wave

$$H = \hbar \nu a^\dagger a - \hbar \Delta |e\rangle\langle e| - \hbar \frac{1}{2} \Omega \sin(\eta(a + a^\dagger) + \phi)(|e\rangle\langle g| + |g\rangle\langle e|)$$

The system Hilbert space is $\mathcal{H}_{TLS} \otimes \mathcal{H}_{L^2(R^1)}$

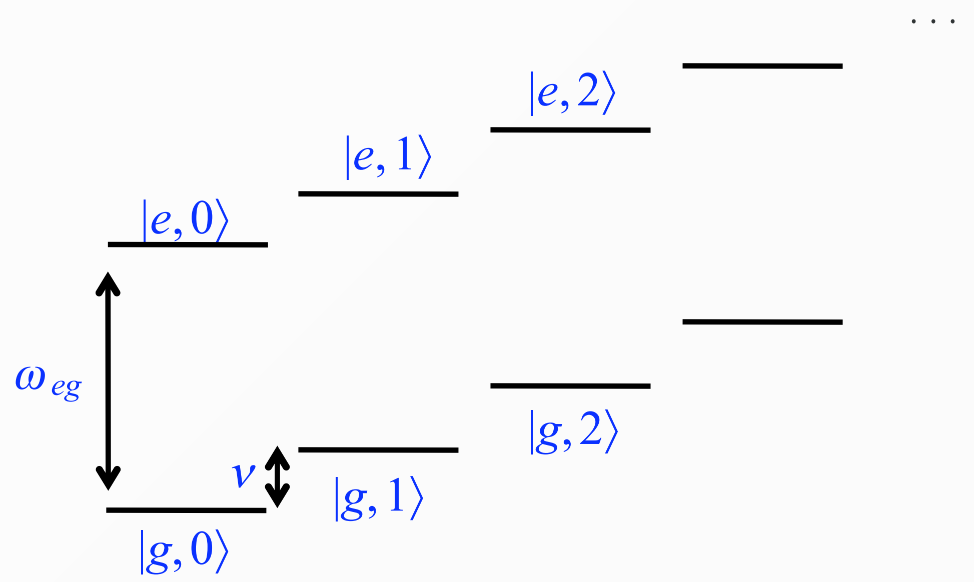
Energy spectrum eigenstates of the bare Hamiltonian H_0

ground state

$$H_0|g\rangle|n\rangle = n\hbar\nu|g\rangle|n\rangle,$$

excited state

$$H_0|e\rangle|n\rangle = \hbar(\omega_{eg} + n\nu)|e\rangle|n\rangle$$



vibrational spectrum + electronic excitation

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Laser induced couplings in a traveling wave configuration

The **matrix elements** coupling the ground states $|g\rangle|n\rangle$ to the excited states $|e\rangle|m\rangle$ are for a running wave:

$$\langle m|\langle e|H_1|g\rangle|n\rangle = -\hbar\frac{1}{2}\Omega\langle m|e^{ik\cdot\hat{x}}|n\rangle = -\hbar\frac{1}{2}\Omega\langle m|e^{i\eta(a+a^\dagger)}|n\rangle$$

In the *Lamb Dicke limit* $\eta \ll 1$ we can expand

$$e^{ik\cdot\hat{x}} \equiv e^{i\eta(a+a^\dagger)} = 1 + i\eta(a+a^\dagger) + \ldots$$

- which in leading order gives the matrix elements

$$\frac{1}{2}\Omega\langle n|e^{ik\cdot\hat{x}}|n\rangle = \frac{1}{2}\Omega(1+O(\eta^2)),$$

$$\langle n+1|e^{ik\cdot\hat{x}}|n\rangle = i\frac{1}{2}\Omega(\eta\sqrt{n+1} + O(\eta^3)),$$

$$\langle n-1|e^{ik\cdot\hat{x}}|n\rangle = i\frac{1}{2}\Omega(\eta\sqrt{n} + O(\eta^3)),$$

(Remark: the real expansion parameter is not η but $\sqrt{n}\,\eta$)

Single Laser-Driven Trapped Ion

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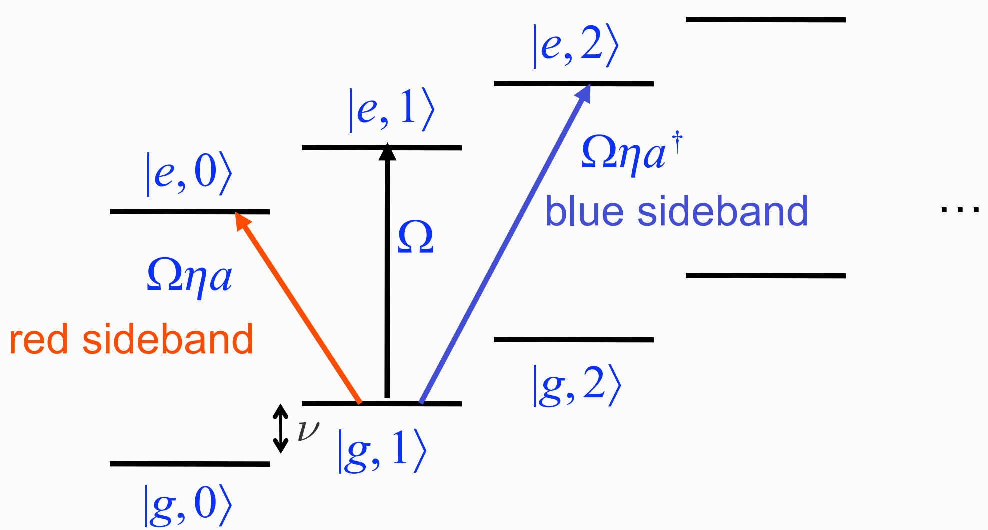
- The **Lamb-Dicke expansion of the traveling wave Hamiltonian** is

$$H = \hbar\nu a^\dagger a - \hbar\Delta|e\rangle\langle e| - \frac{\Omega}{2}\{|e\rangle\langle g|[1 + i\eta(a + a^\dagger) + O(\eta^2)] + \mathrm{h.c.}\}.$$



dominant excitation of the bare transition $|g\rangle|n\rangle \to |e\rangle|n\rangle$ for $\Delta = \omega - \omega_{eg} \approx 0$ with Rabifrequency Ω

two motional sidebands $|g\rangle|n\rangle \to |e\rangle|n \pm 1\rangle$

excited for $\Delta \approx \omega - \omega_{eg} \approx \pm\nu$ with Rabi frequencies $\eta\Omega$

no excitation on the red sideband for the ground state $|g\rangle|0\rangle$

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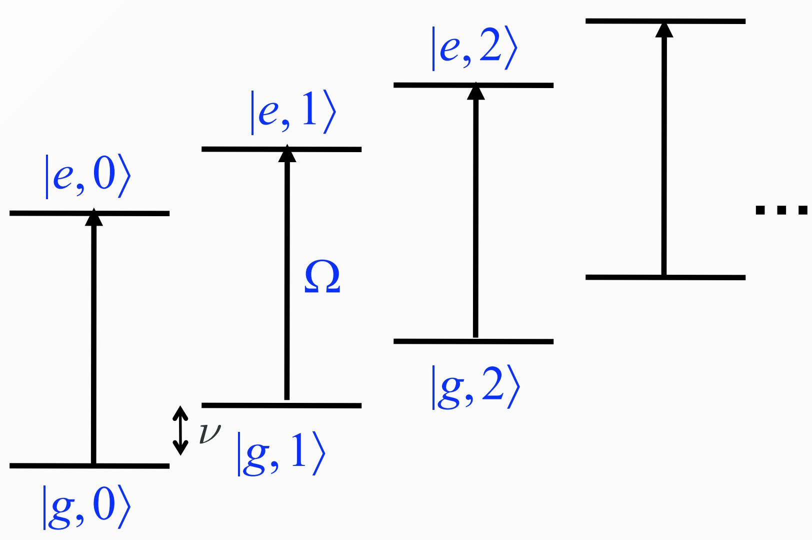
Limiting Hamiltonians:

- *Laser on resonance* $\Delta = \omega - \omega_{eg} \approx 0$ exciting $|g\rangle|n\rangle \to |e\rangle|n\rangle$:

  When we tune the laser close to the atomic transition frequency, the transitions $|g\rangle|n\rangle \to |e\rangle|n\rangle$ will be excited, while for $\eta\Omega \ll \nu$ excitation of the sidebands $|g\rangle|n\rangle \to |e\rangle|n \pm 1\rangle$ is suppressed (because they are off resonant).

  Hamiltonian

  $$H = \hbar\nu a^\dagger a - \hbar\Delta|e\rangle\langle e| - \frac{\Omega}{2}\{|e\rangle\langle g| + \text{h.c.}\}$$



  i.e. the motion is decoupled and we have a TLS

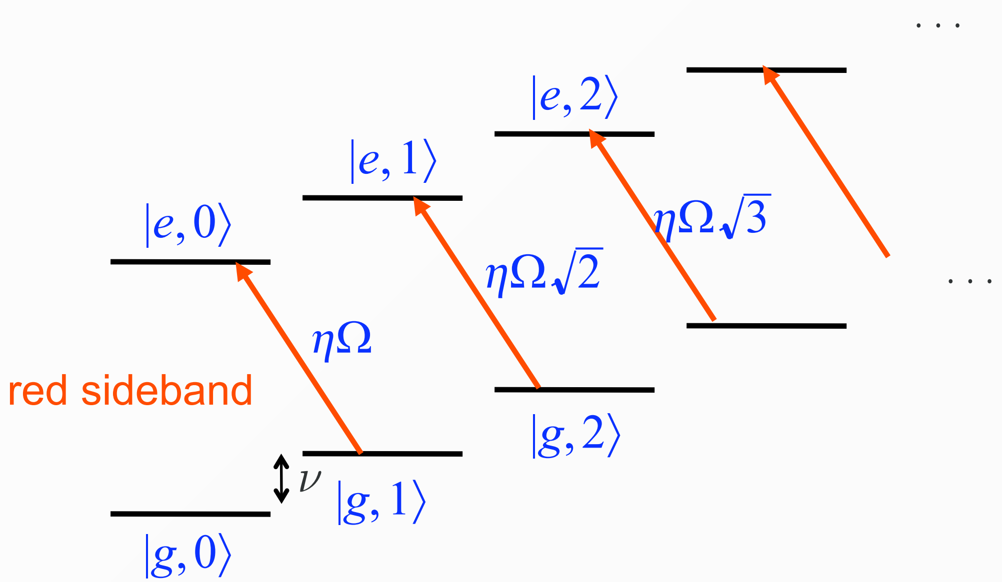
Single Laser-Driven Trapped Ion

- *Laser tuned to the lower motional sideband (red sideband)*
  $\Delta \approx \omega - \omega_{eg} \approx -\nu$ corresponding to $|g\rangle|n\rangle \to |e\rangle|n-1\rangle$.

  For $\Omega \ll \nu$ (a strong condition!) the bare atomic resonance is not excited.

  The Hamiltonian is a *Jaynes Cummings Hamiltonian with RWA*

$$H = \hbar\nu a^\dagger a - \hbar\Delta|e\rangle\langle e| - i\eta\frac{\Omega}{2}\{|e\rangle\langle g|a + \text{h.c.}\}$$



Single Laser-Driven Trapped Ion

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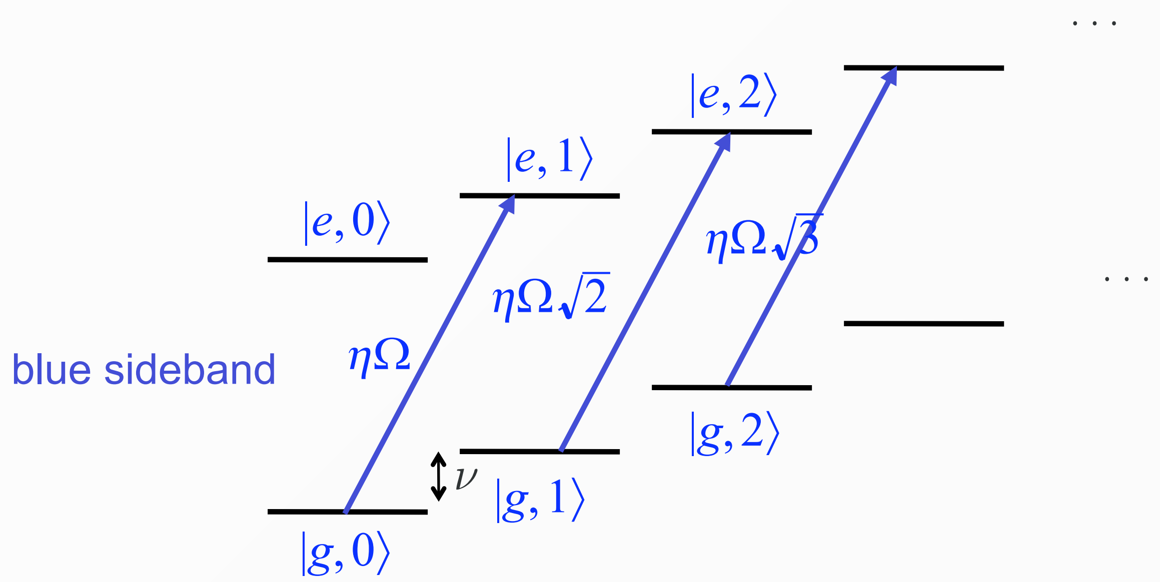
- *Laser tuned to the upper motional sideband (blue sideband)*
  $\Delta \approx \omega - \omega_{eg} \approx +\nu$ corresponding to $|g\rangle|n\rangle \to |e\rangle|n+1\rangle$.

