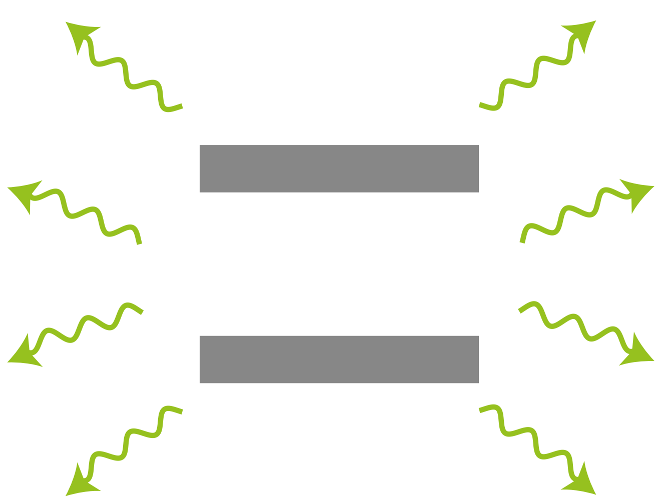
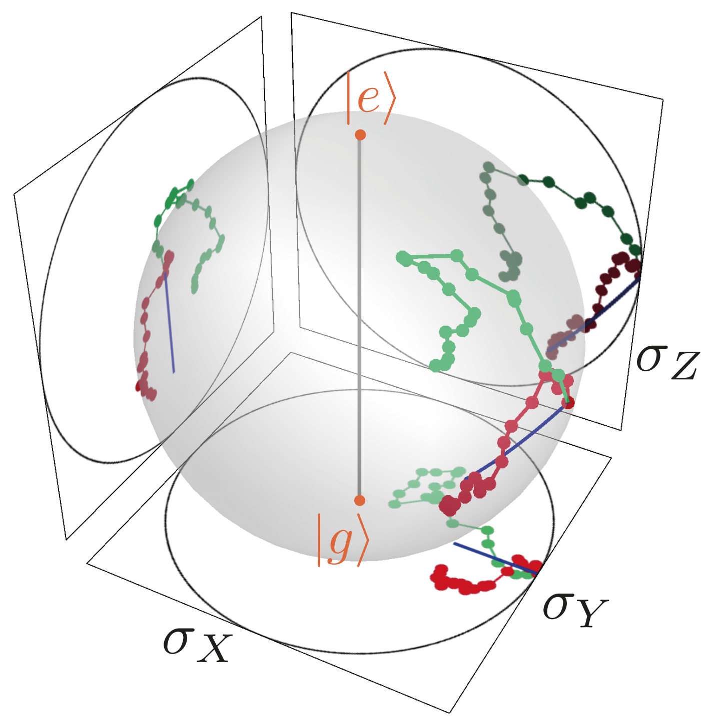
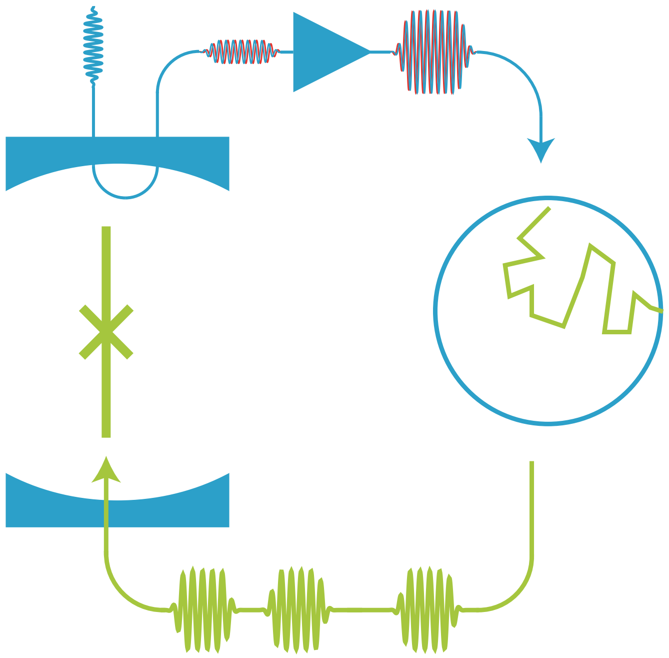
Quantum trajectories and feedback in circuit-QED

Benjamin Huard
Ecole Normale Supérieure de Lyon, France







Quantum laws of evolution

Closed system

$$\partial_t |\psi\rangle = -\frac{i}{\hbar} H |\psi\rangle \qquad \partial_t \rho = -\frac{i}{\hbar}[H, \rho]$$

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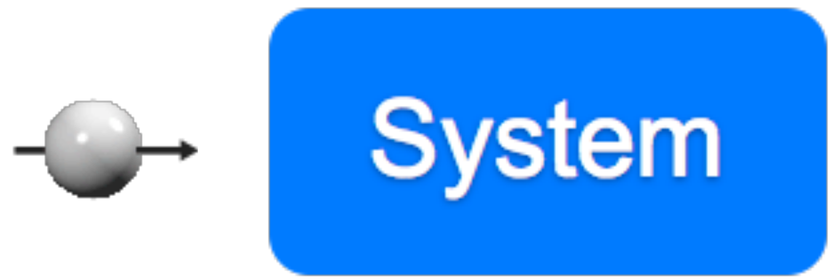
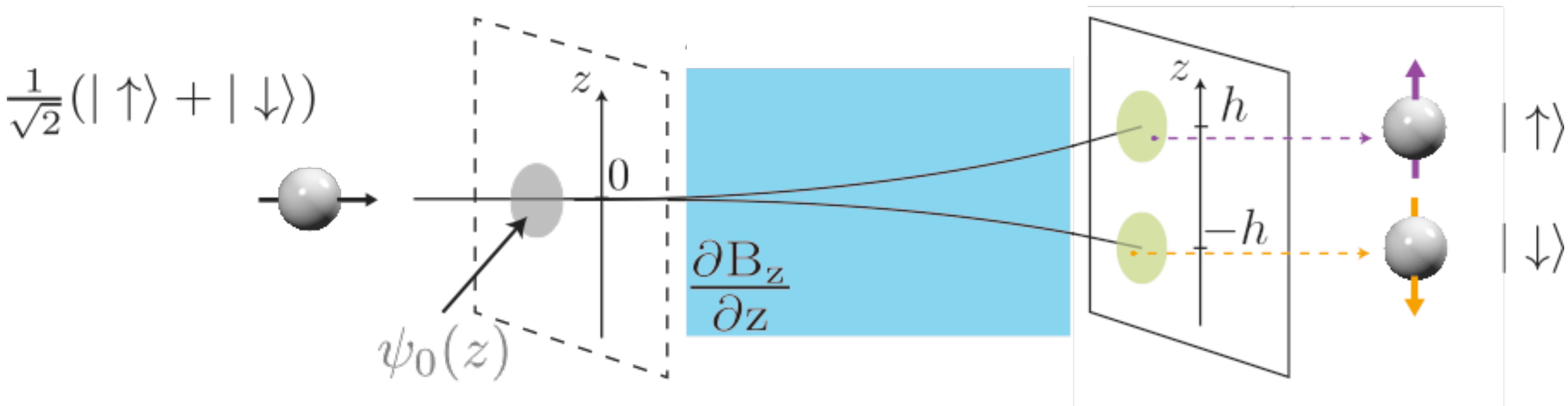


Quantum measurement



Possible outcomes $z \approx h$, $z \approx -h$ with proba $p = \langle\uparrow|\rho|\uparrow\rangle$, $p = \langle\downarrow|\rho|\downarrow\rangle$ and new state is $|\uparrow\rangle$, $|\downarrow\rangle$

Quantum laws of evolution

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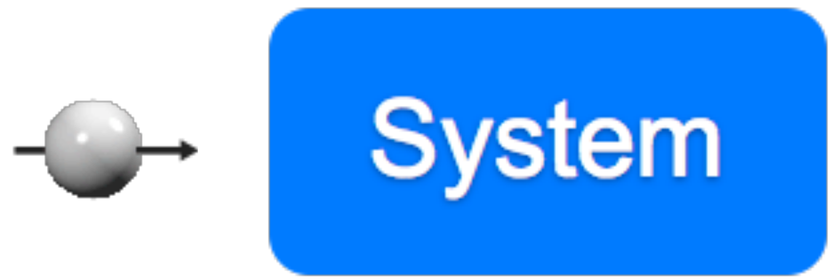
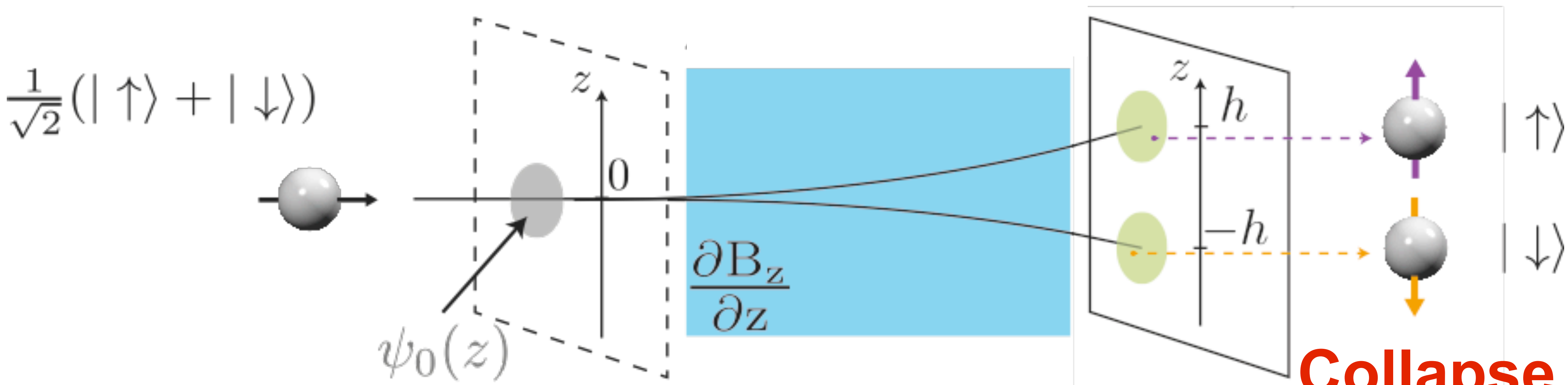
$$\partial_t |\psi\rangle = -\frac{i}{\hbar} H |\psi\rangle \qquad\qquad \partial_t \rho = -\frac{i}{\hbar}[H, \rho]$$



Quantum measurement

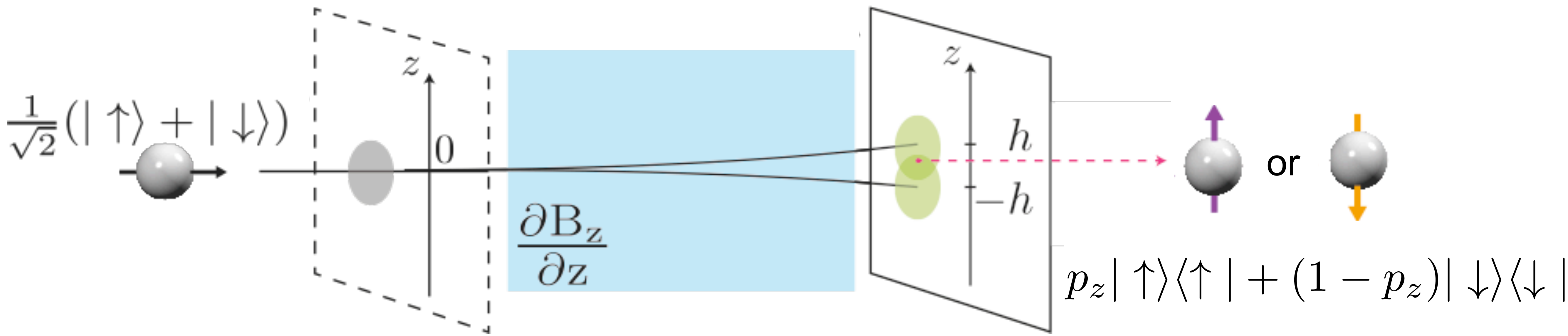


Collapse

Possible outcomes $z \approx h$, $z \approx -h$ with proba $p = \langle \uparrow |\rho| \uparrow \rangle$, $p = \langle \downarrow |\rho| \downarrow \rangle$ and new state is $|\uparrow\rangle$, $|\downarrow\rangle$

Generalized measurement

if position fluctuations are mostly classical



Generalized measurement

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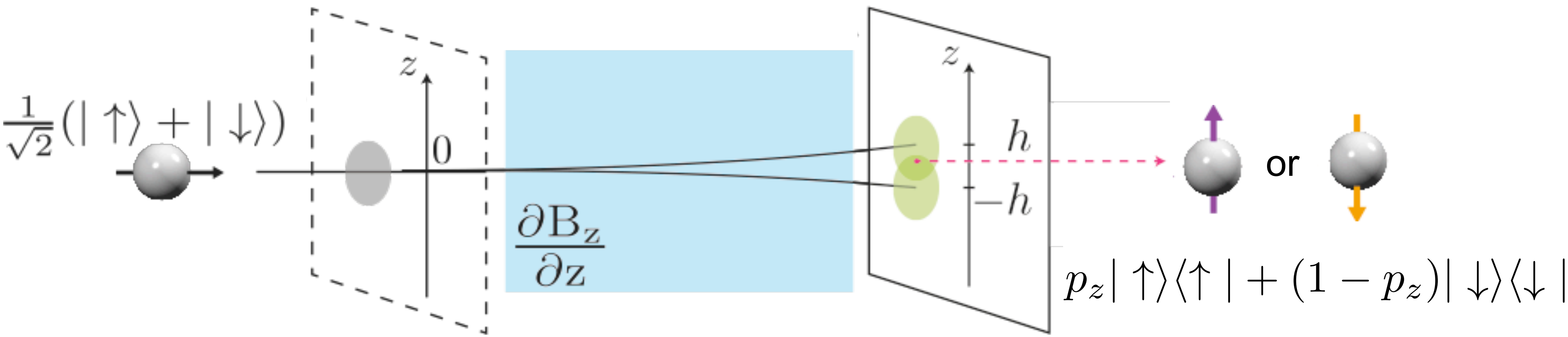
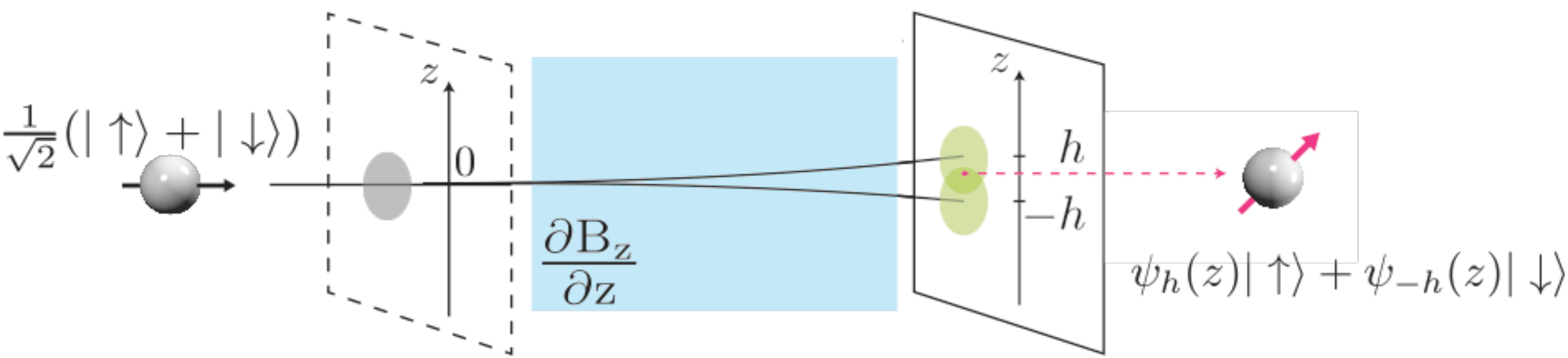


if zero point fluctuations only



Generalized measurement

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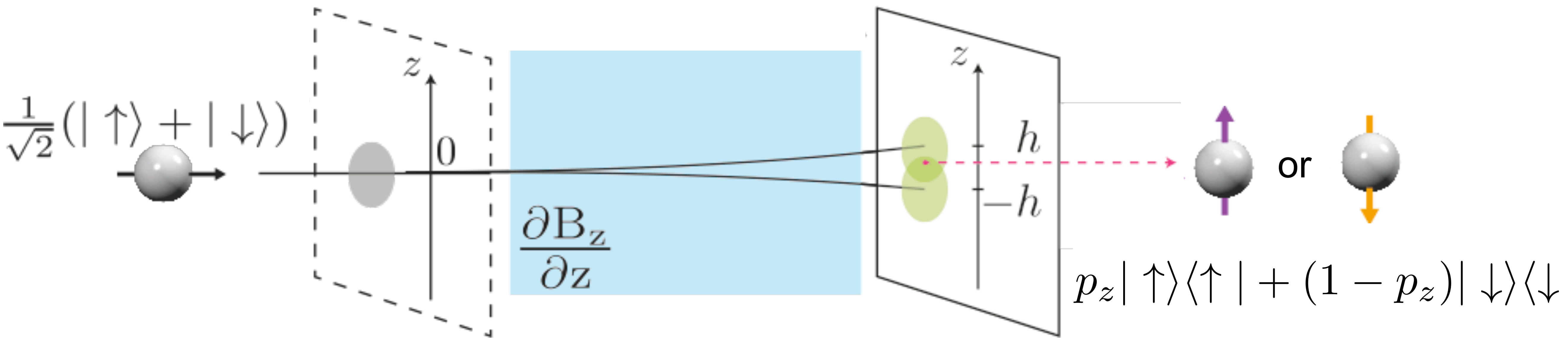
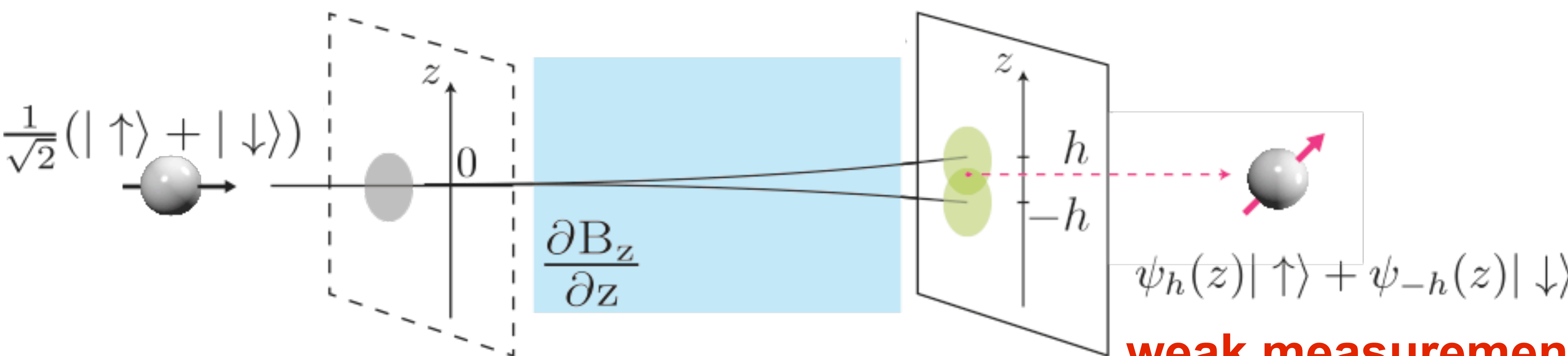
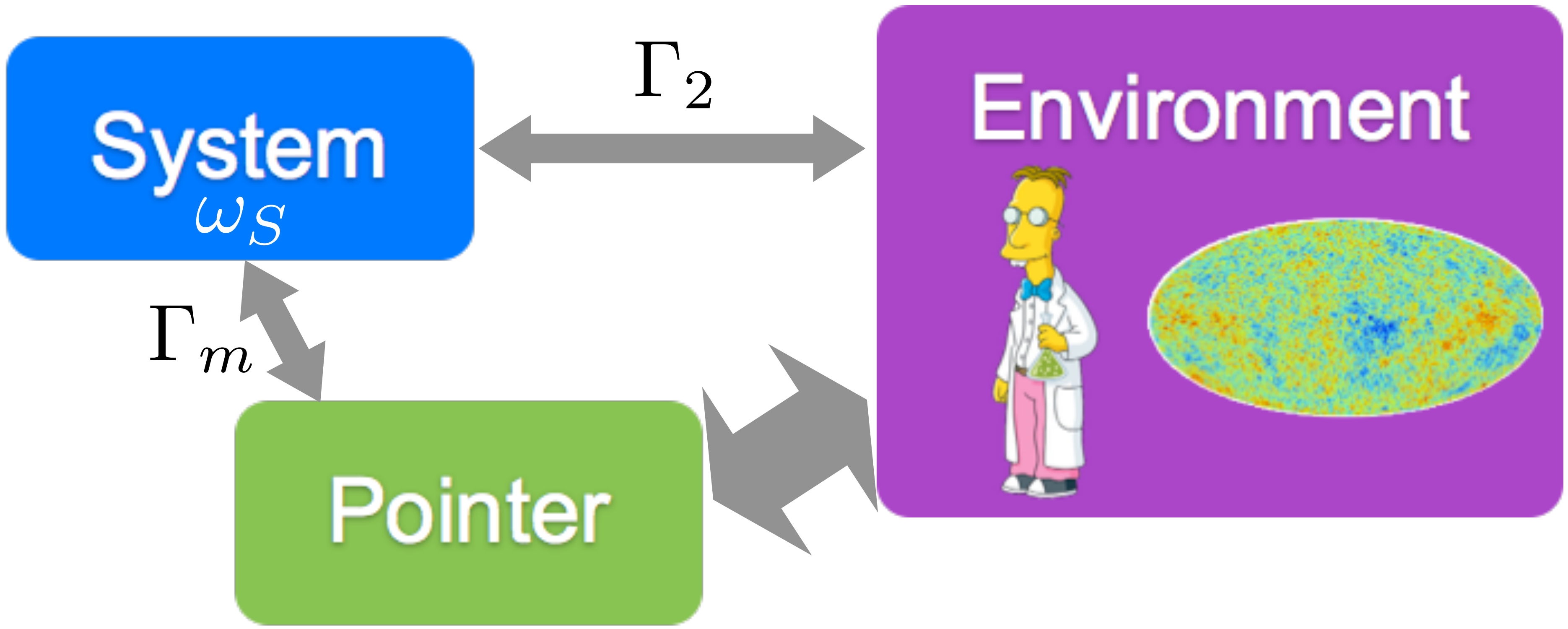


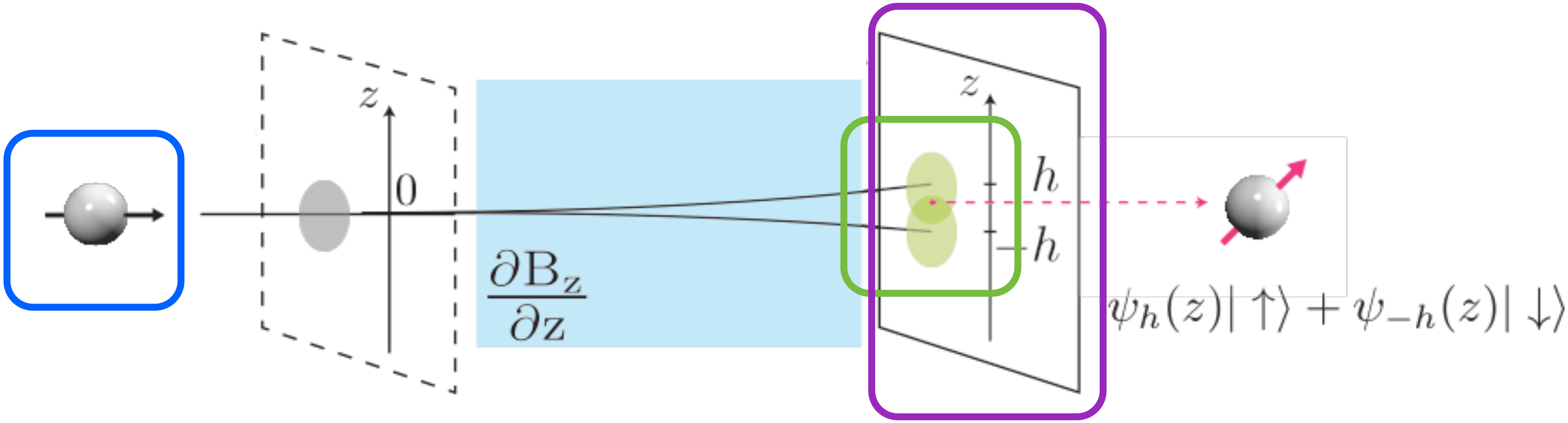
if zero point fluctuations only



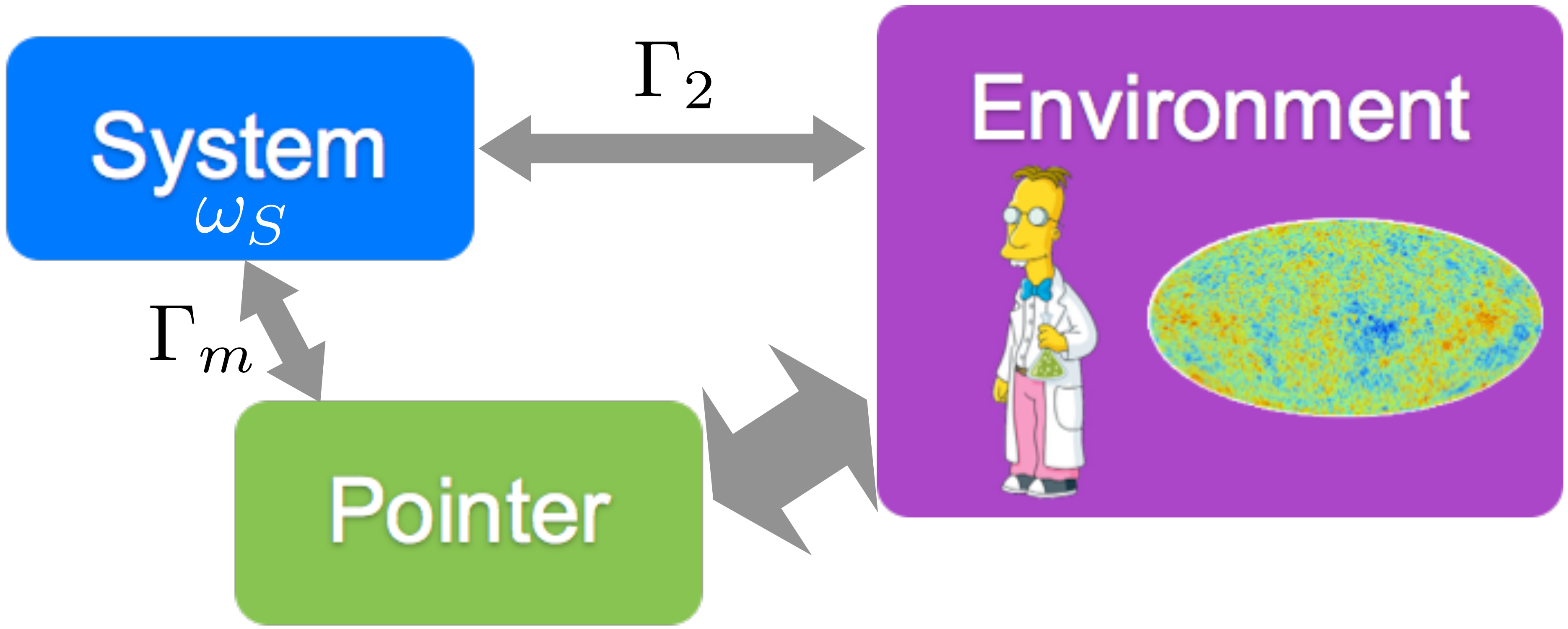
weak measurement
not projection on the sys!

Generalized measurement





Observing measurement backaction



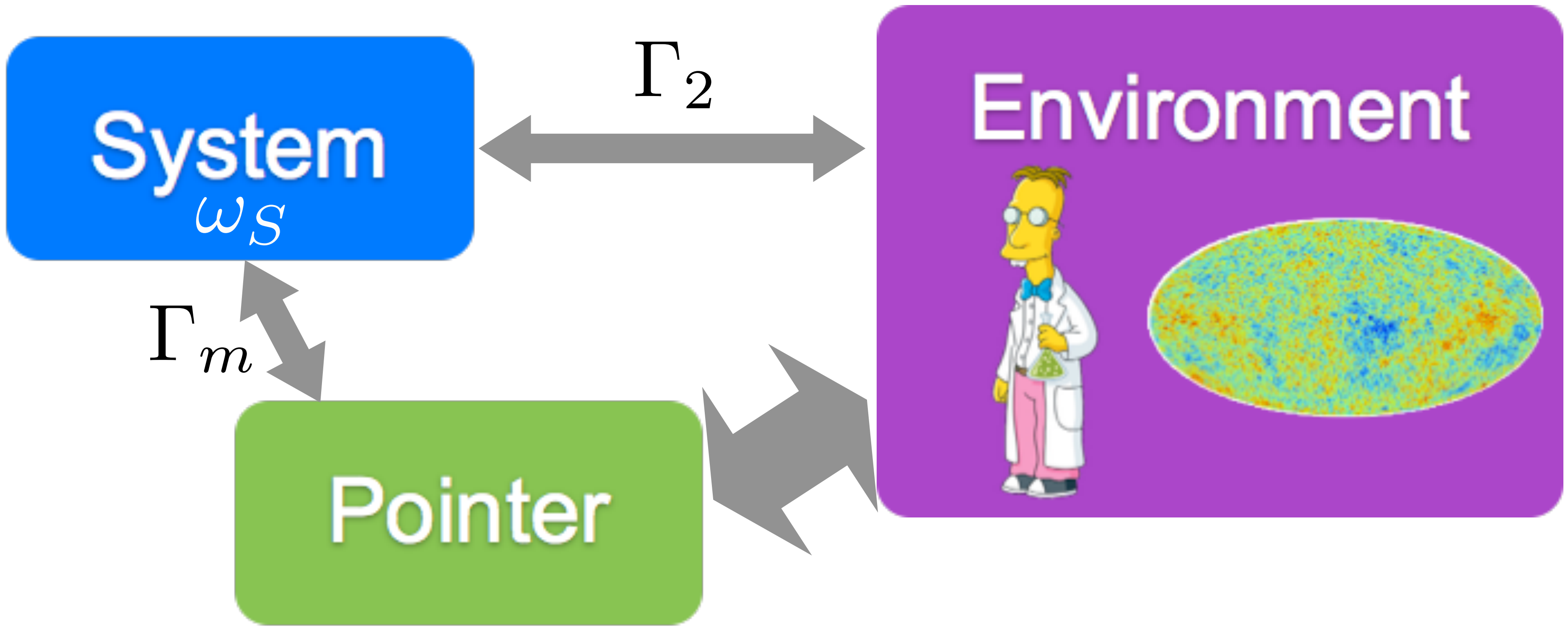
traditional detectors
$\Gamma_m \ll \Gamma_2 \ll \omega_S$

⇒

average on large ensemble of systems (in time or space)

no visible measurement backaction

Observing measurement backaction

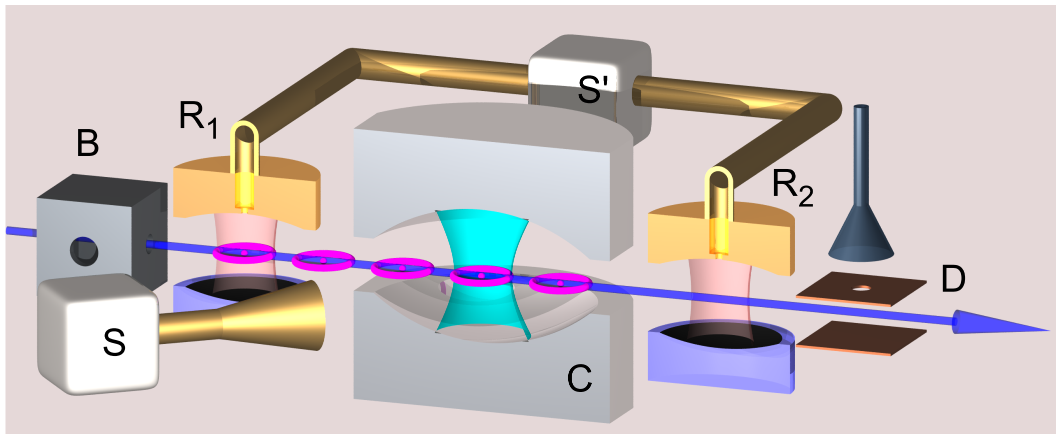


traditional detectors
$\Gamma_m \ll \Gamma_2 \ll \omega_S$
$\Rightarrow$
average on large ensemble of systems (in time or space)
no visible measurement backaction

quantum limited detectors
$\Gamma_2 \lesssim \Gamma_m, \omega_S$
$\Rightarrow$
evolution of single realizations depends on outcomes
visible measurement backaction

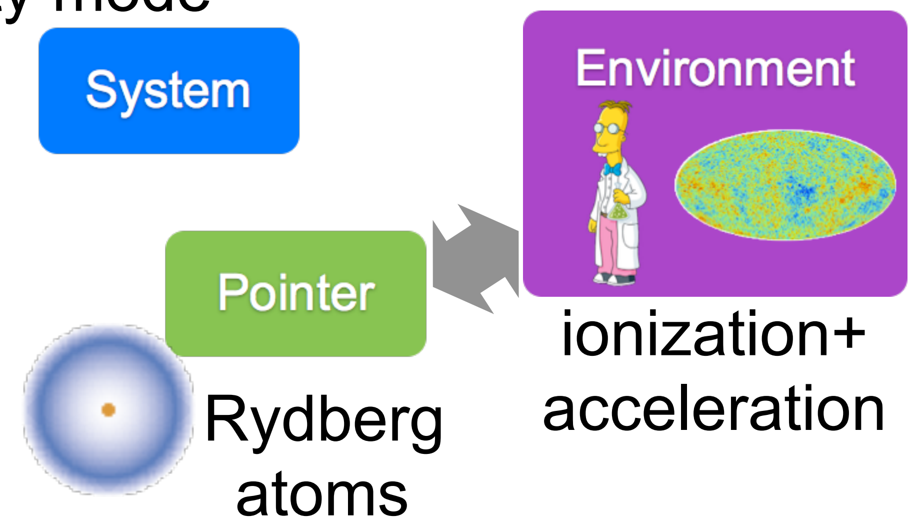
Quantum trajectories already measured in...

Rydberg atoms

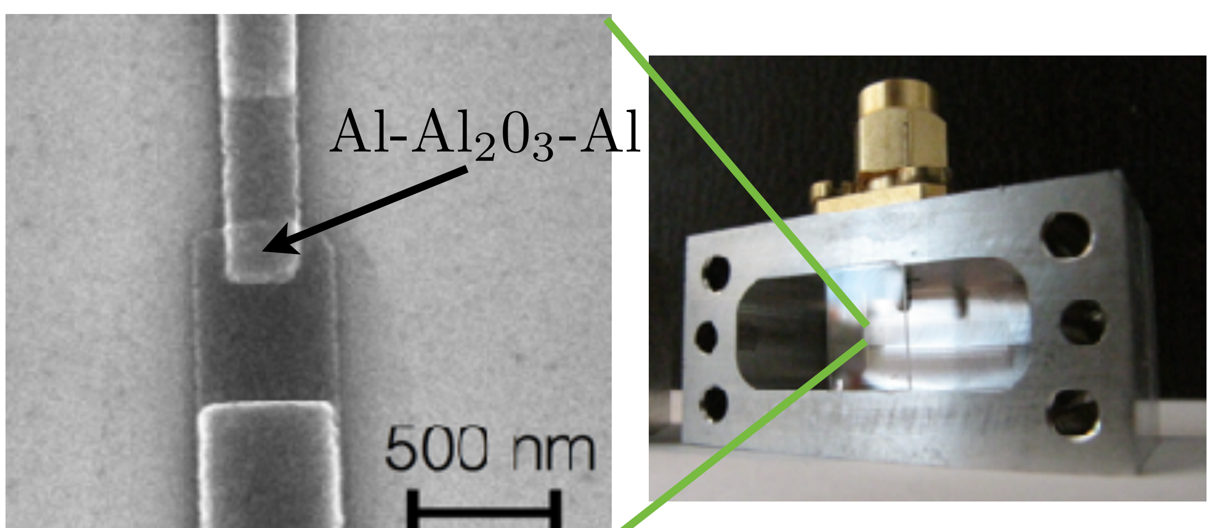


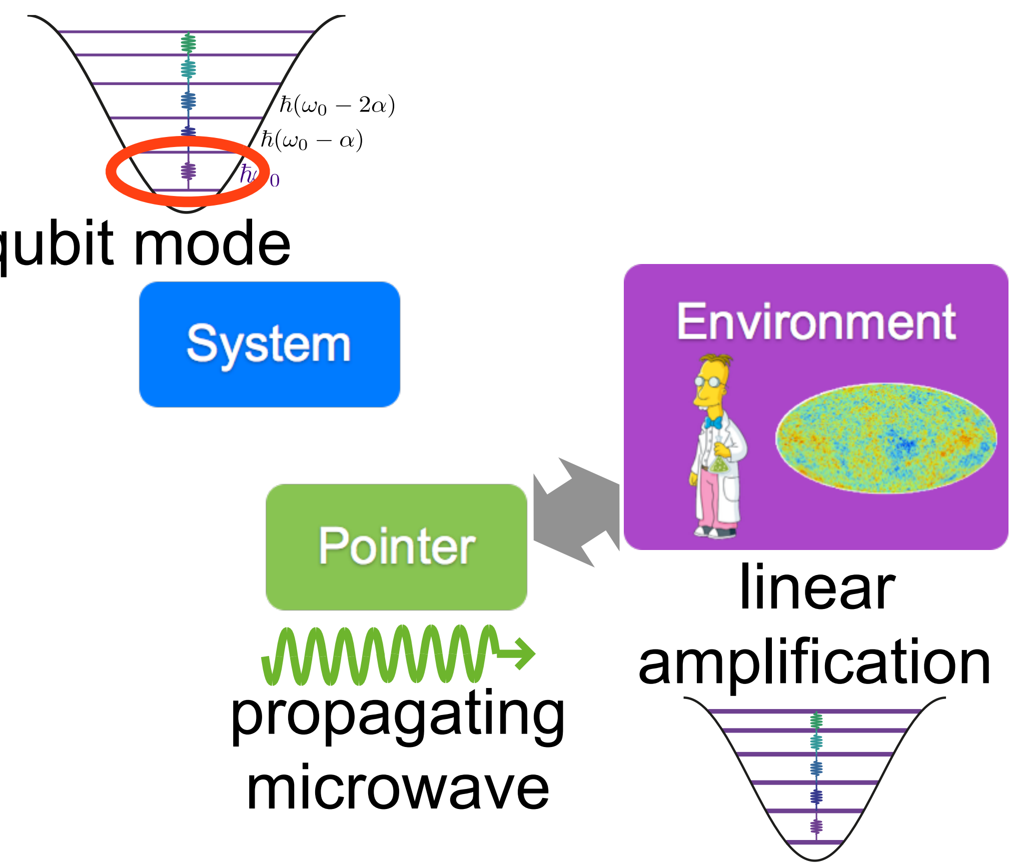
[pic from Haroche group, College de France Paris]

Cavity mode

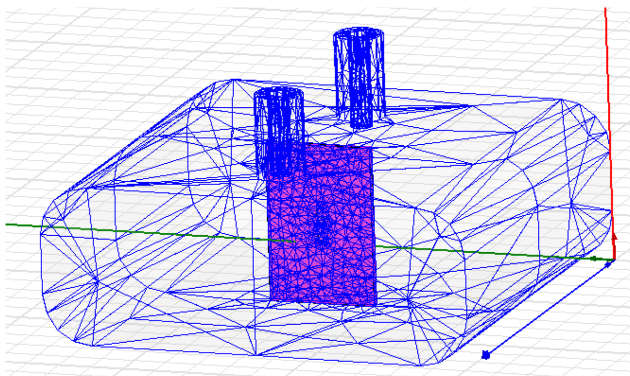


Superconducting circuits





Quantum trajectories and feedback in circuit-QED



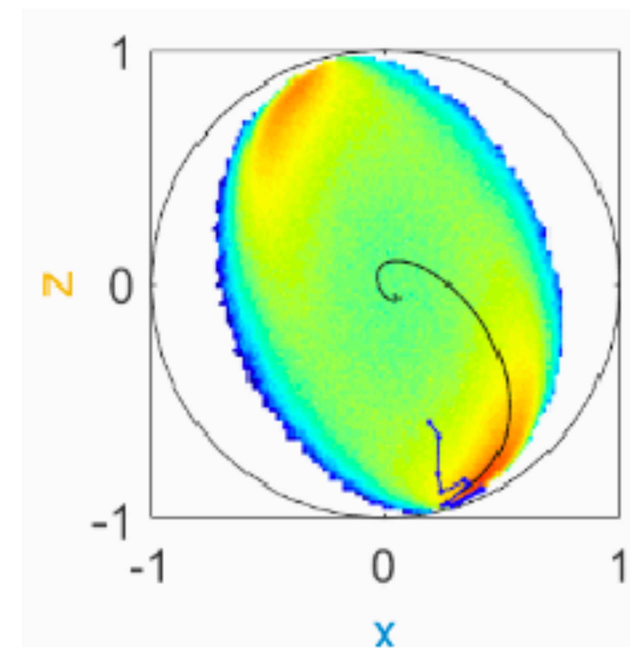
Introduction to circuit-QED

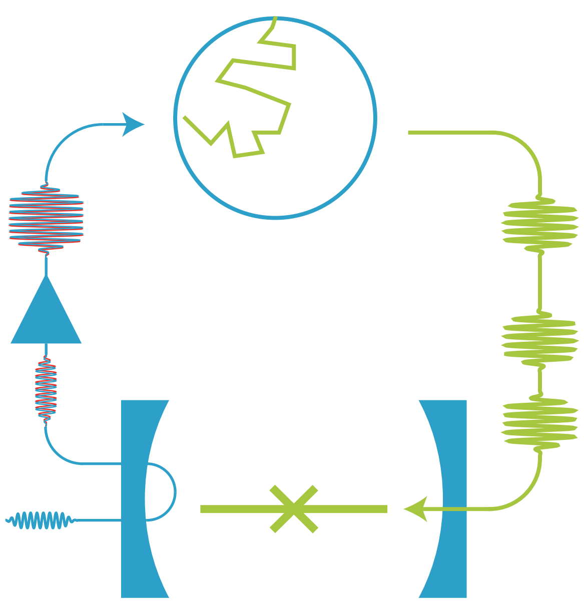
Quantum trajectories in circuit-QED

dispersive measurement

fluorescence measurement

both simultaneously





Measurement based feedback

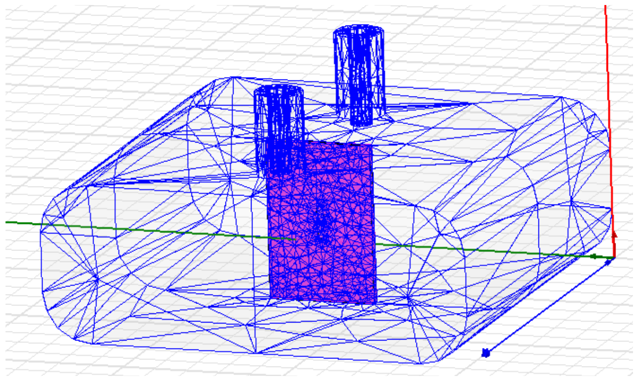
dispersive case

fluorescence case

Post selection in quantum mechanics

$\rho(t), E(t)$

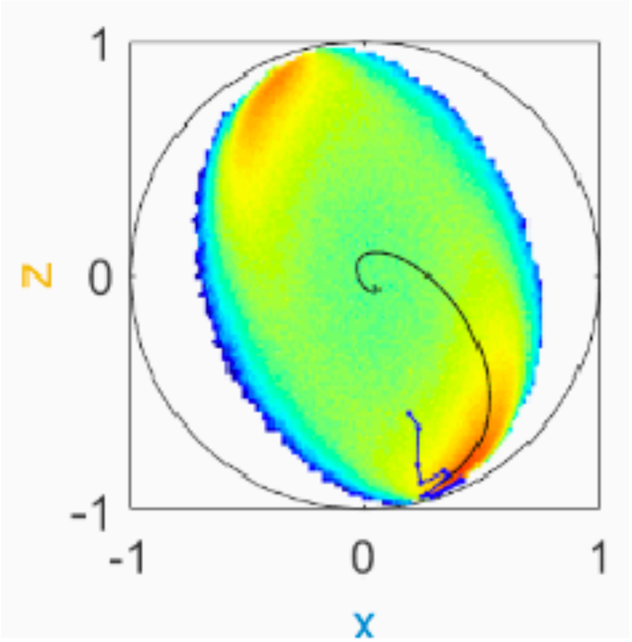
Quantum trajectories and feedback in circuit-QED

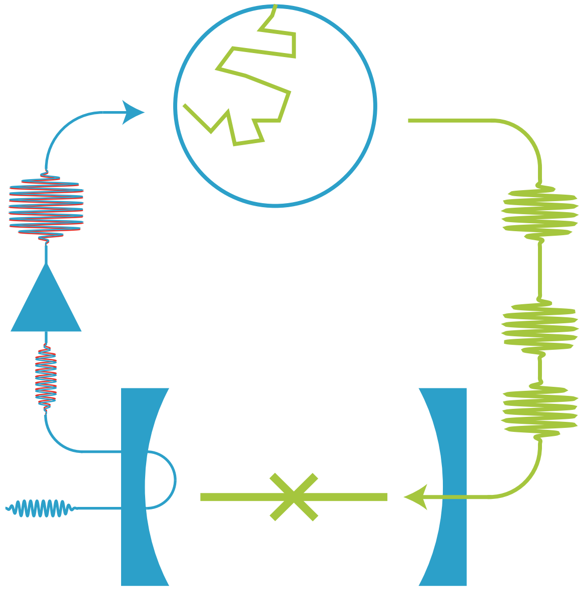


Introduction to circuit-QED

Quantum trajectories in circuit-QED

- dispersive measurement
- fluorescence measurement
- both simultaneously
- generating entanglement





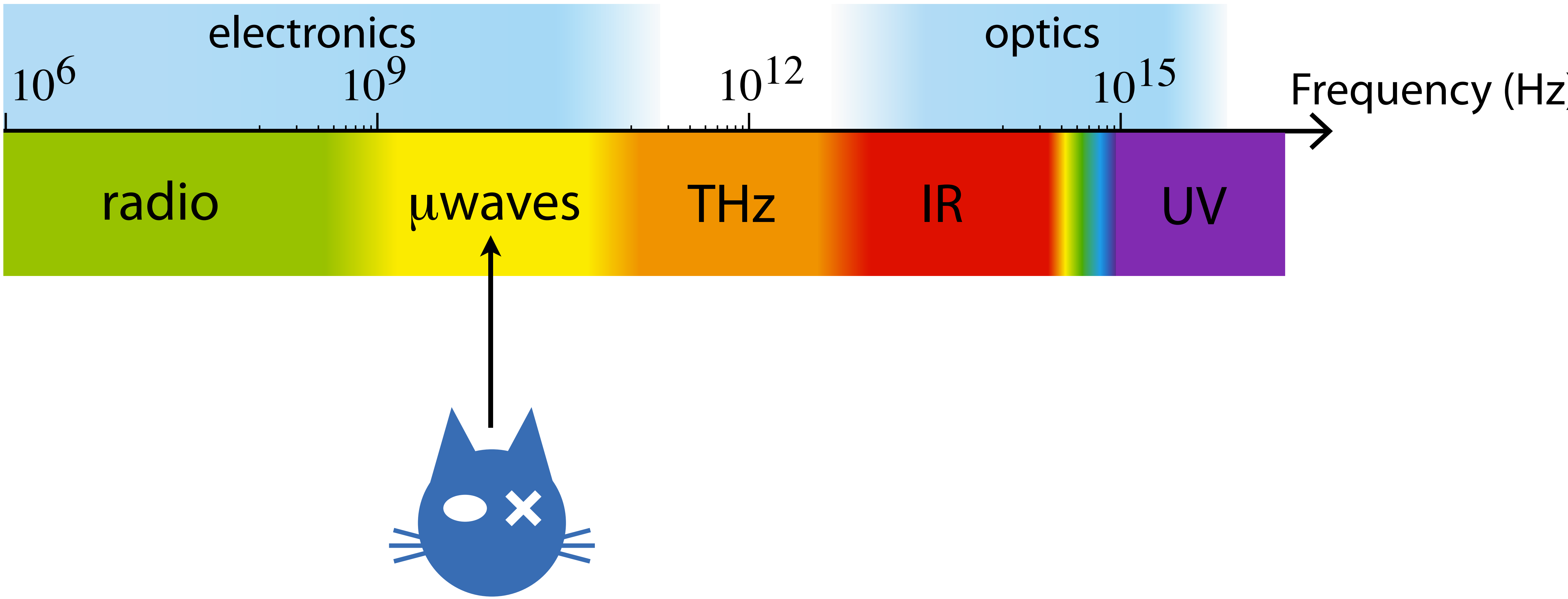
Measurement based feedback

- dispersive case
- fluorescence case

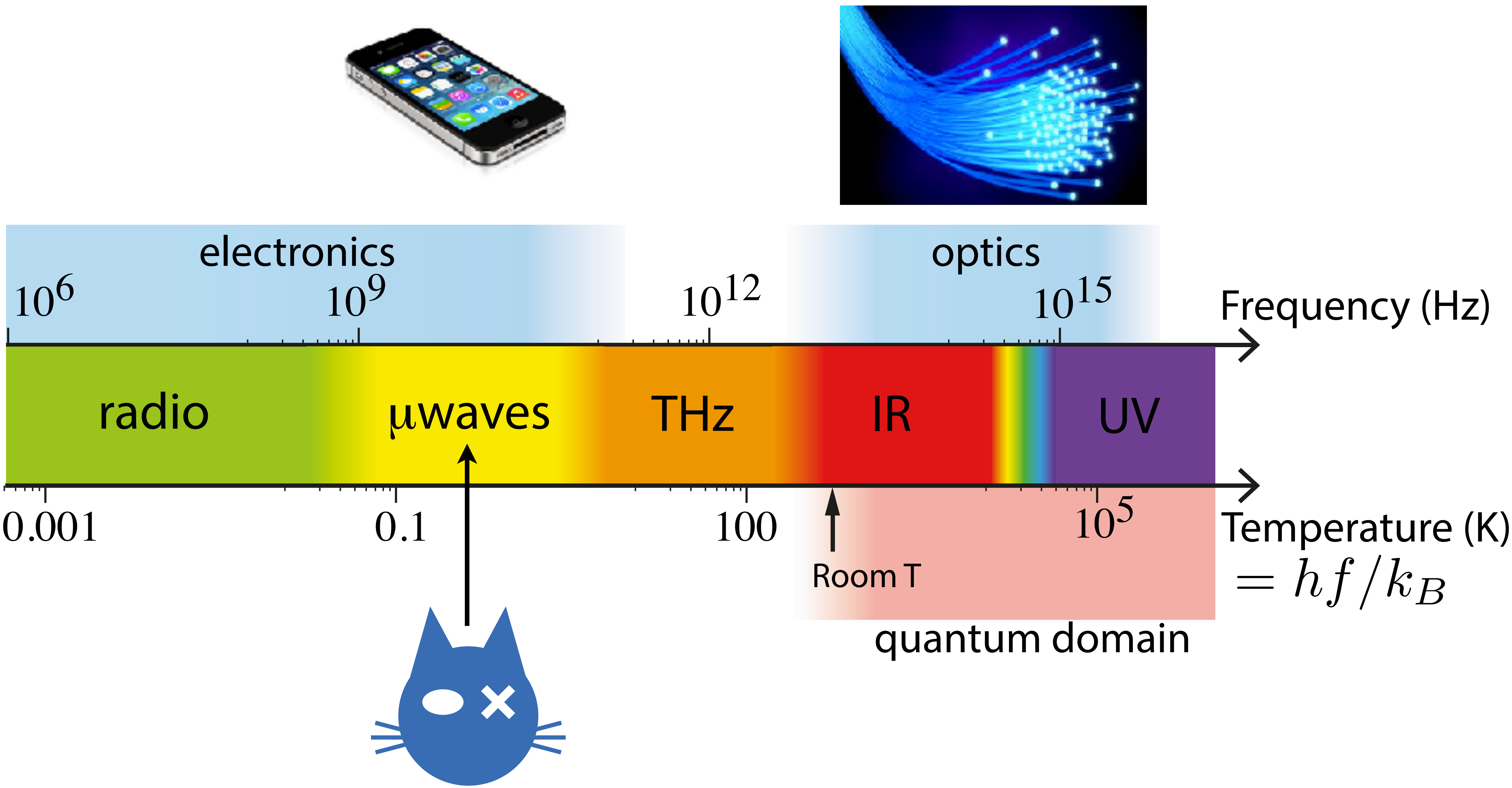
Post selection in quantum mechanics

$\rho(t), E(t)$

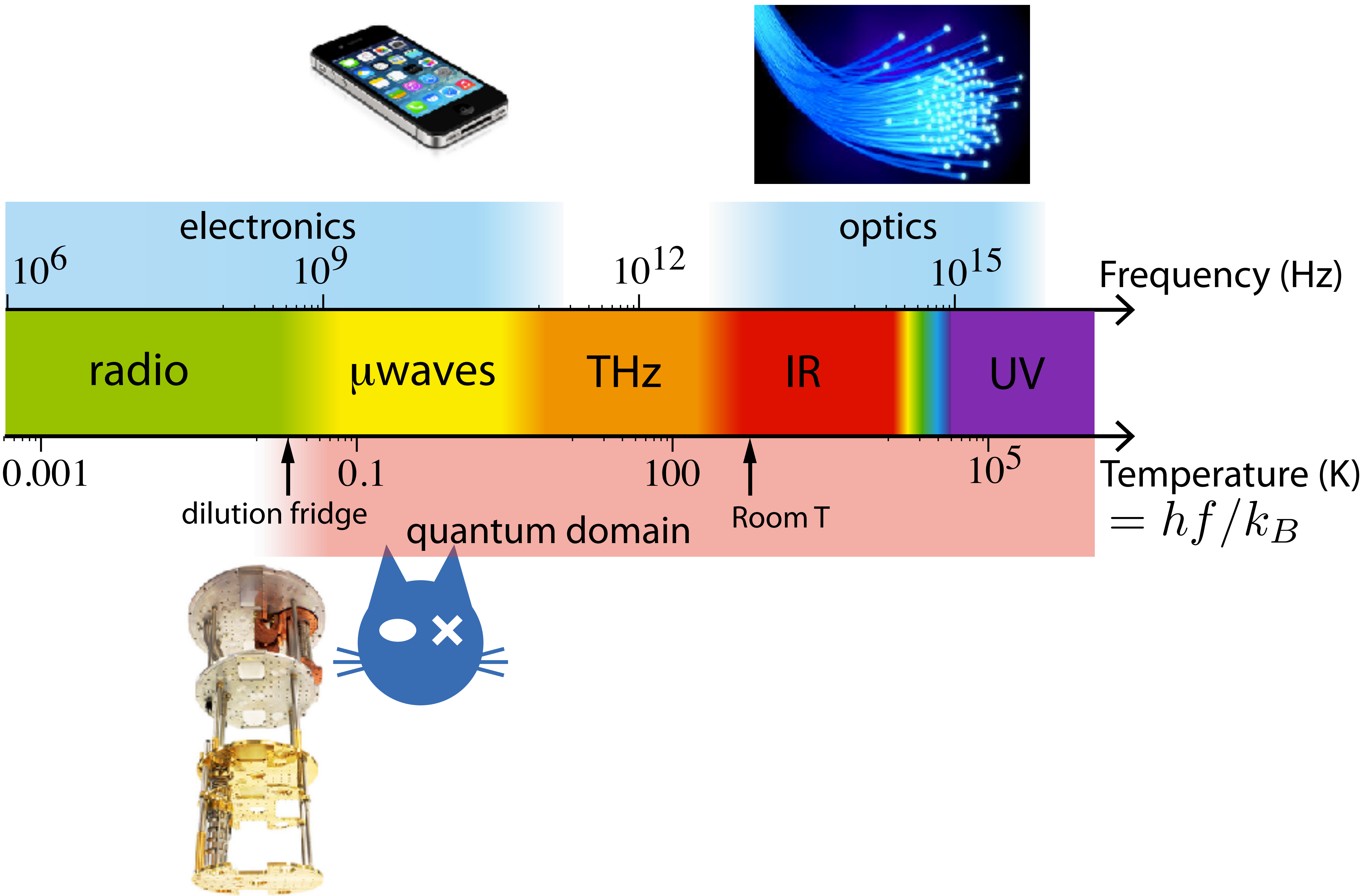
Microwave quantum optics



Microwave quantum optics

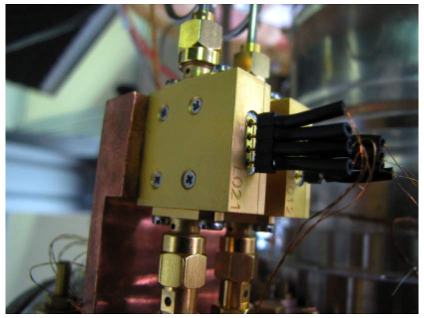


Microwave quantum optics

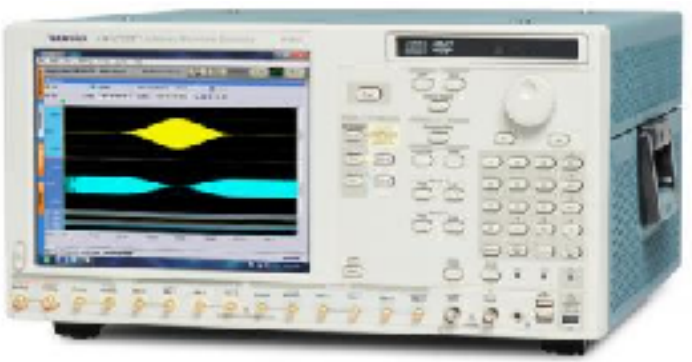


Commercial toolbox for microwave optics

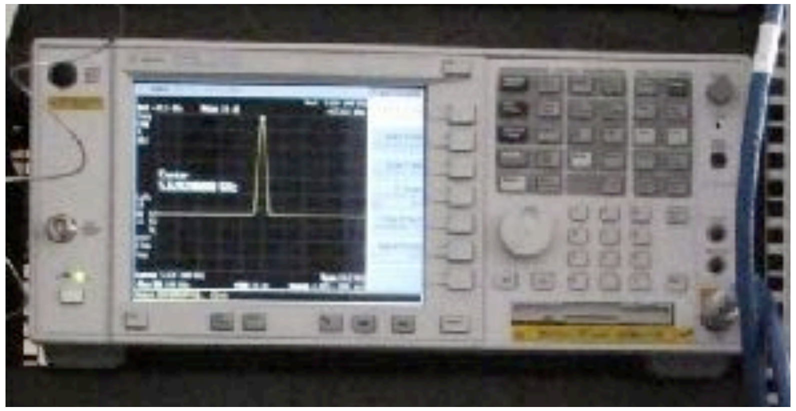
Microwave components

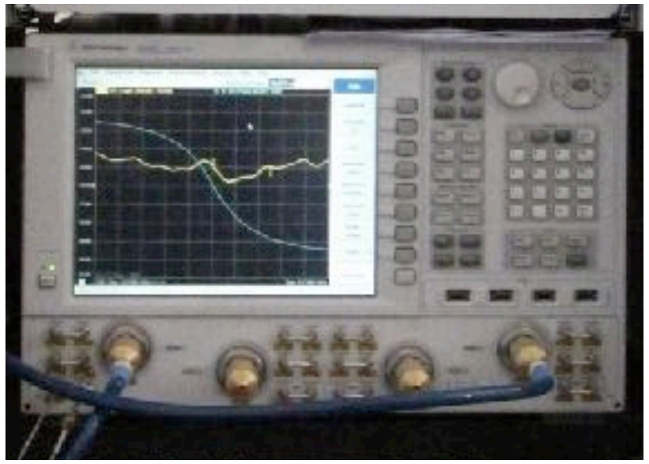


microwave sources



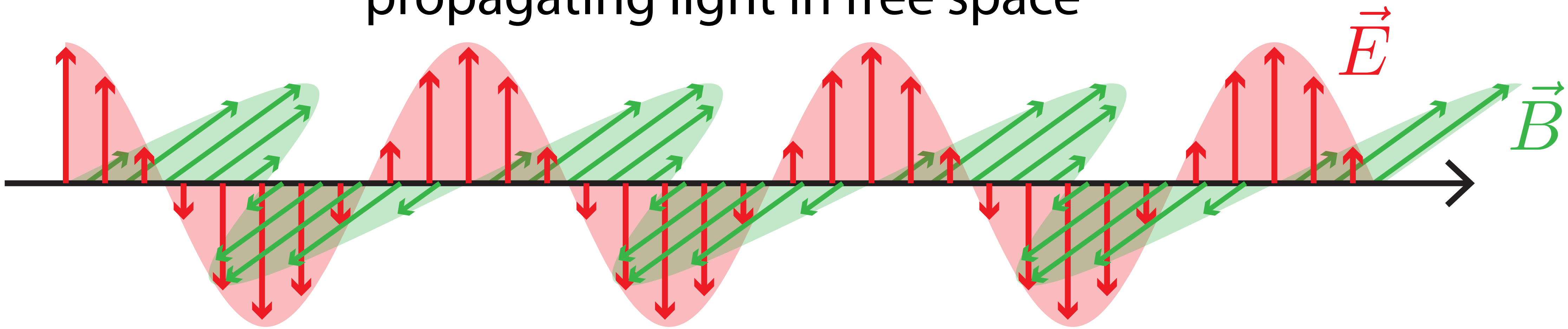
microwave detectors



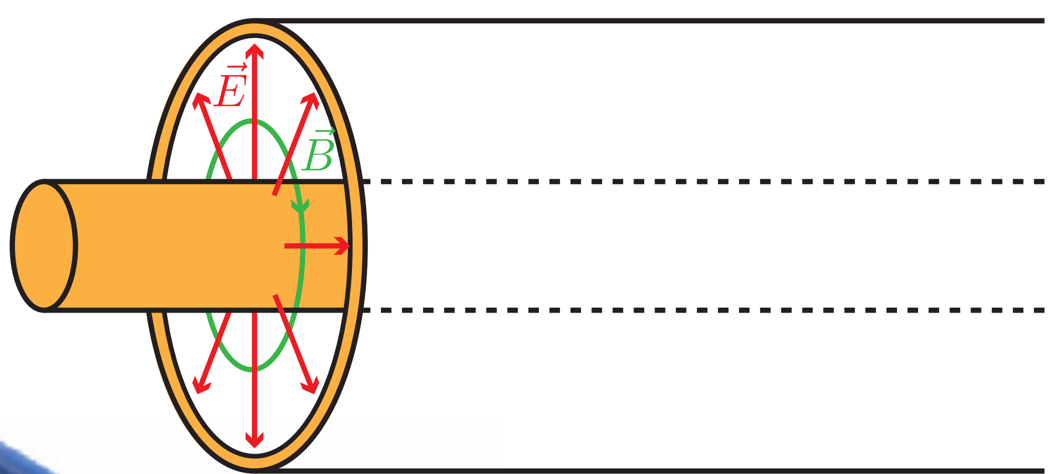


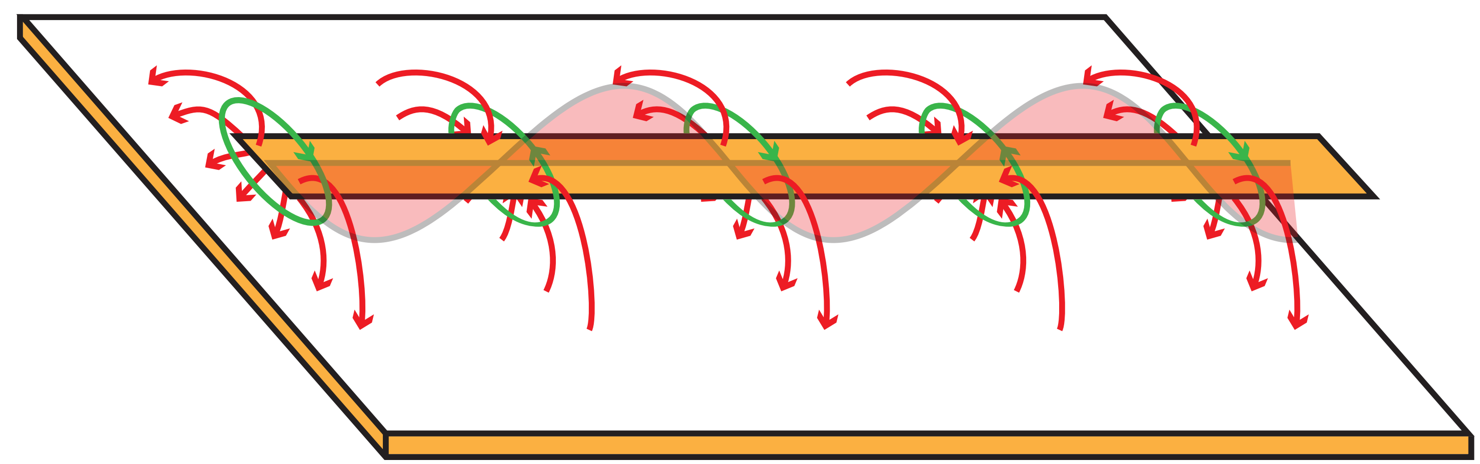
Optics with circuits?

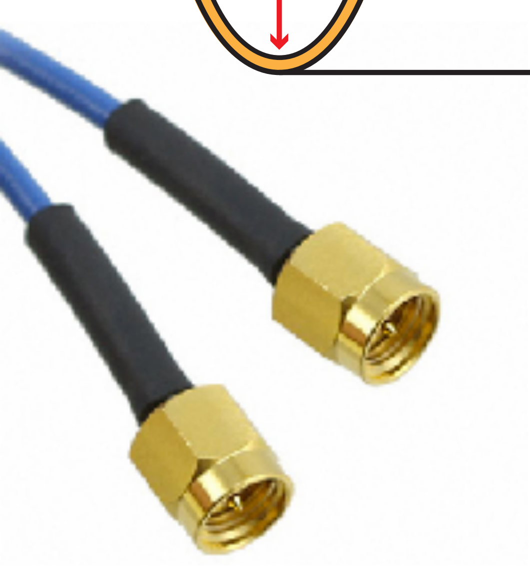
propagating light in free space

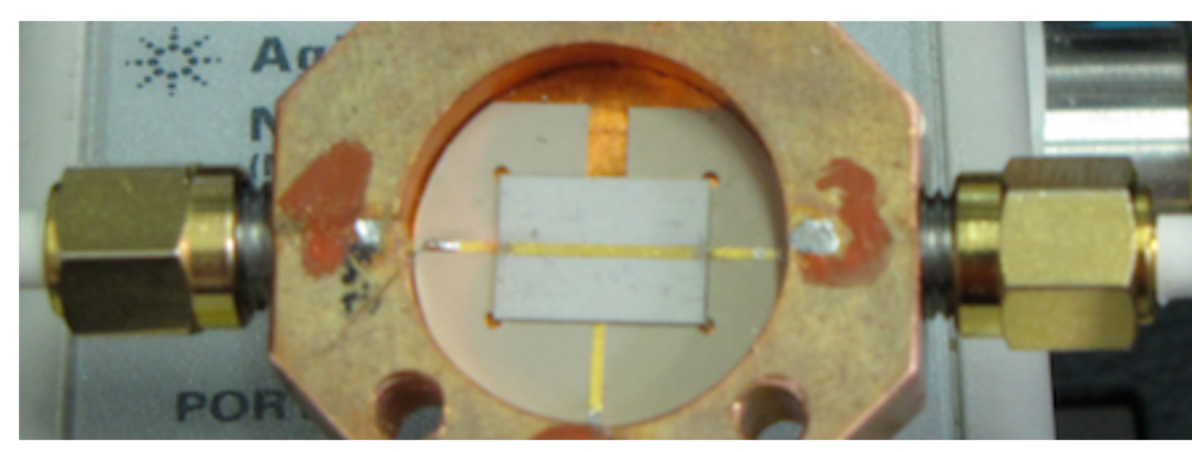


propagating light in waveguides



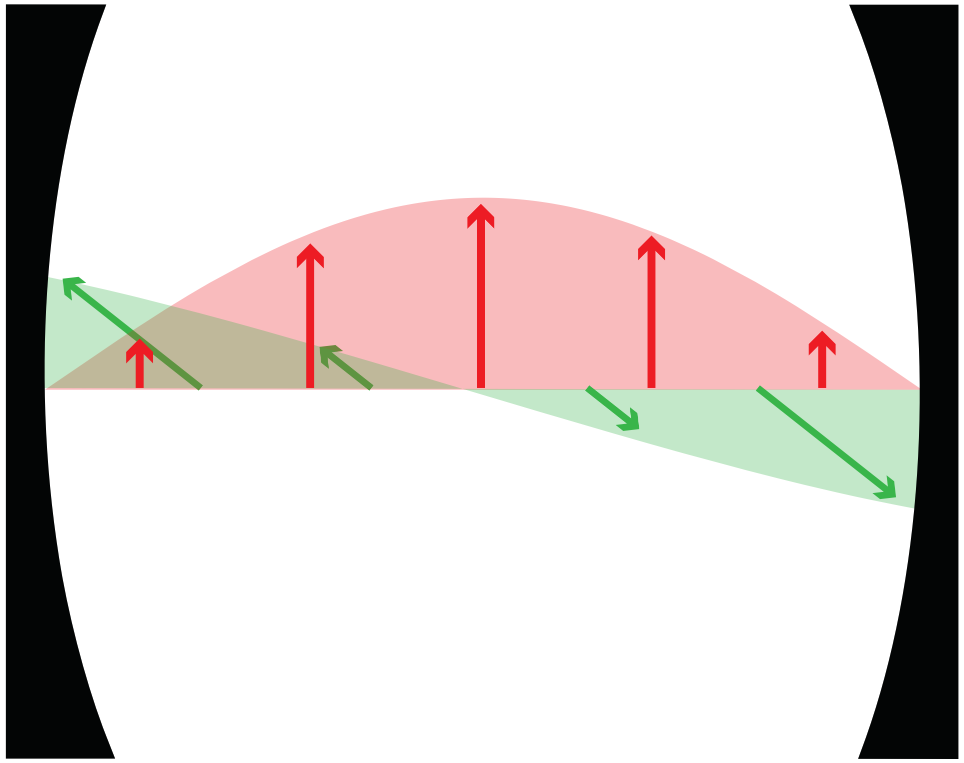


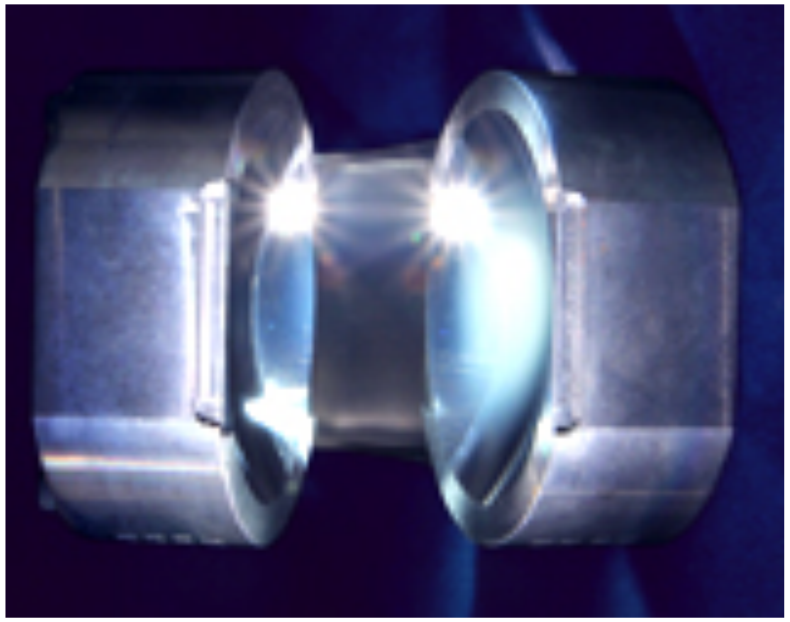




Optics with circuits?

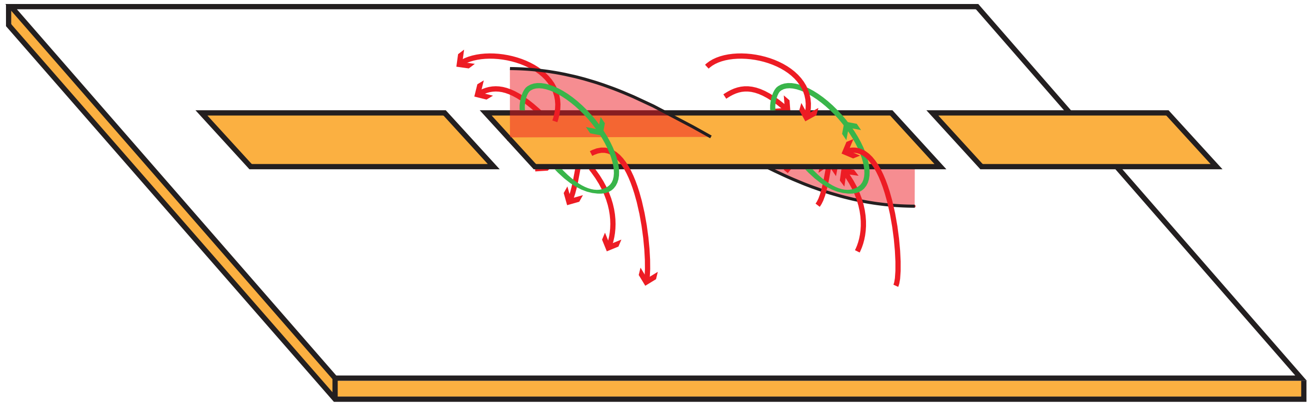
Confined light mode between mirrors



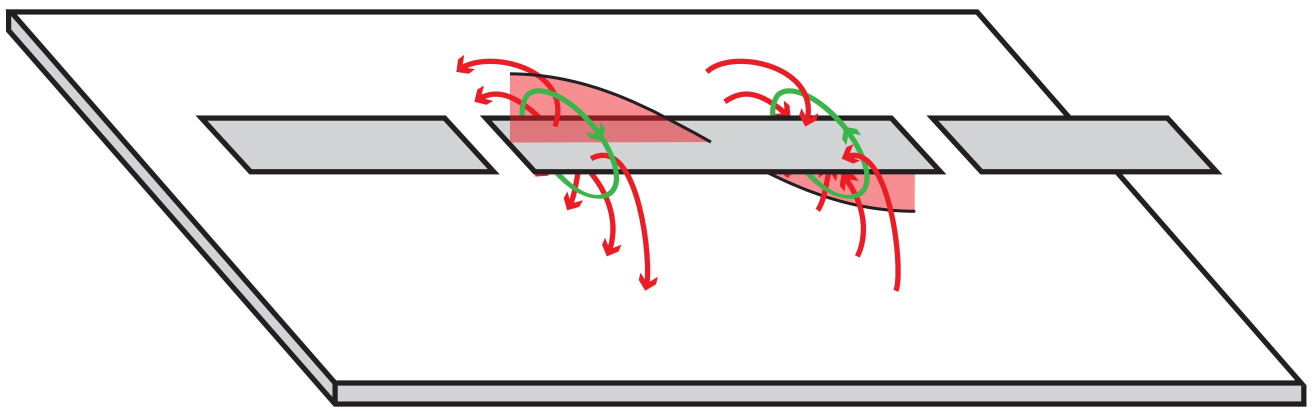


(CQED, LKB, Paris)

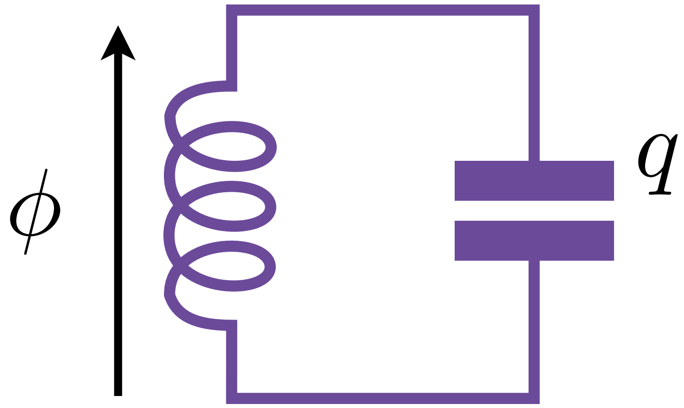
Confined light mode in waveguide



Superconducting circuits



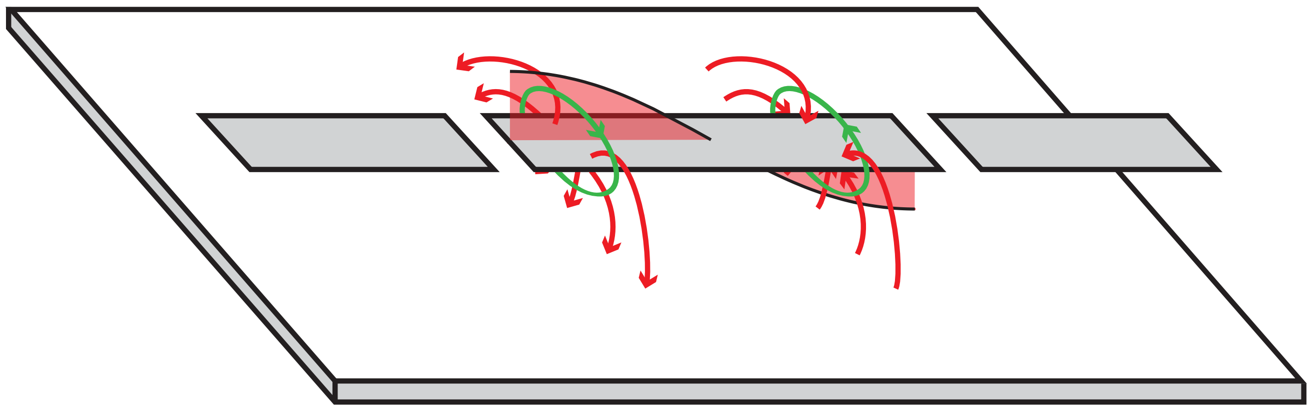
dissipationless LC circuit



$$\hat{H} = \frac{\hat{q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

$$[\hat{\phi}, \hat{q}] = i\hbar$$

Superconducting circuits



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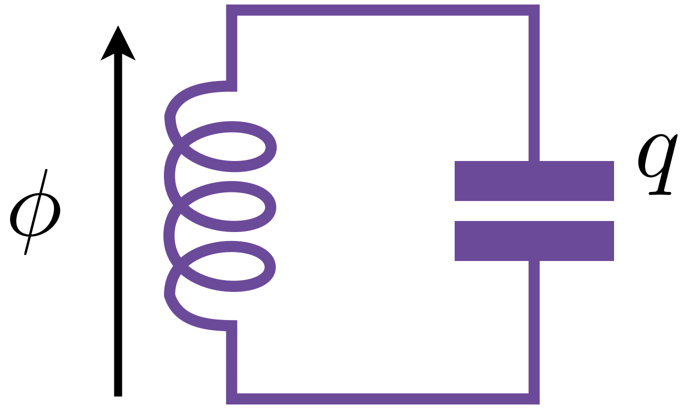


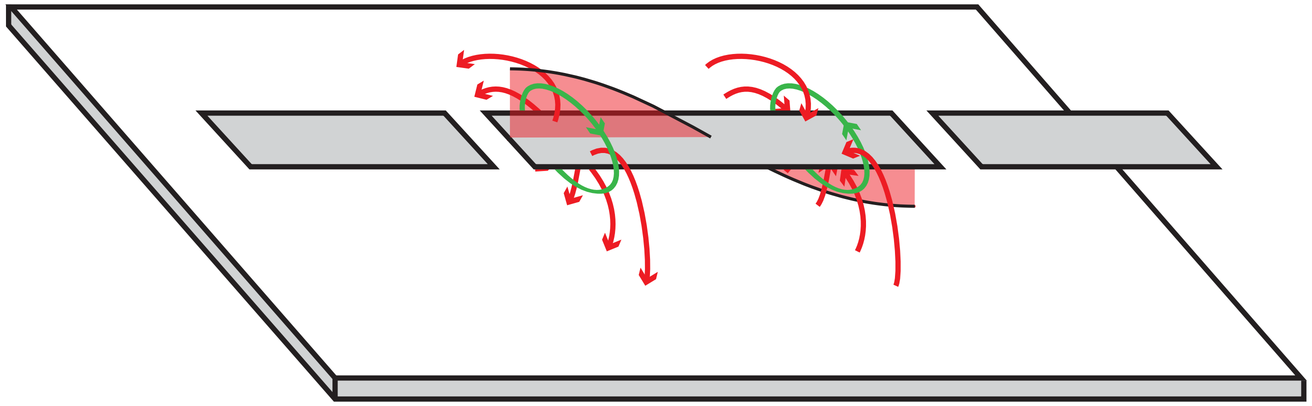
$$\hat{H} = \frac{\hat{q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

$$[\hat{\phi}, \hat{q}] = i\hbar$$

$$\hat{H} = \frac{k\hat{X}^2}{2} + \frac{\hat{P}^2}{2m}$$

$$[\hat{X}, \hat{P}] = i\hbar$$

Superconducting circuits



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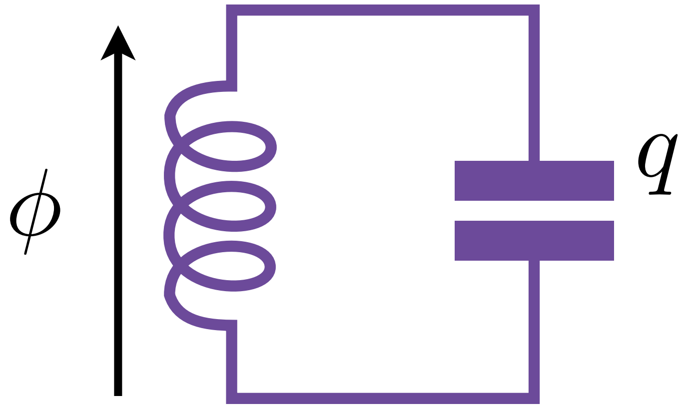


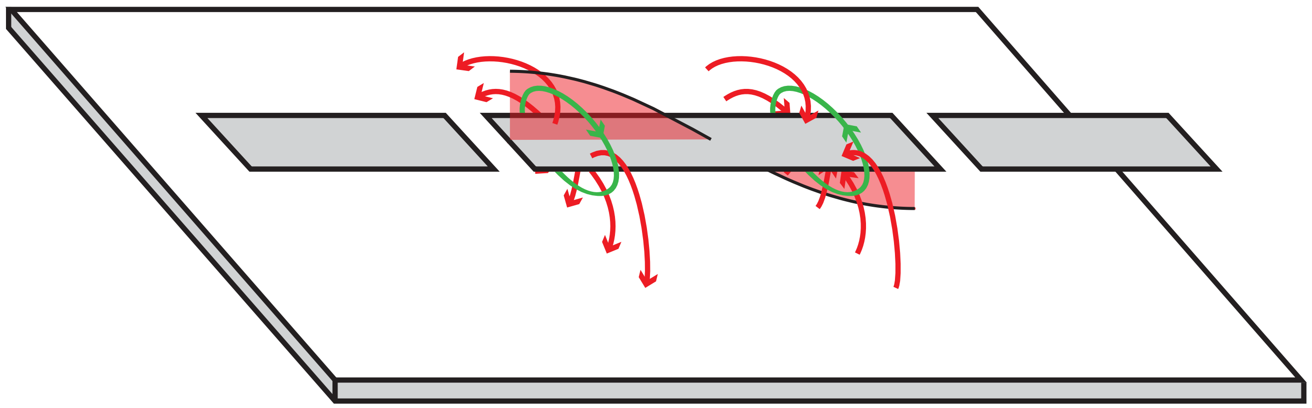
$$\hat{H} = \frac{\hat{q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

$$[\hat{\phi}, \hat{q}] = i\hbar$$

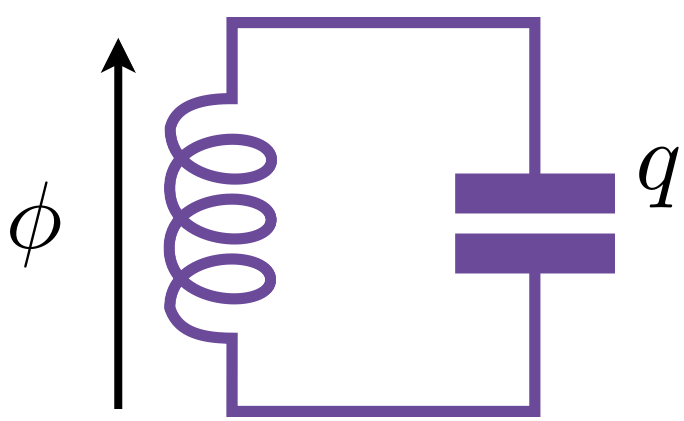
Heisenberg inequality

$$\delta\phi\delta q \geq \frac{\hbar}{2}$$

Superconducting circuits



dissipationless LC circuit



$$\hat{H} = \frac{\hat{q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

$$[\hat{\phi}, \hat{q}] = i\hbar$$

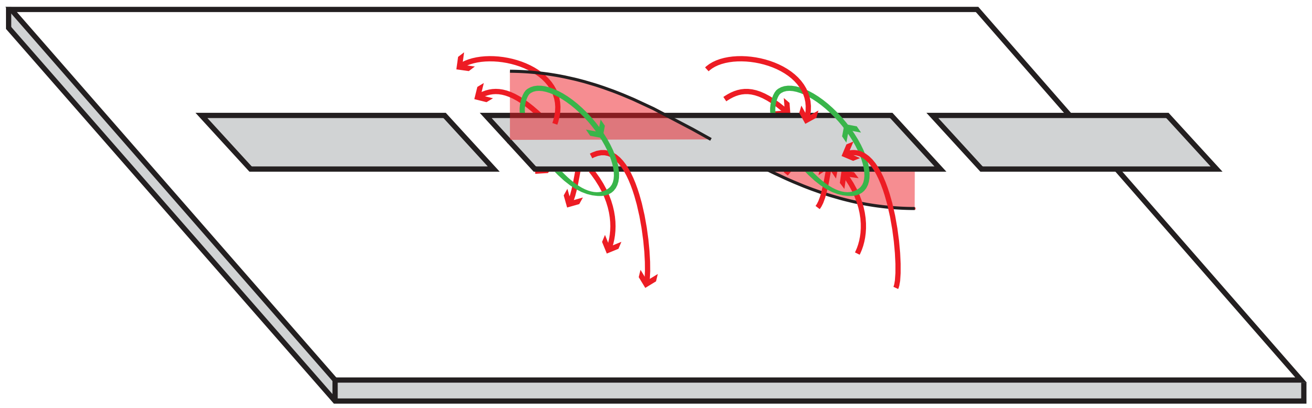
$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Z_0 = \sqrt{\frac{L}{C}}$$

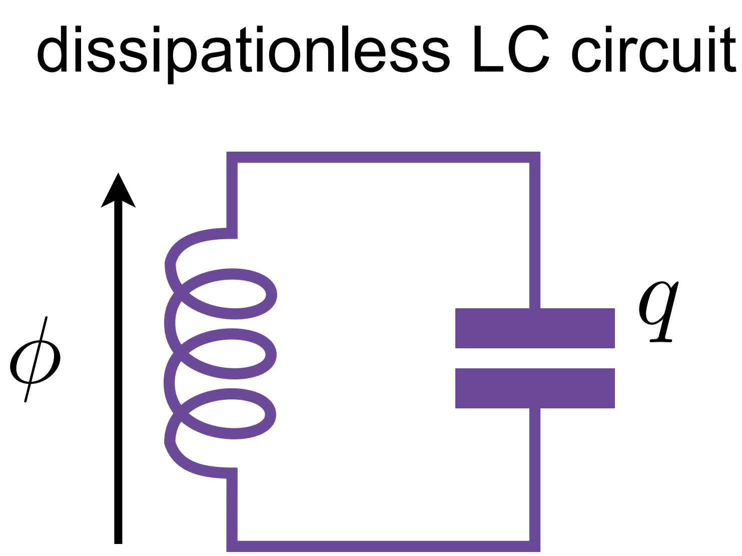
$$\phi_{ZPF} = \sqrt{\frac{\hbar Z_0}{2}} \qquad q_{ZPF} = \sqrt{\frac{\hbar}{2Z_0}}$$

Heisenberg inequality

$$\delta\phi\delta q \geq \frac{\hbar}{2}$$

Superconducting circuits





$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Z_0 = \sqrt{\frac{L}{C}}$$

$$\phi_{ZPF} = \sqrt{\frac{\hbar Z_0}{2}} \qquad q_{ZPF} = \sqrt{\frac{\hbar}{2Z_0}}$$

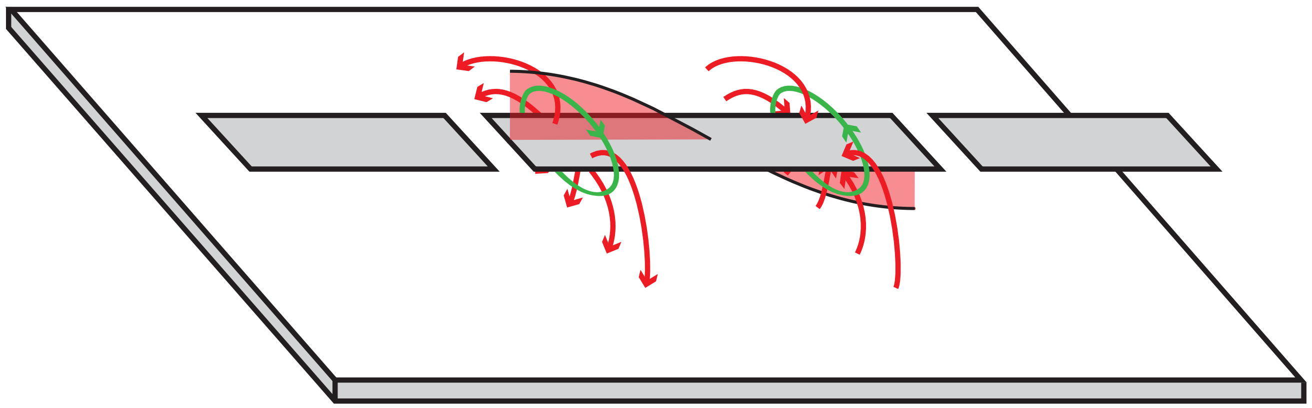
$$\hat{H} = \frac{\hat{q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

$$[\hat{\phi}, \hat{q}] = i\hbar$$

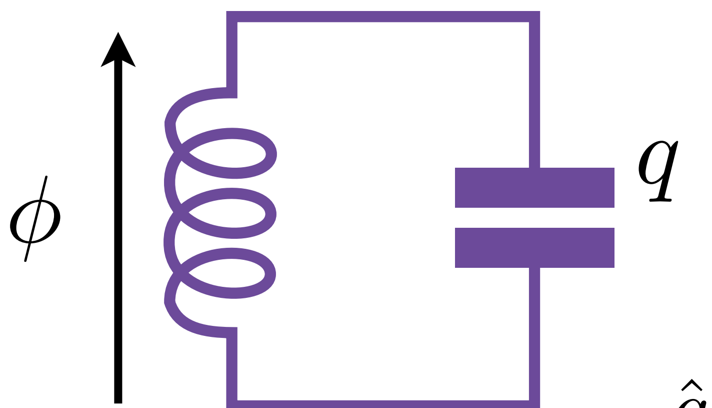
$$\hat{a} = \frac{\hat{\phi}/\phi_{ZPF} + i\hat{q}/q_{ZPF}}{2}$$

$$[\hat{a}, \hat{a}^\dagger] = 1$$

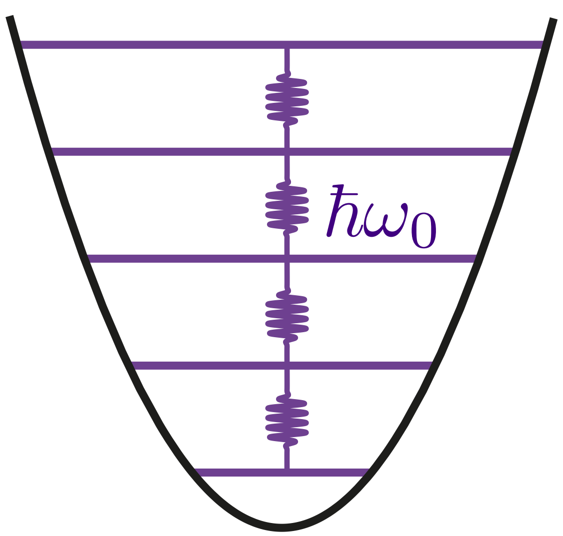
Superconducting circuits



dissipationless LC circuit...