For $\Omega \ll \nu$ (a strong condition!) the bare atomic resonance is not excited.

The Hamiltonian is a “anti”-Jaynes Cummings Hamiltonian with RWA

$$H = \hbar\nu a^\dagger a - \hbar\Delta|e\rangle\langle e| - i\eta\frac{\Omega}{2}\{|e\rangle\langle g|a^\dagger + \mathrm{h.c.}\}$$



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Quantum State Engineering

Statement of the problem

Apply unitary transformations to produce from a given initial (pure) state $|i\rangle$ (which we know how to prepare) a certain final state $|f\rangle$ (which we want to engineer - for whatever reason).

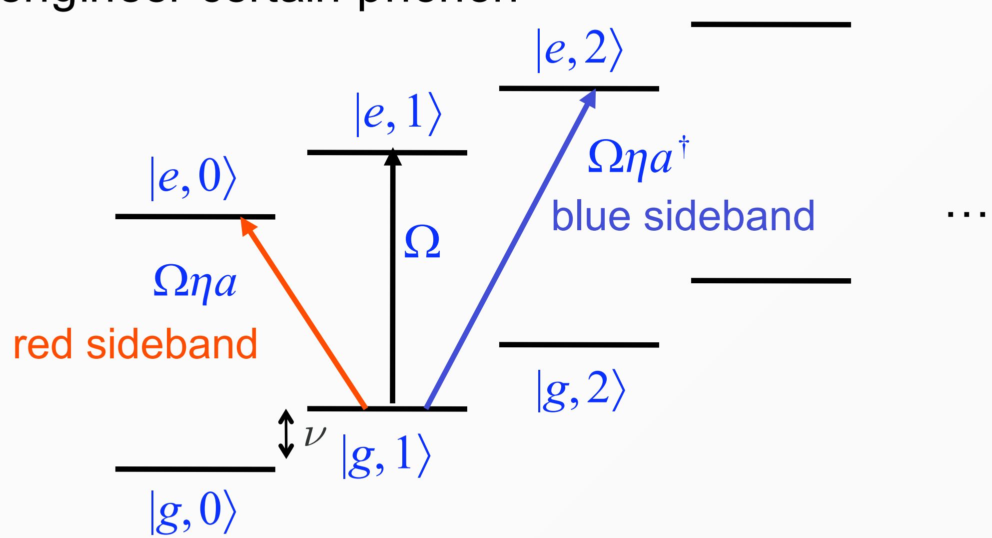
We thus must *design a Hamiltonian*, or a sequence of Hamiltonians, thus that the corresponding time evolution operators give

$$|f\rangle = U|i\rangle \equiv \ldots U_2 U_1 |i\rangle$$

In particular, one can ask the question how to engineer certain phonon (superposition) states

$$|\psi\rangle_{\text{ph}} = \sum_{n=0}^{\infty} c_n |n\rangle$$

using the Hamiltonians we can realize.



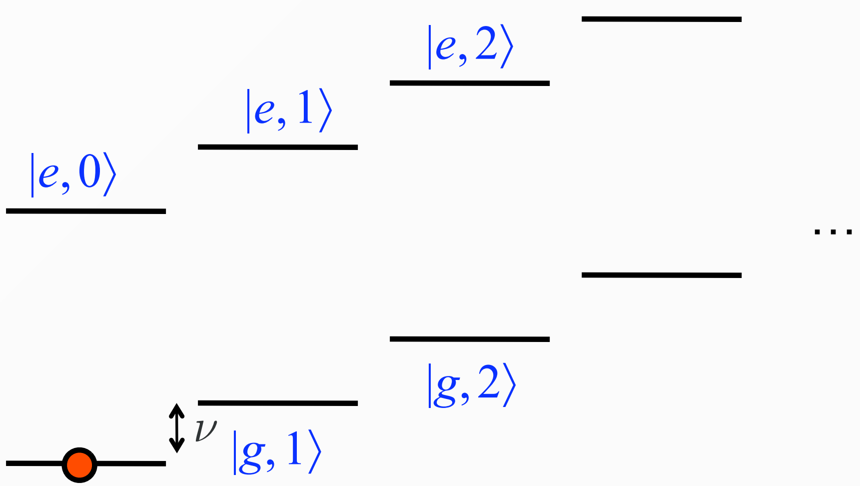
Single Laser-Driven Trapped Ion

Specific relevant examples are:

- Fock states $|n\rangle$
- coherent states $|\alpha\rangle$
- squeezed states $|\alpha, \epsilon\rangle$
- Schrödinger cat states $|\alpha\rangle \pm |-\alpha\rangle$

Initial state: We need a pure state to start with. We assume that the ion can be prepared in the vibrational and atomic ground state using laser cooling techniques (see later).

$$|\Psi(t=0)\rangle \;\; = |g\rangle \otimes |0\rangle$$

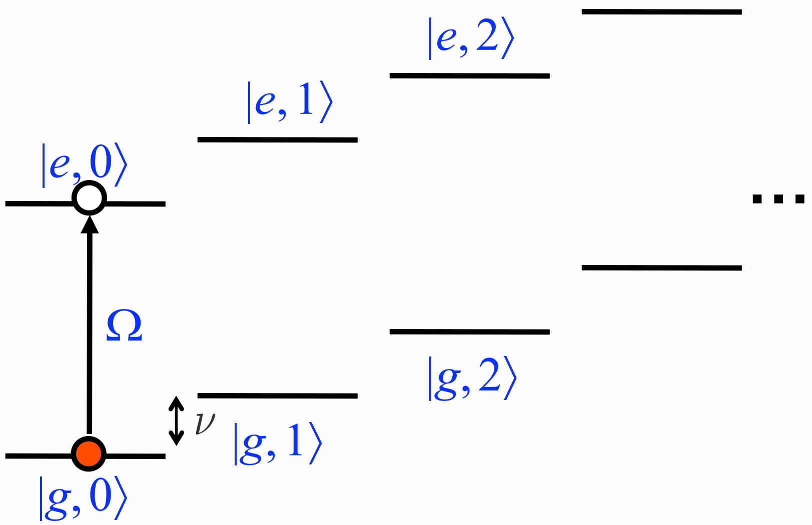


Example 1

Starting from this state we can prepare any superposition

$$|g\rangle \otimes |0\rangle \rightarrow (\alpha|g\rangle + \beta|e\rangle) \otimes |0\rangle$$

by applying an appropriate laser pulse on resonance



The vibrational states are not touched.

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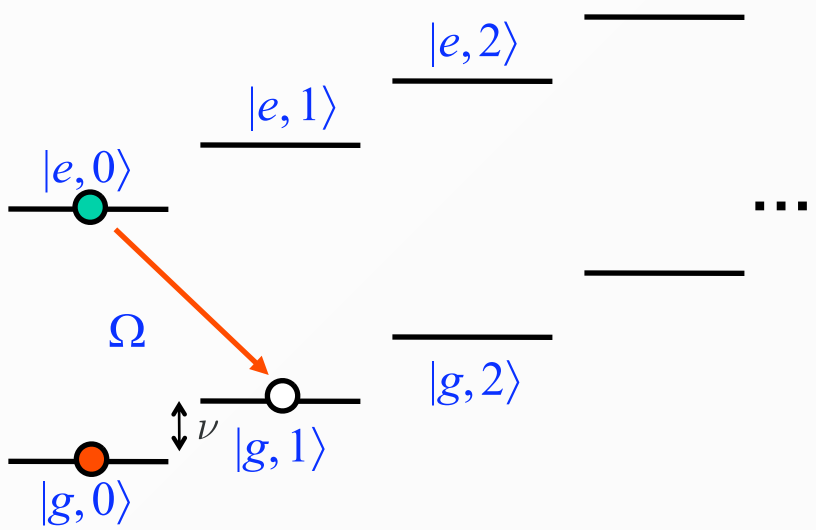
Example 2

We can convert an atomic superposition to a the same superposition of phonon states by applying $\pi -$ laser pulse on the red transition: the state $|g\rangle \otimes |0\rangle$ is not coupled to the laser light, so that $|g\rangle \otimes |0\rangle \rightarrow |g\rangle \otimes |0\rangle$ while $|e\rangle \otimes |0\rangle \rightarrow |g\rangle \otimes |1\rangle$ so that

$$(\alpha|g\rangle + \beta|e\rangle) \otimes |0\rangle \rightarrow |g\rangle \otimes (\alpha|0\rangle + \beta|1\rangle)$$

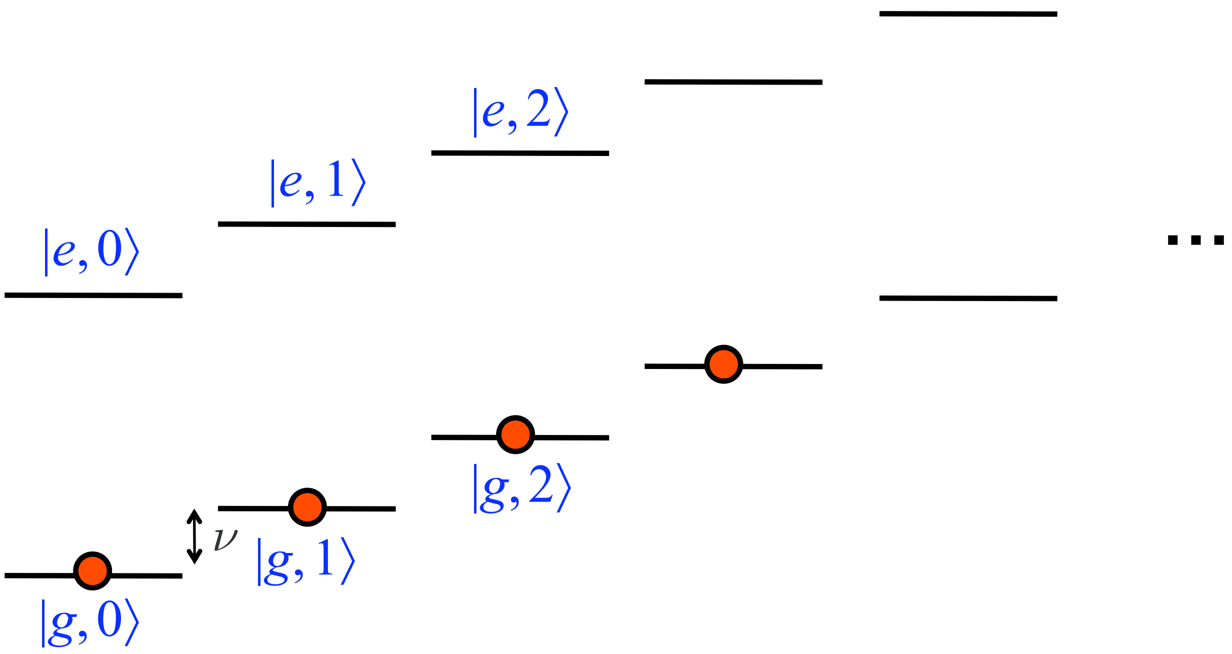


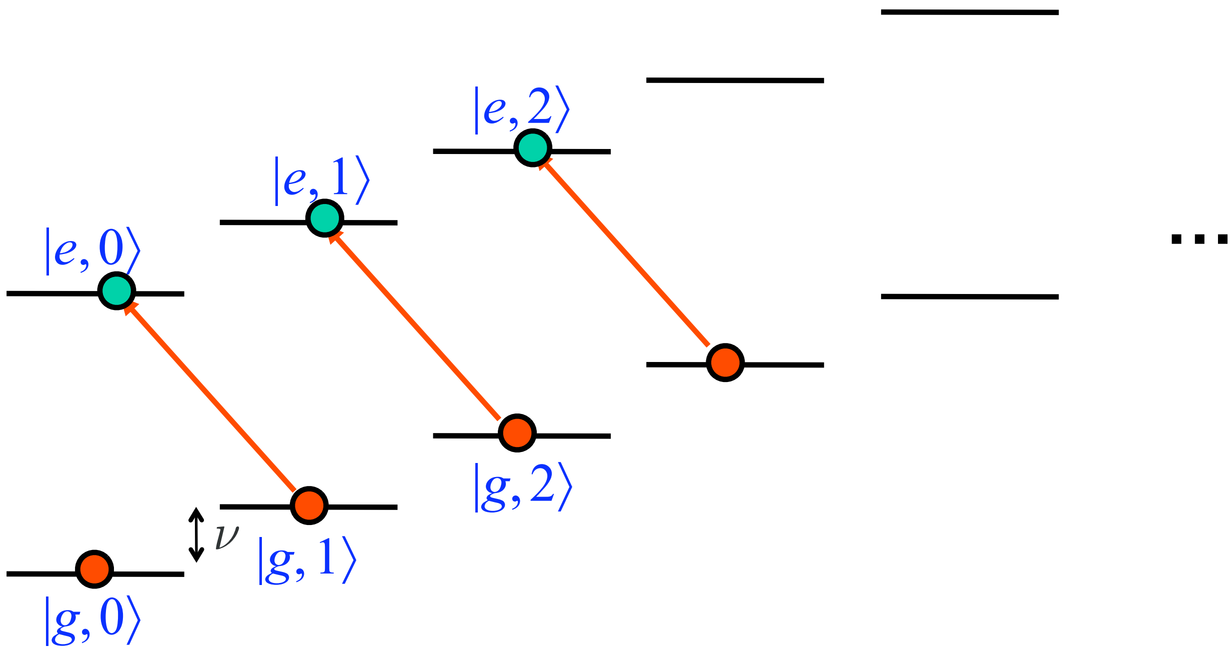
Example 3

we can engineer an arbitrary superposition state of phonon states

$$|g\rangle \otimes |0\rangle \rightarrow |\Psi\rangle = |g\rangle \otimes \sum_{n=0}^{N} c_n |n\rangle$$

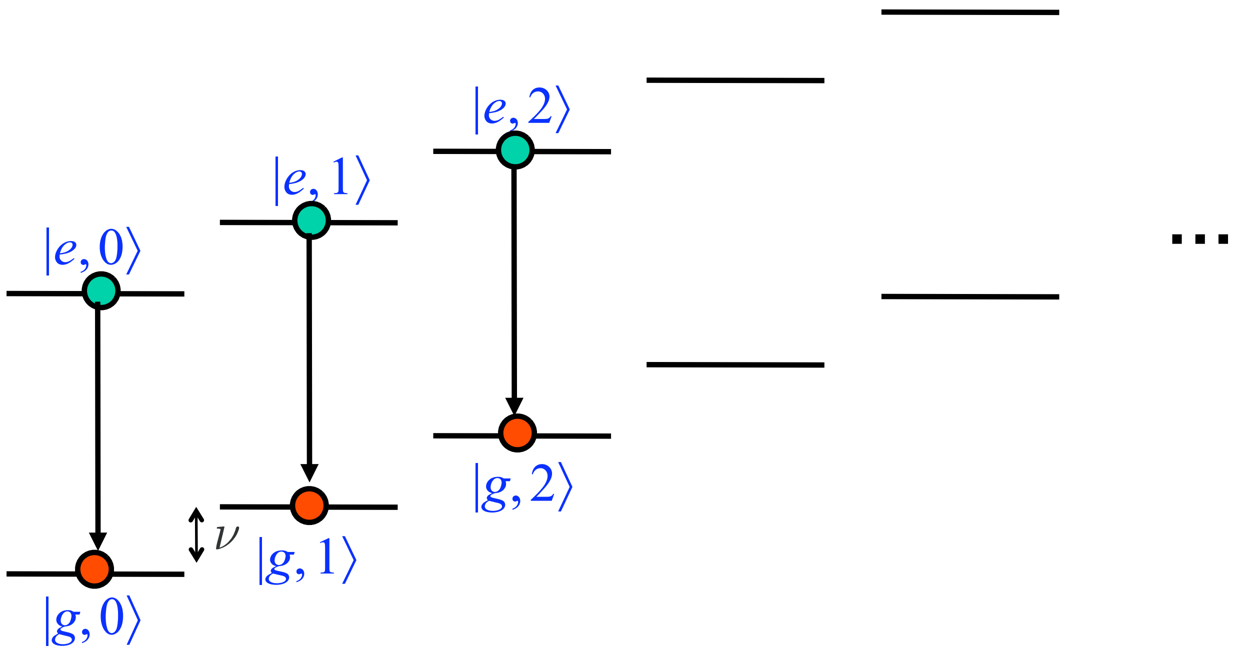
for given coefficients c_n.



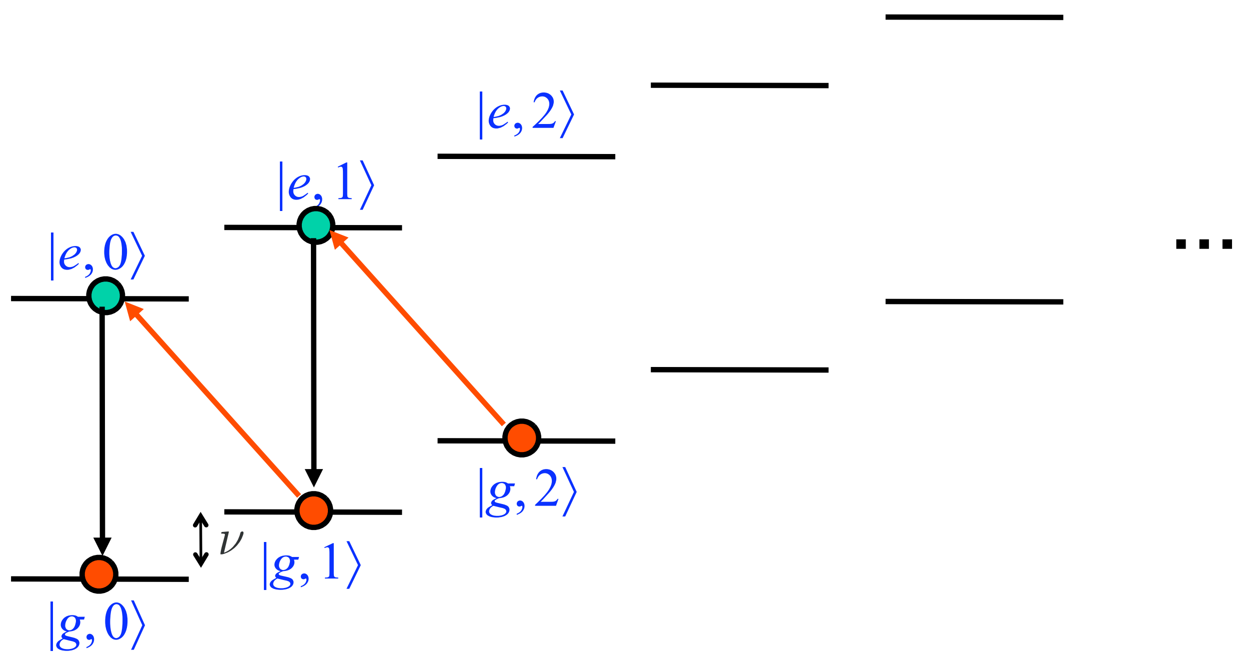


Procedure: Applying a laser on the red sideband we couple the states $|g\rangle|n\rangle \leftrightarrow |e\rangle|n-1\rangle$.

As a first step we apply a π-pulse so that we make the amplitude of $|g\rangle|N\rangle$ equal to zero by transferring the amplitude c_N to $|e\rangle|N-1\rangle$. But we now have a superposition of ground and excited state.



In the second step we apply a resonant laser so that we transform the known! superposition of $|g\rangle|N-1\rangle$, $|e\rangle|N-1\rangle$ to $|g\rangle|N-1\rangle$ with no amplitude left in $|e\rangle|N-1\rangle$. Now we repeat the argument until we have transformed the state to $|g\rangle|0\rangle$.



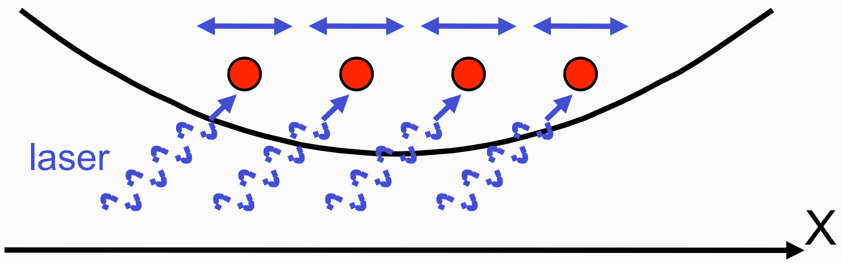
In the second step we apply a resonant laser so that we transform the known! superposition of $|g\rangle|N-1\rangle$, $|e\rangle|N-1\rangle$ to $|g\rangle|N-1\rangle$ with no amplitude left in $|e\rangle|N-1\rangle$. Now we repeat the argument until we have transformed the state to $|g\rangle|0\rangle$.

The inverse transformation produces the desired state starting from the ground state.

Ions in a linear trap

The above model is readily extended to describe a string of N ions in a linear trap

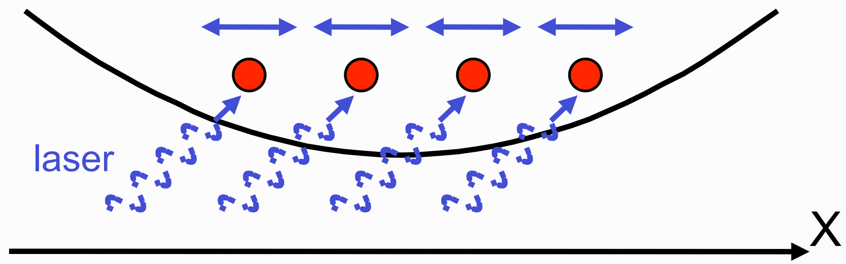
A linear trap corresponds to a confinement of the motion along x,y and z directions in an (anisotropic) harmonic potential of frequencies $\nu \equiv \nu_x \ll \nu_y, \nu_z$.

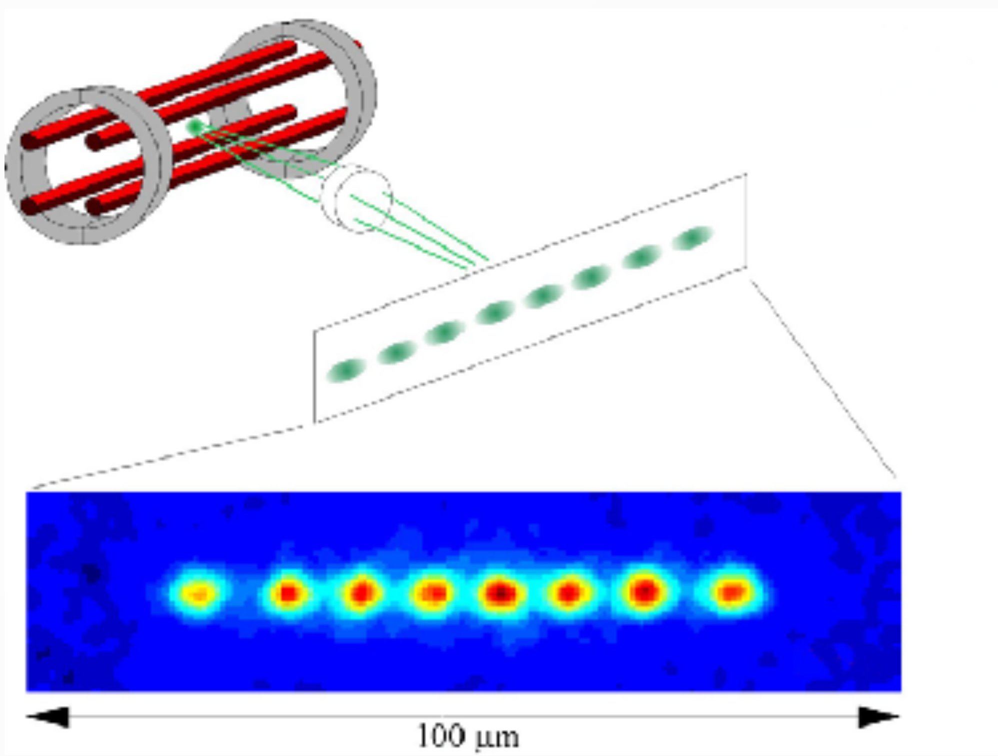


The equilibrium position of the ions will be given by the confining forces of the trapping potential balancing the Coulomb repulsion between the ions.