....canonically quantized



$$\hat{a} = \frac{\hat{\phi}/\phi_{ZPF} + i\hat{q}/q_{ZPF}}{2}$$

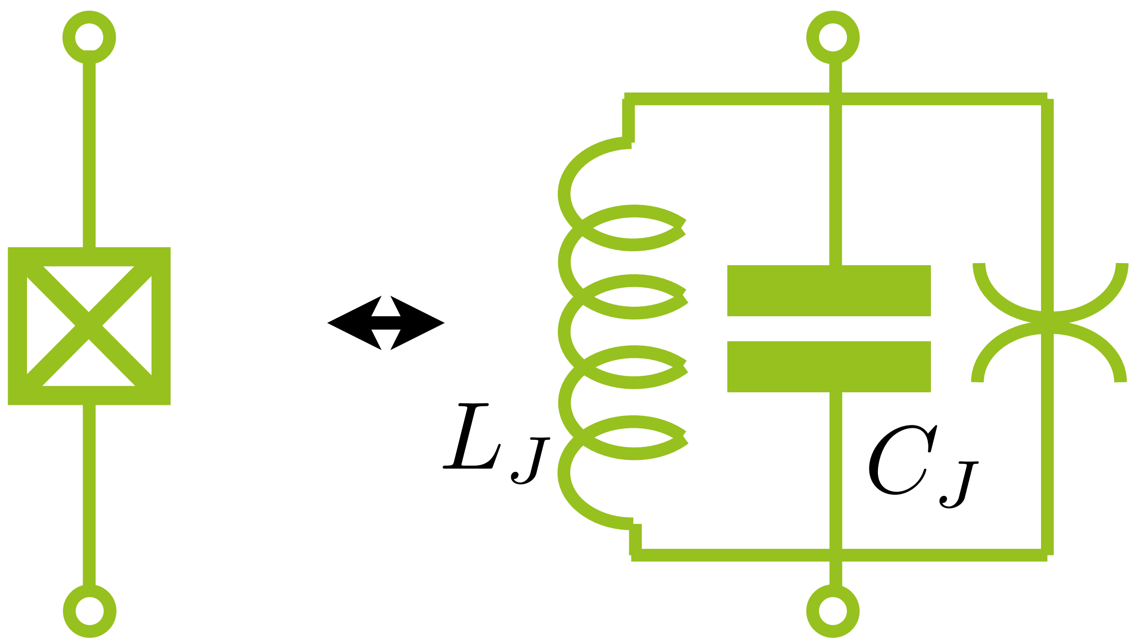
$$\hat{H} = \frac{\hat{q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

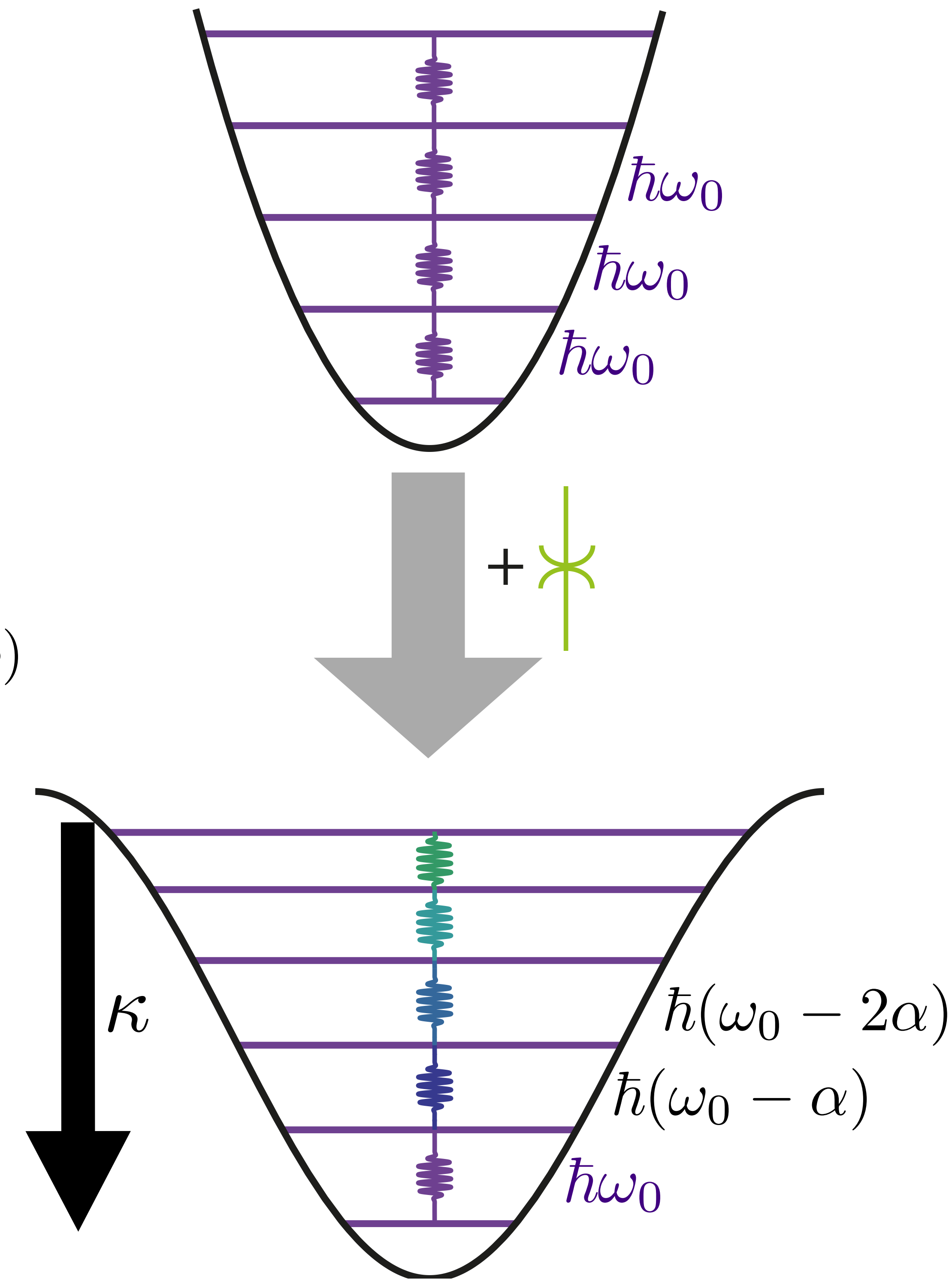
$$[\hat{\phi}, \hat{q}] = i\hbar$$

$$\hat{H} = \hbar\omega_0(\hat{a}^\dagger\hat{a} + \frac{1}{2})$$

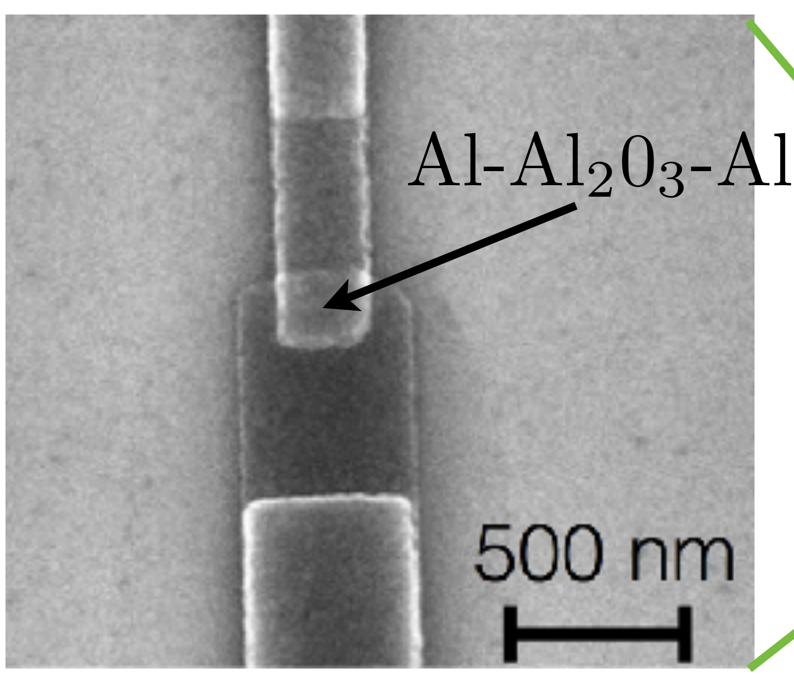
Superconducting circuits

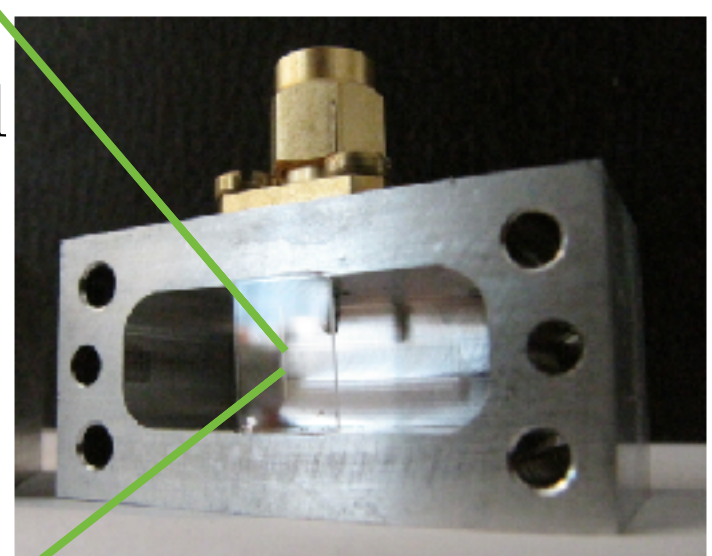
dissipation-less non linear LC circuit





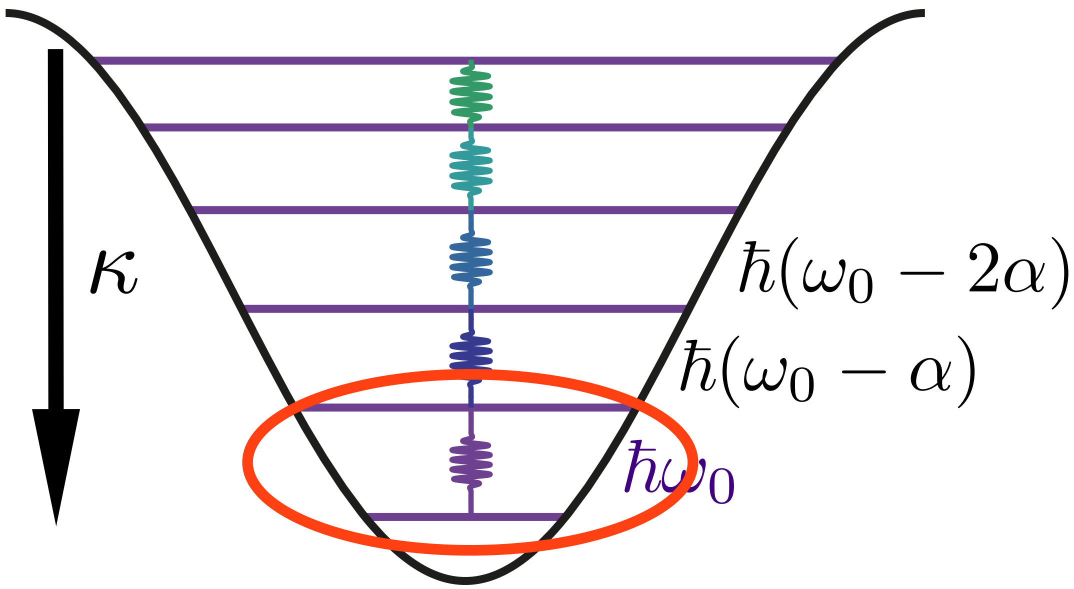
$$\hat{H} = \frac{\hat{q}^2}{2C_J} - E_J \cos\frac{\hat{\phi}}{\hbar/2e} = \frac{\hat{q}^2}{2C_J} + \frac{\hat{\phi}^2}{2L_J} + H_{\mathrm{non-lin}}(\hat{\phi})$$





transitions observed in 1980's [Berkeley & Saclay]
strong coupling regime of CQED in 2004 [Yale]

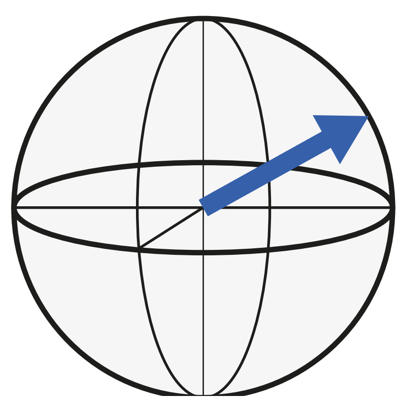
Non-linear superconducting circuits



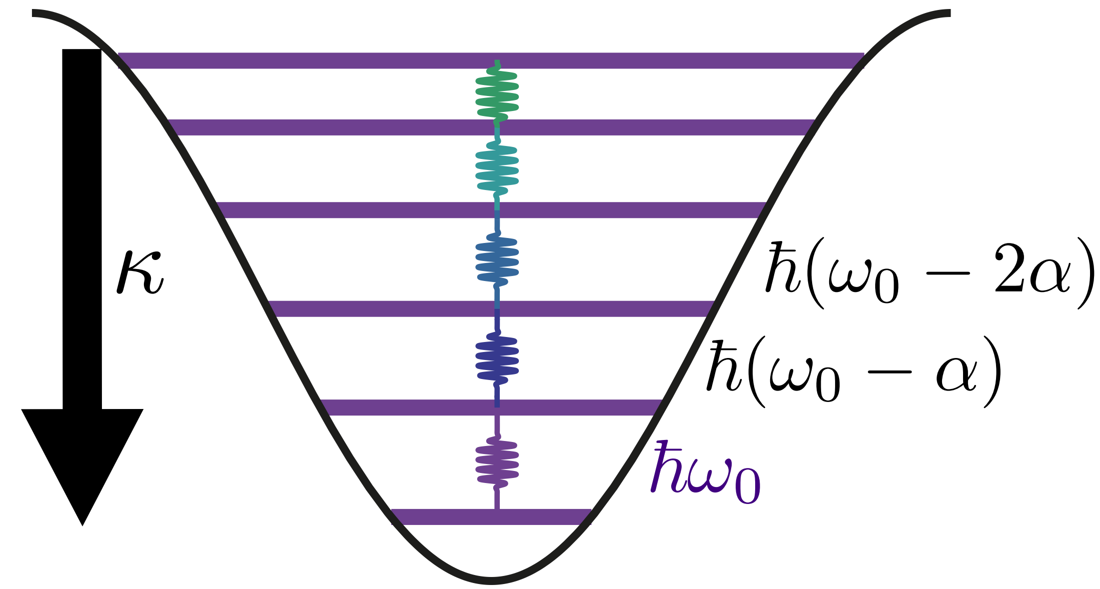
Strongly anharmonic

$\alpha \gg \kappa$

qubit $\hbar\omega\hat{\sigma}_z/2$



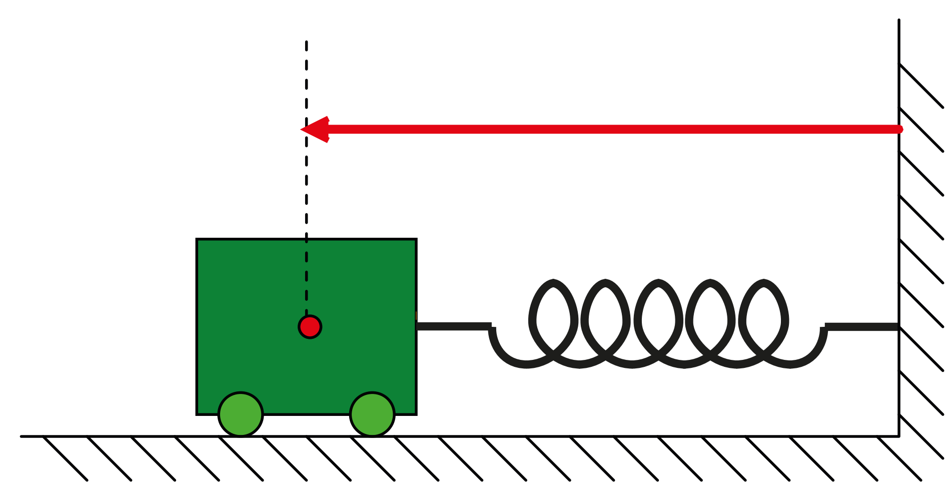
First Rabi oscillations in 1999 [NEC group]
Quantronium in 2002 [Quantronics group]
Charge qubit, phase qubit [Grenoble & others], flux qubit, transmon [Yale, ETH & others], fluxonium, Xmon...



Weakly anharmonic

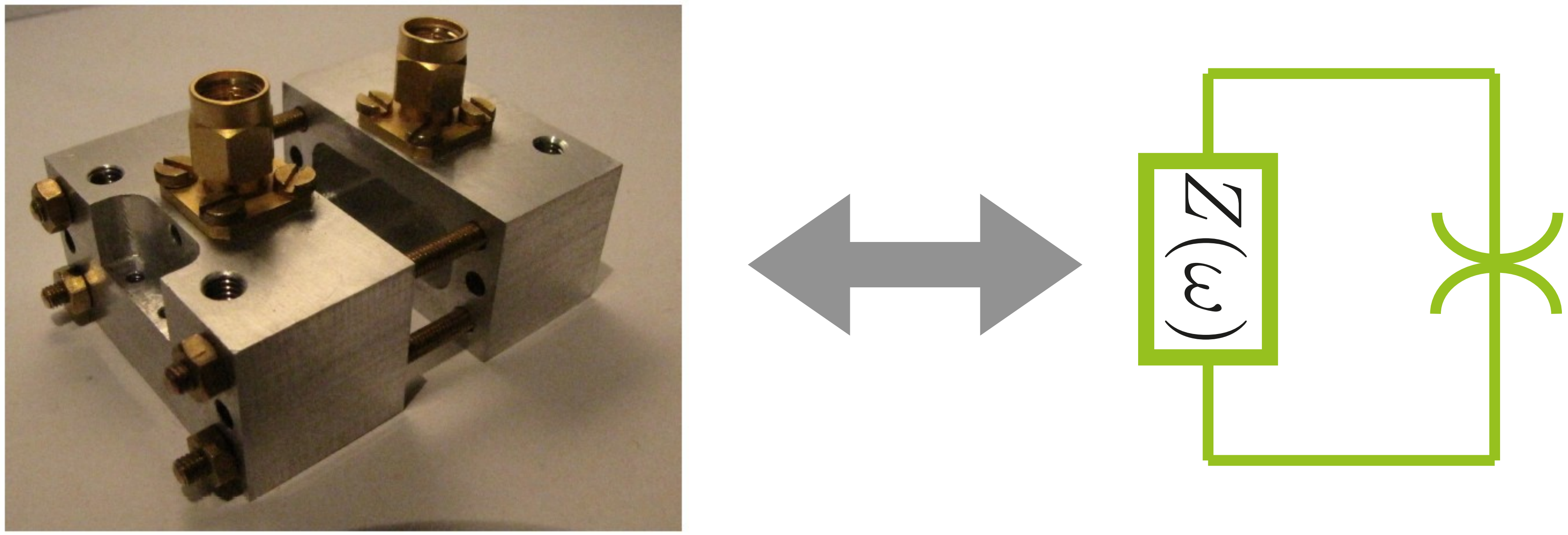
$\alpha \ll \kappa$

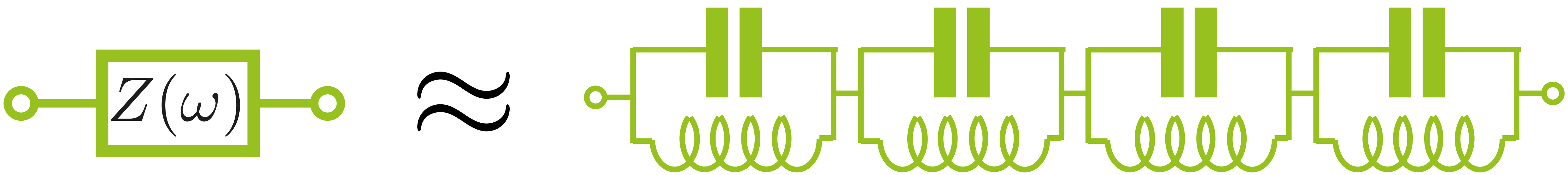
oscillator $\hbar\omega\hat{a}^\dagger\hat{a}$



Parametric amplifiers & squeezing
in 1980's [Bell Labs]

Black Box Quantization





BBQ Recipe [Nigg et al., PRL 2012 (Yale)]

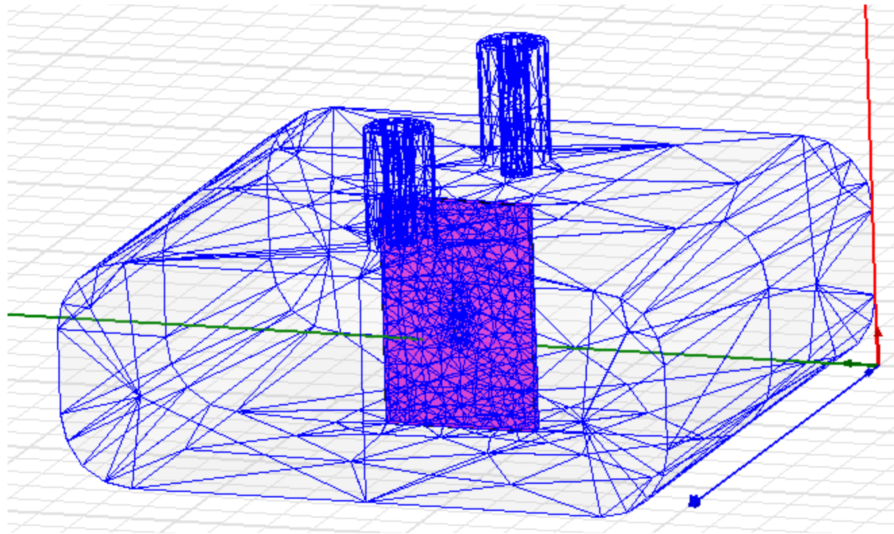
$$\hat{H} = \sum_k \hbar\omega_k + \frac{1}{2}\sum_{k,l} \hbar\chi_{kl}\hat{n}_k\hat{n}_l$$

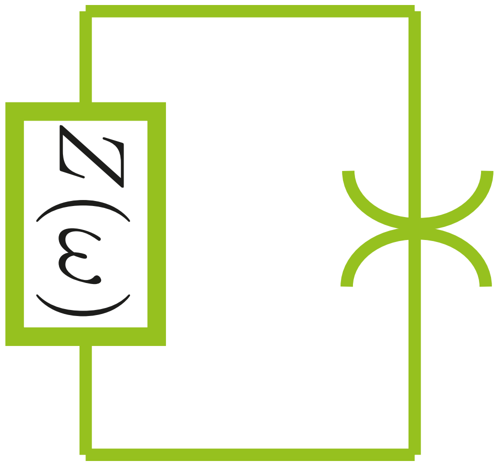
self-Kerr $\chi_{kk} = -\frac{e^2}{2L_J} Z_k^2$

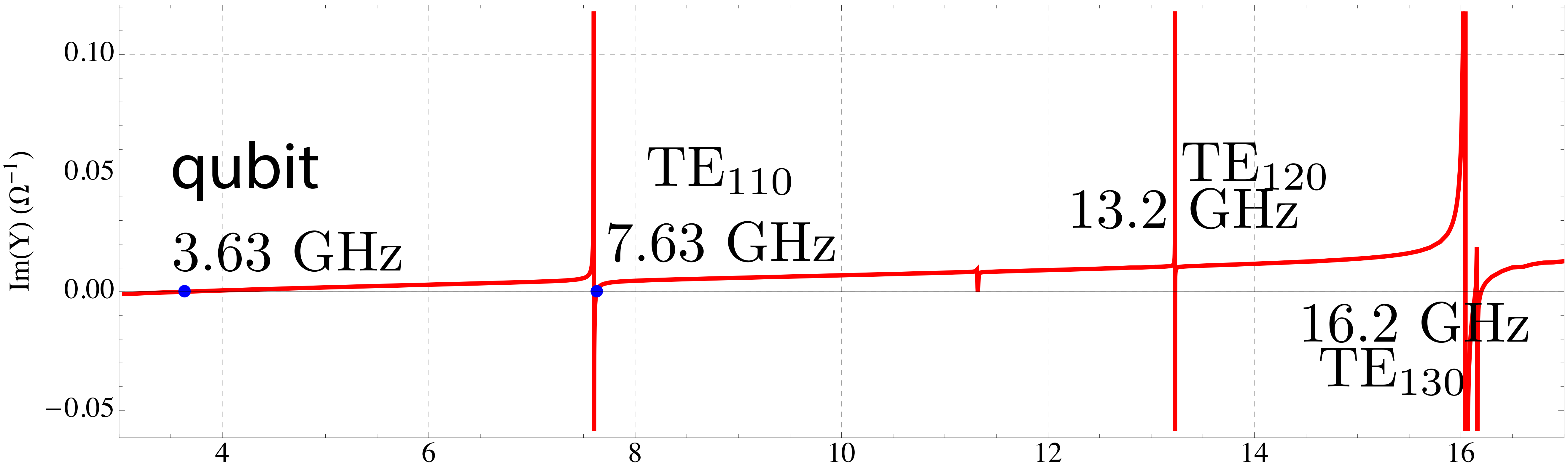
cross-Kerr $\chi_{kl} = -2\sqrt{\chi_{kk}\chi_{ll}}$

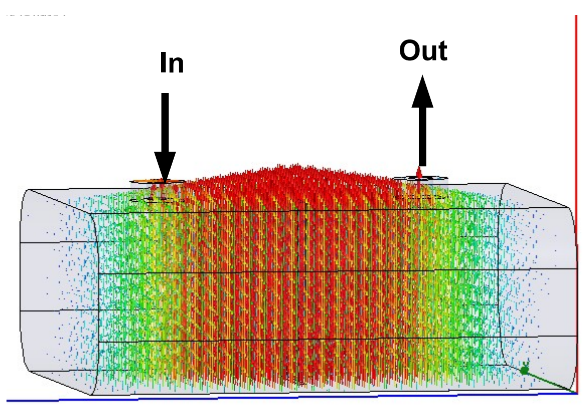
$$Z_k = \frac{2}{\omega_k \mathrm{Im} Y'(\omega_k)}$$

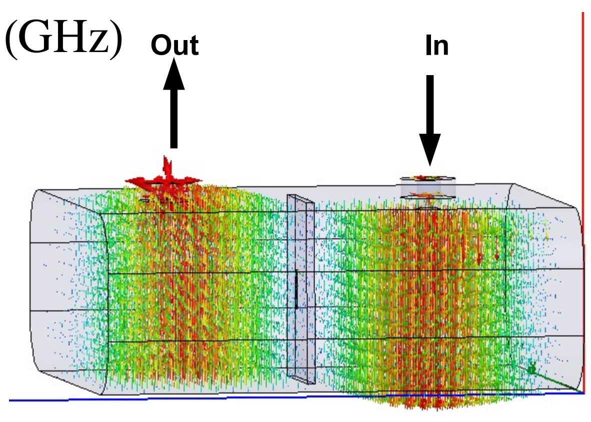
Black Box Quantization

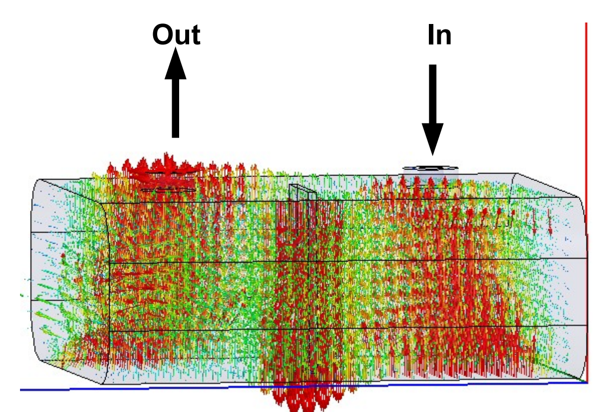




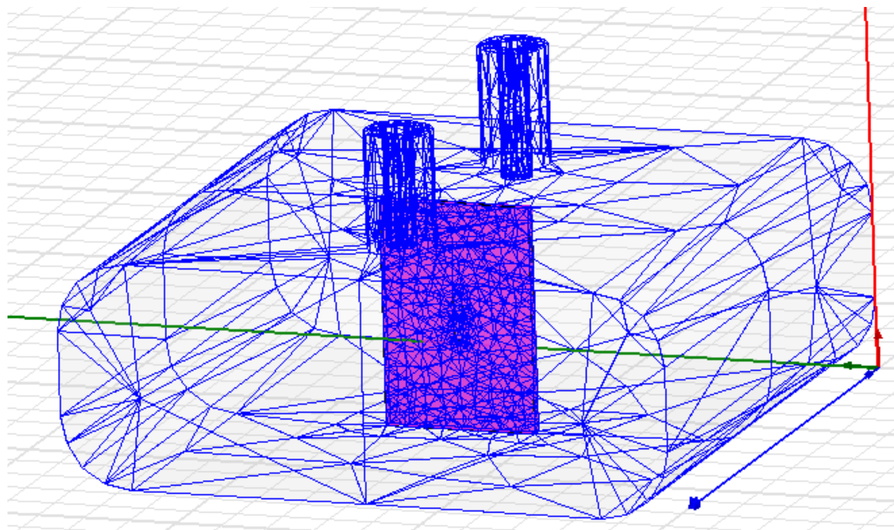


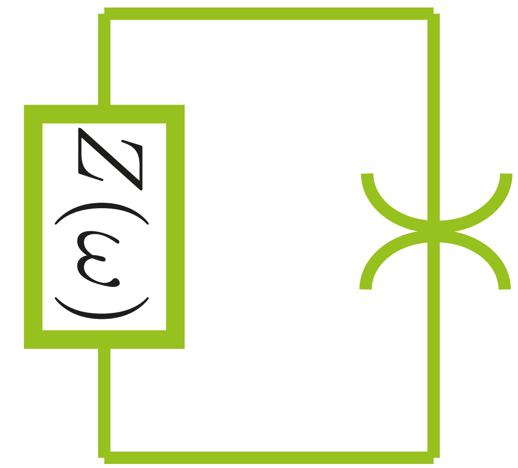


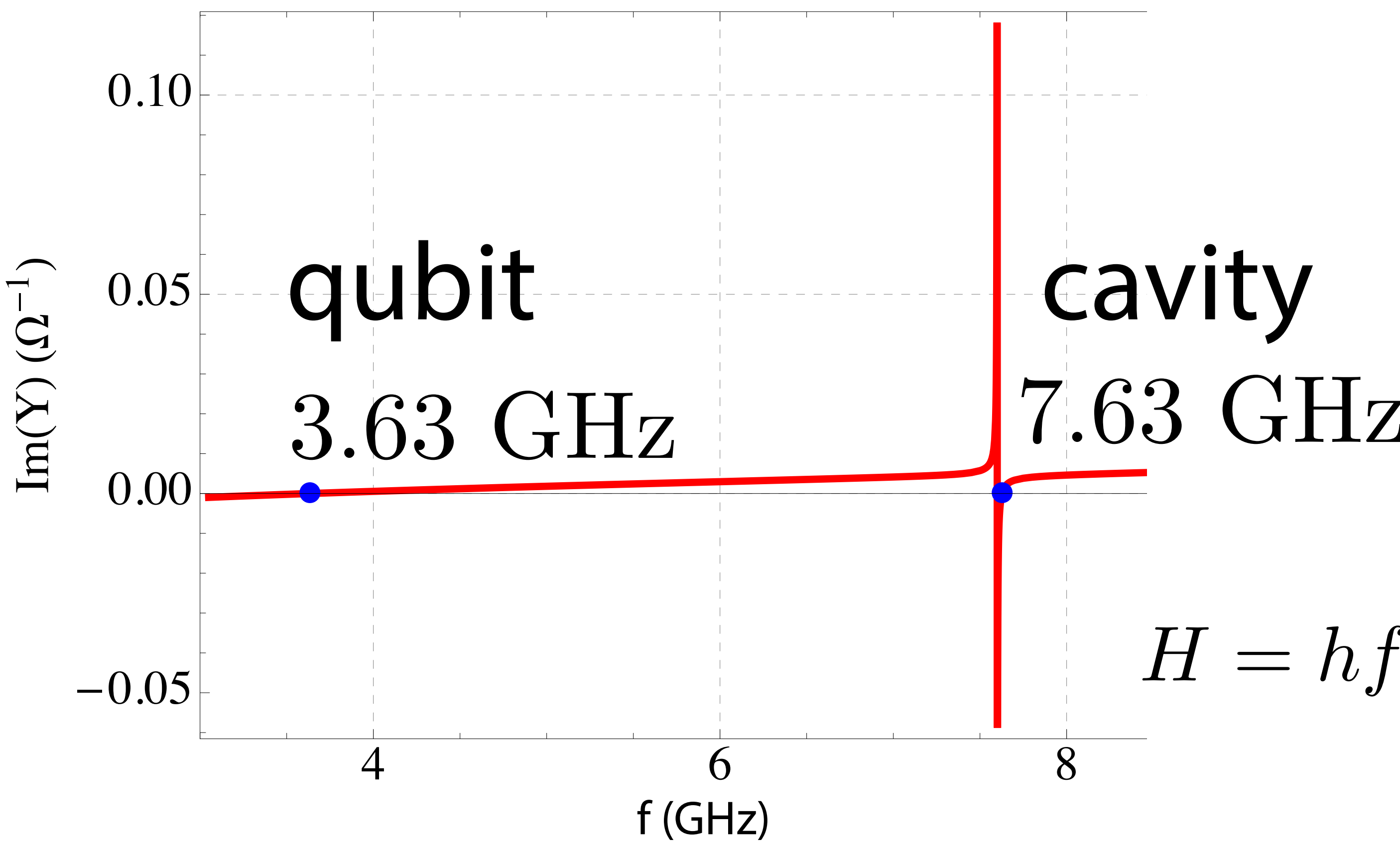




Black Box Quantization







$$\chi_{cq}^{(thy)}/2\pi = -1.4 \text{ MHz}$$

$$\chi_{qq}^{(thy)}/4\pi = -150 \text{ MHz}$$

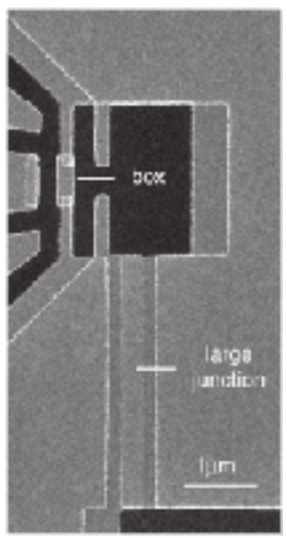
$$H = hf_c a^\dagger a + h(f_q + \chi_{qq} n_q) n_q + h\chi_{cq} n_q a^\dagger a$$

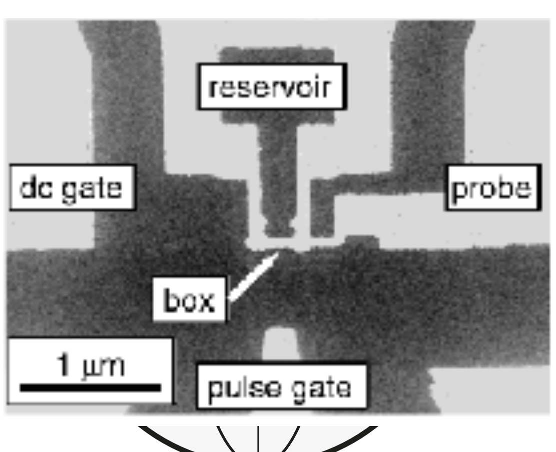
$$H = hf_c a^\dagger a + hf_q \frac{\sigma_Z}{2} + h\chi_{cq}\frac{\sigma_Z}{2} a^\dagger a$$

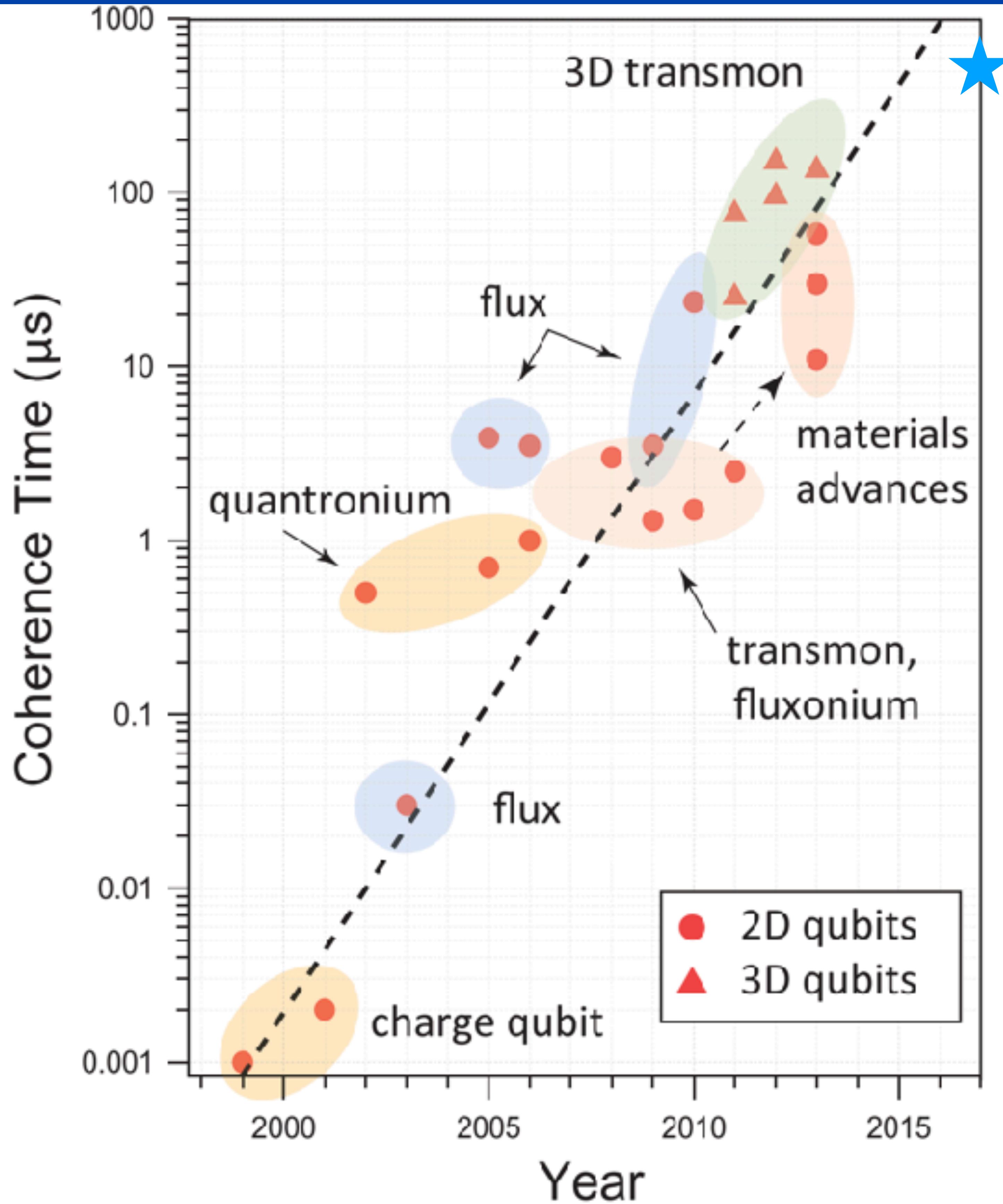
Fast approach using participation of nonlinearities in modes → pyHFSS on Github

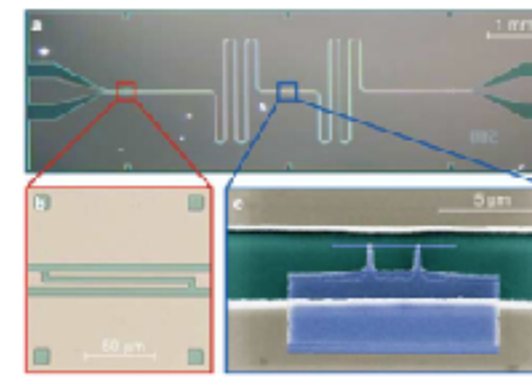
[Minev, Leghtas et al. (Yale) in prep.]

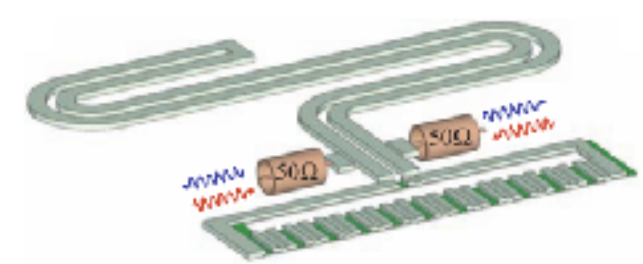
Superconducting qubits











[Oliver and Welander, MRS Bulletin (2013)]

Gate-based quantum computing with superconducting circuits

https://www.research.ibm.com/ibm-q/

IBM
2013
50 qubits

Google
72 qubits
2014

BBN TECHNOLOGIES
2014

ANYON
Computing with Manufactured Quanta™
2014

rigetti
19 qubits
2015

intel
2015

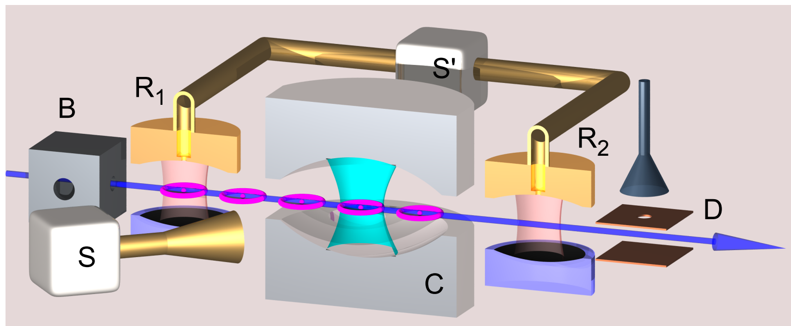
qci
2015

Alibaba Cloud
2015

SeeQC
2017

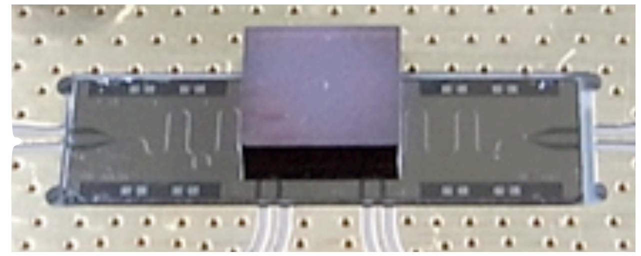
Superconducting circuits with non linear systems

Rydberg atoms



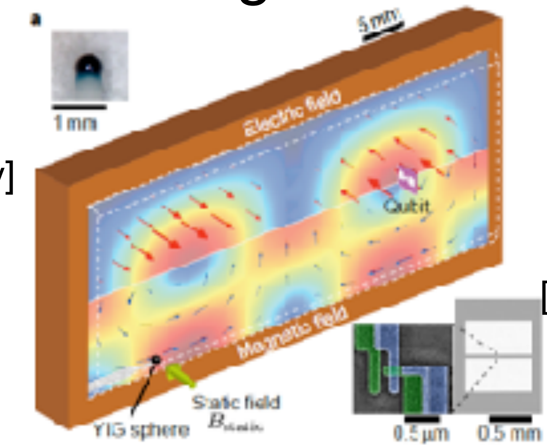
[pic from CQED group, College de France Paris]

NV centers



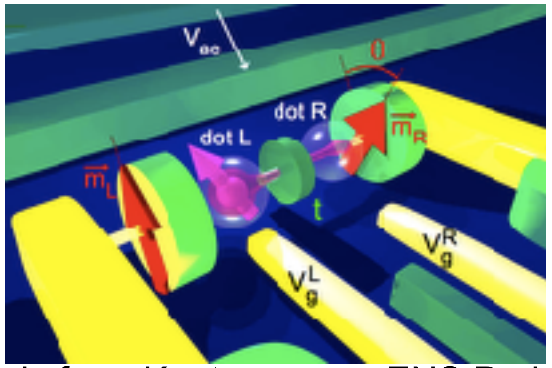
[pic from Quantronics group, CEA Saclay]

Ferromagnetic magnons



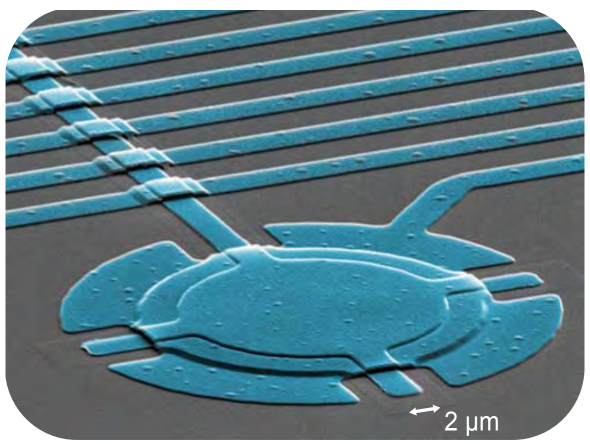
[pic from Nakamura-Usami group, Univ. Tokyo]

Spins in CNT



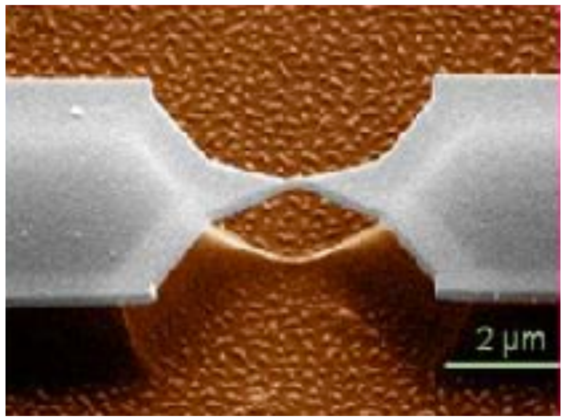
[pic from Kontos group, ENS Paris]

Metallic membrane



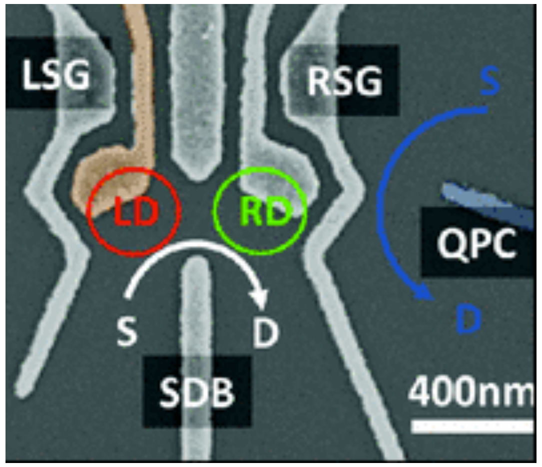
[pic from Lehnert group, JILA Boulder]

Andreev Bound States



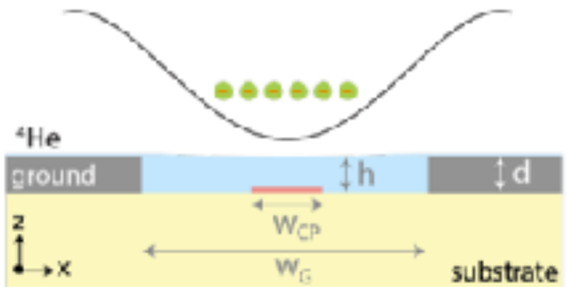
[pic from Quantronics group, CEA Saclay]

Semiconductor quantum dots



[pic from Wallraff group, ETH Zurich]

Electrons on ^{4}He

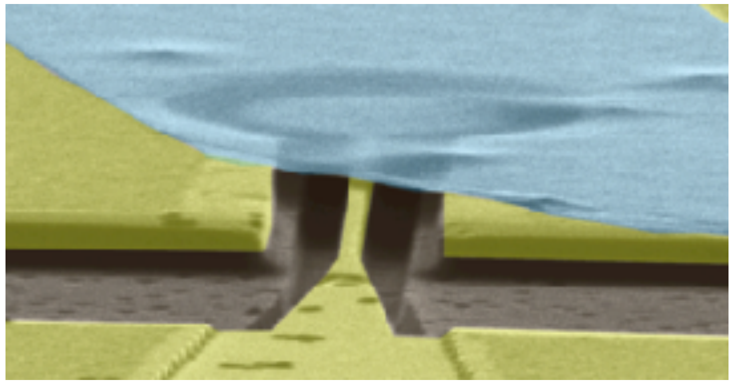


[pic from Shuster group, Chicago]

Propagating phonons

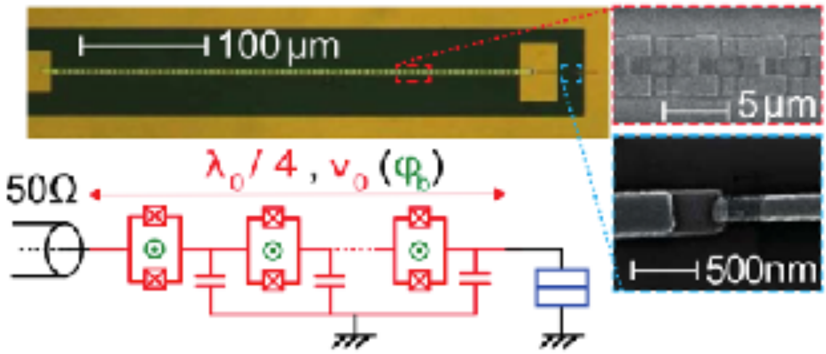
[pic from Delsing group, Chalmers UT]

Graphene membrane



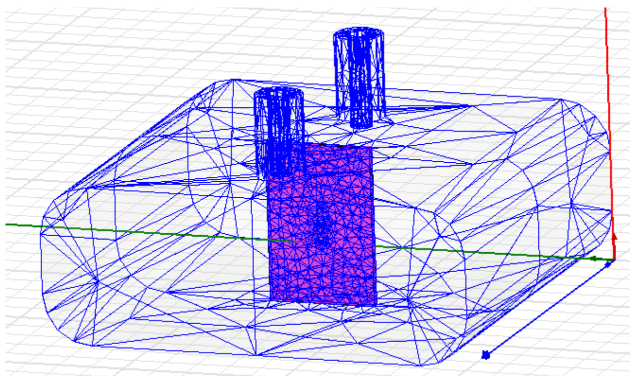
[pic from Steele group, TU Delft]

DC biased junction



[pic from Saclay group]

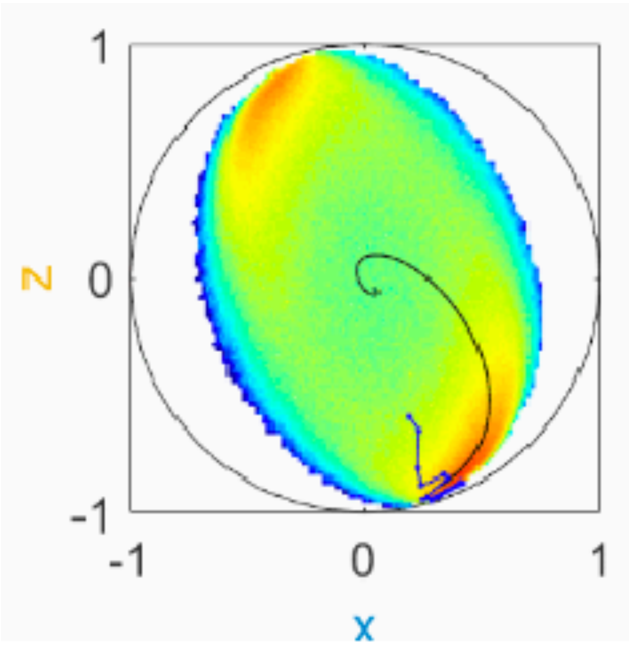
Quantum trajectories and feedback in circuit-QED

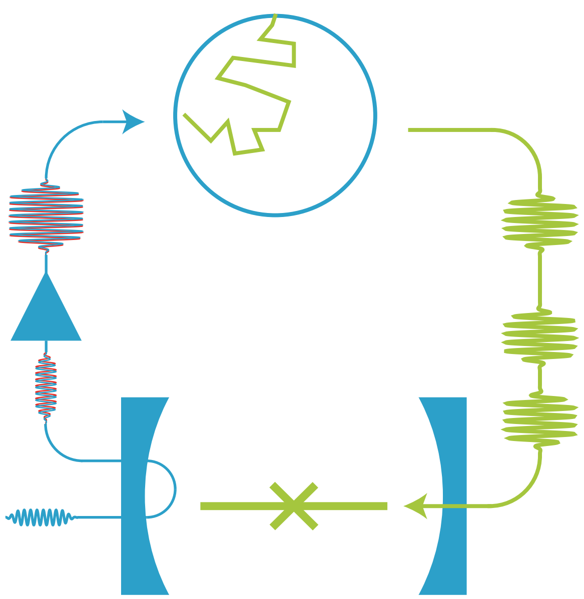


Introduction to circuit-QED

Quantum trajectories in circuit-QED

- dispersive measurement
- fluorescence measurement
- both simultaneously
- generating entanglement





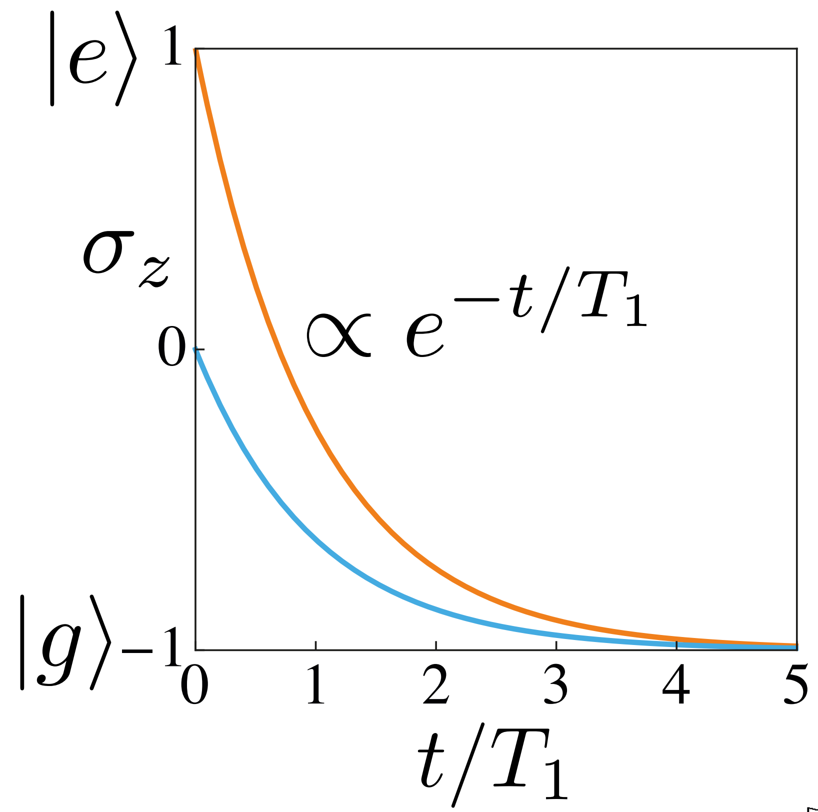
Measurement based feedback

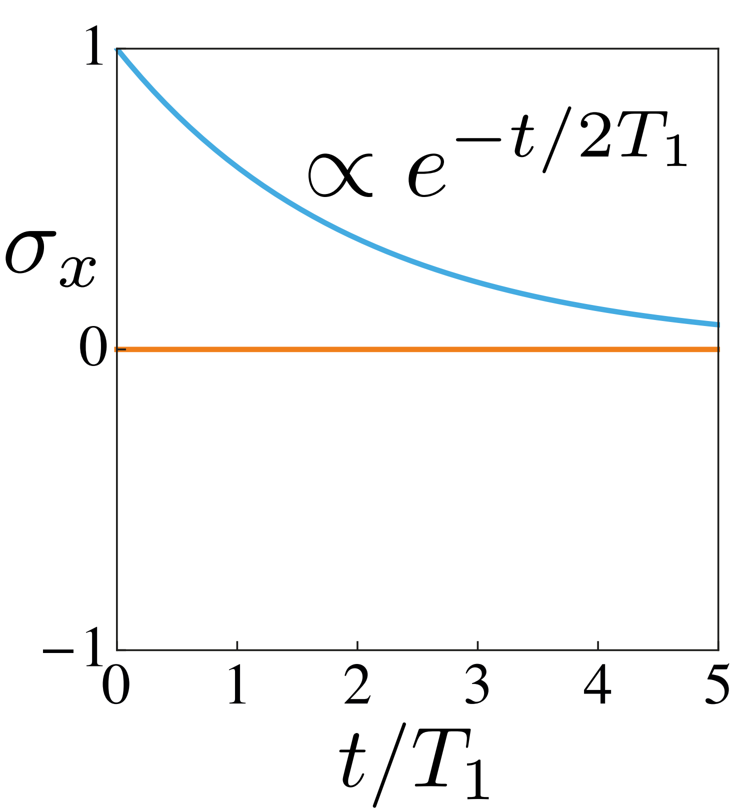
- dispersive case
- fluorescence case

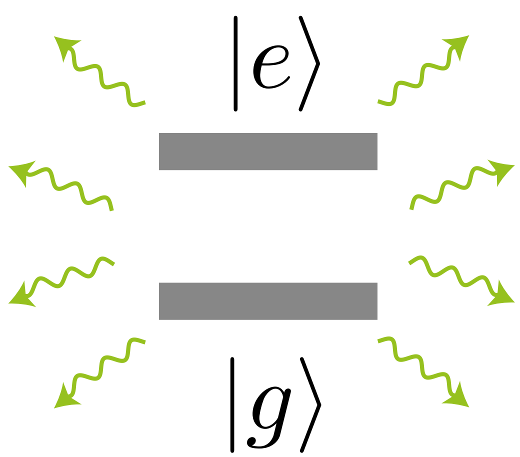
Post selection in quantum mechanics

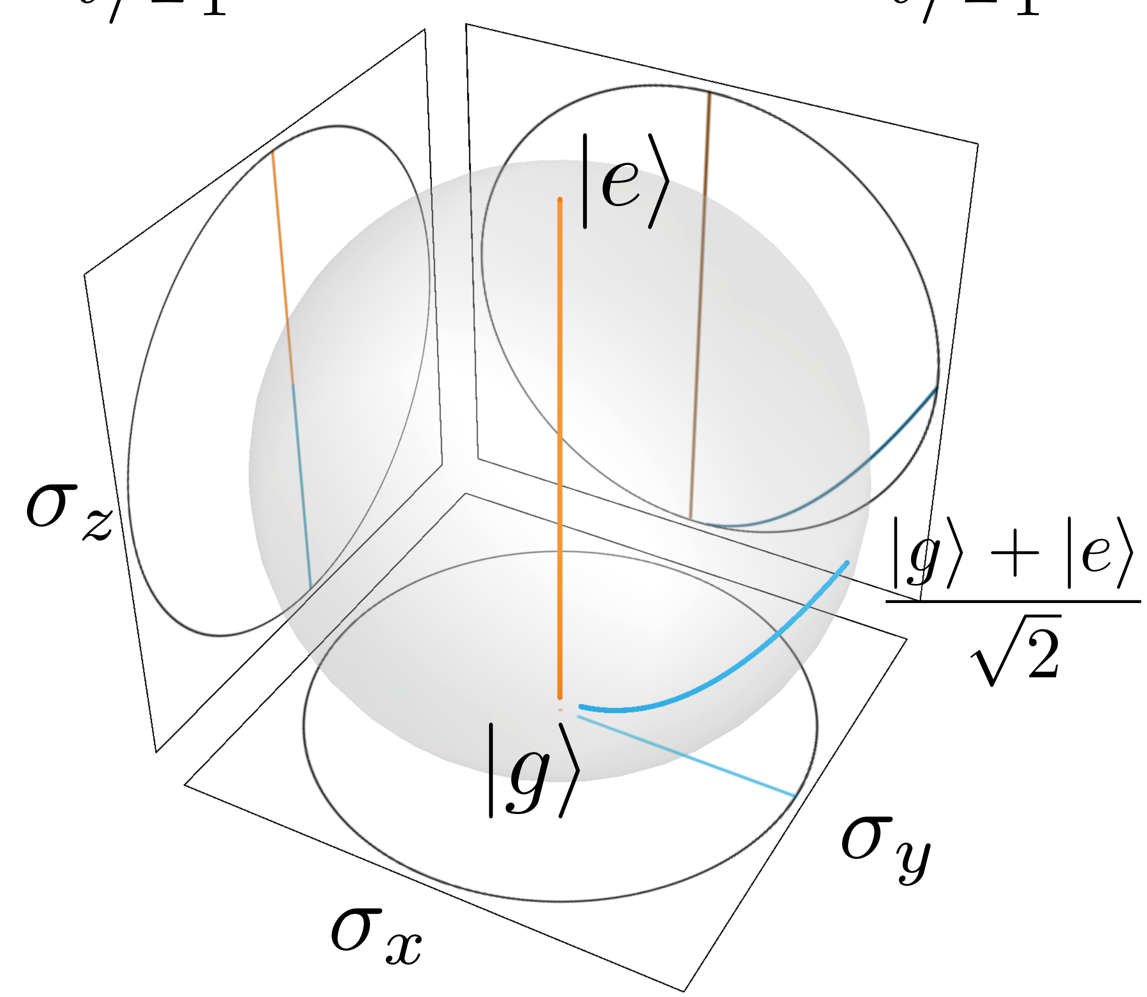
$\rho(t), E(t)$

Ideal quantum jump of an atom

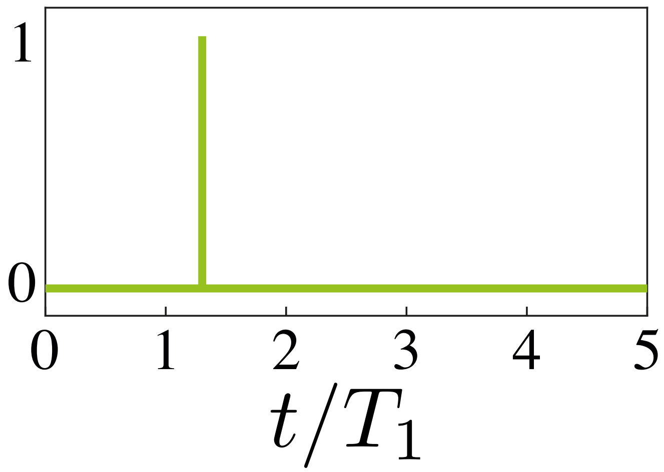


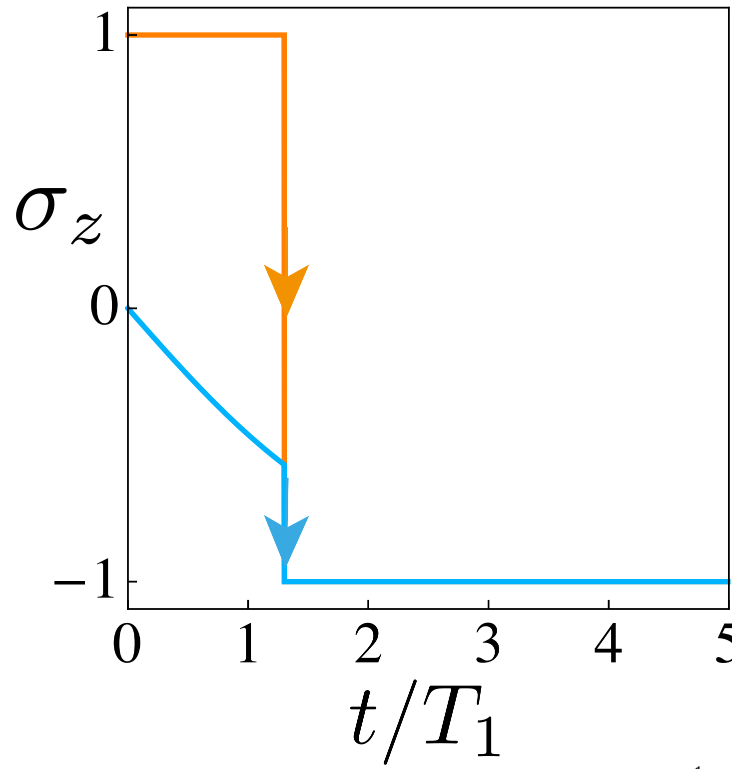


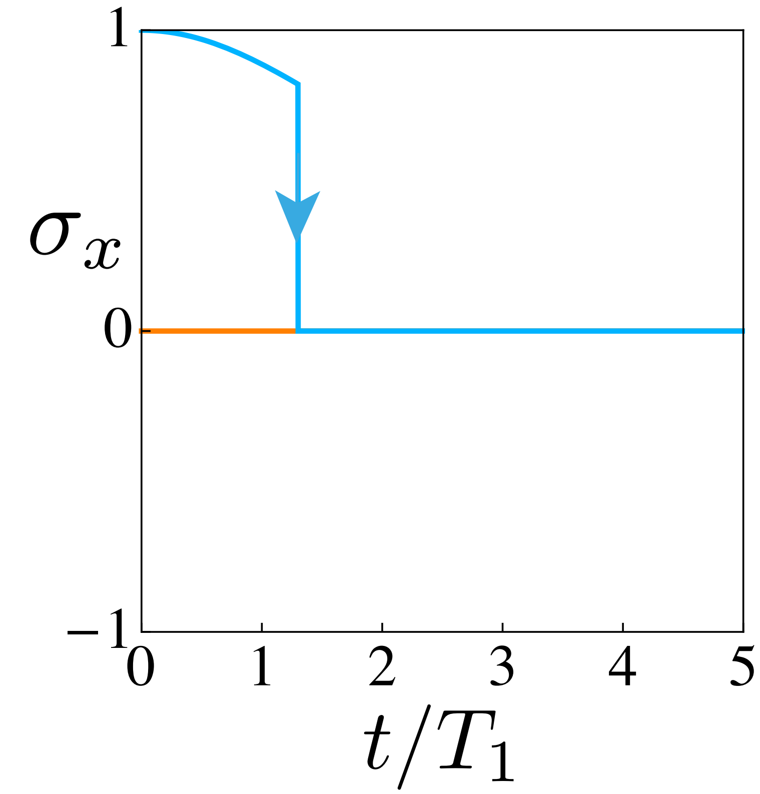


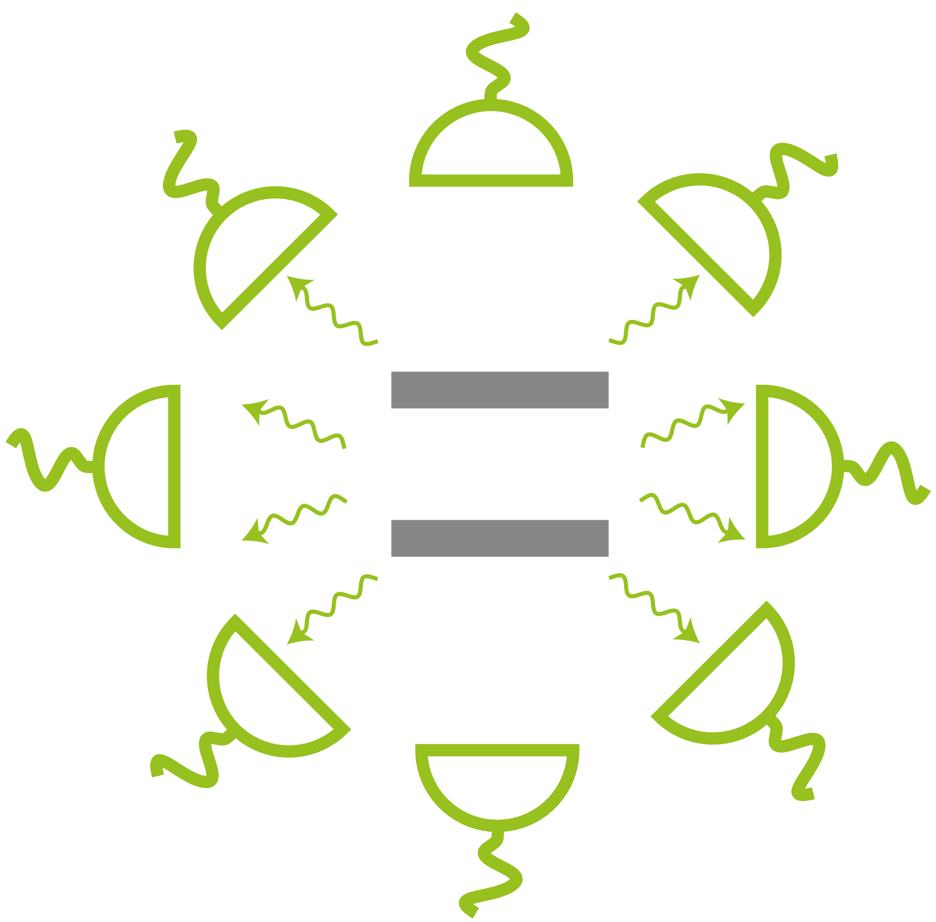


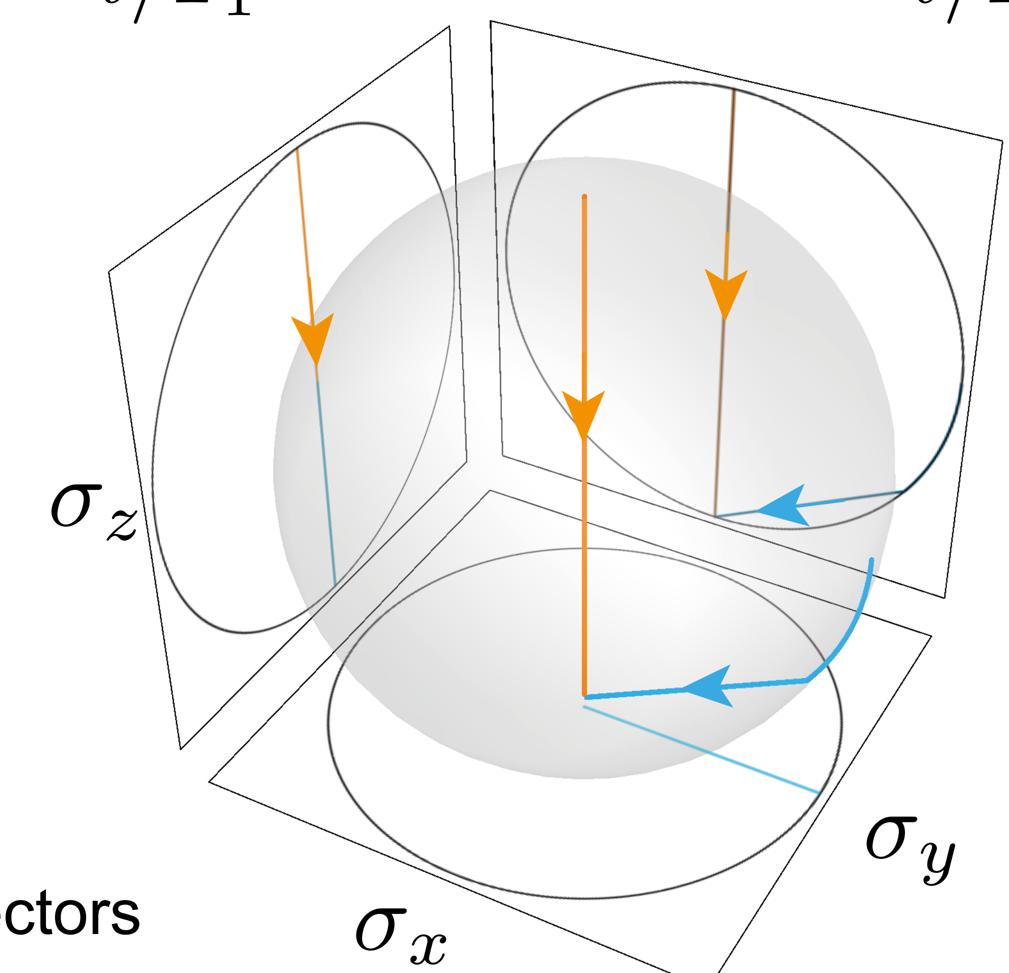
Ideal quantum jump of an atom







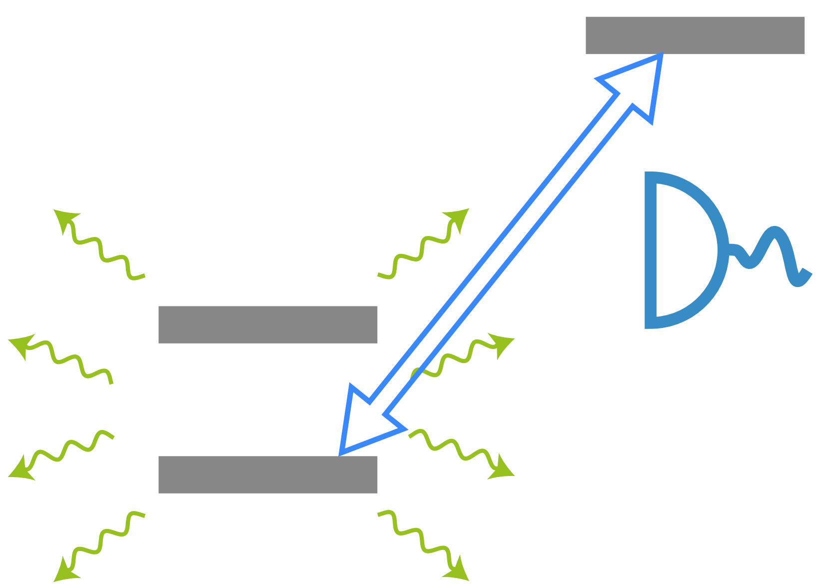




Note: purity of state is 1 only for perfect detectors

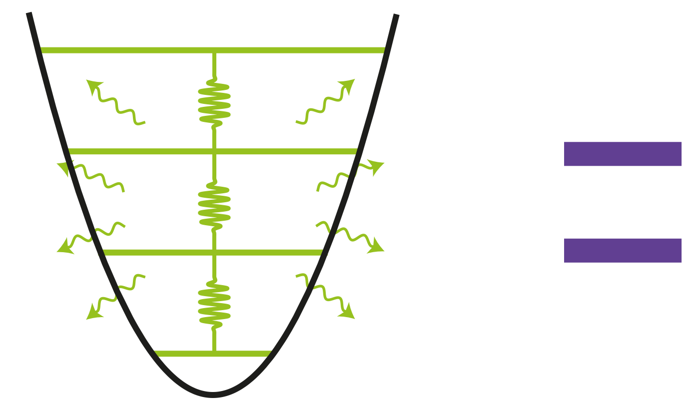
Ideal quantum jump

hard to collect → use an ancillary detector



since 1986 in trapped ions

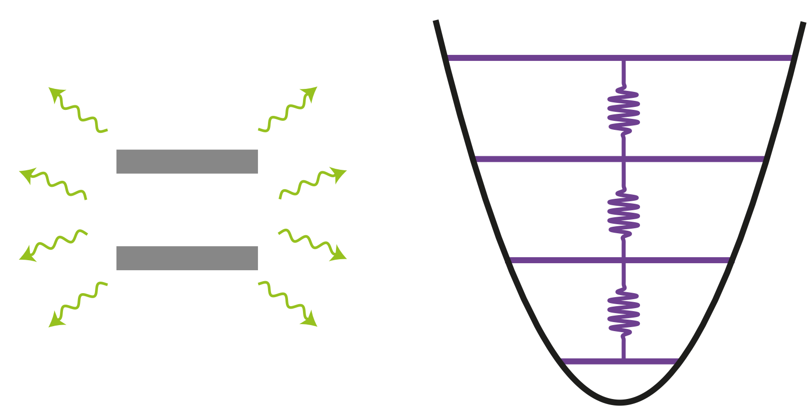
[Wineland group, Boulder
Dehmelt groupe, Seattle
Toschek group, Hambourg]



$$H_{\text{coupl}} = \hbar\chi a^{\dagger} a \frac{\sigma_Z}{2}$$

Rydberg atom probing cavity jumps

[Haroche group, Paris (2007)]



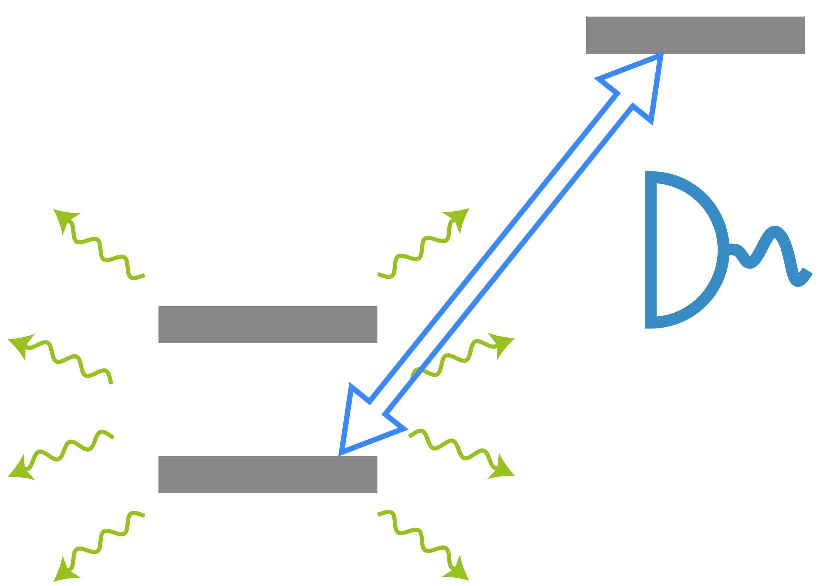
$$H_{\text{coupl}} = \hbar\chi a^{\dagger} a \frac{\sigma_Z}{2}$$

Cavity probing qubit jumps

[Siddiqi group, Berkeley (2011)]

Ideal quantum jump

hard to collect → use an ancillary detector



since 1986 in trapped ions

[Wineland group, Boulder
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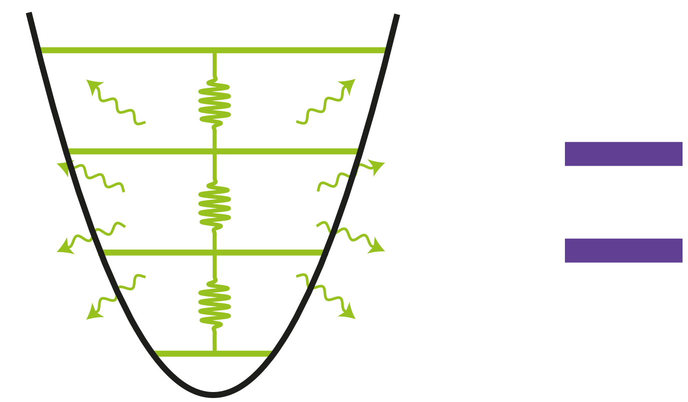

$$H_{\mathrm{coupl}} = \hbar\chi a^\dagger a \frac{\sigma_Z}{2}$$

Rydberg atom probing cavity jumps

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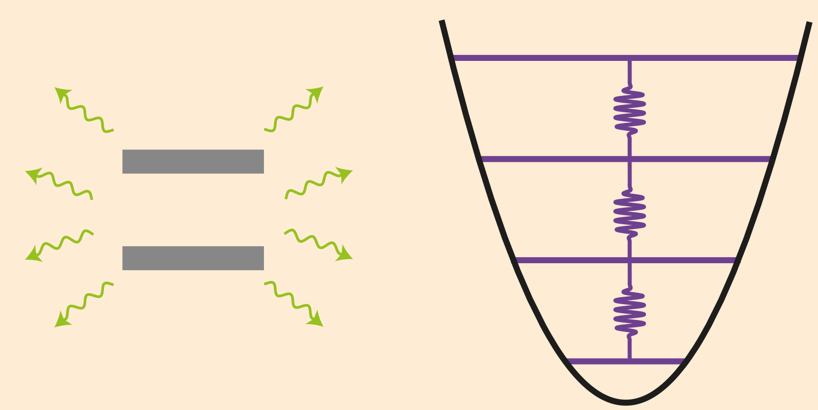

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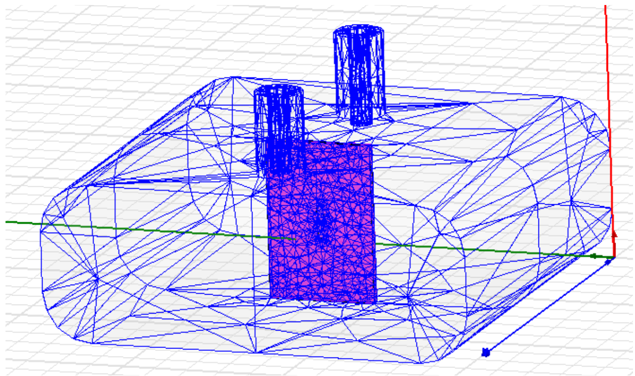
Cavity probing qubit jumps

[Siddiqi group, Berkeley (2011)]

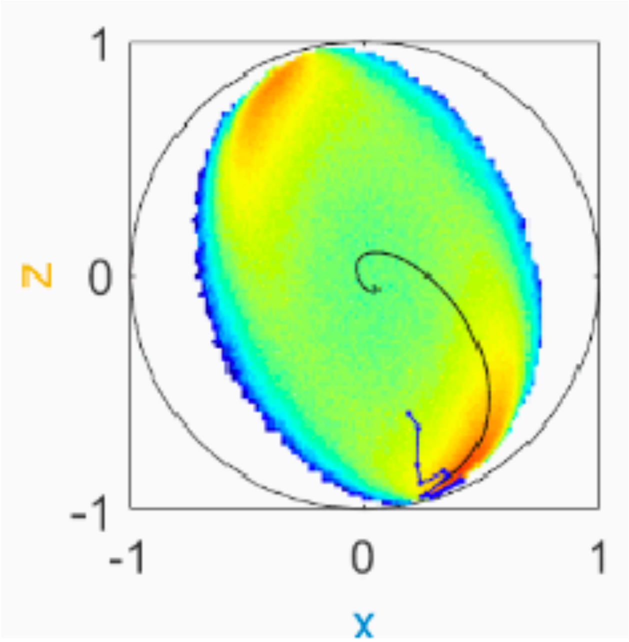
Quantum trajectories and feedback in circuit-QED

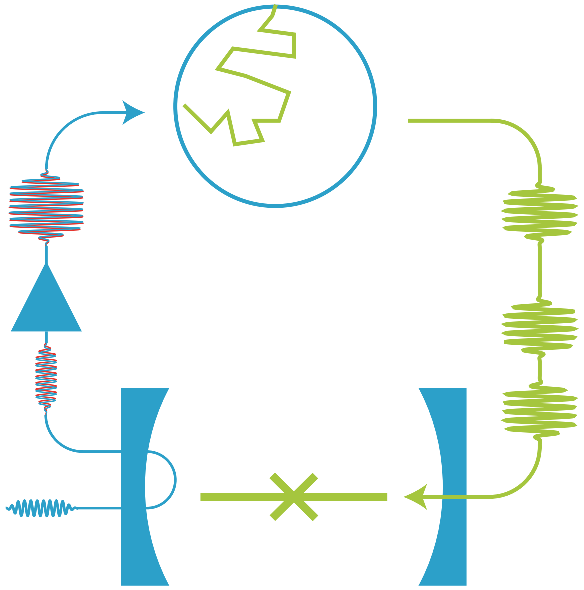


Introduction to circuit-QED

Quantum trajectories in circuit-QED

- **dispersive measurement**
- fluorescence measurement
- both simultaneously
- generating entanglement





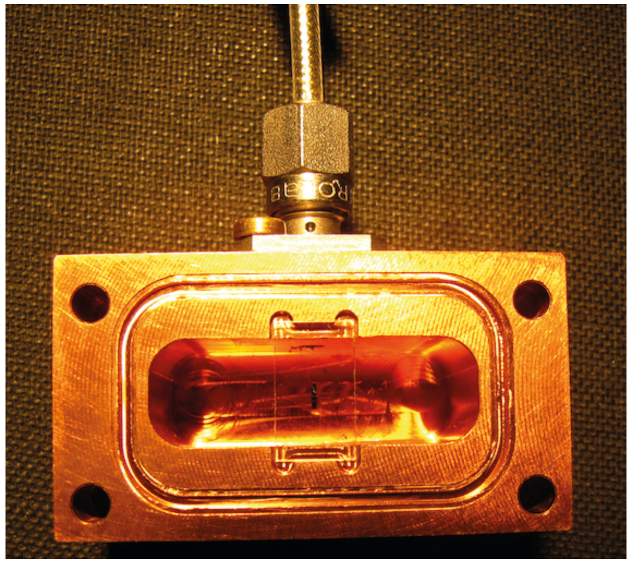
Measurement based feedback

- dispersive case
- fluorescence case

Post selection in quantum mechanics

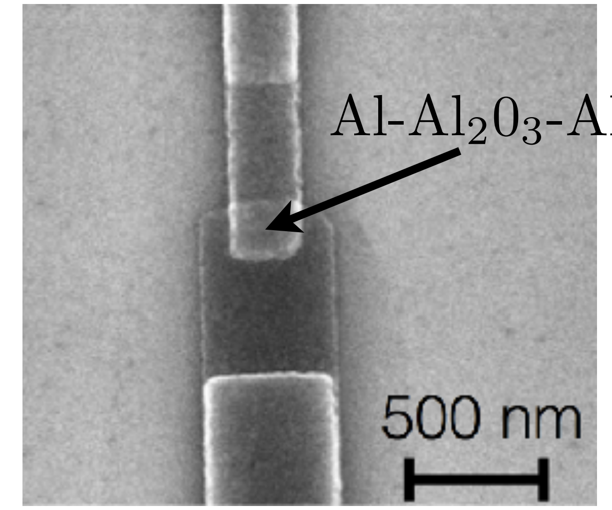
$\rho(t), E(t)$

3D transmon architecture



$$H_{\rm disp} = -h\chi\frac{\sigma_Z}{2}a^\dagger a$$

$$\chi = 4\ \text{MHz}$$

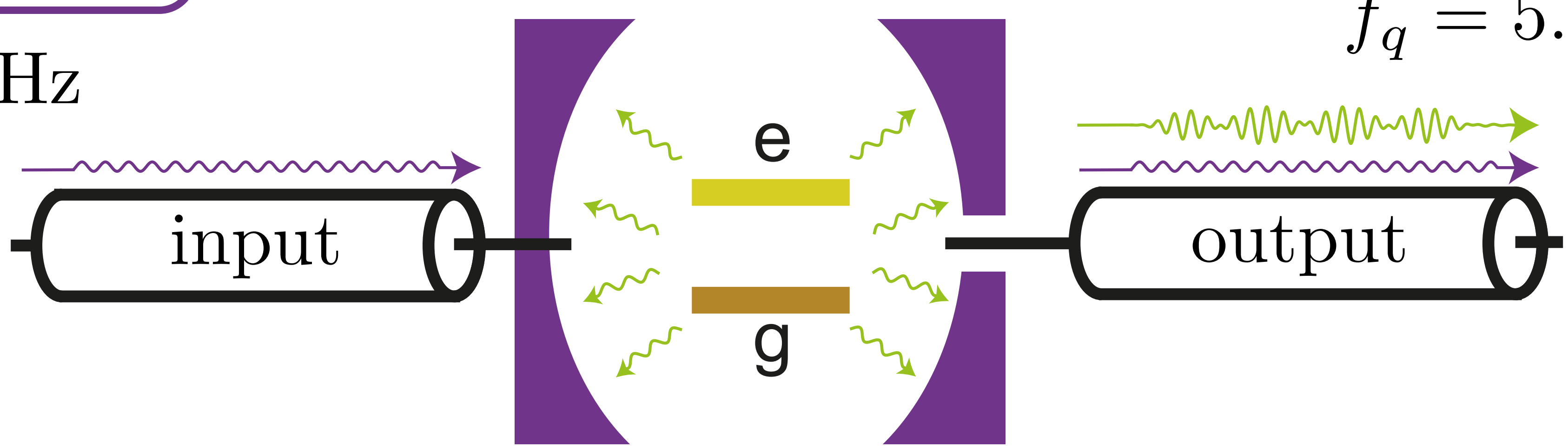


$$H_c = hf_c(a^\dagger a + \frac{1}{2})$$

$$f_c = 7.8\ \text{GHz}$$

$$H_q = hf_q\frac{\sigma_z}{2}$$

$$f_q = 5.4\ \text{GHz}$$

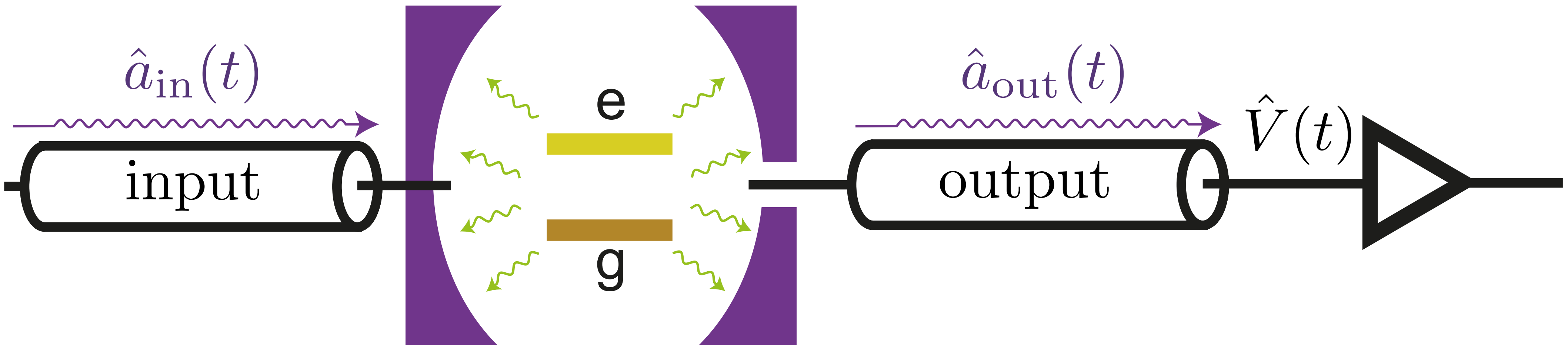


Dispersive Hamiltonian

$$H = hf_c(a^\dagger a + \frac{1}{2}) - h\frac{\chi}{2}\sigma_Z a^\dagger a + hf_q\frac{\sigma_Z}{2}$$

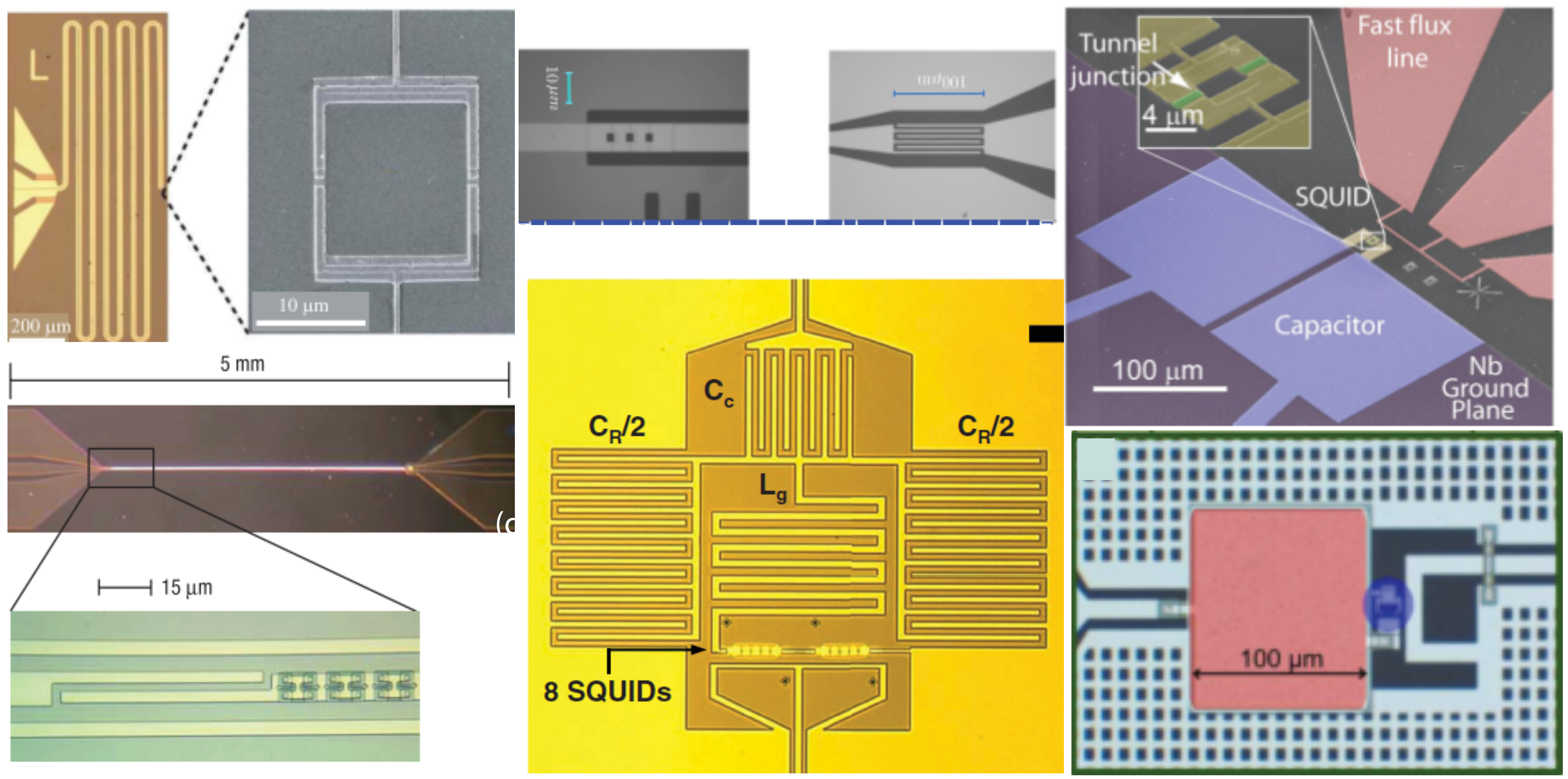
Dispersive Measurement

$$H = hf_q\frac{\sigma_Z}{2} + h(f_c - \frac{\chi}{2}\sigma_z)a^\dagger a$$



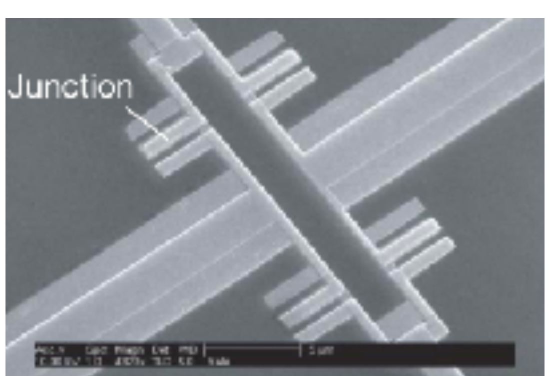
Quantum limited amplifiers

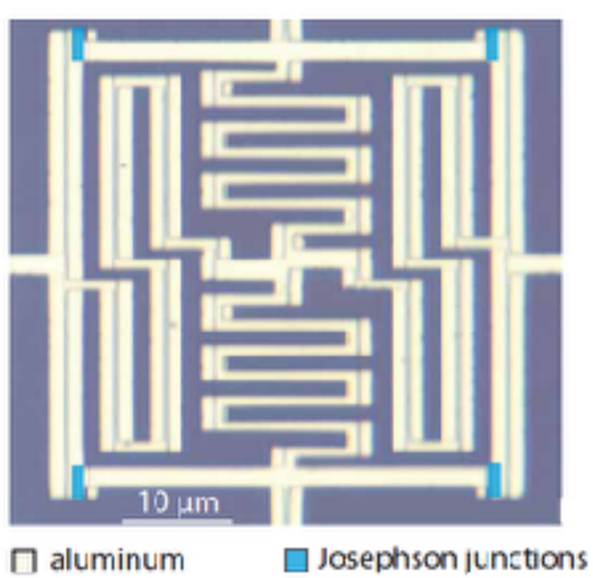
Degenerate amplifiers



(Bell Labs, 1989)
(NEC Tokyo, 2008)
(Boulder, 2008)
(Yale, 2009)
(Zurich, 2011)
(Berkeley, 2011)
(Santa Barbara, 2013)
(Saclay, 2014)

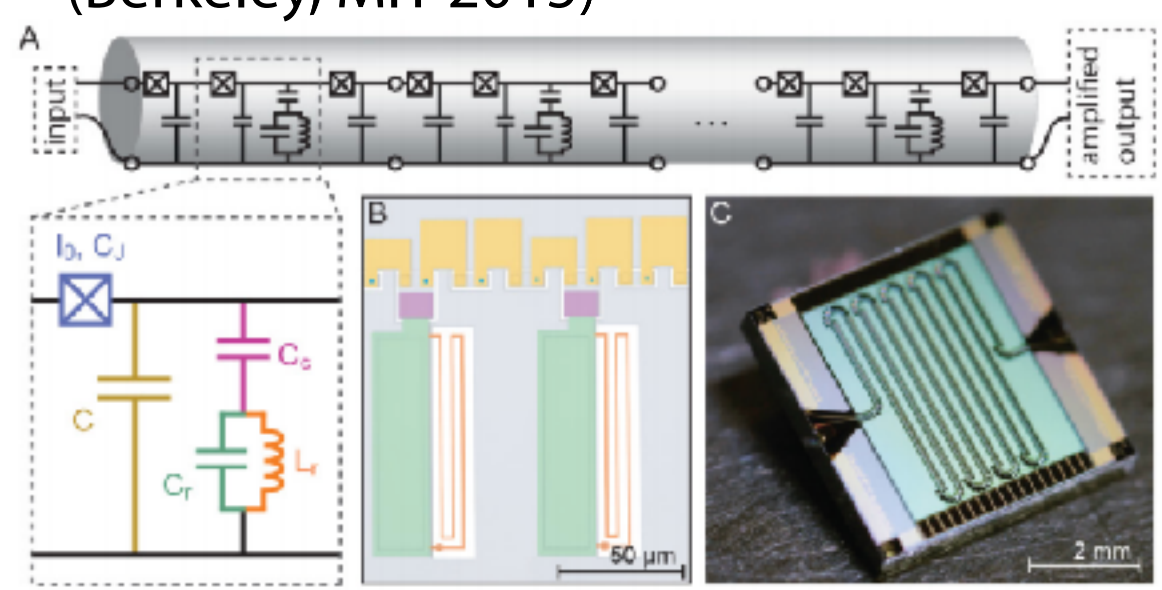
Non degenerate amplifiers





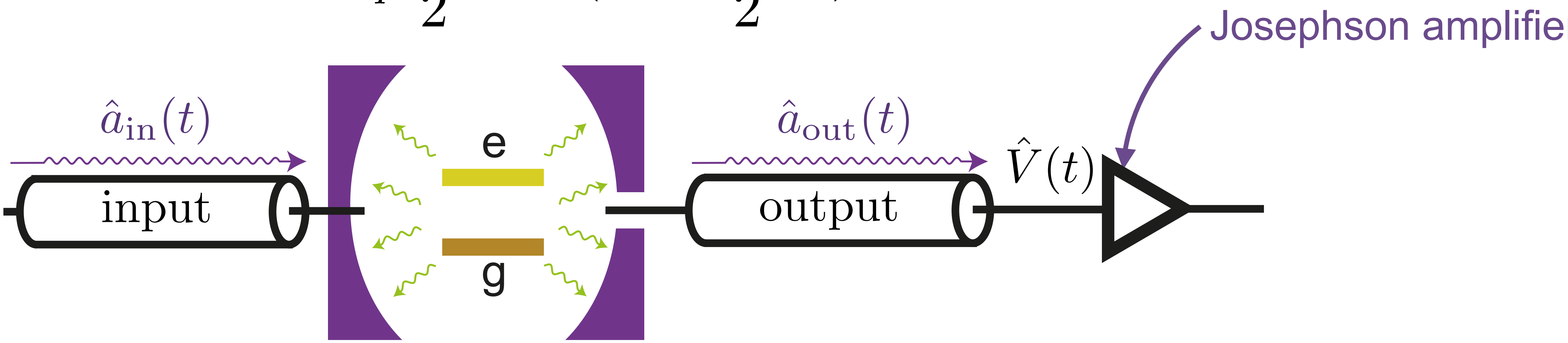
(Yale, 2010)
(ENS Paris, 2012)

Traveling wave amplifier
(Berkeley, MIT 2015)



Dispersive Measurement

$$H = hf_q \frac{\sigma_Z}{2} + h(f_c - \frac{\chi}{2}\sigma_z)a^\dagger a$$



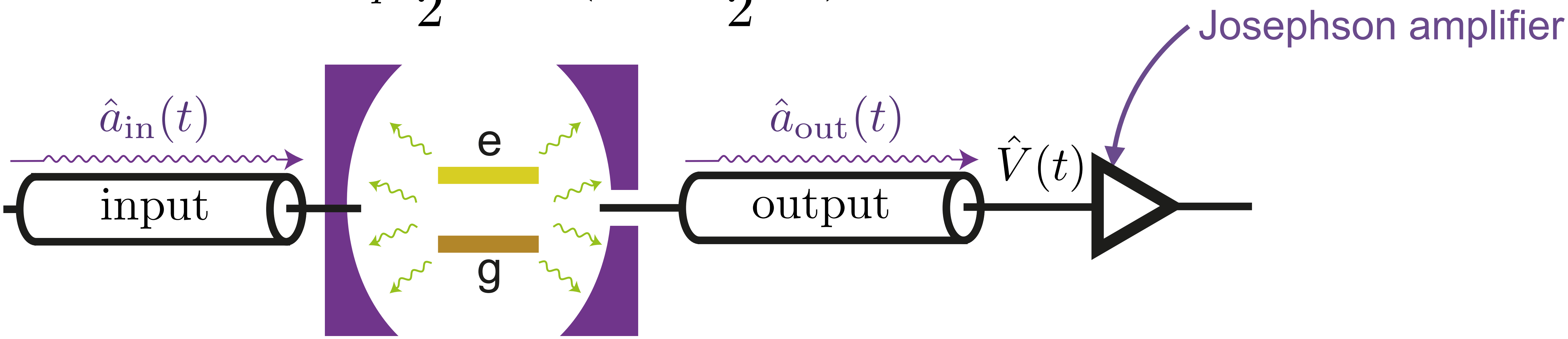
Classically $V(t) = I(t)\cos(2\pi f_c t) + Q(t)\sin(2\pi f_c t)$

$$I_t \to \hat{I}_t \propto \frac{\hat{a}_{\text{out}} + \hat{a}^\dagger_{\text{out}}}{2} = \text{Re}(\hat{a}_{\text{out}})$$

$$Q_t \to \hat{Q}_t \propto \frac{\hat{a}_{\text{out}} - \hat{a}^\dagger_{\text{out}}}{2i} = \text{Im}(\hat{a}_{\text{out}})$$