If the ions have been previously laser cooled in all three dimensions they undergo small oscillations around these equilibrium position. In this case, the motion of the ions is described in terms of normal modes.

String of Laser-Driven Trapped Ions





String of Laser-Driven Trapped Ions

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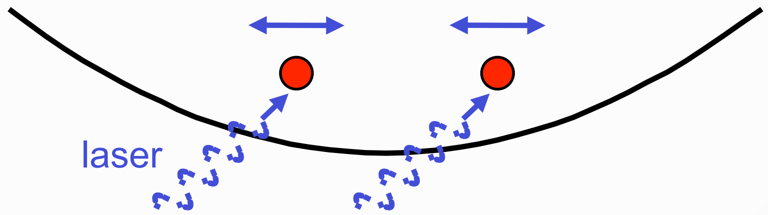
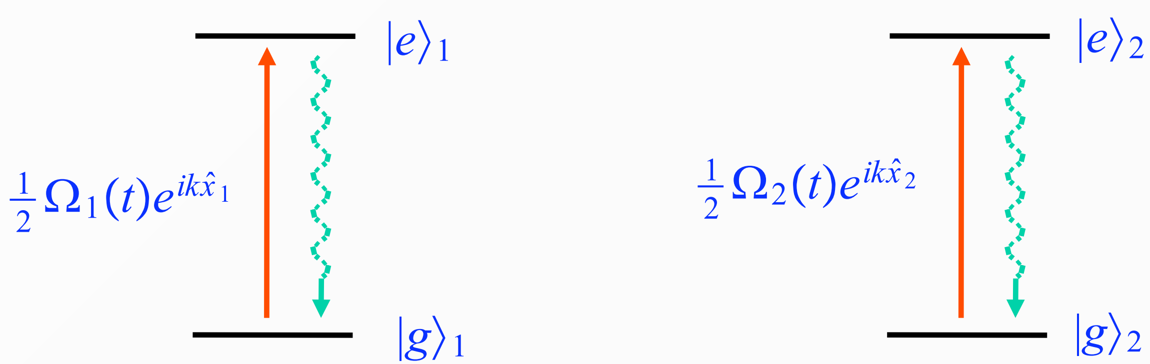
1D model for two ions in a linear trap



$$H = \frac{\hat{p}_1^2}{2M} + \frac{1}{2}M\nu^2\hat{x}_1^2 + \frac{\hat{p}_2^2}{2M} + \frac{1}{2}M\nu^2\hat{x}_2^2 + \frac{e^2}{4\pi\epsilon_0|\hat{x}_1 - \hat{x}_2|}$$
$$+ \hbar\omega_{1eg}|e\rangle_{11}\langle e| + \hbar\omega_{2eg}|e\rangle_{22}\langle e|$$
$$+ \left[\frac{1}{2}\Omega_1(t)e^{ik\hat{x}_1 - i\omega t}|e\rangle_{11}\langle g| + \text{h.c.}\right] + \left[\frac{1}{2}\Omega_2(t)e^{ik\hat{x}_2 - i\omega t}|e\rangle_{22}\langle g| + \text{h.c.}\right]$$



String of Laser-Driven Trapped Ions

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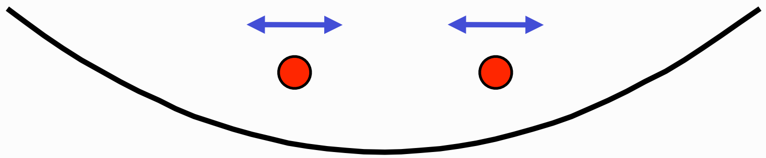
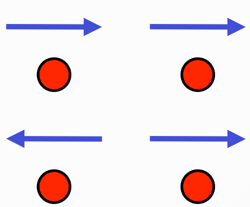
1D model for two ions in a linear trap



center-of-mass (COM) and relative coordinates

$$\hat{X} = \tfrac{1}{2}(\hat{x}_1 + \hat{x}_2) \qquad \hat{x}_1 = \hat{X} - \tfrac{1}{2}\hat{x}$$
$$\hat{x} = \hat{x}_2 - \hat{x}_1 \qquad \hat{x}_2 = \hat{X} + \tfrac{1}{2}\hat{x}$$

$$H = \left[\frac{\hat{P}^2}{2(2M)} + \frac{1}{2}(2M)\nu^2\hat{X}^2\right] + \left[\frac{\hat{p}^2}{2(M/2)} + \frac{1}{2}(\frac{1}{2}M)\nu^2\hat{x}^2 + \frac{e^2}{4\pi\epsilon_0|\hat{x}|}\right]$$
$$+\hbar\omega_{1eg}|e\rangle_{11}\langle e| + \hbar\omega_{2eg}|e\rangle_{22}\langle e|$$
$$+\left[\frac{1}{2}\Omega_1(t)e^{ik\hat{X}}e^{-ik\frac{1}{2}\hat{x}}e^{-i\omega t}|e\rangle_{11}\langle g| + \text{h.c.}\right] + \left[\frac{1}{2}\Omega_2(t)e^{ik\hat{X}}e^{+ik\frac{1}{2}\hat{x}}e^{-i\omega t}|e\rangle_{22}\langle g| + \text{h.c.}\right]$$



center of mass mode ν

stretchmode $\sqrt{3}\,\nu$

for N ions the separation remains with increasing N

String of Laser-Driven Trapped Ions

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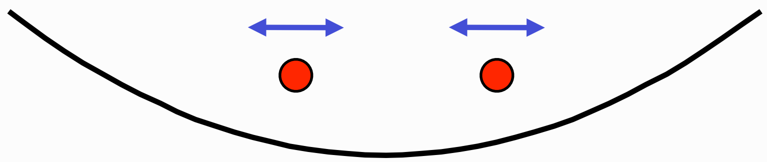
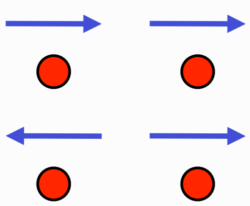
1D model for two ions in a linear trap



center-of-mass (COM) and relative coordinates

$$
\begin{aligned}
H = \; & \nu a^\dagger_{\rm cm} a_{\rm cm} + \sqrt{3}\,\nu a^\dagger_{\rm r} a_{\rm r} \\
& - \Delta_1 |e\rangle_{1\,1}\langle e| - \Delta_2 |e\rangle_{2\,2}\langle e| \\
& + \left[\frac{1}{2} \Omega_1(t) e^{-i\eta_{\rm cm}(a_{\rm cm}+a^\dagger_{\rm cm})} e^{-i\eta_{\rm r}(a_{\rm r}+a^\dagger_{\rm r})} |e\rangle_{1\,1}\langle g| + \text{h.c.} \right] \\
& + \left[\frac{1}{2} \Omega_2(t) e^{-i\eta_{\rm cm}(a_{\rm cm}+a^\dagger_{\rm cm})} e^{+i\eta_{\rm r}(a_{\rm r}+a^\dagger_{\rm r})} |r\rangle_{2\,2}\langle g| + \text{h.c.} \right].
\end{aligned}
$$



center of mass mode ν

stretchmode $\sqrt{3}\nu$

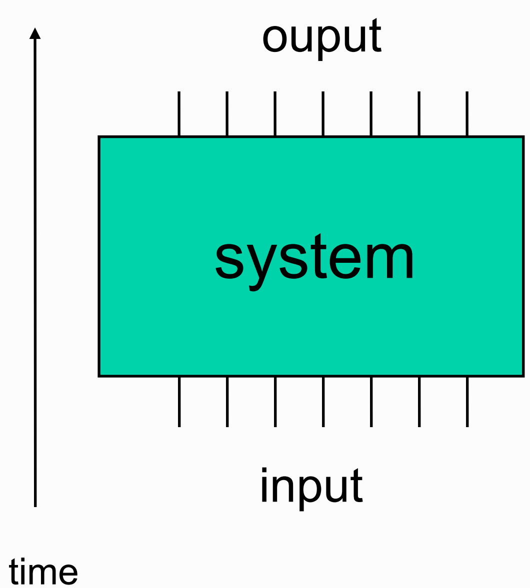
for N ions the separation remains with increasing N

Background Material

Basic Quantum Computing

Quantum Computing

- computing as a physical process



- quantum computing

$|\psi_{\text{out}}\rangle$ — state measurement

$|\psi_{\text{out}}\rangle = \hat{U}|\psi_{\text{in}}\rangle$ — unitary evolution

$|\psi_{\text{in}}\rangle$ — preparation

- no coupling to environment (decoherence)

Quantum Memory

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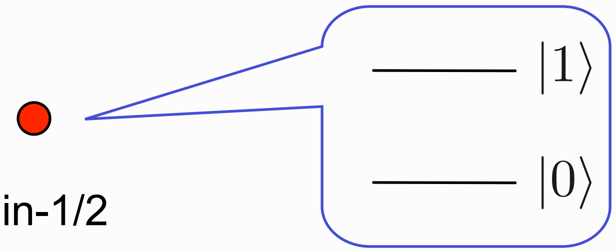
- Qubit:



spin-1/2

$$\alpha|0\rangle + \beta|1\rangle \in \mathcal{H}_2$$

superposition

- Quantum register

N spin-1/2 systems

$$|\Psi\rangle = c_{000}|000\rangle + c_{001}|001\rangle + \ldots + c_{111}|111\rangle$$

entangled state

- quantum parallelism
- interference of computational paths
  (+ cleverness) = quantum algorithms

Quantum Gates

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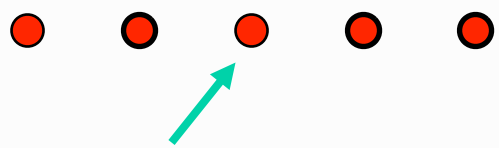
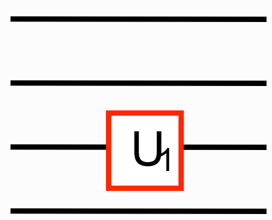
$$|\psi_{\text{in}}\rangle \longrightarrow |\psi_{\text{out}}\rangle = \hat{U}|\psi_{\text{in}}\rangle$$ with U unitary

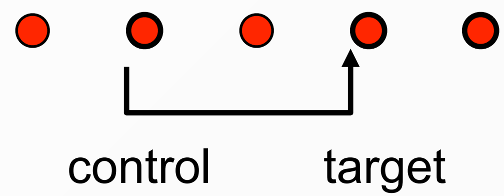
- Single qubit gate



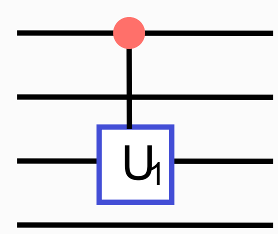
$\hat{U}_1 =$ rotation of a single qubit



- **Two qubit gate**



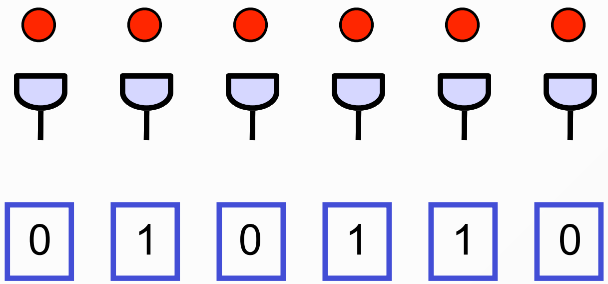
$$\hat{U}_{1+2} = |0\rangle_1\langle 0| \otimes \hat{1}_2 + |1\rangle_1\langle 1| \otimes \hat{U}_1$$



- A general unitary transformation can be decomposed into single bit rotations and a universal two-bit quantum gate

3. Read out

- state measurement



String of Laser-Driven Trapped Ions

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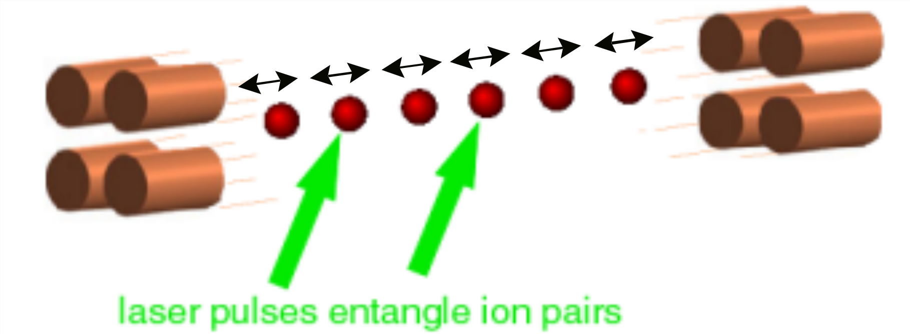
Ion Trap Quantum Computer '95

- Cold ions in a linear trap



Qubits: internal atomic states

1-qubit gates: addressing ions with a laser

2-qubit gates: entanglement via exchange of phonons of quantized collective mode

- State vector

$$|\Psi\rangle = \sum c_x |x_{N-1}, \ldots , x_0\rangle_{\text{atom}} \quad |0\rangle_{\text{phonon}}$$

quantum register   databus

- QC as a time sequence of laser pulses
- Read out by quantum jumps

String of Laser-Driven Trapped Ions

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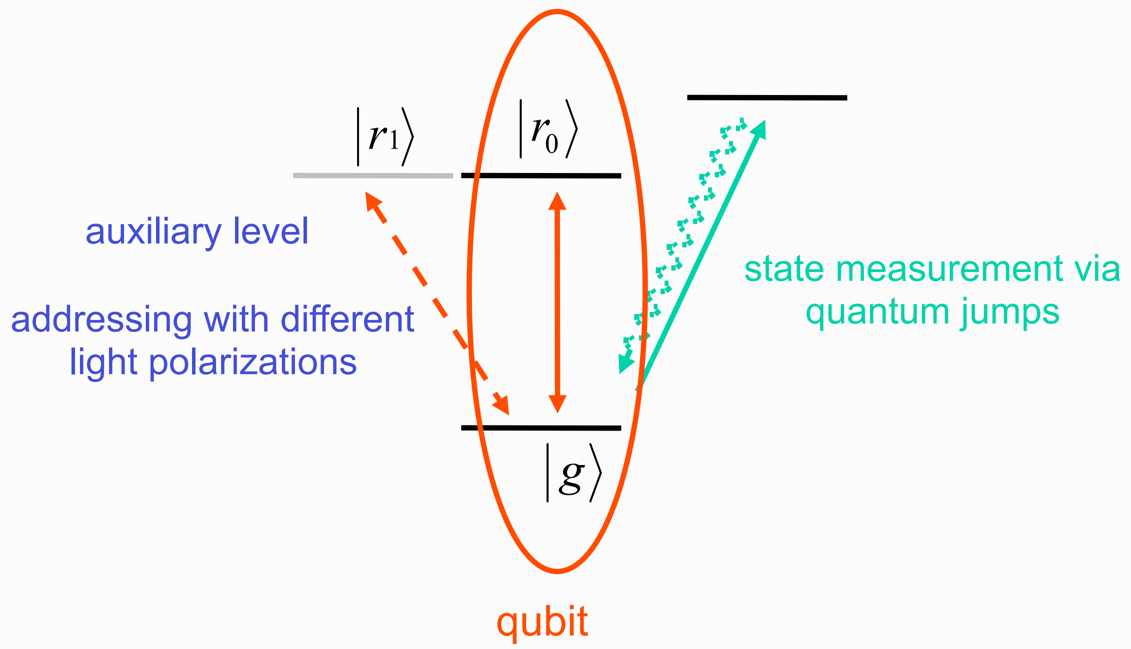
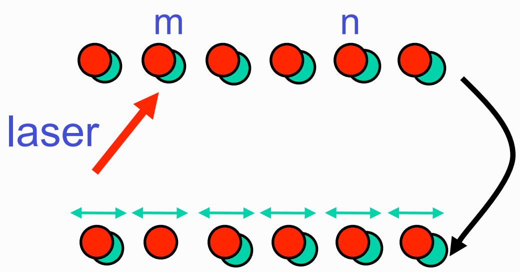
Level scheme

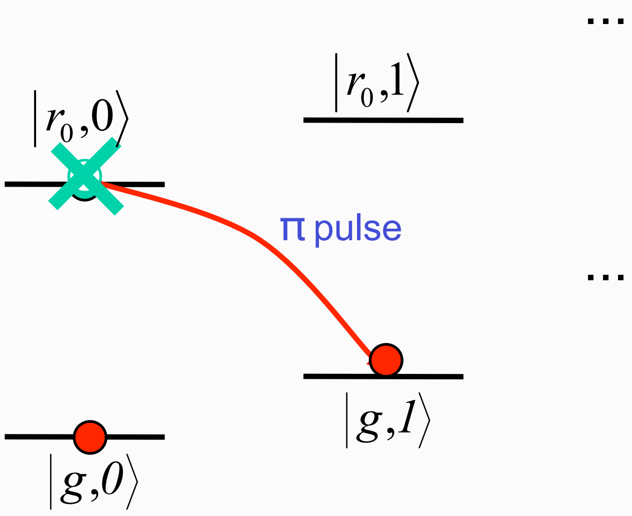


Two-qubit phase gate

- step 1: swap first qubit to phonon

first atom: m





$$\begin{array}{ccc} & \hat{U}_m^{\pi,0} & \\ |g\rangle_m|0\rangle & \longrightarrow & |g\rangle_m|0\rangle \\ |r\rangle_m|0\rangle & \longrightarrow & -i|g\rangle_m|1\rangle \end{array}$$

String of Laser-Driven Trapped Ions

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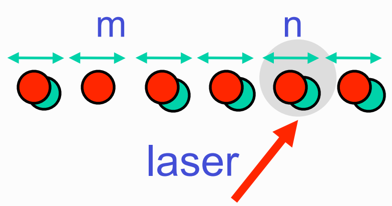
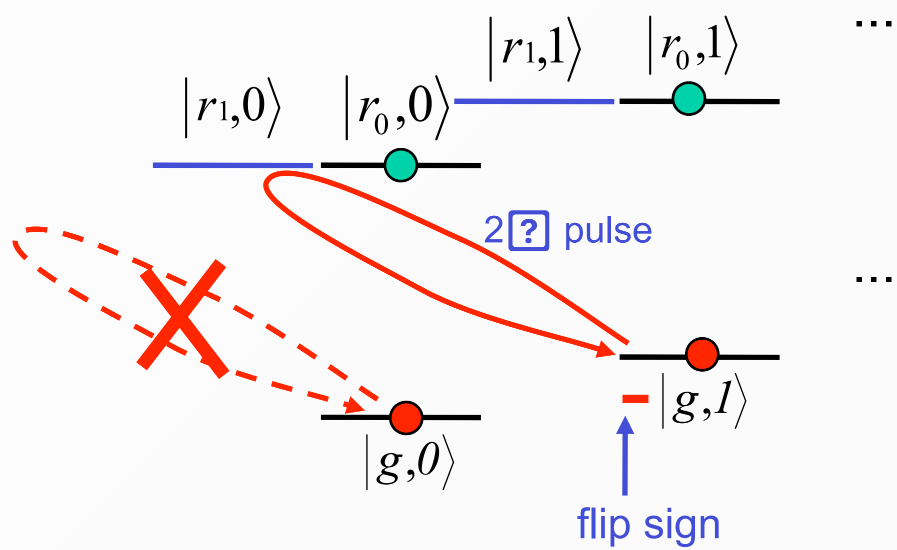
- step 2: conditional sign change

second atom: n





$$\begin{array}{rcr} & \hat{U}_n^{2\pi,1} & \\ |g\rangle_m|g\rangle_n|0\rangle & \longrightarrow & |g\rangle_m|g\rangle_n|0\rangle \\ |g\rangle_m|r\rangle_n|0\rangle & \longrightarrow & |g\rangle_m|r\rangle_n|0\rangle \\ -i|g\rangle_m|g\rangle_n|1\rangle & \longrightarrow & i|g\rangle_m|g\rangle_n|1\rangle \\ -i|g\rangle_m|r\rangle_n|1\rangle & \longrightarrow & -i|g\rangle_m|r\rangle_n|1\rangle \end{array}$$

String of Laser-Driven Trapped Ions

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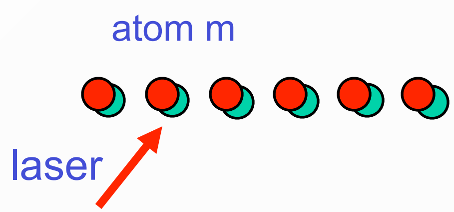
- step 3: swap phonon back to first qubit



$$|g\rangle_m \qquad \begin{array}{rcl} & \hat{U}_m^{\pi,0} & \\ |g\rangle_n|0\rangle & \longrightarrow & |g\rangle_m|g\rangle_n \\ |r\rangle_n|0\rangle & \longrightarrow & |g\rangle_m|r\rangle_n \\ i|g\rangle_n|1\rangle & \longrightarrow & |r\rangle_m|g\rangle_n \\ -i|r\rangle_n|1\rangle & \longrightarrow & -|r\rangle_m|r\rangle_n \end{array} \qquad |0\rangle$$

String of Laser-Driven Trapped Ions

- summary: we have a phase gate between atom m and n

$$
\begin{aligned}
|g\rangle|g\rangle\ |0\rangle &\longrightarrow \quad |g\rangle|g\rangle\ |0\rangle, \\
|g\rangle|r_0\rangle\ |0\rangle &\longrightarrow \quad |g\rangle|r_0\rangle\ |0\rangle, \\
|r_0\rangle|g\rangle\ |0\rangle &\longrightarrow \quad |r_0\rangle|g\rangle\ |0\rangle, \\
|r_0\rangle|r_0\rangle|0\rangle &\longrightarrow -|r_0\rangle|r_0\rangle\ |0\rangle.
\end{aligned}
$$

phonon mode returned to initial state

$$|\epsilon_1\rangle|\epsilon_2\rangle \to (-1)^{\epsilon_1 \epsilon_2}|\epsilon_1\rangle|\epsilon_2\rangle \quad (\epsilon_{1,2} = 0, 1)$$

Rem.: this idea translates immediately to CQED

String of Laser-Driven Trapped Ions

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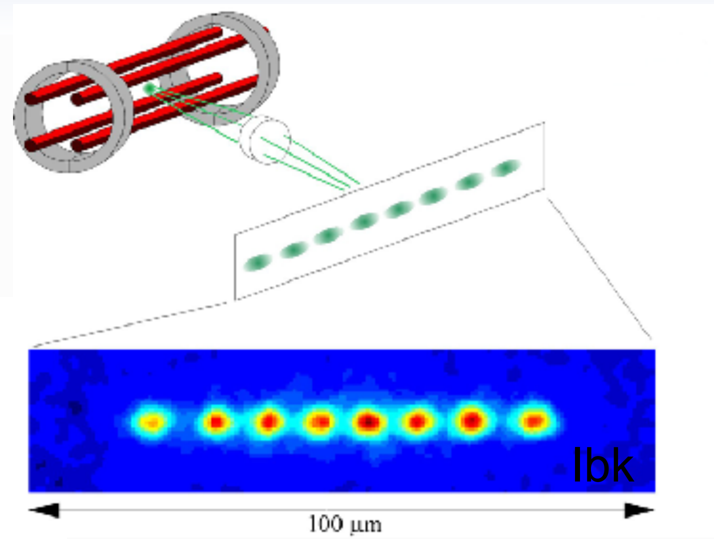
Quantum gates with ions: Nature March 2003