Dispersive Measurement

$$H = hf_q \frac{\sigma_Z}{2} + h(f_c - \frac{\chi}{2}\sigma_z)a^\dagger a$$



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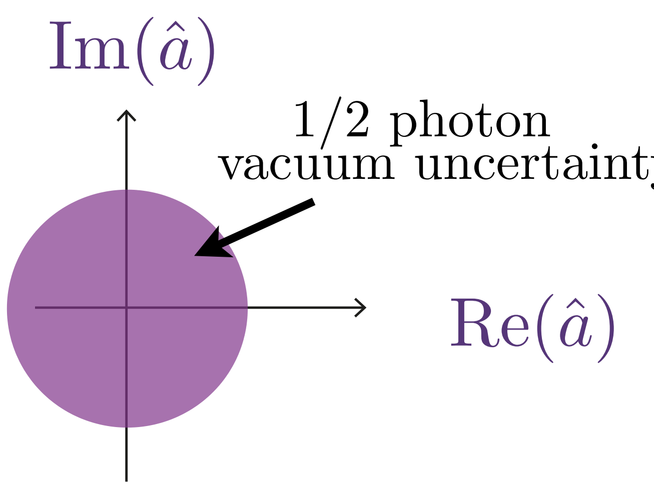
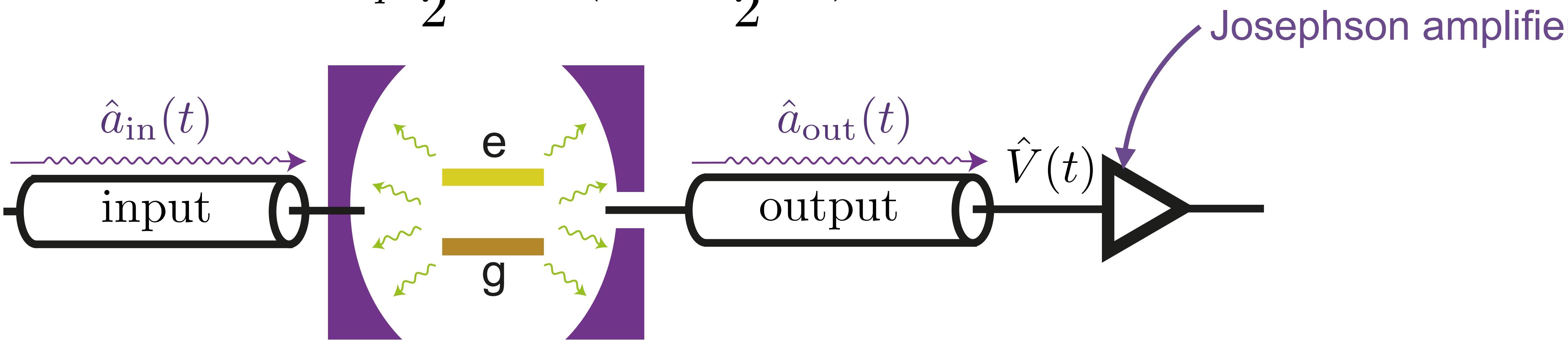


Zero-point fluctuations $|0\rangle$

Dispersive Measurement

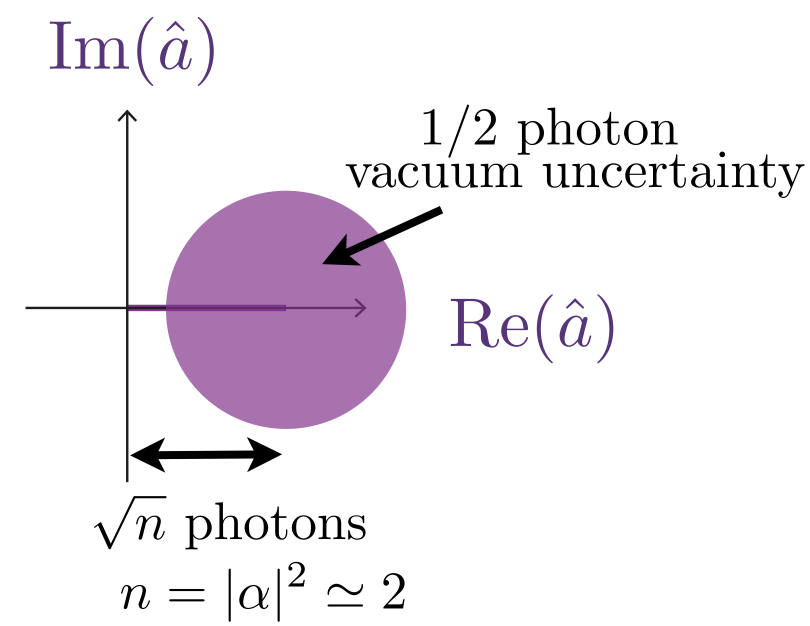
$$H = hf_q \frac{\sigma_Z}{2} + h(f_c - \frac{\chi}{2}\sigma_z)a^\dagger a$$



Classically $V(t) = I(t)\cos(2\pi f_c t) + Q(t)\sin(2\pi f_c t)$

$$I_t \rightarrow \hat{I}_t \propto \frac{\hat{a}_{\text{out}} + \hat{a}^\dagger_{\text{out}}}{2} = \text{Re}(\hat{a}_{\text{out}})$$

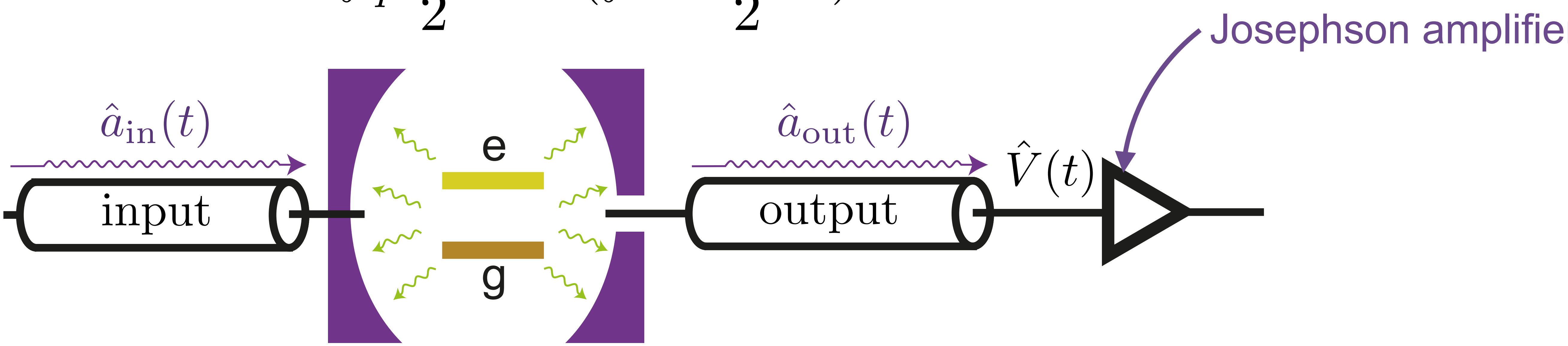
$$Q_t \rightarrow \hat{Q}_t \propto \frac{\hat{a}_{\text{out}} - \hat{a}^\dagger_{\text{out}}}{2i} = \text{Im}(\hat{a}_{\text{out}})$$

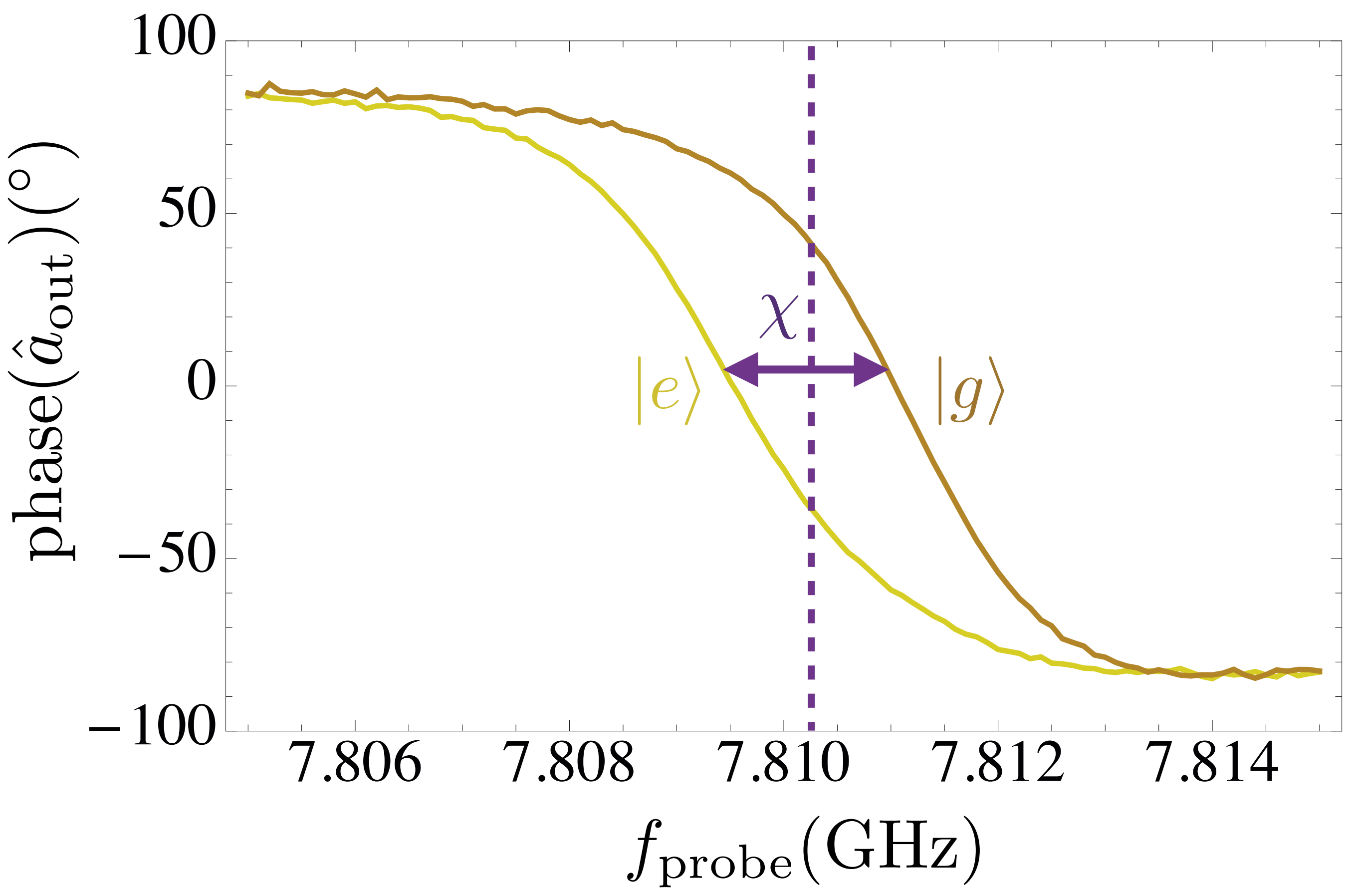


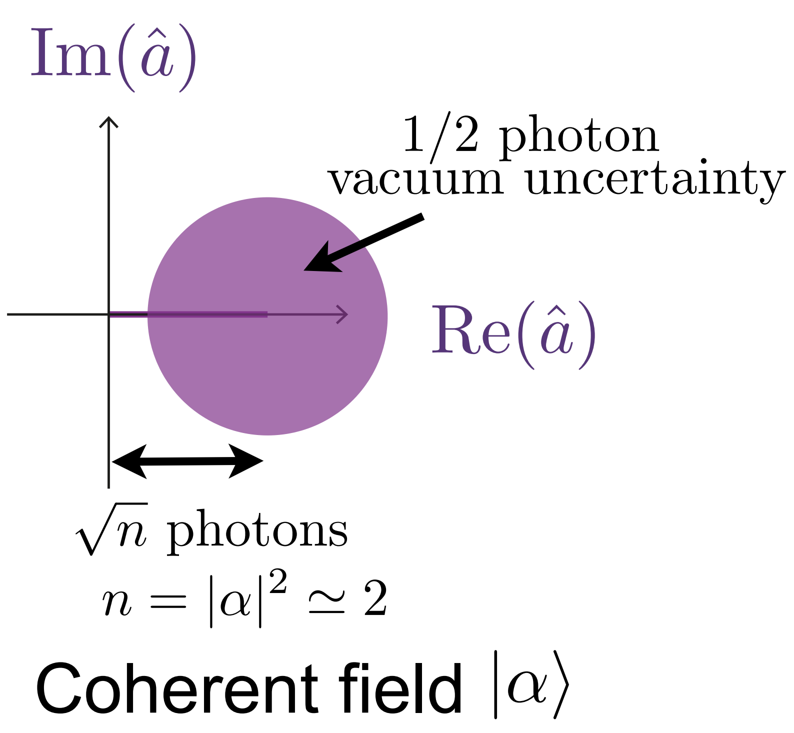
Coherent field $|\alpha\rangle$

Dispersive Measurement

$$H = hf_q \frac{\sigma_Z}{2} + h(f_c - \frac{\chi}{2}\sigma_z)a^\dagger a$$

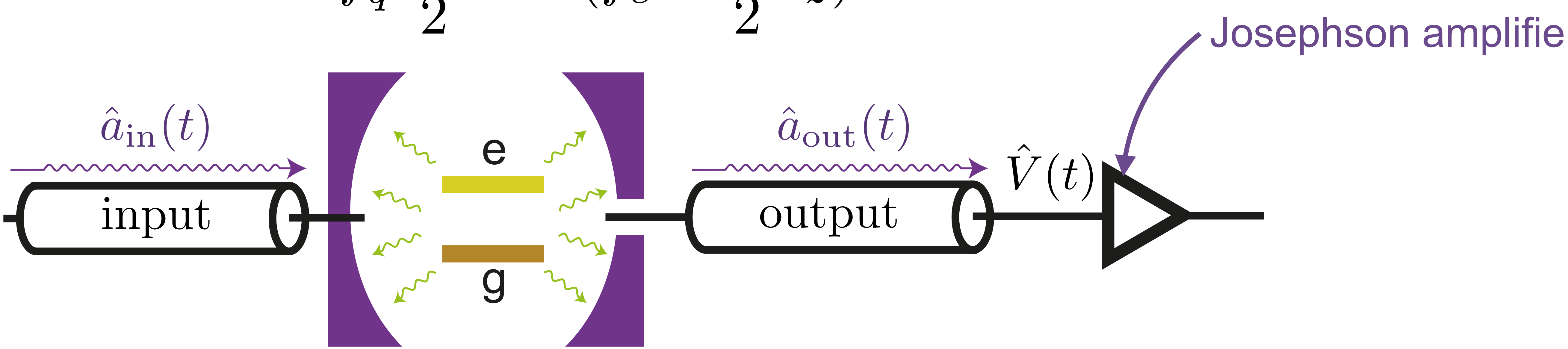






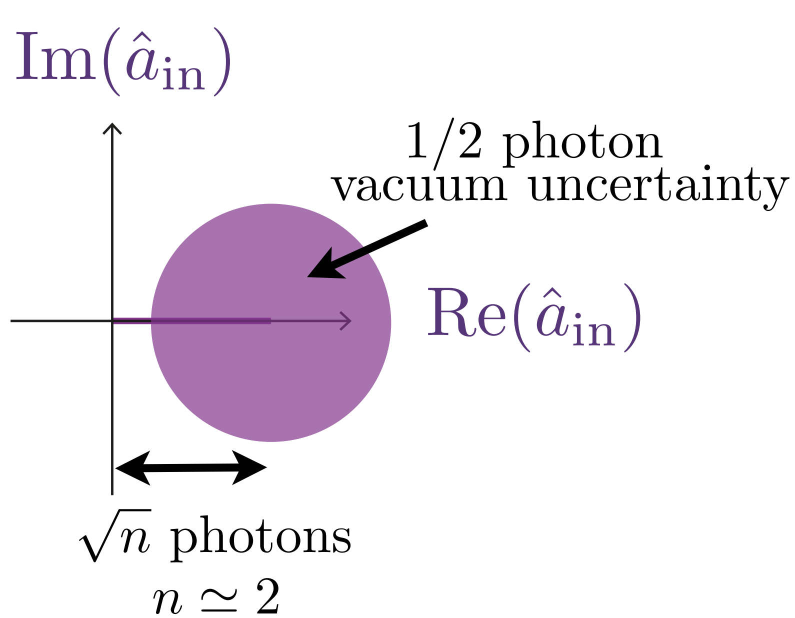
Dispersive Measurement

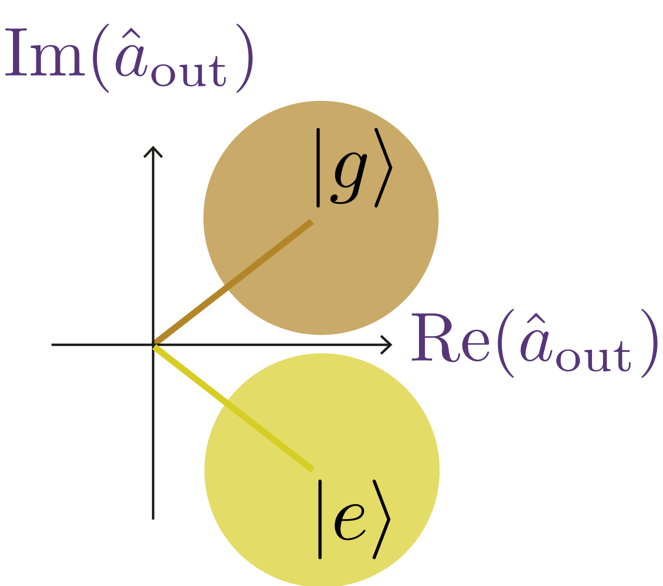
$$H = hf_q \frac{\sigma_Z}{2} + h(f_c - \frac{\chi}{2}\sigma_z)a^\dagger a$$



field going in ...

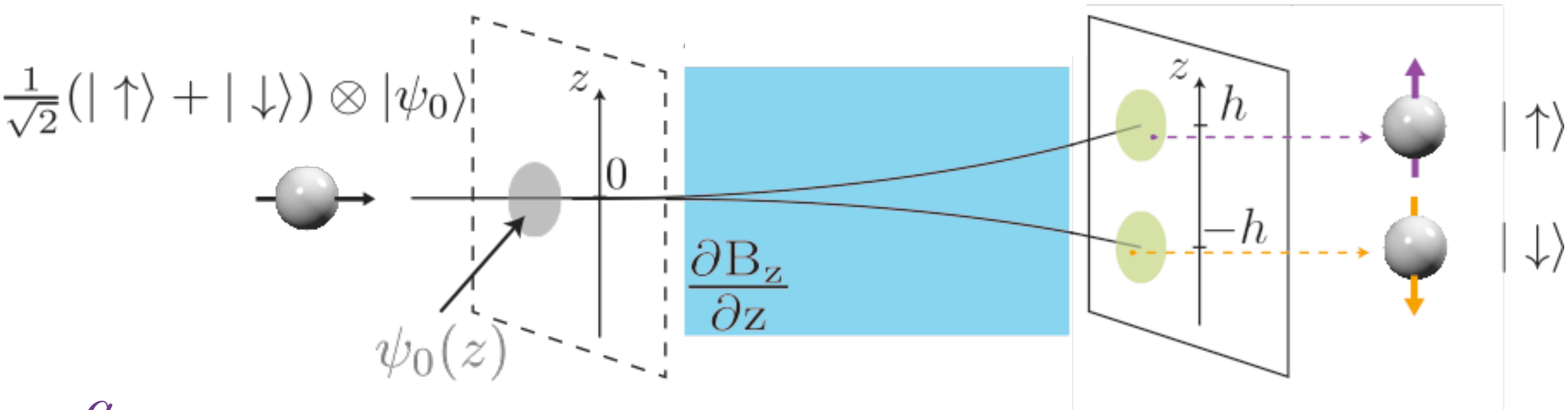
... field coming out

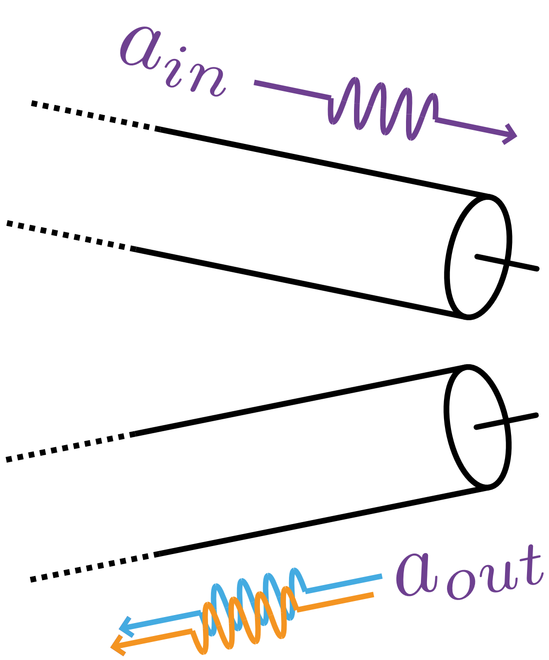


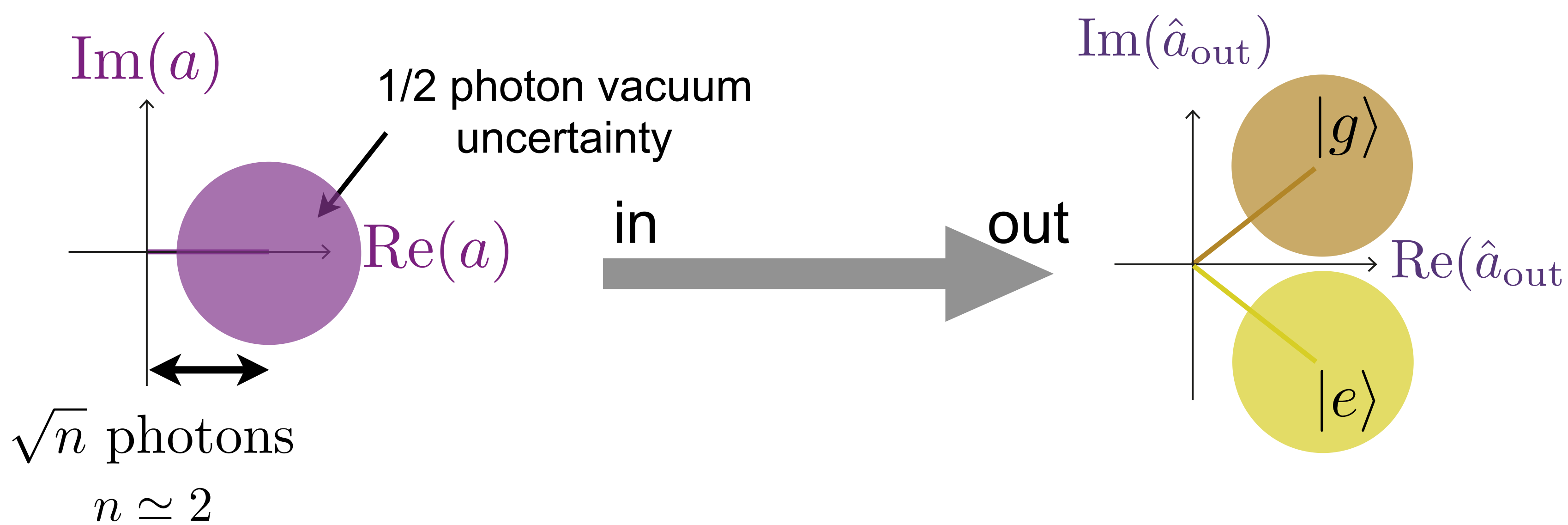


measuring $\mathrm{Im}(\hat{a}_{\rm out})$ ⟶ Strong QND measurement

Dispersive measurement

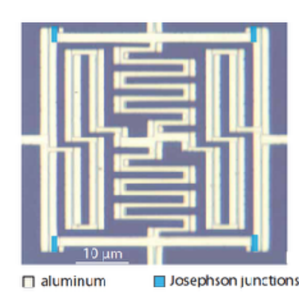






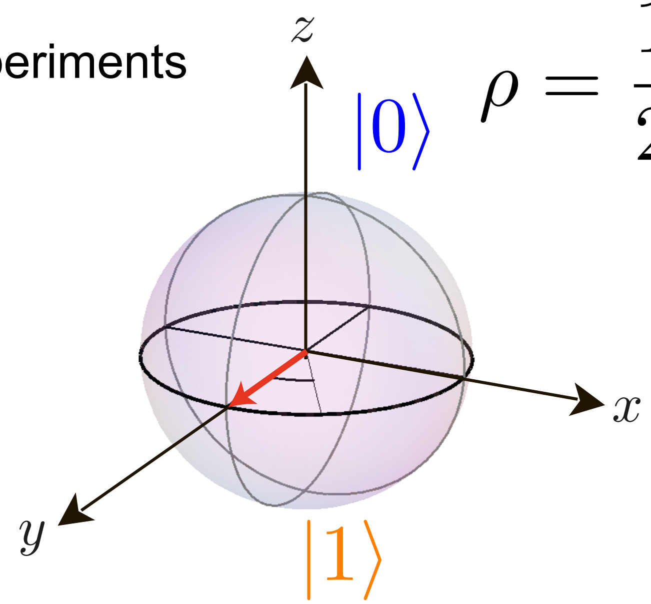
Phase encodes qubit state

measurement uses a non-degenerate quantum limited amplifier
[Roch et al., PRL 2012]

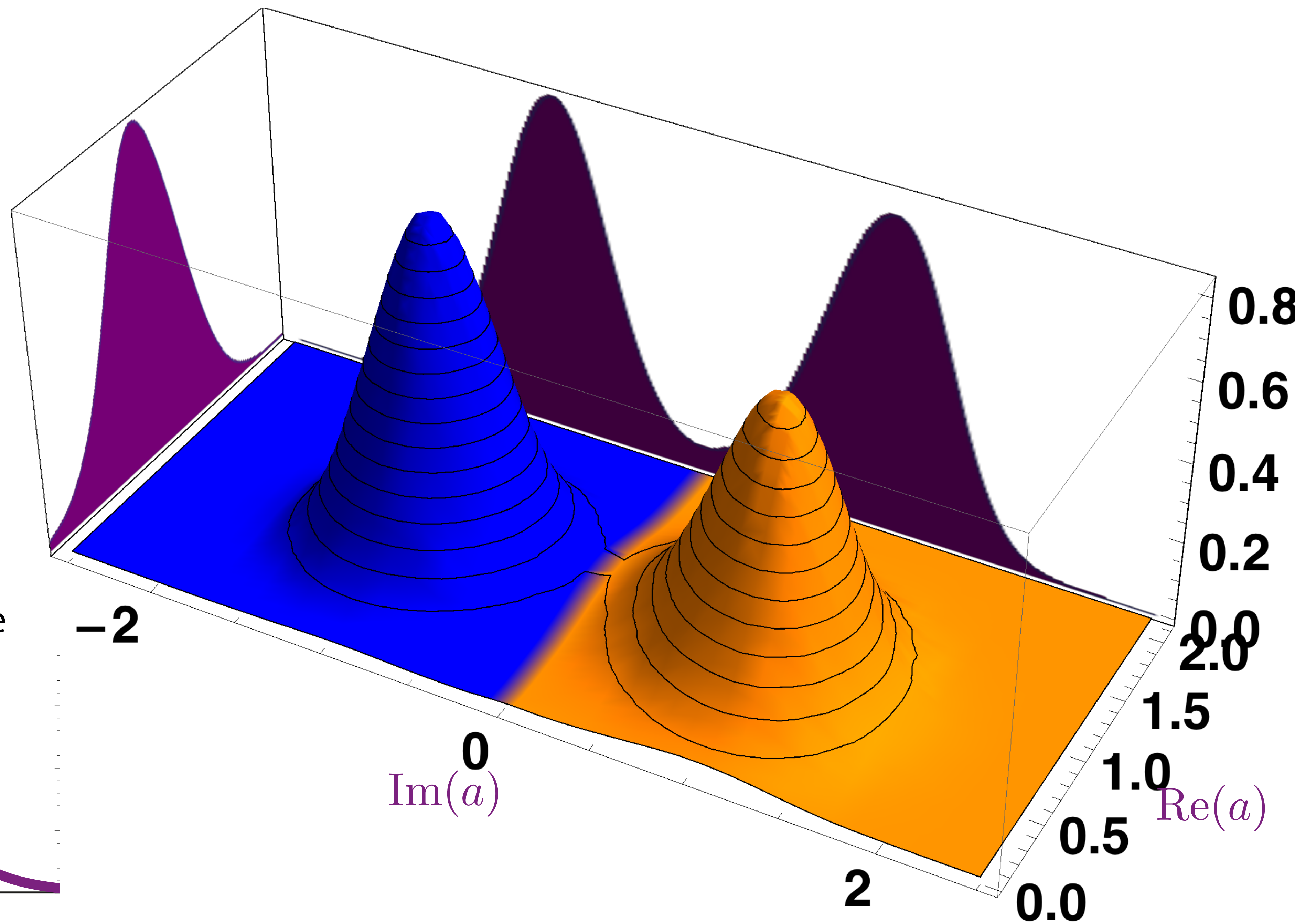


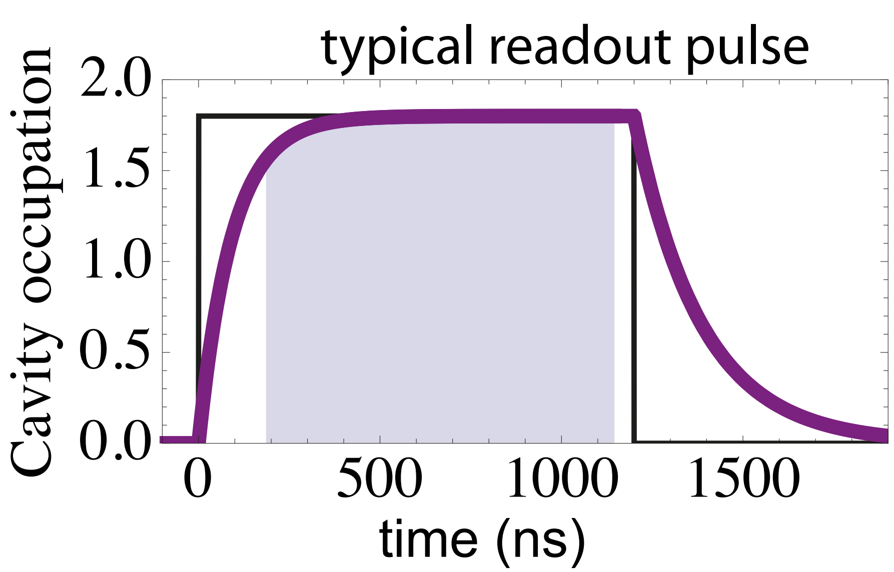
Single shot qubit state readout

10^6 experiments



$$\rho = \frac{1}{2}(\mathbf{1} + x\sigma_x + y\sigma_y + z\sigma_z)$$

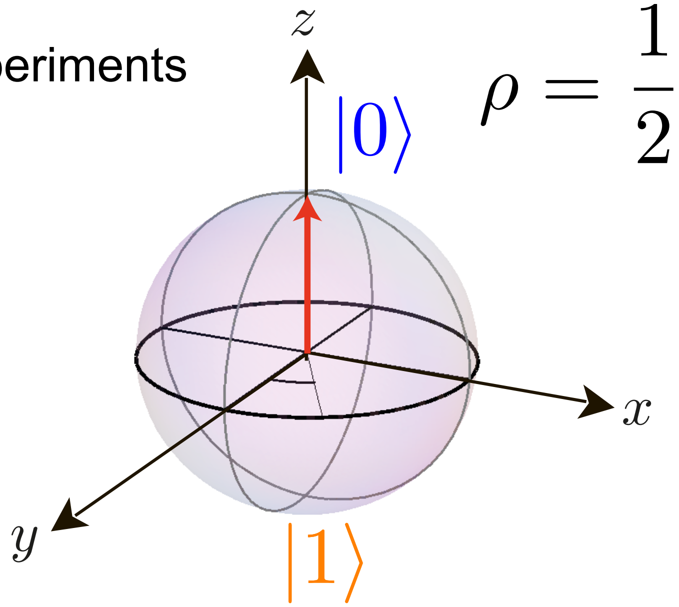




[Campagne-Ibarcq et al., PRX 2013 (ENS Paris)]

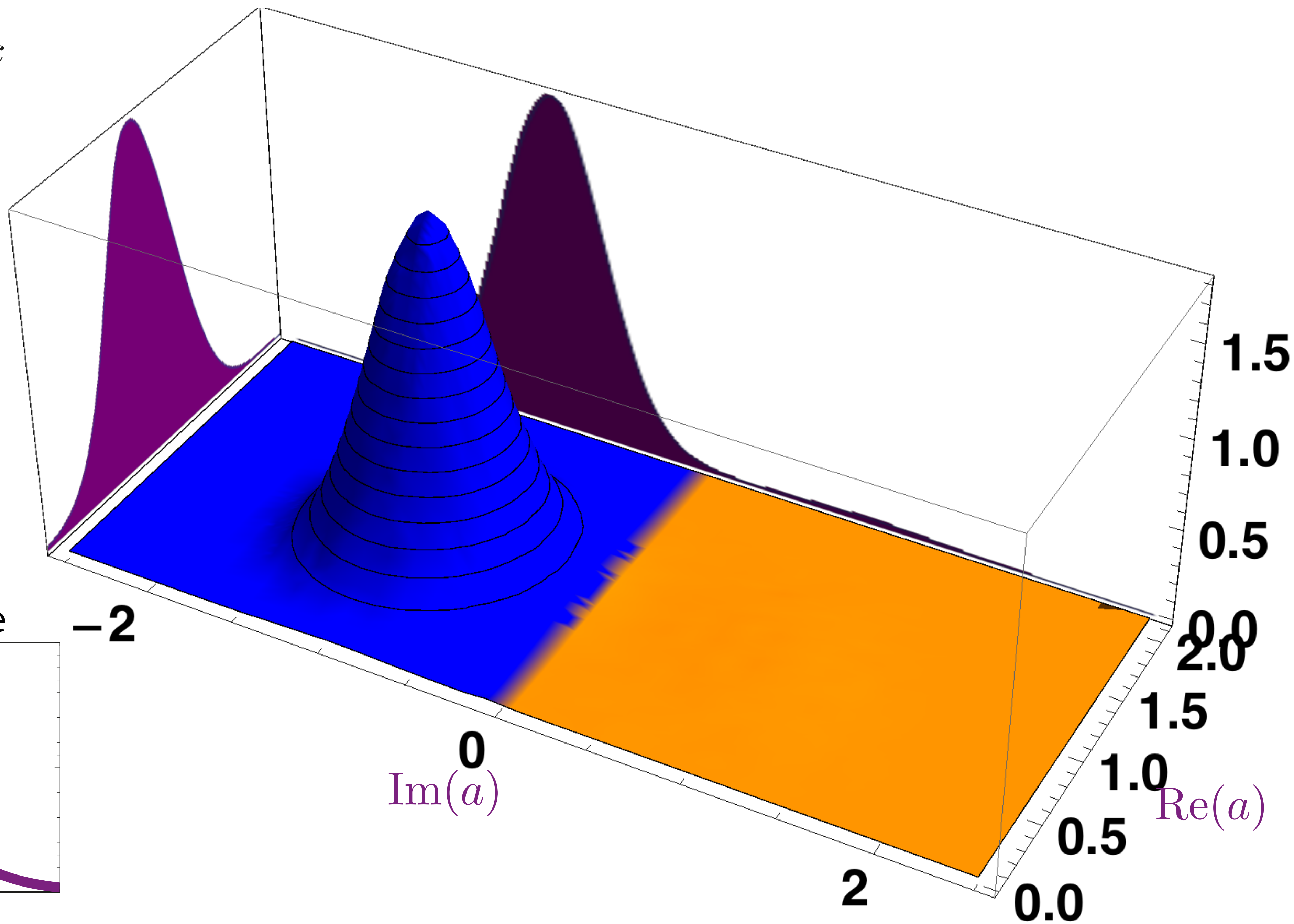
Single shot qubit state readout

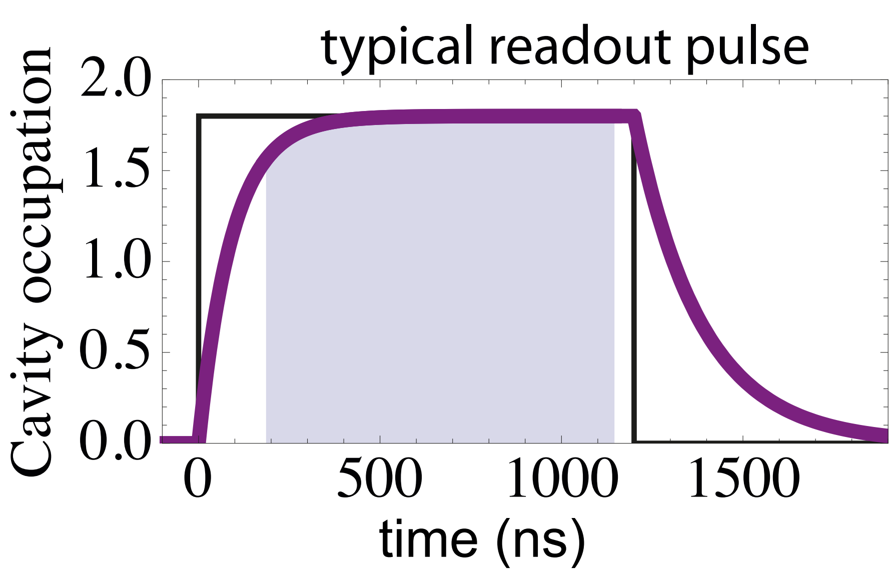
10^6 experiments



$$\rho = \frac{1}{2}(\mathbf{1} + x\sigma_x + y\sigma_y + z\sigma_z)$$

P(Im(a)>0)=2.5%

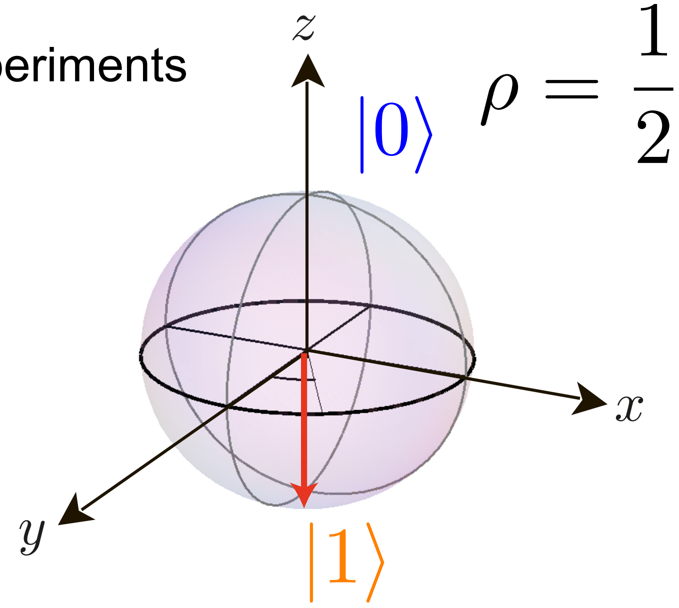




[Campagne-Ibarcq et al., PRX 2013 (ENS Paris)]

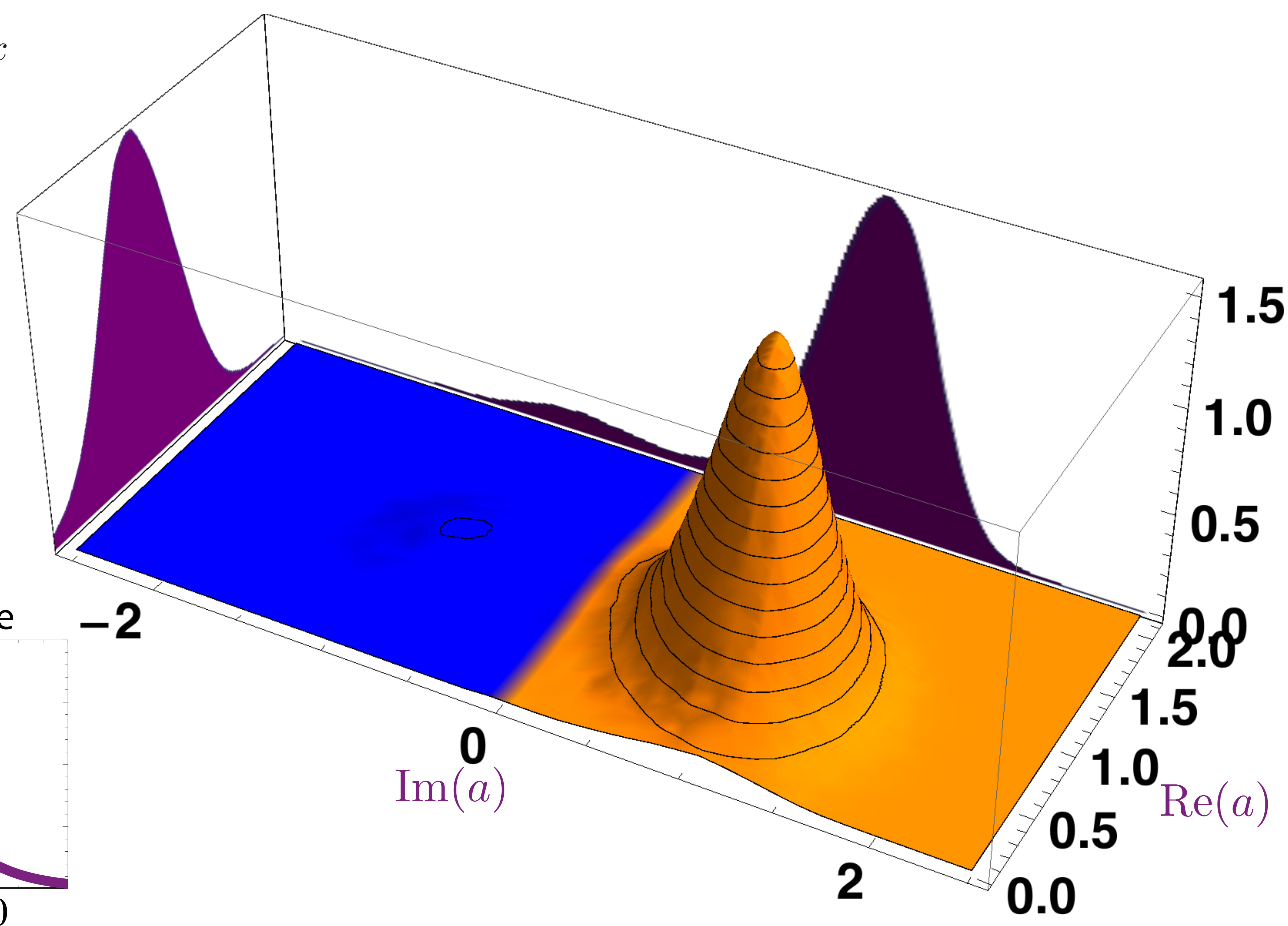
Single shot qubit state readout

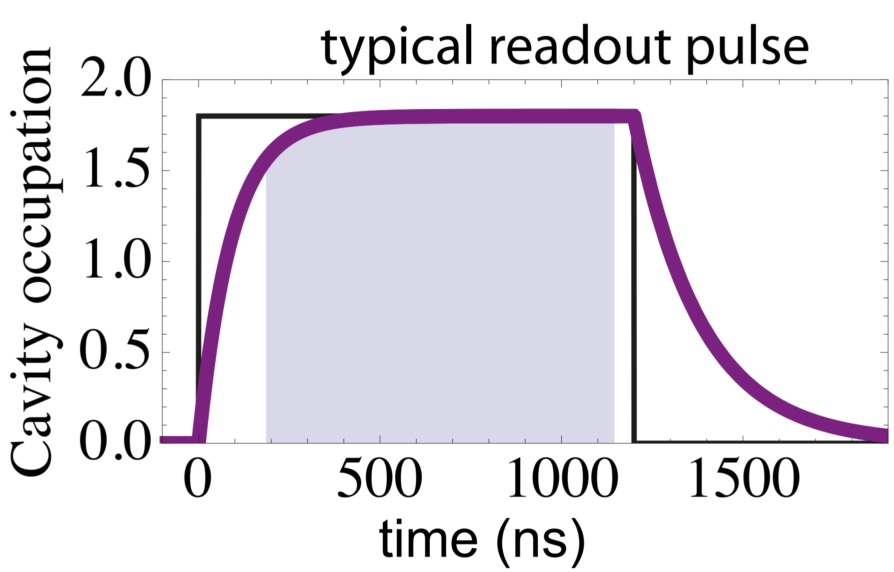
10^6 experiments



$$\rho = \frac{1}{2}(\mathbf{1} + x\sigma_x + y\sigma_y + z\sigma_z)$$

P(Im(a)>0)=93.5%

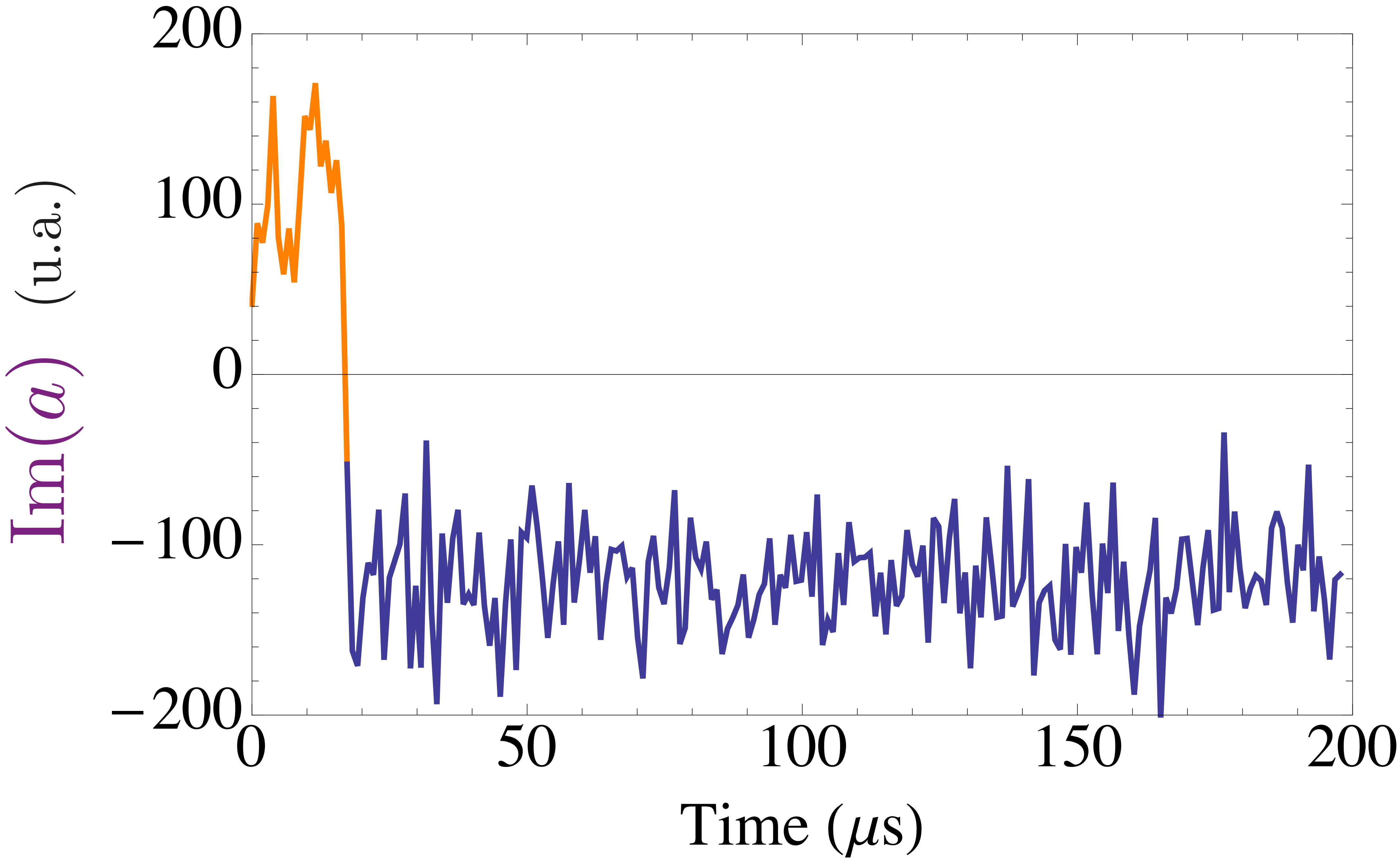




[Campagne-Ibarcq et al., PRX 2013 (ENS Paris)]

Quantum jumps

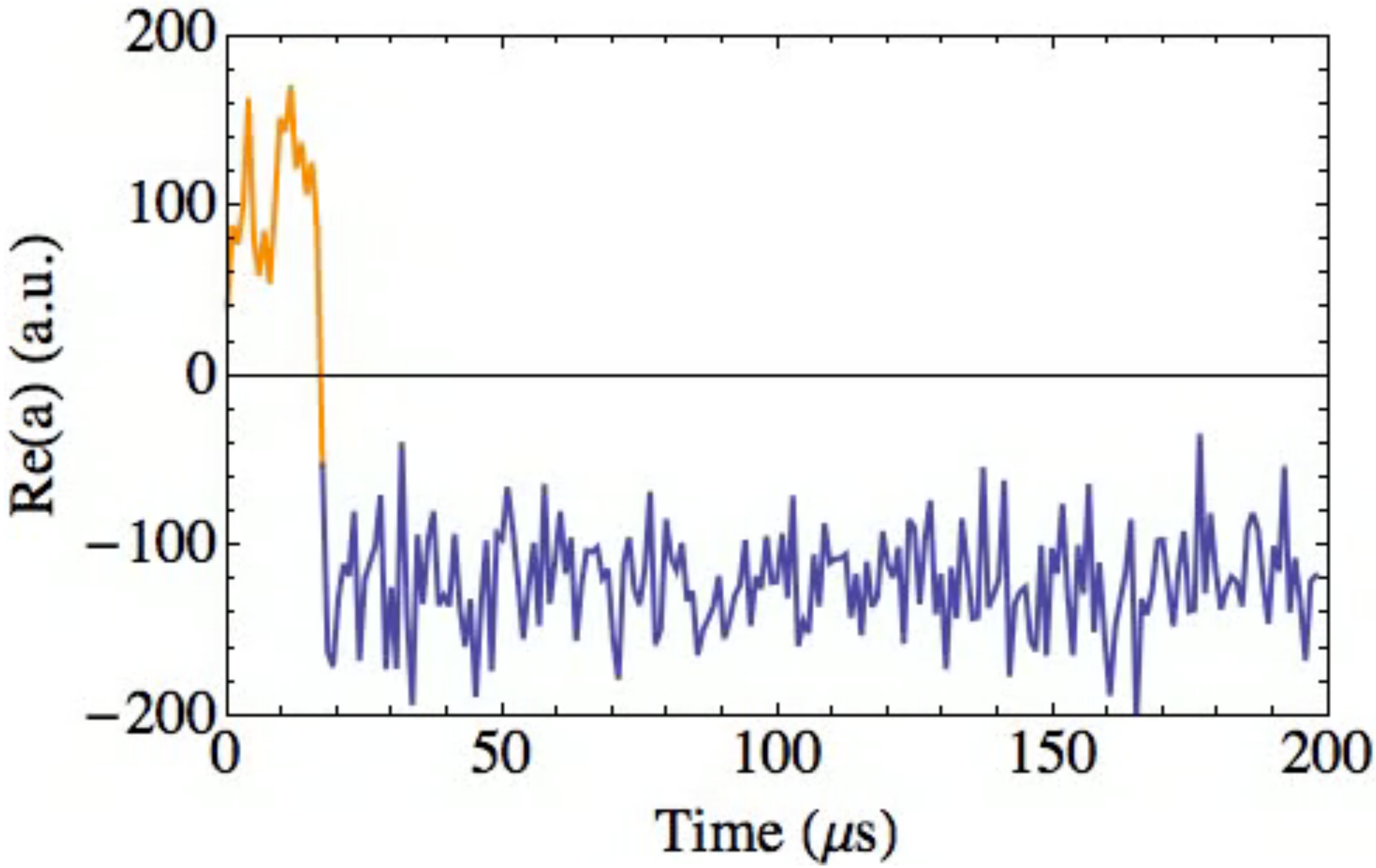
prepare $|1\rangle$ and continuous measurement at 1.8 photons



similar to [Vijay et al., PRL 2011 (Berkeley)]

Quantum jumps

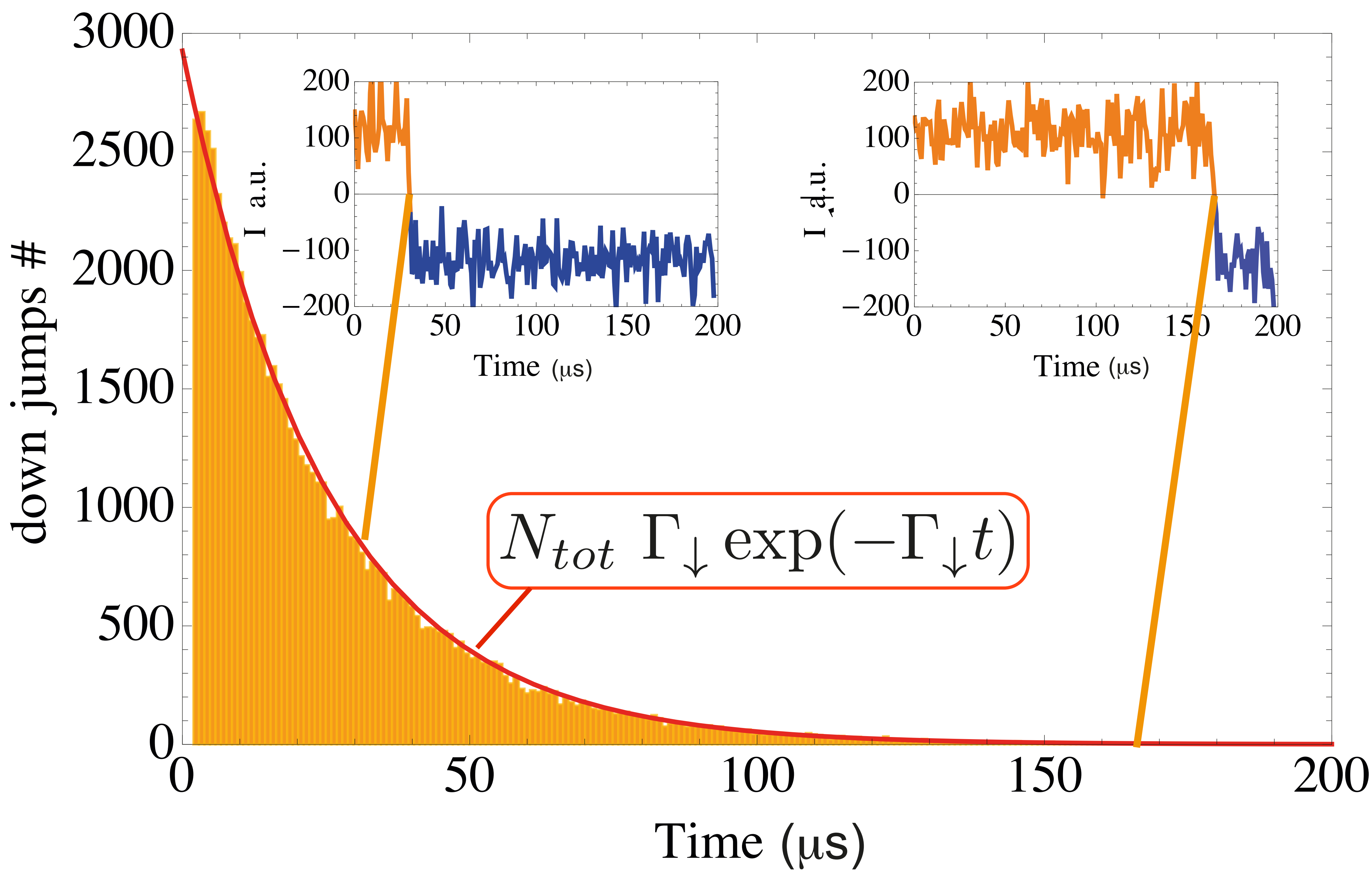
prepare $|1\rangle$ and continuous measurement at 1.8 photons



similar to [Vijay et al., PRL 2011 (Berkeley)]

Quantum jumps

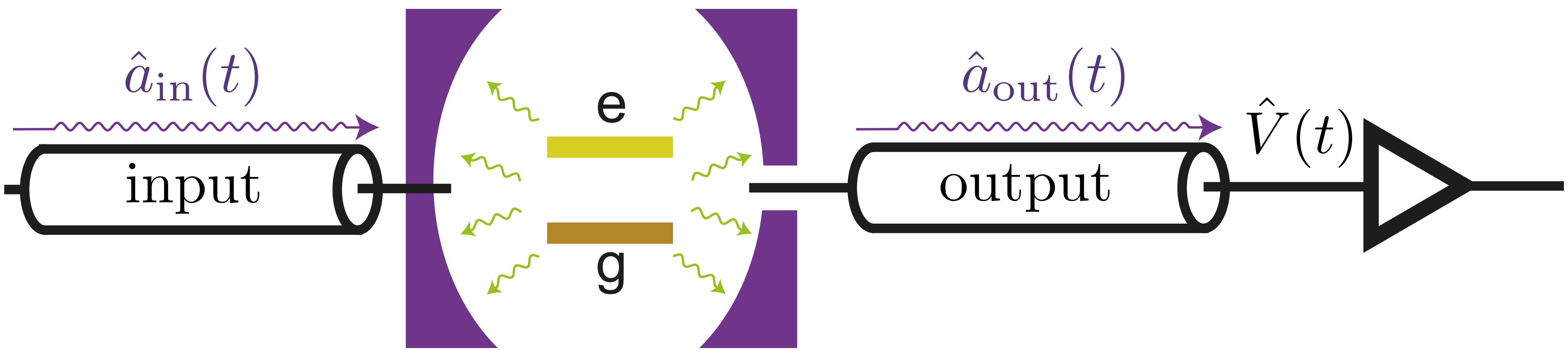
continuous measurement at 1.8 photons



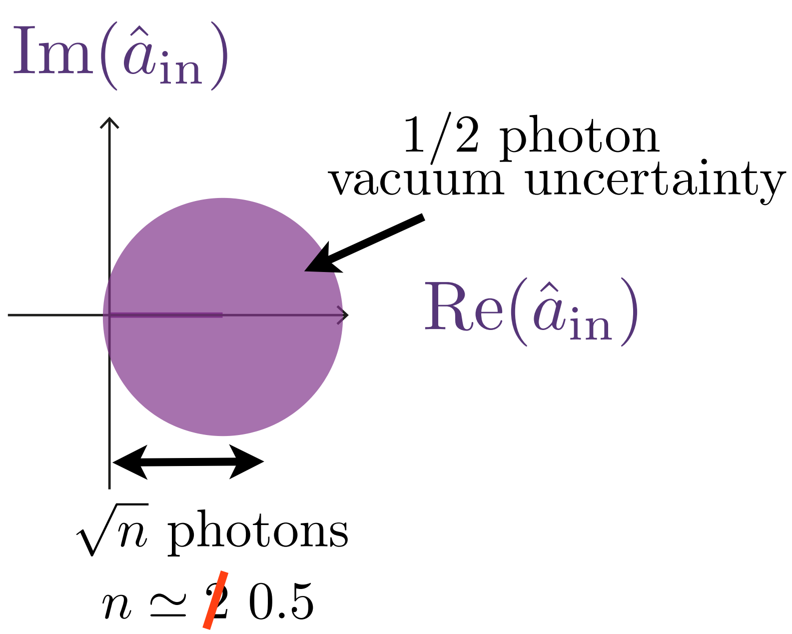
$$\frac{1}{\Gamma\downarrow} \simeq T_1 = 26\ \mu\text{s}$$

[Campagne-Ibarcq et al., PRX 2013 (ENS Paris)]

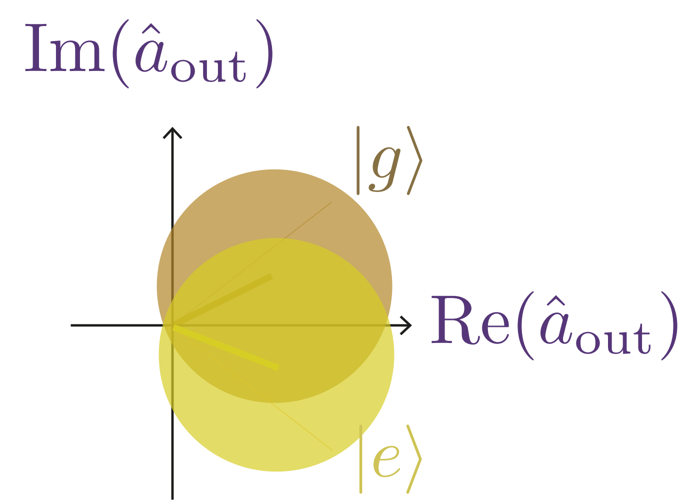
Weak measurement



field going in ...



... field coming out



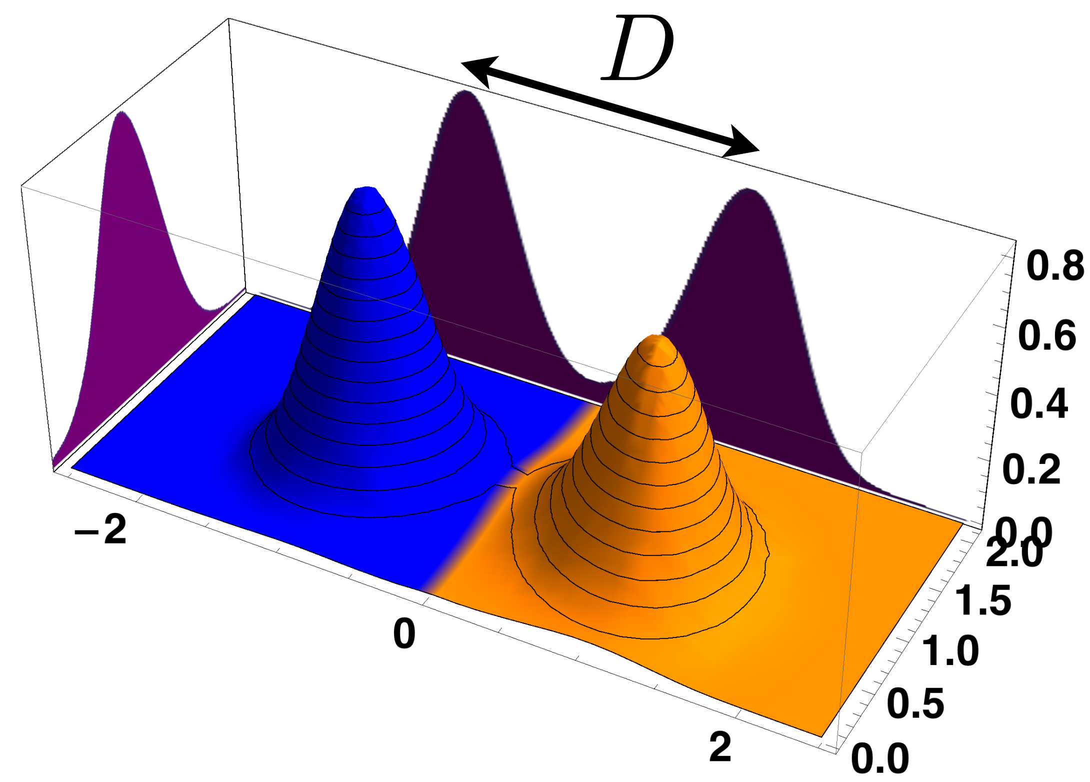
measuring $\text{Im}(\hat{a}_{\text{out}})$ ⟶ Weak QND measurement

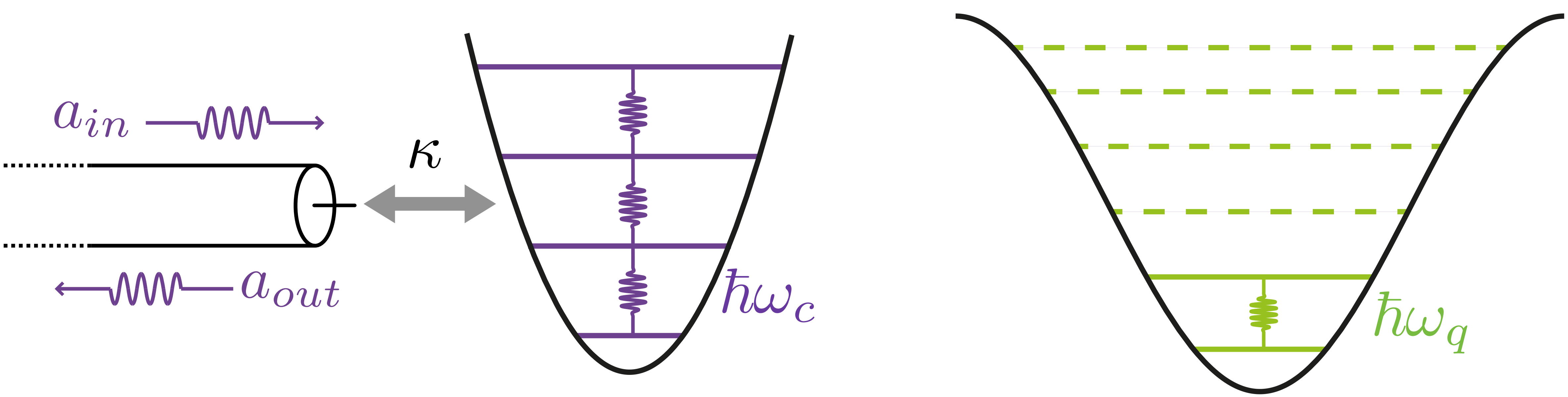
Measurement rate

$$\Gamma_m = 2n_{ph}\frac{\chi^2}{\kappa} = \kappa\frac{D^2}{2}$$

Acquired bits per second $\frac{\Gamma_m}{\ln(2)}$

if unread, extra dephasing Γ_m

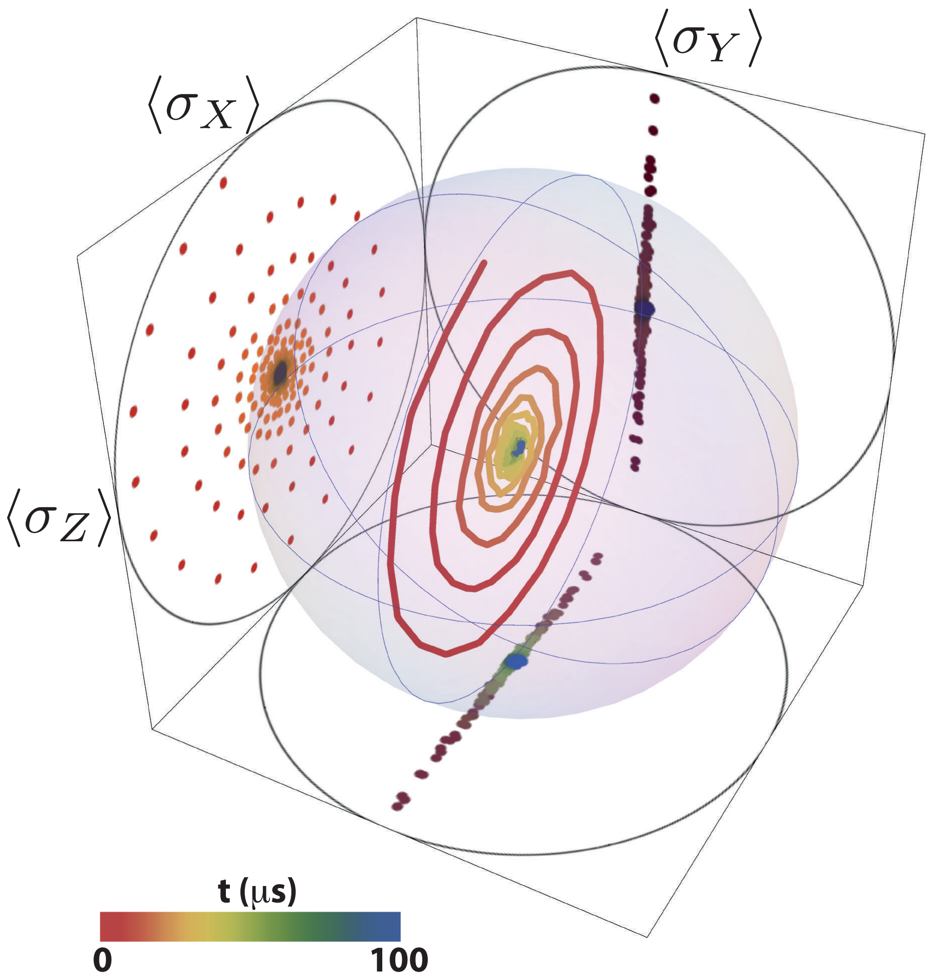




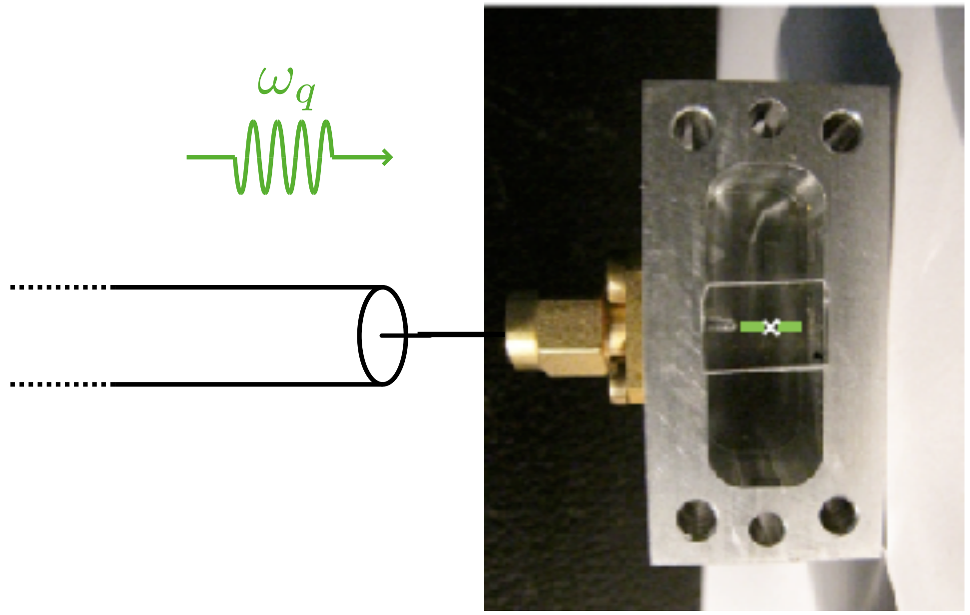
$$H_{\mathrm{coupl}} = \hbar\chi a^\dagger a\frac{\sigma_Z}{2}$$

[Clerk *et al.*, RMP 2010]

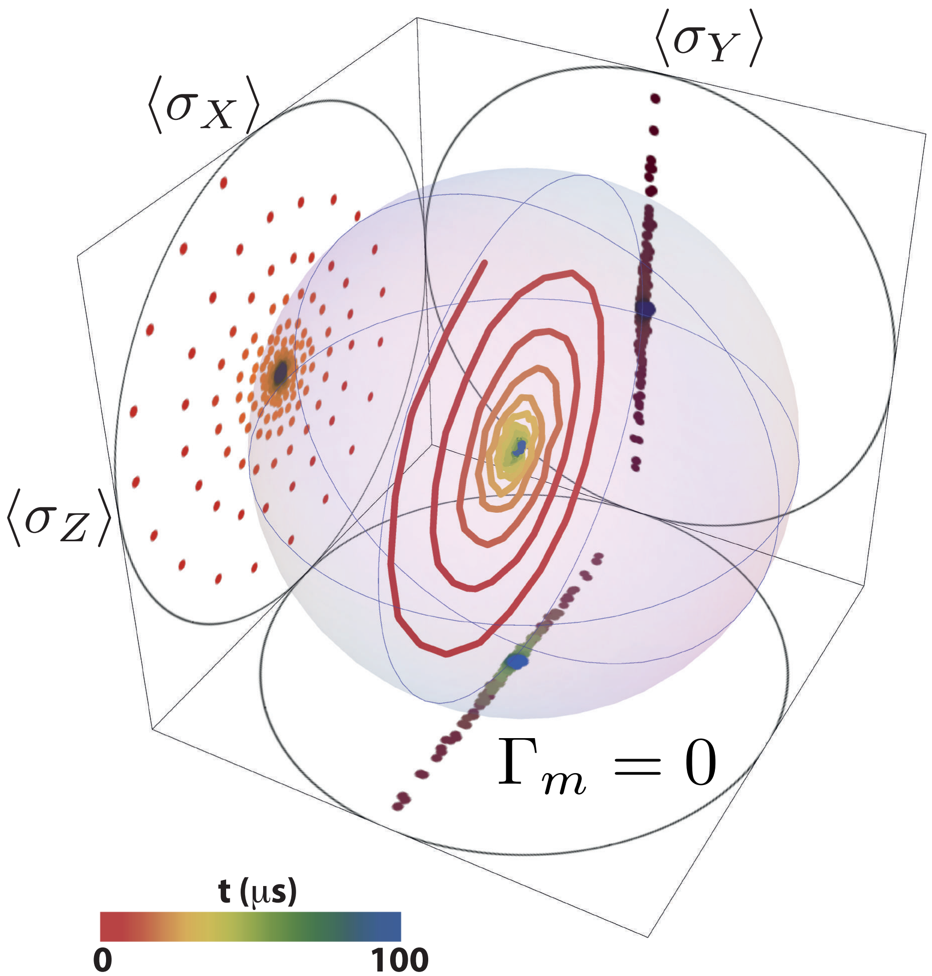
Zeno effect on Rabi oscillations



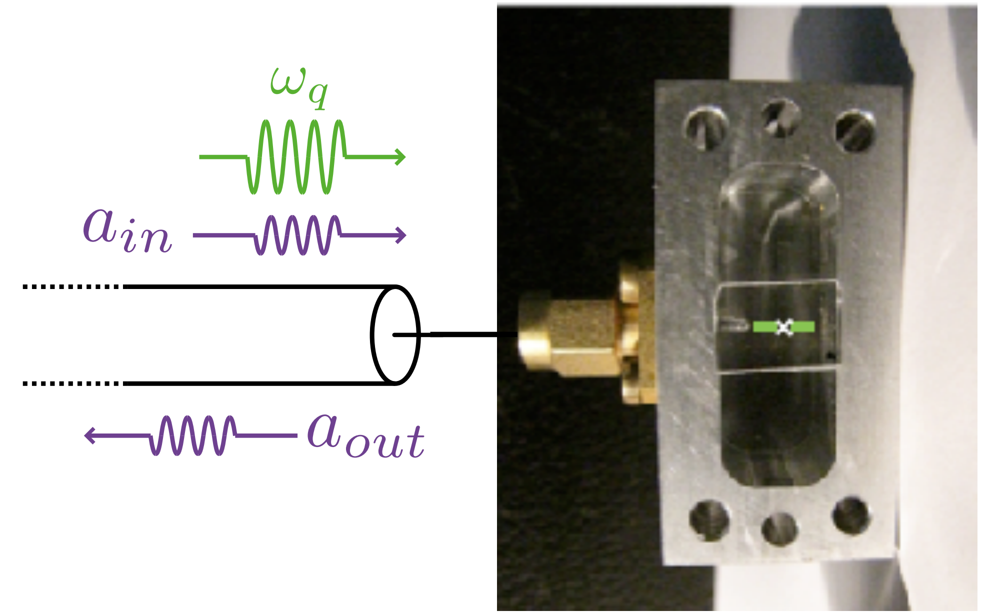
[Campagne-Ibarcq et al., PRX 2013 (ENS Paris)]

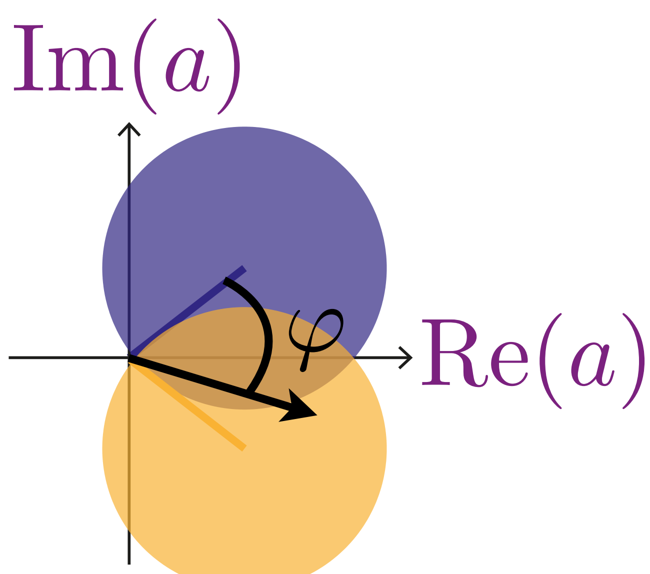


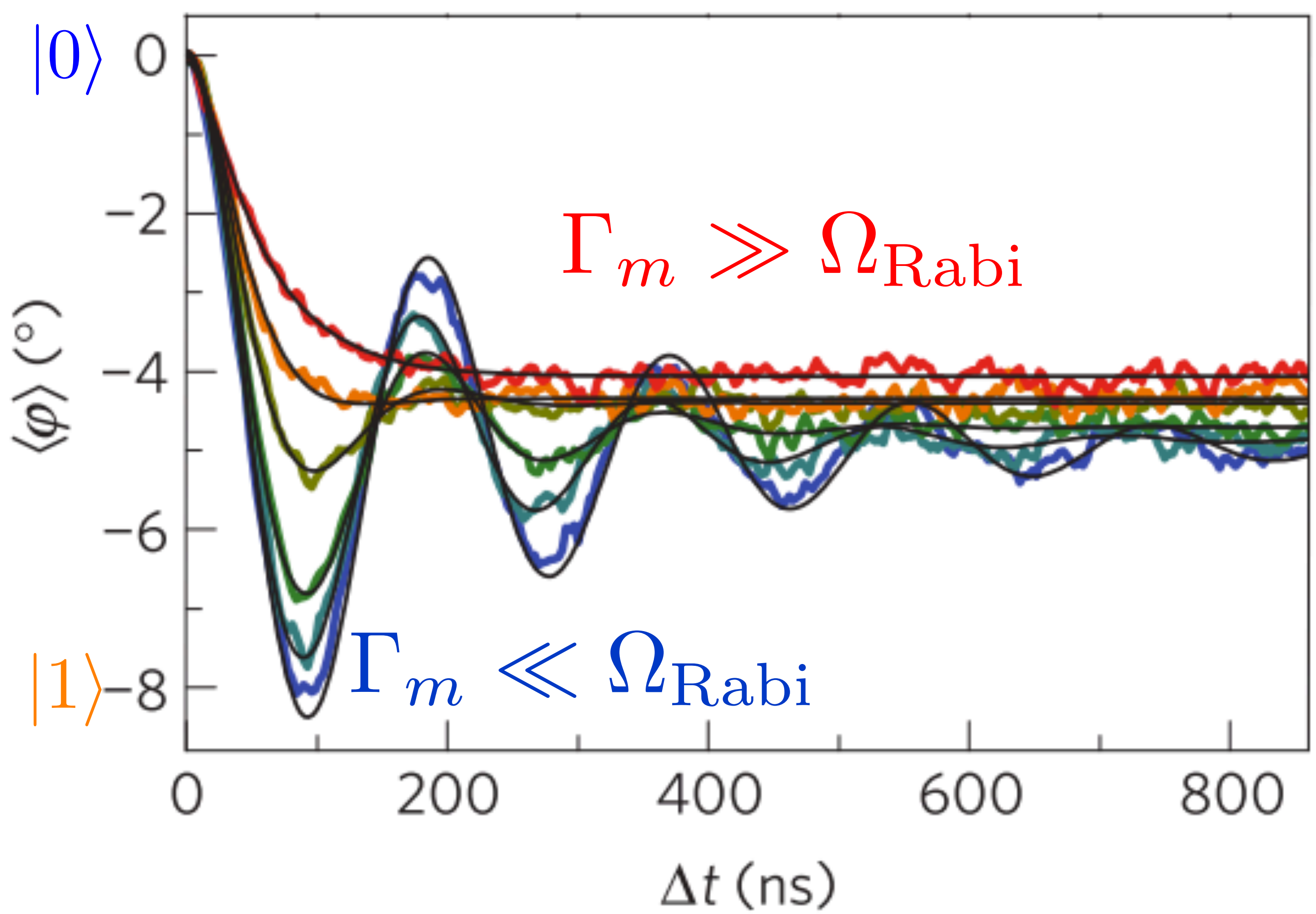
Zeno effect on Rabi oscillations



[Campagne-Ibarcq et al., PRX 2013 (ENS Paris)]





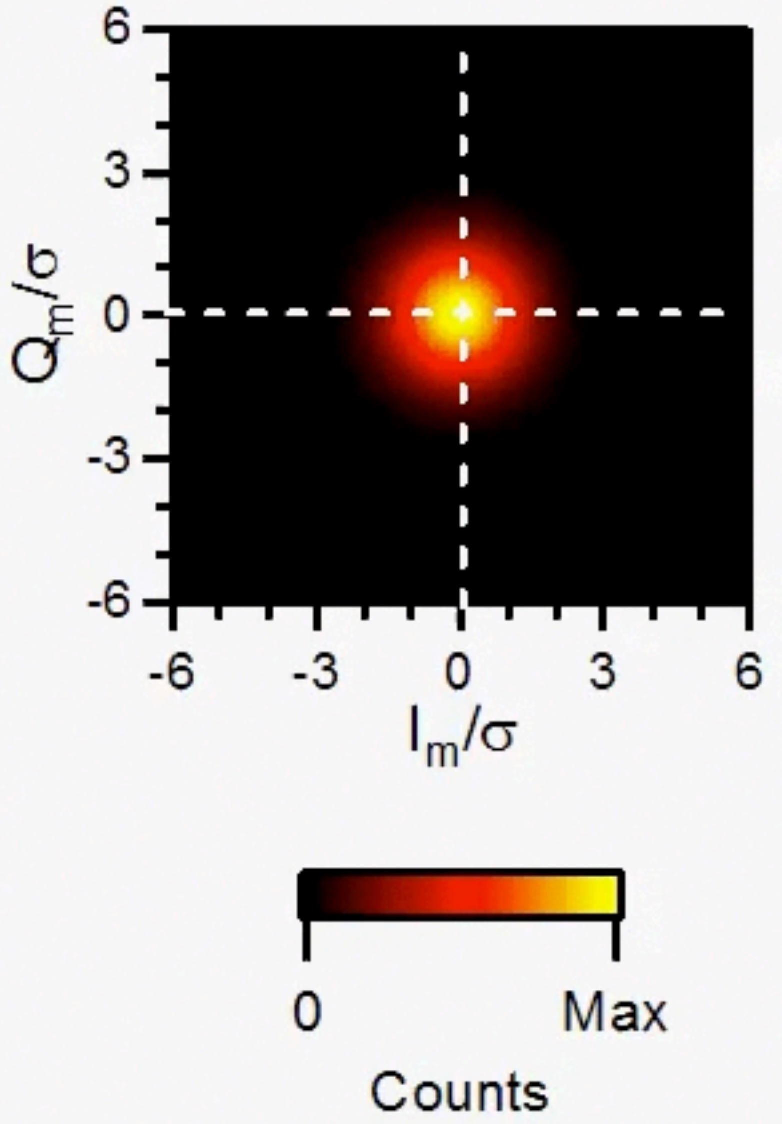


[Palacios-Laloy *et al.*, Nat. Phys. 2010 (Saclay)]

Measurement backaction

Cavity Drive = 0.0e+00 photons

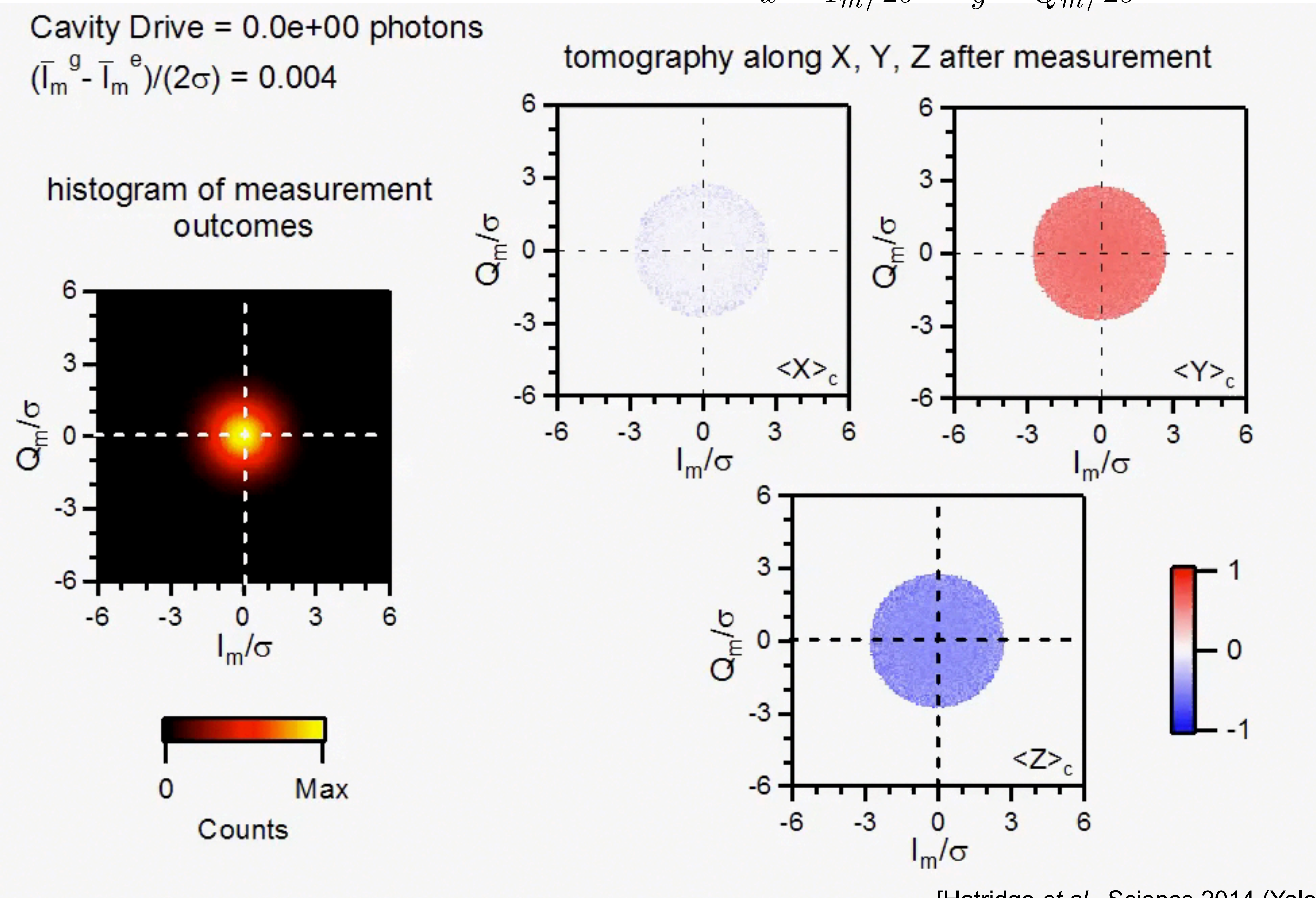
histogram of measurement outcomes



[Hatridge *et al.*, Science 2014 (Yale)]

$$M_{x+iy} = \frac{1}{\sqrt{\pi}} e^{-(y-y_0)^2} \left[e^{-(x+x_0)^2} e^{2ix_0 y} |g\rangle\langle g| + e^{-(x-x_0)^2} e^{-2ix_0 y} |e\rangle\langle e| \right]$$

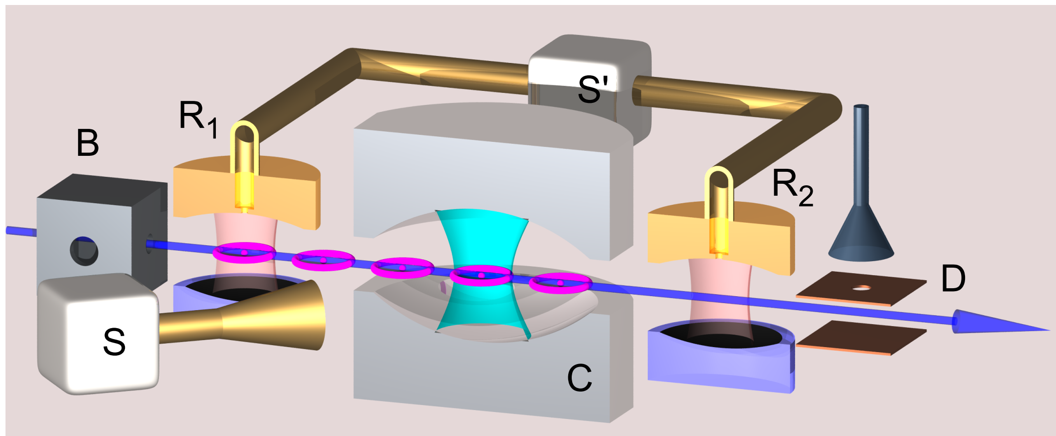
$$x = I_m/2\sigma \qquad y = Q_m/2\sigma$$



[Hatridge *et al.*, Science 2014 (Yale)]

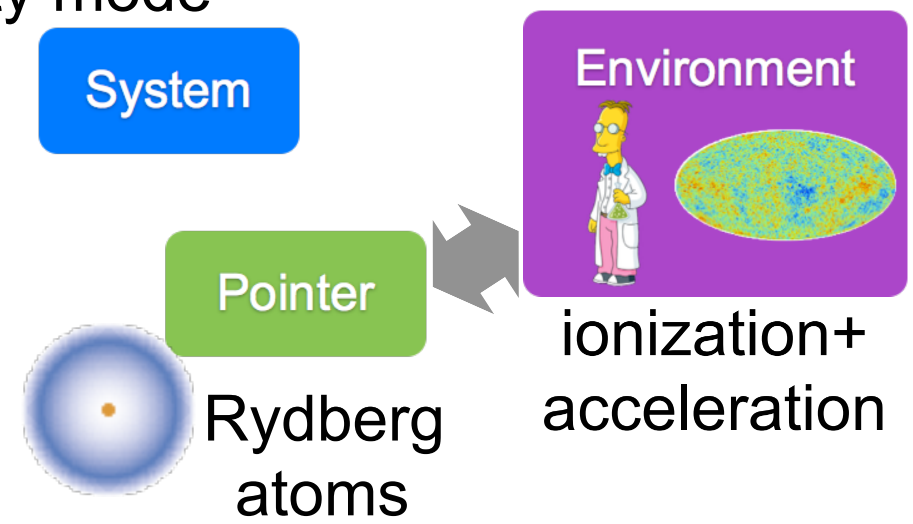
Quantum trajectories already measured in...