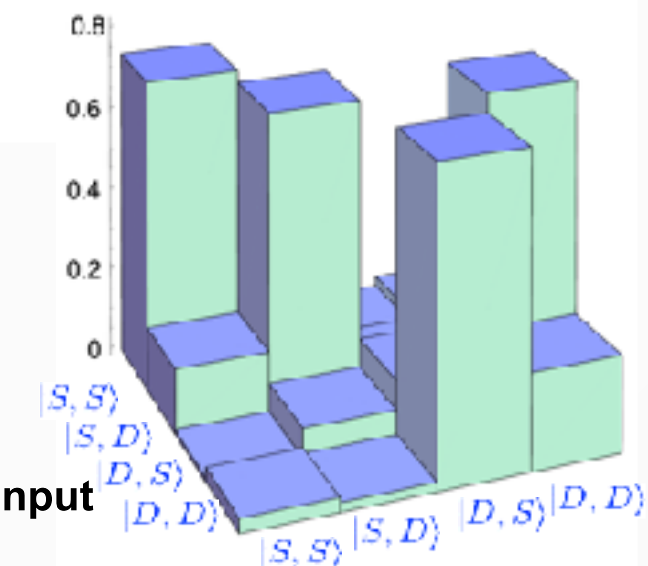


Realization of the Cirac–Zoller controlled-NOT quantum gate

Ferdinand Schmidt-Kaler, Hartmut Häffner, Mark Riebe, Stephan Gulde, Gavin P. T. Lancaster, Thomas Deuschle, Christoph Becher, Christian F. Roos, Jürgen Eschner & Rainer Blatt

Institut für Experimentalphysik, Universität Innsbruck, Technikerstraße 25, A-6020 Innsbruck, Austria

truth table CNOT



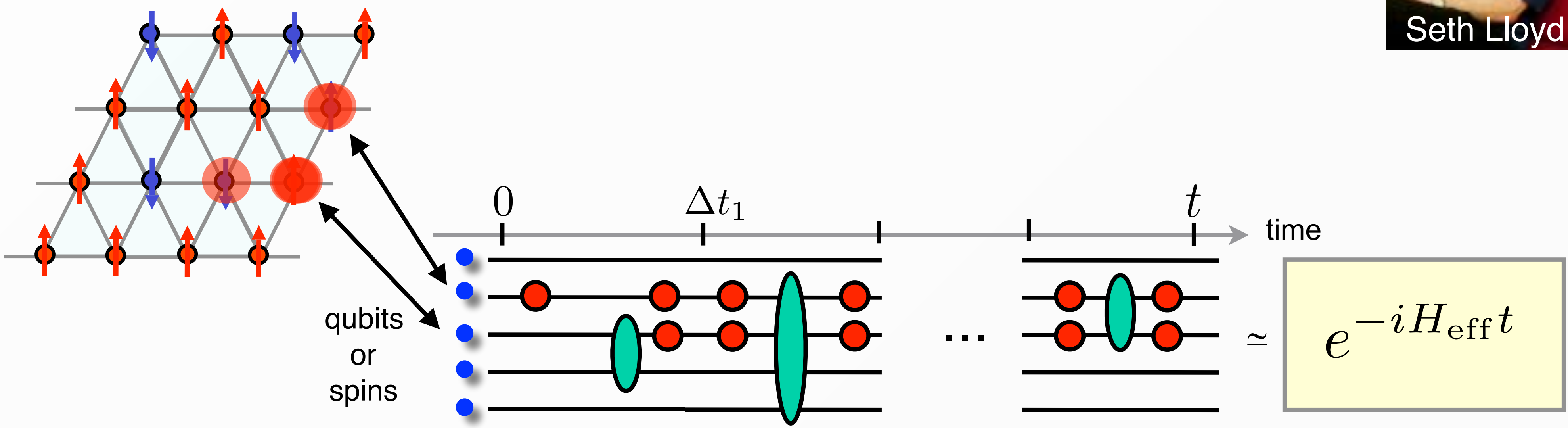
Experimental demonstration of a robust, high-fidelity geometric two ion-qubit phase gate

D. Leibfried*†, B. DeMarco*, V. Meyer*, D. Lucas*‡, M. Barrett*, J. Britton*, W. M. Itano*, B. Jelenković*§, C. Langer*, T. Rosenband* & D. J. Wineland*

Where we are today …

Digital and Analog Quantum Simulation with Trapped Ions

Digital Quantum Simulation



idea: approximate time evolution by a stroboscopic sequence of gates

$$U(t) \equiv e^{-iHt/\hbar} = e^{-iH\Delta t_n/\hbar} \cdots e^{-iH\Delta t_1/\hbar}$$

Trotter expansion:

$$e^{-iH\Delta t/\hbar} \simeq \underbrace{e^{-iH_1\Delta t/\hbar}}_{\text{first term}} \underbrace{e^{-iH_2\Delta t/\hbar}}_{\text{second term}} \underbrace{e^{\frac{1}{2}\frac{(\Delta t)^2}{\hbar^2}[H_1,H_2]}}_{\text{Trotter errors for non-commuting terms}}$$

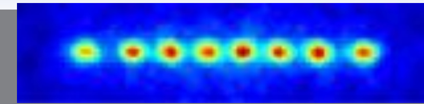
first term

second term

Trotter errors for non-commuting terms

$$H = \boxed{-J\sigma_1^z\sigma_2^z} + \boxed{B(\sigma_1^x + \sigma_2^x)}$$

SCIENCE VOL 334 7 OCTOBER 2011



REPORTS

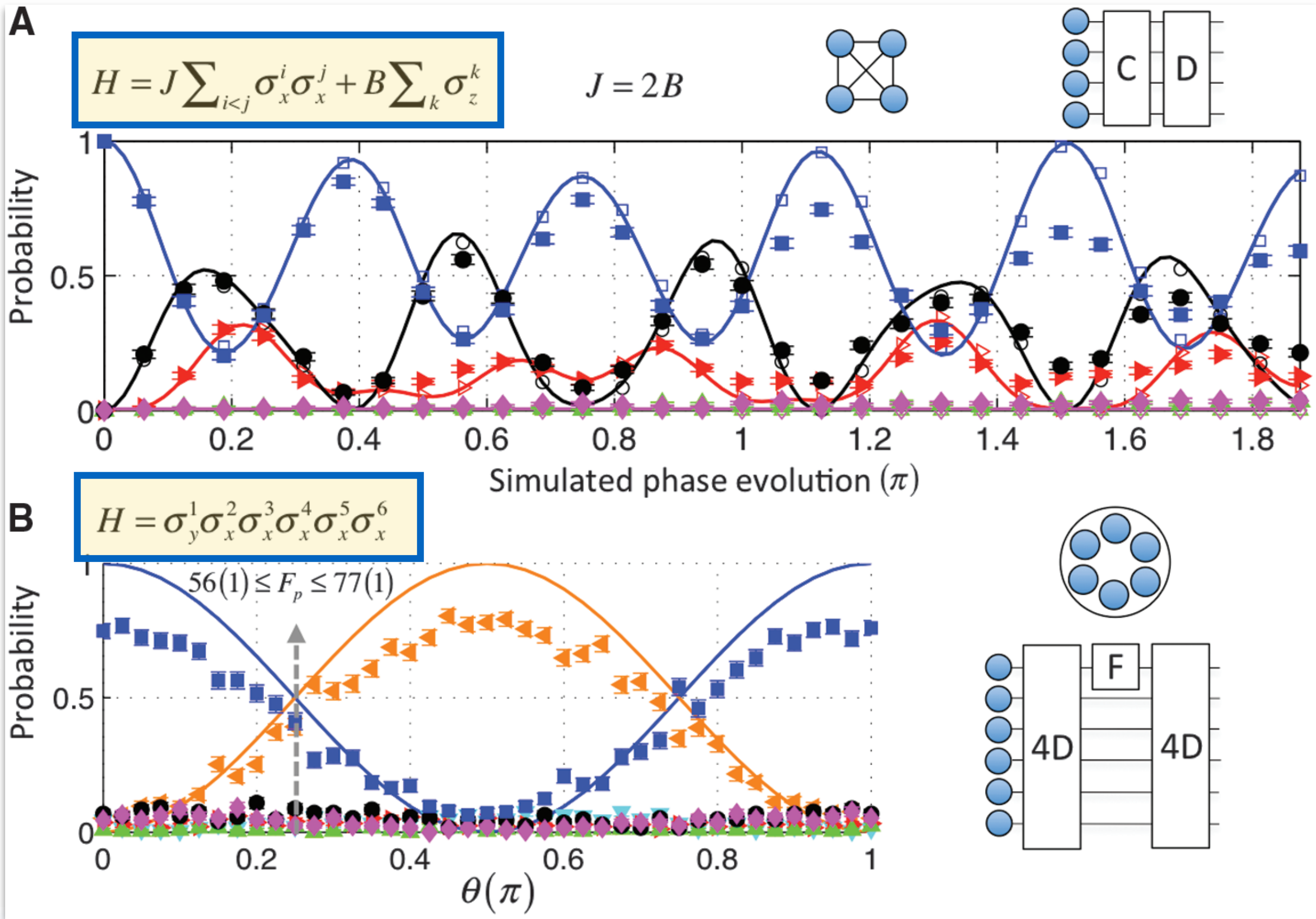
Universal Digital Quantum Simulation with Trapped Ions

B. Lanyon

C. Roos

B. P. Lanyon,[1,2*] C. Hempel,[1,2] D. Nigg,[2] M. Müller,[1,3] R. Gerritsma,[1,2] F. Zähringer,[1,2] P. Schindler,[2] J. T. Barreiro,[2] M. Rambach,[1,2] G. Kirchmair,[1,2] M. Hennrich,[2] P. Zoller,[1,3] R. Blatt,[1,2] C. F. Roos[1,2]

4 & 6 Spins



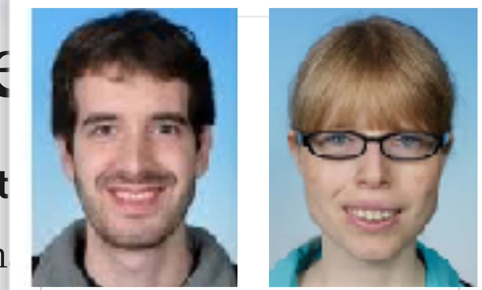
remarks:

- scalability (?)
- error correction (?)

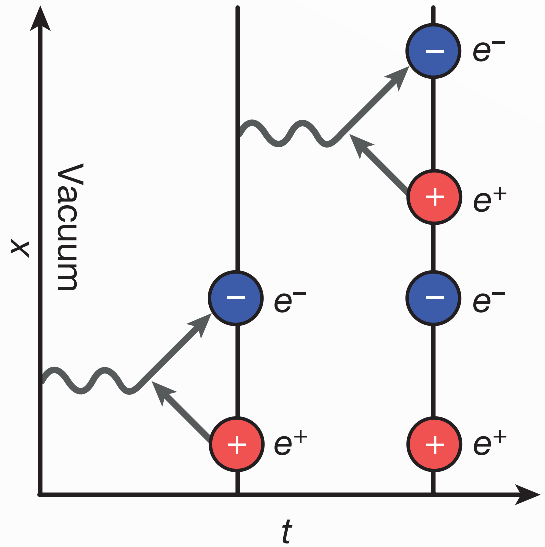
Real-time dynamics of lattice gauge the
few-qubit quantum computer

doi:10.1038/nat

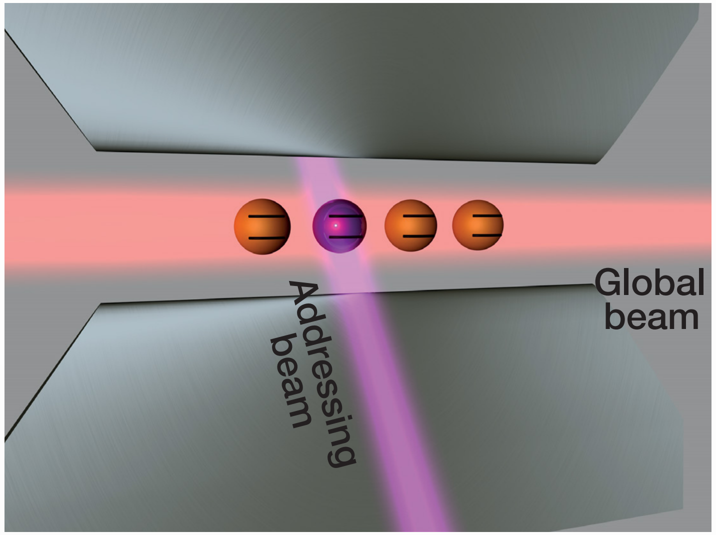
Esteban A. Martinez[1]*, Christine A. Muschik[2,3]*, Philipp Schindler[1], Daniel Nigg[1], Alexander Erh
Philipp Hauke[2,3], Marcello Dalmonte[2,3], Thomas Monz[1], Peter Zoller[2,3] & Rainer Blatt[1,2]


E. Martinez C. Muschik

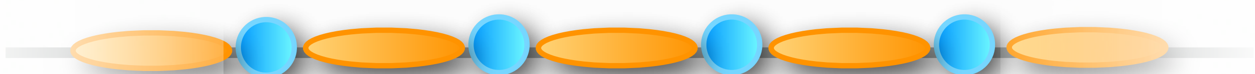
Schwinger pair production



ion trap quantum computer



Schwinger Model: 1+1D QED

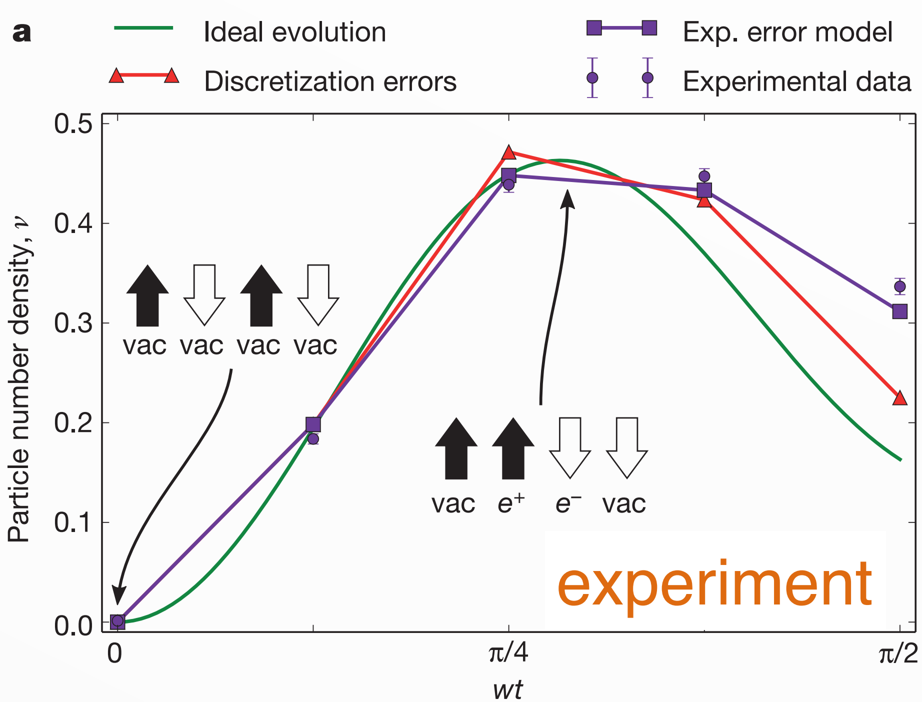


$$\hat{H}_{\text{lat}} = -iw\sum_{n=1}^{N-1}\left[\hat{\Phi}_n^{\dagger} e^{i\hat{\theta}_n}\hat{\Phi}_{n+1} - \text{h.c.}\right] + J\sum_{n=1}^{N-1}\hat{L}_n^2 + m\sum_{n=1}^{N}(-1)^n\hat{\Phi}_n^{\dagger}\hat{\Phi}_n$$

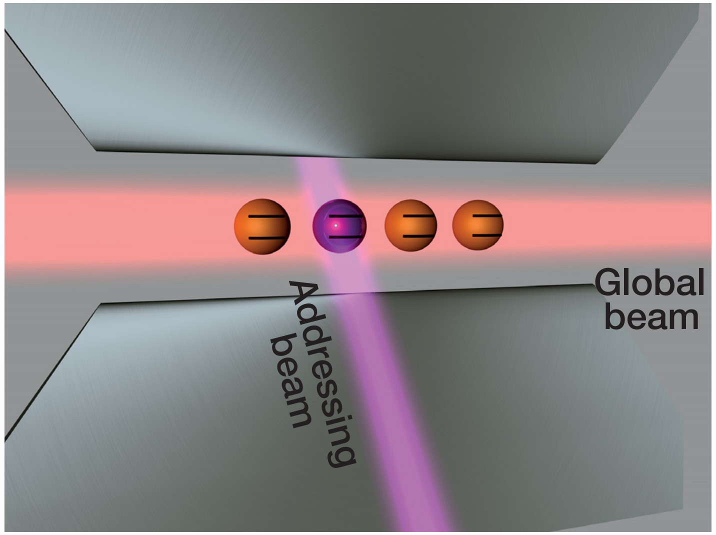
Kogut-Susskind Hamiltonian (Wilson LGT)

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

Schwinger pair production



ion trap quantum computer



Digital Quantum Simulation of an Exotic Spin Model

- obtained after integrating ... gauge field

220 quantum gates

Analog Quantum Simulation

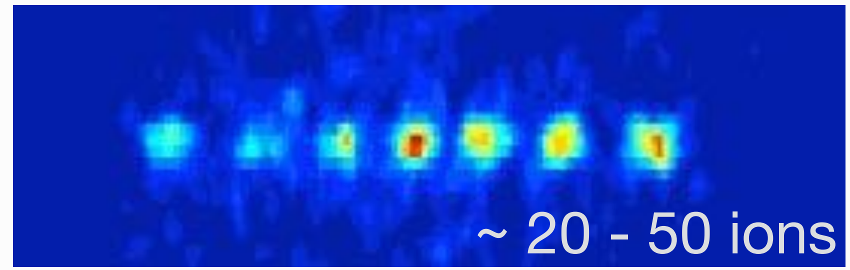
- **spin models**

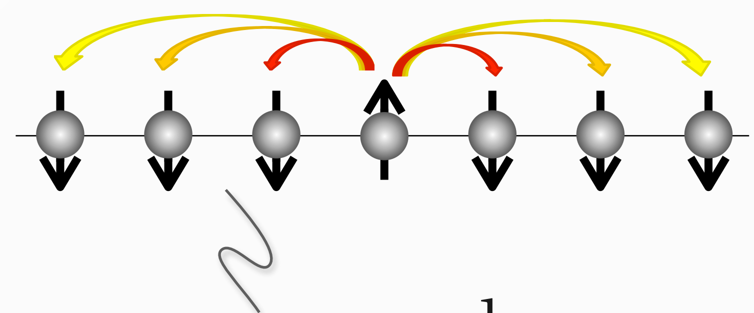
$$H = \hbar \sum_{i,j} J_{ij} \sigma_x^{(i)} \sigma_x^{(j)} + \hbar B \sum_i \sigma_z^{(i)}$$

K. Kim, C. Monroe et al., Nature (2010)

P Jurcevic, BP Lanyon, R Blatt, C. Roos et al. (2014)

- **trapped ions**

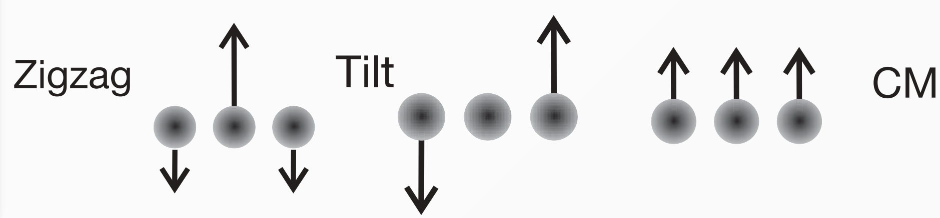




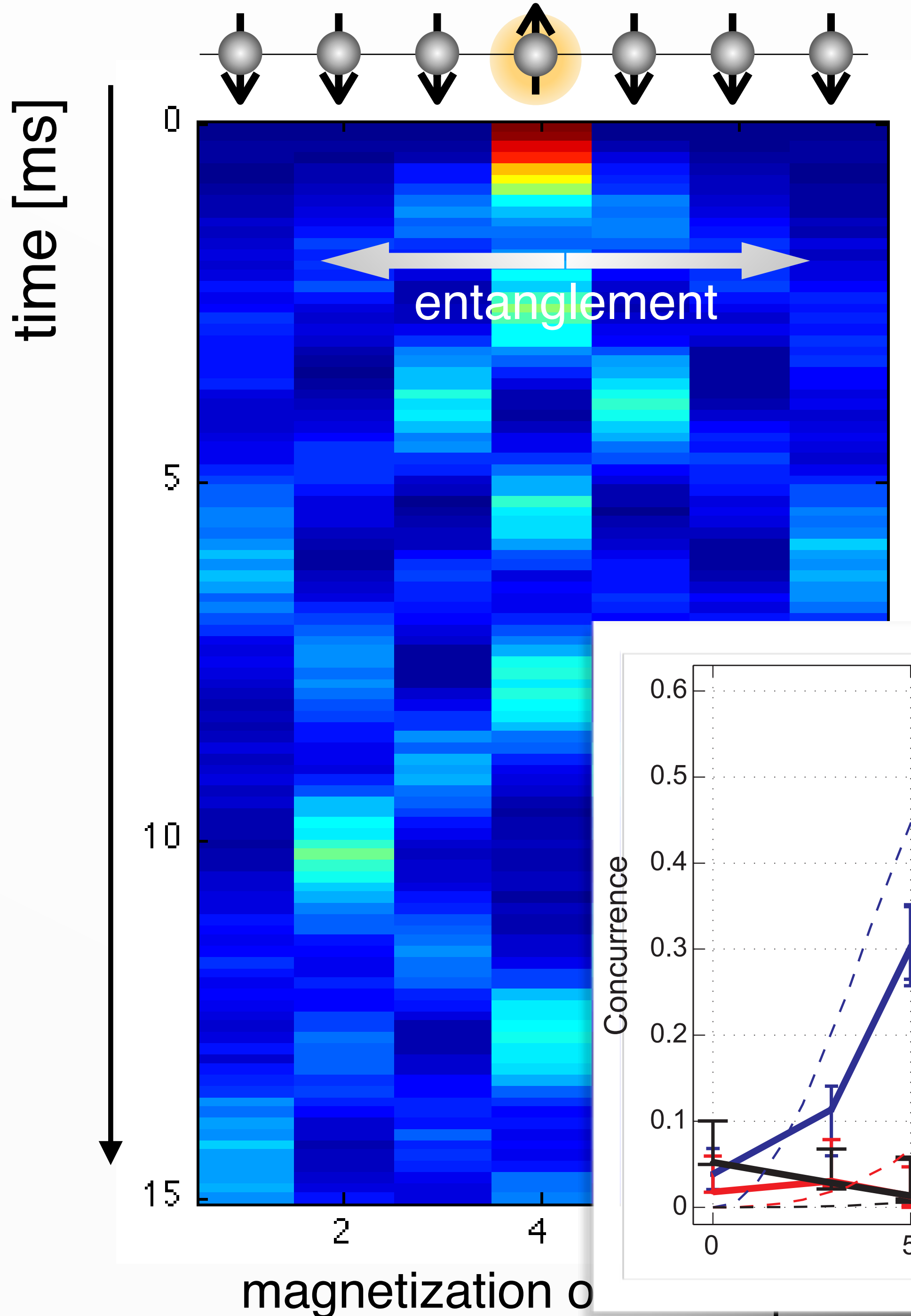
$$J_{ij} \sim \frac{1}{|i-j|^{\alpha}} \qquad 0 \ldots 3$$

tunable range interaction

- We “build” a quantum system with desired Hamiltonian & *controllable parameters*, e.g. Hubbard models of atoms in optical lattices