Rydberg atoms

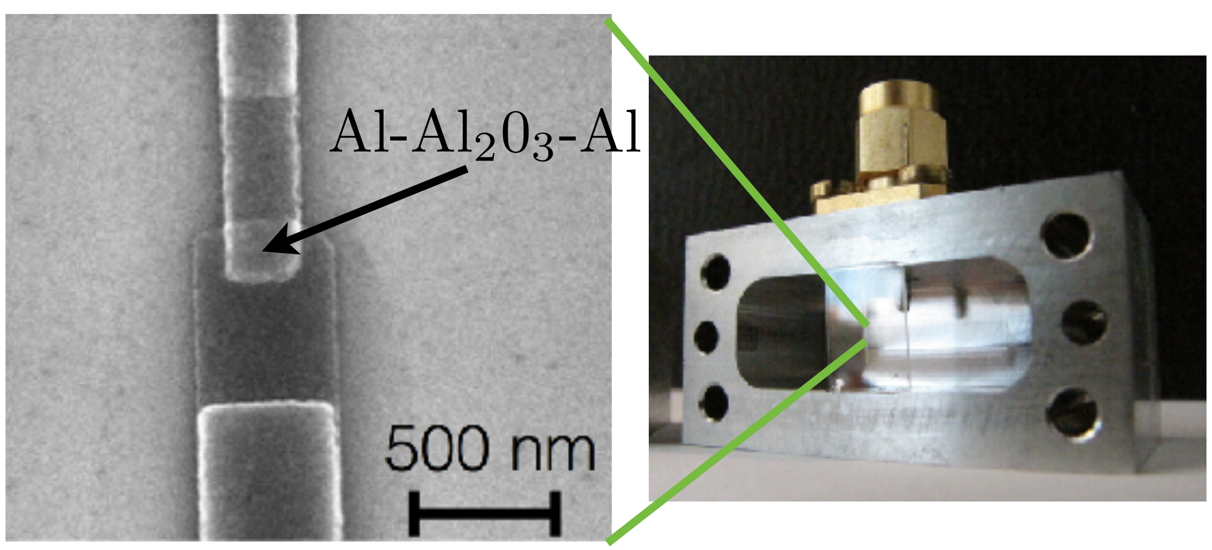


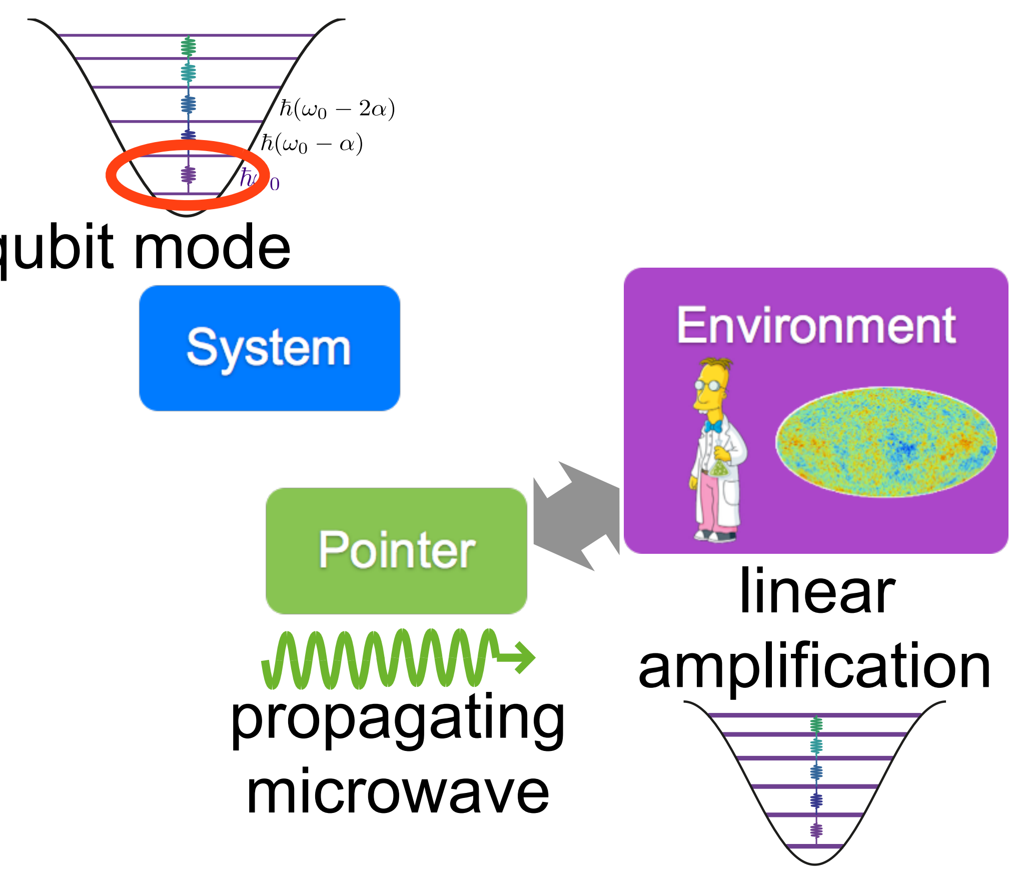
[pic from Haroche group, College de France Paris]

Cavity mode



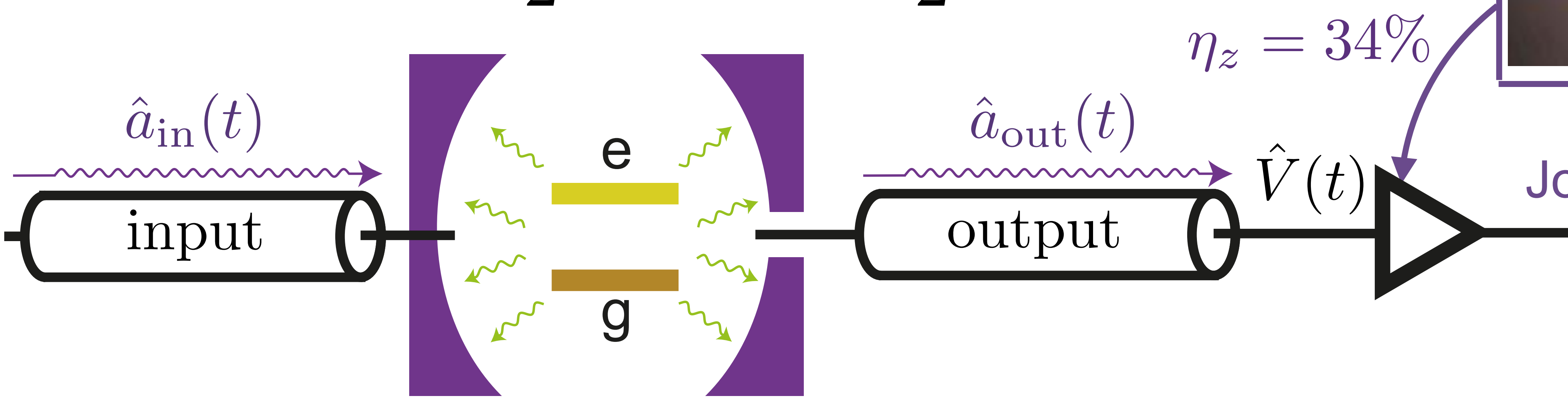
Superconducting circuits

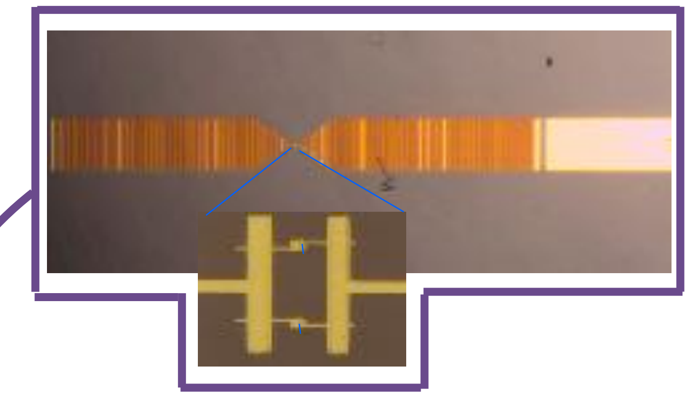




Dispersive Measurement

$$H = hf_q \frac{\sigma_Z}{2} + h(f_c - \frac{\chi}{2}\sigma_z)a^\dagger a$$





Josephson amplifier

[Kamal *et al.*, PRB 2009]

jump operator $L_z = \sqrt{\frac{\Gamma_d}{2}}\sigma_z$

$$dw_t = \sqrt{2\eta_z\Gamma_d}\left\langle\sigma_z\right\rangle_{\rho_t} dt + dW_{t,3}$$

average outcome

noise (Wiener)

Wiener Process

$$\mathbb{E}(dW_{t,i}) = 0$$

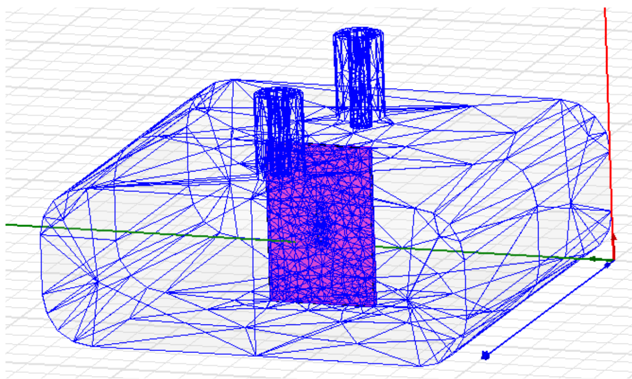
$$dW_{t,i}{}^2 = dt$$

$\{dw_t\} \xrightarrow{\text{stochastic master equation}} \rho_t^{\text{A}}$

we will trajectories see later

[Murch *et al.*, Nature 2013]
[Hatridge *et al.*, Science 2013]

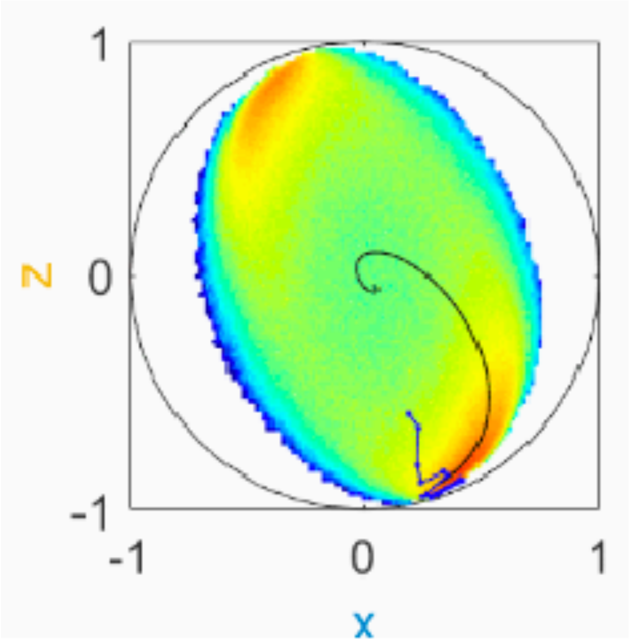
Quantum trajectories and feedback in circuit-QED

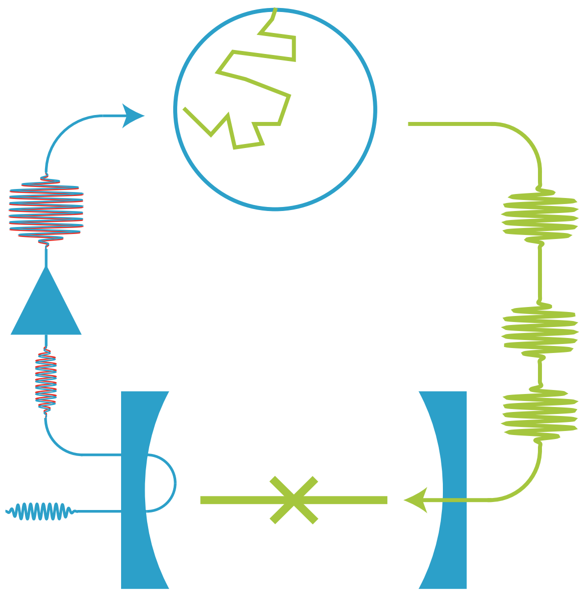


Introduction to circuit-QED

Quantum trajectories in circuit-QED

- dispersive measurement
- **fluorescence measurement**
- both simultaneously
- generating entanglement





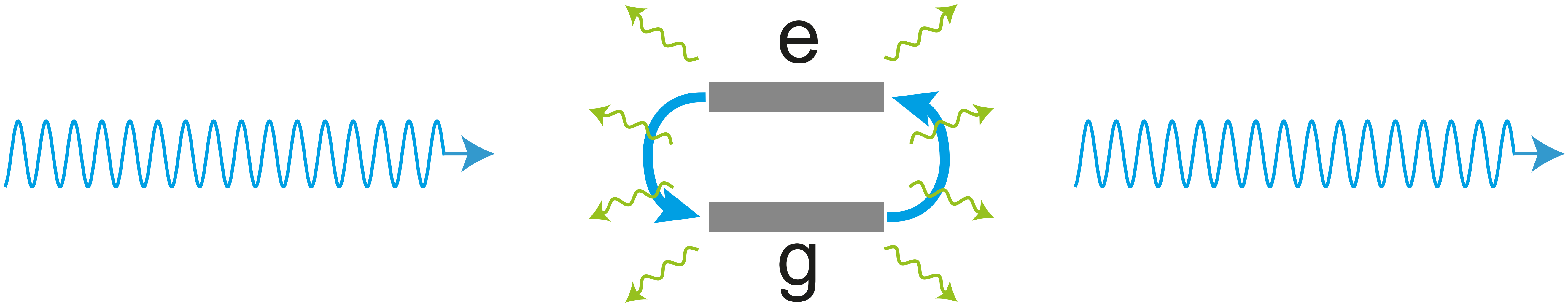
Measurement based feedback

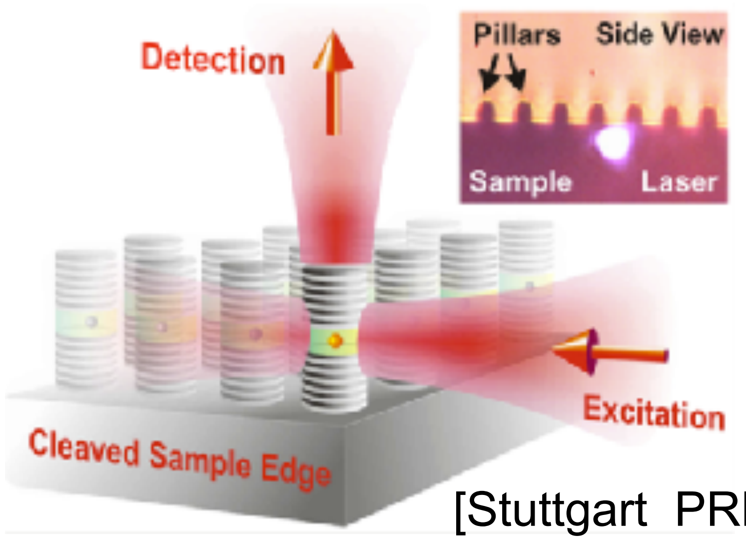
- dispersive case
- fluorescence case

Post selection in quantum mechanics

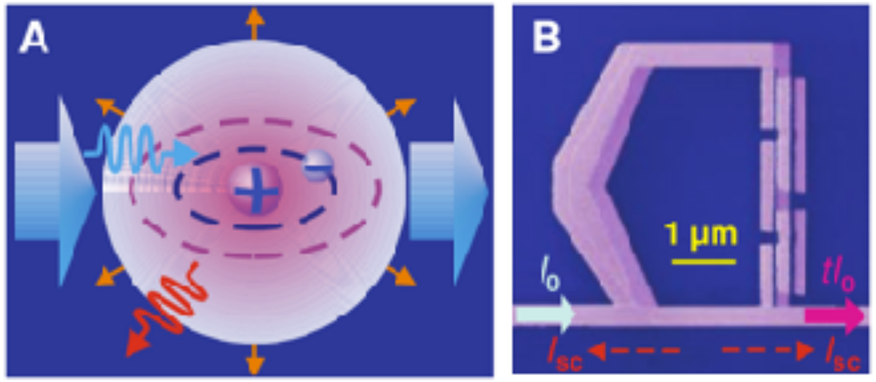
$\rho(t), E(t)$

Resonance fluorescence

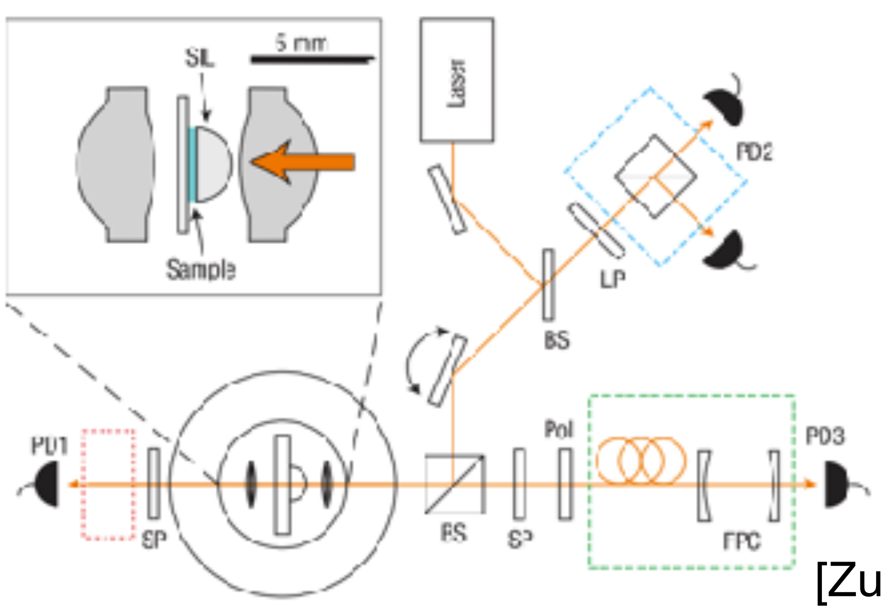




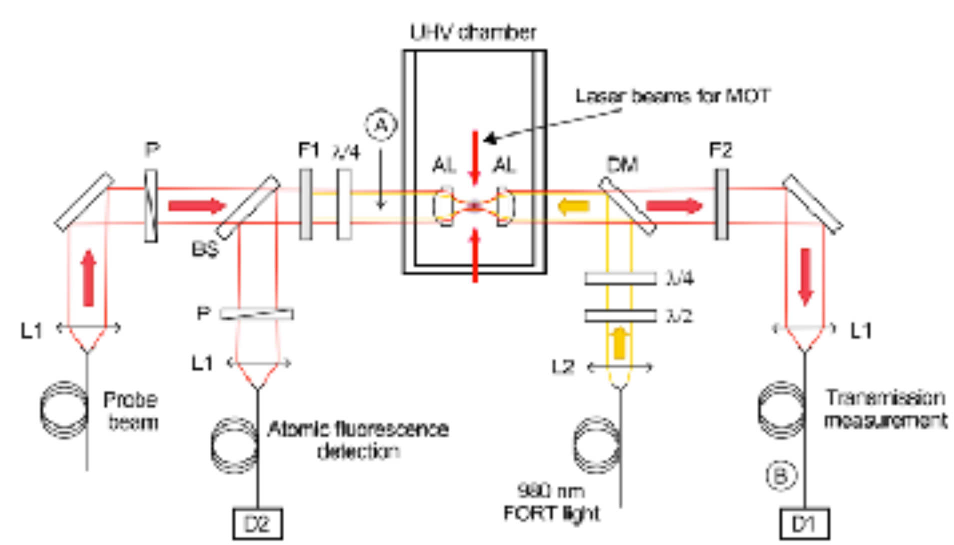
[Stuttgart PRL 2009]



[Tsukuba Science 2010]

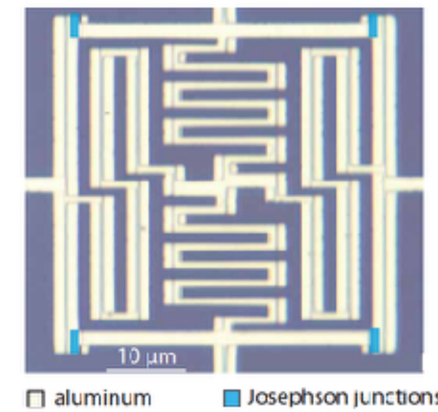


[Zurich Nat. Phys. 2007]

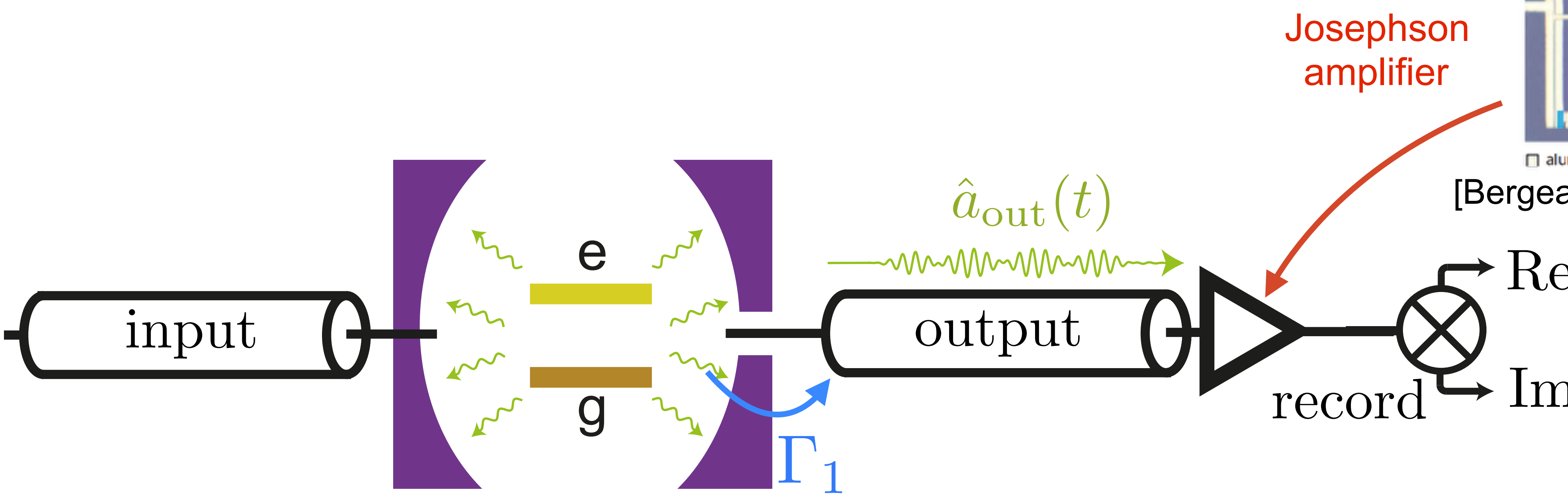


[Singapore Nat. Phys. 2008]

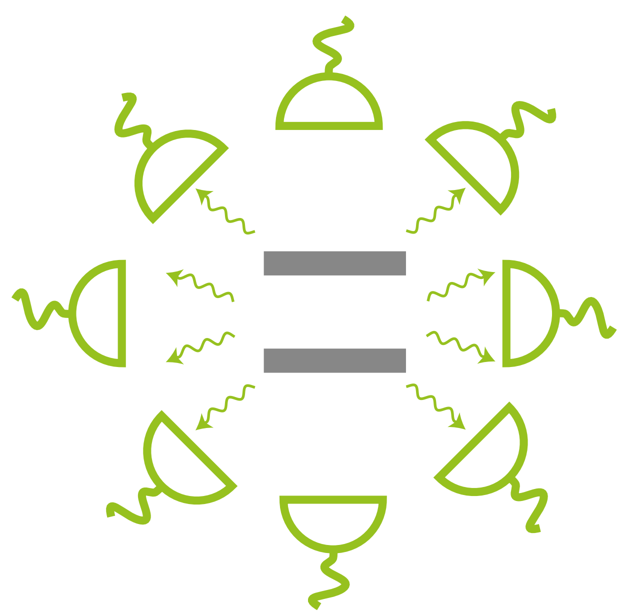
Fluorescence Measurement



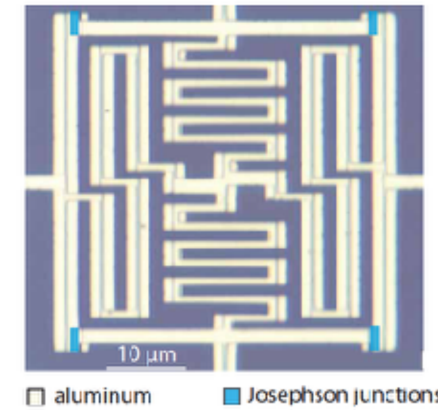
[Bergeal *et al.*, Nature 2010]



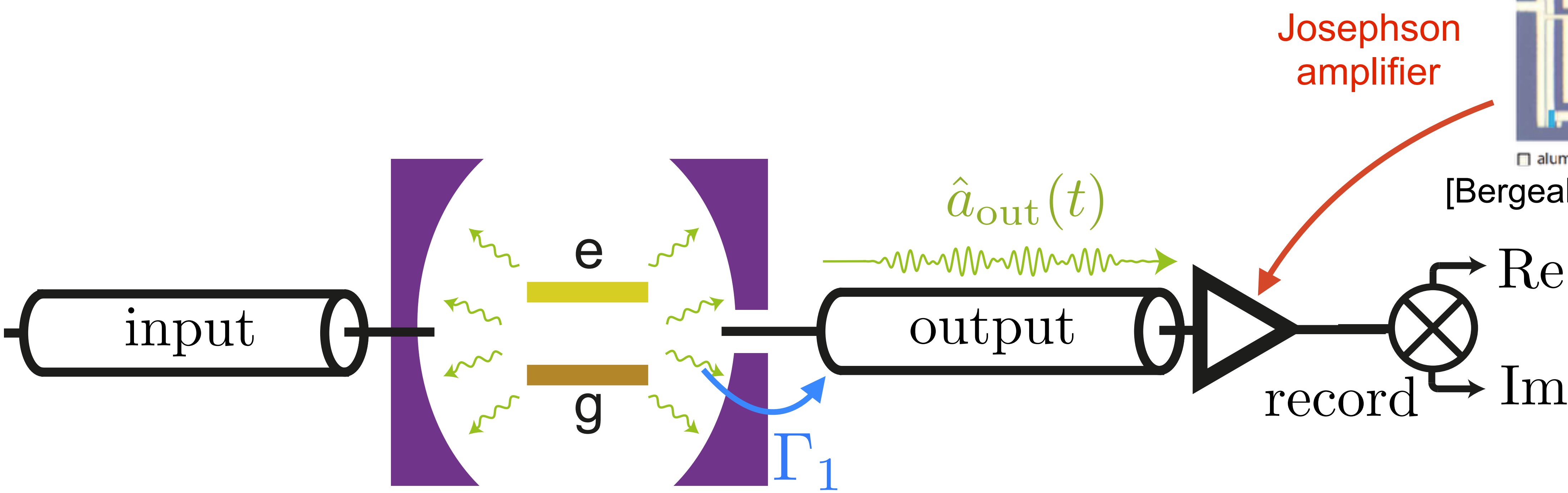
simular to our
gedanken experiment



Fluorescence Measurement



[Bergeal *et al.*, Nature 2010]

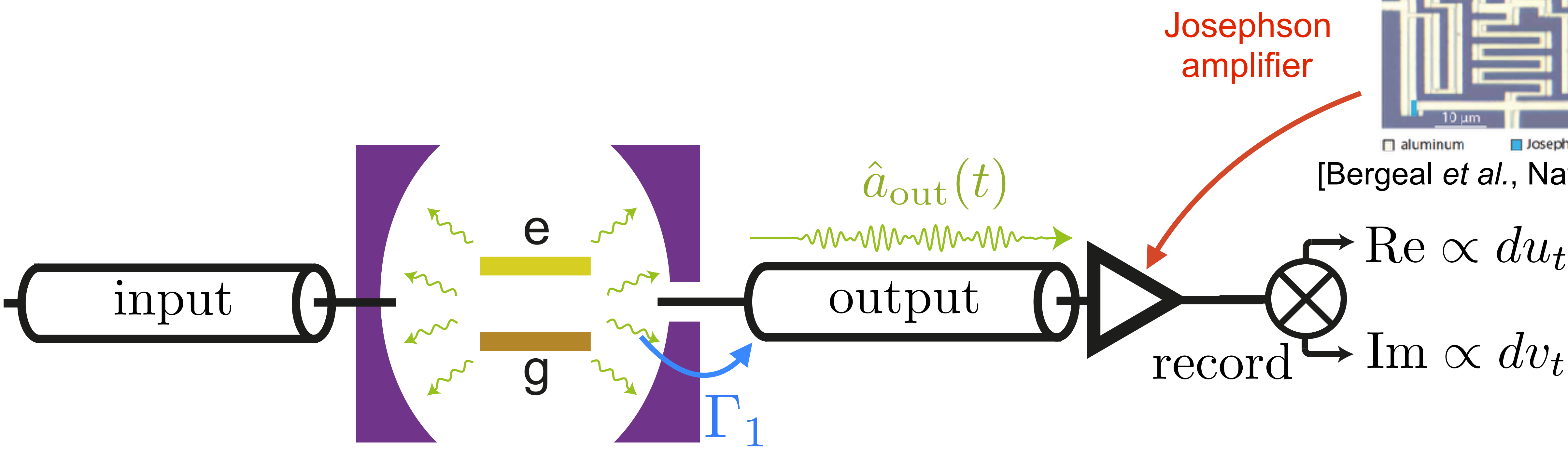


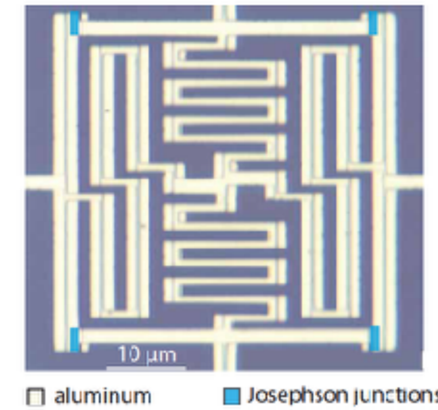
mean signal $$\langle \hat{a}_{\mathrm{out}} \rangle \propto \sqrt{\Gamma_1} \langle \sigma_- \rangle$$

jump operator $$\propto \sigma_- = |g\rangle \langle e| = \frac{\sigma_x - i\sigma_y}{2}$$

$$\Gamma_1 = (12.5\ \mu\mathrm{s})^{-1}$$

Fluorescence Measurement



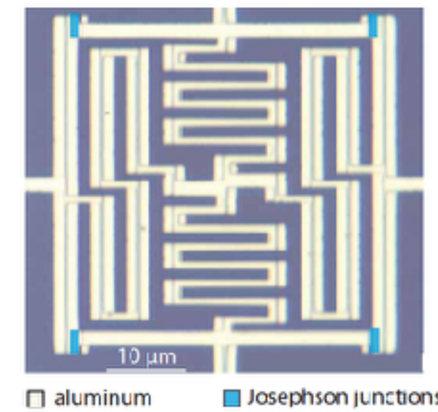


[Bergeal *et al.*, Nature 2010]

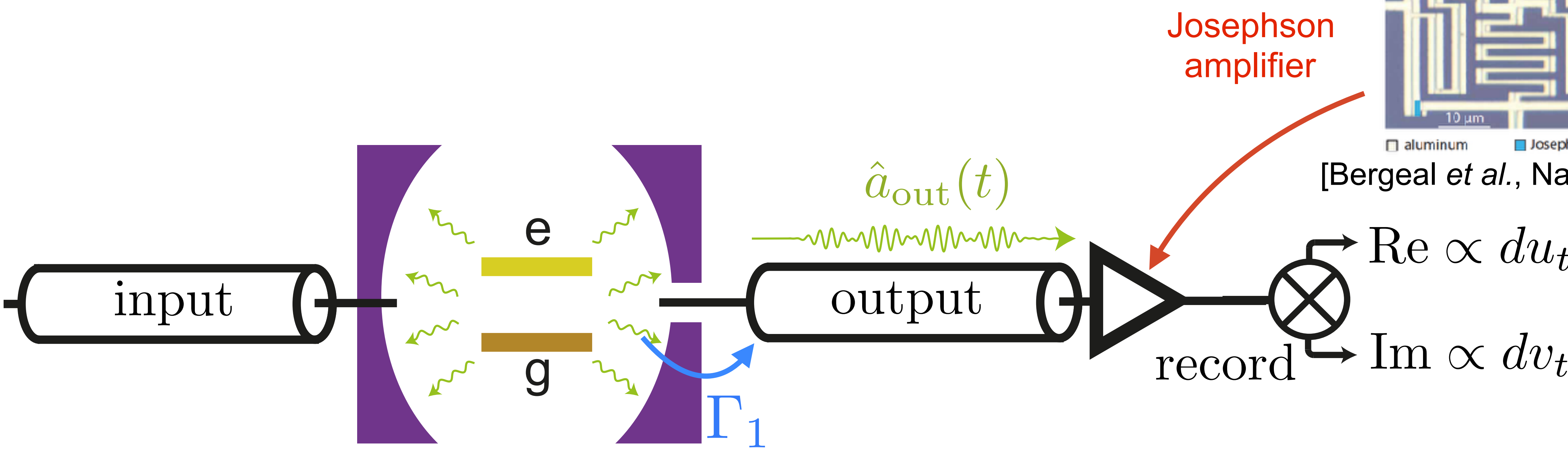
$$du_t = \sqrt{\frac{\eta \Gamma_1}{2}} \langle \sigma_X \rangle_{\rho_t} dt + dW_{t,1}$$

$$dv_t = \sqrt{\frac{\eta \Gamma_1}{2}} \langle \sigma_Y \rangle_{\rho_t} dt + dW_{t,2}$$

Fluorescence Measurement



[Bergeal *et al.*, Nature 2010]



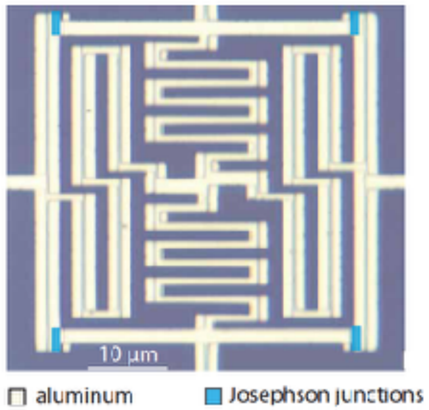
$$du_t = \sqrt{\frac{\eta\Gamma_1}{2}} \langle \sigma_X \rangle_{\rho_t} dt + dW_{t,1}$$

$$dv_t = \sqrt{\frac{\eta\Gamma_1}{2}} \langle \sigma_Y \rangle_{\rho_t} dt + dW_{t,2}$$

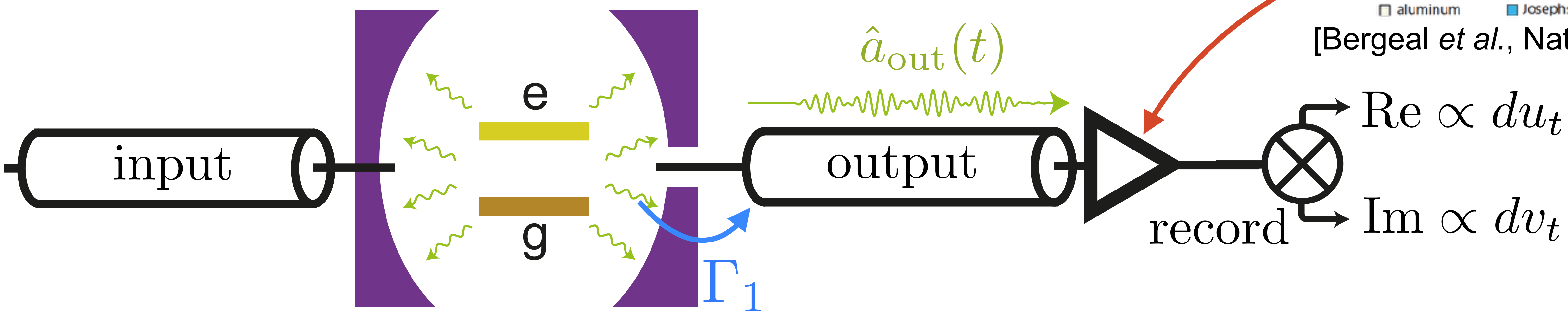
average outcome

noise (Wiener)

Fluorescence Measurement



[Bergeal *et al.*, Nature 2010]



$$du_t = \sqrt{\frac{\eta \Gamma_1}{2}} \langle \sigma_X \rangle_{\rho_t} dt + dW_{t,1}$$

$$dv_t = \sqrt{\frac{\eta \Gamma_1}{2}} \langle \sigma_Y \rangle_{\rho_t} dt + dW_{t,2}$$

average outcome

noise (Wiener)

$$\{du_t, dv_t\} \xrightarrow{\text{stochastic master equation}} \rho_t^{\rm B}$$

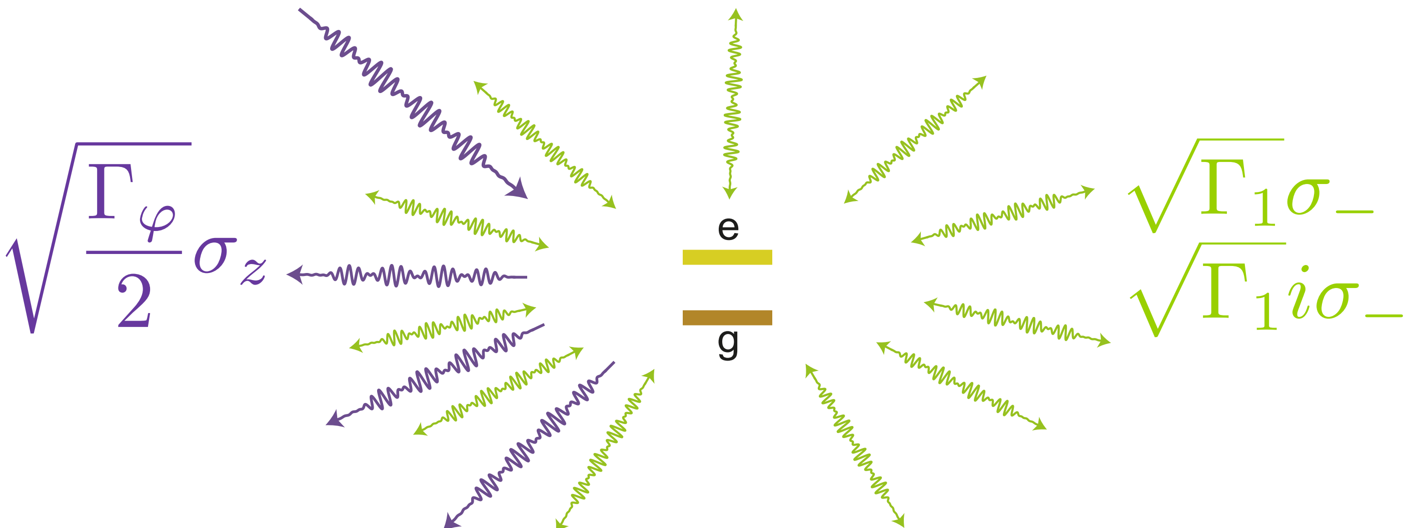
[Campagne-Ibarcq *et al.*, PRX 2016]
[Naghiloo *et al.*, Nat. Comm. 2016]
[Ficheux *et al.*, Nat. Comm. in press]

Quantum Trajectories - SME

Stochastic Master Equation for a continuous and weak measurement

$$d\rho_t = -\frac{i}{\hbar}[H, \rho_t]dt + \sum_{i=1}^{m} \mathcal{D}_i(\rho_t)dt$$

Decoherence $\mathcal{D}_i(\rho_t) = L_i \rho_t L_i^\dagger - \frac{1}{2}\rho_t L_i^\dagger L_i - \frac{1}{2} L_i^\dagger L_i \rho_t$



$L_i \rightarrow$ jump operator

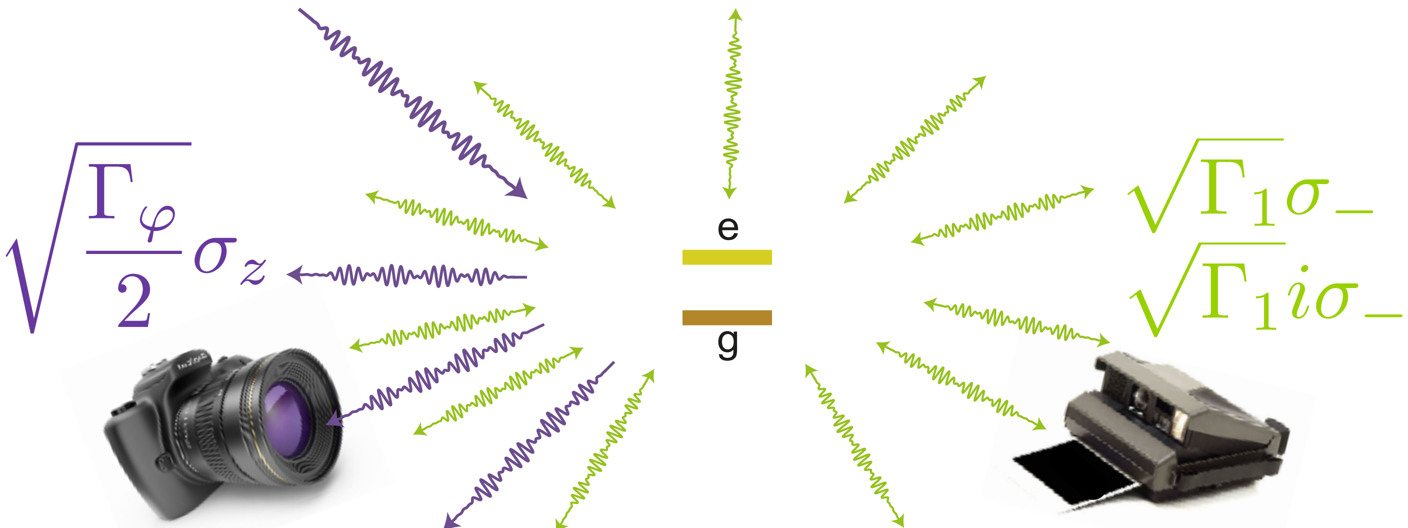
Quantum Trajectories - SME

Stochastic Master Equation for a continuous and weak measurement

$$d\rho_t = -\frac{i}{\hbar}[H, \rho_t]dt + \sum_{i=1}^{m} \mathcal{D}_i(\rho_t)dt + \sum_{i=1}^{m} \sqrt{\eta_i}\mathcal{M}_i(\rho_t)dW_{t,i}$$

Decoherence $\mathcal{D}_i(\rho_t) = L_i\rho_t L_i^\dagger - \frac{1}{2}\rho_t L_i^\dagger L_i - \frac{1}{2}L_i^\dagger L_i \rho_t$

Innovation $\mathcal{M}_i(\rho_t) = L_i\rho_t + \rho_t L_i^\dagger - \mathrm{Tr}(L_i\rho_t + \rho_t L_i^\dagger)\rho_t$



$L_i \rightarrow$ jump operator

$\eta_i \rightarrow$ measurement efficiency

Quantum Trajectories - SME

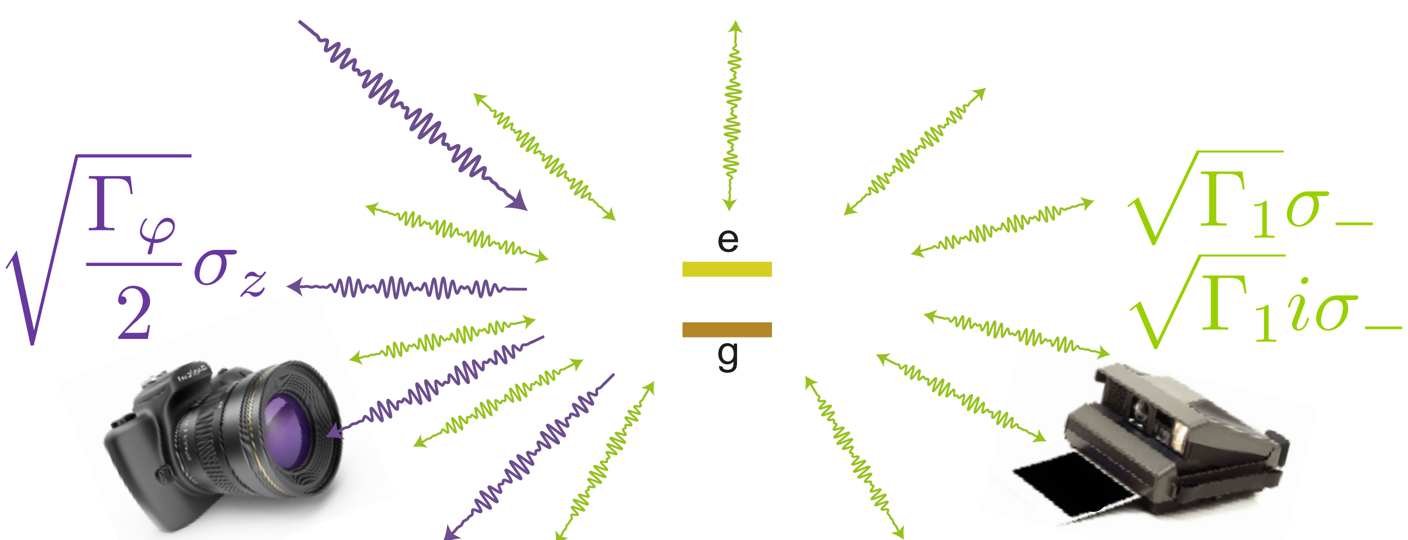
Stochastic Master Equation for a continuous and weak measurement

$$d\rho_t = -\frac{i}{\hbar}[H, \rho_t]dt + \sum_{i=1}^{m} \mathcal{D}_i(\rho_t)dt + \sum_{i=1}^{m} \sqrt{\eta_i}\mathcal{M}_i(\rho_t)dW_{t,i}$$

Decoherence $\mathcal{D}_i(\rho_t) = L_i\rho_t L_i^\dagger - \frac{1}{2}\rho_t L_i^\dagger L_i - \frac{1}{2}L_i^\dagger L_i \rho_t$

Innovation $\mathcal{M}_i(\rho_t) = L_i\rho_t + \rho_t L_i^\dagger - \mathrm{Tr}(L_i\rho_t + \rho_t L_i^\dagger)\rho_t$

Measurement records $dy_t^i = \sqrt{\eta_i}\mathrm{Tr}(L_i\rho_t + \rho_t L_i^\dagger)dt + dW_{t,i}$



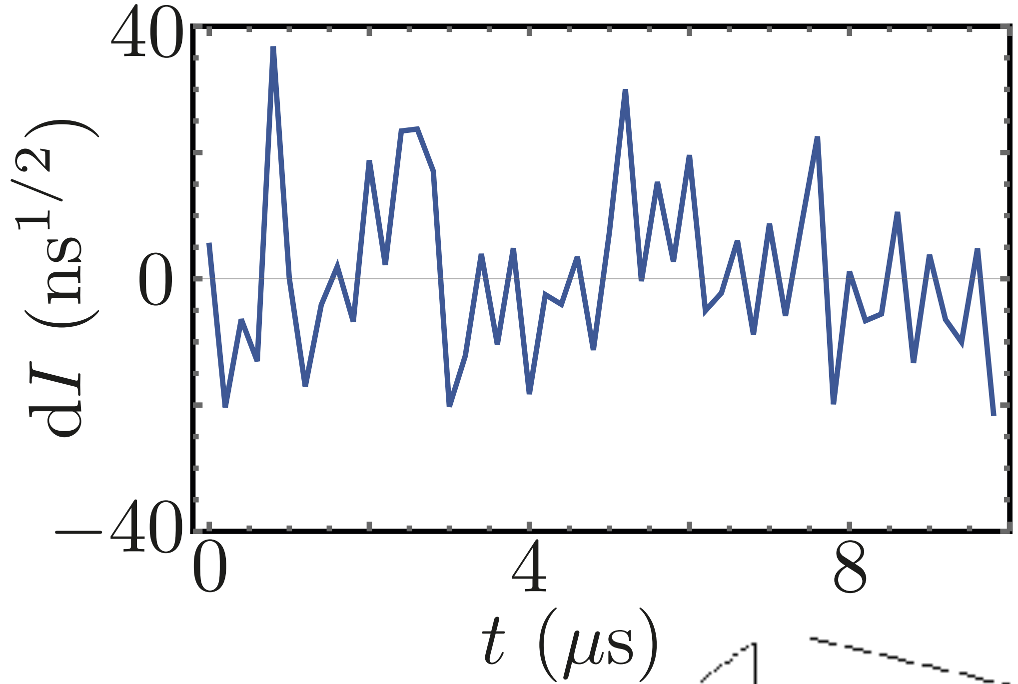
$L_i \rightarrow$ jump operator

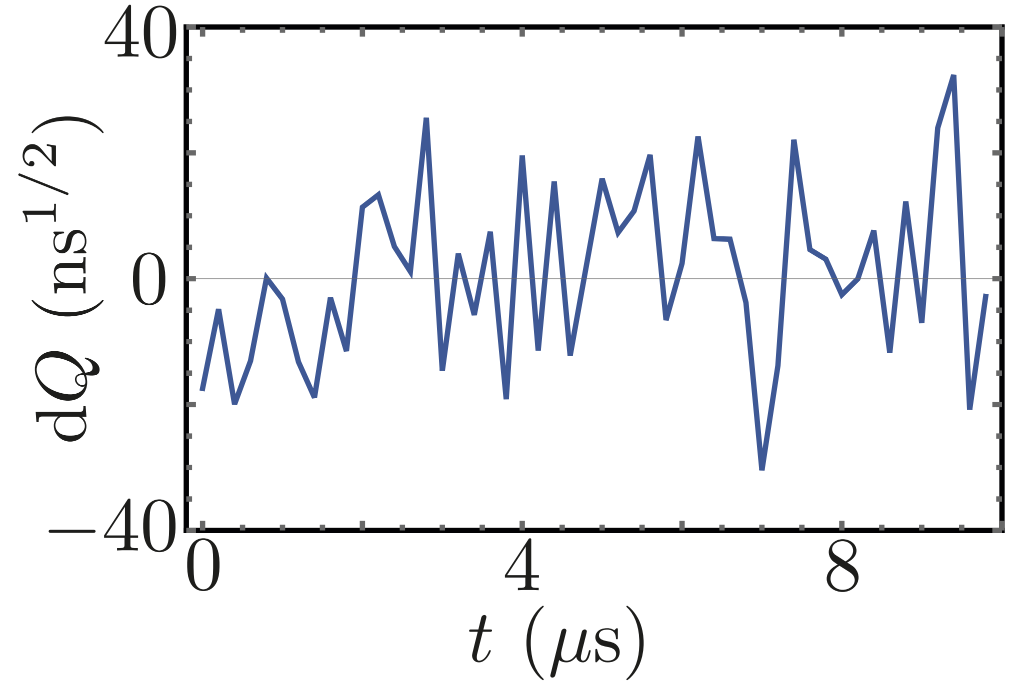
$\eta_i \rightarrow$ measurement efficiency

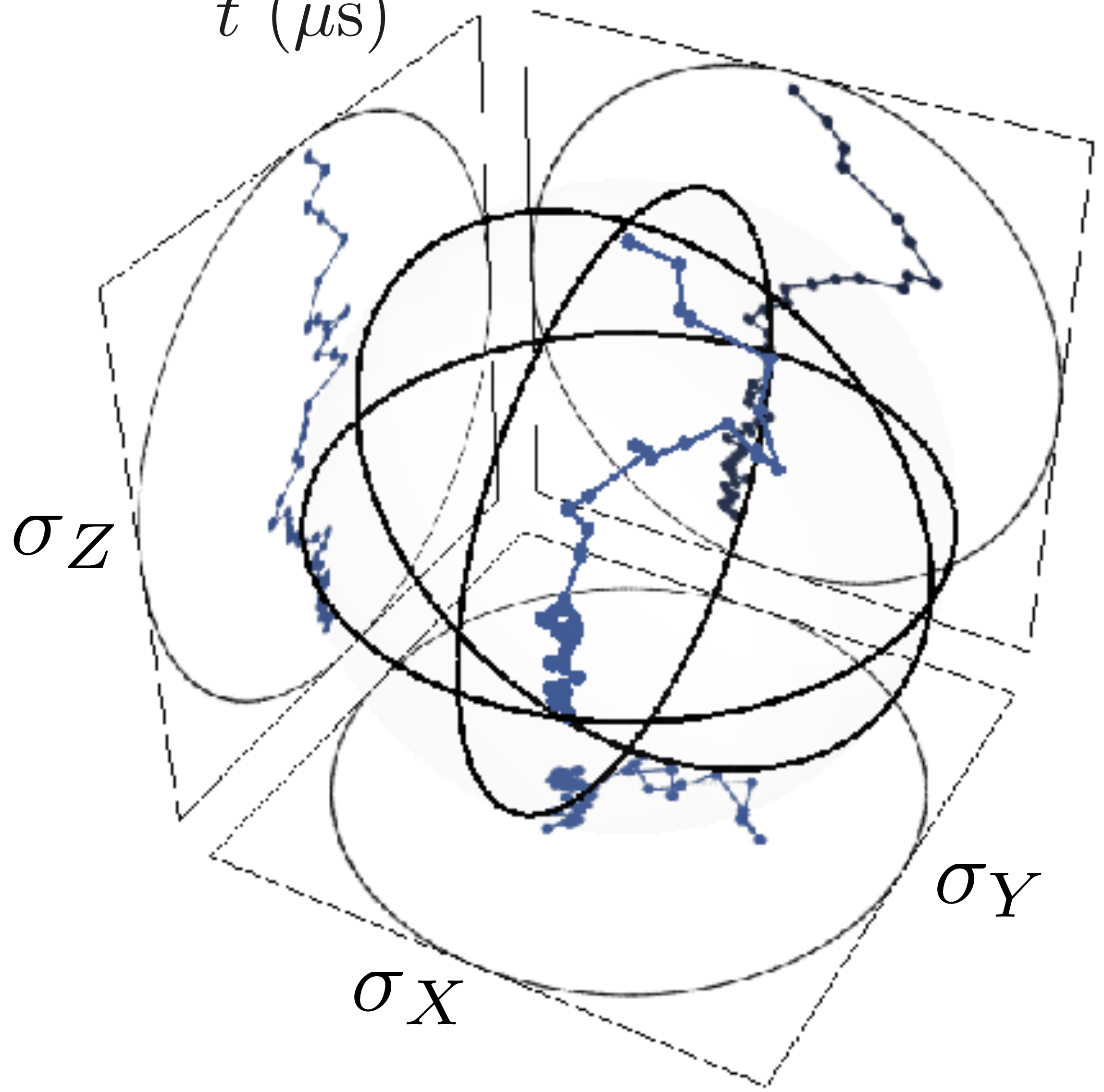
$dt \rightarrow$ limited by amplifier bandwith

Quantum trajectory

corrected for JPC low-pass filter →





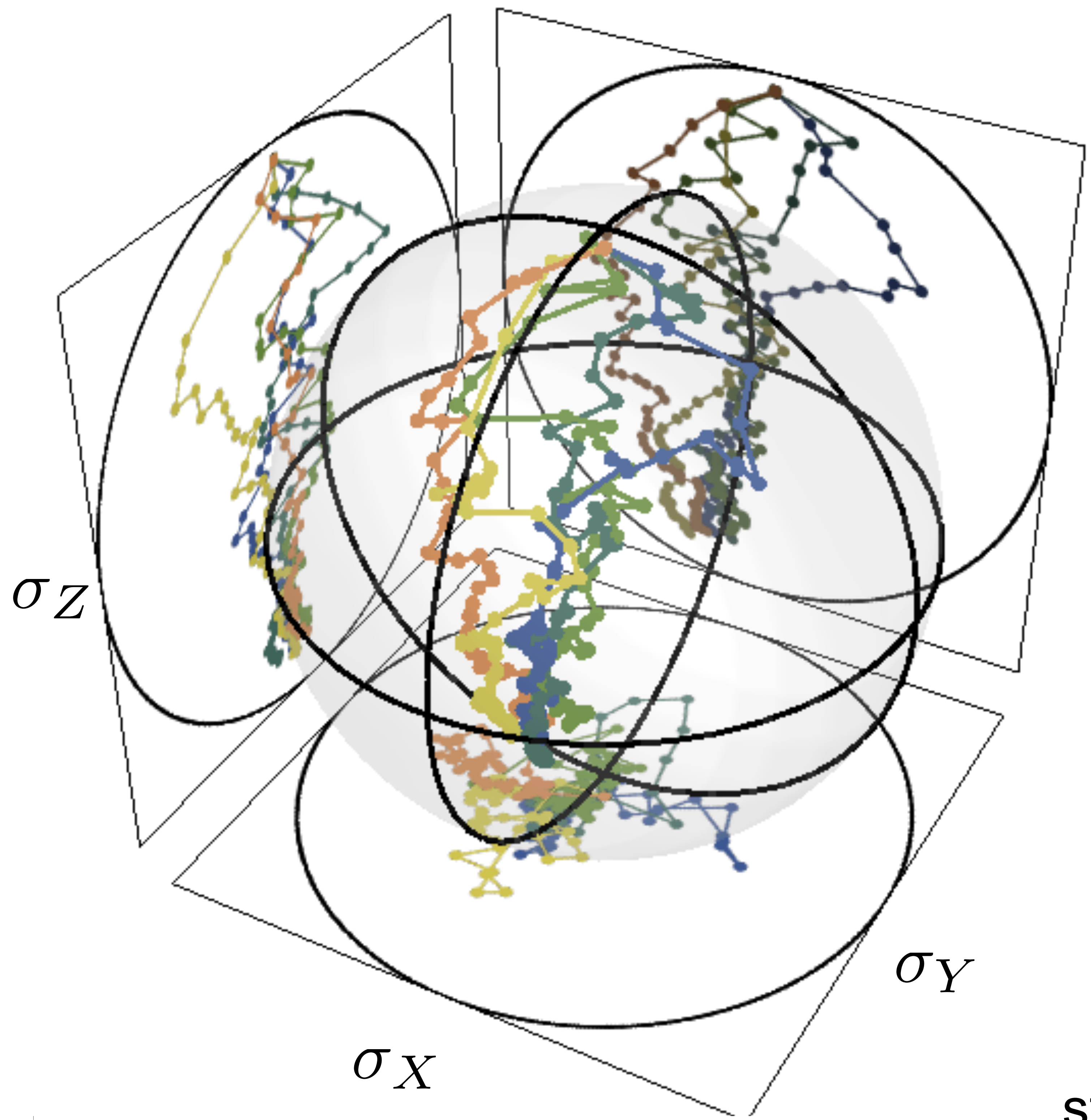


start from $|e\rangle$ at $t = 0$

5 Quantum trajectories

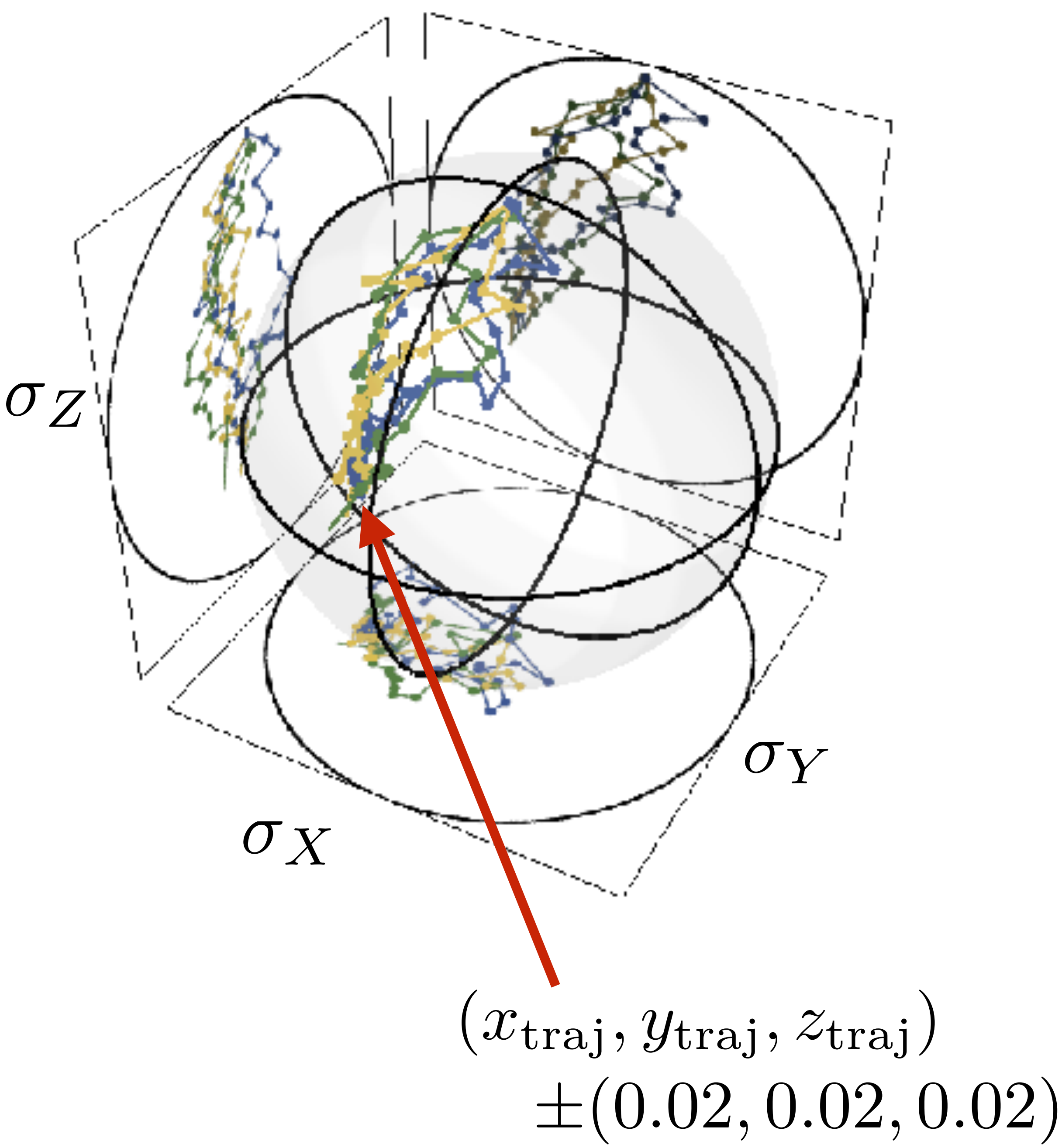
$T_{\mathrm{traj}} = 10\ \mu\mathrm{s}$

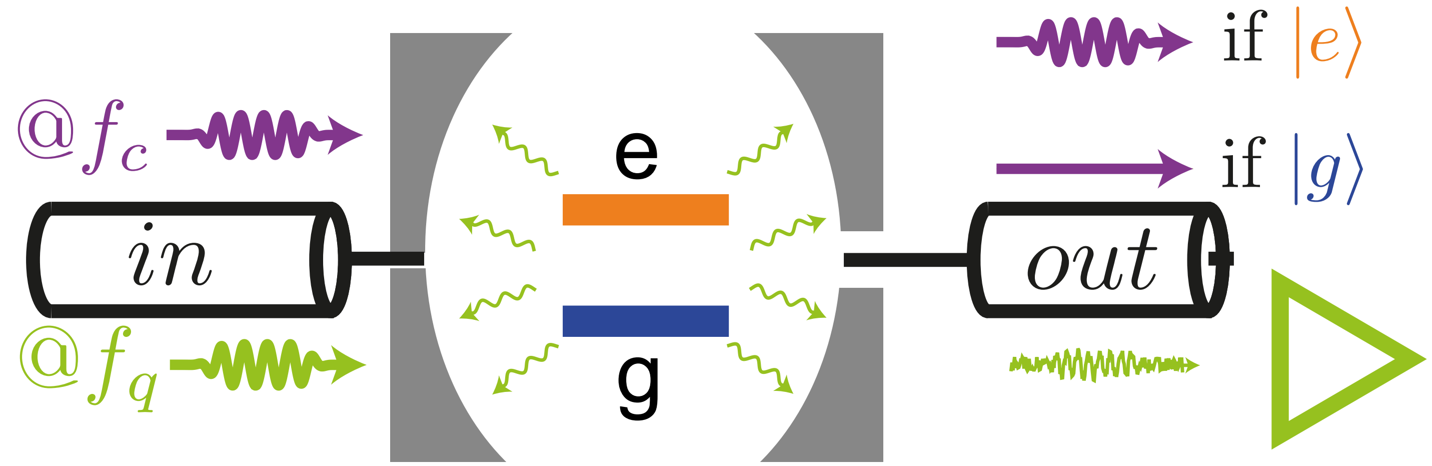
$T_1 = 4\ \mu\mathrm{s}$

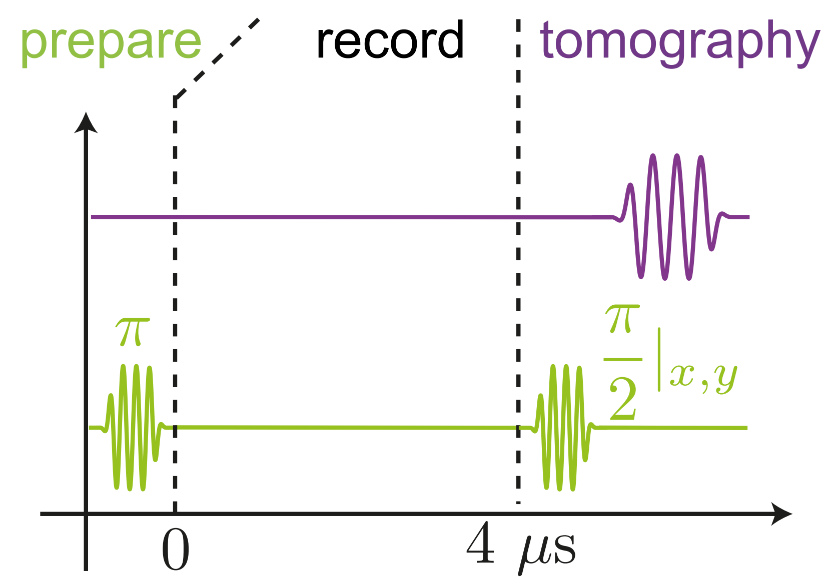


start from $|e\rangle$
at $t = 0$

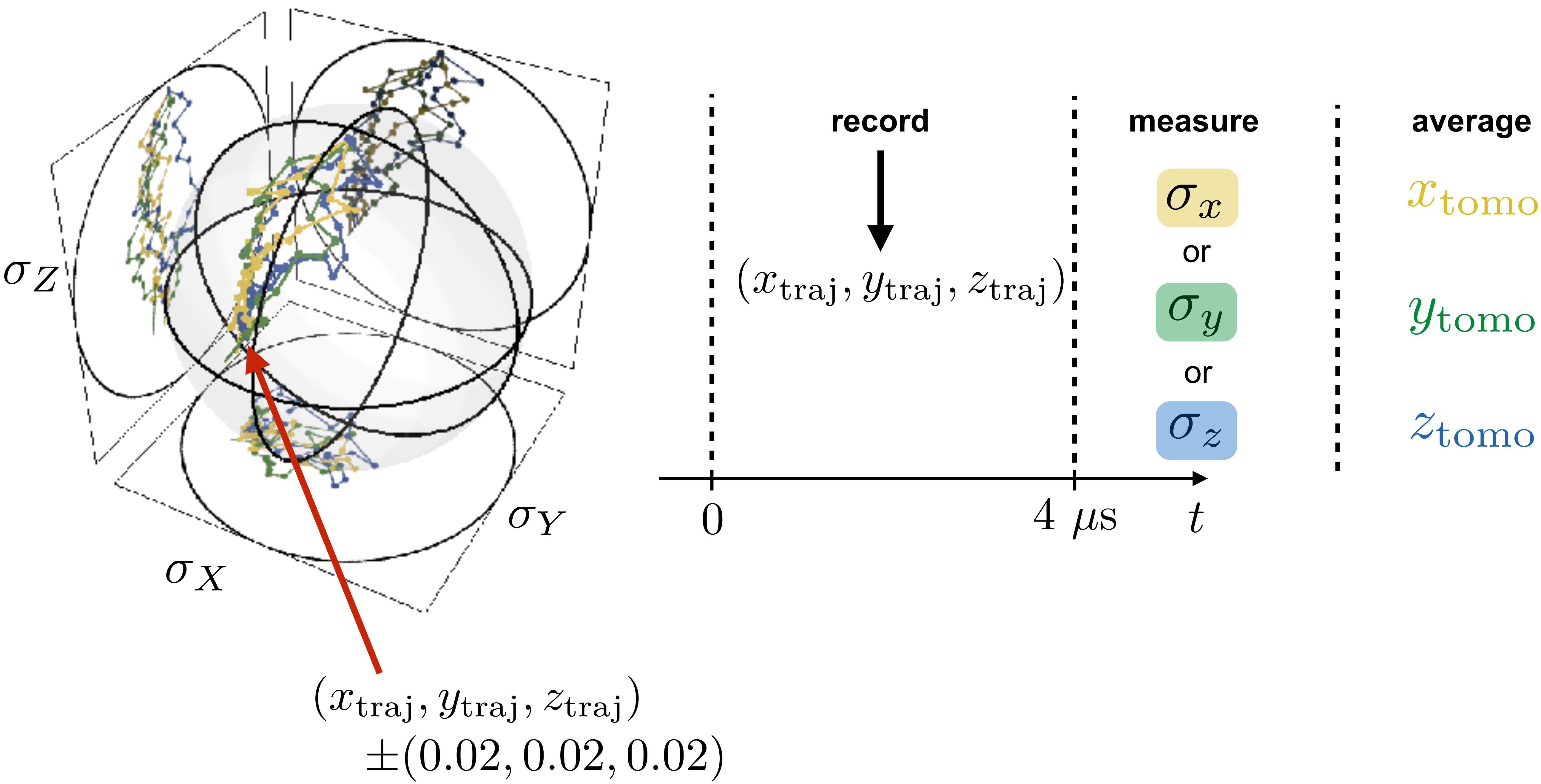
Trajectories vs tomography





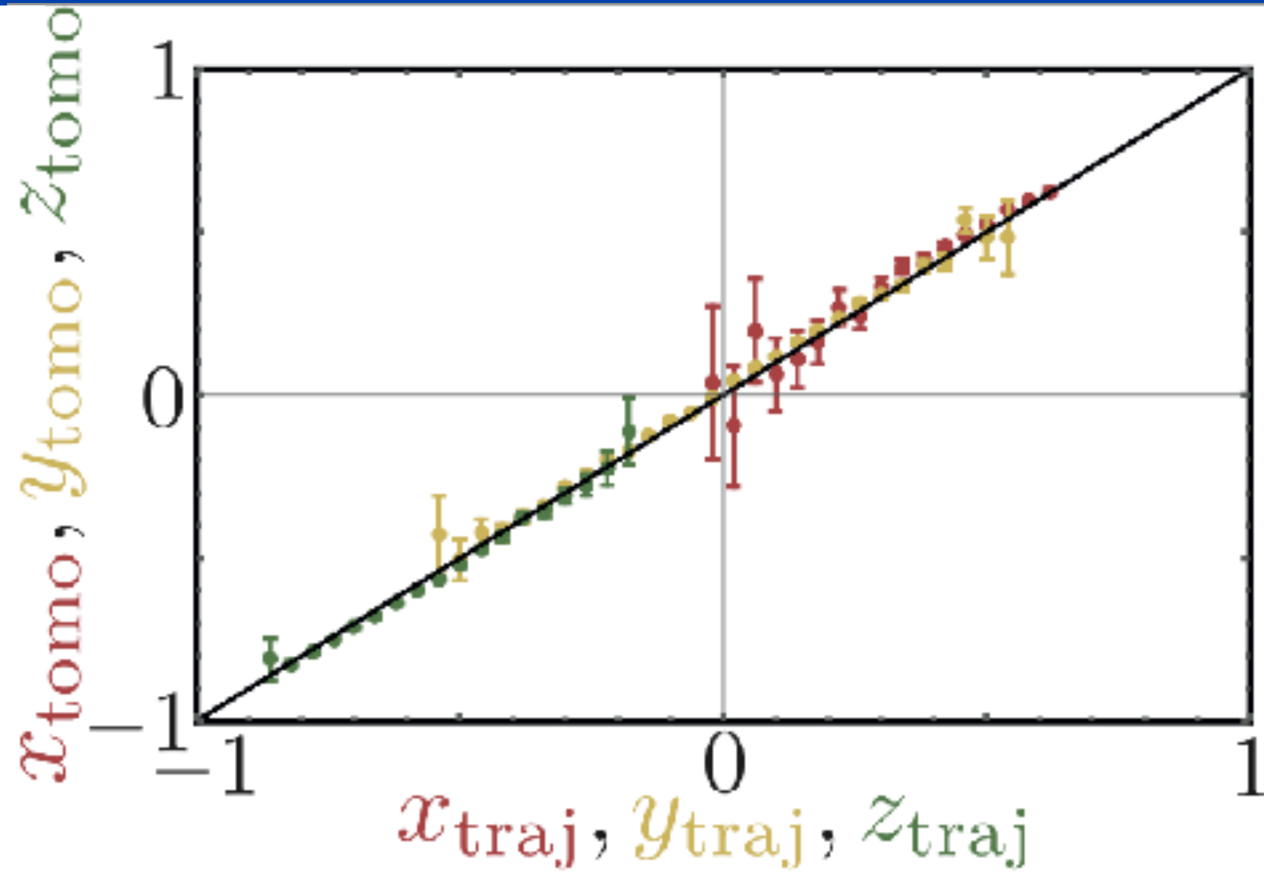


Trajectories vs tomography



Trajectories vs tomography

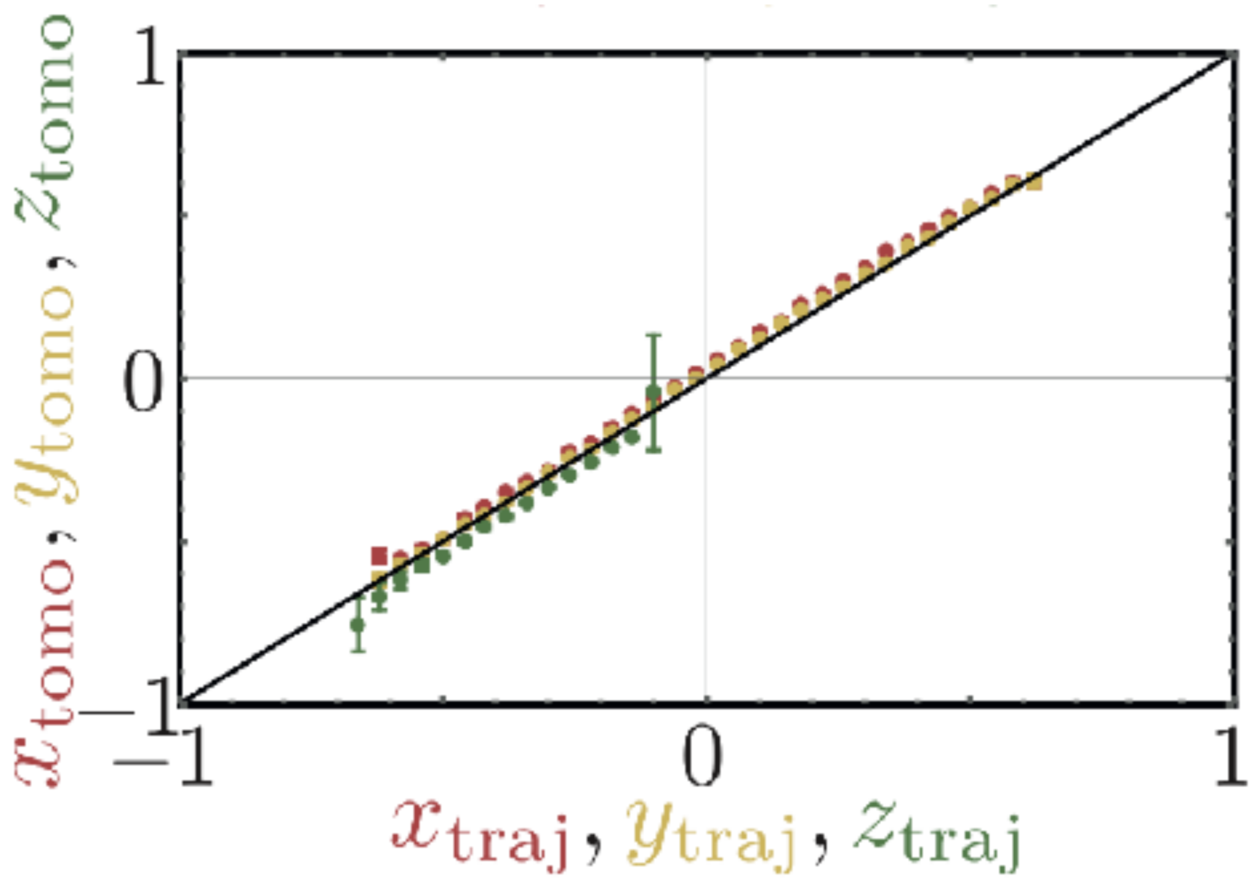
from $\frac{|g\rangle + |e\rangle}{\sqrt{2}}$



after $4\ \mu\mathrm{s}$

$\eta = 24\%$

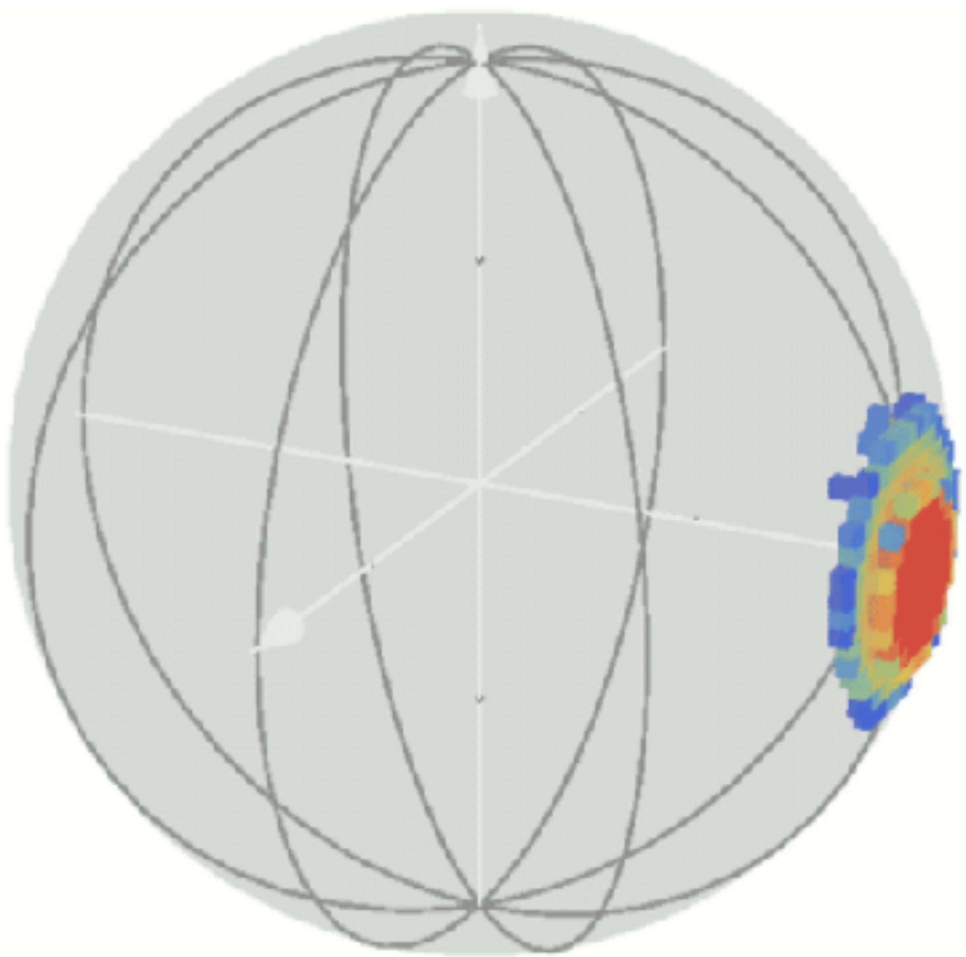
from $|e\rangle$

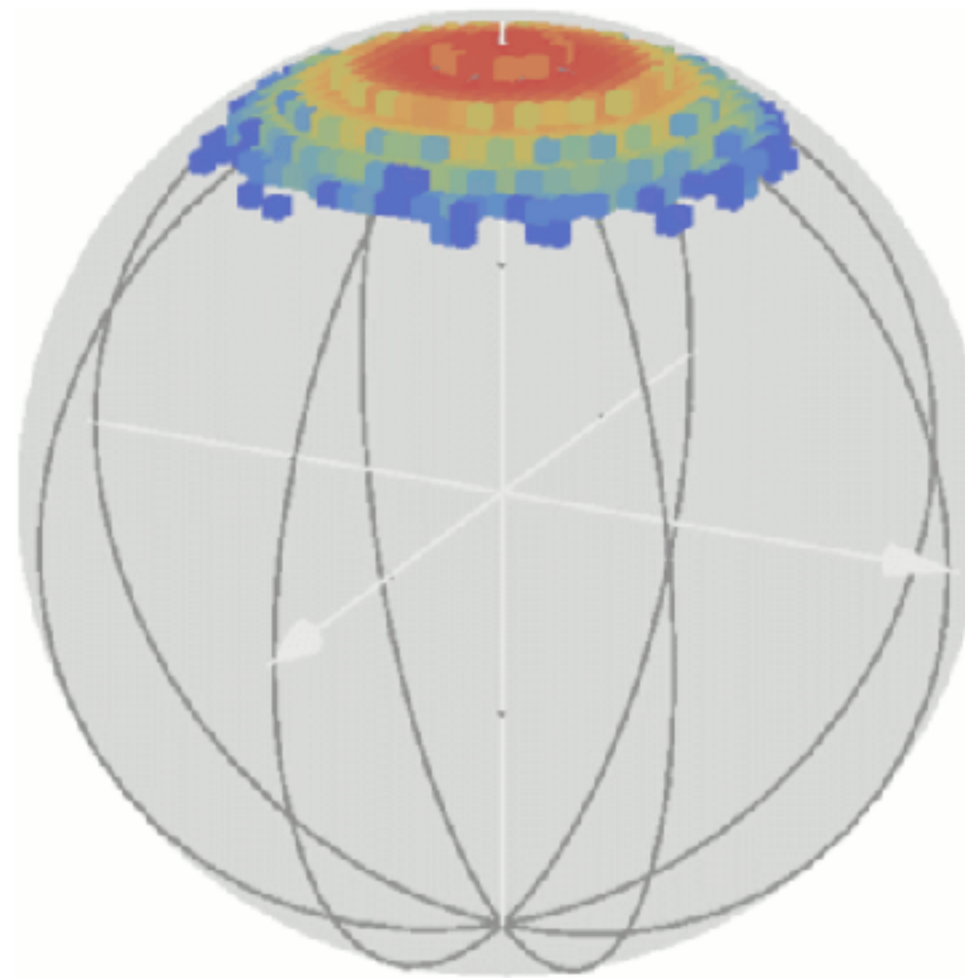


Statistics of relaxation trajectories

start in $|+x\rangle = \frac{|g\rangle + |e\rangle}{\sqrt{2}}$

start in $|e\rangle$





10^6 experiments

$\mathbb{P}(\rho_t)$

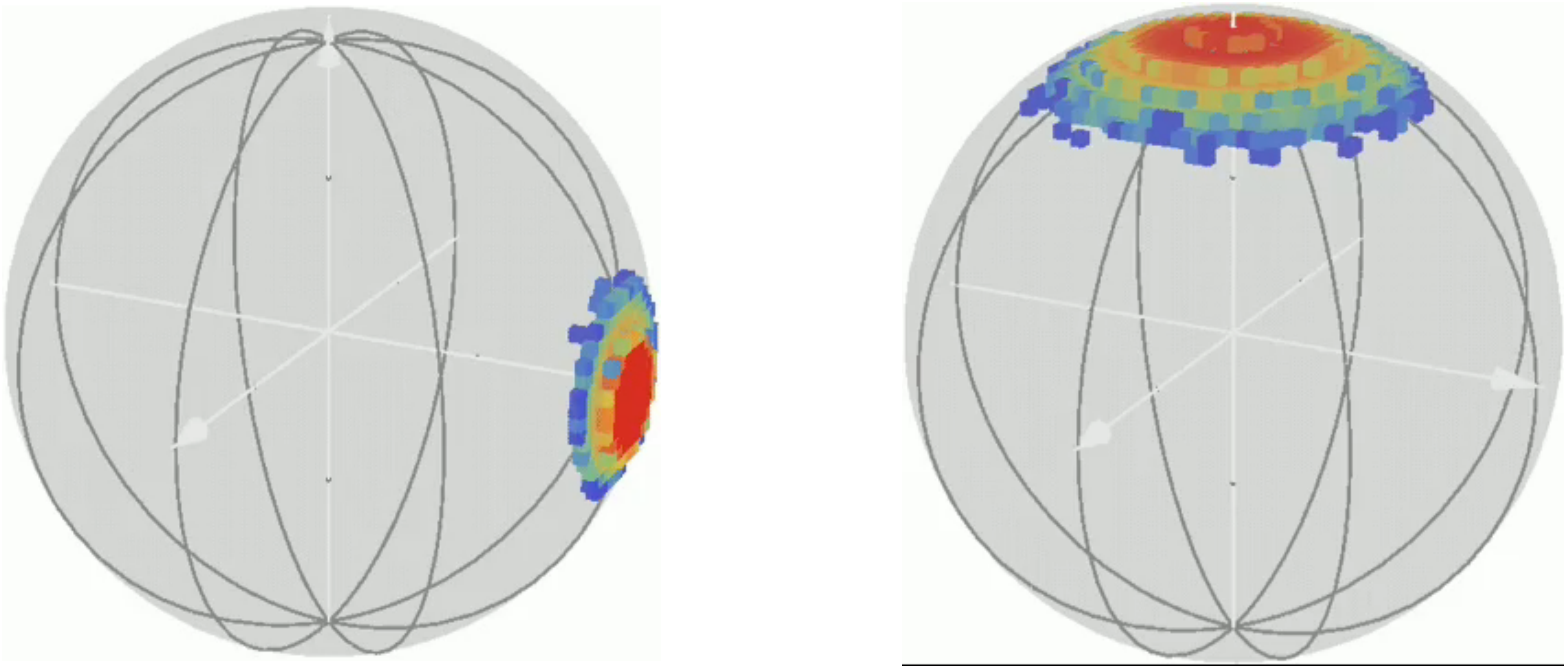
trajectories per pixel

[Campagne-Ibarcq *et al.*, PRX 2016]

Statistics of relaxation trajectories

start in $|+x\rangle = \frac{|g\rangle + |e\rangle}{\sqrt{2}}$

start in $|e\rangle$

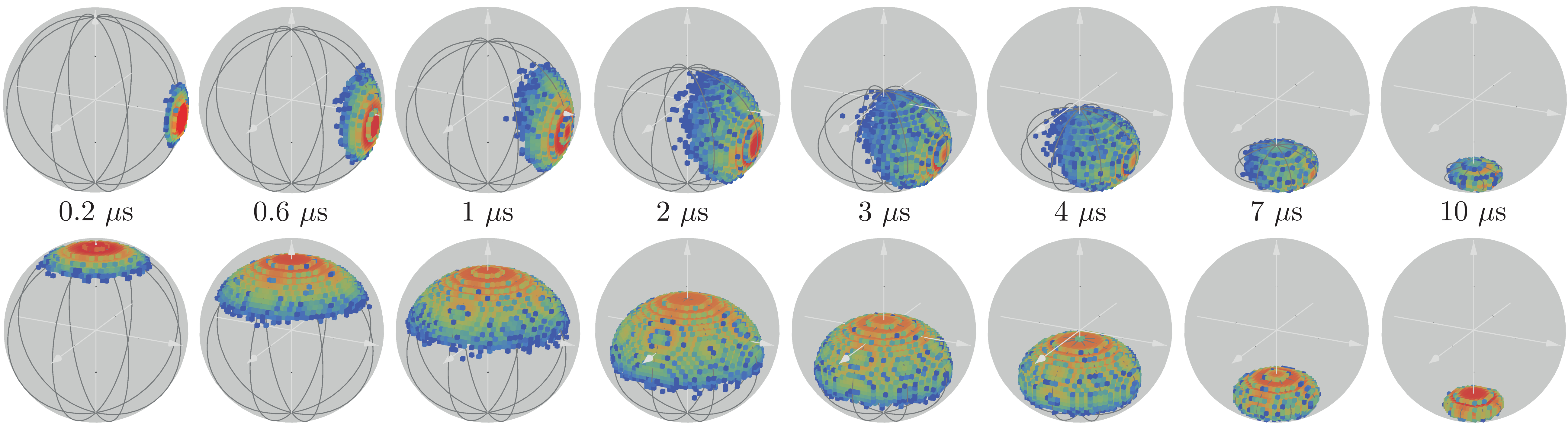


10^6 experiments

$\mathbb{P}(\rho_t)$

[Campagne-Ibarcq *et al.*, PRX 2016]

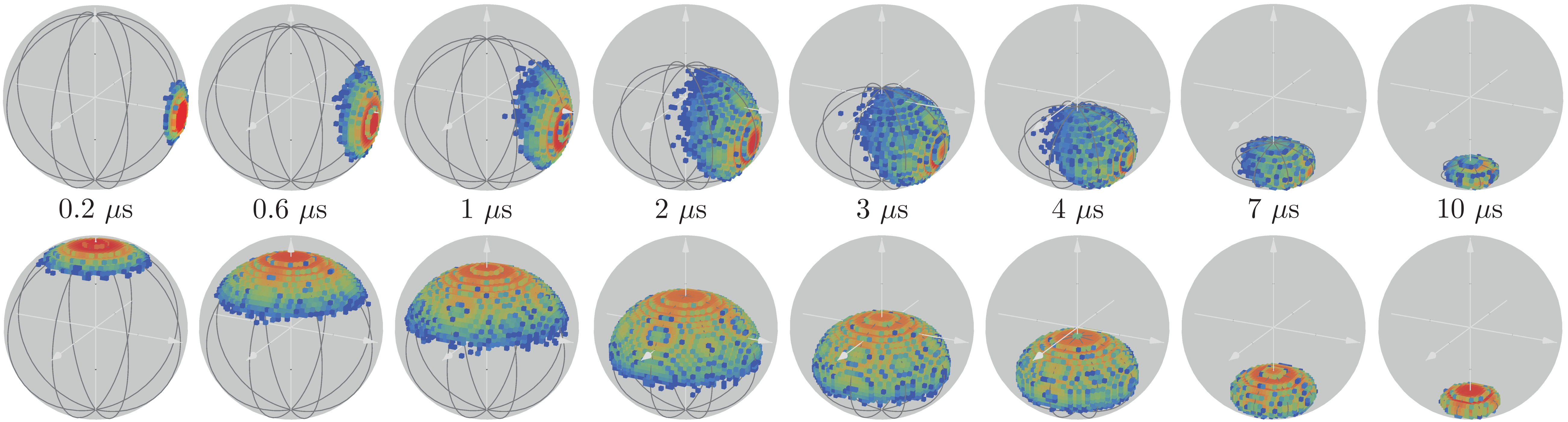
Statistics of trajectories



[Campagne-Ibarcq *et al.*, PRX 2016]

[Jordan, Chantasri, Rouchon, BH., arxiv:1511:06677]

Statistics of trajectories



[Campagne-Ibarcq *et al.*, PRX 2016]

[Jordan, Chantasri, Rouchon, BH., arxiv:1511:06677]

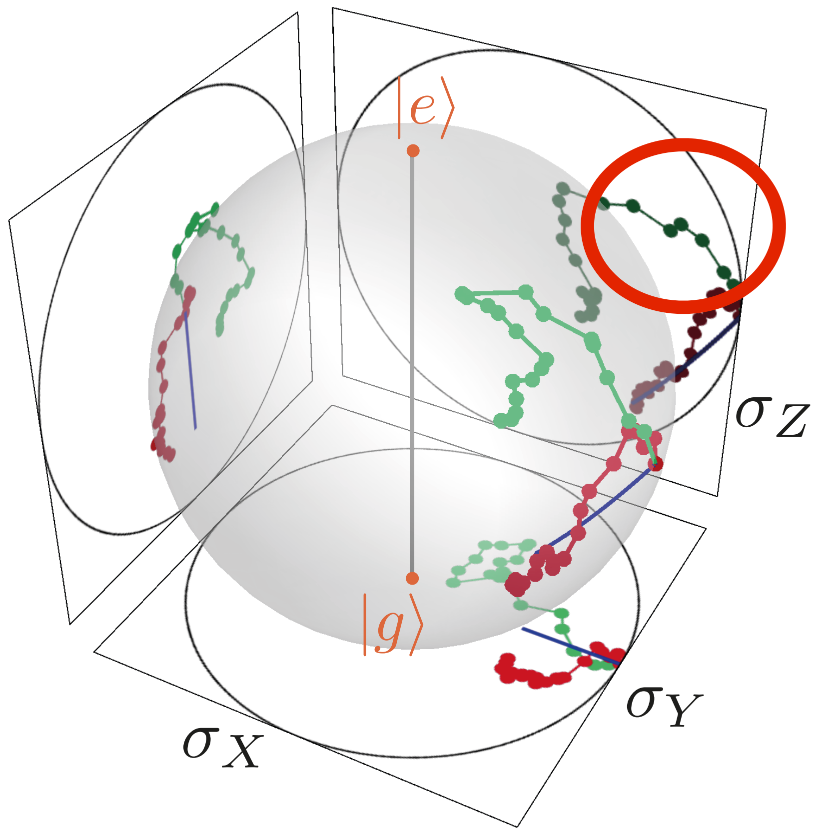
equation of the spheroid

$$\alpha(x^2 + y^2) + \alpha^2 \left(z + 1 - \frac{1}{\alpha}\right)^2 = 1$$

parameter $\alpha(t) = \eta + [\alpha(0) - \eta]e^{\Gamma_1 t}$

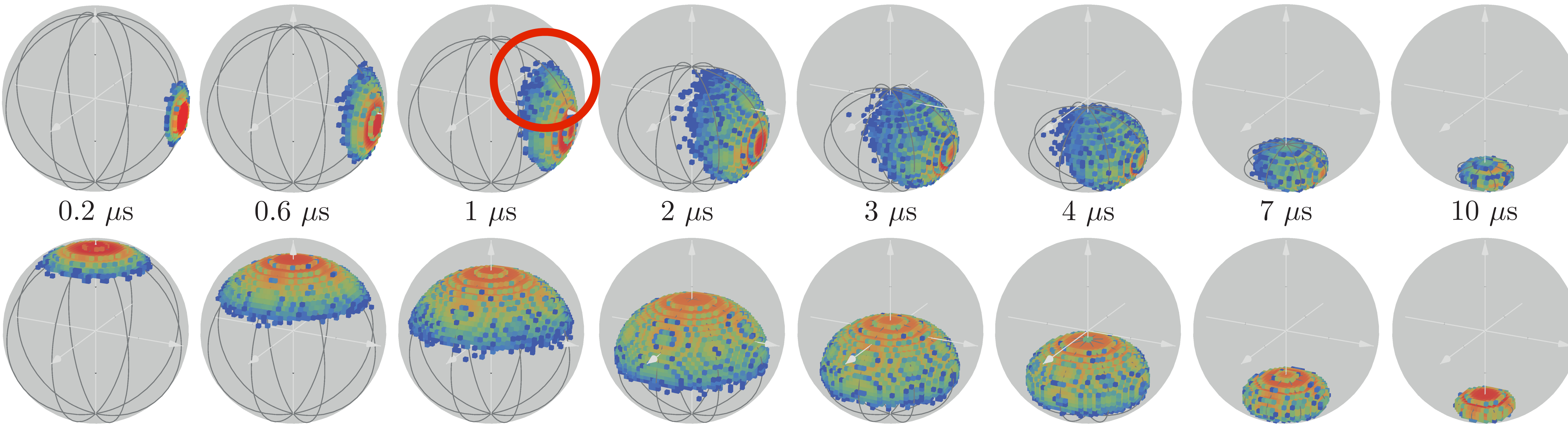
[A.Sarlette and P.Rouchon, Communications in Mathematical Physics 2016]

Counterintuitive trajectories



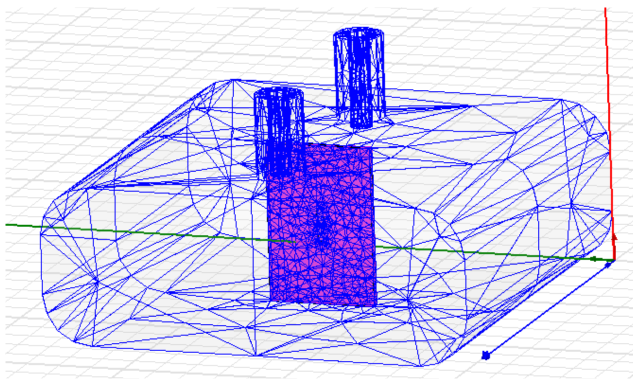
Energy expectation can **increase** due to the backaction of measuring spontaneously emitted photons

[Bolund and Mölmer, PRA 2014]



[Campagne-Ibarcq *et al.*, PRX 2016]
[Jordan, Chantasri, Rouchon, BH., arxiv:1511:06677]

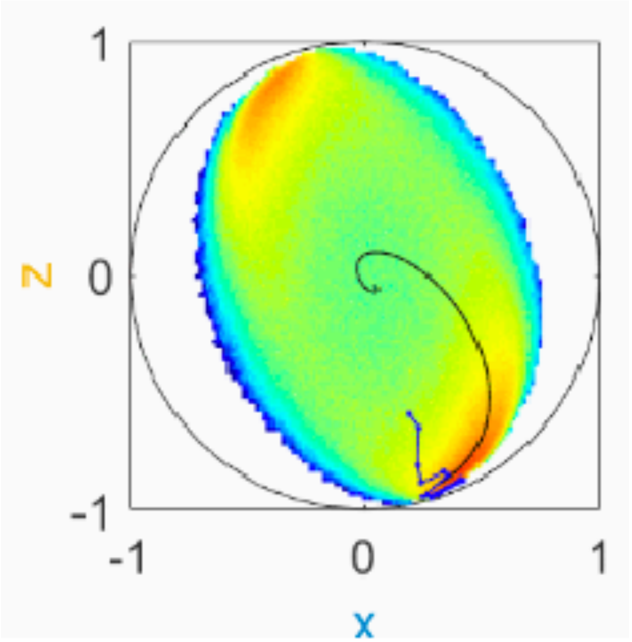
Quantum trajectories and feedback in circuit-QED

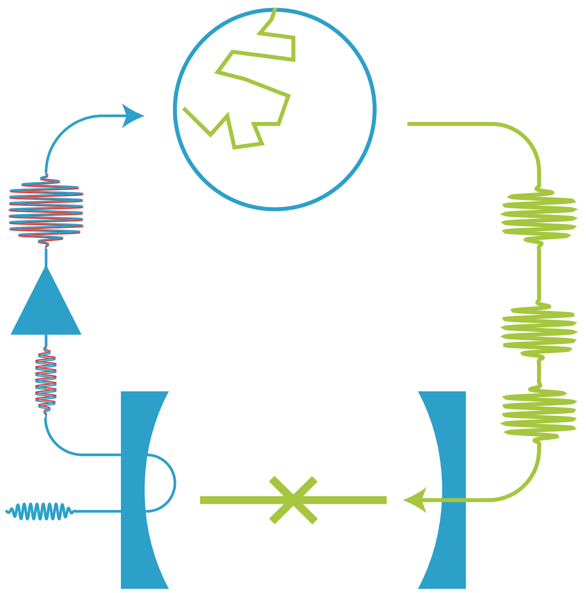


Introduction to circuit-QED

Quantum trajectories in circuit-QED

- dispersive measurement
- fluorescence measurement
- **both simultaneously**
- generating entanglement





Measurement based feedback

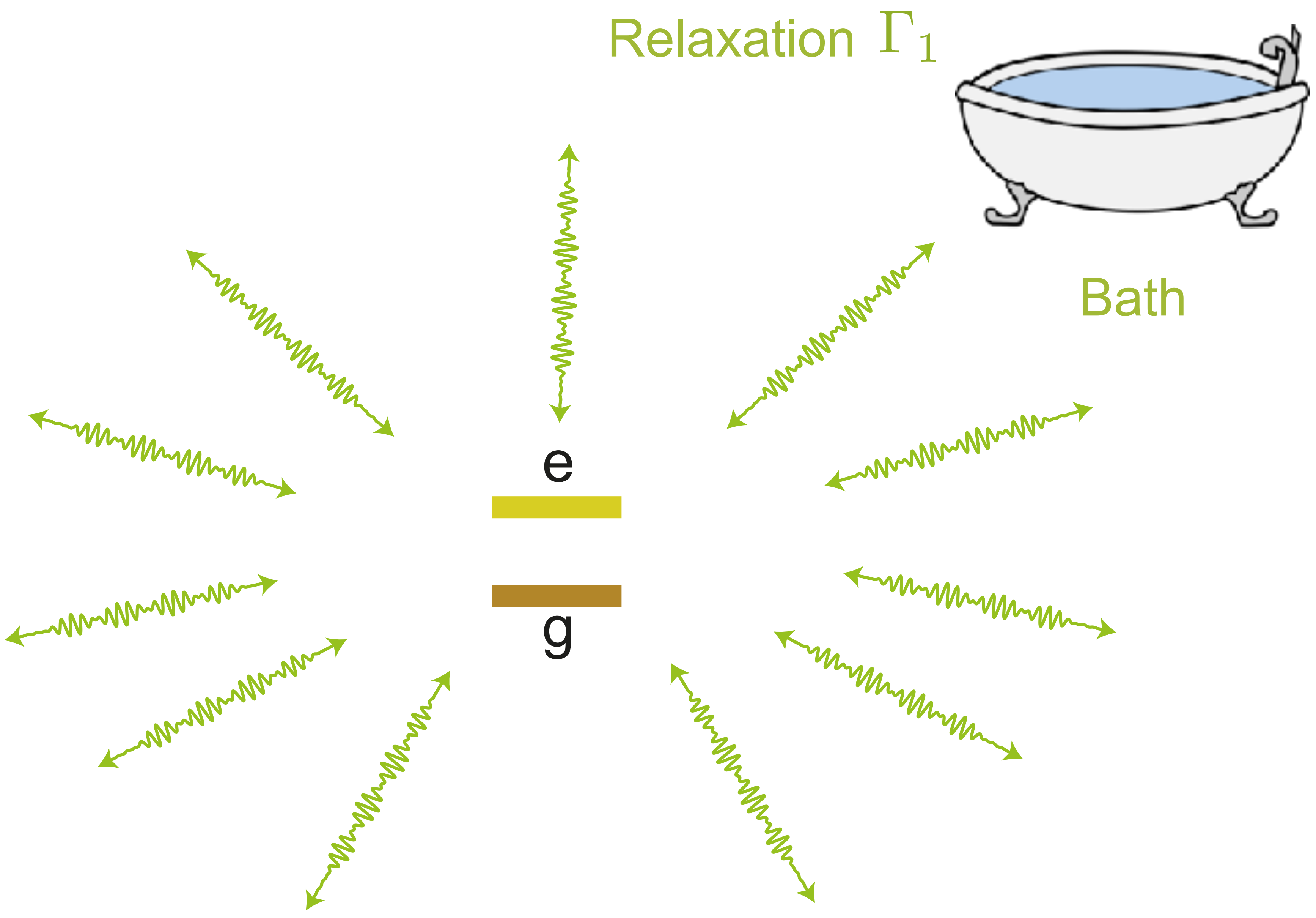
- dispersive case
- fluorescence case

Post selection in quantum mechanics

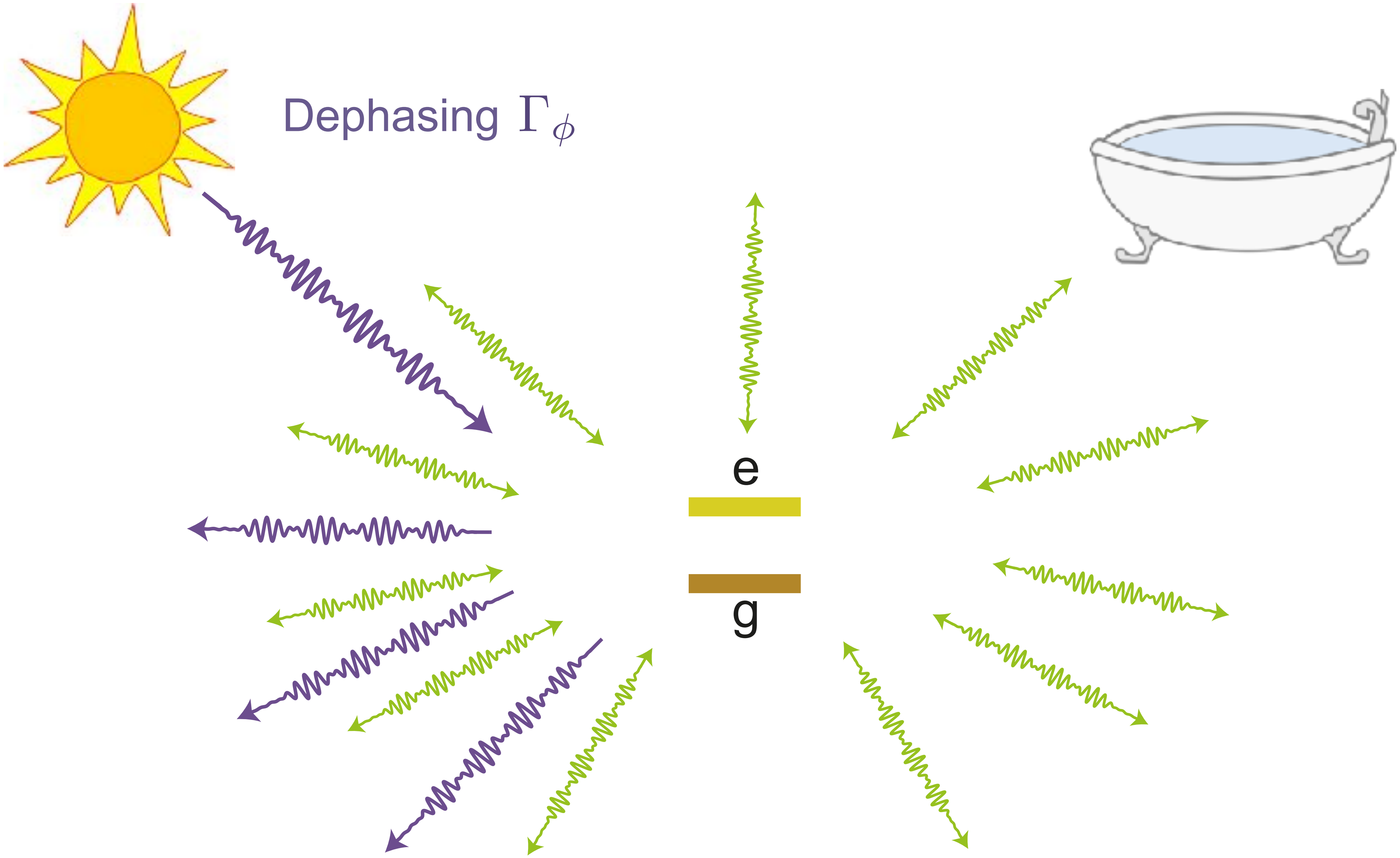
$$\rho(t), E(t)$$

Decoherence channels of a qubit

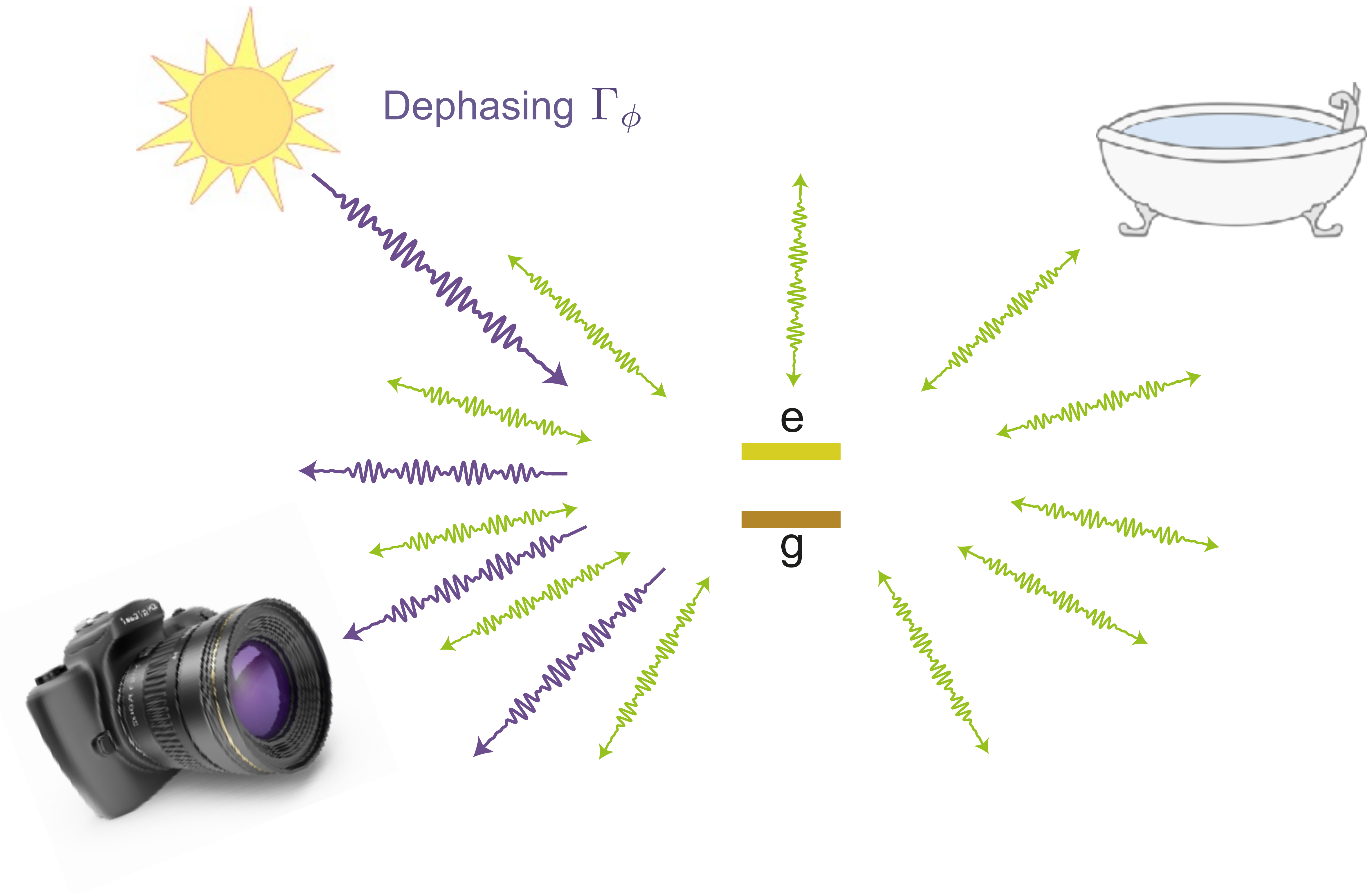
Decoherence channels of a qubit



Decoherence channels of a qubit

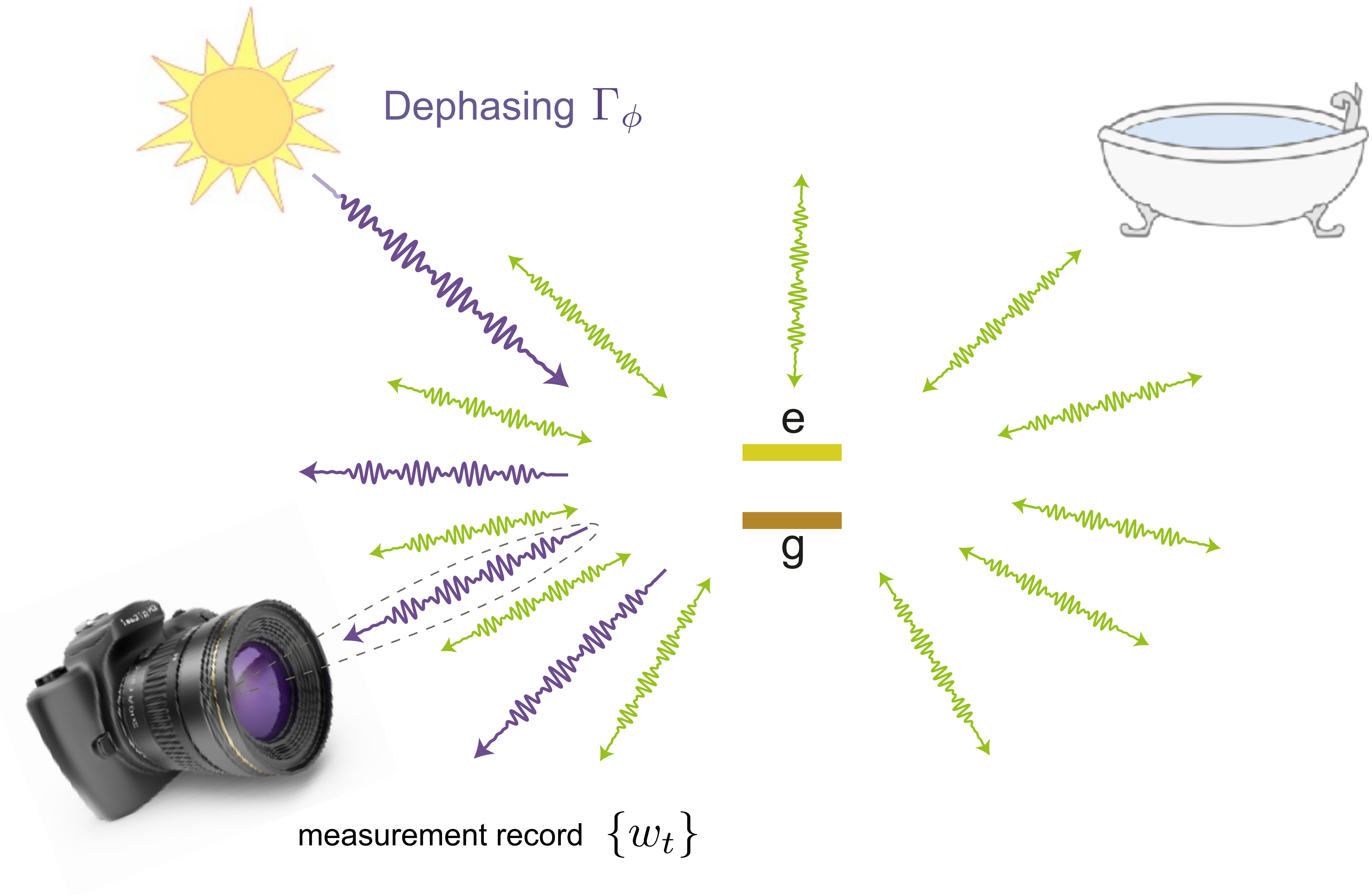


Measurement of decoherence channels of a qubit



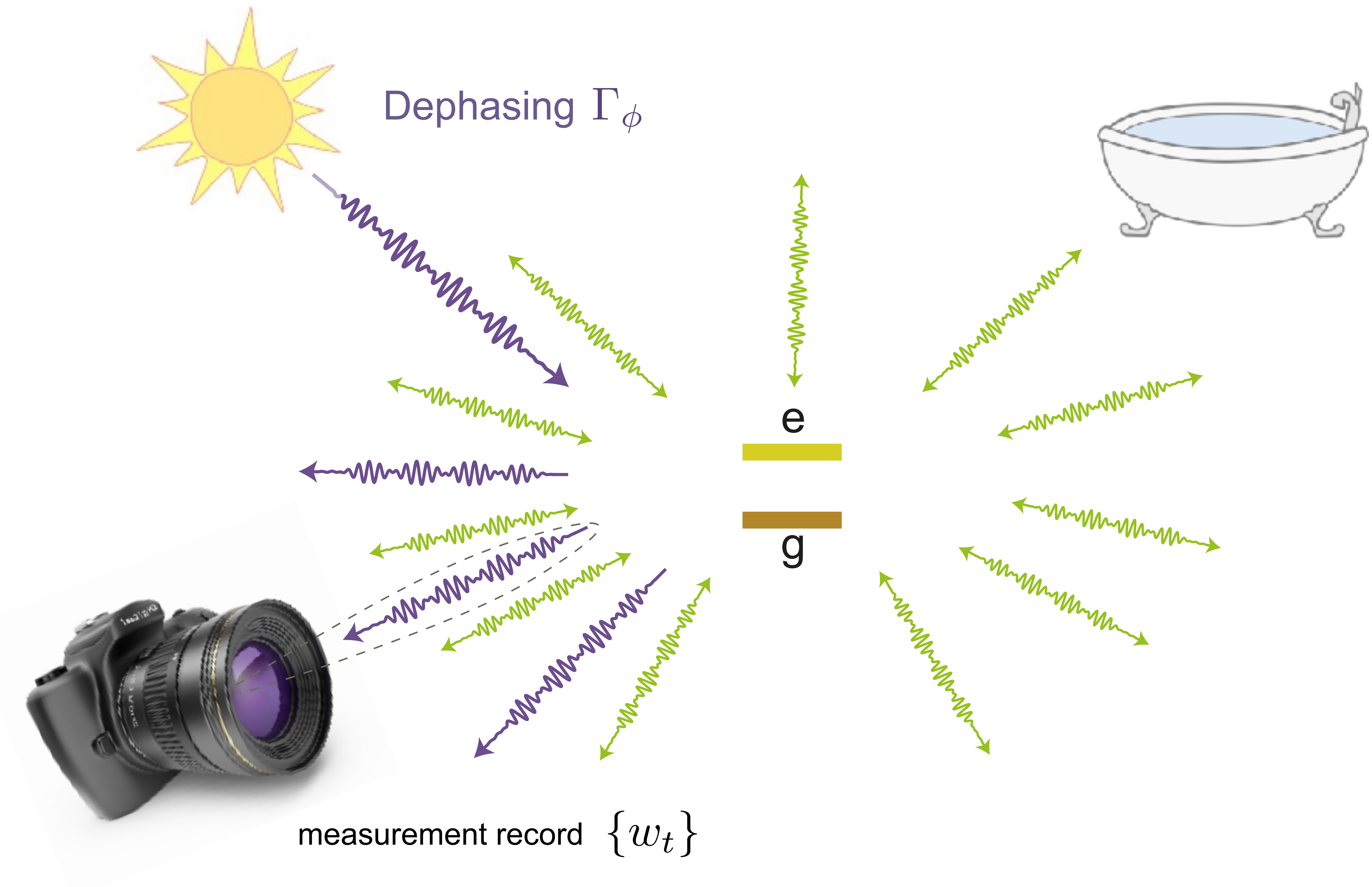
measurement A

Measurement of decoherence channels of a qubit



measurement A

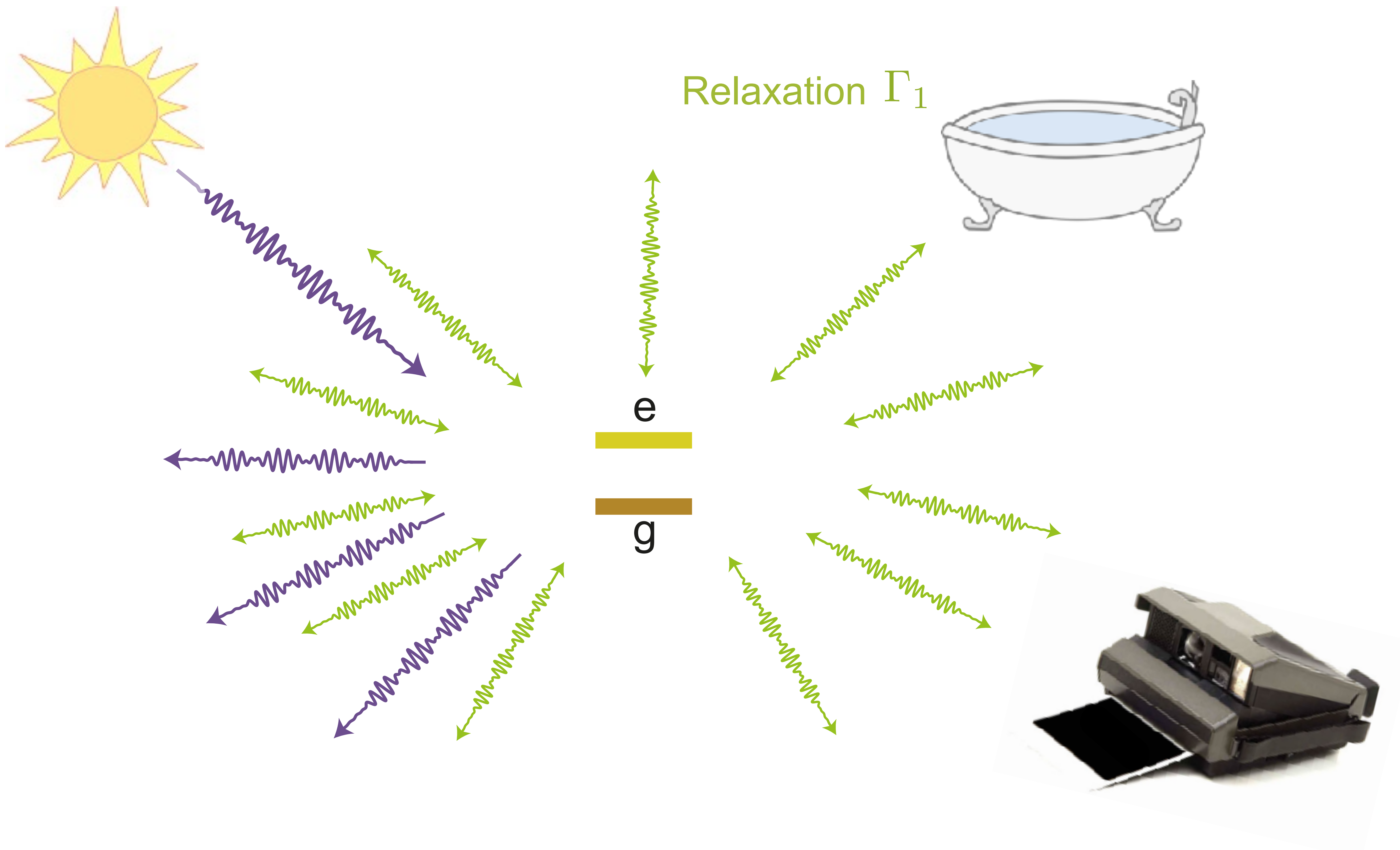
Measurement of decoherence channels of a qubit



Quantum trajectory = $\{\rho_t^{\mathrm{A}}\}$

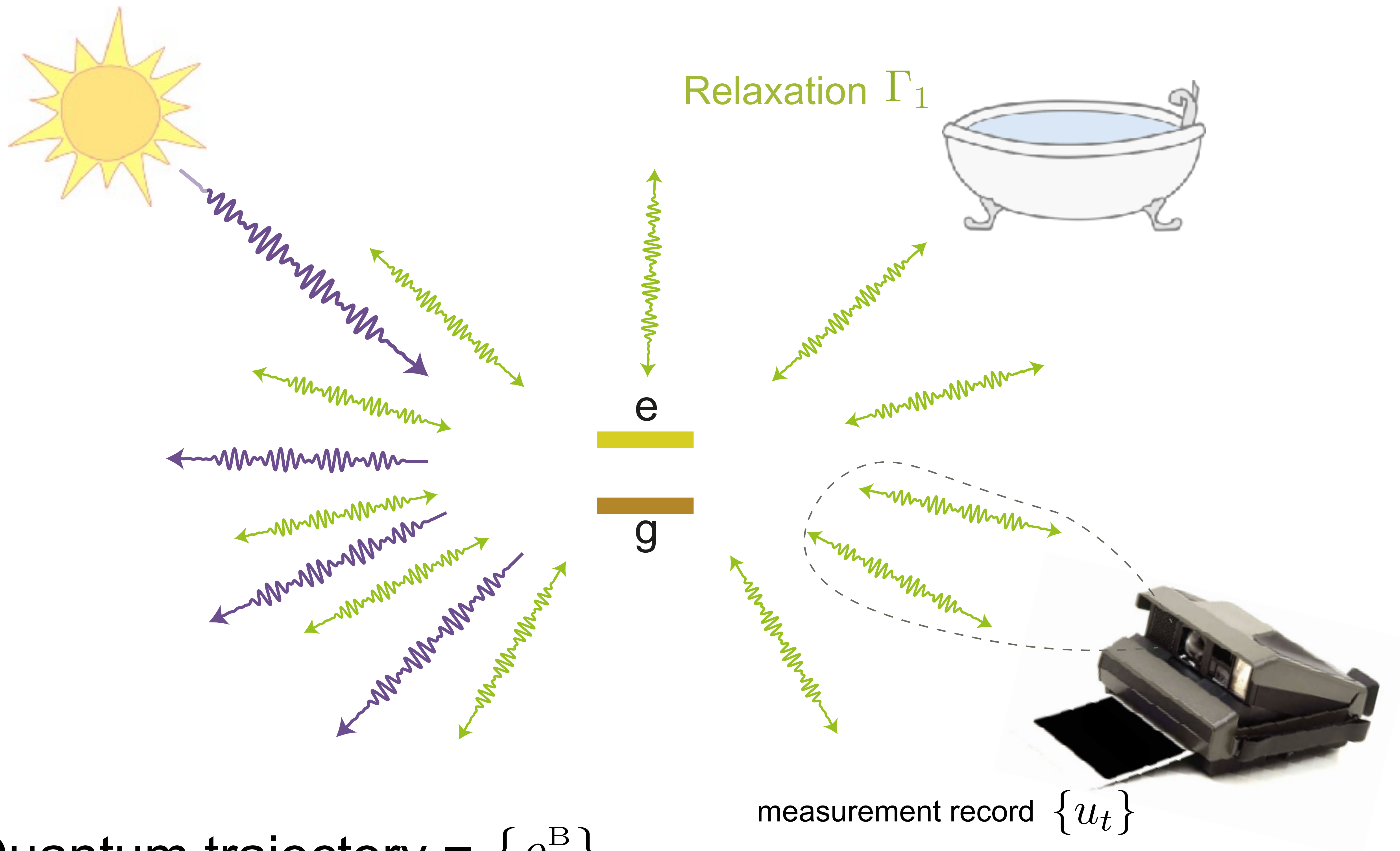
measurement A

Measurement of decoherence channels of a qubit



measurement B

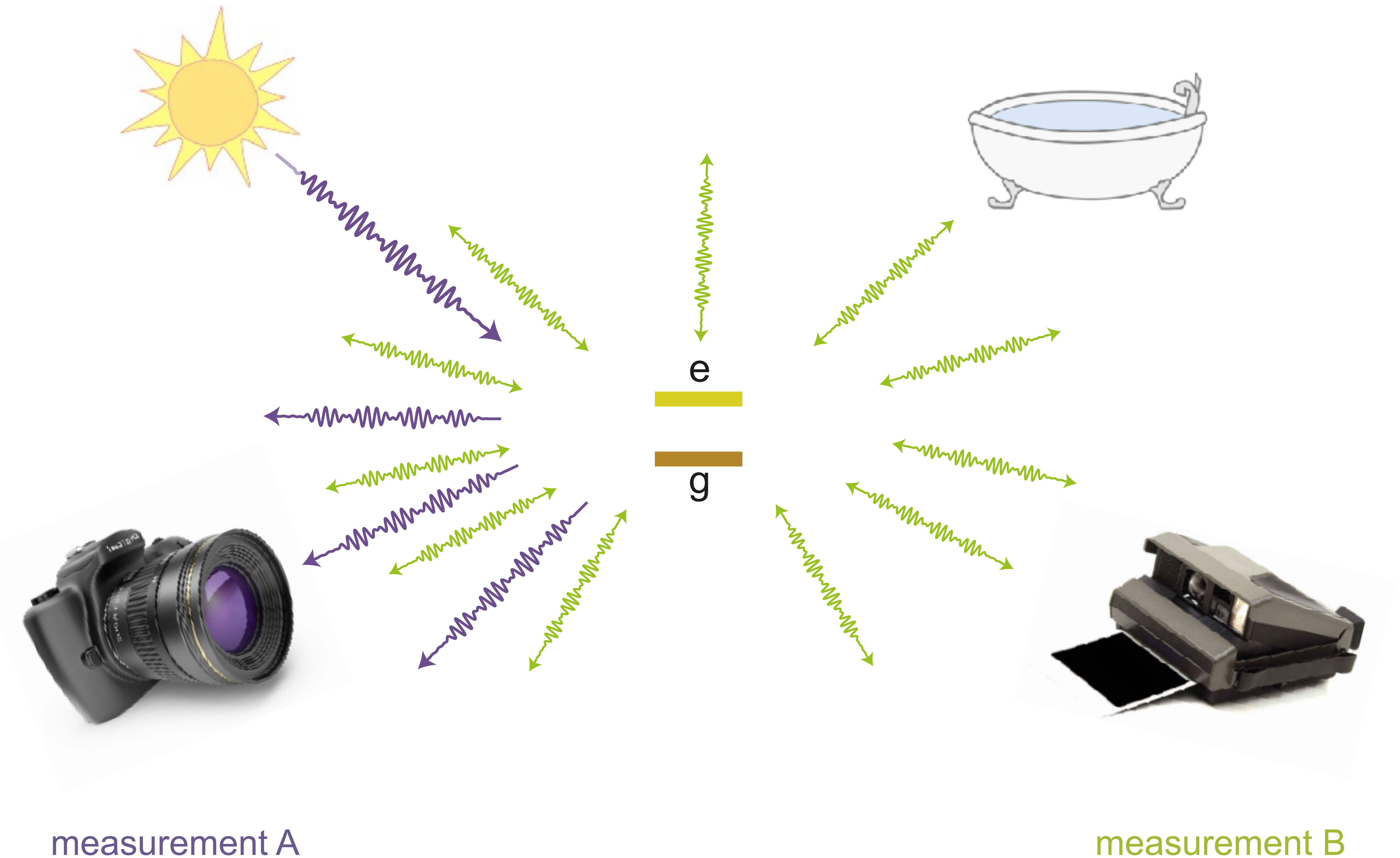
Measurement of decoherence channels of a qubit



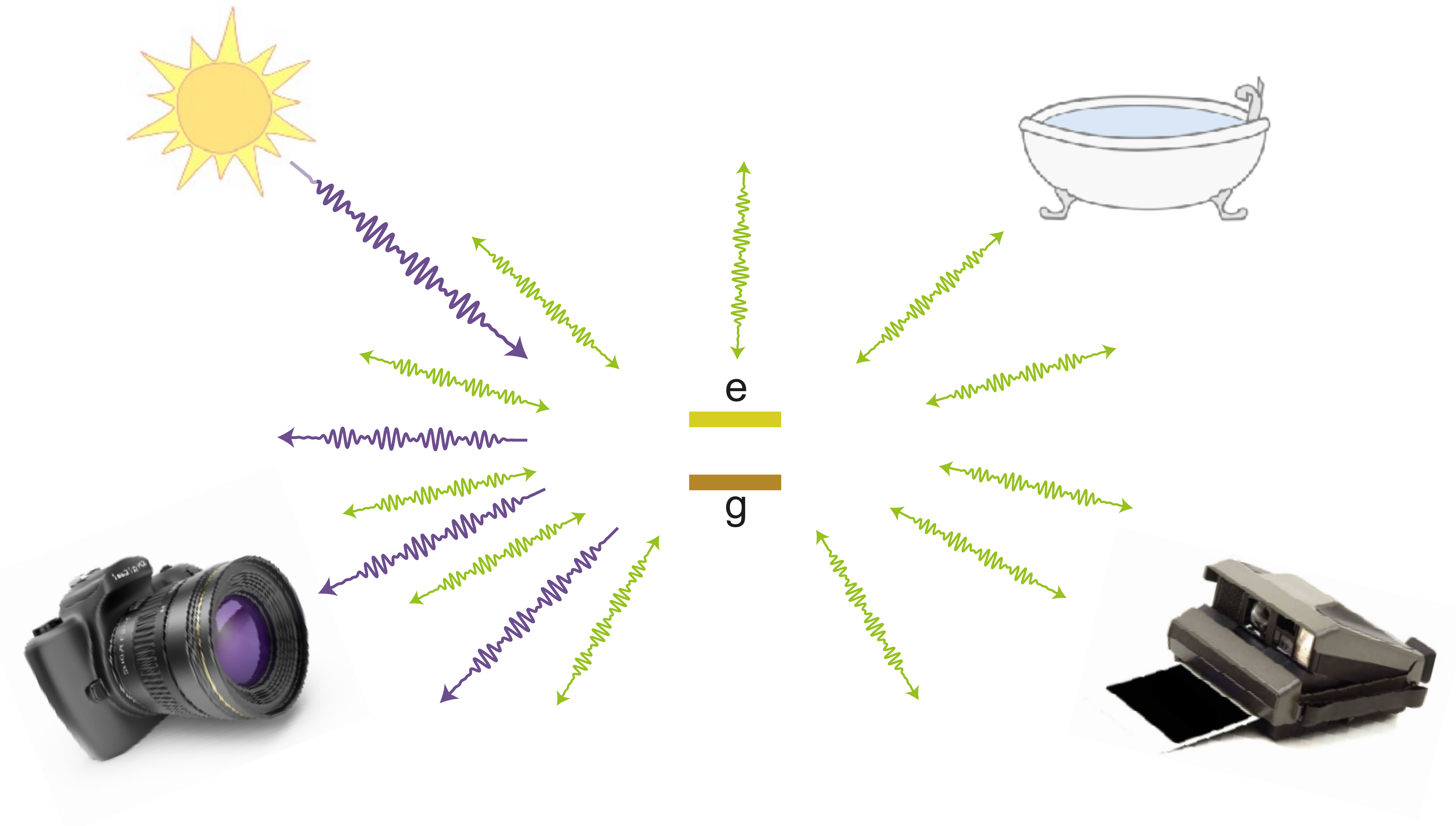
Quantum trajectory = $\{\rho_t^{\mathrm{B}}\}$

measurement B

Measurement of decoherence channels of a qubit

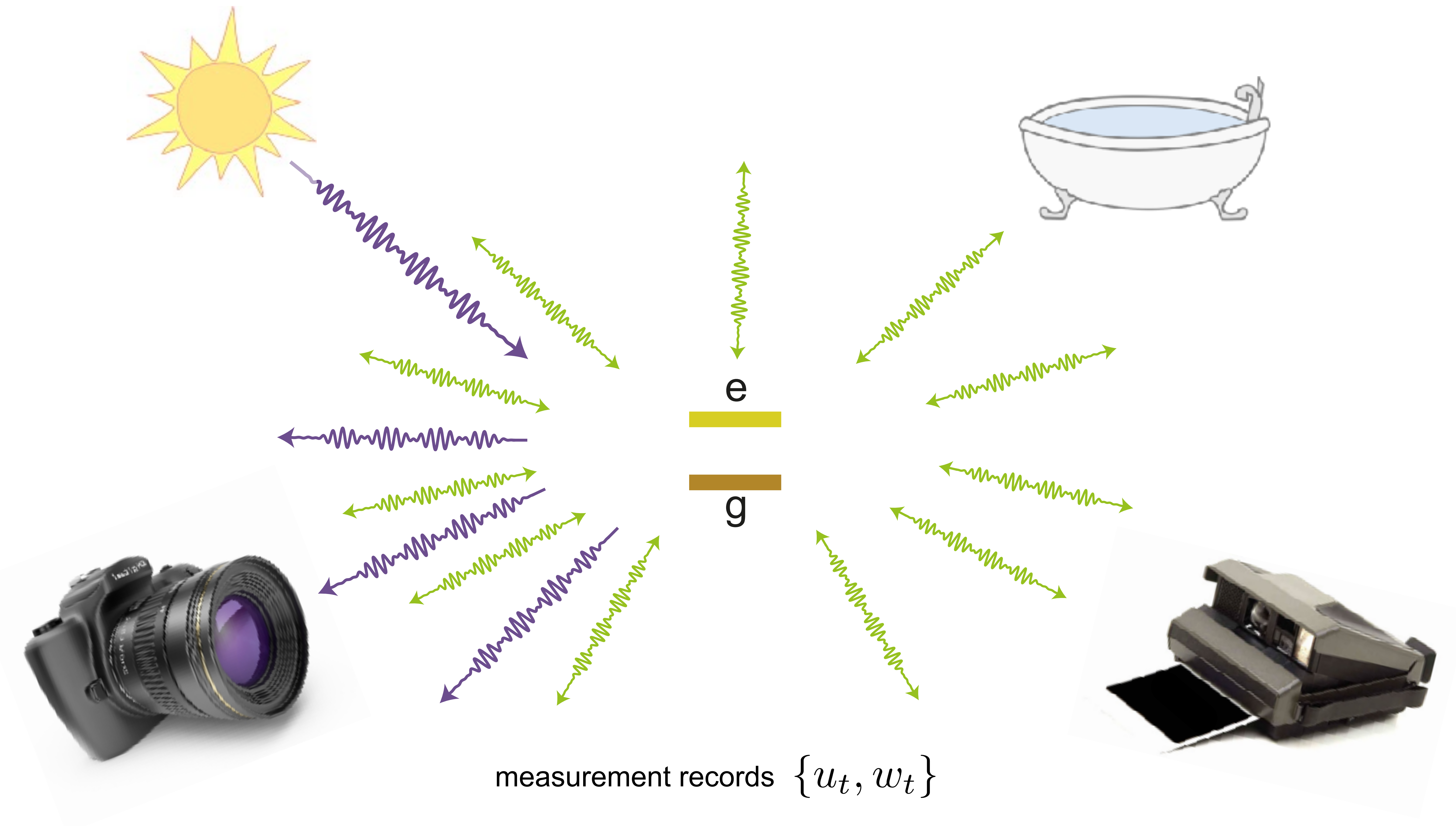


Measurement of decoherence channels of a qubit



measurement A ⚠ Incompatible measurements measurement B

Measurement of decoherence channels of a qubit



measurement records $\{u_t, w_t\}$

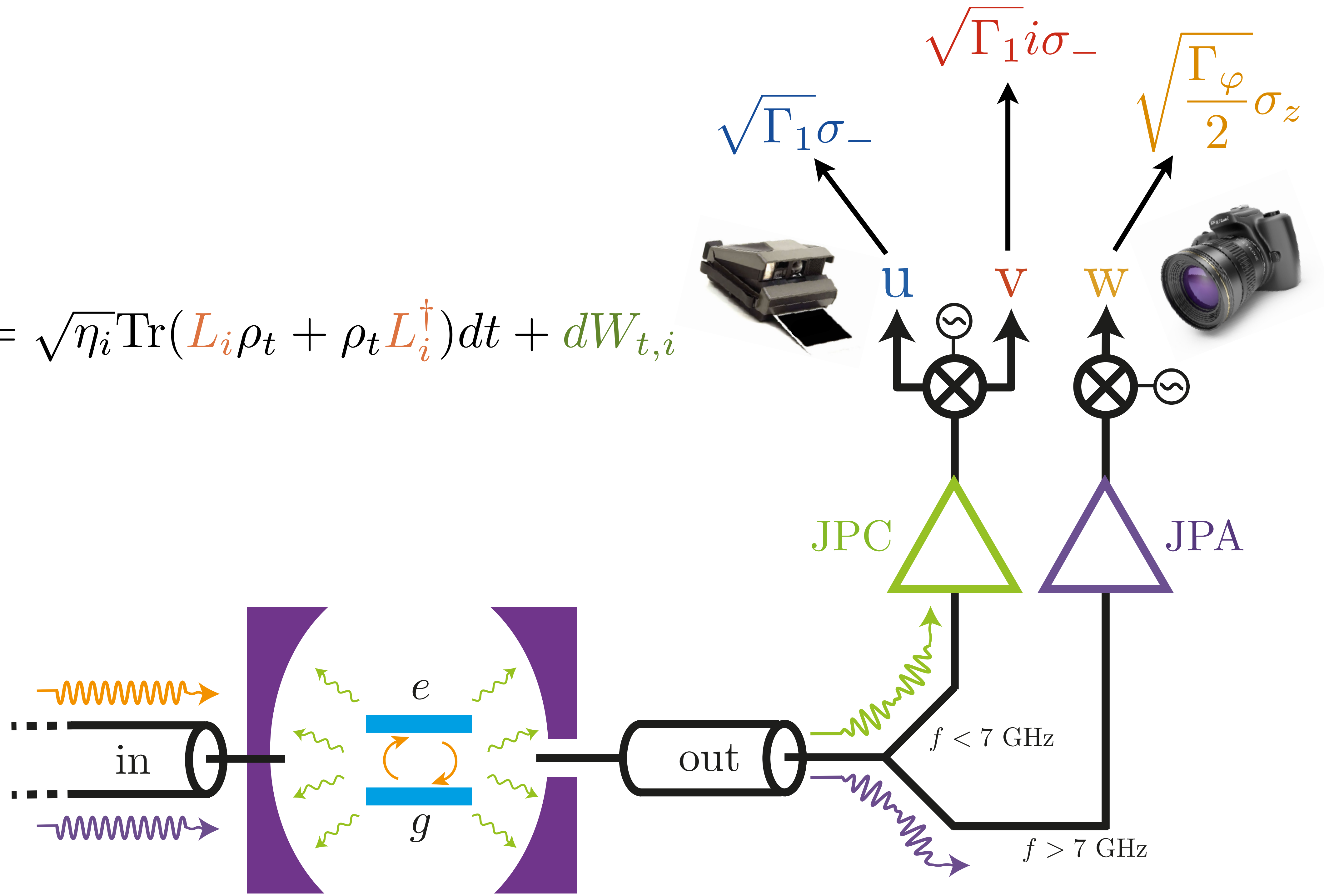
Quantum trajectory = $\{\rho_t^{\mathrm{A+B}}\}$

measurement A

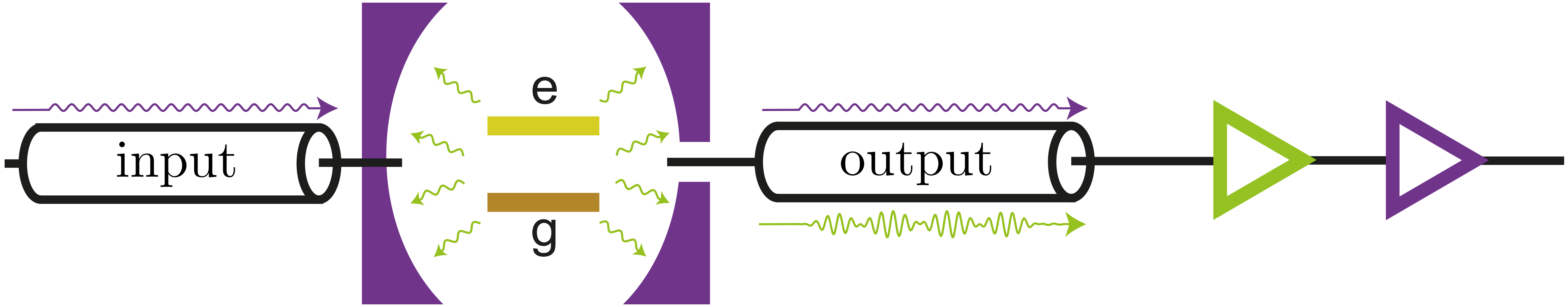
measurement B

Measurement setup

$$dy_t^i = \sqrt{\eta_i}\mathrm{Tr}(L_i\rho_t + \rho_t L_i^\dagger)dt + dW_{t,i}$$



Records of simultaneous X, Y and Z



full measurement records

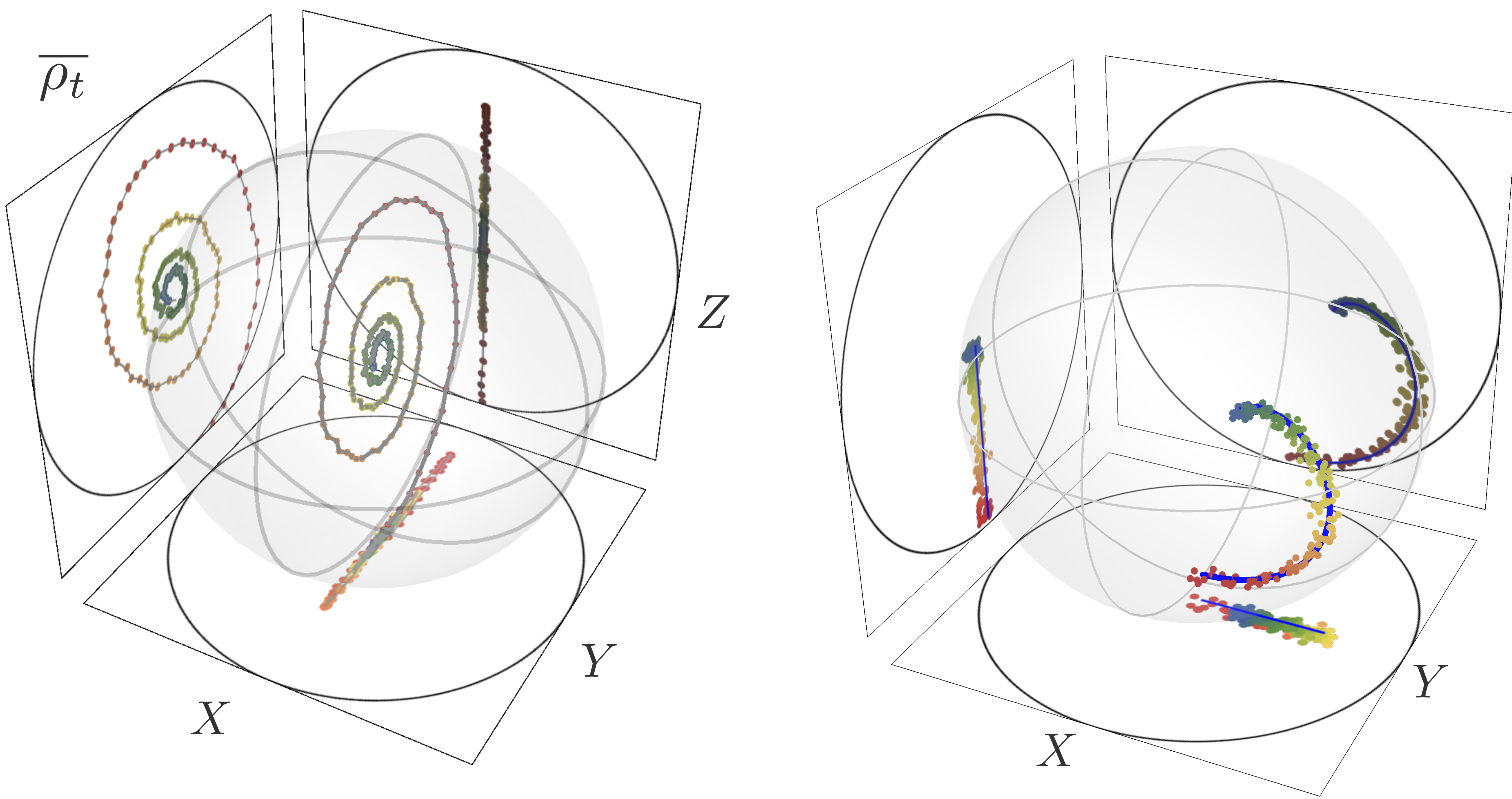
$$du_t = \sqrt{\eta_{\text{fluo}}\Gamma_1/2}\,\langle\sigma_X\rangle_{\rho_t}\,dt + dW_{t,1}$$
$$dv_t = \sqrt{\eta_{\text{fluo}}\Gamma_1/2}\,\langle\sigma_Y\rangle_{\rho_t}\,dt + dW_{t,2}$$
$$dw_t = \sqrt{2\eta_{\text{disp}}\Gamma_d}\,\langle\sigma_Z\rangle_{\rho_t}\,dt + dW_{t,3}$$

average outcome

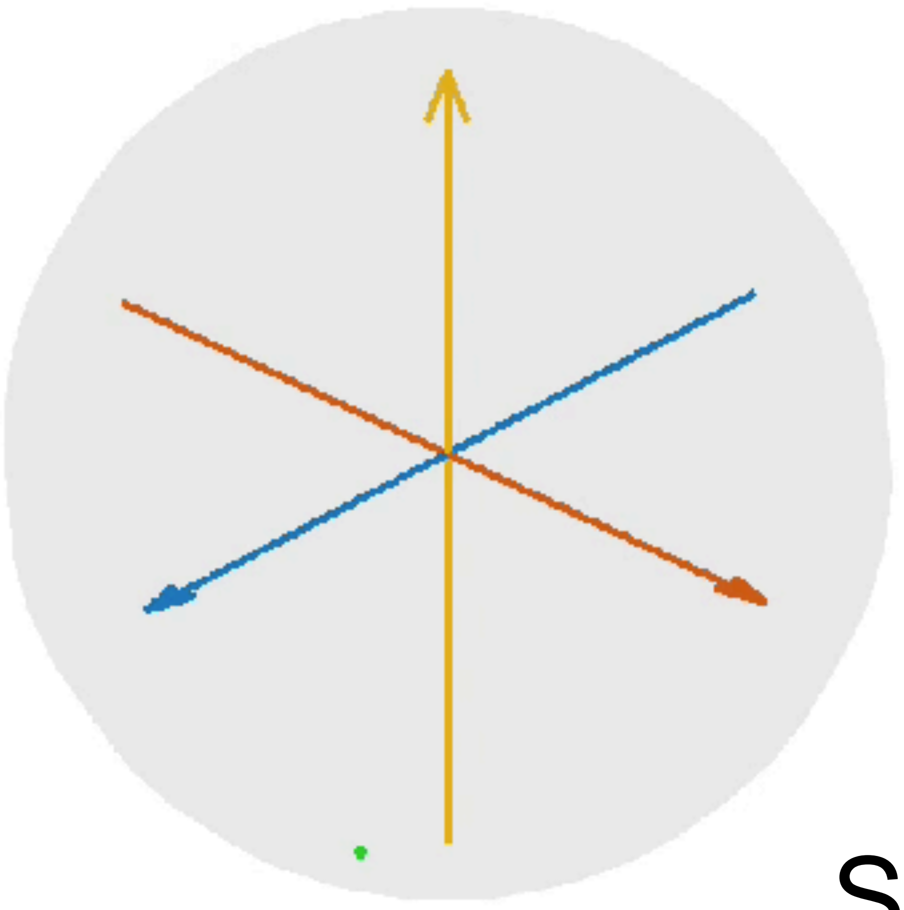
noise (Wiener)

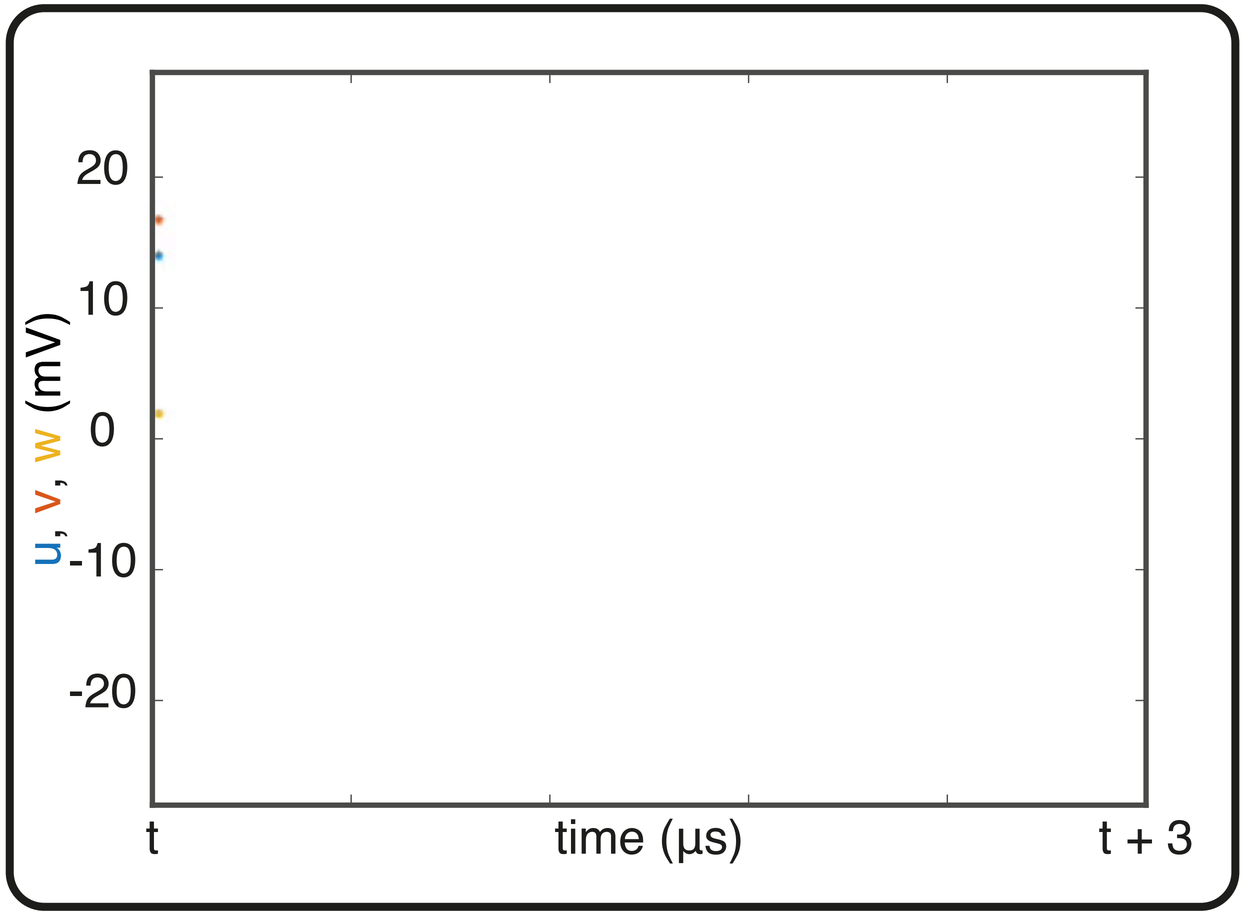
raw averaging directly gives Bloch vector

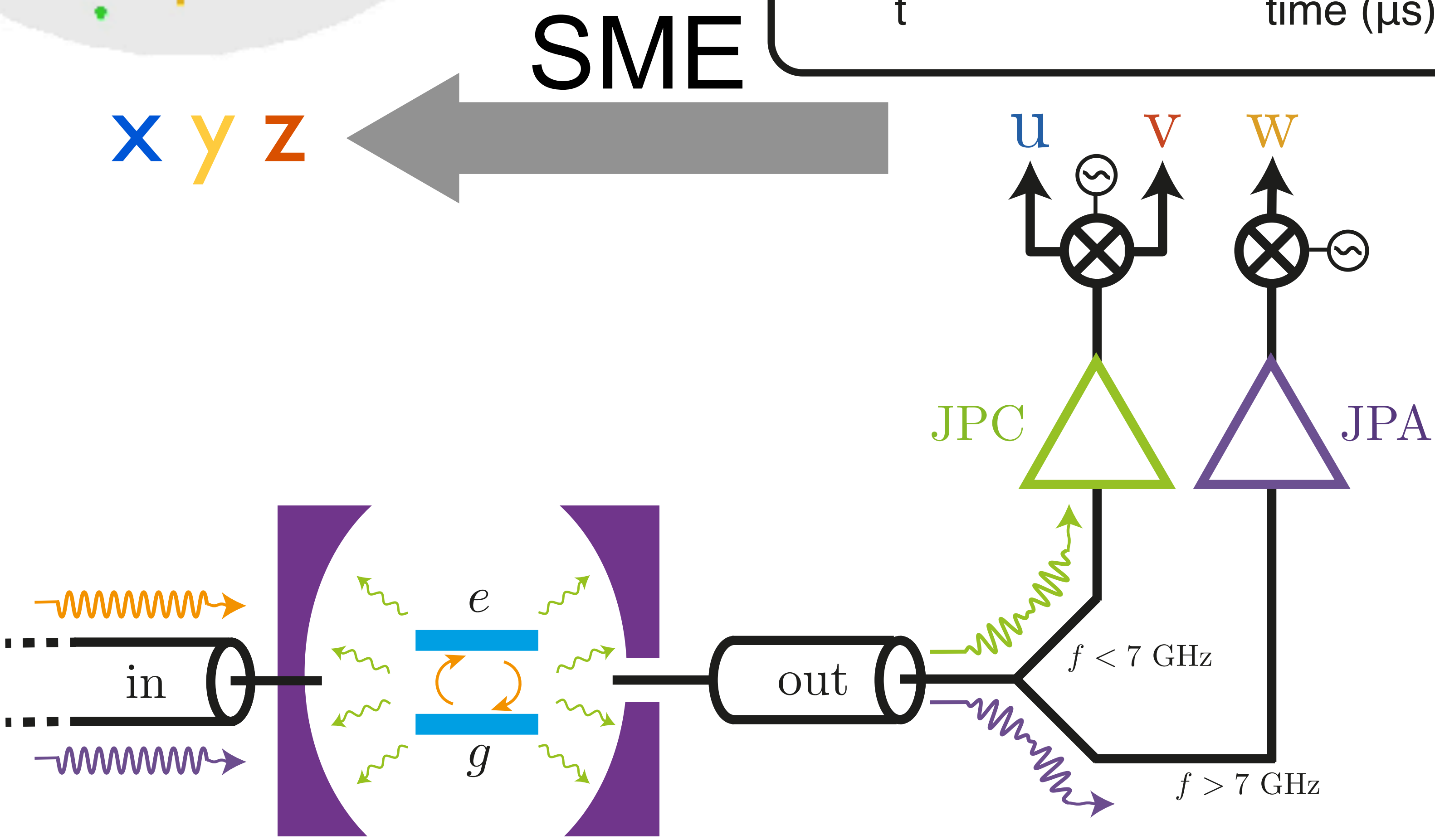
Average trajectory



One quantum trajectory







η_{fluo} = 14 %

η_{disp} = 34 %

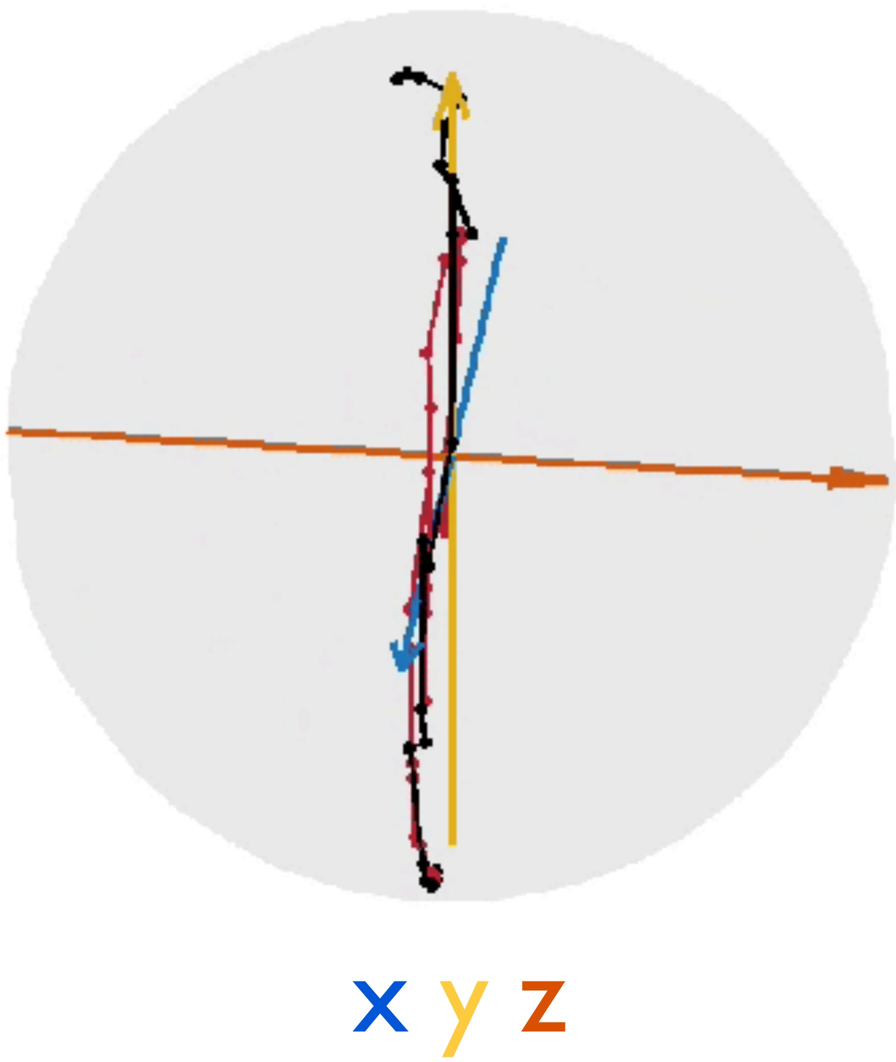
T_1 = 15.0 µs

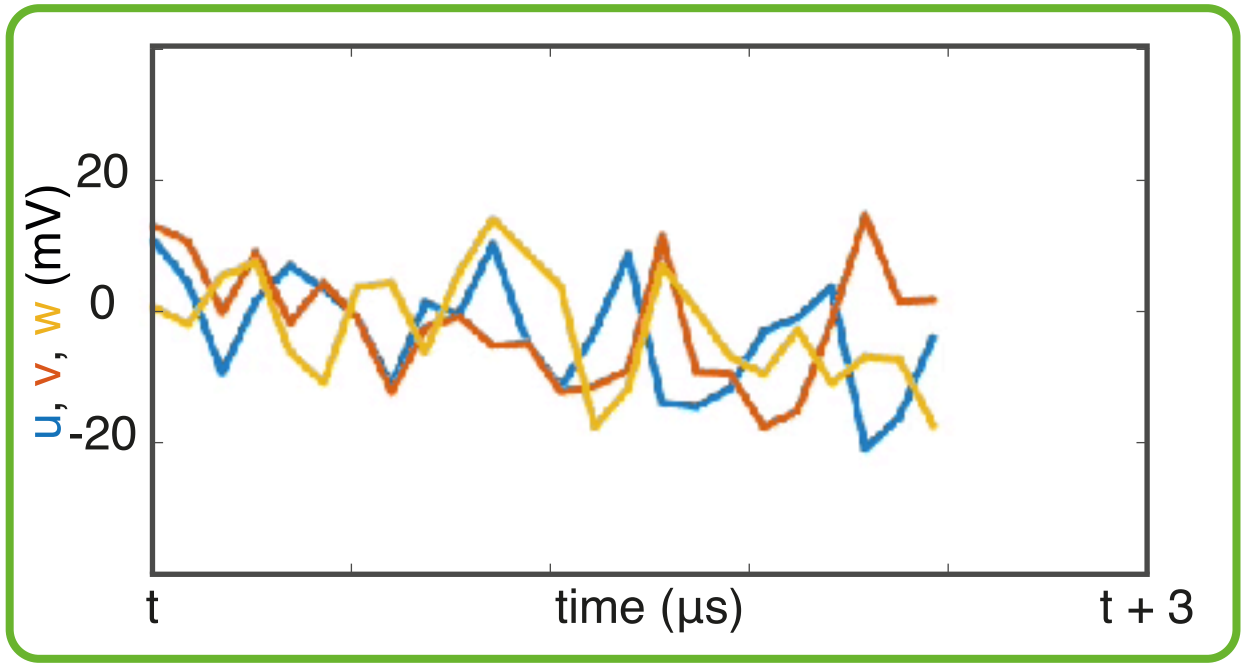
T_2 = 11.2 µs

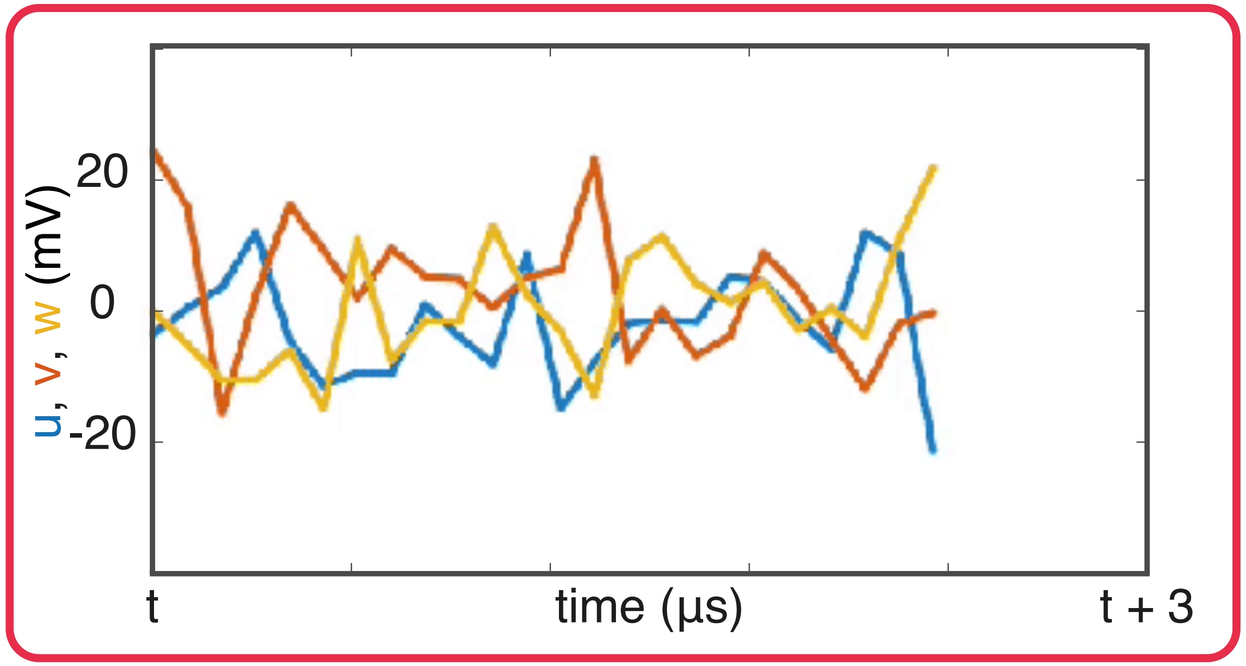
T_d = 0.9 µs

T_R = 5.2 µs

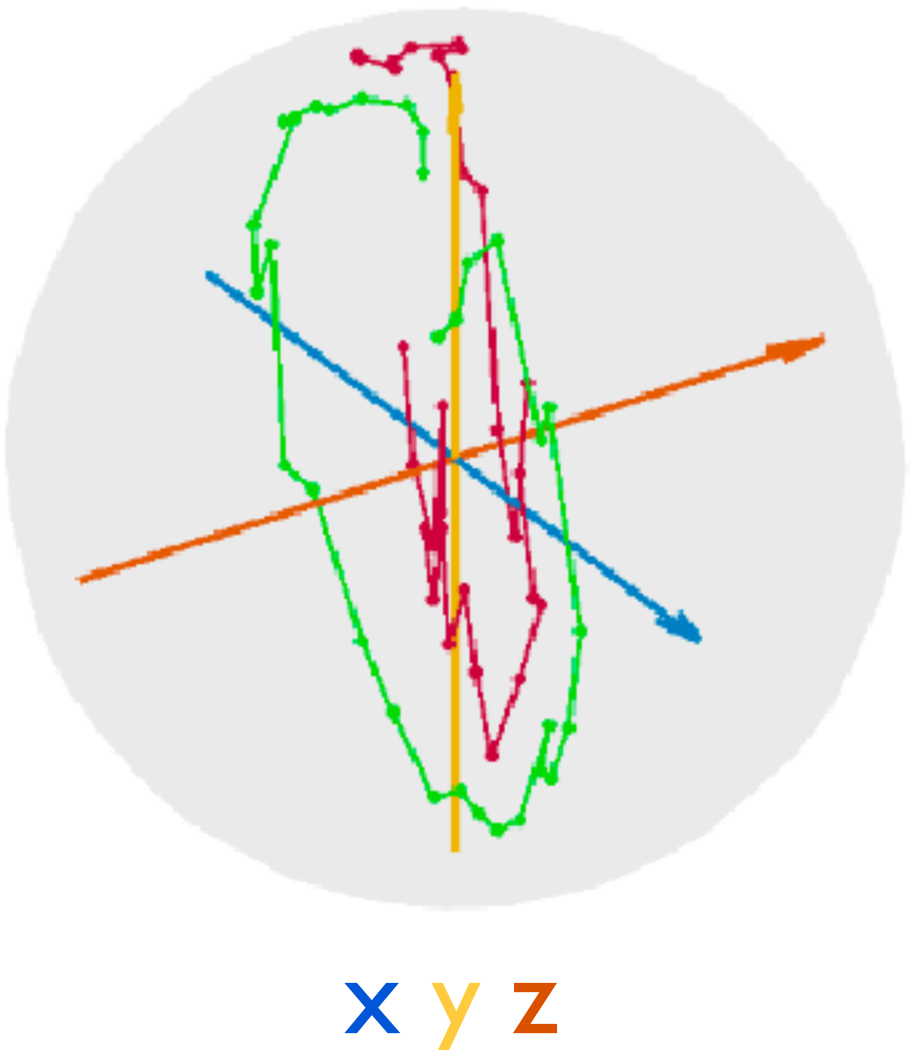
Two quantum trajectories

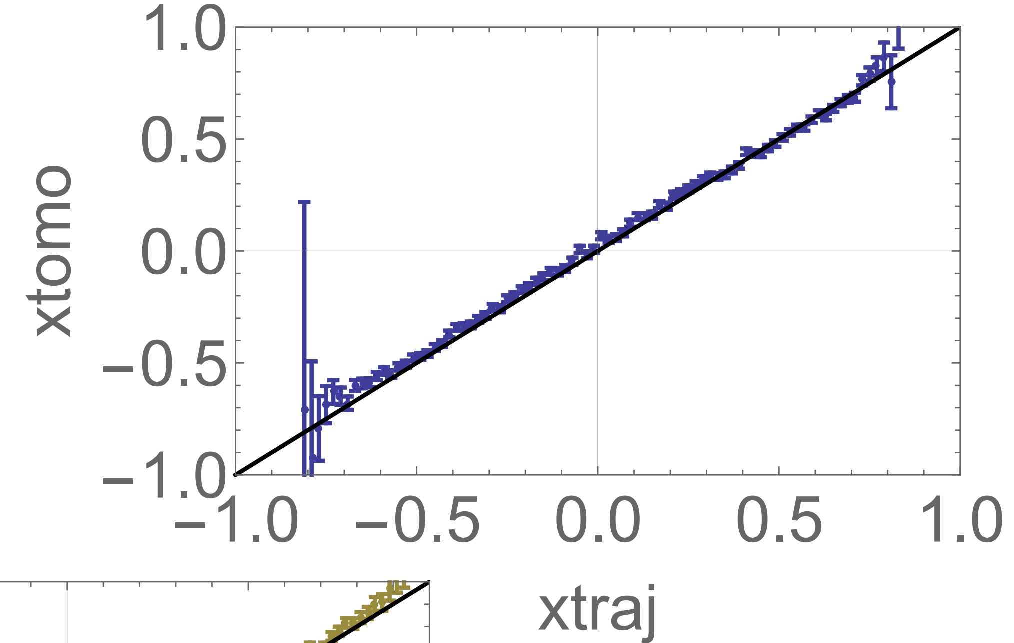


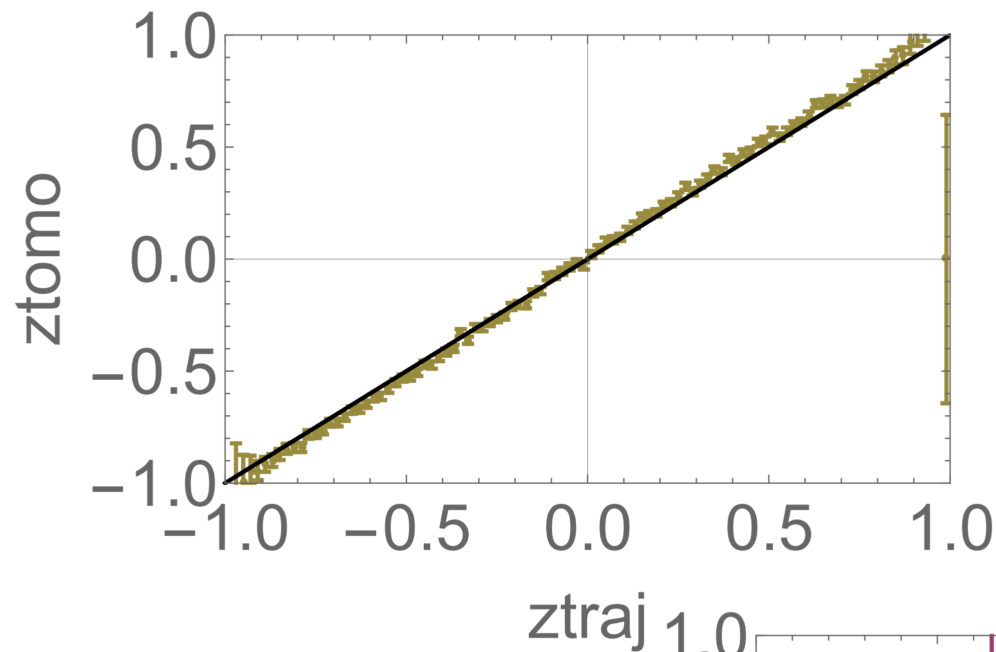




Control trajectories by tomography

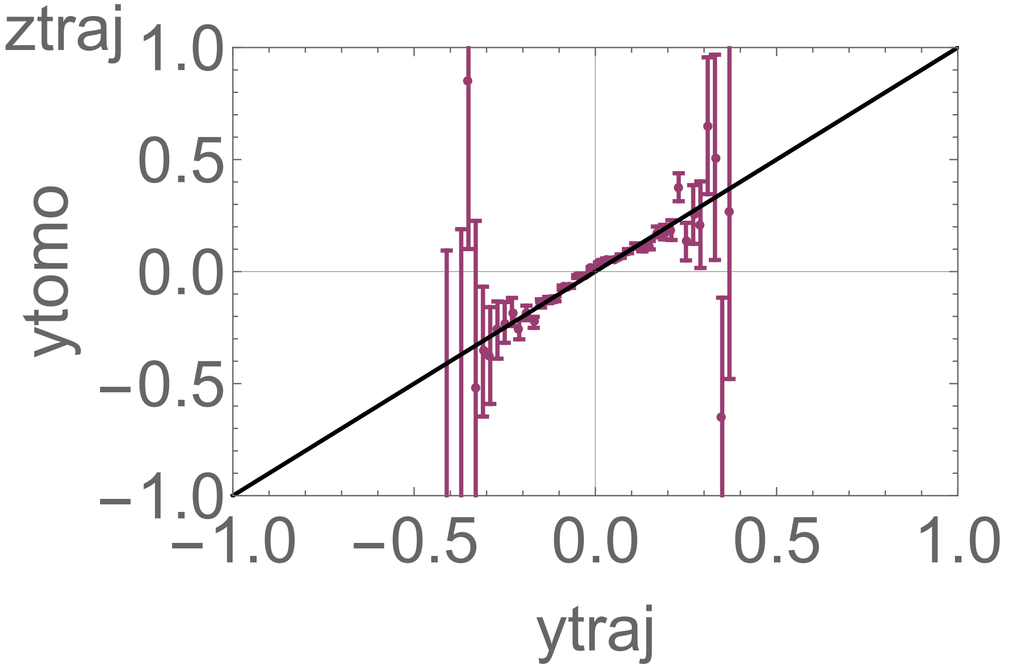






η_{fluo} = 14 %

η_{disp} = 34 %



T_1 = 15.0 μs

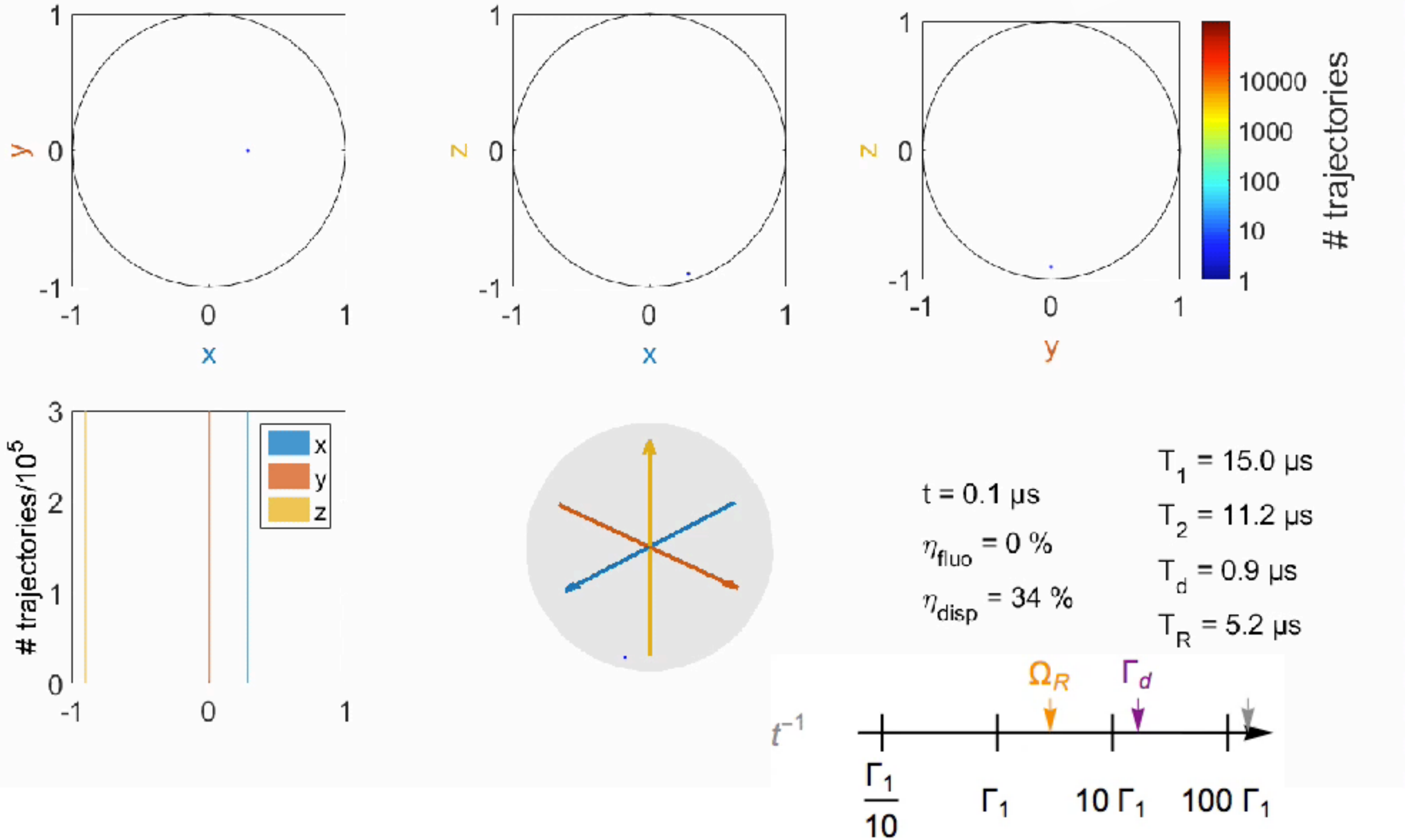
T_2 = 11.2 μs

T_d = 0.9 μs

T_R = 5.2 μs

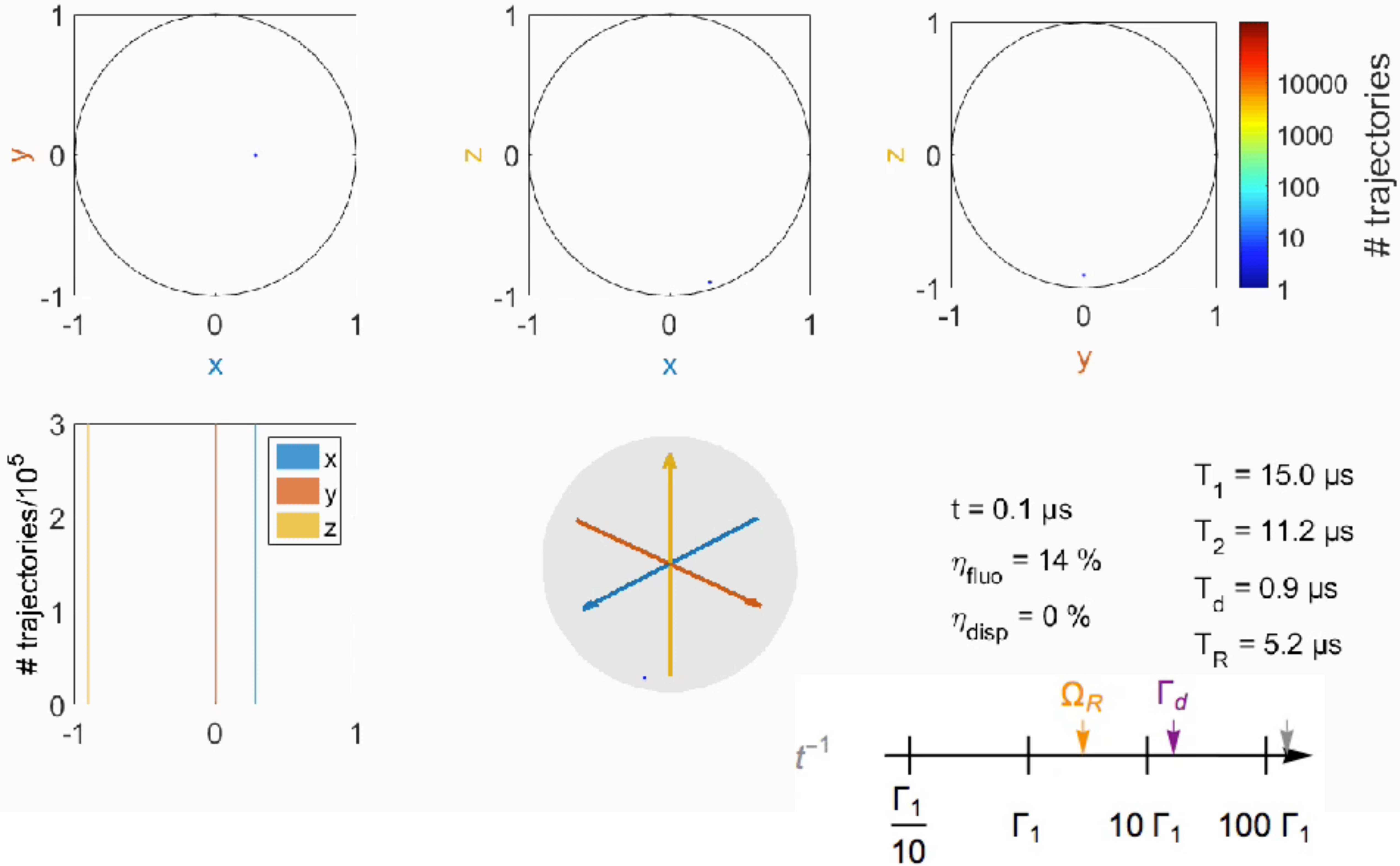
statistics in the Zeno regime

σ_Z measurement only



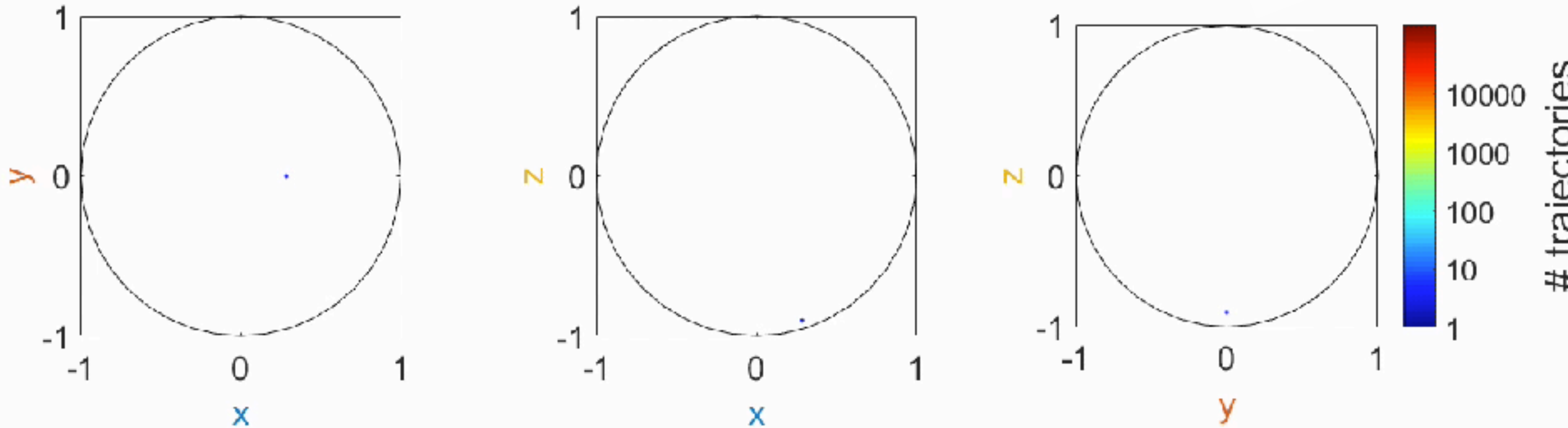
statistics in the Zeno regime

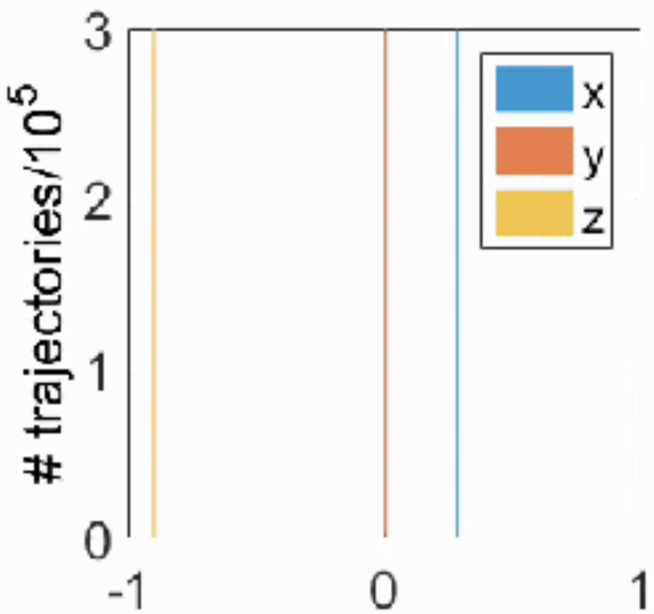
σ_- measurement only

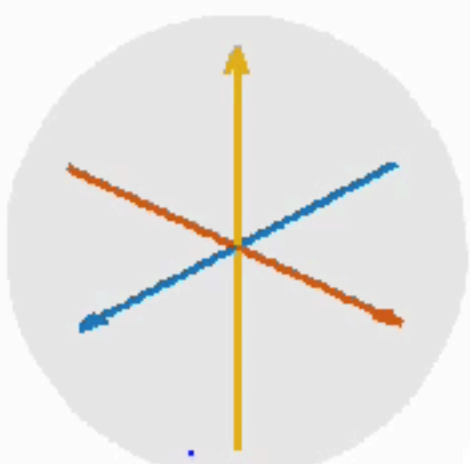


statistics in the Zeno regime

σ_- and σ_Z measurements at the same time







t = 0.1 μs

η_{fluo} = 14 %

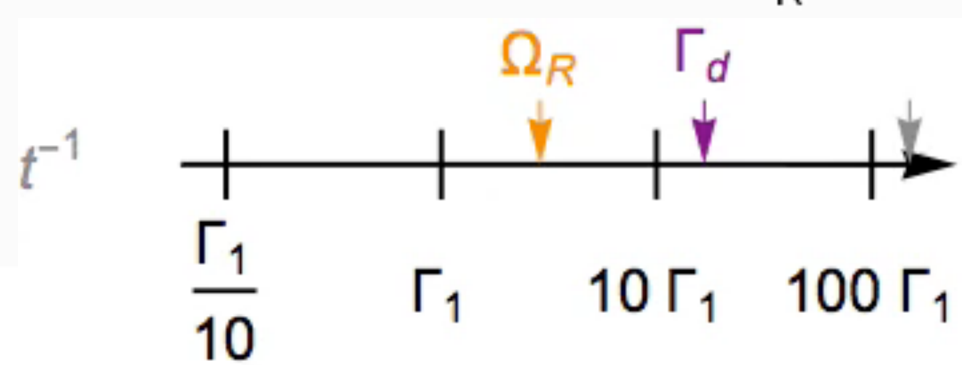
η_{disp} = 34 %

T_1 = 15.0 μs

T_2 = 11.2 μs

T_d = 0.9 μs

T_R = 5.2 μs

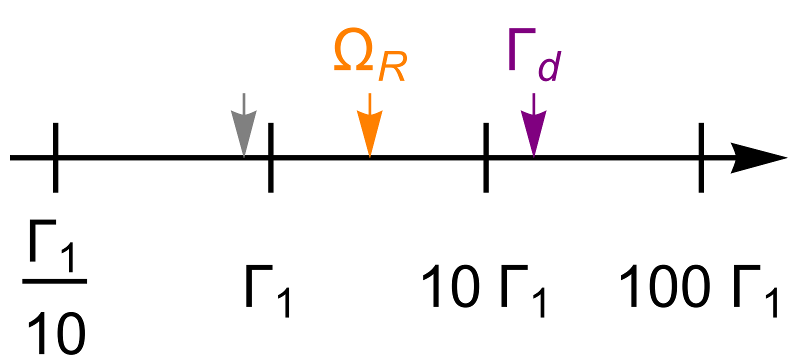


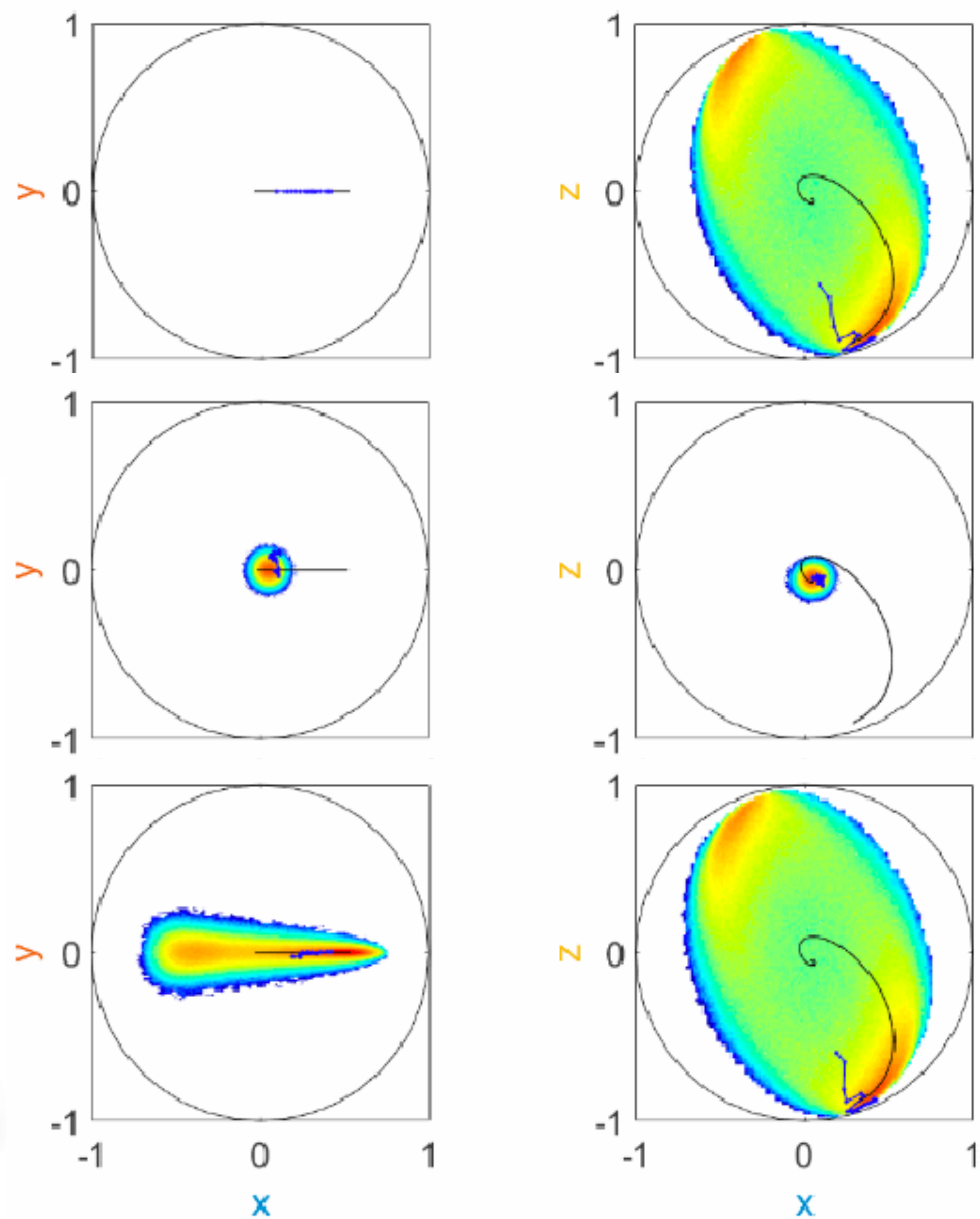
statistics in the Zeno regime

σ_Z measurement only

σ_- measurement only

σ_- and σ_Z measurements



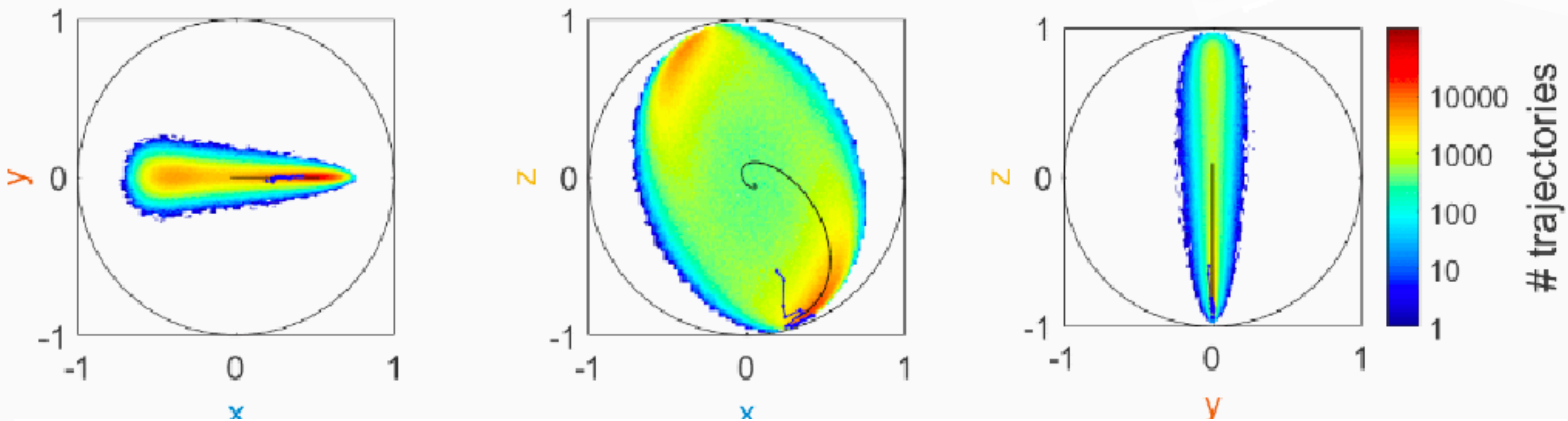


[Ficheux *et al.*, Nat. Comm. in press]

statistics in the Zeno regime

σ_- and σ_Z measurements at the same time

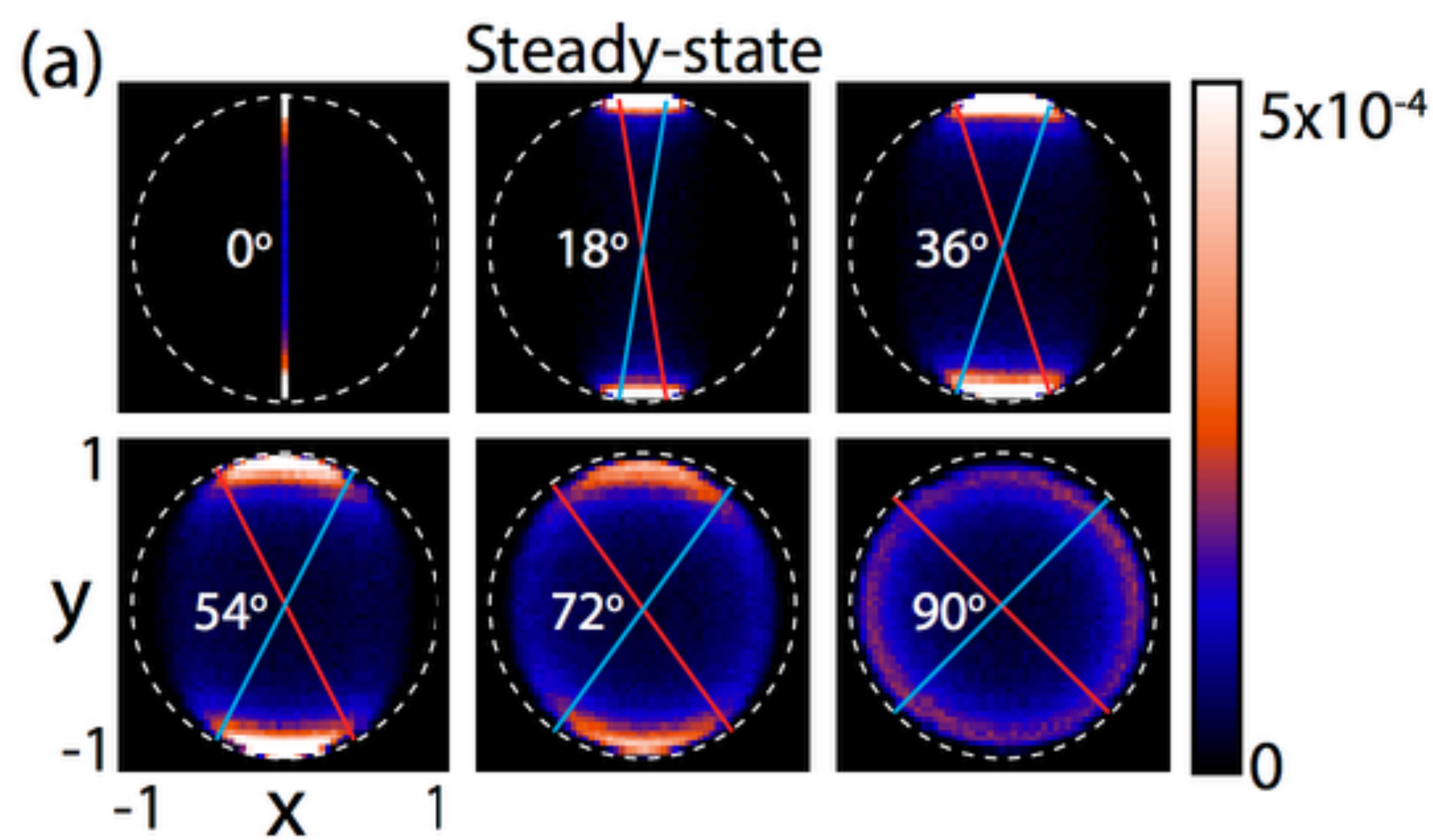
[Ficheux *et al.*, Nat. Comm. in press]



differs from the case of

σ_x and σ_y

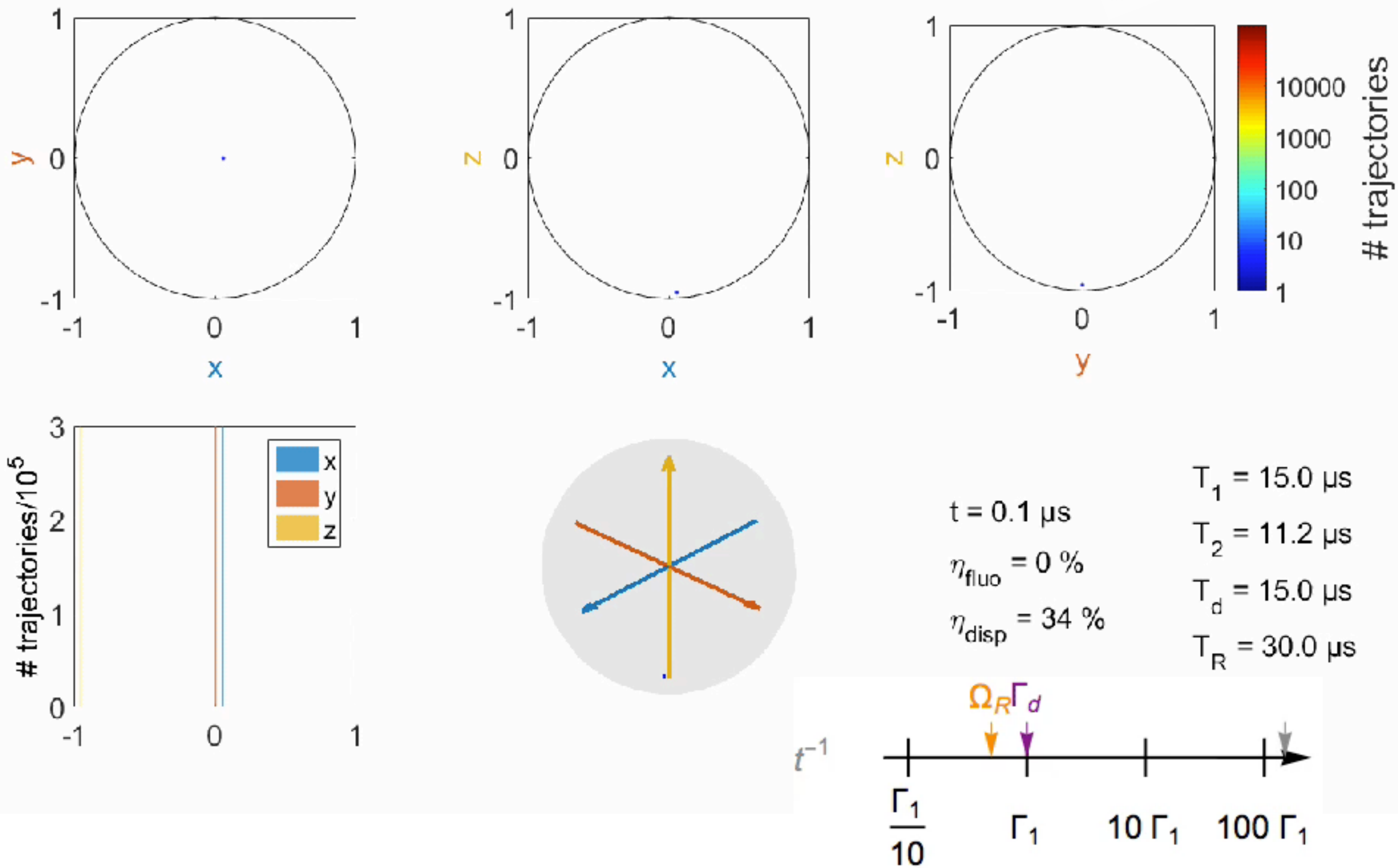
measurement



[Hacohen-Gourgy *et al.*, Nature 2016]