Magnon propagation

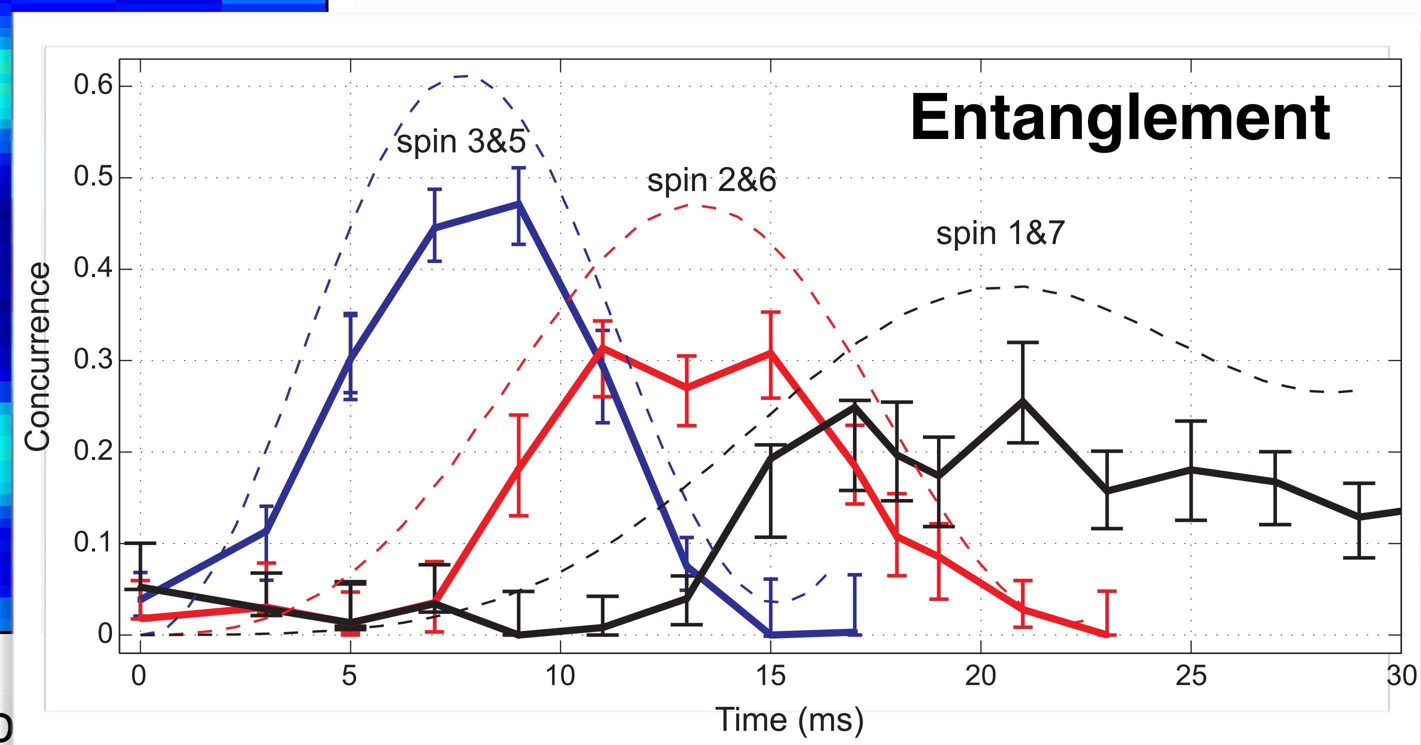


$$\breve{H}_{XY} = \sum_{i<j} J_{ij}(\sigma^+\sigma^- + \sigma^-\sigma^+).$$

✓ Light-cone-like spreading of entanglement

✓ breakdown of the quantum speed-limit due to long range interactions

[exp & theory indistinguishable]



P Jurcevic, et al., PZ, R Blatt & CF Roos, Nature (2014); related work by C. Monroe group deleted

Tomorrow …

A Short Preview

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What is Quantum Optics?

… a Short Tour & Overview

[Experiment + Theory]

- Quantum Properties of Light
- ⇨ [Quantum] Interaction of Light & Matter

control

… a Few Examples

Ex 4: Quantum Metrology with Cold Atoms

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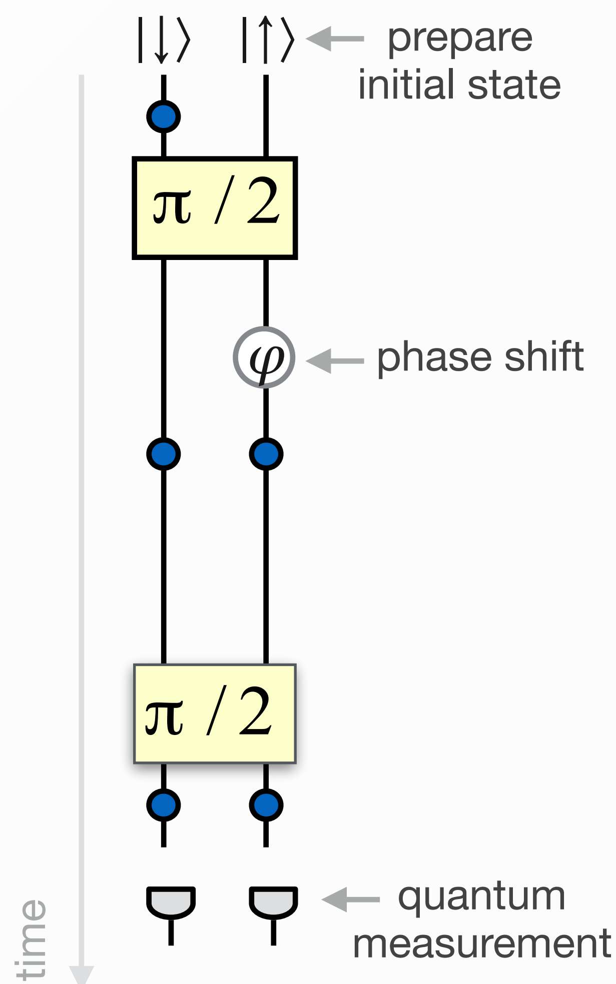
… measurements beyond the Standard Quantum Limit

Ramsey interferometer



N - independent atoms

product state $\sim |\downarrow_1\rangle \dots |\downarrow_N\rangle$

N - entangled atoms

GHZ $\sim |\downarrow_1 \dots \downarrow_N\rangle + |\uparrow_1 \dots \uparrow_N\rangle$

or: spin squeezed state

coherent control

input states

$$\delta\varphi \sim \frac{1}{\sqrt{N}}$$

standard quantum noise limit

$$\delta\varphi \sim \frac{1}{N}$$

Heisenberg limit

parameter estimation of phase shift

[Fisher information, Cramer-Rao bound etc.]

quantum optical systems allow measurements beyond SQL

Tutorial: J Huang et al., arxiv1308.6092

Ex 5: Cavity QED & Quantum Networks

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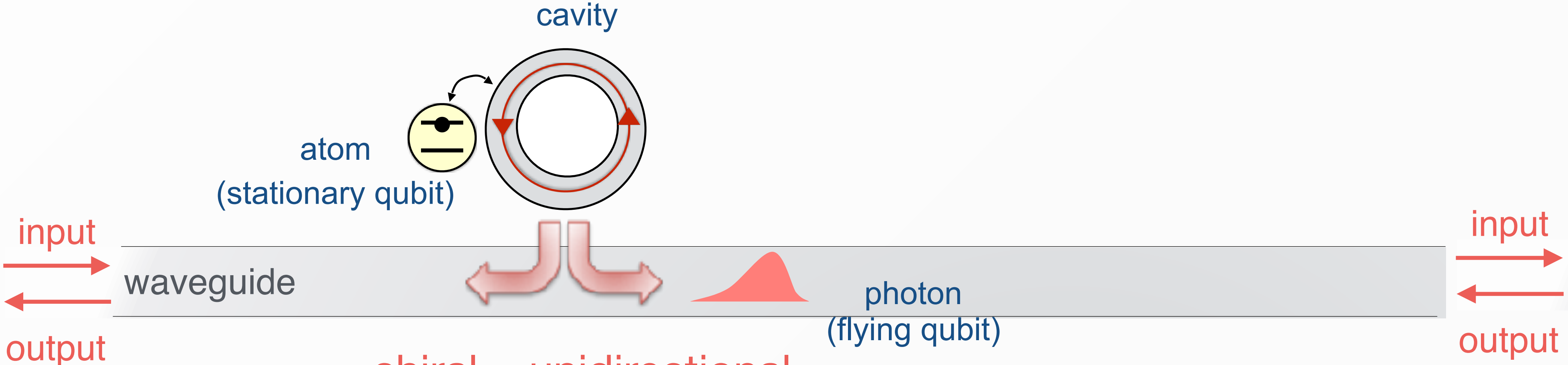
- CQED & light-matter interface



chiral = unidirectional

open quantum system

For a review see: P. Lodahl, A Rauschenbeutel, P.Z. et al., Nature 2017

Ex 5: Cavity QED & Quantum Networks

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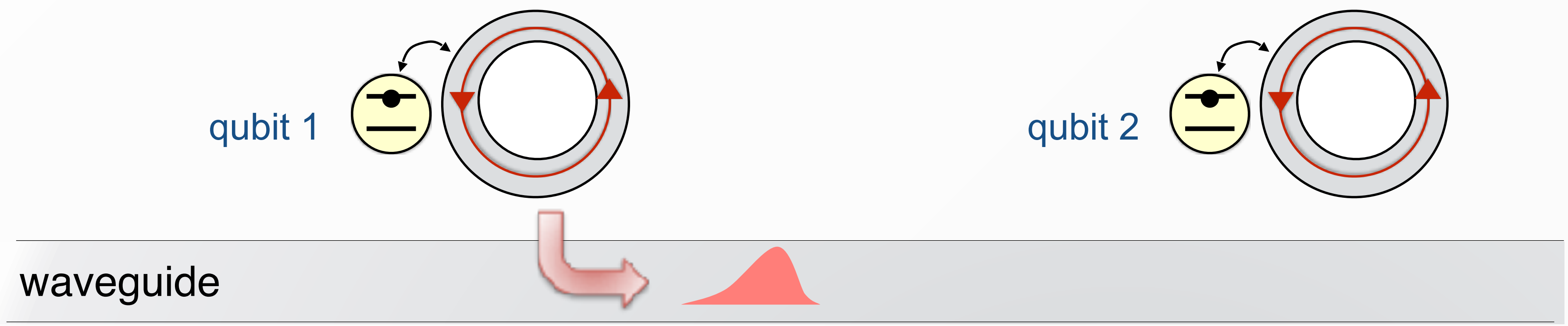
- CQED & light-matter interface



- Quantum State Transfer

$$\left(\alpha|g\rangle_1+\beta|e\rangle_1\right)|0\rangle_p|g\rangle_1 \rightarrow |g\rangle_1\left(\alpha|0\rangle_p+\beta|1\rangle_p\right)|g\rangle_1 \rightarrow |g\rangle_1|0\rangle_p\left(\alpha|g\rangle_2+\beta|e\rangle_2\right)$$

qubit 1 — wavepacket in waveguide — qubit 2

fidelity of transfer ~ quantum control problem

… see Lecture 4

Ex 5: Cavity QED & Quantum Networks

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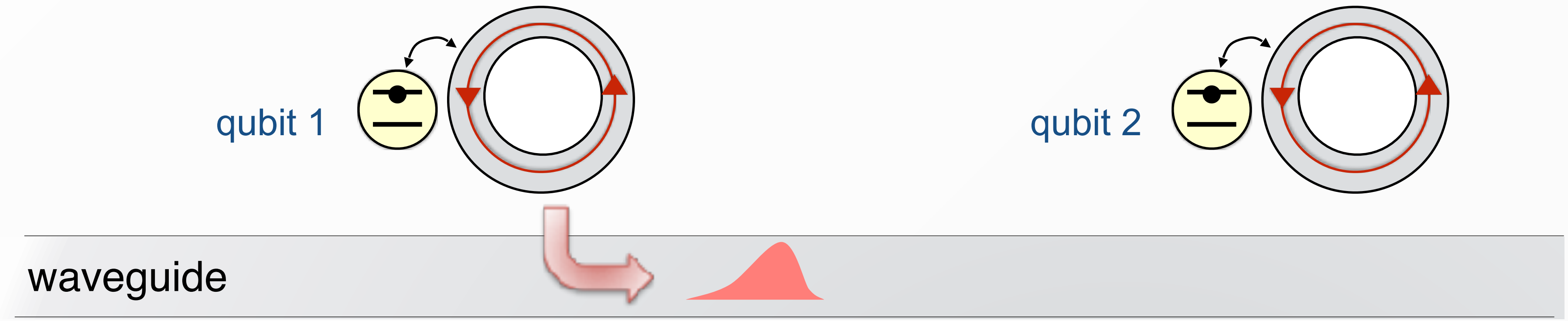
- CQED & light-matter interface



- Quantum Networks