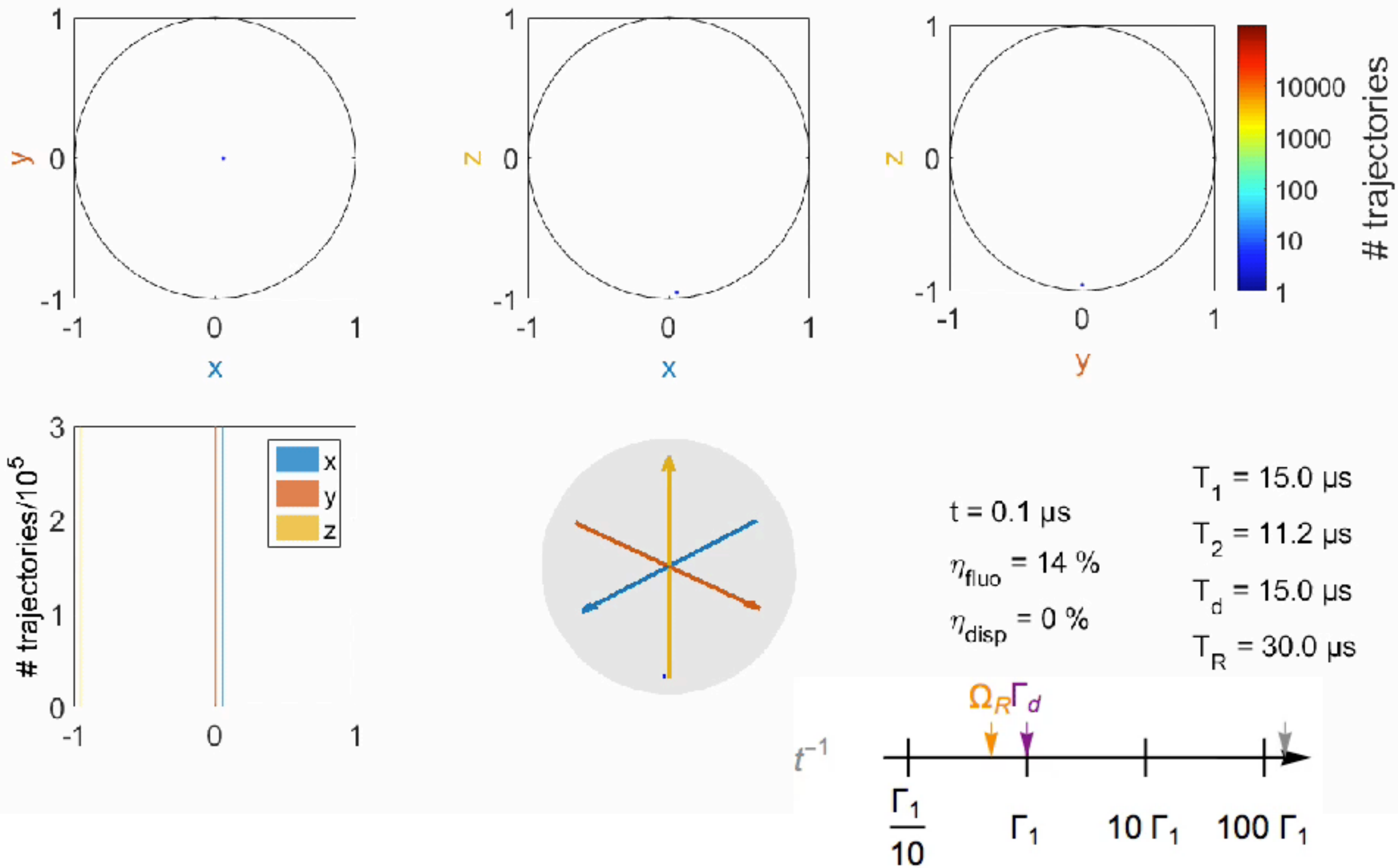
statistics with weak measurement and slow rotation

σ_Z measurement only



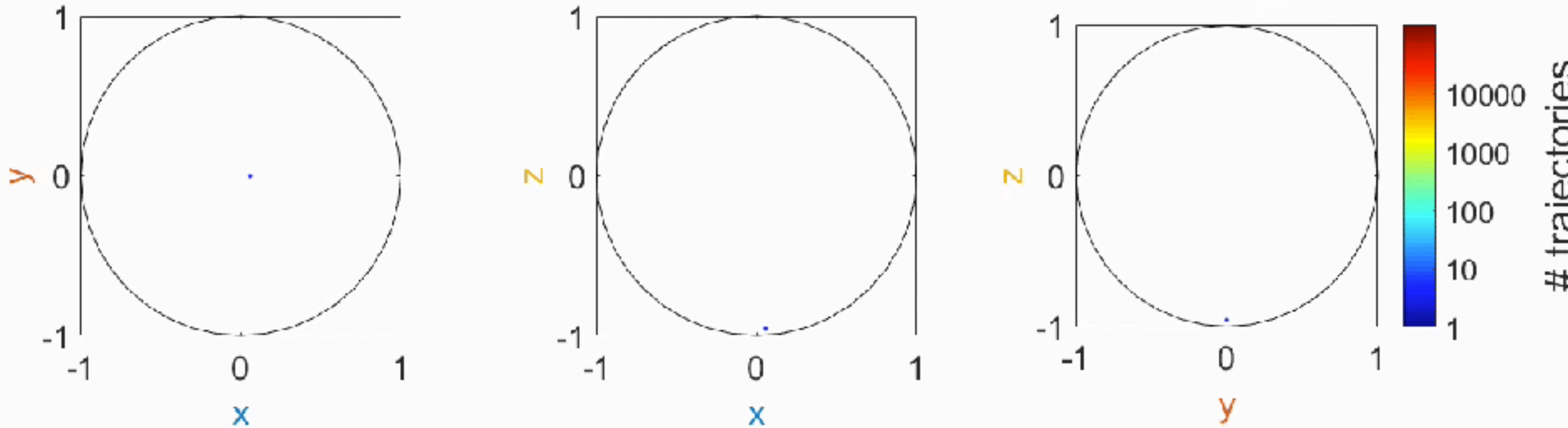
statistics with weak measurement and slow rotation

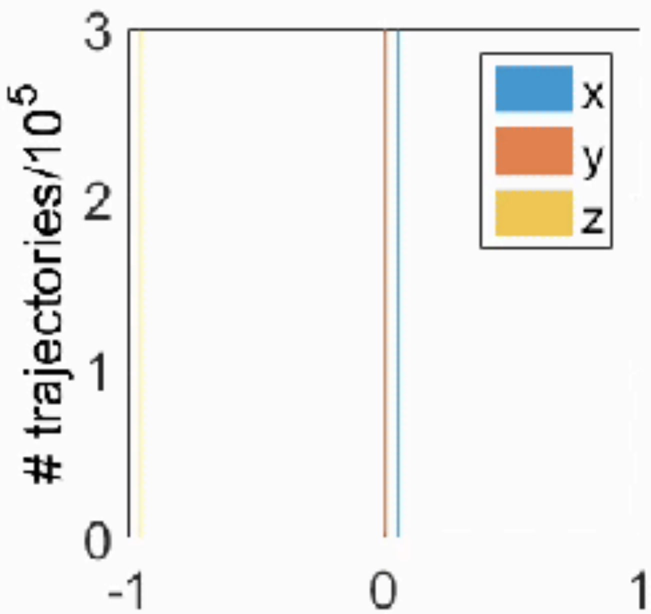
σ_- measurement only

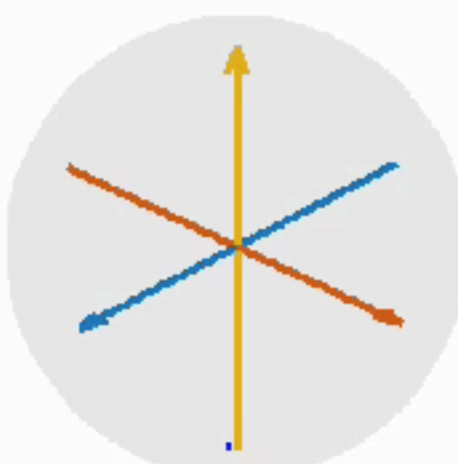


statistics with weak measurement and slow rotation

σ_- and σ_Z measurements at the same time







t = 0.1 µs

η_{fluo} = 14 %

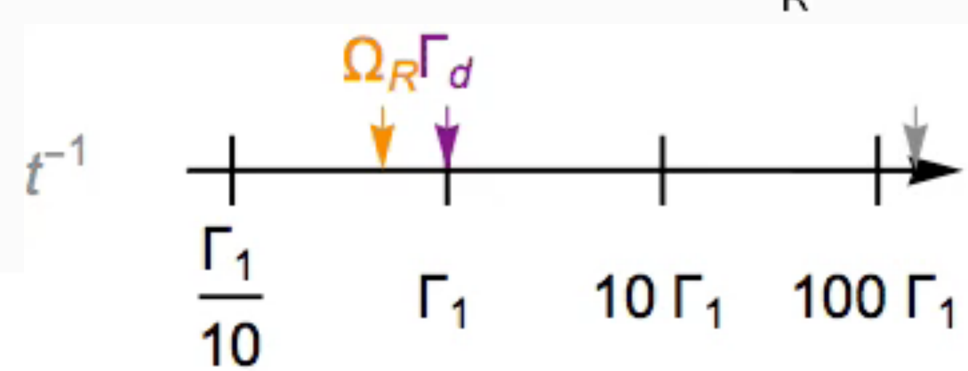
η_{disp} = 34 %

T_1 = 15.0 µs

T_2 = 11.2 µs

T_d = 15.0 µs

T_R = 30.0 µs

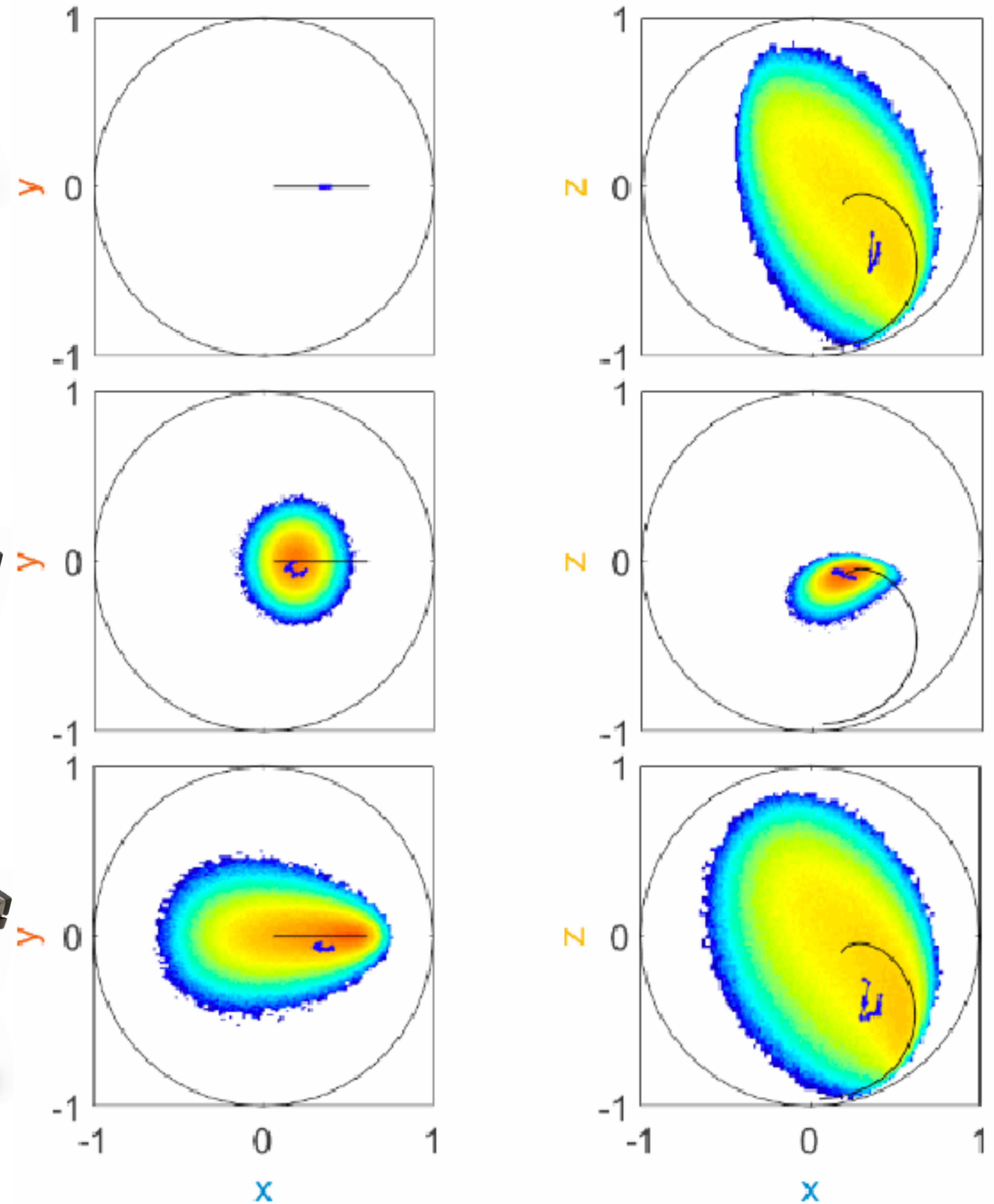


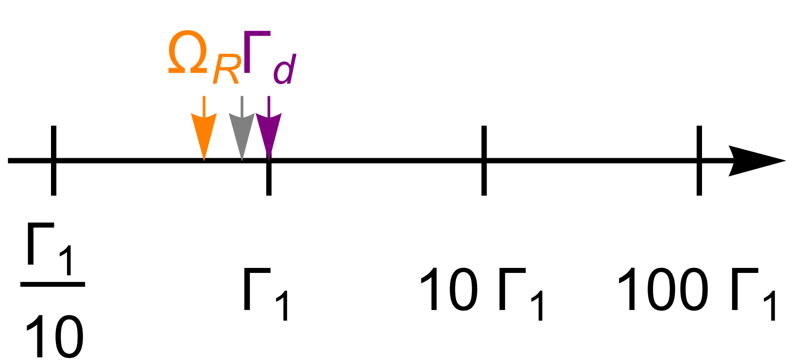
statistics with weak measurement and slow rotation

σ_Z measurement only

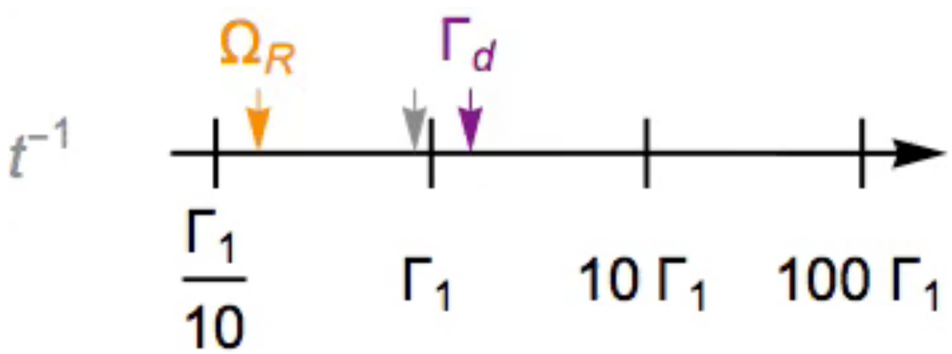
σ_- measurement only

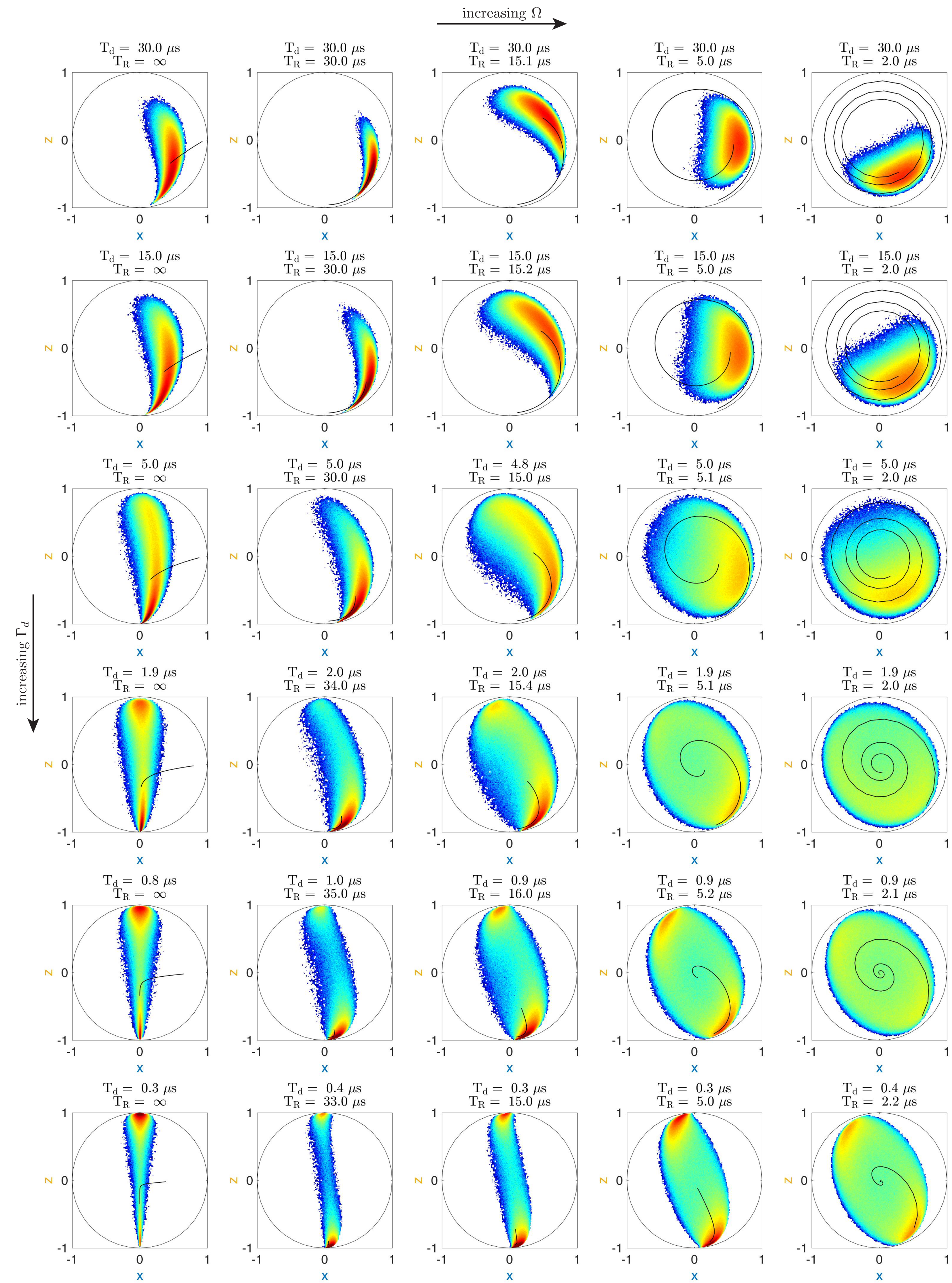
σ_- and σ_Z measurements





Any configuration

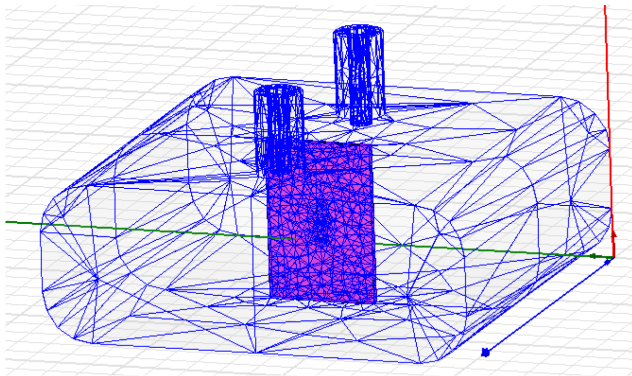




http://www.physinfo.fr

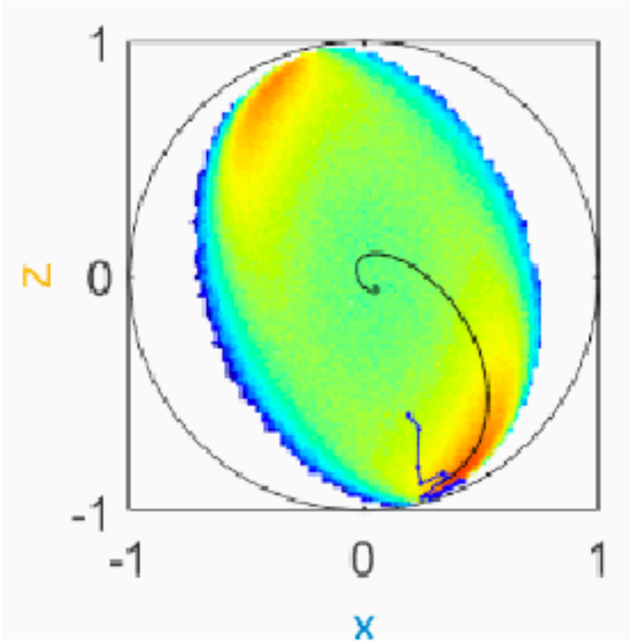
[Ficheux *et al.*, Nat. Comm. in press]

Quantum trajectories and feedback in circuit-QED

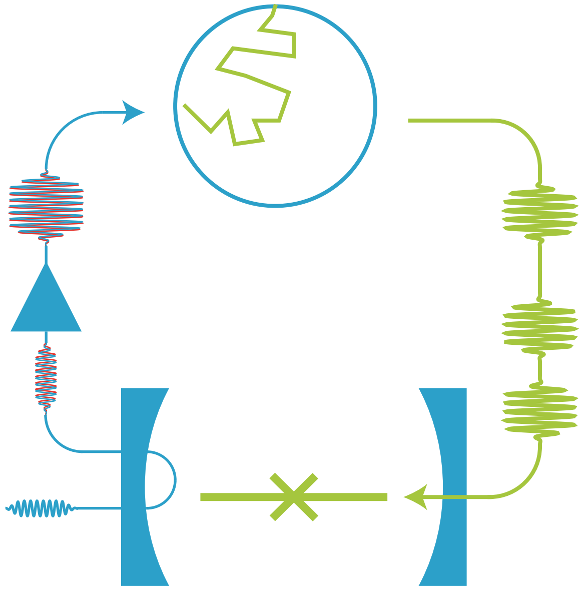


Introduction to circuit-QED

Quantum trajectories in circuit-QED



- dispersive measurement
- fluorescence measurement
- both simultaneously
- **generating entanglement**



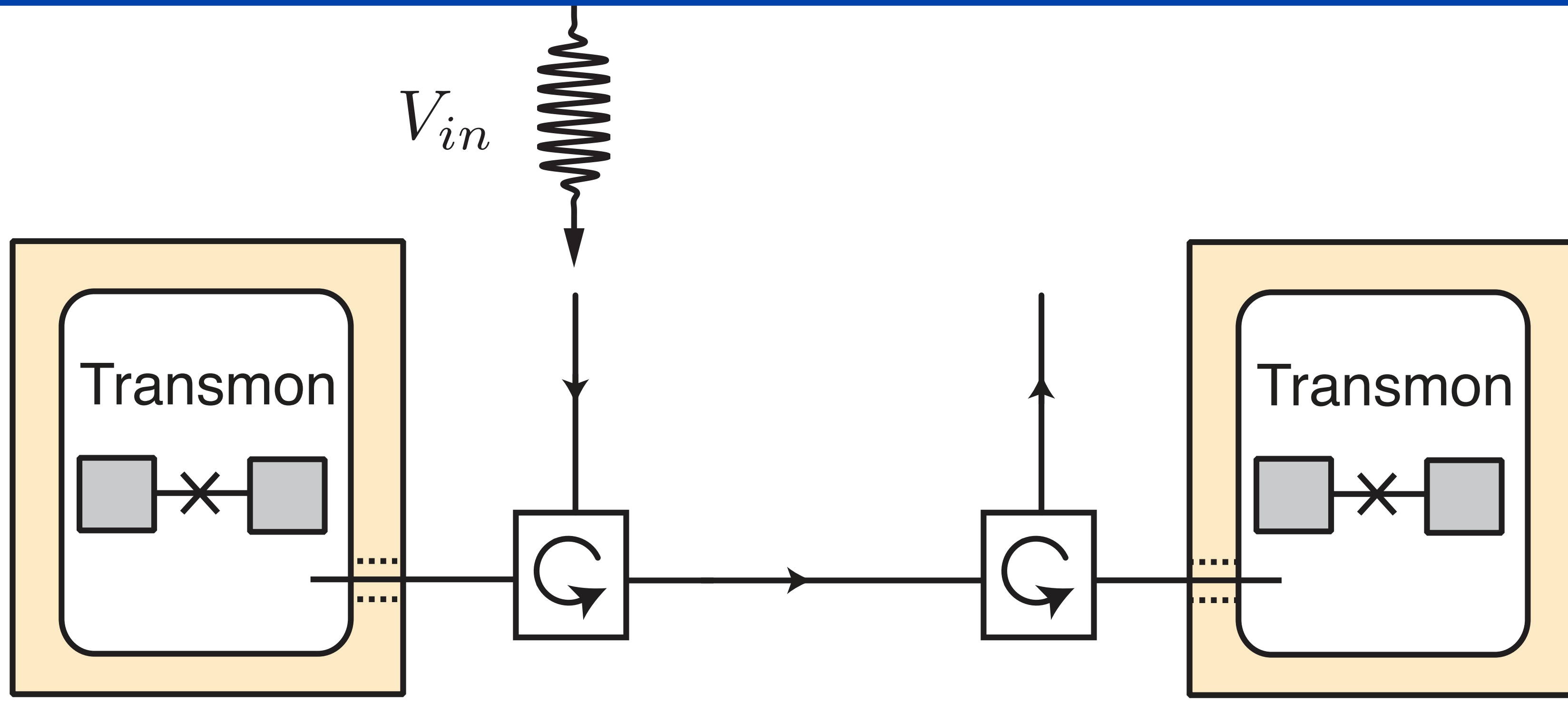
Measurement based feedback

- dispersive case
- fluorescence case

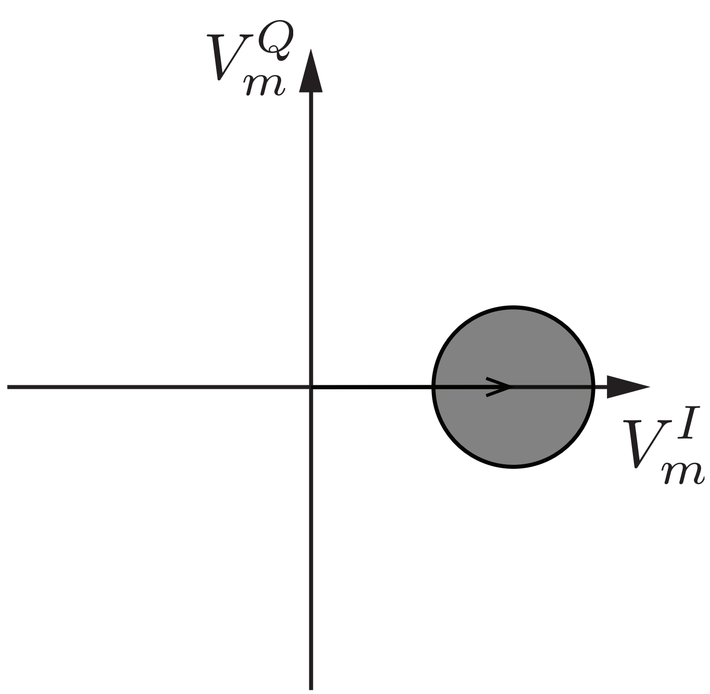
Post selection in quantum mechanics

$\rho(t), E(t)$

Cascade approach to entanglement generation

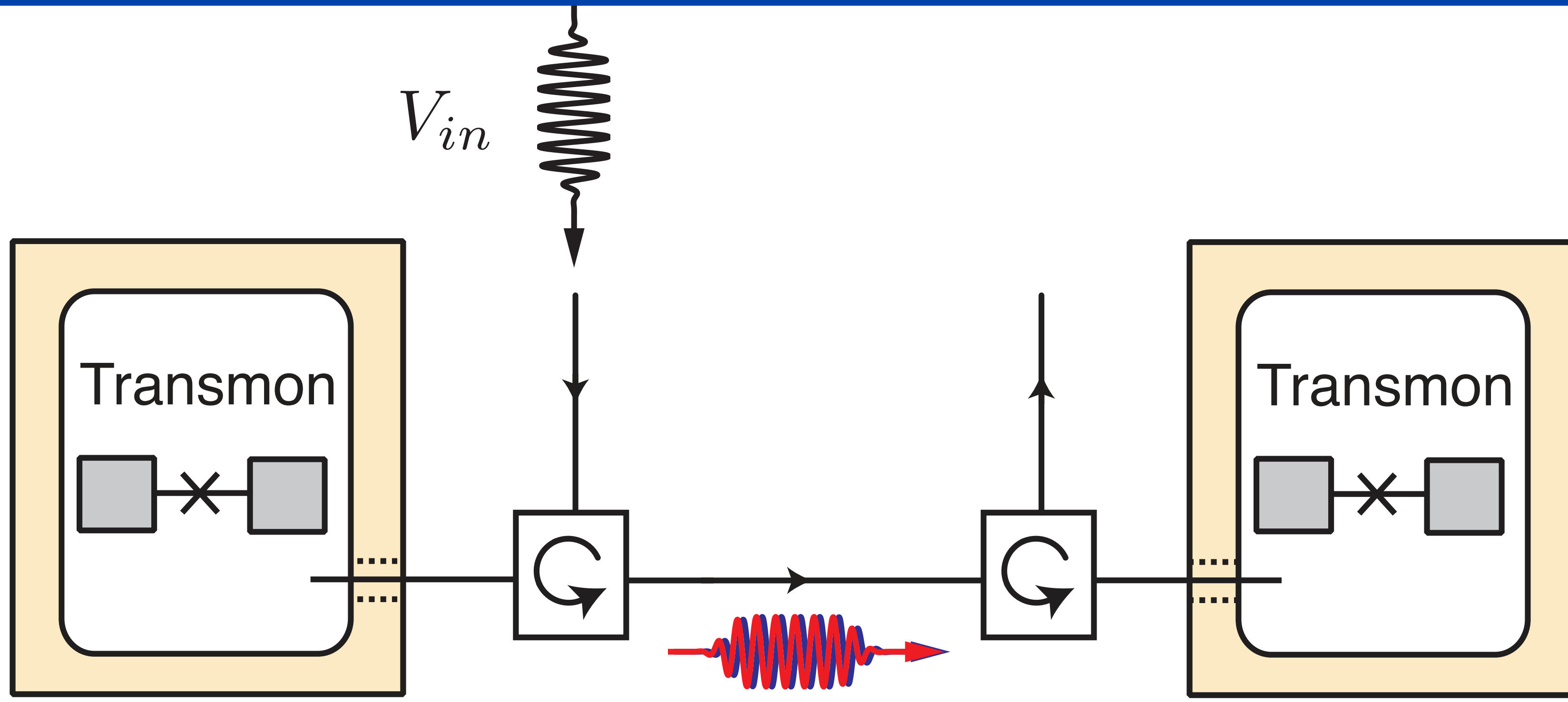


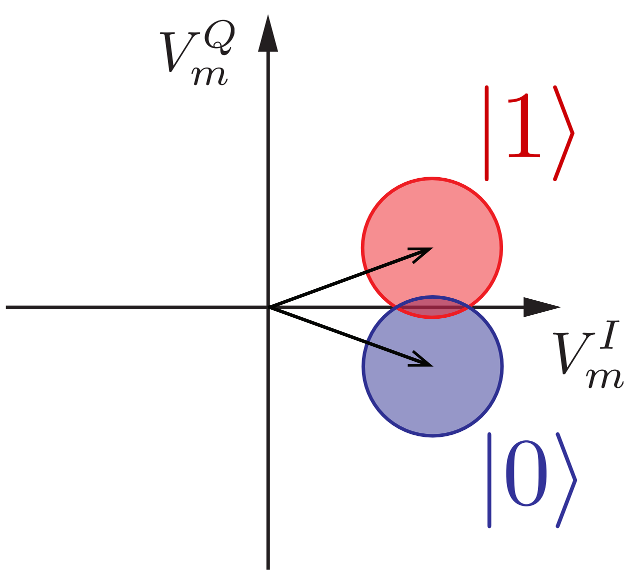
[Risté et al., Nature 2013 (Delft)]
[Roch et al., PRL 2014 (Berkeley)]
[Chantasri et al., PRX 2016 (Berkeley+Rochester)]
[Dickel et al., PRB 2018 (Delft)]

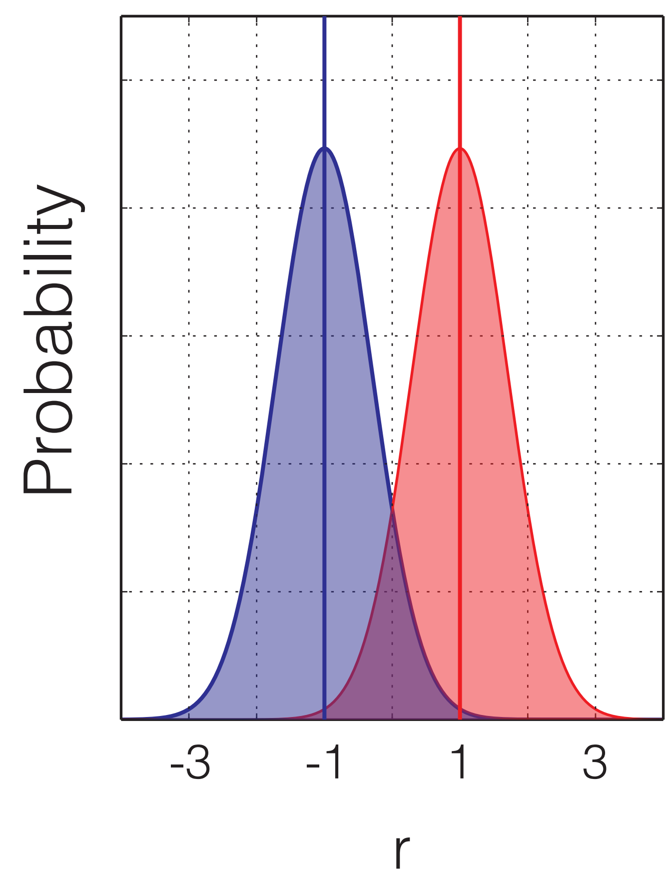


[Slide courtesy of N. Roch]

Cascade approach to entanglement generation





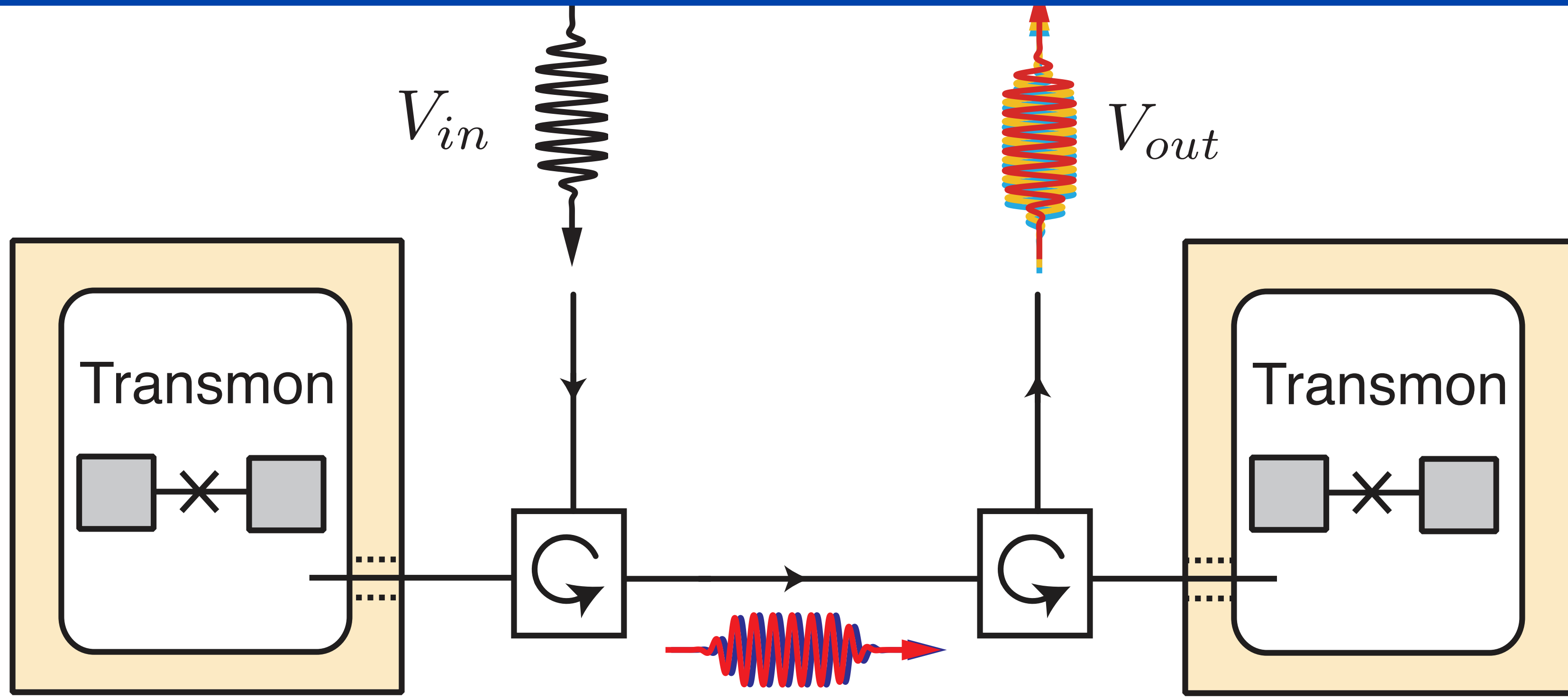


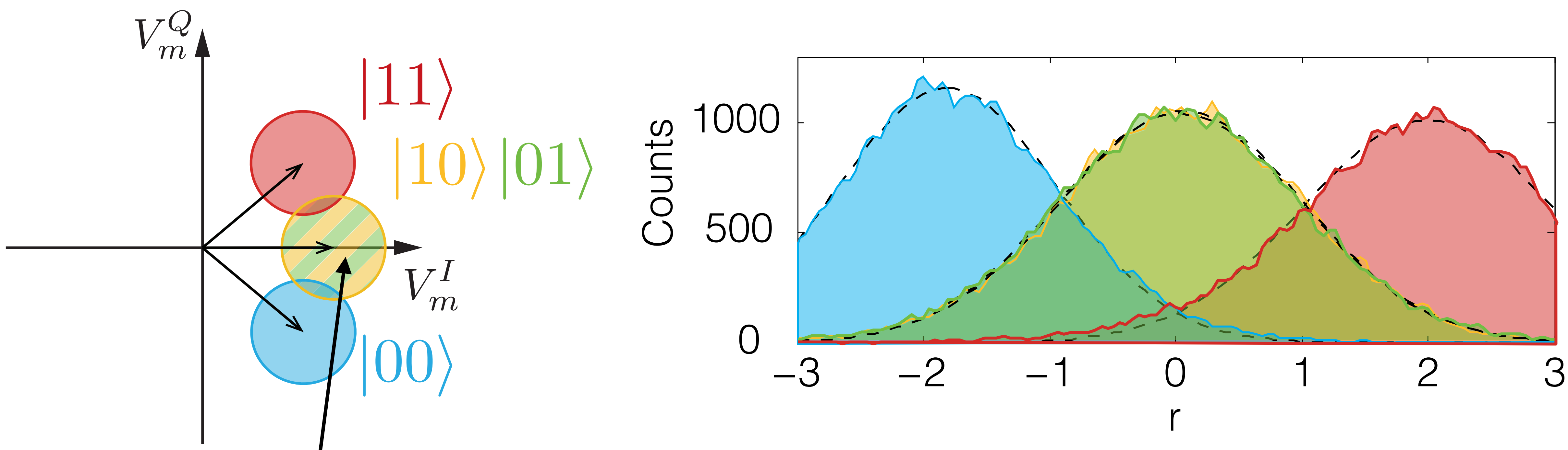
$$V_m = \frac{1}{\Delta t}\int_0^{\Delta t} V_{out}(t)dt$$

$$r = 2V_m^Q/\Delta V$$

[Slide courtesy of N. Roch]

Cascade approach to entanglement generation

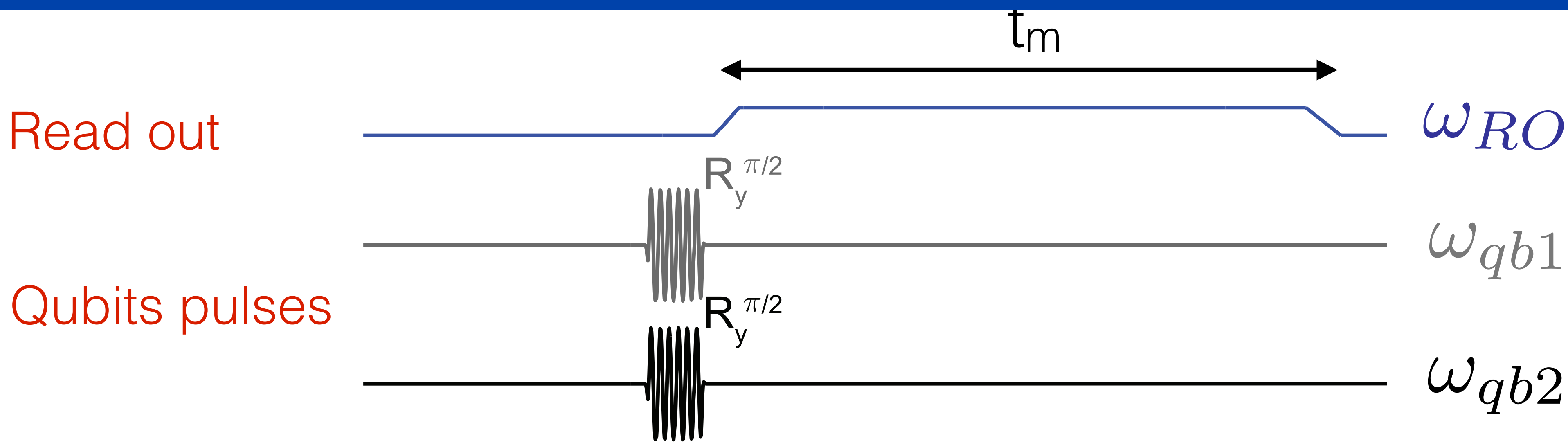




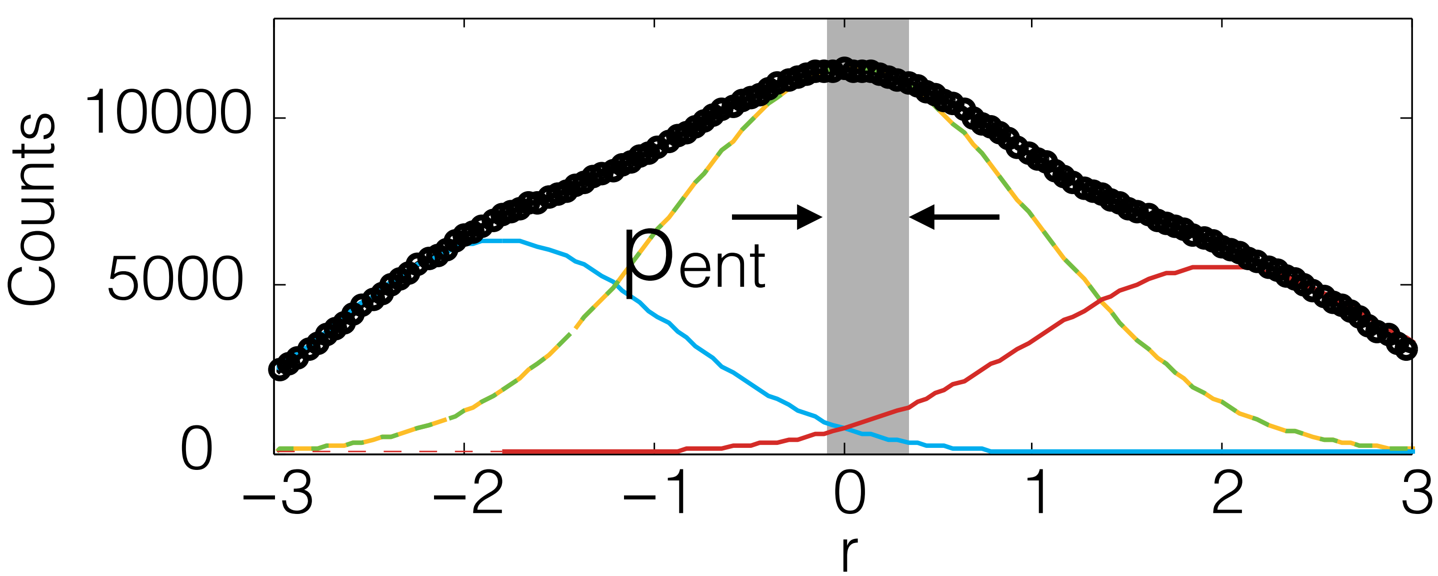
outcome here → projects the qubit pair onto $\mathrm{span}(|10\rangle, |01\rangle)$

[Slide courtesy of N. Roch]

Measurement induced entanglement



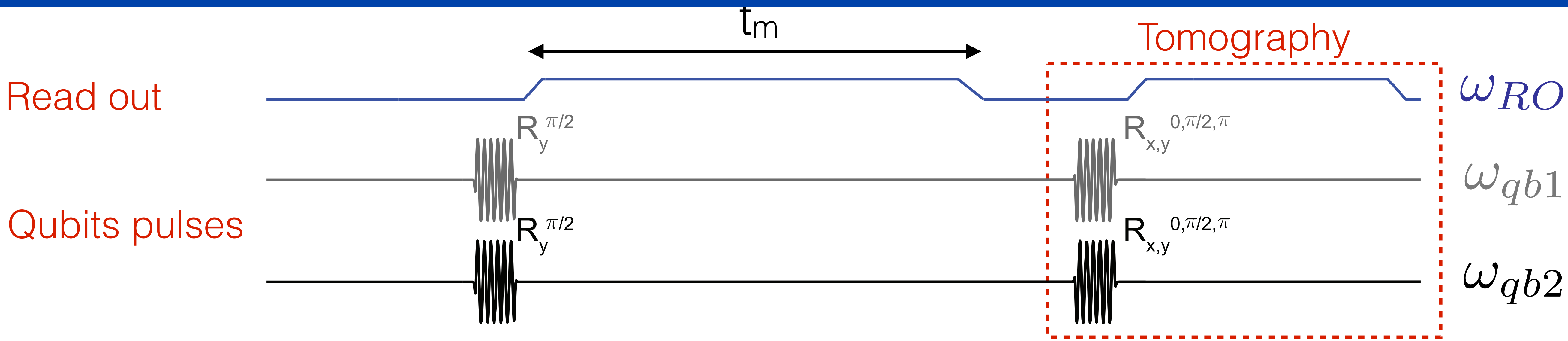
$$|\psi_i\rangle = (|00\rangle + |01\rangle + |10\rangle + |11\rangle)/\,2$$



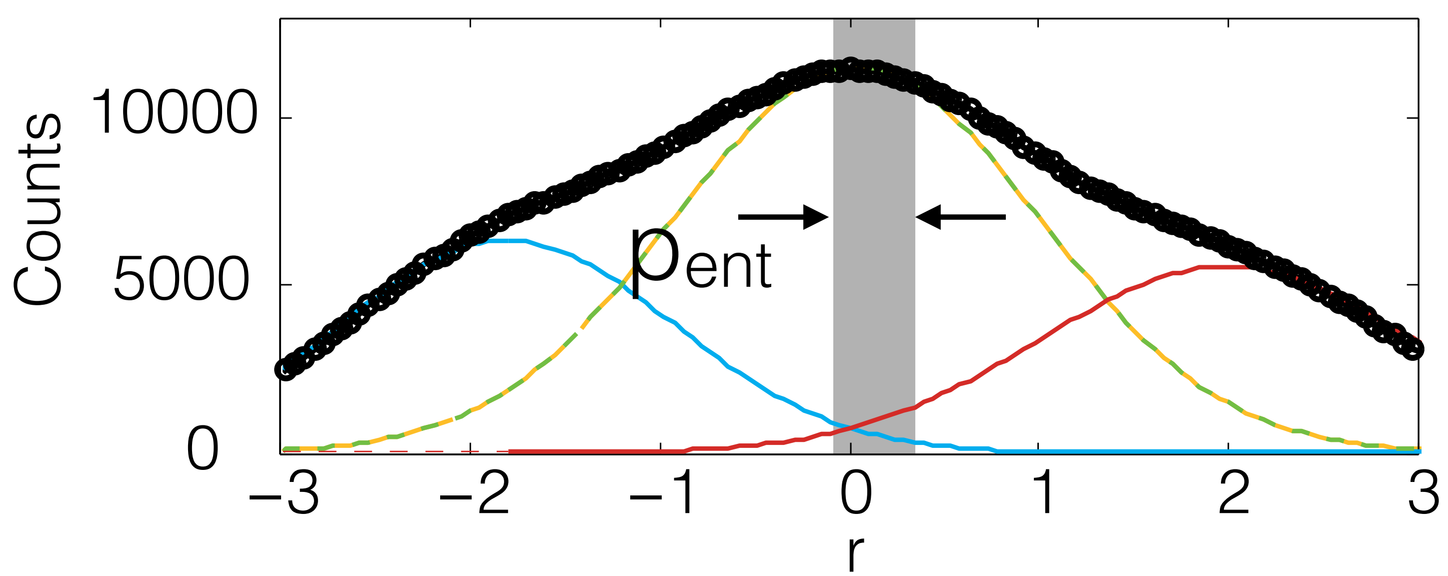
p_{ent}=10%

[Slide courtesy of N. Roch]

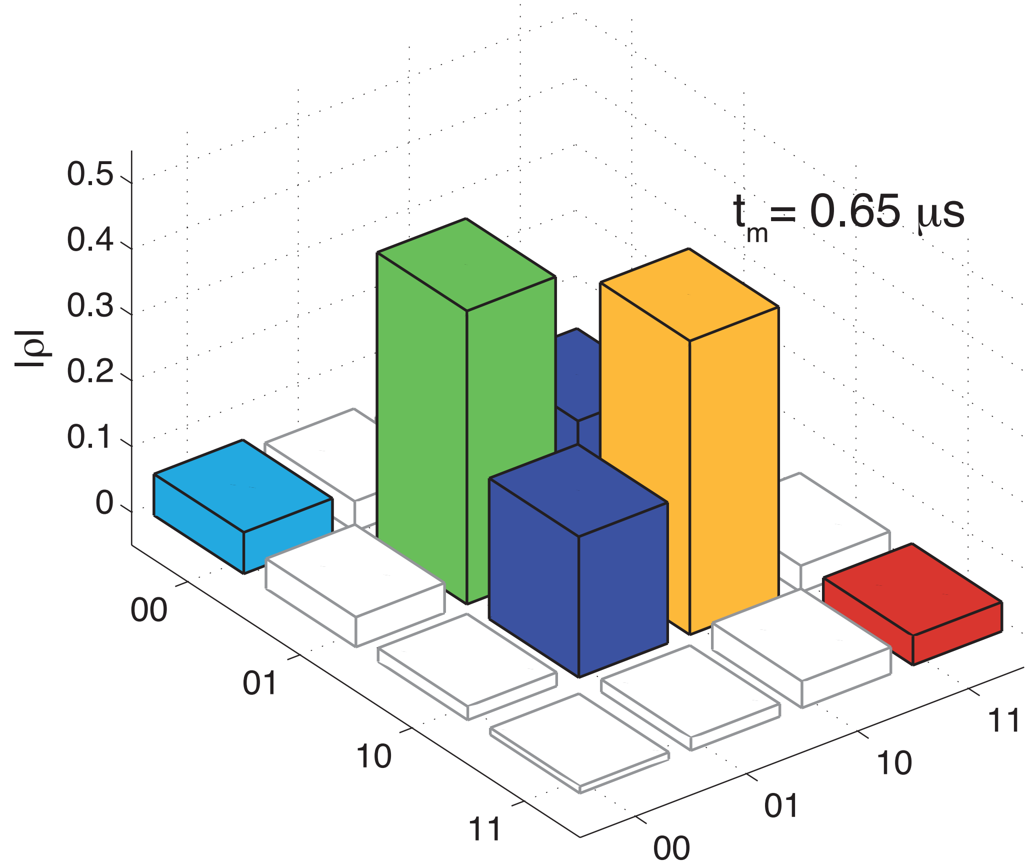
Measurement induced entanglement



$$|\psi_i\rangle = (|00\rangle + |01\rangle + |10\rangle + |11\rangle)/2$$



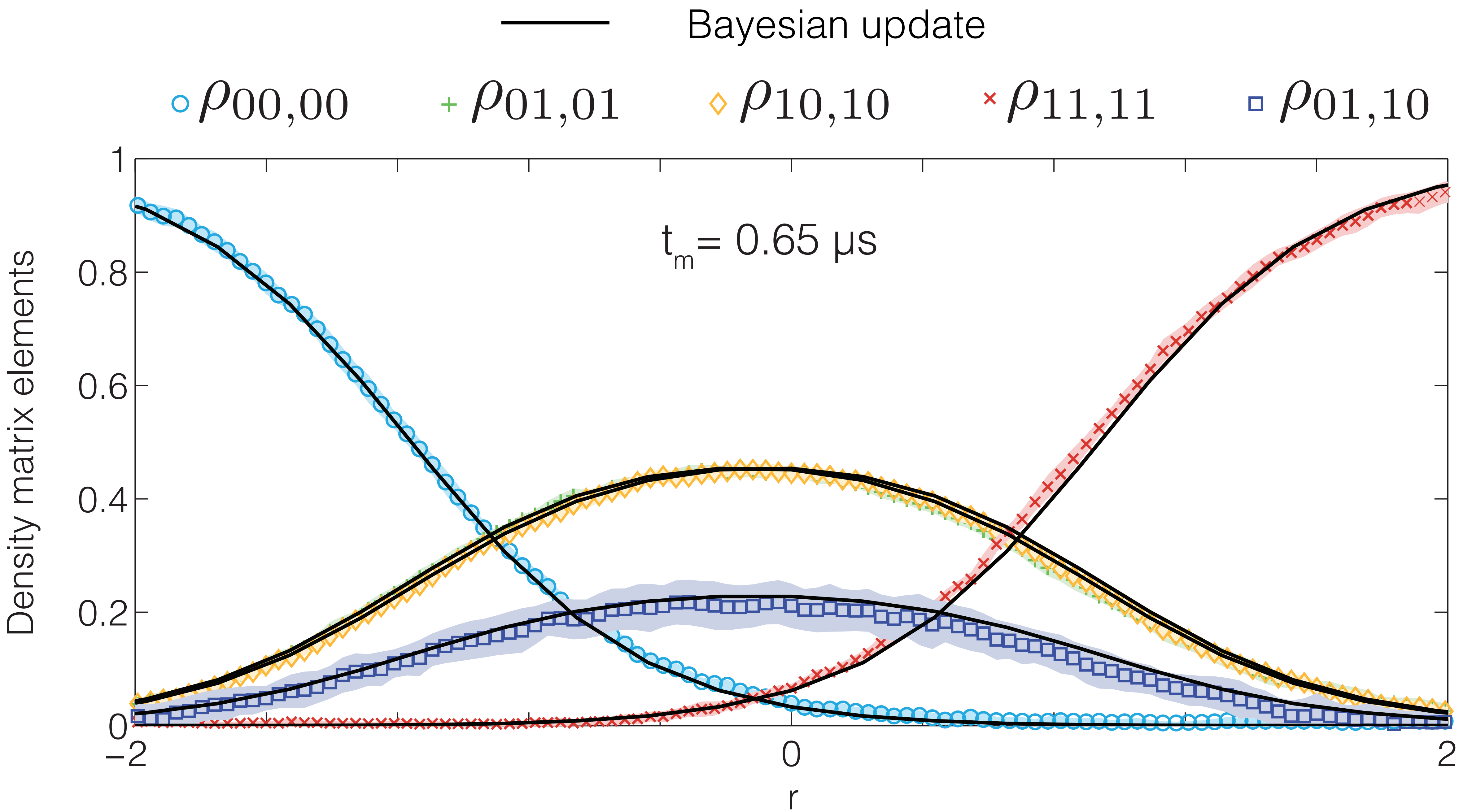
p_{ent}=10%



$\mathcal{C} = 0.35$

[Slide courtesy of N. Roch]

Conditional tomography



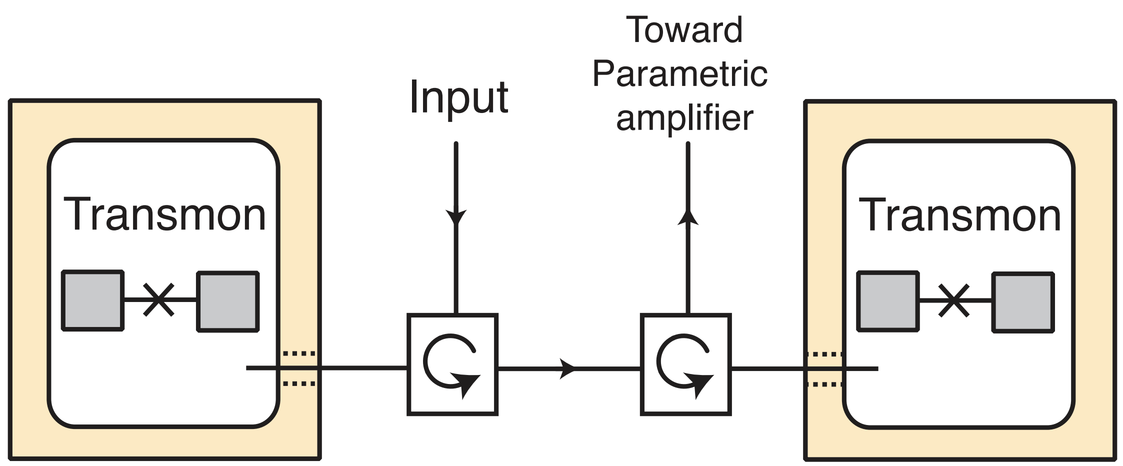
Bayes rule:

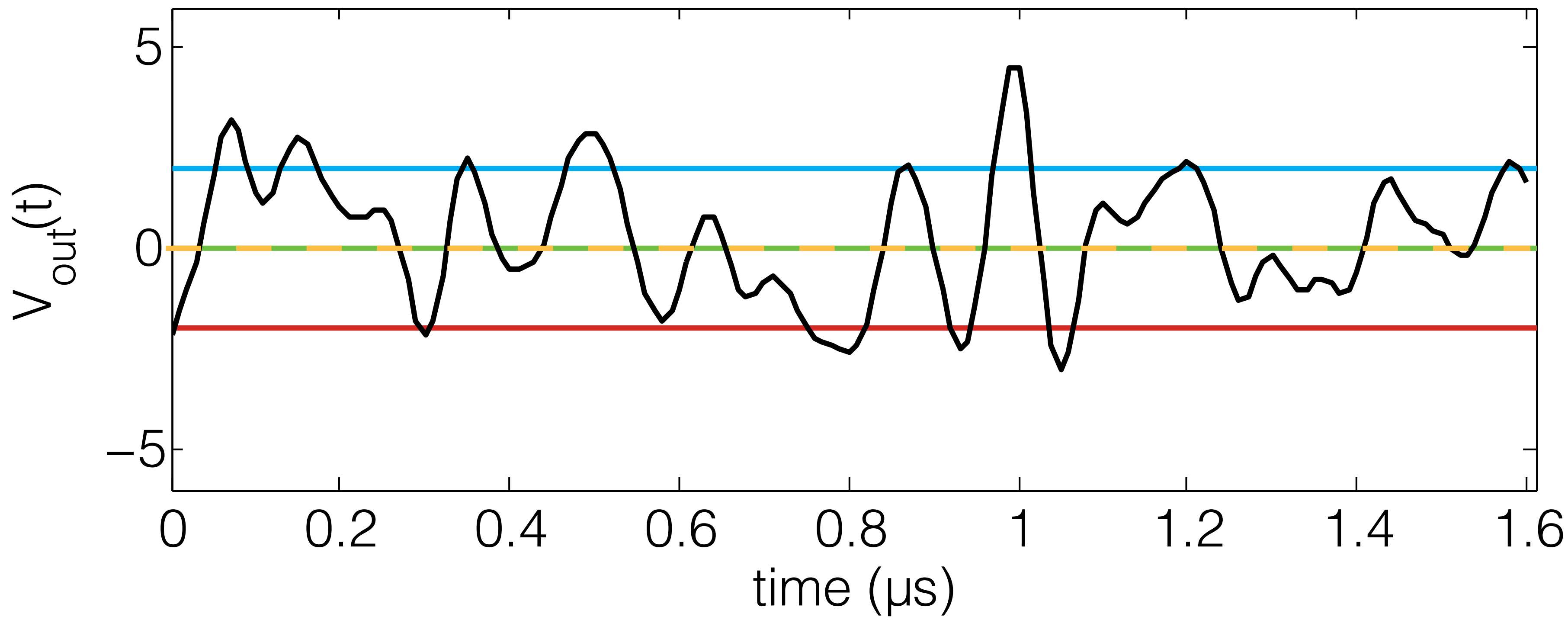
$$p(|ij\rangle | r) = \frac{p(|ij\rangle) p(r||ij\rangle)}{p(r)}$$

→ Mapping of r onto ρ

[Slide courtesy of N. Roch]

Quantum trajectories for 2 qubits

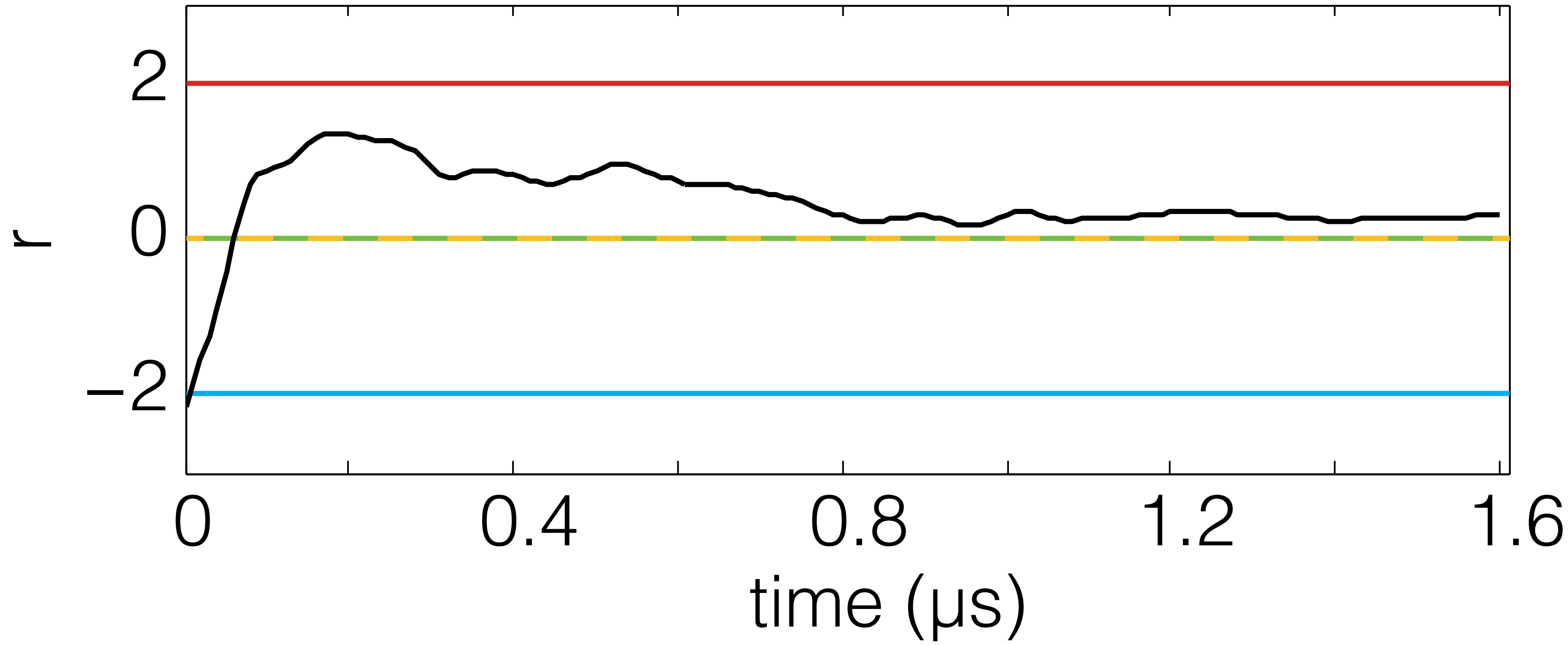




Single time trace

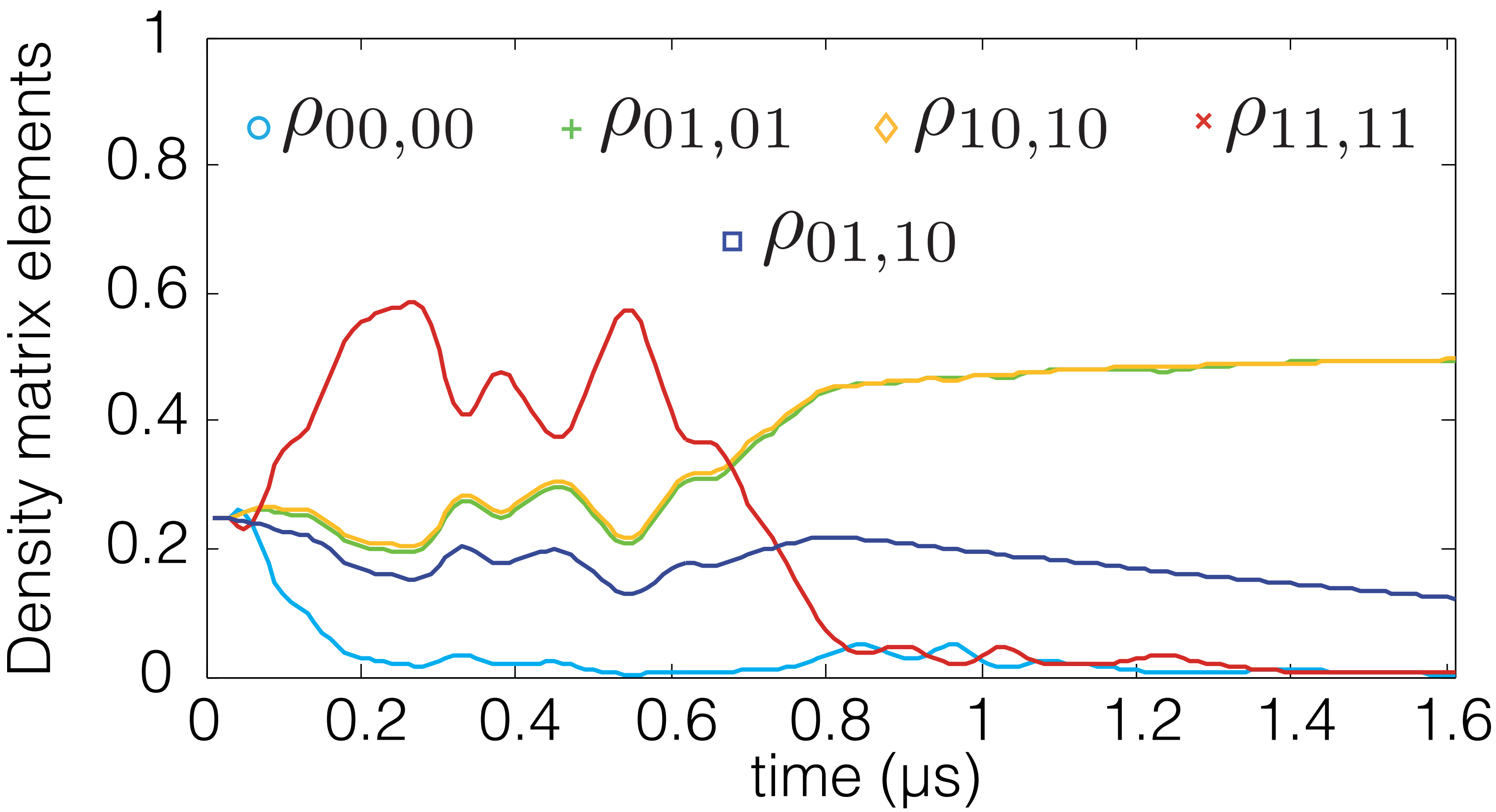
[Slide courtesy of N. Roch]

Quantum trajectories for 2 qubits



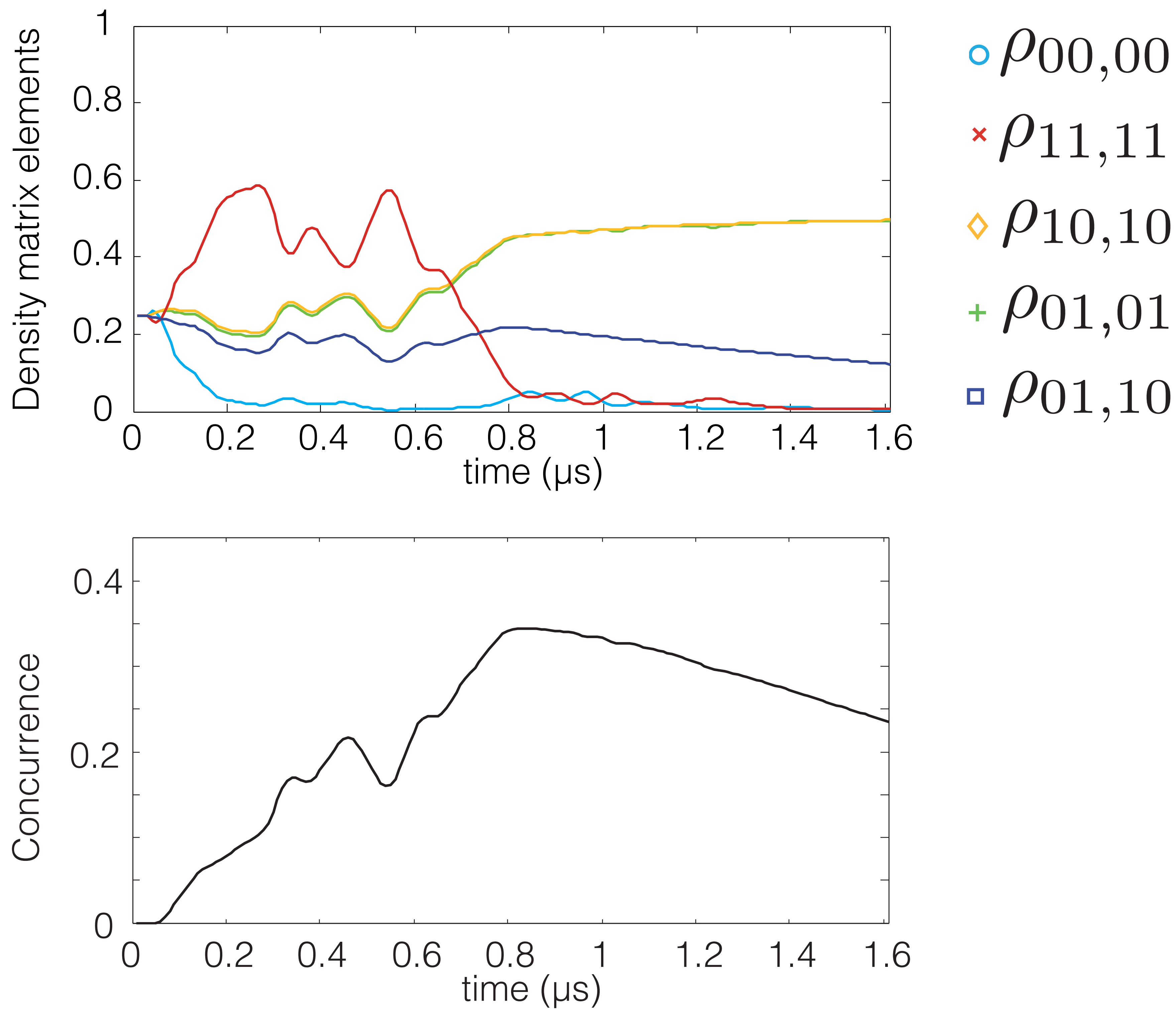
for each point:

$$p(|ij\rangle|r) = \frac{p(|ij\rangle)p(r||ij\rangle)}{p(r)}$$



[Slide courtesy of N. Roch]

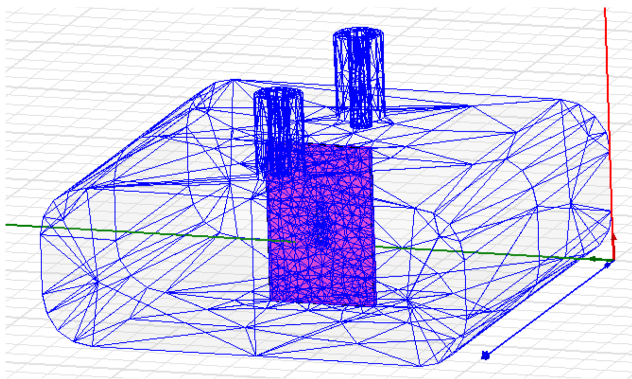
Quantum trajectories for 2 qubits



$$\mathcal{C} = \max(0, |\rho_{01,10}| - \sqrt{\rho_{00,00}\rho_{11,11}})$$

[Slide courtesy of N. Roch]

Quantum trajectories and feedback in circuit-QED



Introduction to circuit-QED

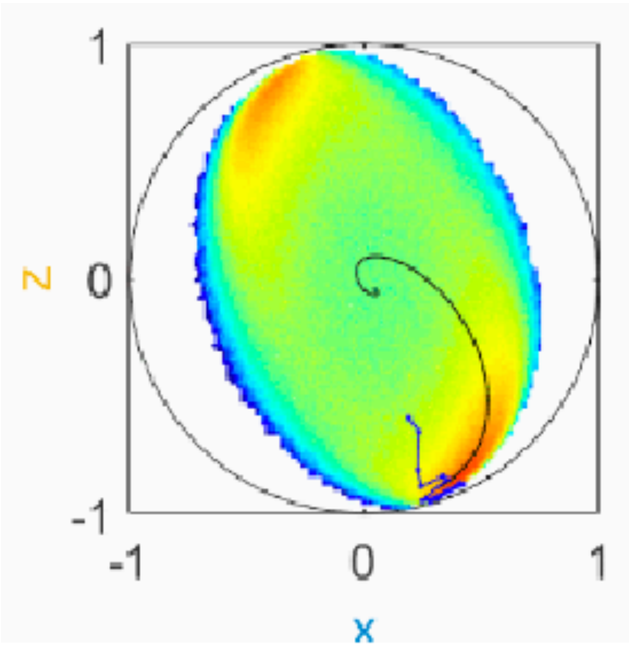
Quantum trajectories in circuit-QED

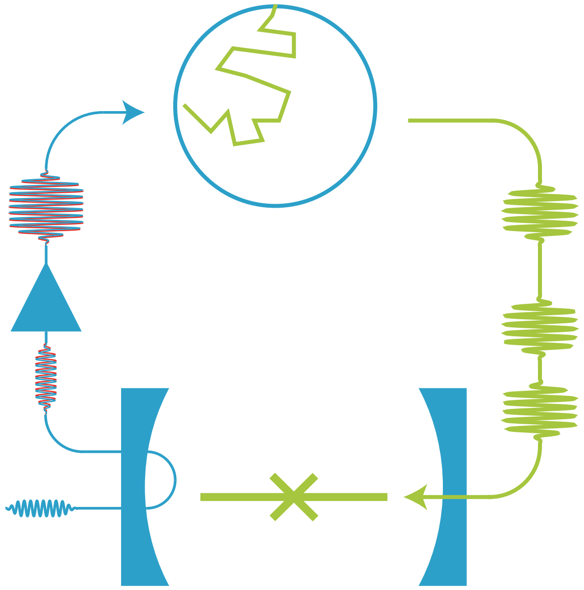
dispersive measurement

fluorescence measurement

both simultaneously

generating entanglement





Measurement based feedback

dispersive case

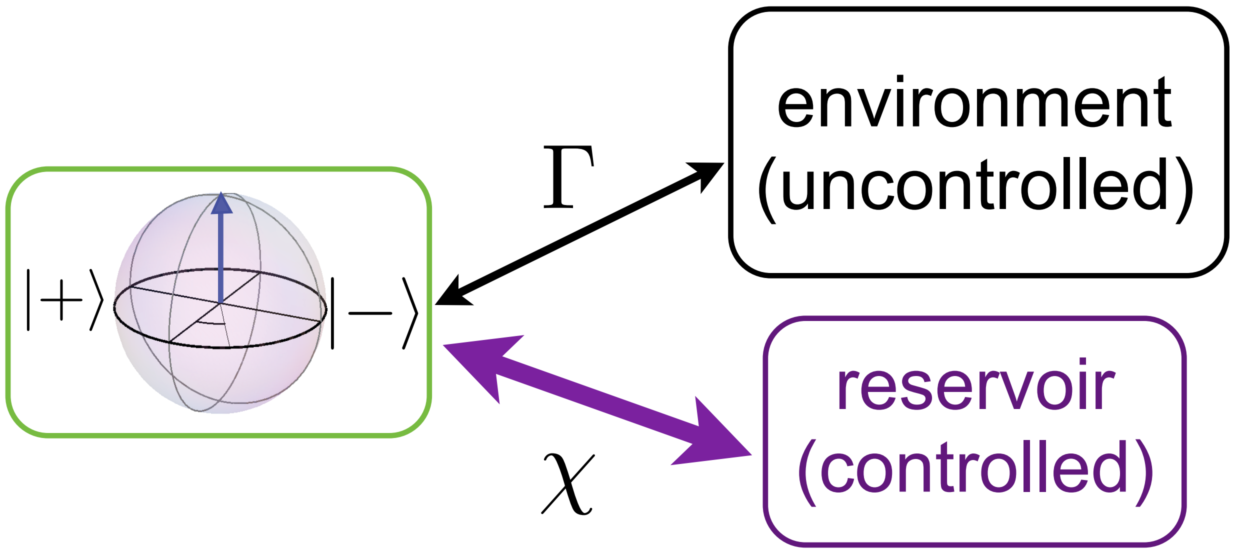
fluorescence case

Post selection in quantum mechanics

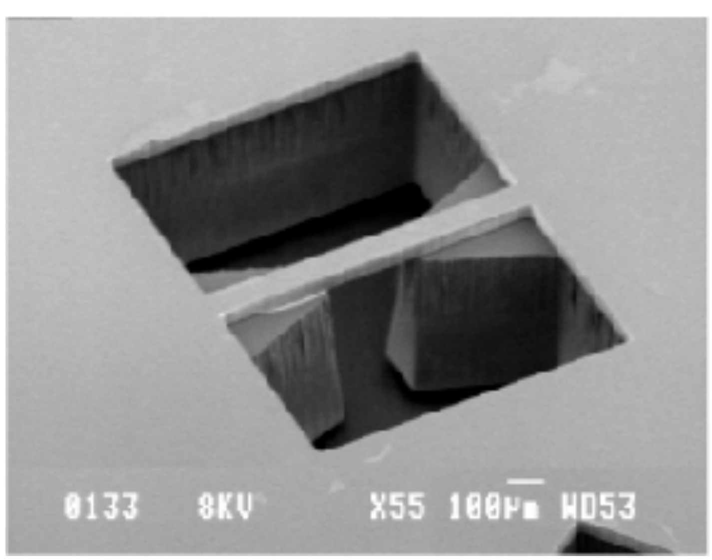
$\rho(t), E(t)$

How to preserve an arbitrary state?

Reservoir engineering $\chi \gg \Gamma$



Side band cooling of a mirror

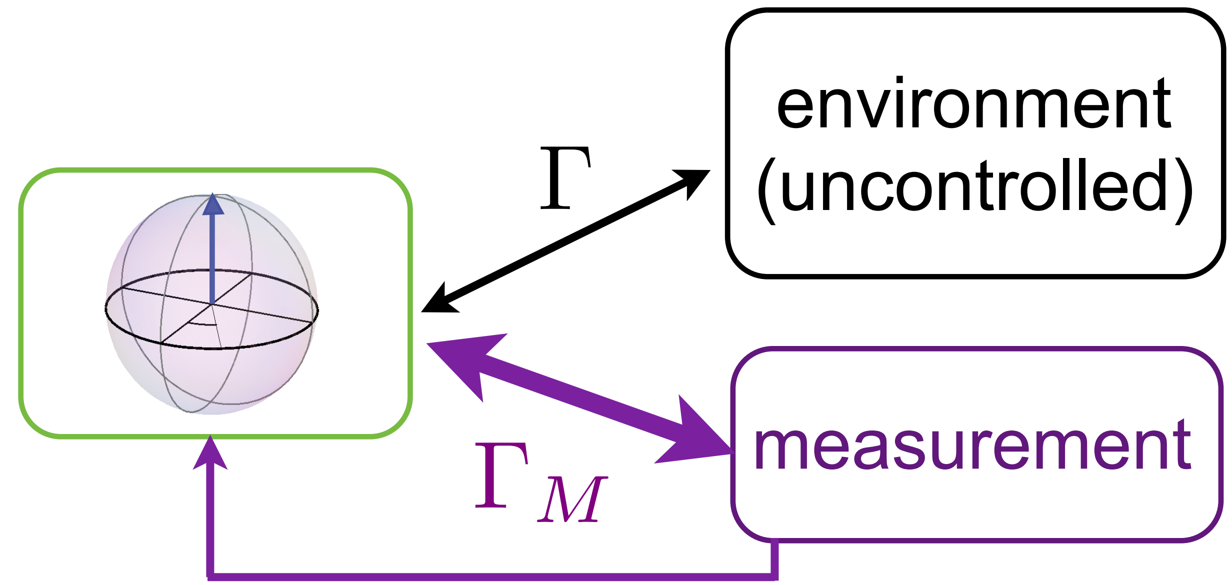


(Pinard et al., LKB, 1999)

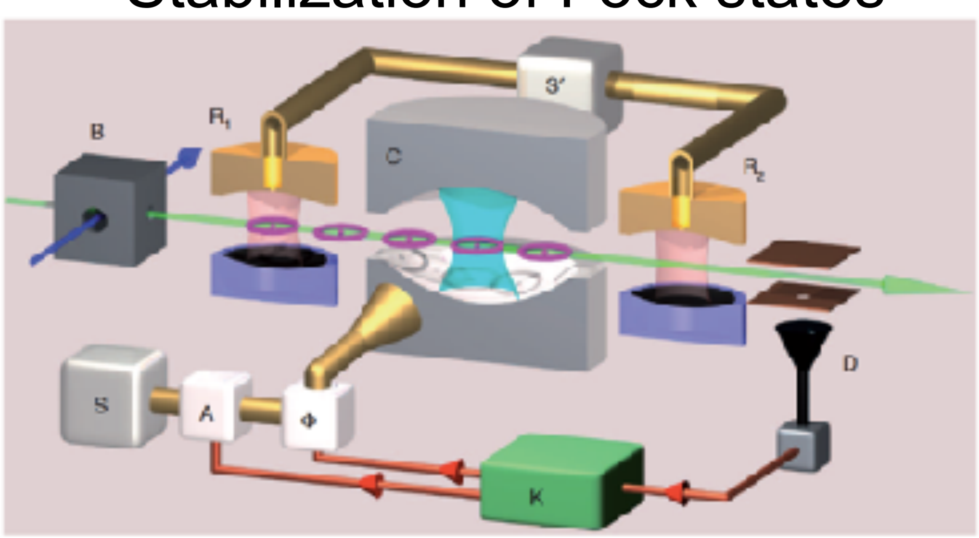
$\frac{1}{\sqrt{2}}(|g\rangle + |e\rangle)$ (Siddiqi group, Berkeley, 2012)

Measurement based feedback

$\Gamma_M \gg \Gamma$



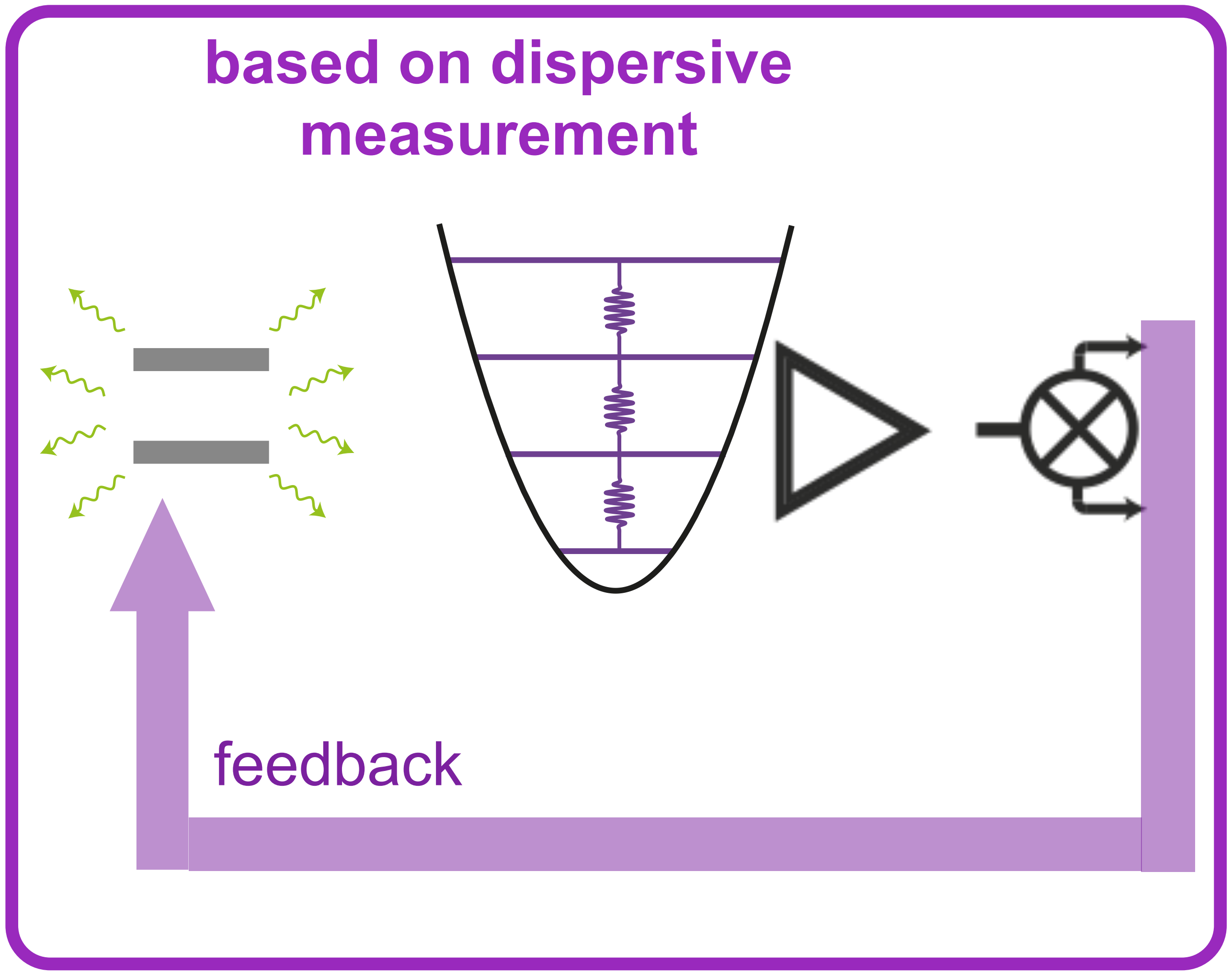
Stabilization of Fock states



(Haroche, LKB, 2011)

And in circuit QED?

Measurement based feedback

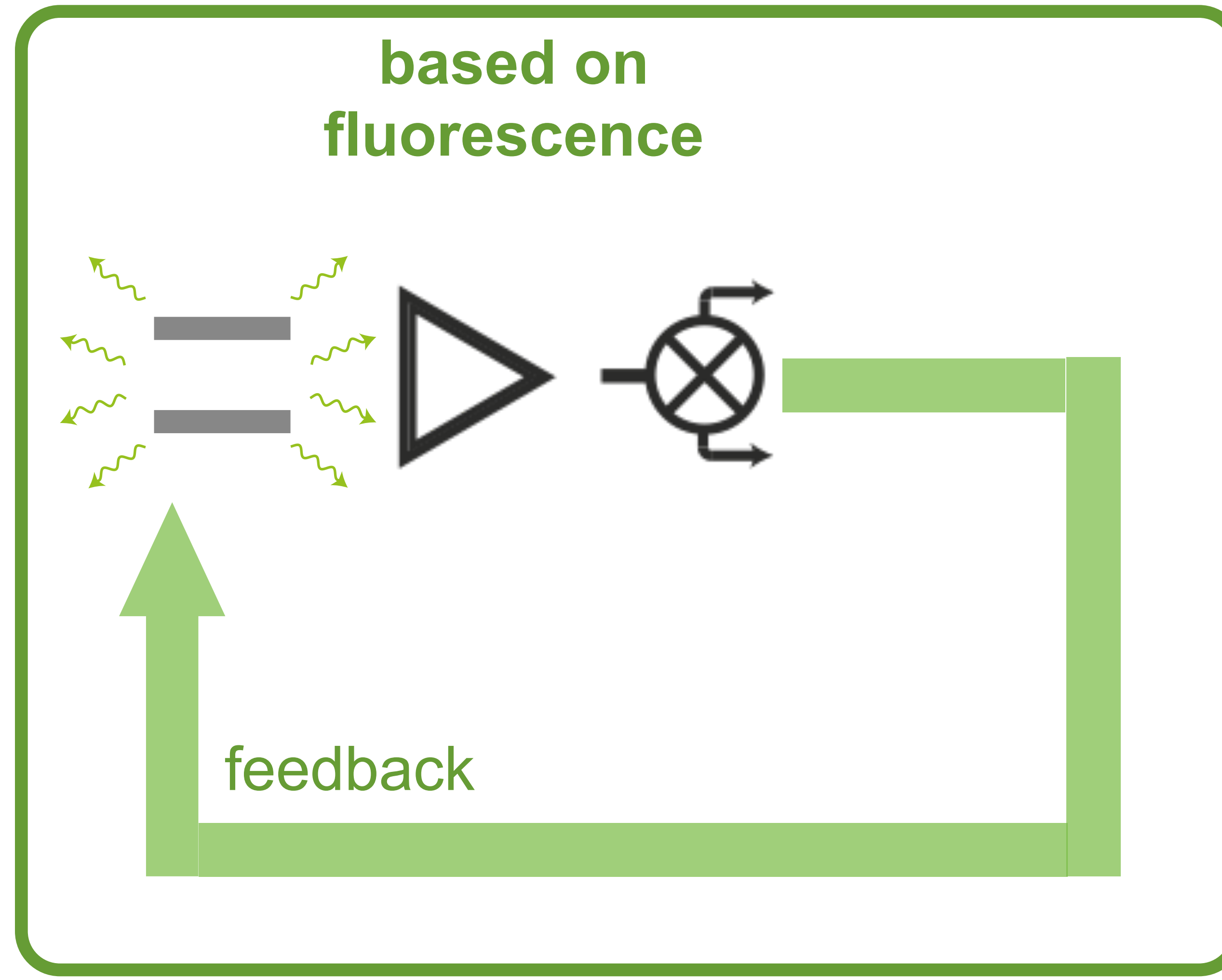


[Vijay et al., Nature 2012 (Berkeley)]

[Ristè et al., PRL 2012 (Delft)]

[Campagne-Ibarcq et al., PRX 2013 (ENS Paris)]

...now standard technique

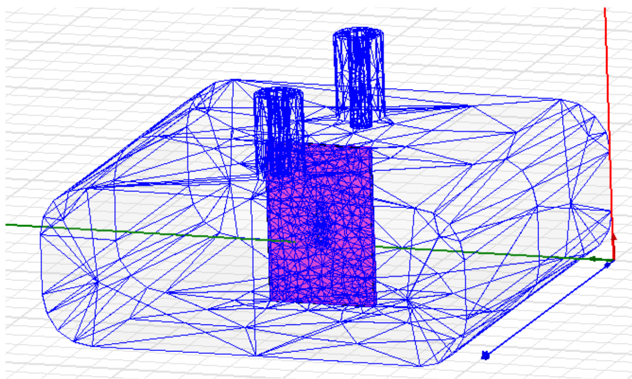


compatible with dispersive feedback

converges in T1 for any state

here continuous feedback qualitatively more efficient than stroboscopic

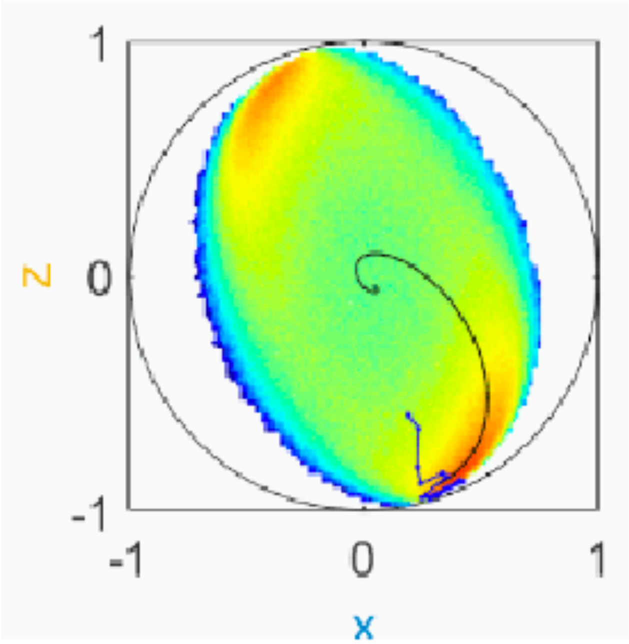
Quantum trajectories and feedback in circuit-QED

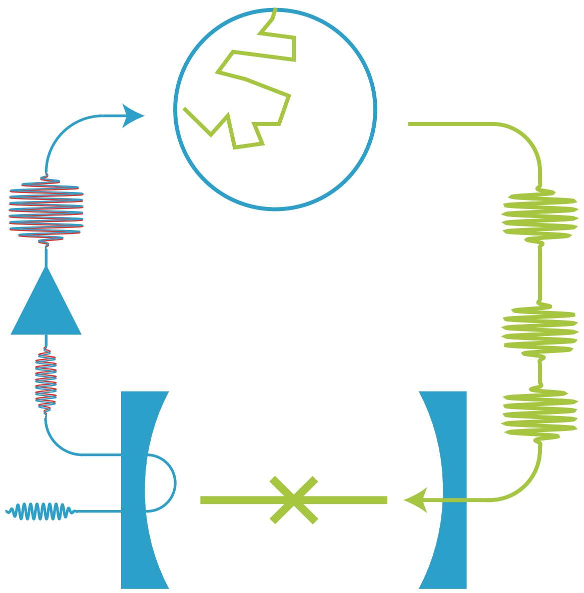


Introduction to circuit-QED

Quantum trajectories in circuit-QED

- dispersive measurement
- fluorescence measurement
- both simultaneously
- generating entanglement





Measurement based feedback

- **dispersive case**
- fluorescence case

Post selection in quantum mechanics

$\rho(t), E(t)$

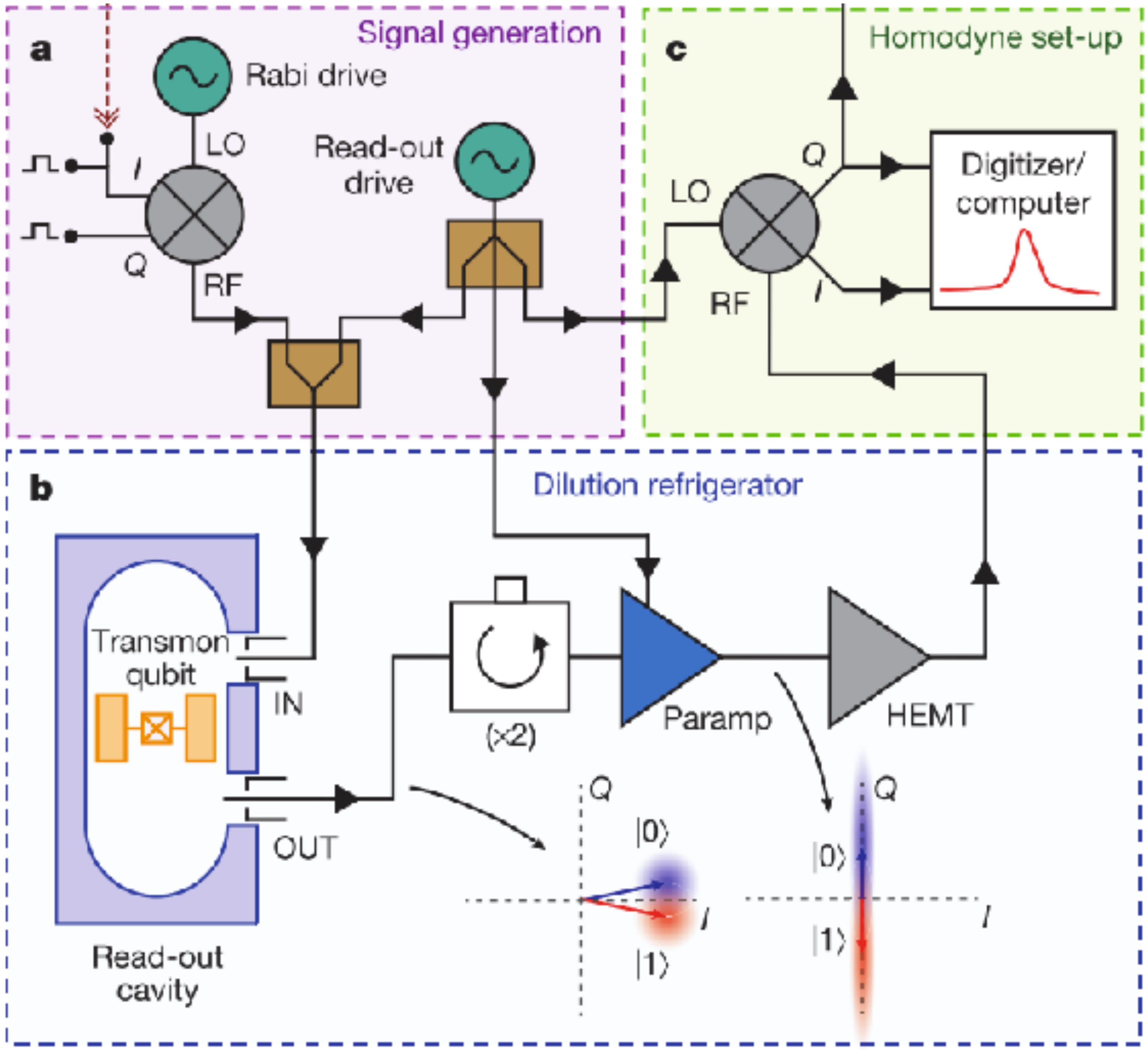
Analog feedback

Stabilize Rabi oscillations against decoherence

$$H = \hbar \frac{\Omega(t)}{2} \sigma_X$$

$$Q_{out}(t) = \cos(\Omega_0 t + \theta)$$

goal is $\theta = 0$



[Vijay et al., Nature 2012, Berkeley]

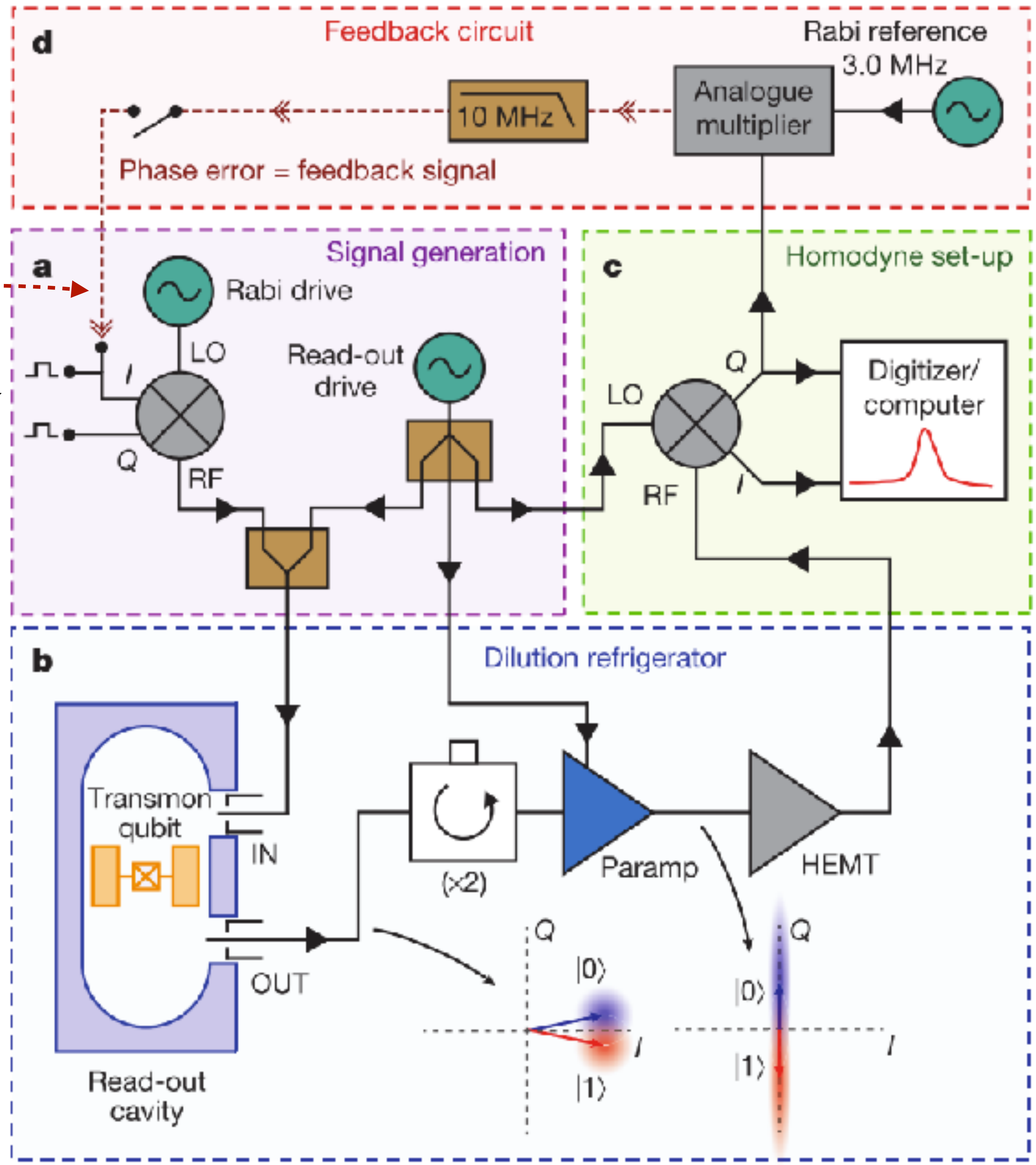
Analog feedback

Stabilize Rabi oscillations against decoherence

$$H = \hbar \frac{\Omega(t)}{2} \sigma_X$$

$$\Omega(t) = \Omega_0 + 2\lambda \sin(\Omega_0 t) Q_{out}(t)$$

$$Q_{out}(t) = \cos(\Omega_0 t + \theta)$$



[Vijay et al., Nature 2012, Berkeley]

Analog feedback

Stabilize Rabi oscillations against decoherence

$$H = \hbar\frac{\Omega(t)}{2}\sigma_X$$

$$\Omega(t) = \Omega_0 + 2\lambda \sin(\Omega_0 t) Q_{out}(t)$$

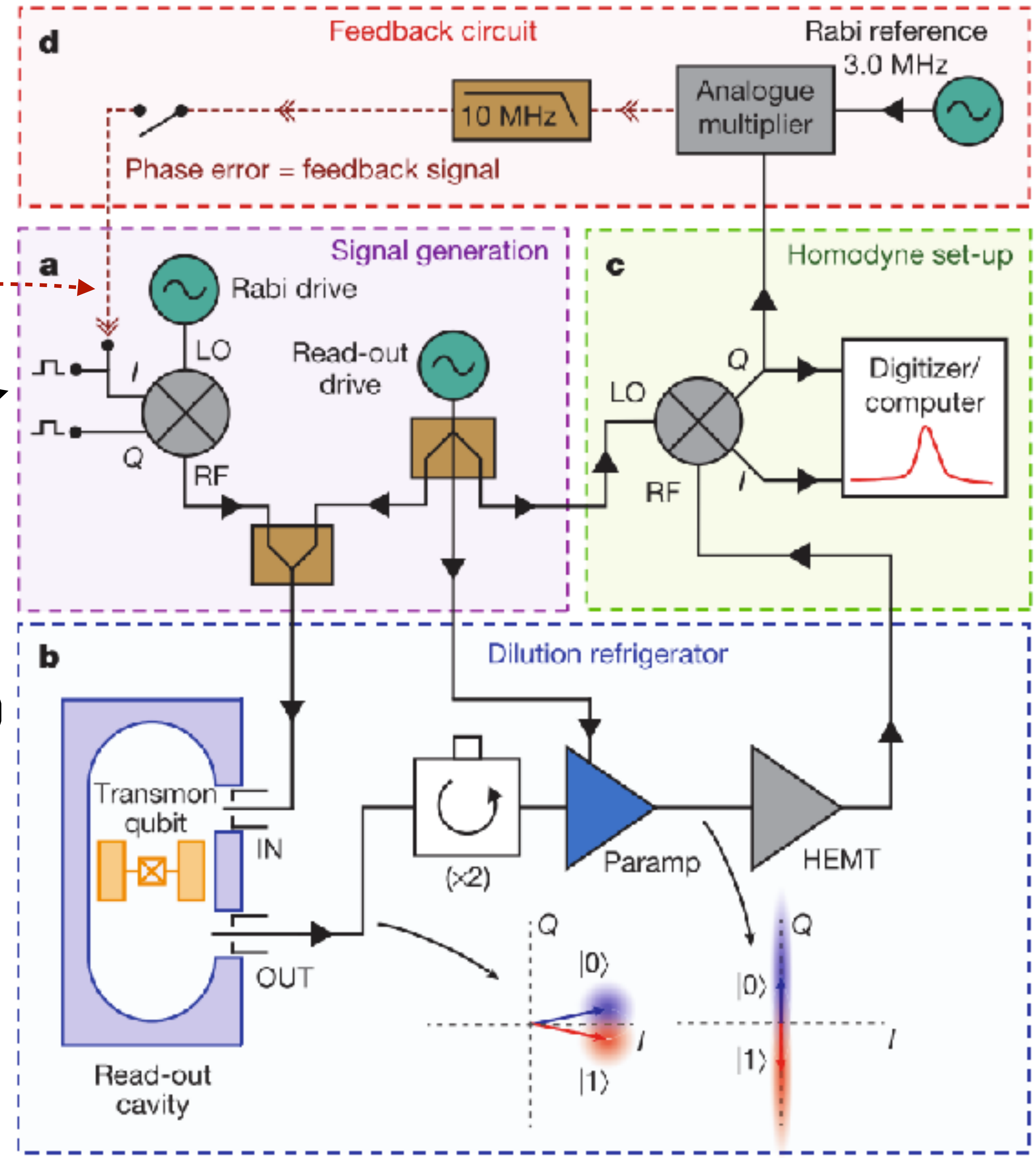
$$Q_{out}(t) = \cos(\Omega_0 t + \theta)$$

$$\Omega(t) = \Omega_0 + \lambda \sin(\theta) + \lambda \sin(2\Omega_0 t + \theta)$$

$$\lambda < 0$$

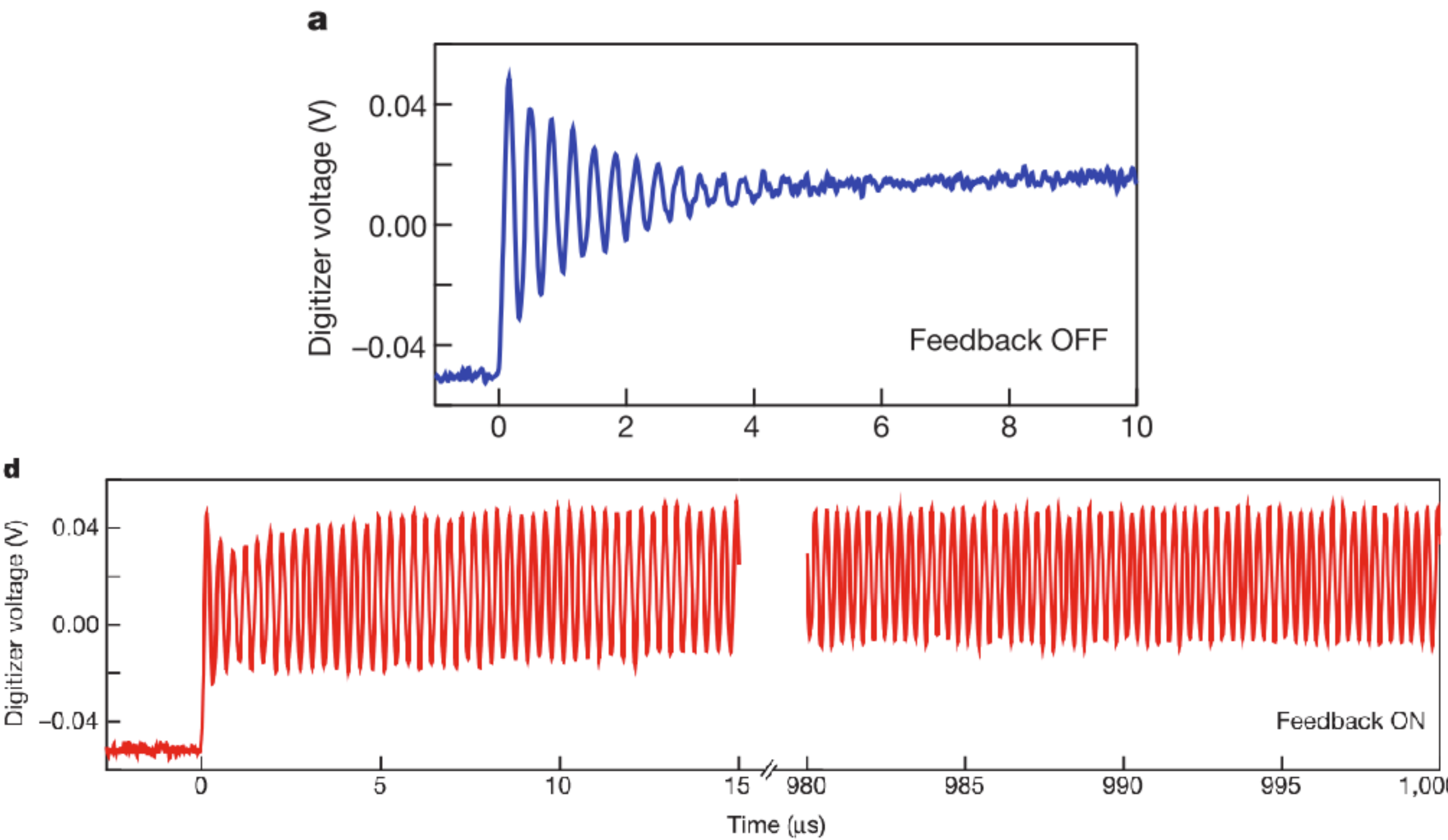
$\theta < 0 \Rightarrow$ late so rotation accelerates

$\theta > 0 \Rightarrow$ in advance so rotation slows down



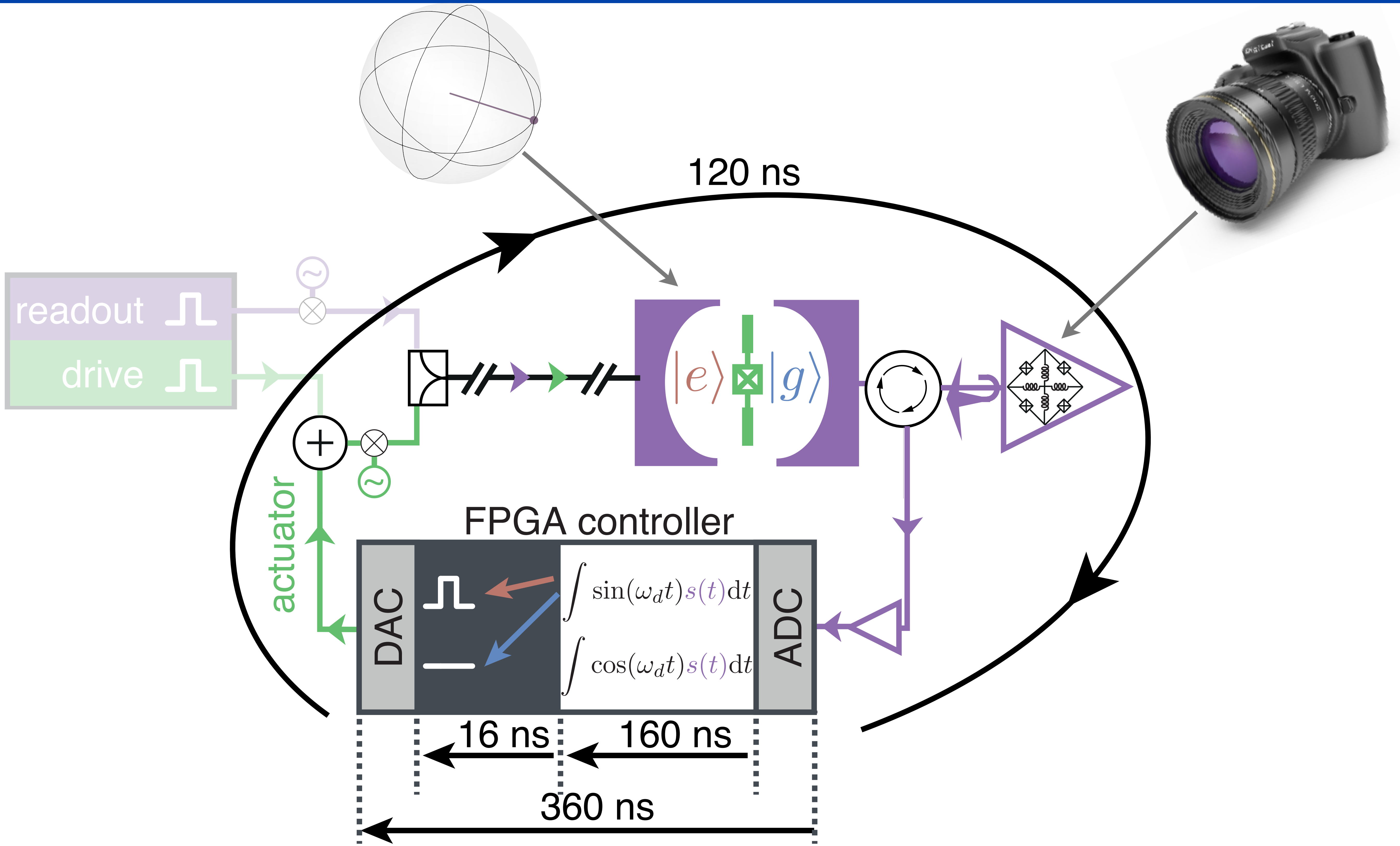
[Vijay et al., Nature 2012, Berkeley]

Analog feedback



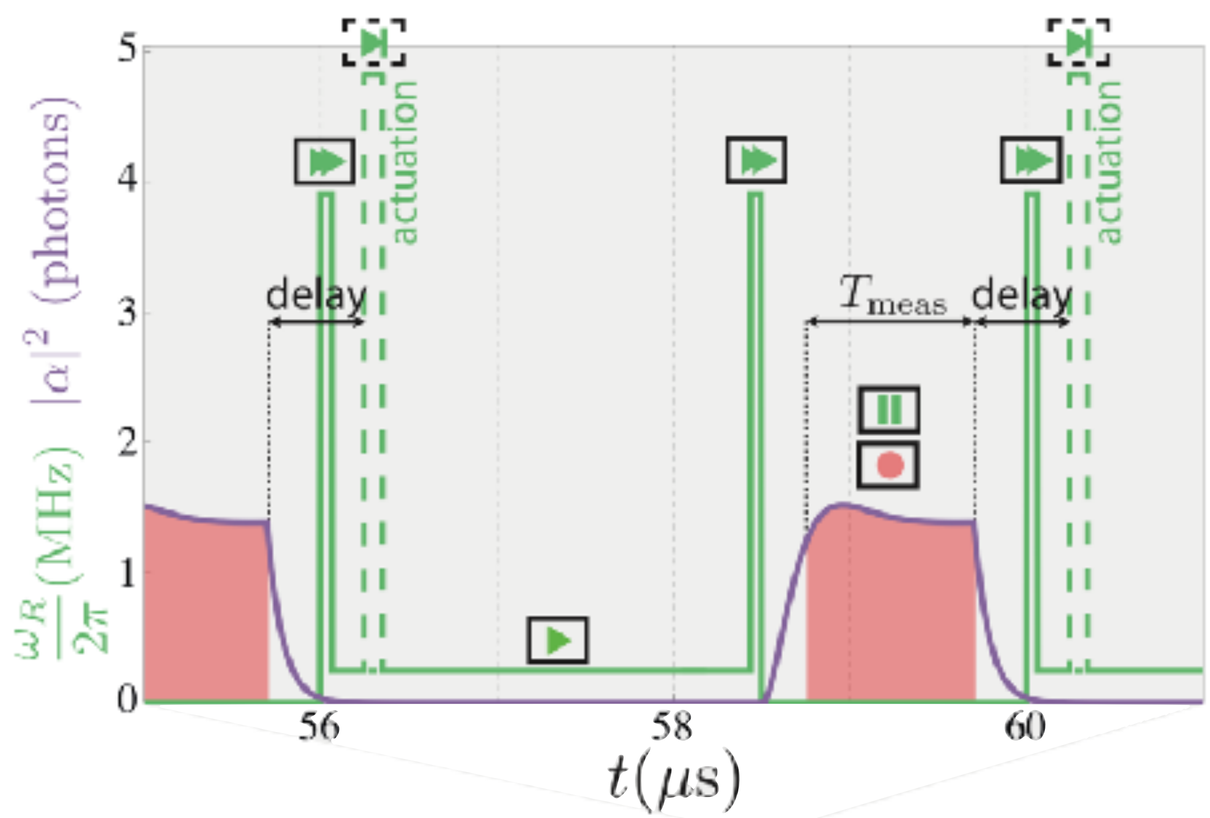
[Vijay et al., Nature 2012, Berkeley]

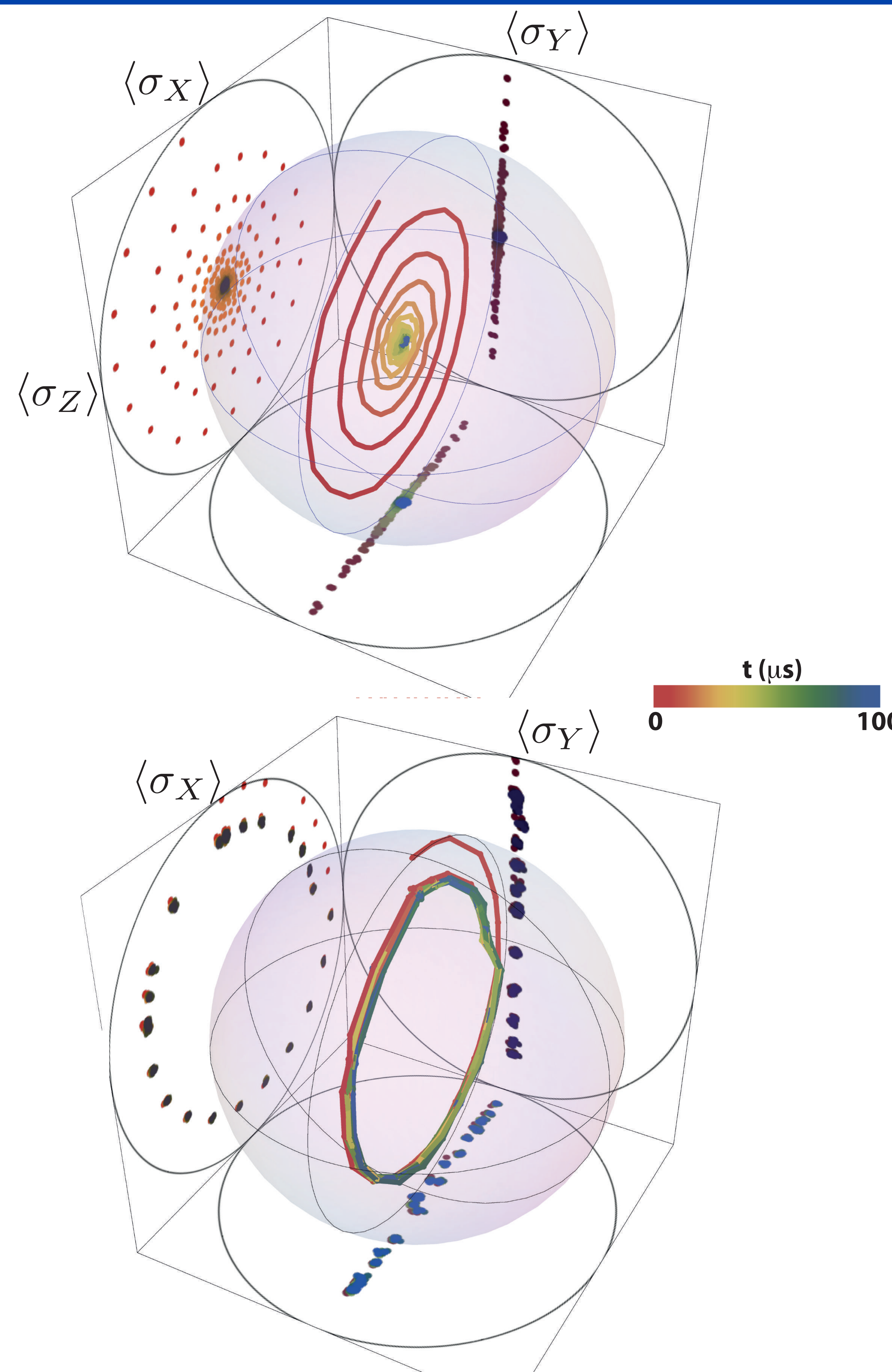
Digital feedback and variable measurement strength



[Campagne-Ibarcq et al., PRX 2013]

Digital feedback





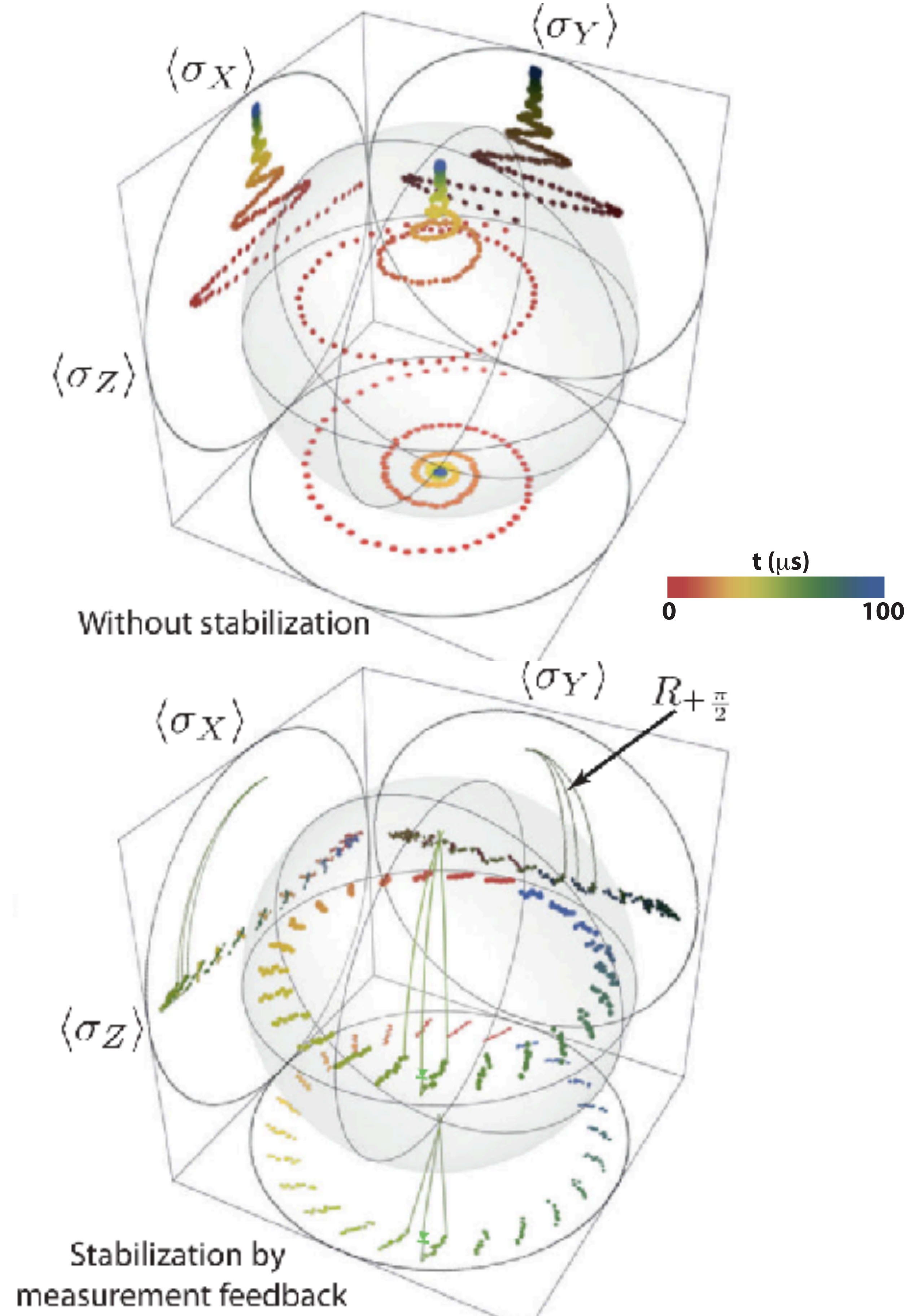
Stabilization of Rabi and Ramsey oscillations
[Campagne-Ibarcq et al., PRX 2013 (ENS Paris)]

77% average Bloch vector length
vs 45% in constant measurement strength
[Vijay et al., Nature 2012 (Berkeley)]

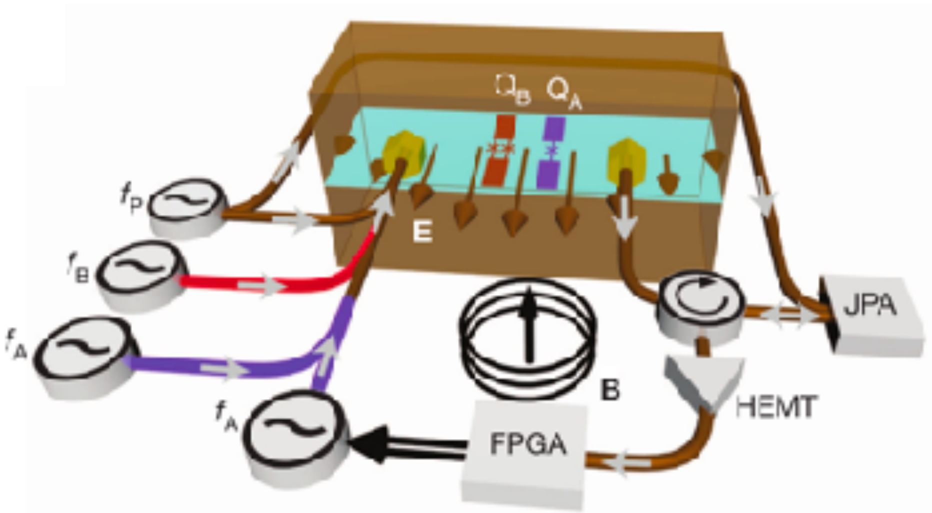
Reset by feedback
[Ristè et al., PRL 2012 (Delft)]

Digital feedback for Ramsey oscillations

Stabilization of Rabi and Ramsey oscillations
[Campagne-Ibarcq et al., PRX 2013 (ENS Paris)]



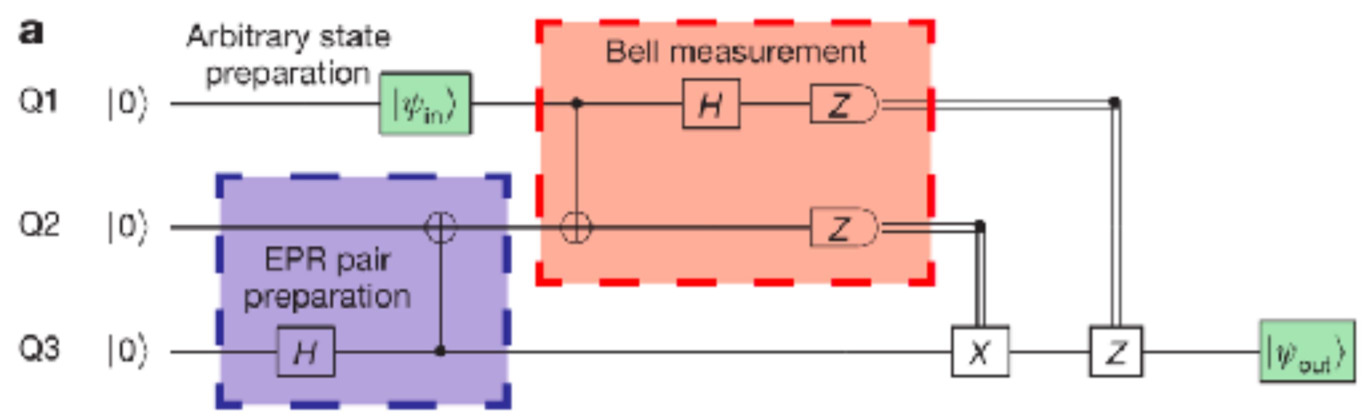
Other applications of dispersive measurement feedback



Entanglement stabilization

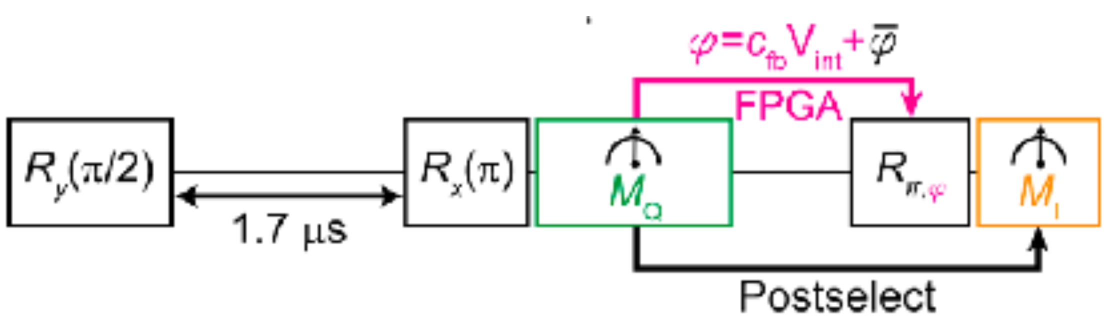
[Risté et al., Nature 2013 (Delft)]

[Liu et al., PRX 2016 (Yale)]



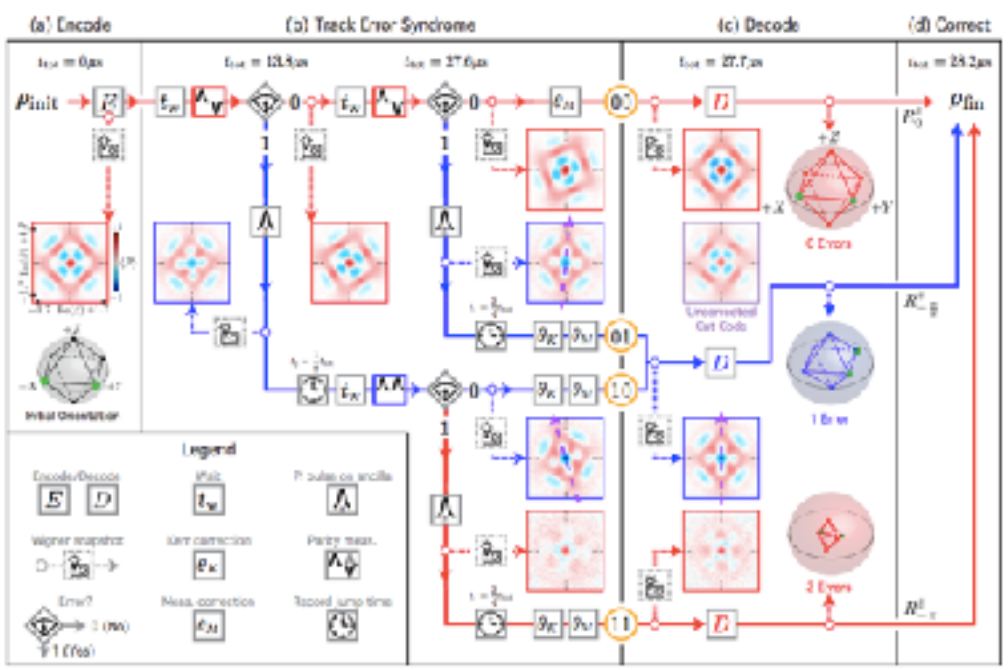
Teleportation using feed forward

[Steffen et al., Nature 2013 (Zurich)]



Canceling the dephasing induced by the measurement

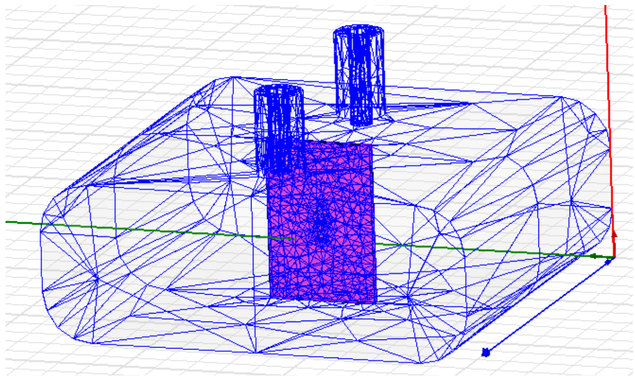
[de Lange et al., PRL 2014 (Delft)]



Parity measurement for quantum error correction

[Offek et al., Nature 2016 (Yale)]

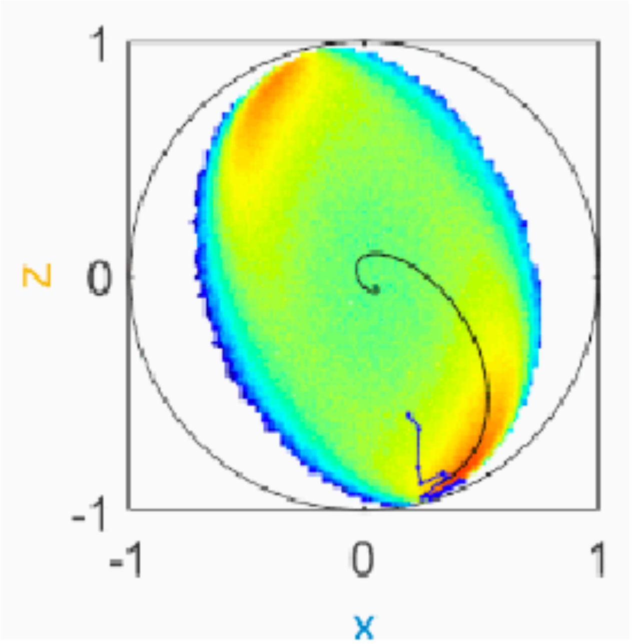
Quantum trajectories and feedback in circuit-QED

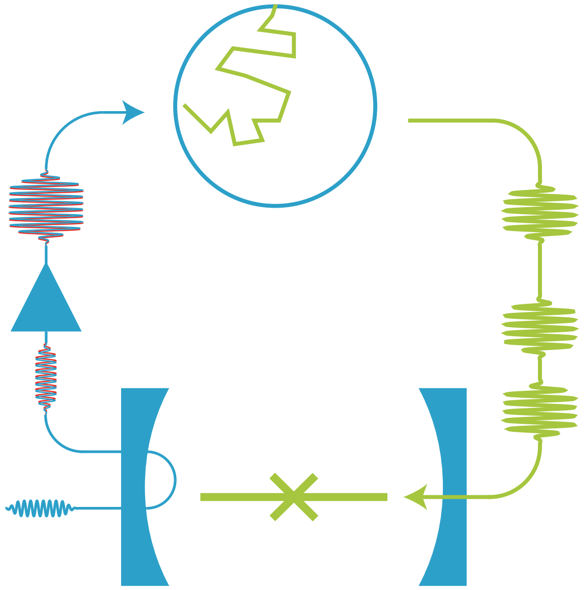


Introduction to circuit-QED

Quantum trajectories in circuit-QED

- dispersive measurement
- fluorescence measurement
- both simultaneously
- generating entanglement





Measurement based feedback

- dispersive case
- **fluorescence case**

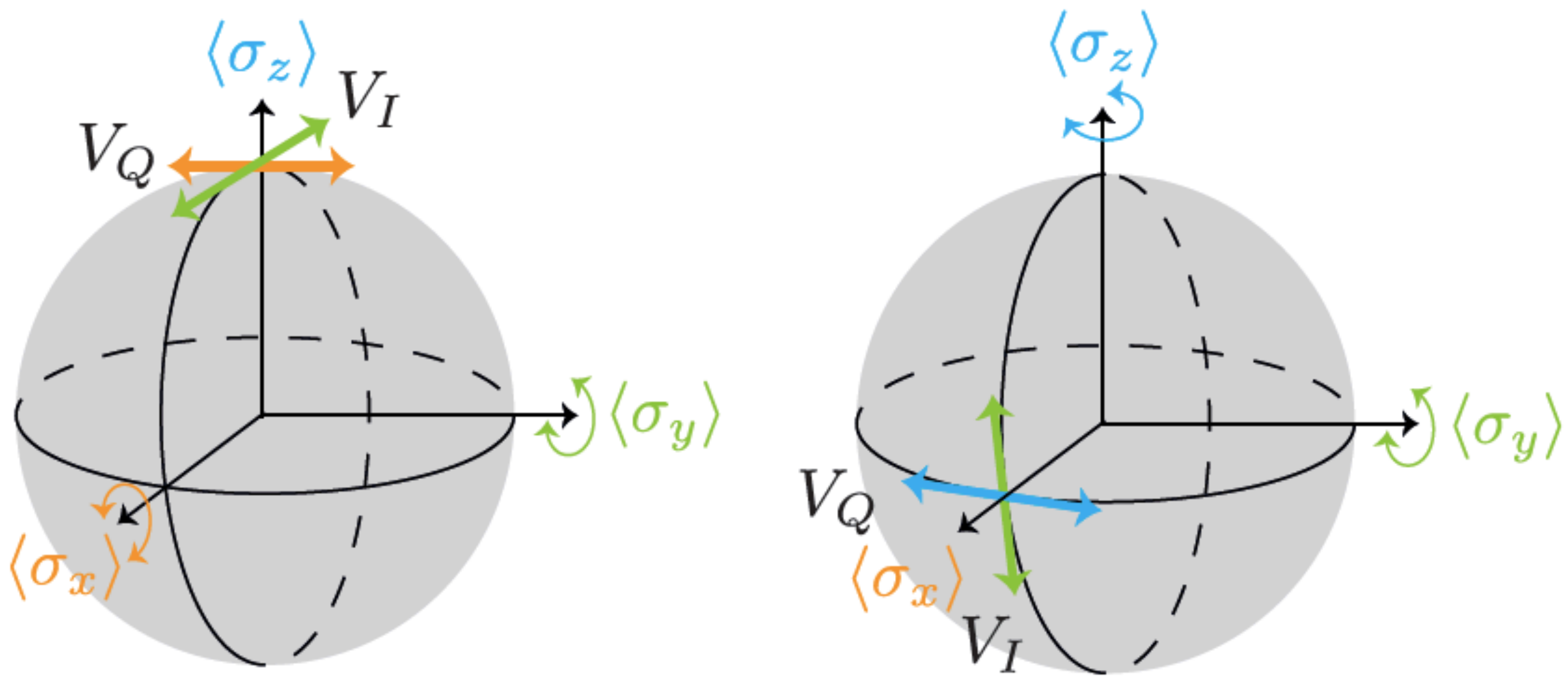
Post selection in quantum mechanics

$\rho(t), E(t)$

Fluorescence based feedback

stabilize target $\cos\frac{\theta}{2}|e\rangle + \sin\frac{\theta}{2}e^{i\varphi}|g\rangle$

compensate stochastic kicks due to fluorescence?



use 3 rotation axes and Markovian feedback

[Campagne-Ibarcq *et al.*, PRL (2016)]

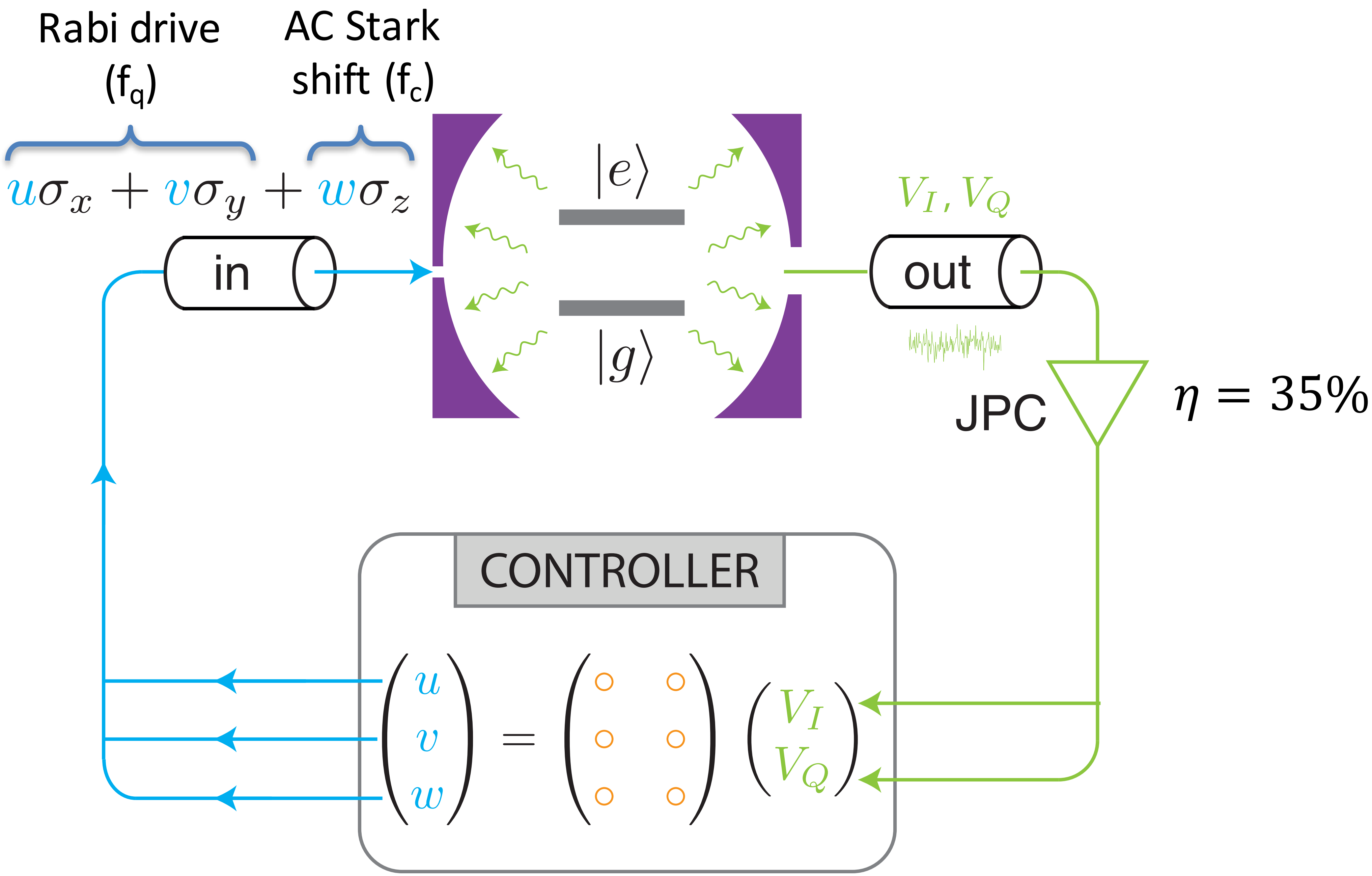
previous proposals

south hemisphere only [Hofmann, Mahler, Hess, PRA (1998)]

every state but equator [Wang, Wiseman, PRA (2001)]

Fluorescence based feedback

stabilize target $\cos\frac{\theta}{2}|e\rangle + \sin\frac{\theta}{2}e^{i\varphi}|g\rangle$



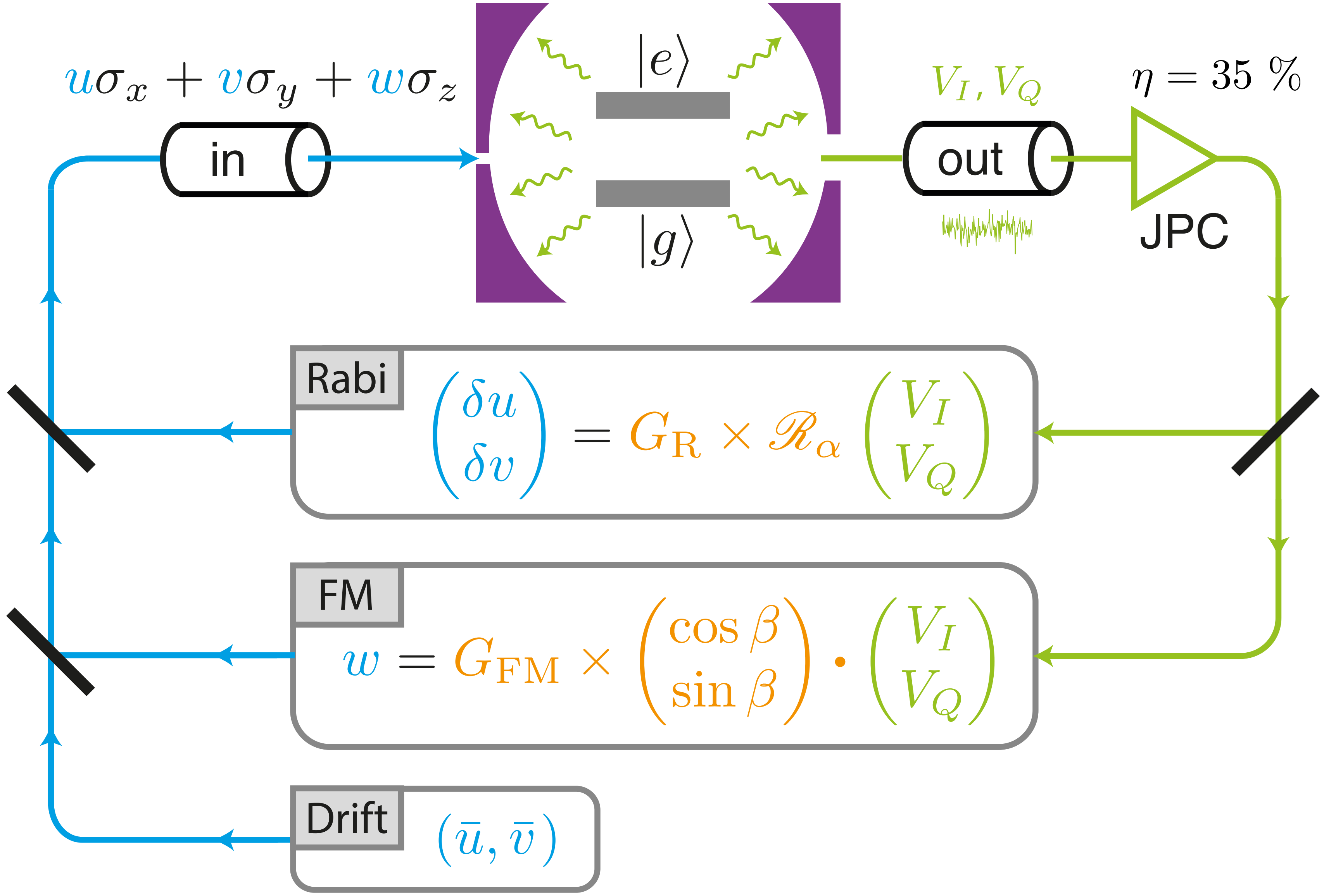
multi inputs and multi output Markovian feedback

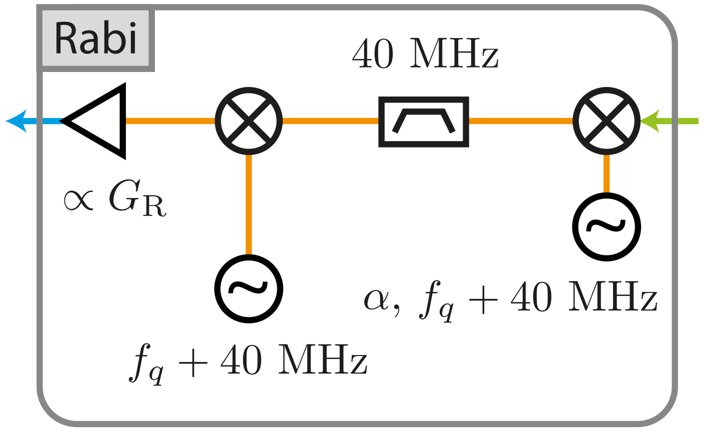
[Campagne-Ibarcq *et al.*, PRL (2016)]

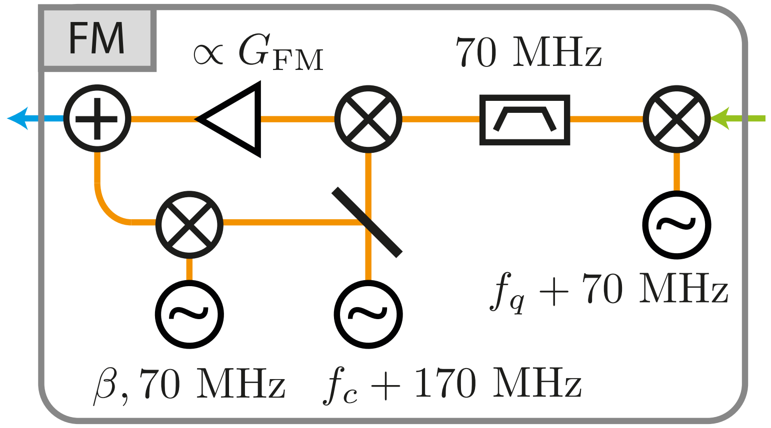
Fluorescence based feedback

stabilize target $\cos\frac{\theta}{2}|e\rangle + \sin\frac{\theta}{2}e^{i\varphi}|g\rangle$

$$\begin{cases} G_\mathrm{R} = \sqrt{\frac{\gamma_1}{8\eta}}\,(1+\cos\theta)\,, & \alpha = \pi/2 \\ G_\mathrm{FM} = \sqrt{\frac{\gamma_1}{8\eta}}\,\sin\theta, & \beta = \varphi - \pi/2 \\ -\frac{\bar{u}}{\sin\varphi} = \frac{\bar{v}}{\cos\varphi} = \frac{\gamma_1}{8\eta}\,(\cos\theta-\eta)\sin\theta & \end{cases}$$





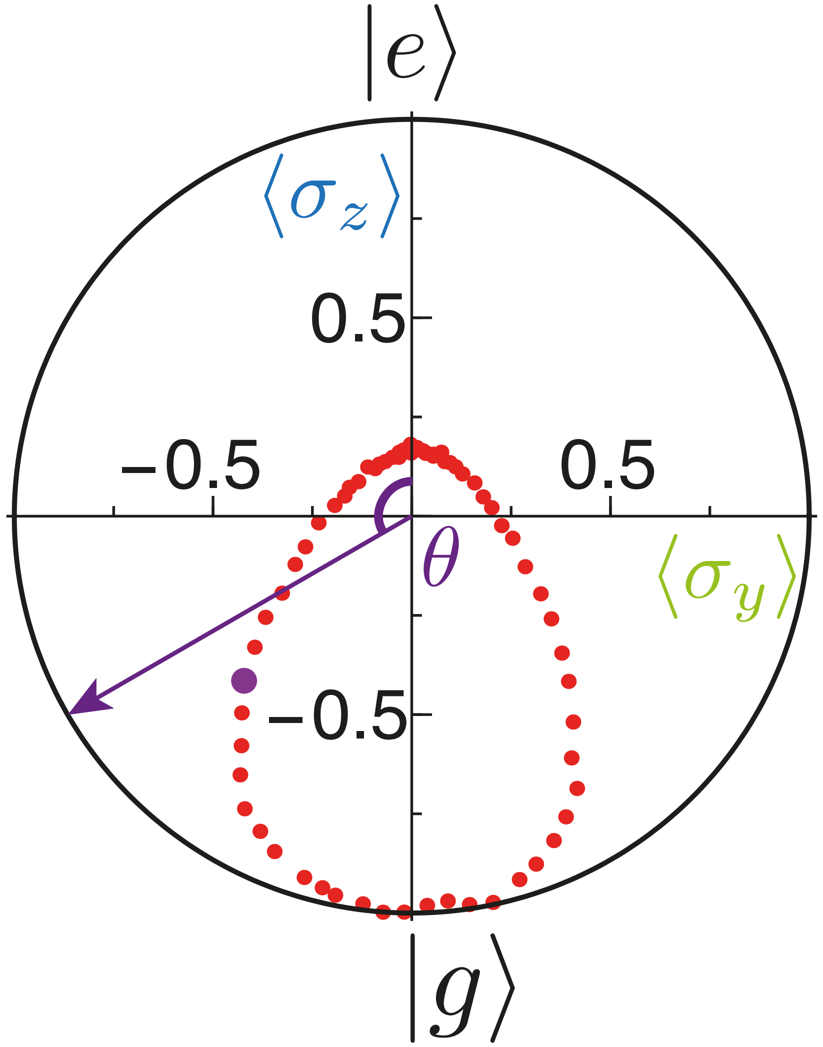


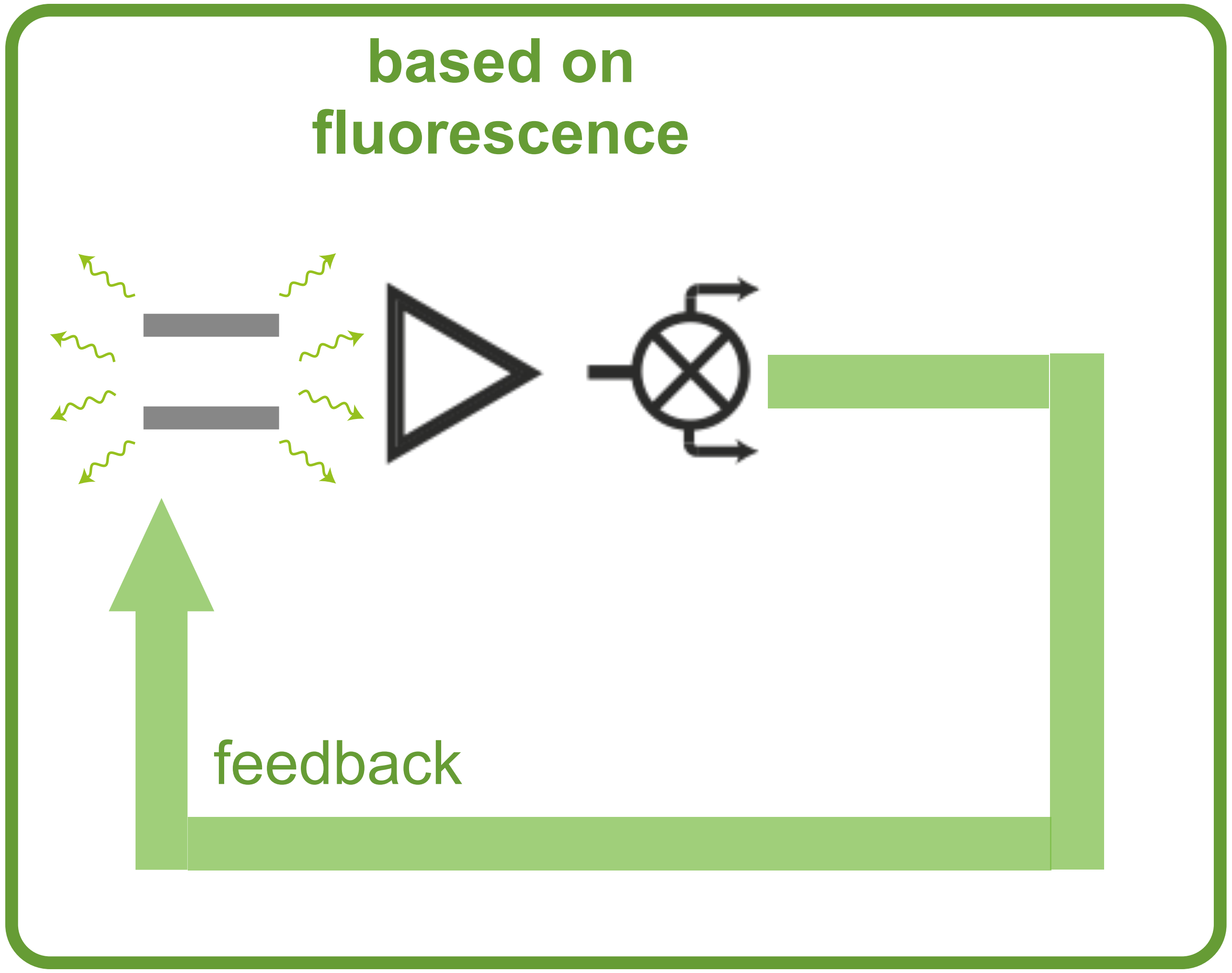
[Campagne-Ibarcq *et al.*, PRL (2016)]

Fluorescence based feedback

stabilize target $\cos\frac{\theta}{2}|e\rangle + \sin\frac{\theta}{2}e^{i\varphi}|g\rangle$

Stabilization of any state

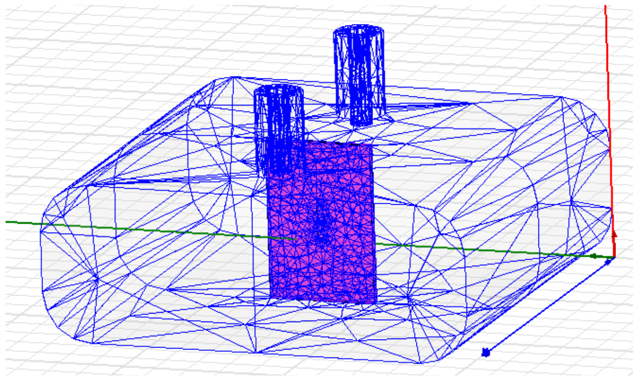




continuous measurement based feedback
with **multi inputs and multi outputs**
in the quantum regime

[Campagne-Ibarcq *et al.*, PRL (2016)]

Quantum trajectories and feedback in circuit-QED



Introduction to circuit-QED

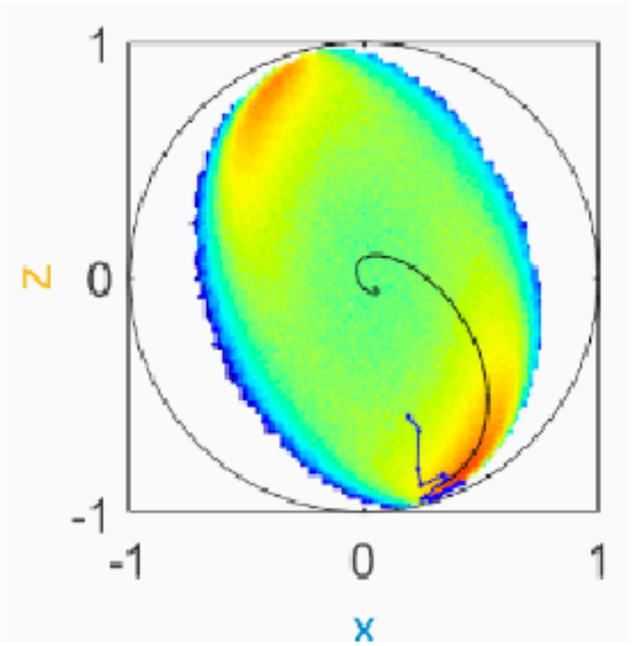
Quantum trajectories in circuit-QED

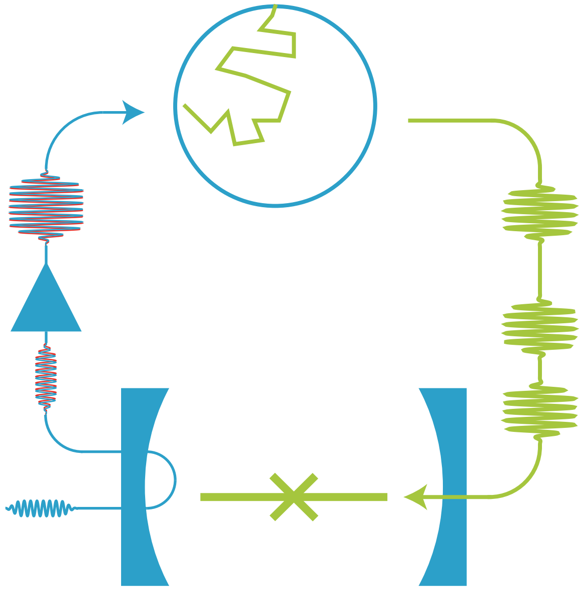
dispersive measurement

fluorescence measurement

both simultaneously

generating entanglement





Measurement based feedback

dispersive case

fluorescence case

Post selection in quantum mechanics

$\rho(t), E(t)$

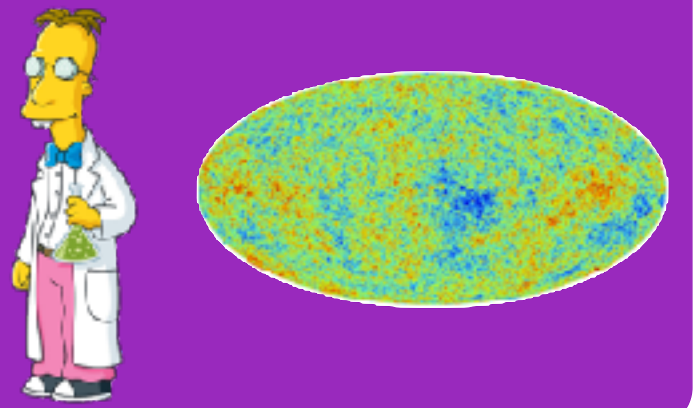
Generalized measurements

System

To predict the statistics of measurement outcomes at time t, all there is to know about the past is encoded in

$$\rho(\mathbf{t})$$

Environment



For each possible outcome m, there is a measurement operator $\hat{M}_m$ acting on the qubit such that

$$\mathcal{P}(m) = \mathrm{Tr}\left[\hat{M}_m \rho(\mathbf{t}) \hat{M}_m^\dagger\right]$$

the new state becomes

$$\frac{\hat{M}_m \rho(\mathbf{t}) \hat{M}_m^\dagger}{\mathcal{P}(m)}$$

last example: $\hat{M}_0 = |0\rangle\langle 0| \quad \hat{M}_1 = |1\rangle\langle 1|$

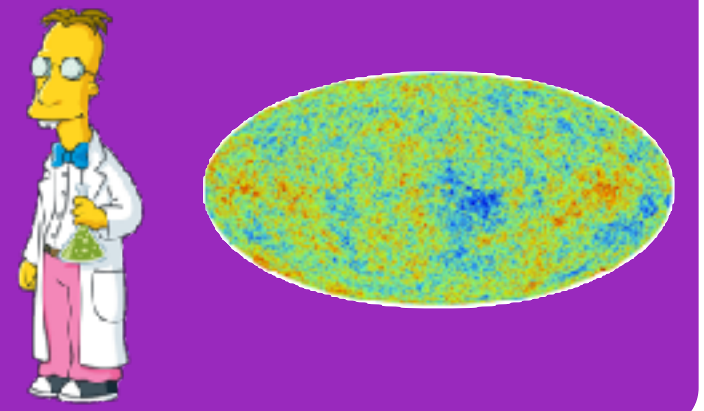
Generalized measurements

System

To predict the statistics of measurement outcomes at time t, all there is to know about the past is encoded in

$$\rho(\mathbf{t})$$

Environment



For each possible outcome m, there is a measurement operator $\hat{M}_m$ acting on the qubit such that

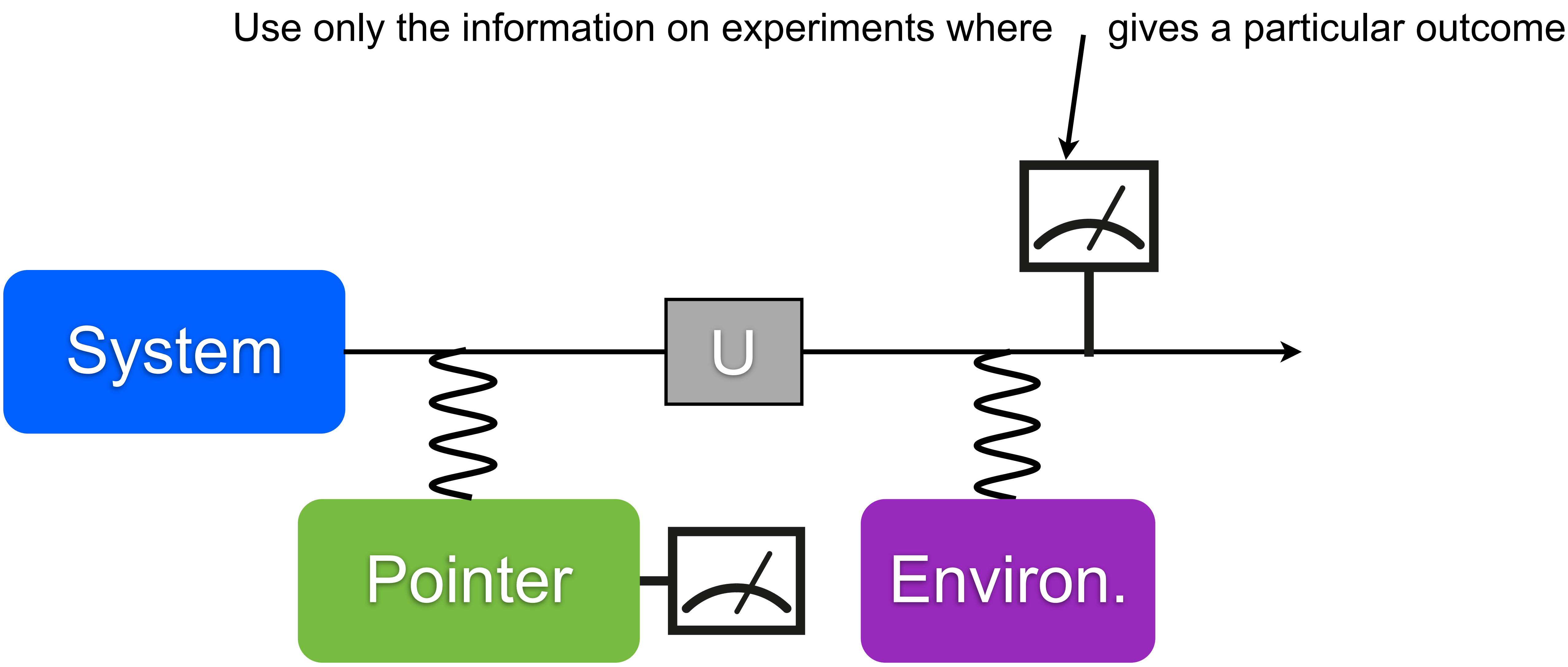
$$\mathcal{P}(m) = \mathrm{Tr}\left[\hat{M}_m \rho(\mathbf{t}) \hat{M}_m^\dagger\right]$$

the new state becomes

$$\frac{\hat{M}_m \rho(\mathbf{t}) \hat{M}_m^\dagger}{\mathcal{P}(m)}$$

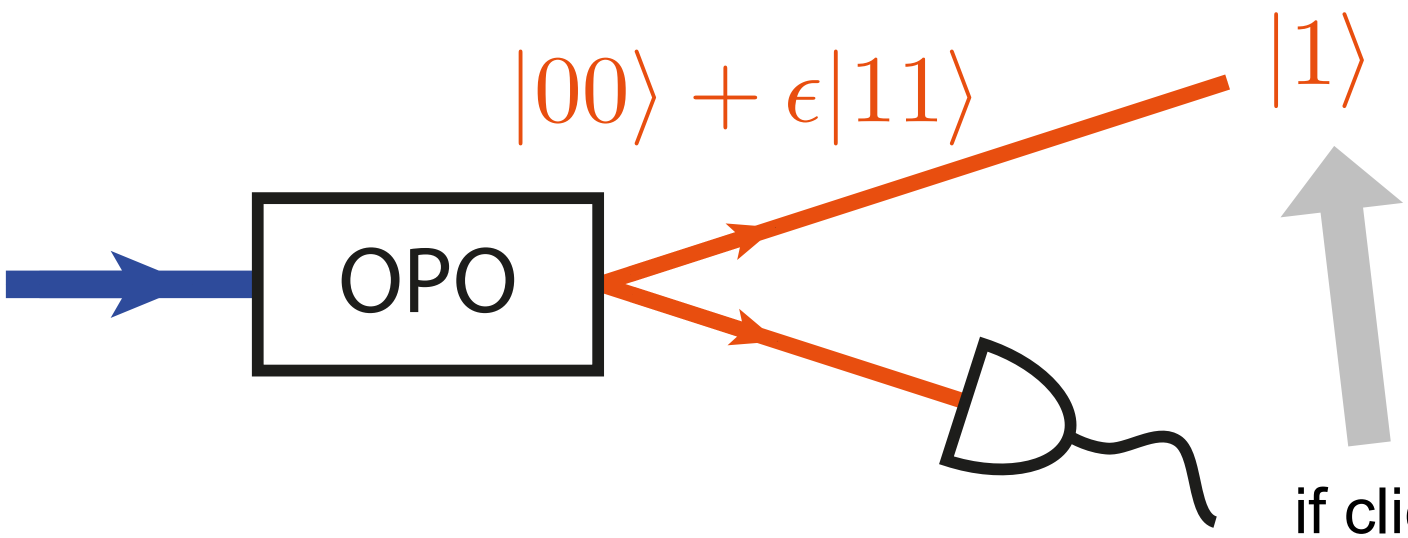
What if we also know the result of a measurement in the future?

Post-selection on quantum systems

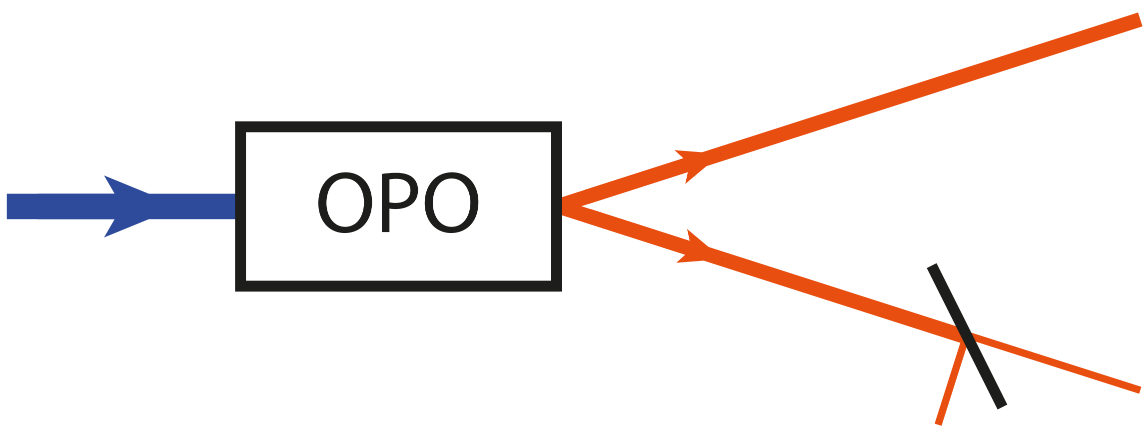


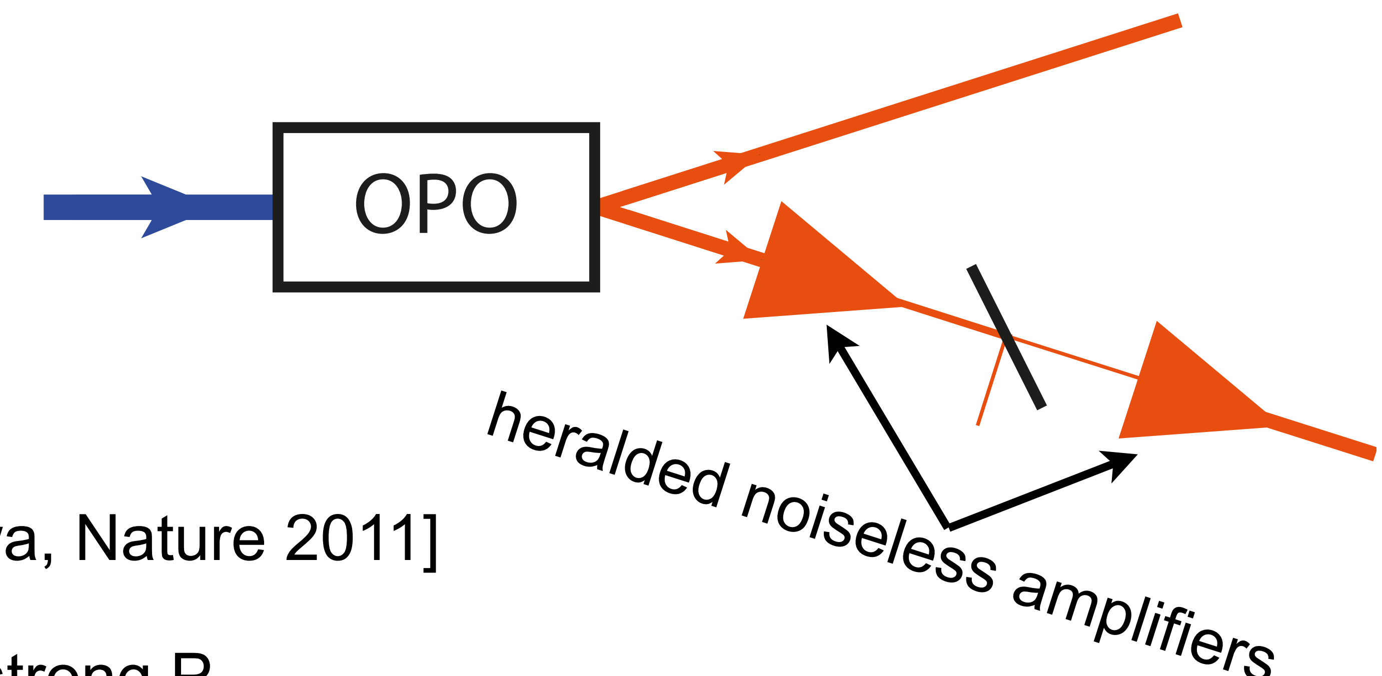
Use of post-selection

Single photon generation from an OPO [Rochester, PRL 1985]

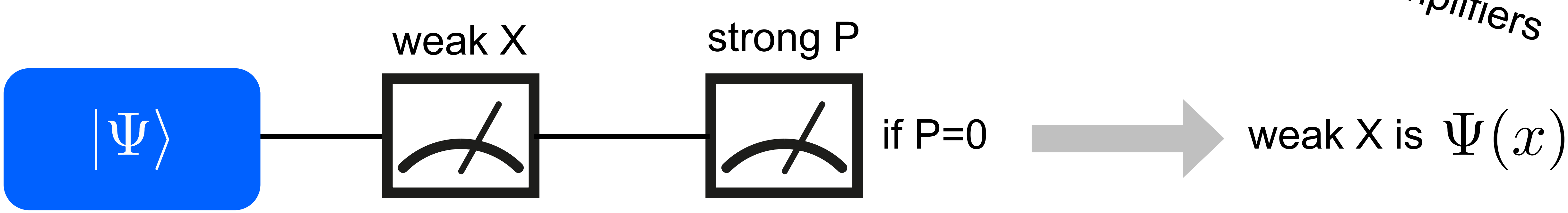


Compensate for losses in Q communication [Olomouc, PRL 2012]





Direct « probing of wavefunctions » [Ottawa, Nature 2011]



Generalized measurements

System

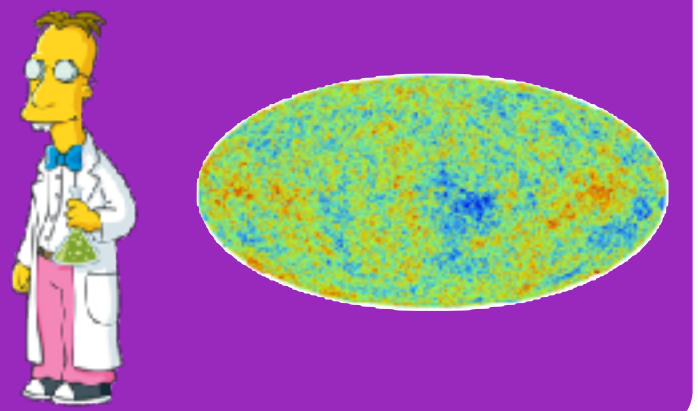
To predict the statistics of measurement outcomes at time t, all there is to know about the **past** is encoded in

$$\rho(\mathbf{t})$$

To predict the statistics of measurement outcomes at time t, all there is to know about the **future** is encoded in

$$\mathbf{E}(\mathbf{t})$$

Environment



For each possible outcome m, there is a measurement operator $\hat{M}_m$ acting on the qubit such that

$$\mathcal{P}(m) = \frac{1}{\mathcal{N}} \mathrm{Tr}\left[\hat{M}_m \rho(\mathbf{t}) \hat{M}_m^\dagger \mathbf{E}(\mathbf{t})\right]$$

$$\mathcal{N} = \sum_m \mathrm{Tr}\left[\hat{M}_m \rho(\mathbf{t}) \hat{M}_m^\dagger \mathbf{E}(\mathbf{t})\right]$$

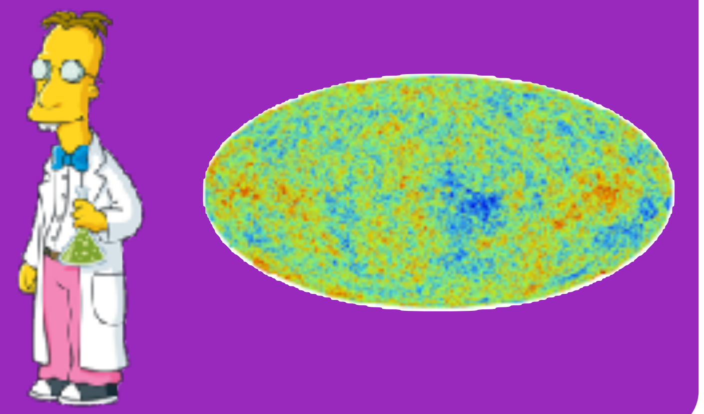
example: $\mathbf{E}(\mathbf{t}) = 0.8|0\rangle\langle 0| + 0.2|1\rangle\langle 1|$

[Wiseman, PRA 2002] [Tsang, PRA 2009]
[Gammelmark, Julsgaard, Mølmer, PRL 2013]

Generalized measurements

System

Environment



To predict the statistics of measurement outcomes at time t, all there is to know about the **past** is encoded in

$$\rho(\mathbf{t})$$

To predict the statistics of measurement outcomes at time t, all there is to know about the **future** is encoded in

$$\mathbf{E}(\mathbf{t})$$

For each possible outcome m, there is a measurement operator $\hat{M}_m$ acting on the qubit such that

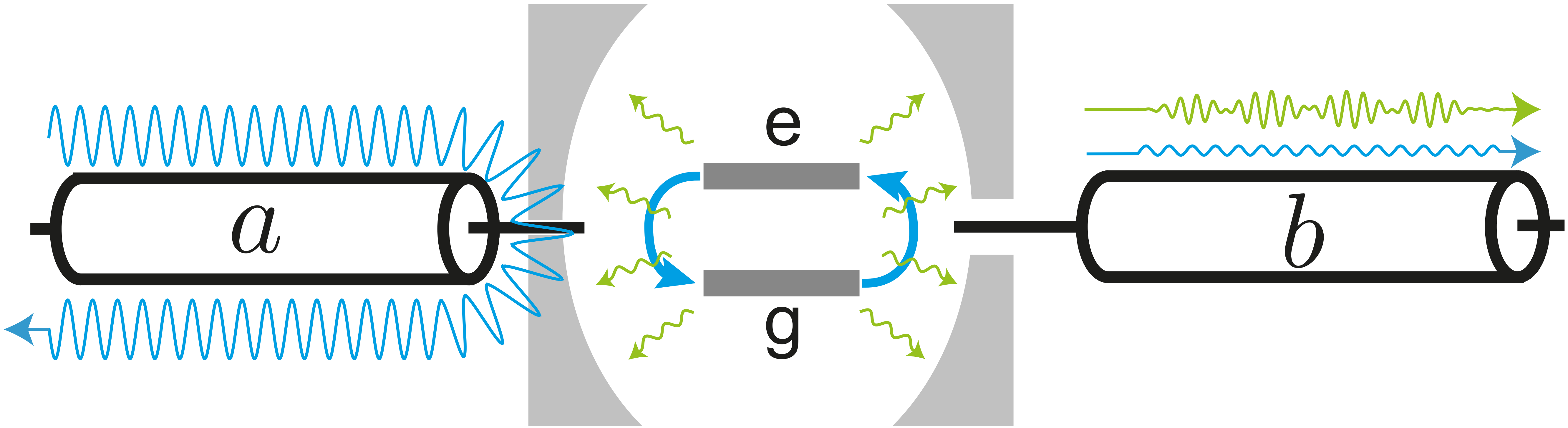
$$\mathcal{P}(m) = \frac{1}{\mathcal{N}} \mathrm{Tr}\left[\hat{M}_m \rho(\mathbf{t}) \hat{M}_m^\dagger \mathbf{E}(\mathbf{t})\right]$$

$$\mathcal{N} = \sum_m \mathrm{Tr}\left[\hat{M}_m \rho(\mathbf{t}) \hat{M}_m^\dagger \mathbf{E}(\mathbf{t})\right]$$

Test on the simplest open quantum system...

[Wiseman, PRA 2002] [Tsang, PRA 2009]
[Gammelmark, Julsgaard, Mølmer, PRL 2013]

Resonance fluorescence in time domain



$\nu_{\rm cav} \approx 8$ GHz $\quad \nu_{\rm q} \approx 5$ GHz $\quad \Gamma_b \approx 300$ kHz

$$\langle b_{out} \rangle = \langle b_{out} \rangle_0 - \sqrt{\gamma_{1b}} \langle \sigma_- \rangle$$

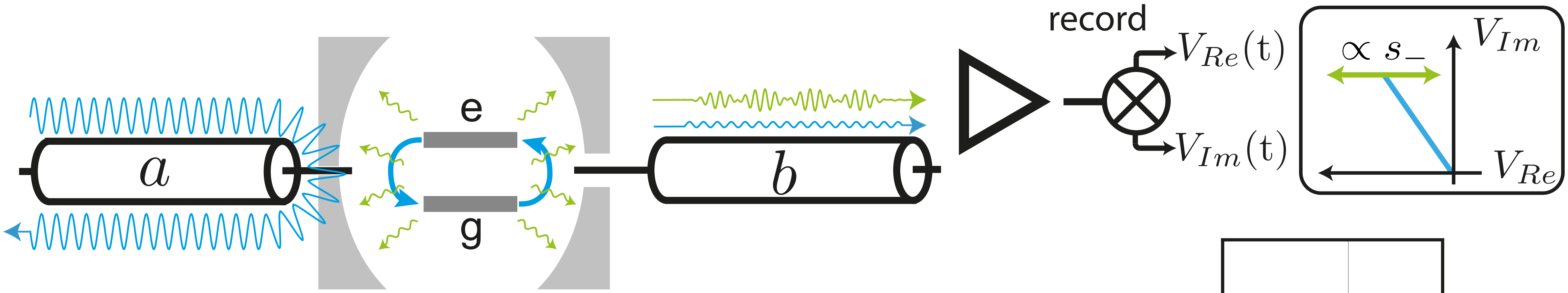
leak through cavity

spontaneous emission into b line

$$\sigma_- = |g\rangle\langle e| = \frac{\sigma_x - i\sigma_y}{2}$$

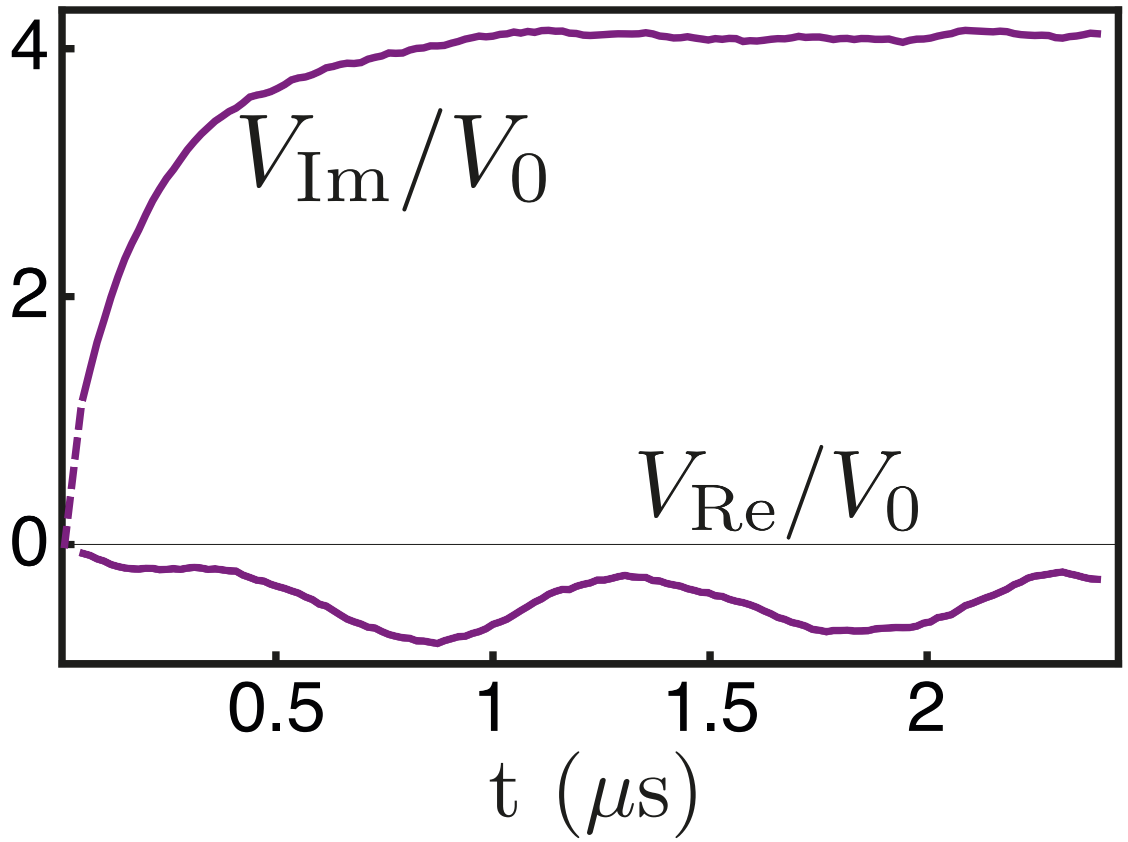
$$\gamma_{1b} \approx \frac{1}{92\ \mu s}$$

Resonance fluorescence in time domain

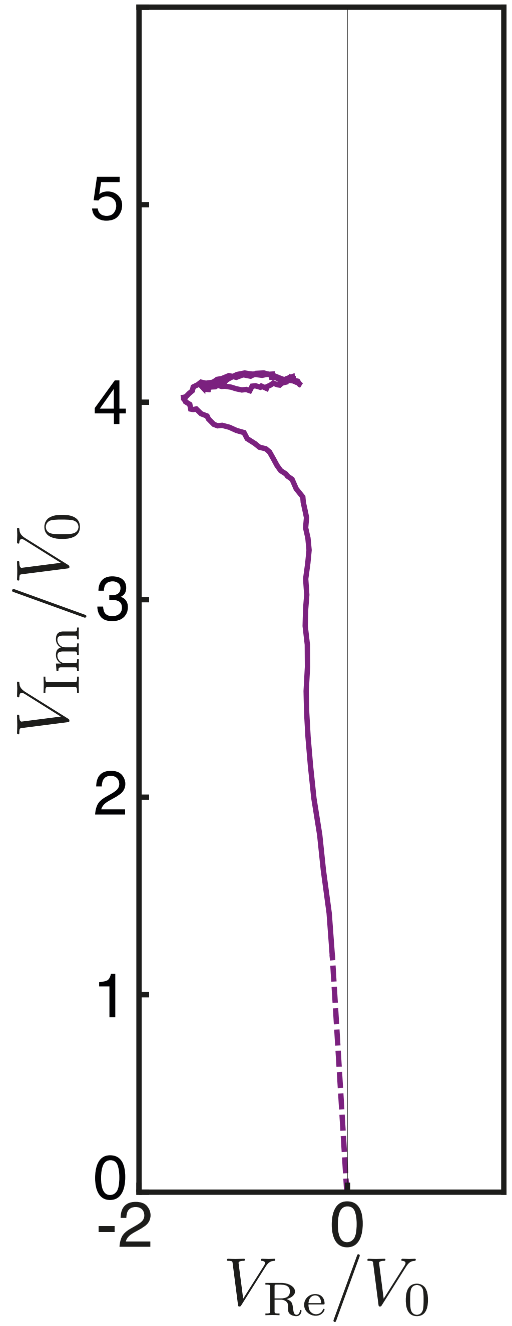


$$\overline{V_{\mathrm{Re}}}(t) = \overline{V_{\mathrm{Re}}^{(0)}}(t) - V_0 \mathrm{Re}\langle \sigma_- \rangle$$

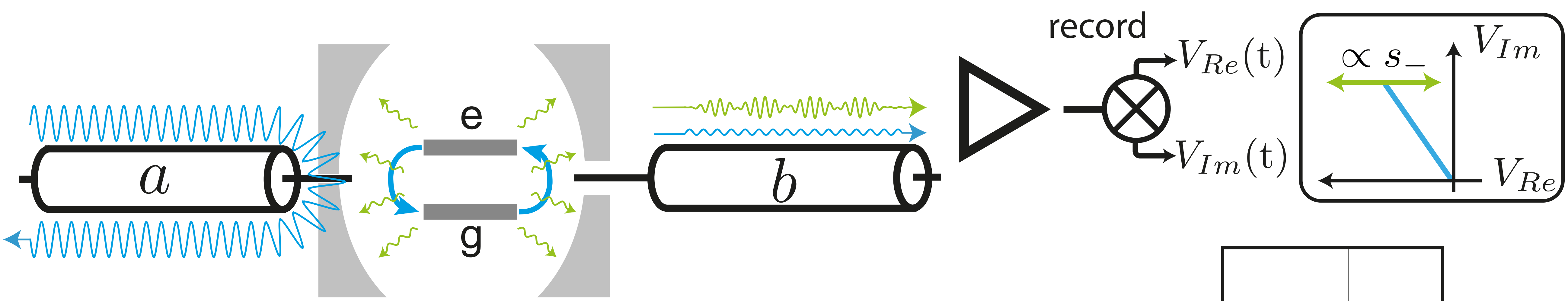
$$\overline{V_{\mathrm{Im}}}(t) = \overline{V_{\mathrm{Im}}^{(0)}}(t) - V_0 \mathrm{Im}\langle \sigma_- \rangle$$



qubit starts in $|g\rangle$

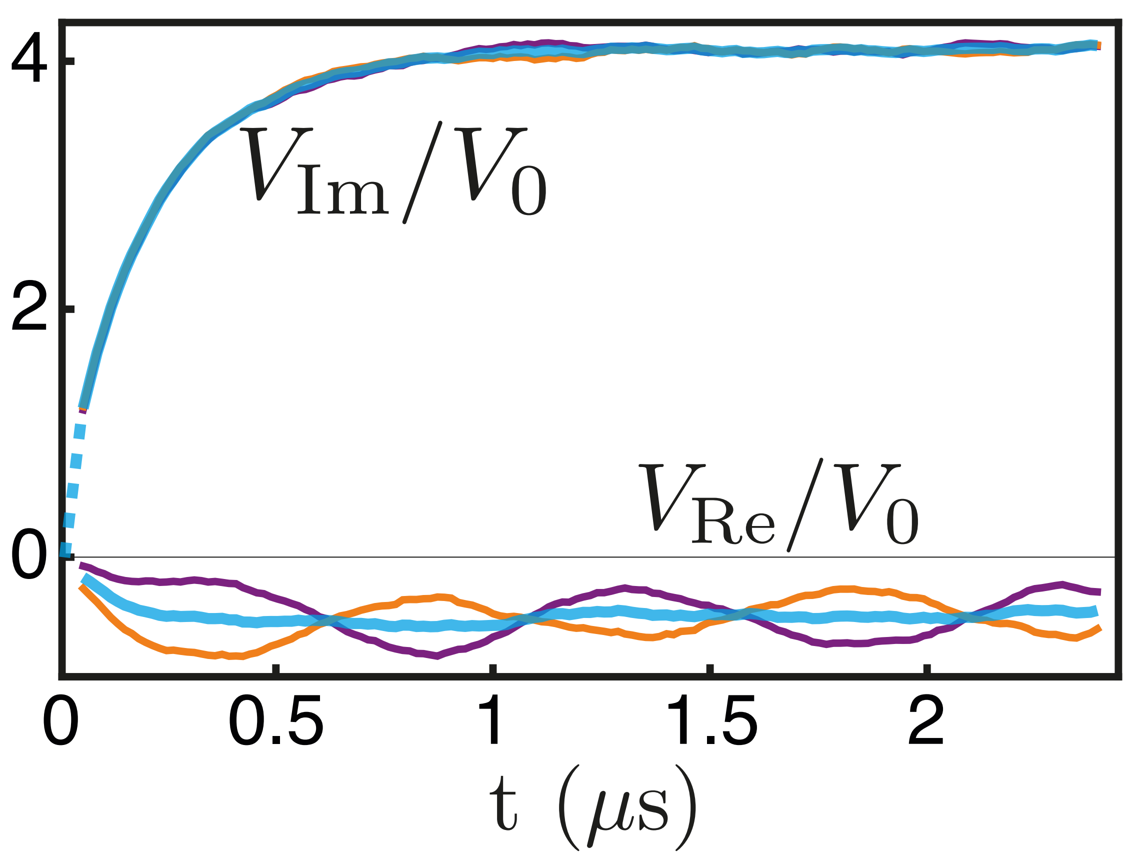


Resonance fluorescence in time domain



$$\overline{V_{\mathrm{Re}}}(t) = \overline{V_{\mathrm{Re}}^{(0)}}(t) - V_0 \mathrm{Re}\langle \sigma_- \rangle$$

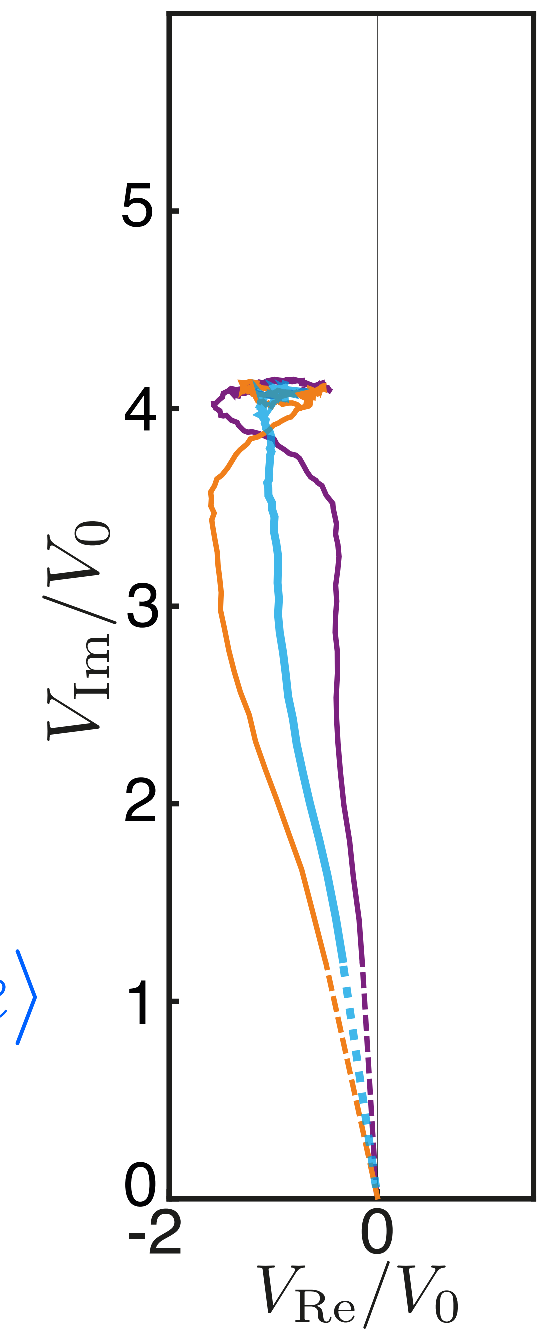
$$\overline{V_{\mathrm{Im}}}(t) = \overline{V_{\mathrm{Im}}^{(0)}}(t) - V_0 \mathrm{Im}\langle \sigma_- \rangle$$



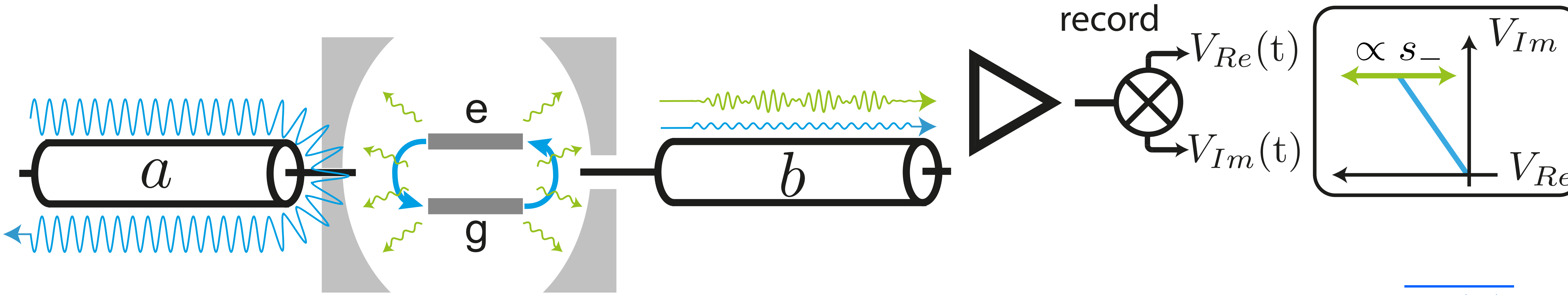
qubit starts in $|g\rangle$

qubit starts in $|e\rangle$

qubit starts in $|g\rangle$ or $|e\rangle$



Resonance fluorescence in time domain



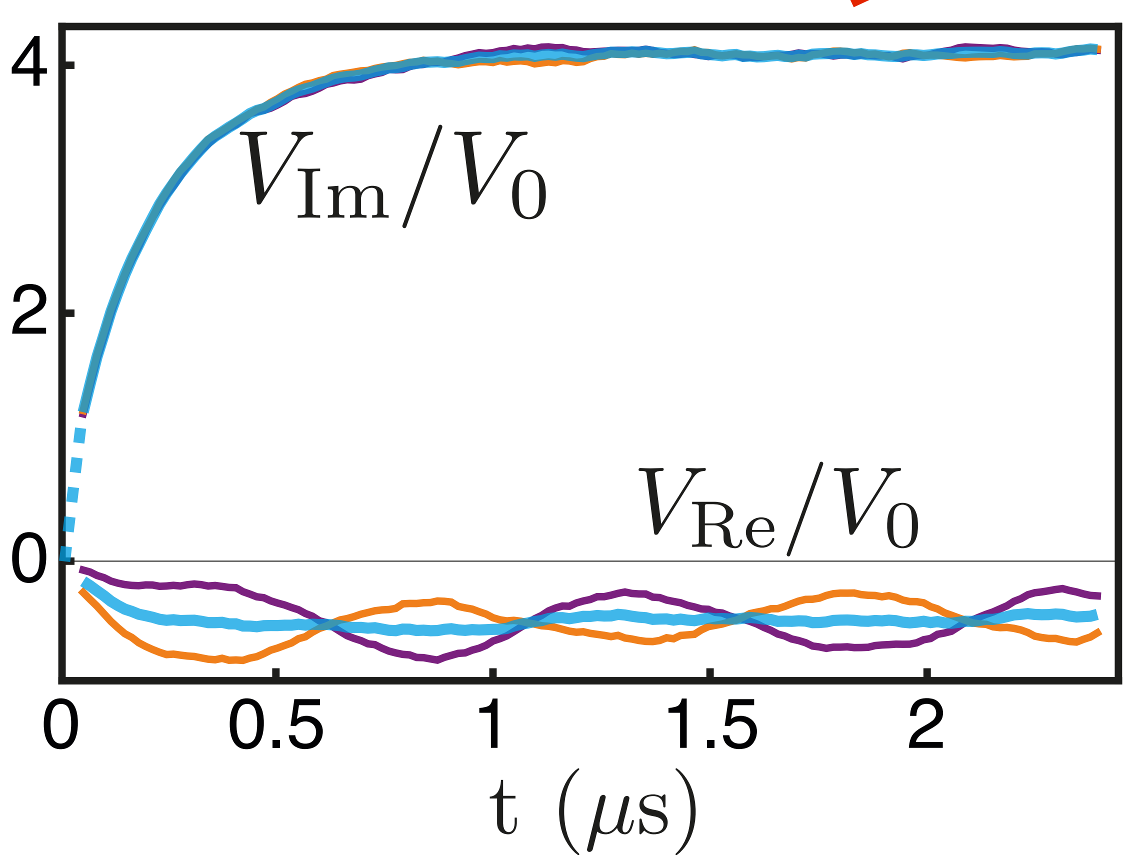
$$\overline{V_{\text{Re}}}(t) = \overline{V_{\text{Re}}^{(0)}}(t) - V_0 \text{Re}\langle\sigma_-\rangle$$

$$s_-(t) \equiv \frac{V_{\text{Re}}(t) - \overline{V_{\text{Re}}^{(0)}}(t)}{V_0}$$

$$\overline{V_{\text{Im}}}(t) = \overline{V_{\text{Im}}^{(0)}}(t) - V_0 \text{Im}\langle\sigma_-\rangle$$

if qubit driven around Y

$$\sigma_- = |g\rangle\langle e| = \frac{\sigma_x - i\sigma_y}{2}$$

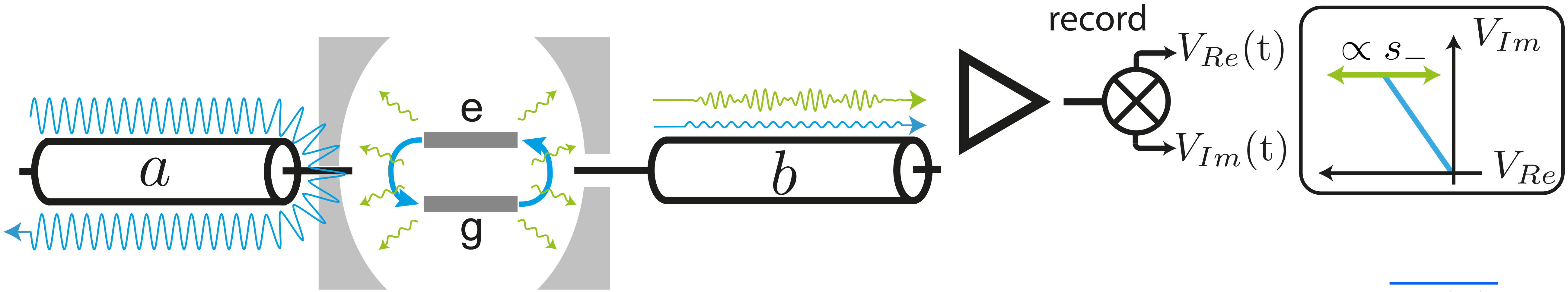


qubit starts in $|g\rangle$

qubit starts in $|e\rangle$

qubit starts in $|g\rangle$ or $|e\rangle$

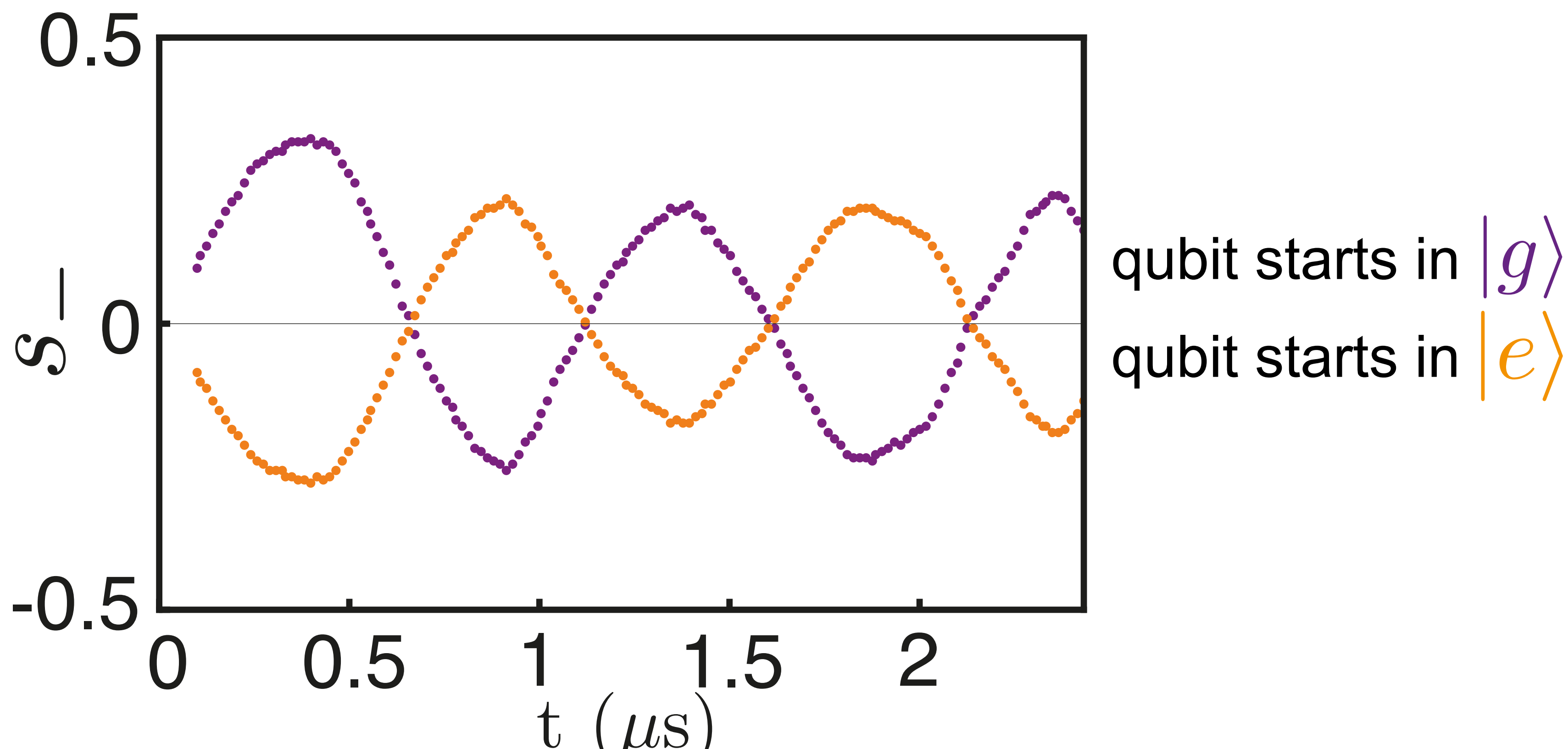
Resonance fluorescence in time domain



$$\overline{V_{\mathrm{Re}}}(t) = \overline{V_{\mathrm{Re}}^{(0)}}(t) - V_0 \mathrm{Re}\langle\sigma_-\rangle \qquad s_-(t) \equiv \frac{V_{\mathrm{Re}}(t) - \overline{V_{\mathrm{Re}}^{(0)}}(t)}{V_0}$$

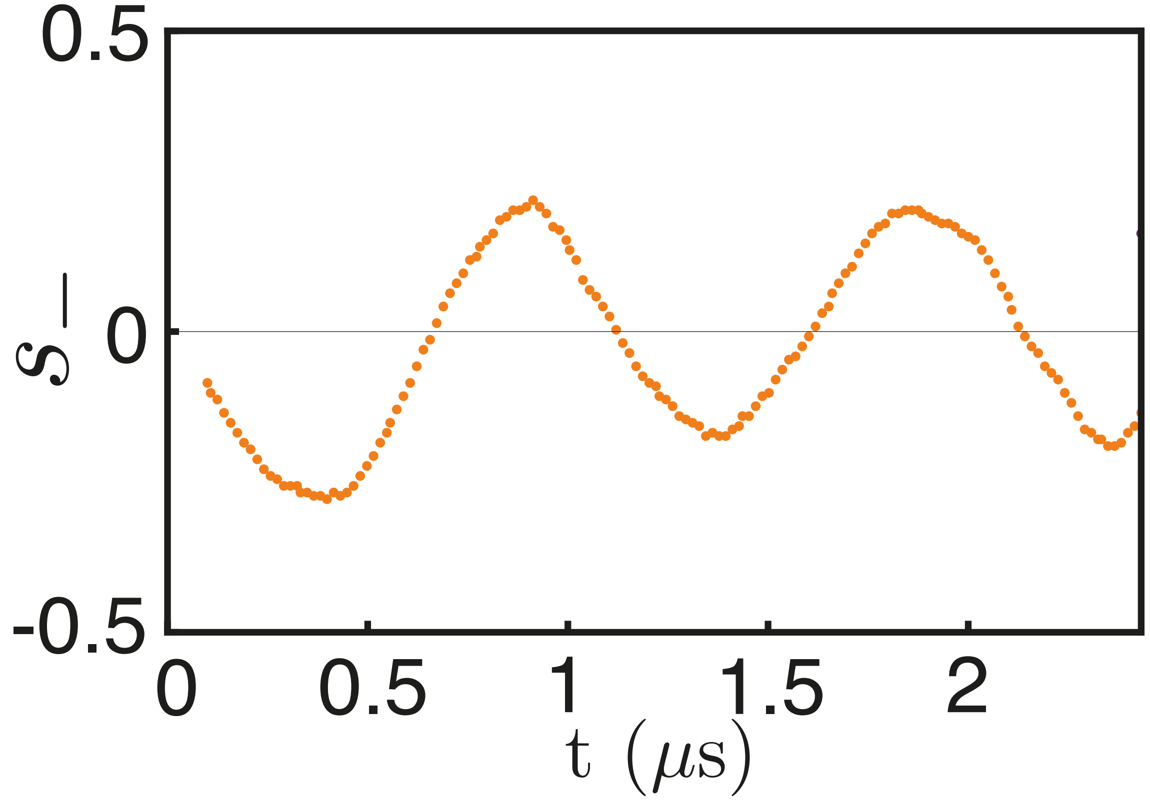
$$\overline{V_{\mathrm{Im}}}(t) = \overline{V_{\mathrm{Im}}^{(0)}}(t) - V_0 \mathrm{Im}\langle\sigma_-\rangle$$

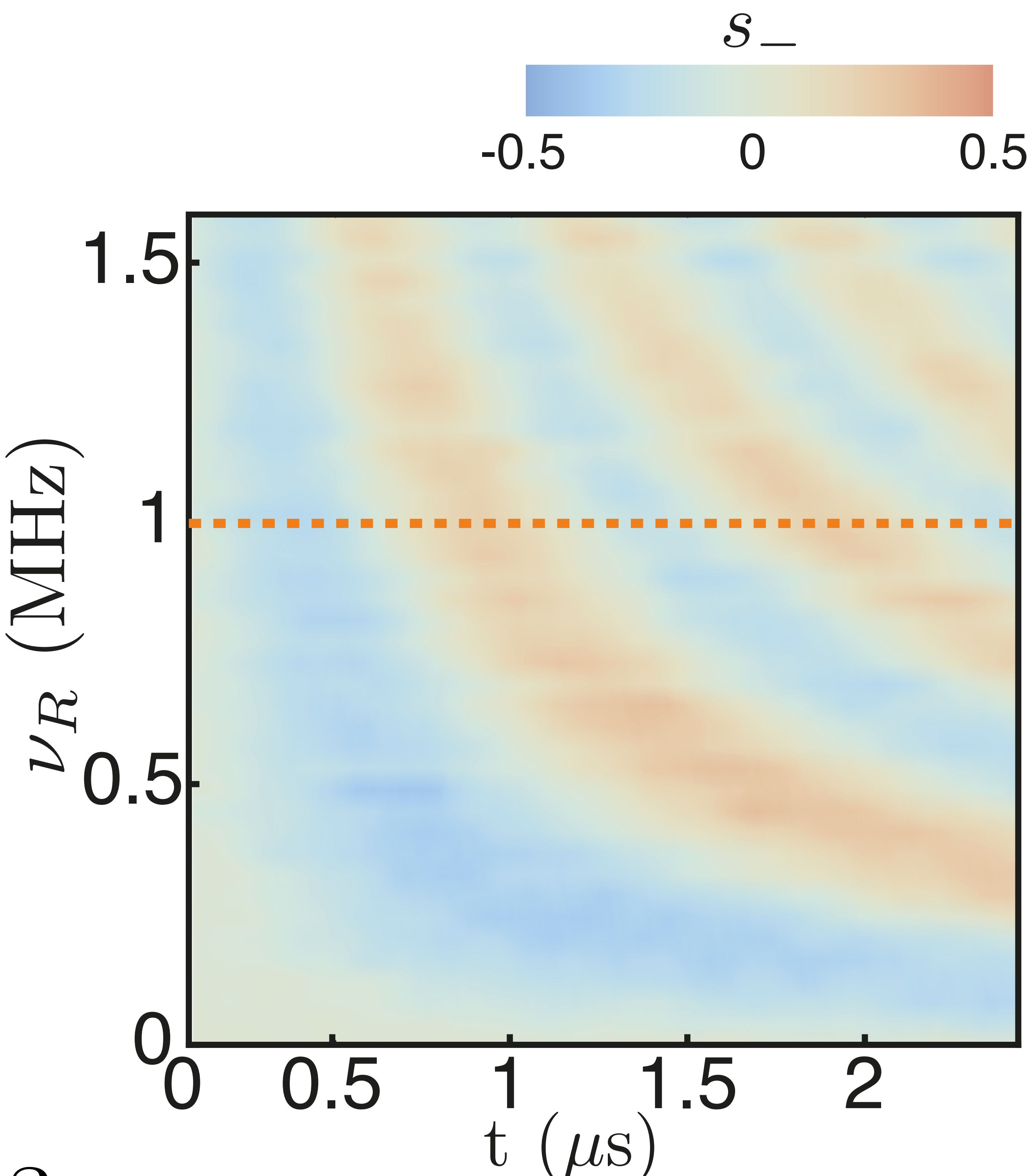
if qubit driven around Y



Resonance fluorescence in time domain

$$s_-(t) \equiv \frac{V_{\mathrm{Re}}(t) - \overline{V_{\mathrm{Re}}^{(0)}(t)}}{V_0}$$

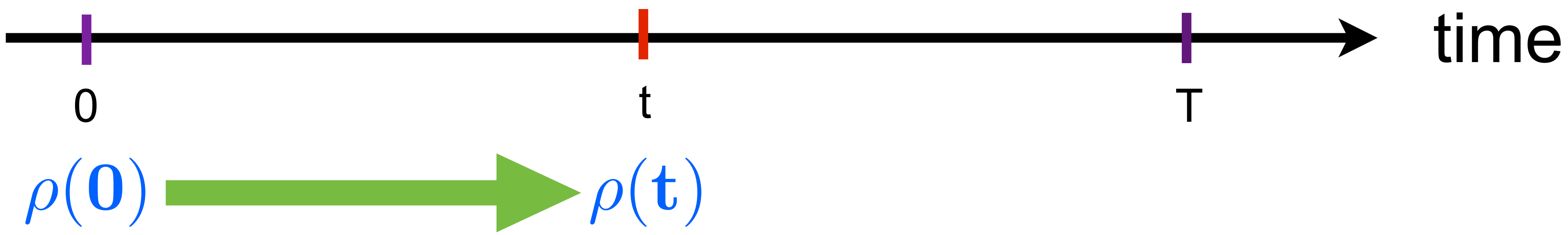




$$\overline{s_-}(t) = \mathrm{Re}\langle\sigma_-(t)\rangle?$$

Master equation

$$\mathrm{Re}\langle\sigma_-(t)\rangle = \frac{\langle\sigma_x(t)\rangle}{2} = \frac{\mathrm{Tr}(\sigma_x\rho(\mathbf{t}))}{2}$$



$$\rho(\mathbf{0}) = 0.85|e\rangle\langle e| + 0.15|g\rangle\langle g|$$ Preparation

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}\left[\tilde{H}, \rho\right] + \gamma_1\left(\sigma_-\rho\sigma_+ - \frac{1}{2}\left[\sigma_+\sigma_-\rho + \rho\sigma_+\sigma_-\right]\right)$$ Dynamics

$$\tilde{H} = \frac{1}{2}h\nu_q\sigma_z + \frac{1}{2}h\nu_R\sigma_y$$

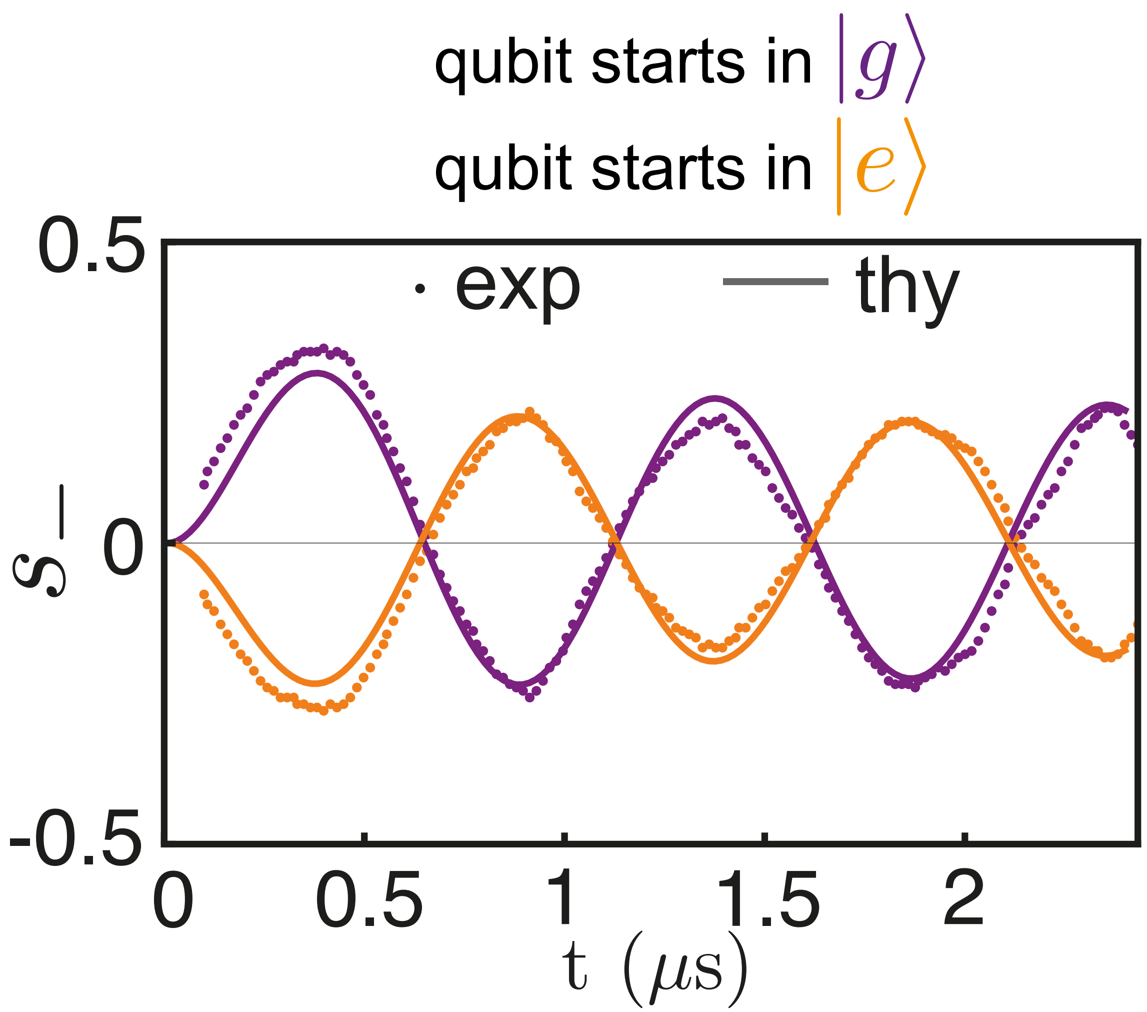
drive

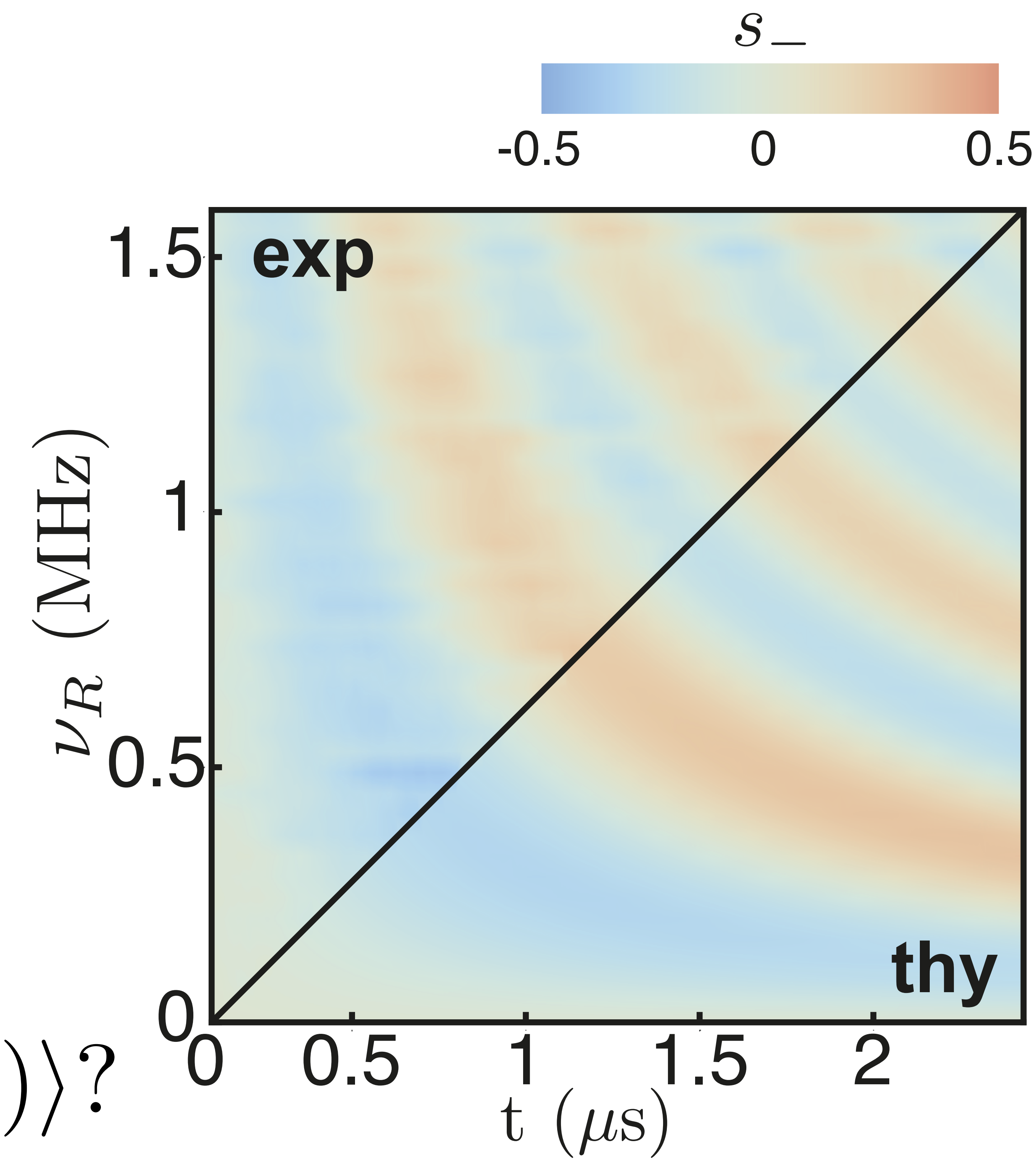
qubit relaxation $\frac{1}{\gamma_1} = 16\ \mu\mathrm{s}$

[Lindblad 1976]

Resonance fluorescence in time domain

$$s_-(t) \equiv \frac{V_{\mathrm{Re}}(t) - \overline{V_{\mathrm{Re}}^{(0)}}(t)}{V_0}$$

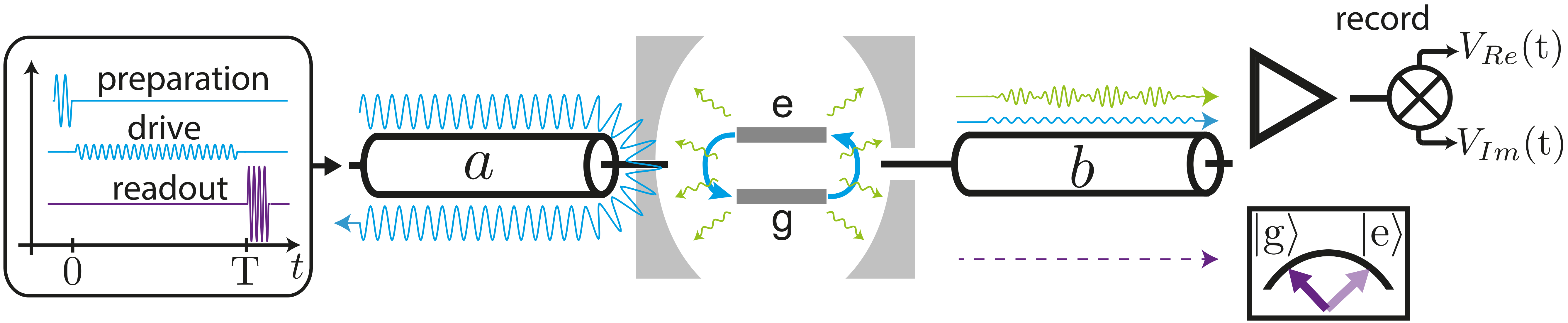




$$\overline{s_-}(t) = \mathrm{Re}\langle\sigma_-(t)\rangle?$$

Yes if considering the 1.6 MHz detector bandwidth

Scheme to get information on a future measurement

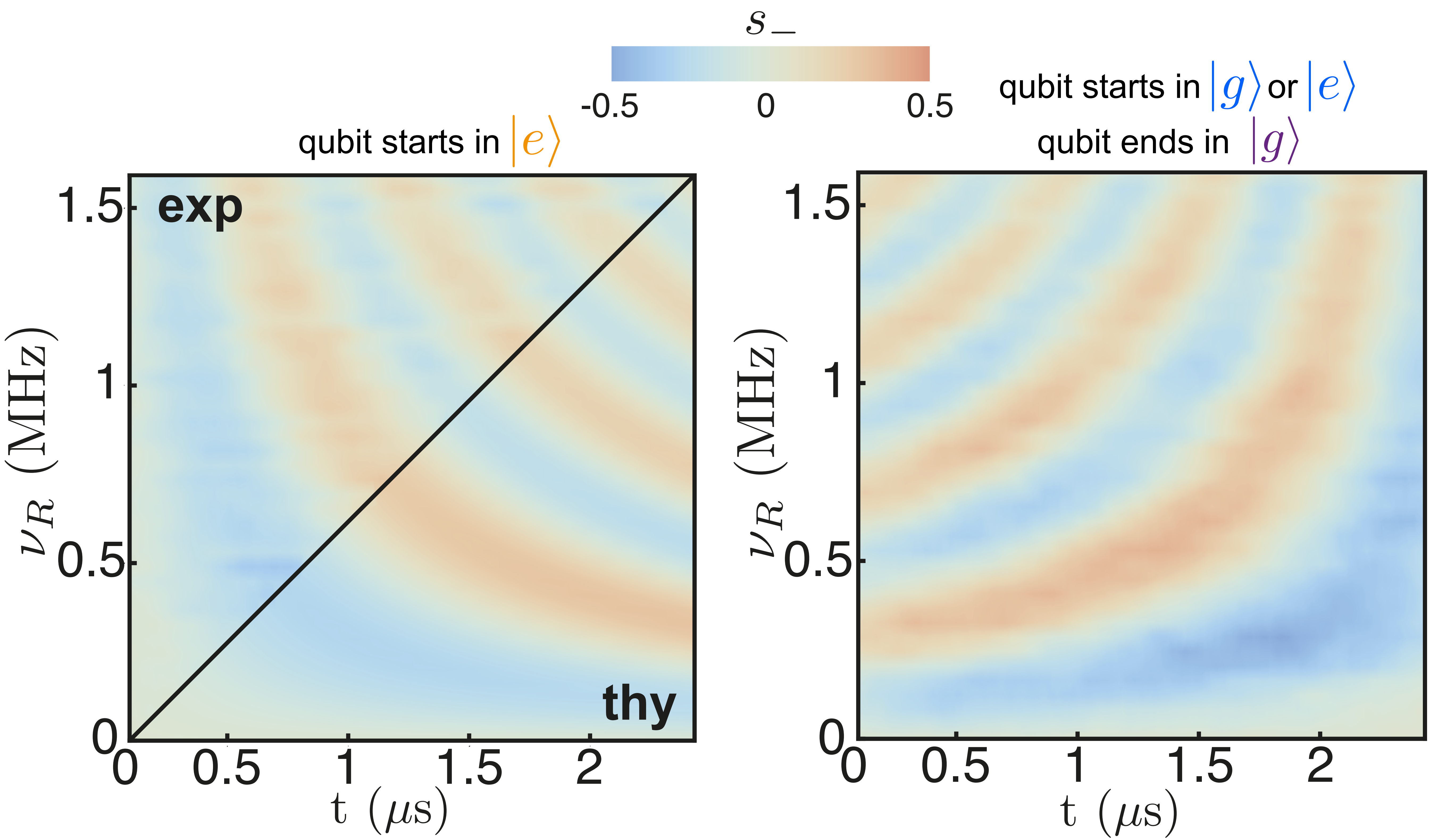


At time T=2.5 μs, projective measurement of σ_z

High power readout at f_{cav}

efficient (about 96%)
not QND
need to wait before repeating

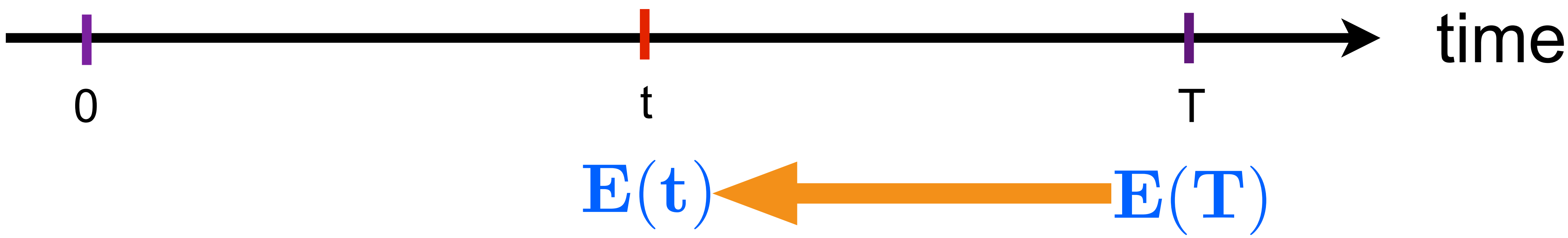
Comparison between preparation and postselection



Time reversed fluorescence traces

Master equation back in time

$$\mathrm{Re}\langle\sigma_-(t)\rangle = \frac{\langle\sigma_x(t)\rangle}{2} = \frac{\mathrm{Tr}(\mathbf{E}(\mathbf{t})\sigma_x)}{2\mathrm{Tr}\mathbf{E}(\mathbf{t})}$$



Post-selection

$$\mathbf{E}(\mathbf{T}) = 0.96|g\rangle\langle g| + 0.04|e\rangle\langle e|$$

Dynamics

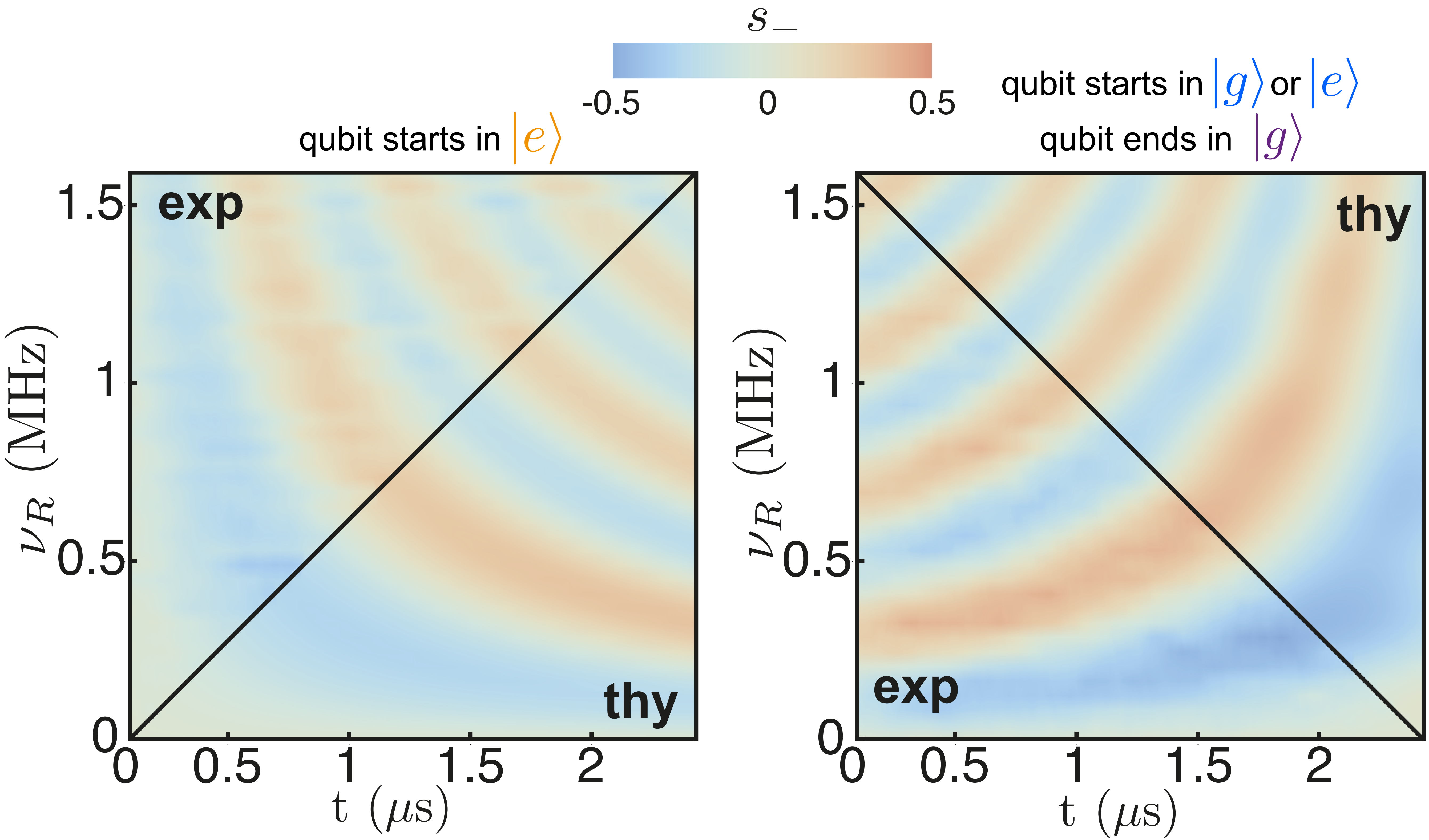
$$\frac{d\mathbf{E}}{dt} = -\frac{i}{\hbar}\left[\tilde{H}, \mathbf{E}\right] - \gamma_1\left(\sigma_+\mathbf{E}\sigma_- - \frac{1}{2}\left[\sigma_+\sigma_-\mathbf{E} + \mathbf{E}\sigma_+\sigma_-\right]\right)$$

[Wiseman, PRA 2002]

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}\left[\tilde{H}, \rho\right] + \gamma_1\left(\sigma_-\rho\sigma_+ - \frac{1}{2}\left[\sigma_+\sigma_-\rho + \rho\sigma_+\sigma_-\right]\right)$$

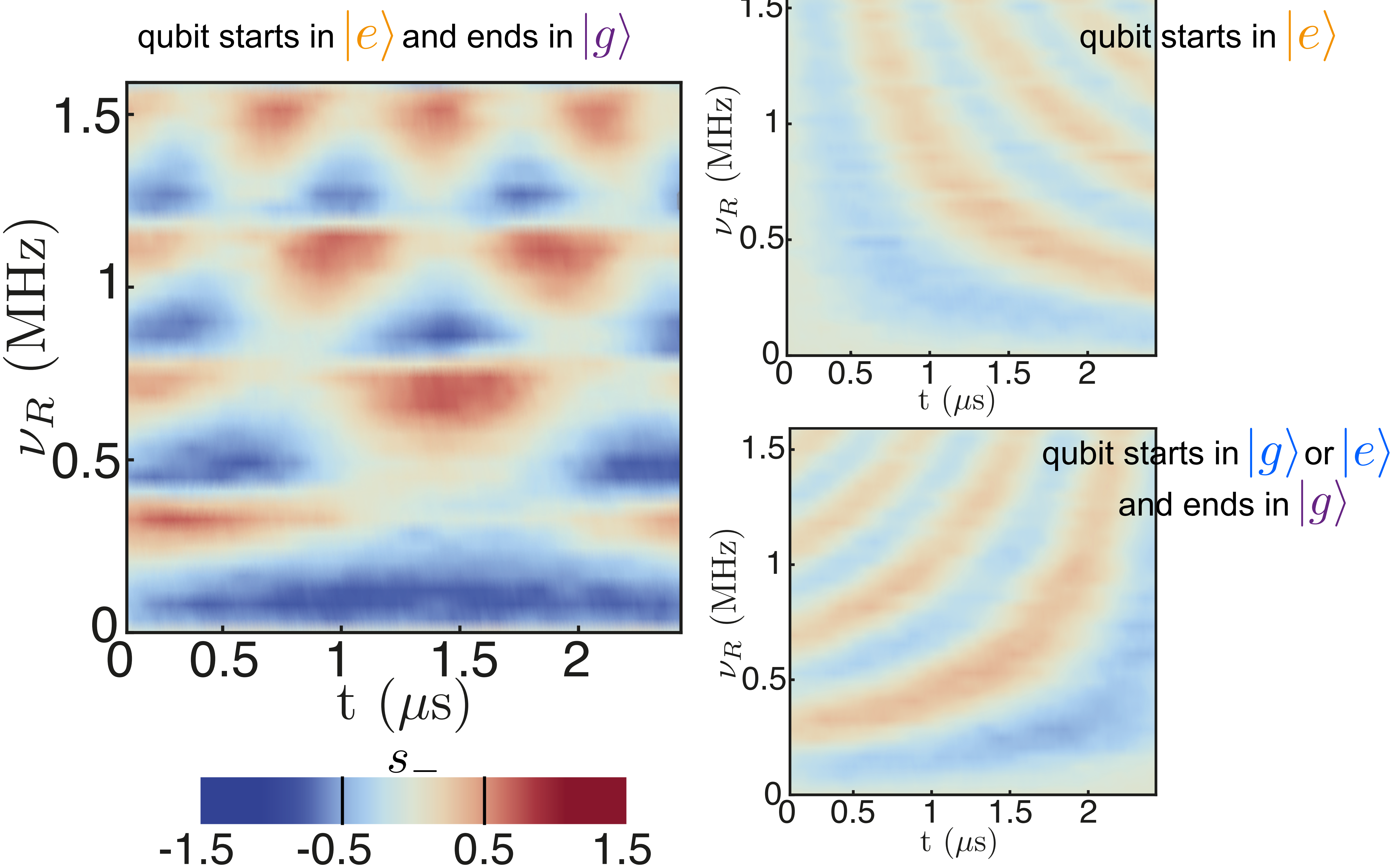
A time-symmetric formulation of quantum mechanics
[Aharonov, Popescu, Tollaksen, Physics Today 2010]

Comparison between preparation and postselection



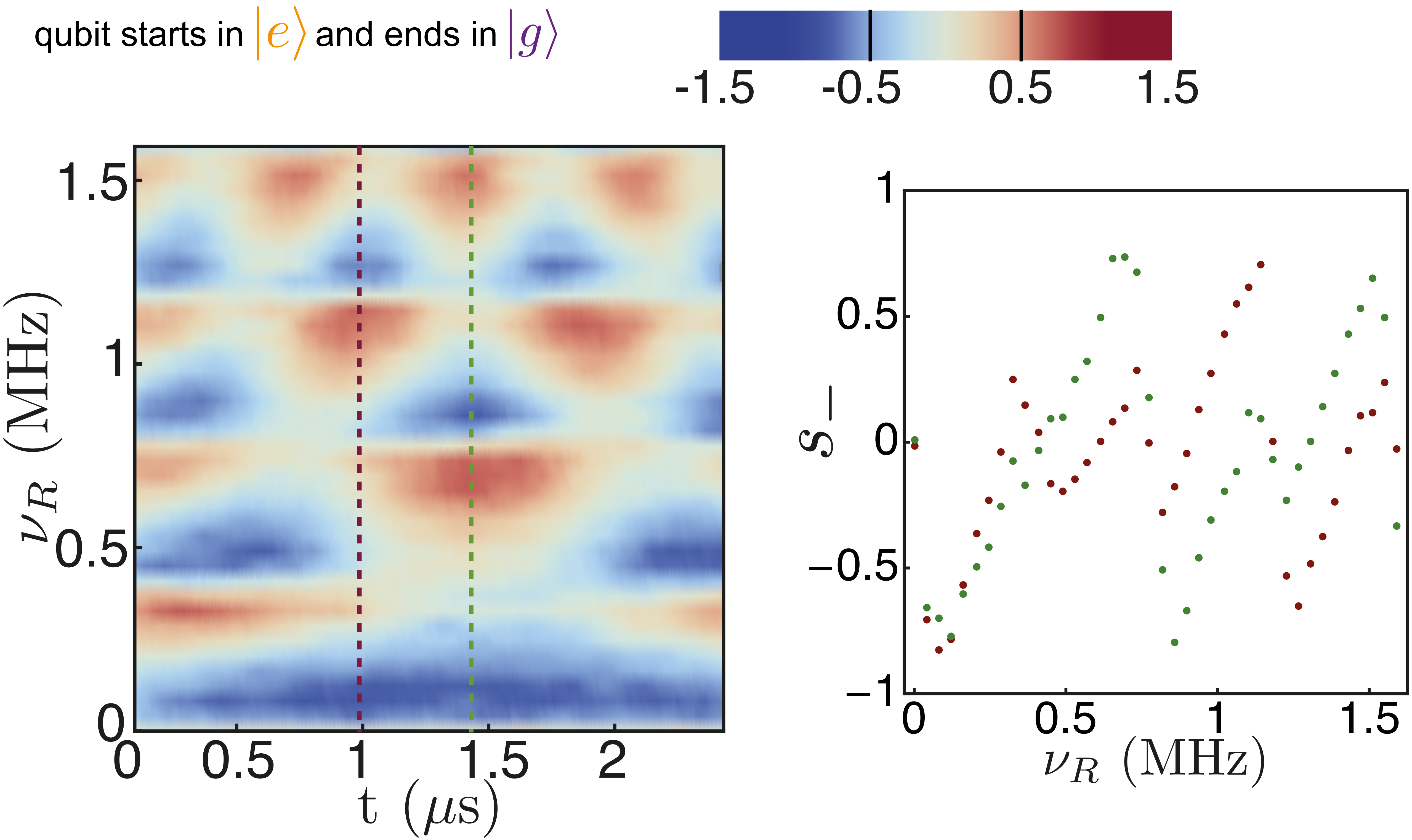
With both preparation and post-selection?

Interferences between past and future



Interferences between past and future oscillations

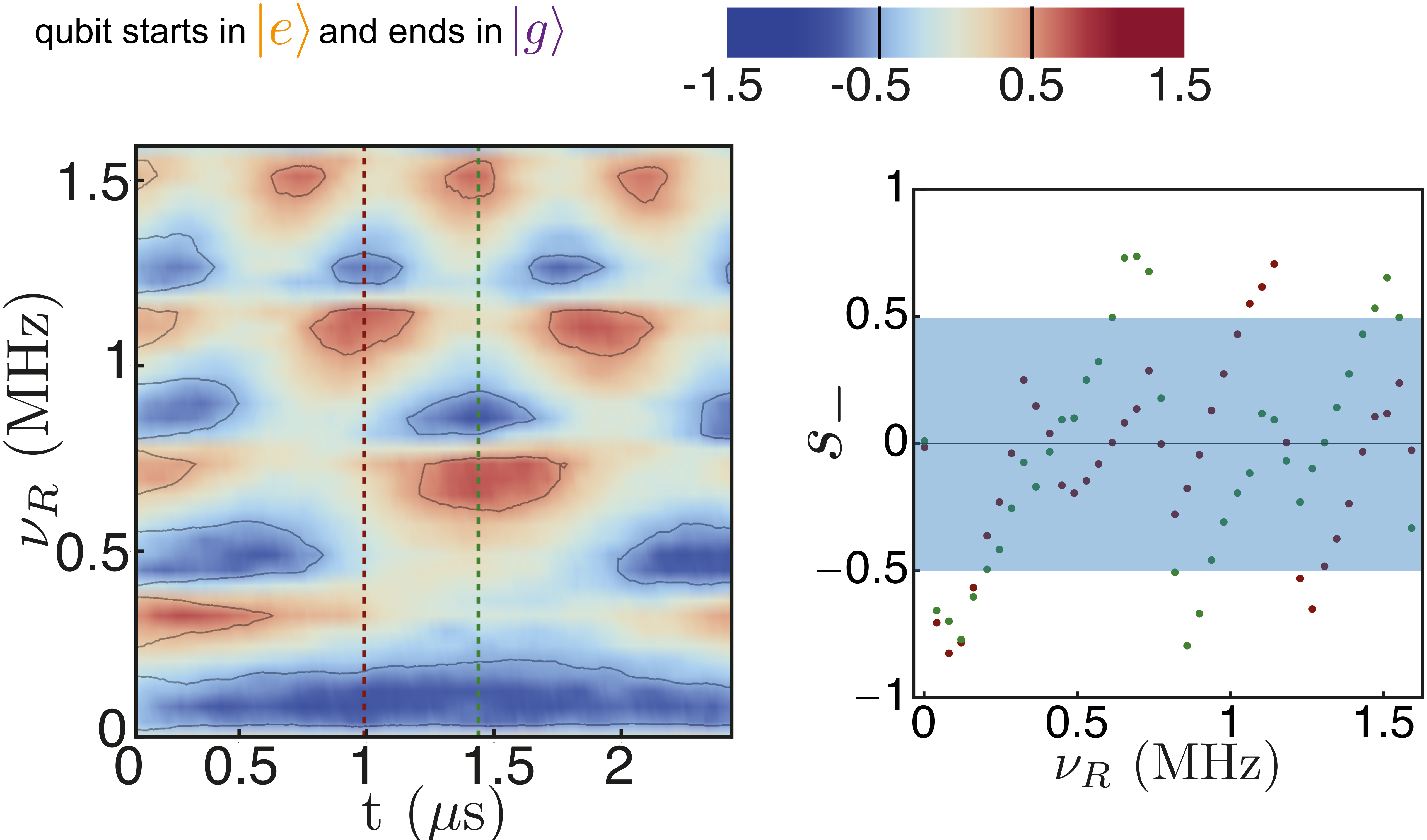
Interferences between past and future



Interferences between past and future oscillations

Interferences between past and future

qubit starts in $|e\rangle$ and ends in $|g\rangle$



Out of bound conditional averages $|\mathrm{Re}\langle\sigma_-(t)\rangle| = \dfrac{|\langle\sigma_x(t)\rangle|}{2} \leq 0.5$

Weak values

VOLUME 60, NUMBER 14 PHYSICAL REVIEW LETTERS 4 APRIL 1988

How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100

Yakir Aharonov, David Z. Albert, and Lev Vaidman

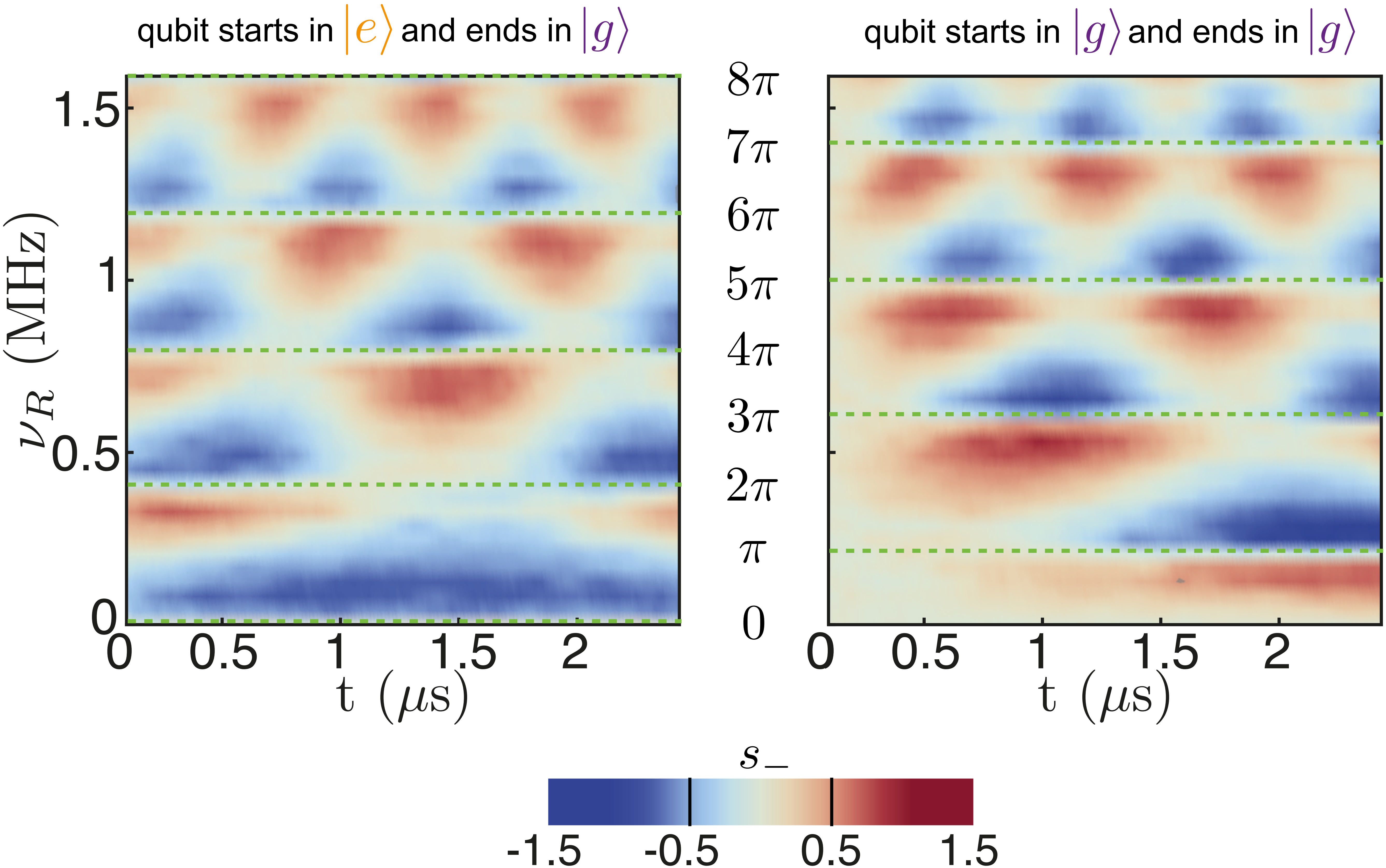
Physics Department, University of South Carolina, Columbia, South Carolina 29208, and School of Physics and Astronomy, Tel-Aviv University, Ramat Aviv 69978, Israel

(Received 30 June 1987)

We have found that the usual measuring procedure for preselected and postselected ensembles of quantum systems gives unusual results. Under some natural conditions of weakness of the measurement, its result consistently defines a new kind of value for a quantum variable, which we call the weak value. A description of the measurement of the weak value of a component of a spin for an ensemble of preselected and postselected spin-$\frac{1}{2}$ particles is presented.

First seen in Q optics in 1991 by Richie, Story, Hulet

Interferences between past and future

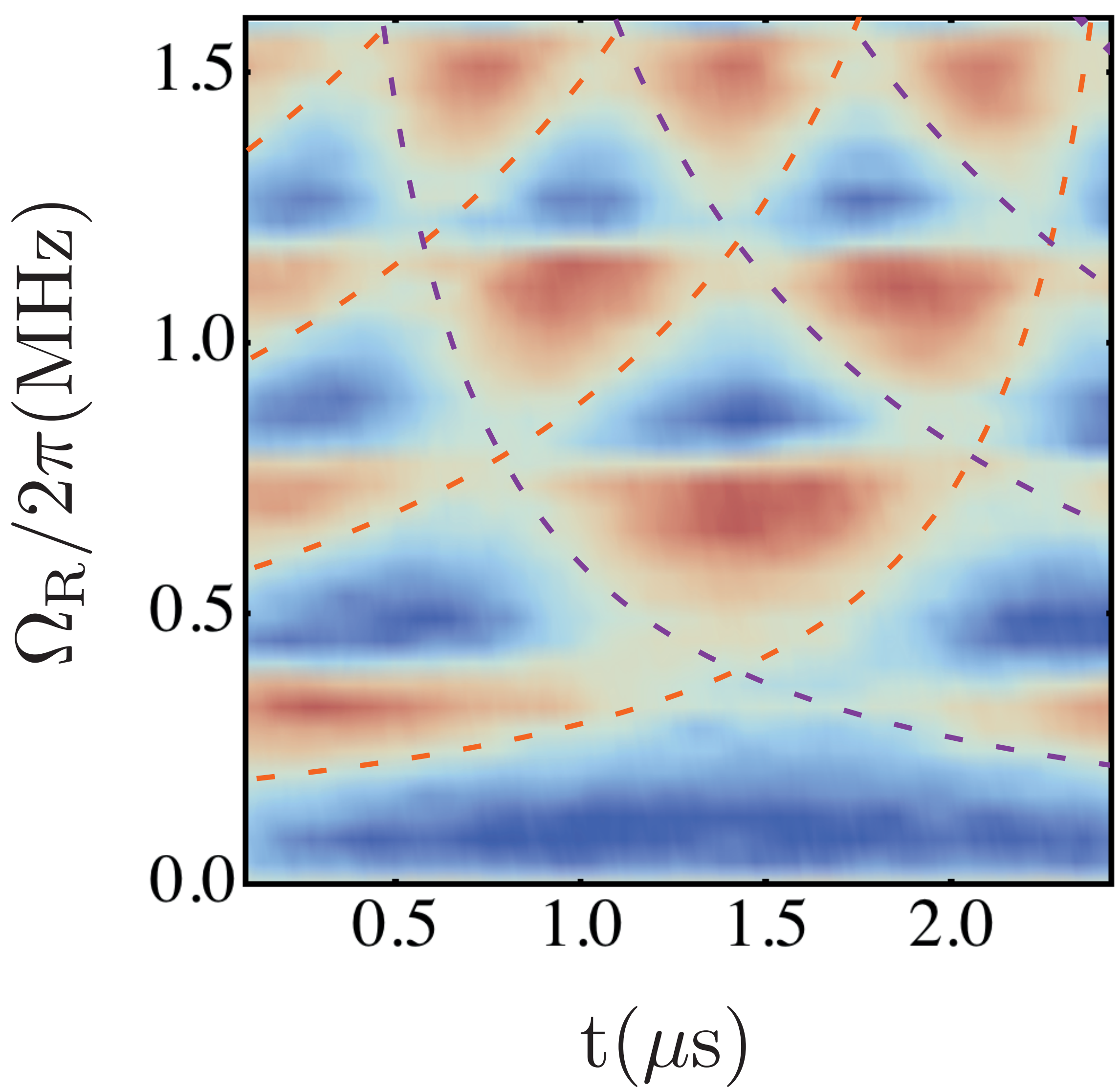


Characteristic features

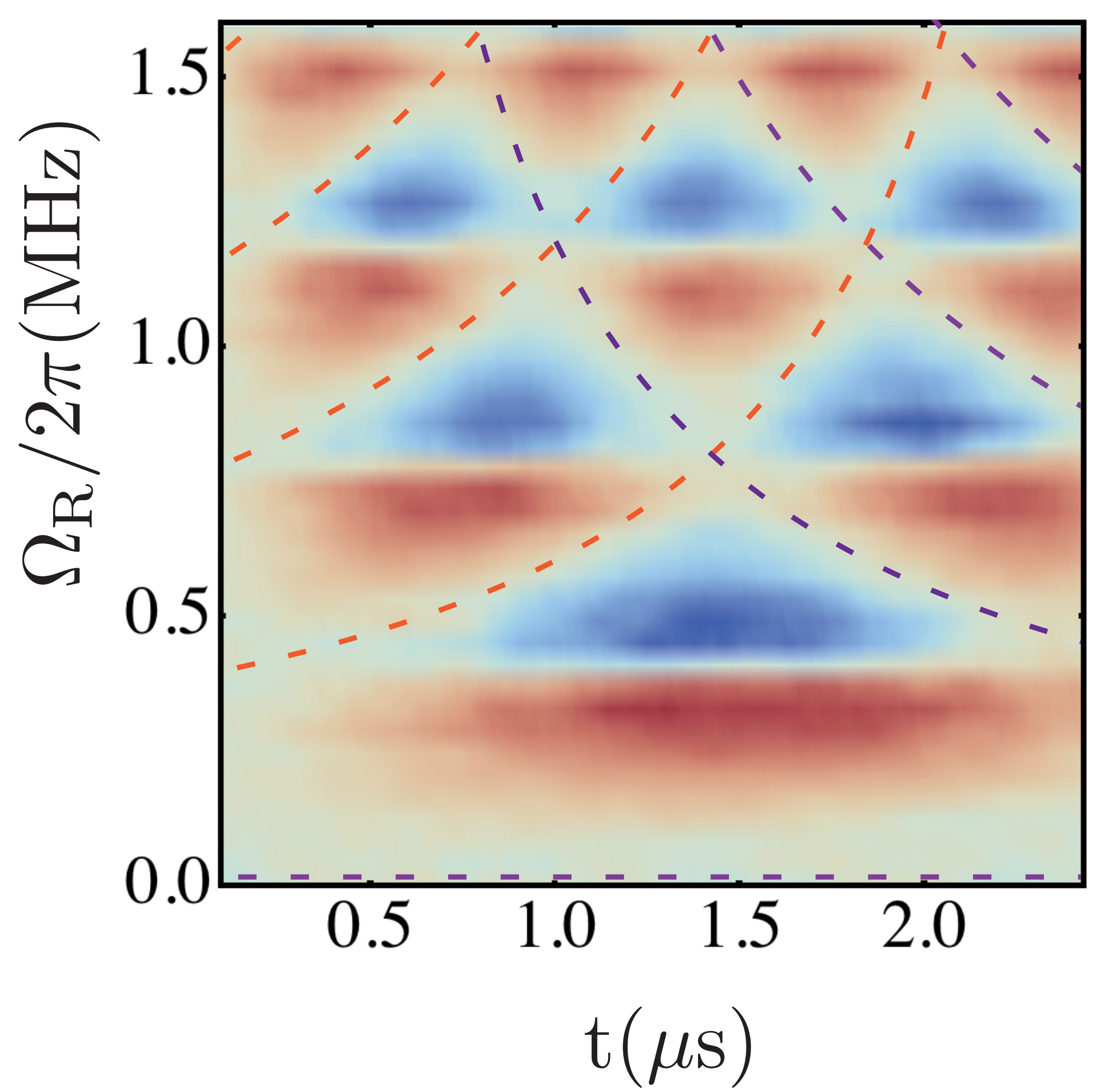
Past information indicates qubit in |g> $\rho(t) = |g\rangle\langle g|$

Future information indicates qubit in |e> $E(t) = |e\rangle\langle e|$

$|e\rangle \rightarrow |g\rangle$



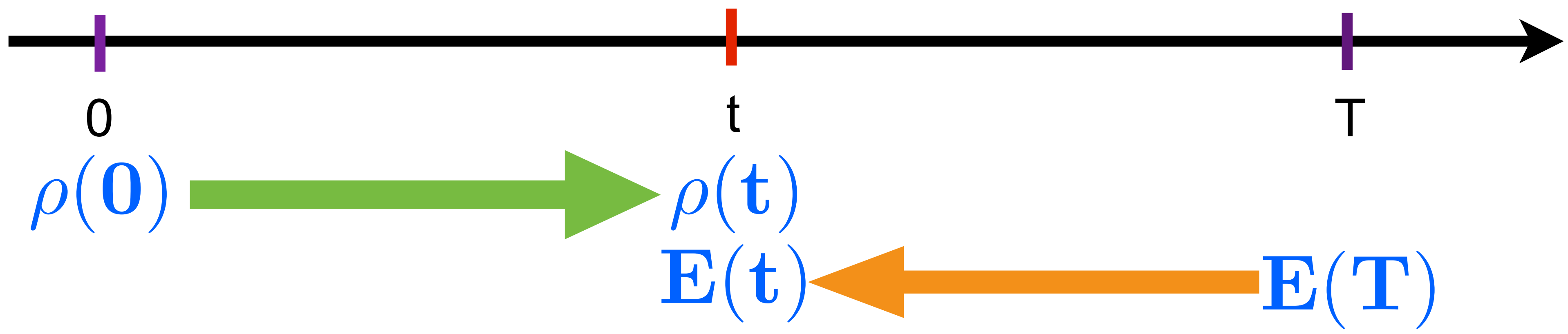
$|g\rangle \rightarrow |e\rangle$



$$\sigma_-\rho(t) = 0 \text{ or } E(t)\sigma_- = 0 \Rightarrow \overline{s_-(t)} = 0$$

Generalized master equation

$$\mathrm{Re}\langle\sigma_-(t)\rangle_w = \mathrm{Re}\left[\frac{\mathrm{Tr}(\mathbf{E}(\mathbf{t})\sigma_-\rho(\mathbf{t}))}{\mathrm{Tr}\rho(\mathbf{t})\mathbf{E}(\mathbf{t})}\right]$$



$$\rho(\mathbf{0}) = 0.85|e\rangle\langle e| + 0.15|g\rangle\langle g|$$

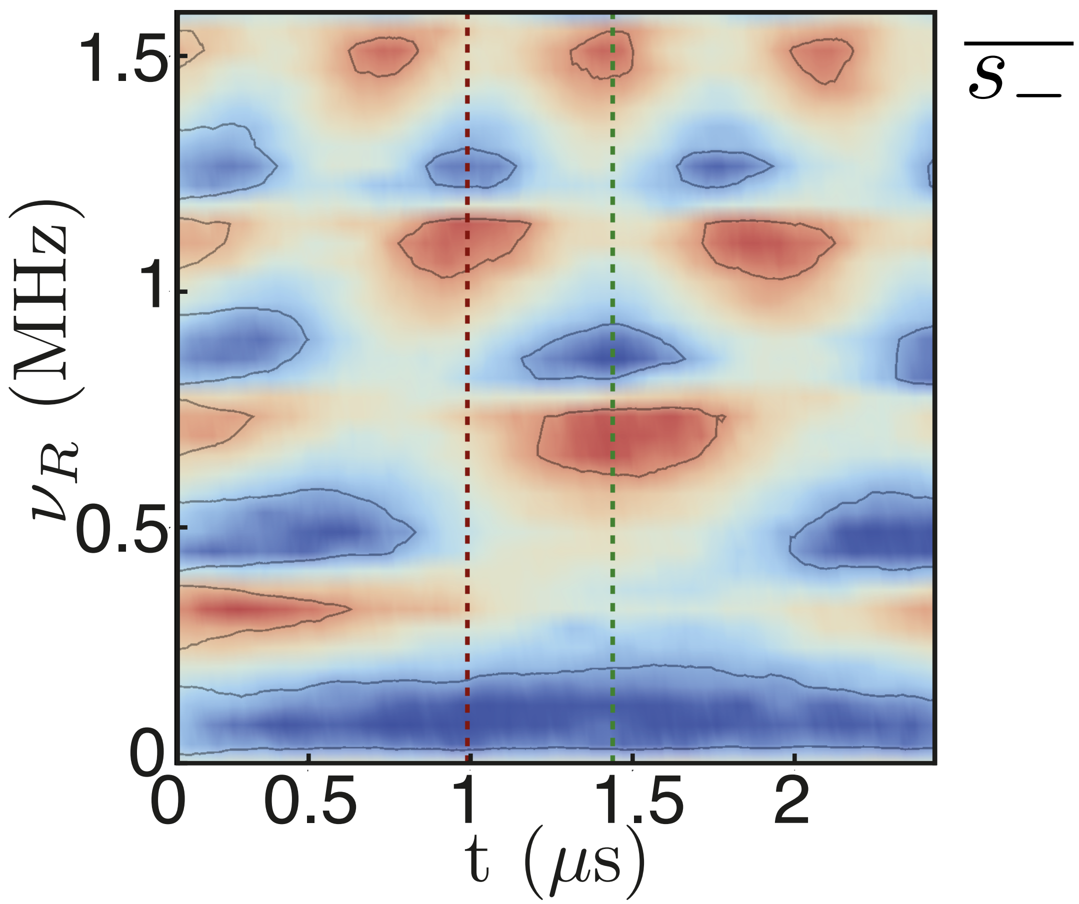
$$\frac{d\rho}{dt} = -\frac{i}{\hbar}\left[\tilde{H},\rho\right] + \gamma_1\left(\sigma_-\rho\sigma_+ - \frac{1}{2}\left[\sigma_+\sigma_-\rho + \rho\sigma_+\sigma_-\right]\right)$$

$$\mathbf{E}(\mathbf{T}) = 0.96|g\rangle\langle g| + 0.04|e\rangle\langle e|$$

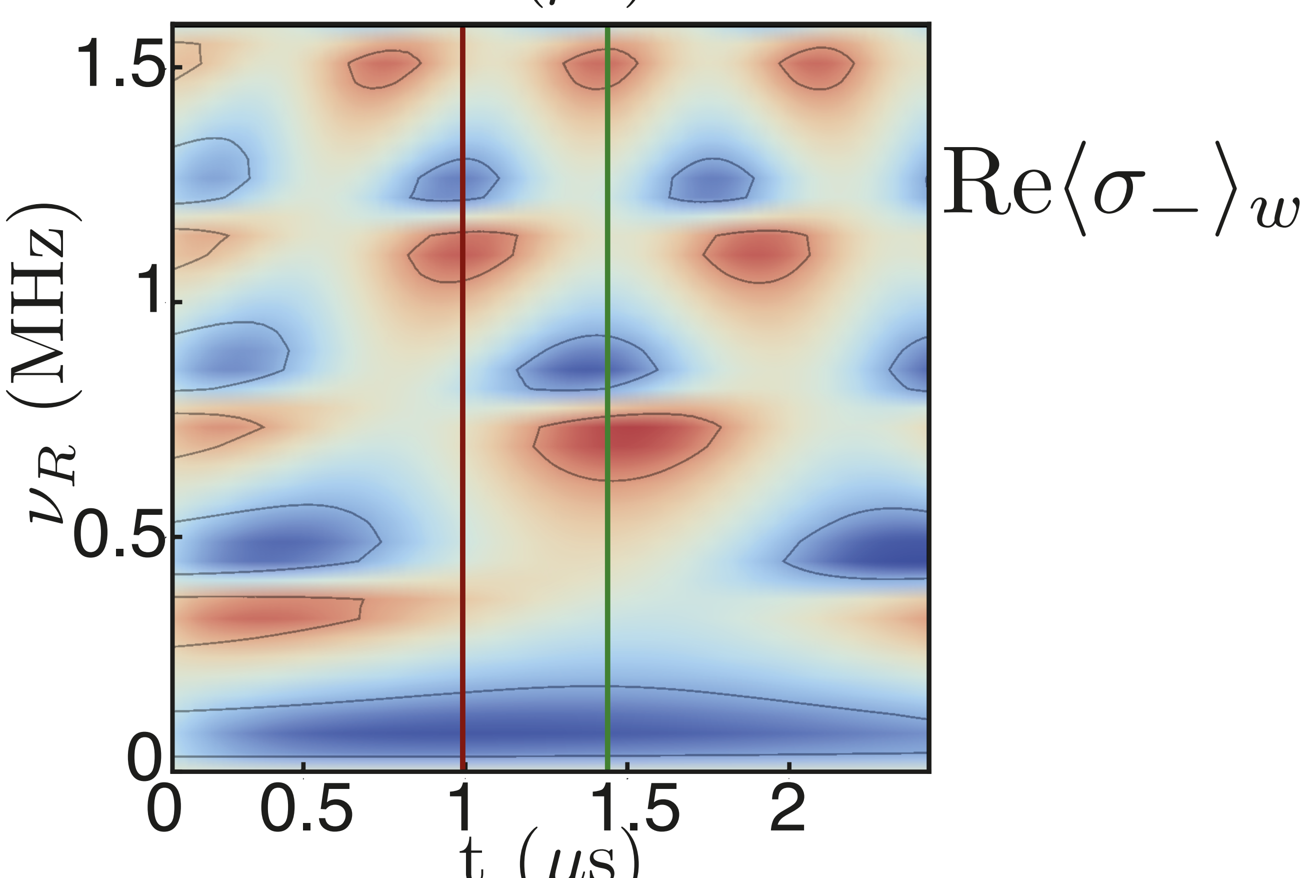
$$\frac{d\mathbf{E}}{dt} = -\frac{i}{\hbar}\left[\tilde{H},\mathbf{E}\right] - \gamma_1\left(\sigma_+\mathbf{E}\sigma_- - \frac{1}{2}\left[\sigma_+\sigma_-\mathbf{E} + \mathbf{E}\sigma_+\sigma_-\right]\right)$$

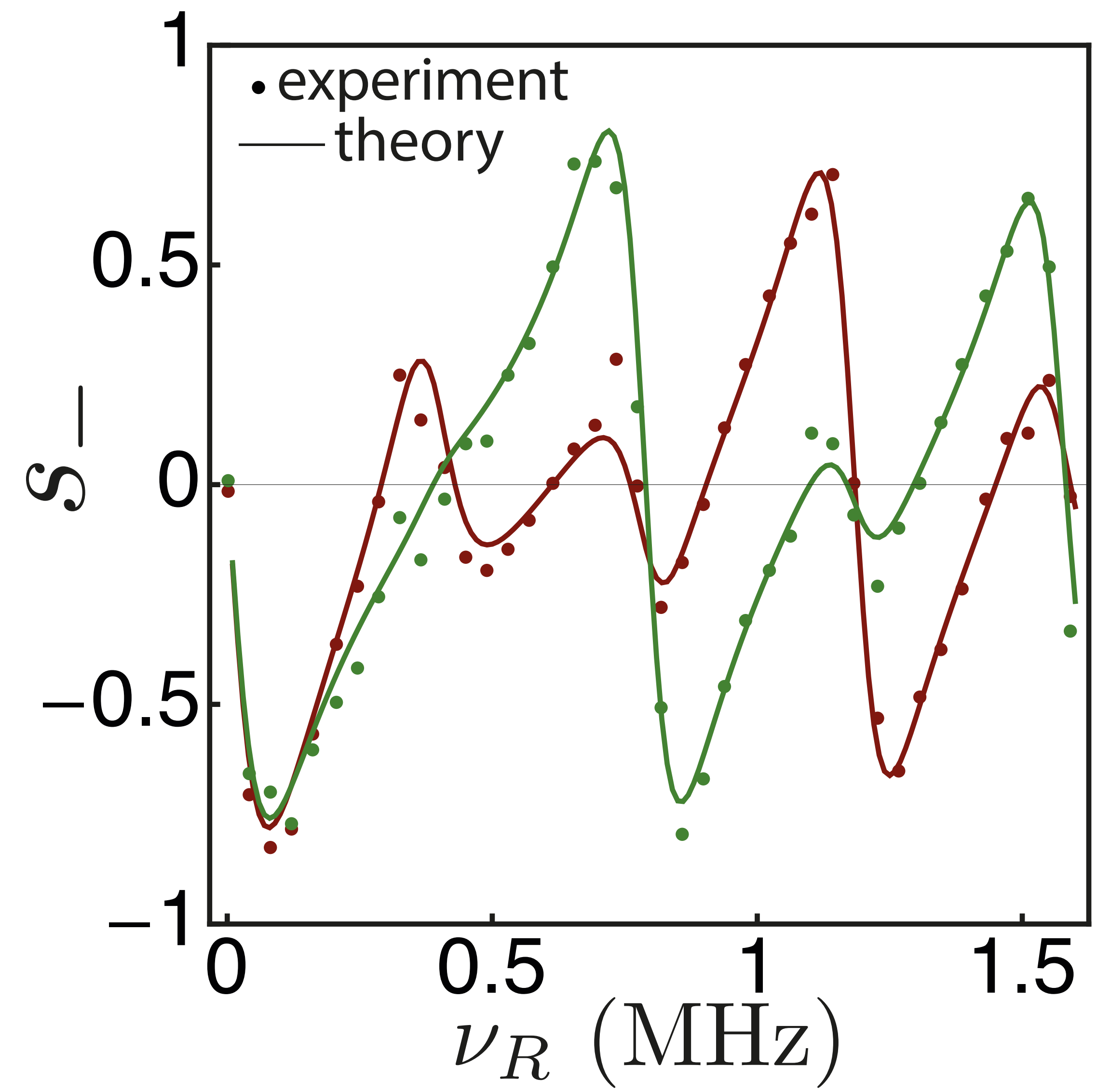
[Wiseman, PRA 2002]

Comparison with generalized theory



qubit starts in $|e\rangle$ and ends in $|g\rangle$

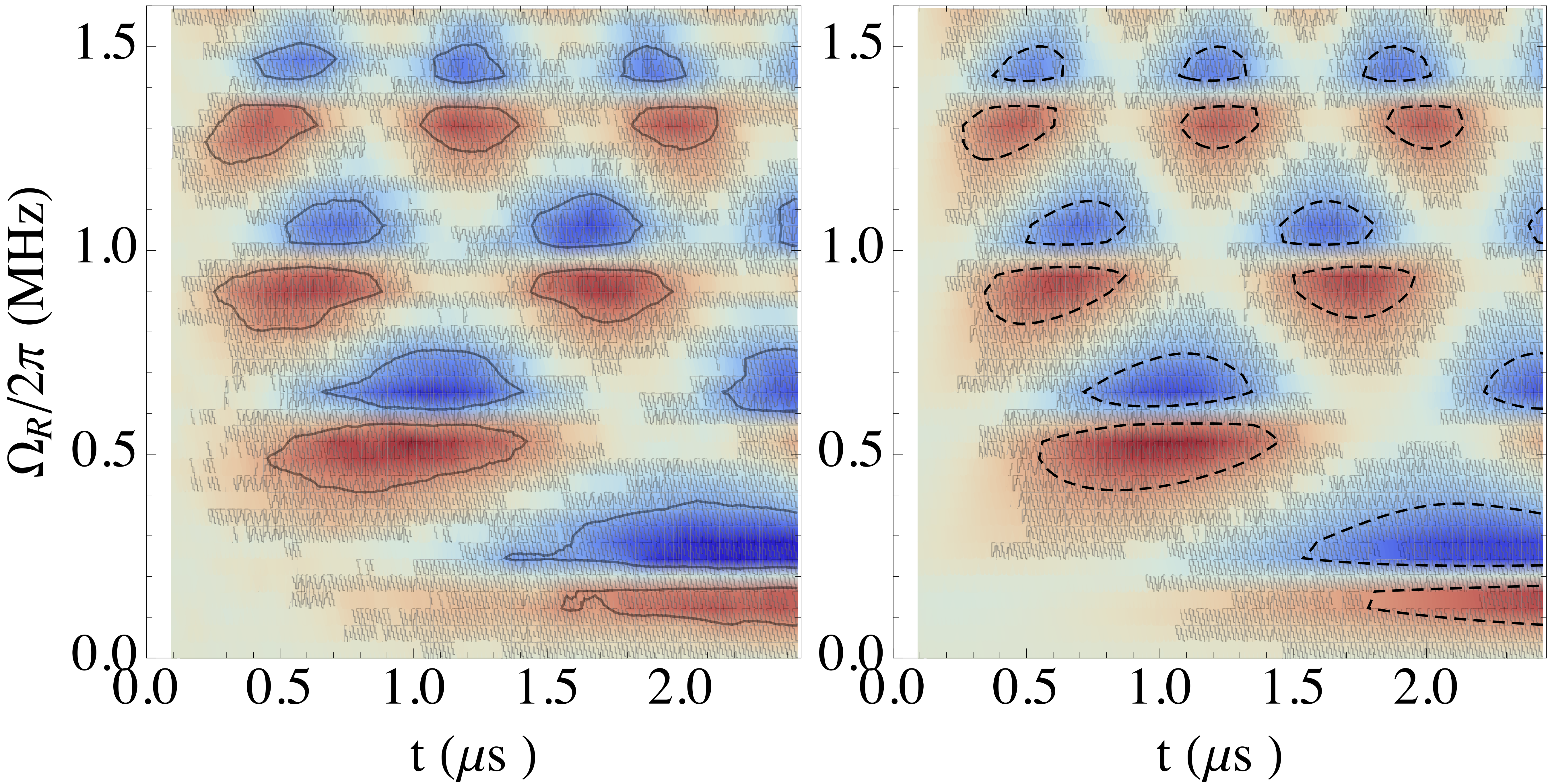




Good agreement!

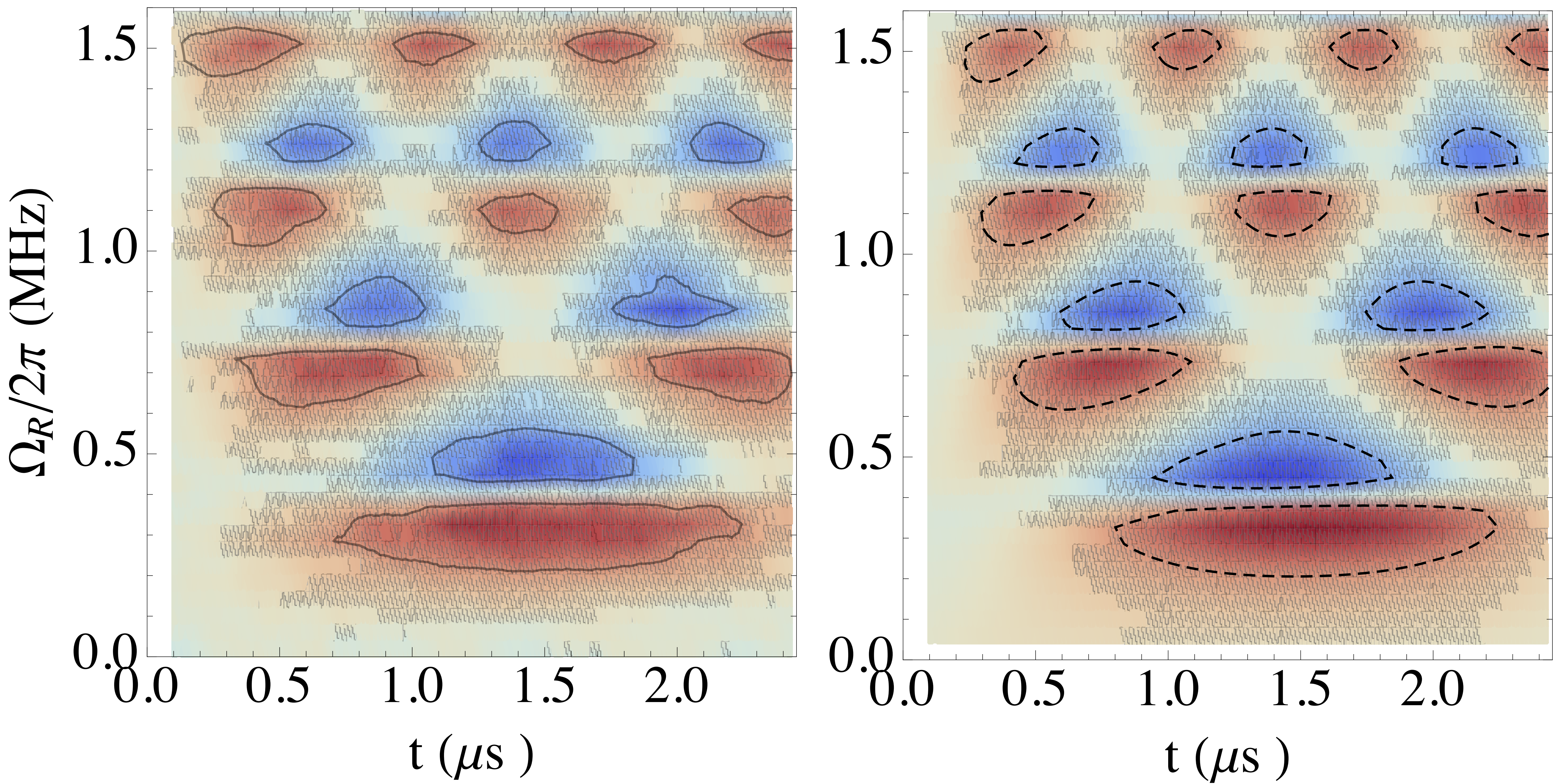
Comparison with generalized theory

qubit starts in $|g\rangle$ and ends in $|g\rangle$



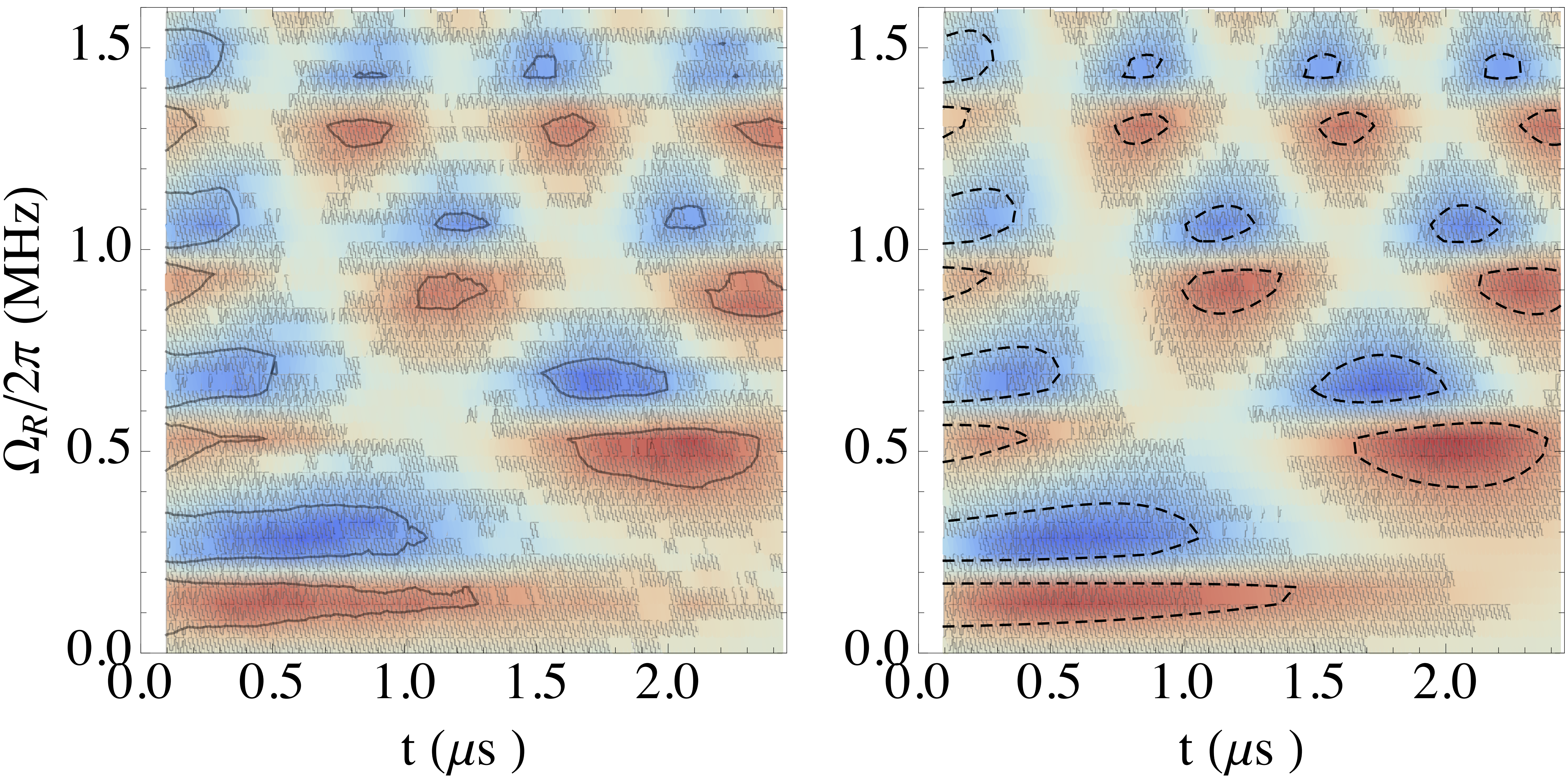
Comparison with generalized theory

qubit starts in $|g\rangle$ and ends in $|e\rangle$

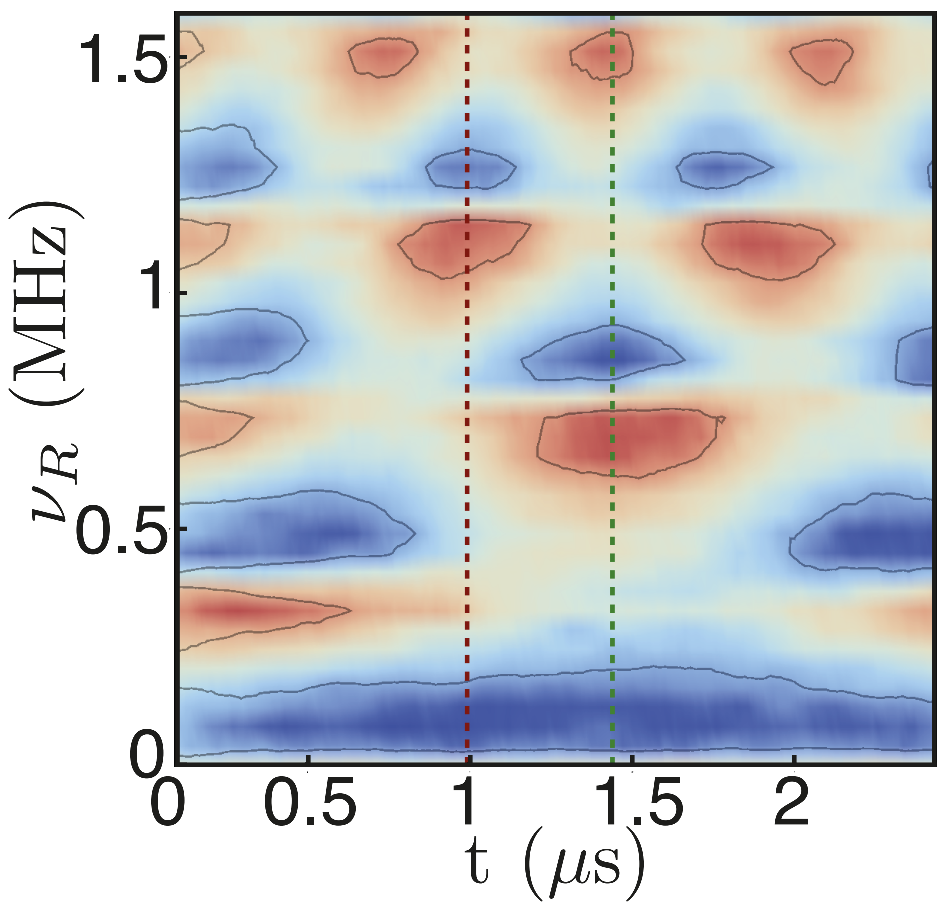


Comparison with generalized theory

qubit starts in $|e\rangle$ and ends in $|e\rangle$

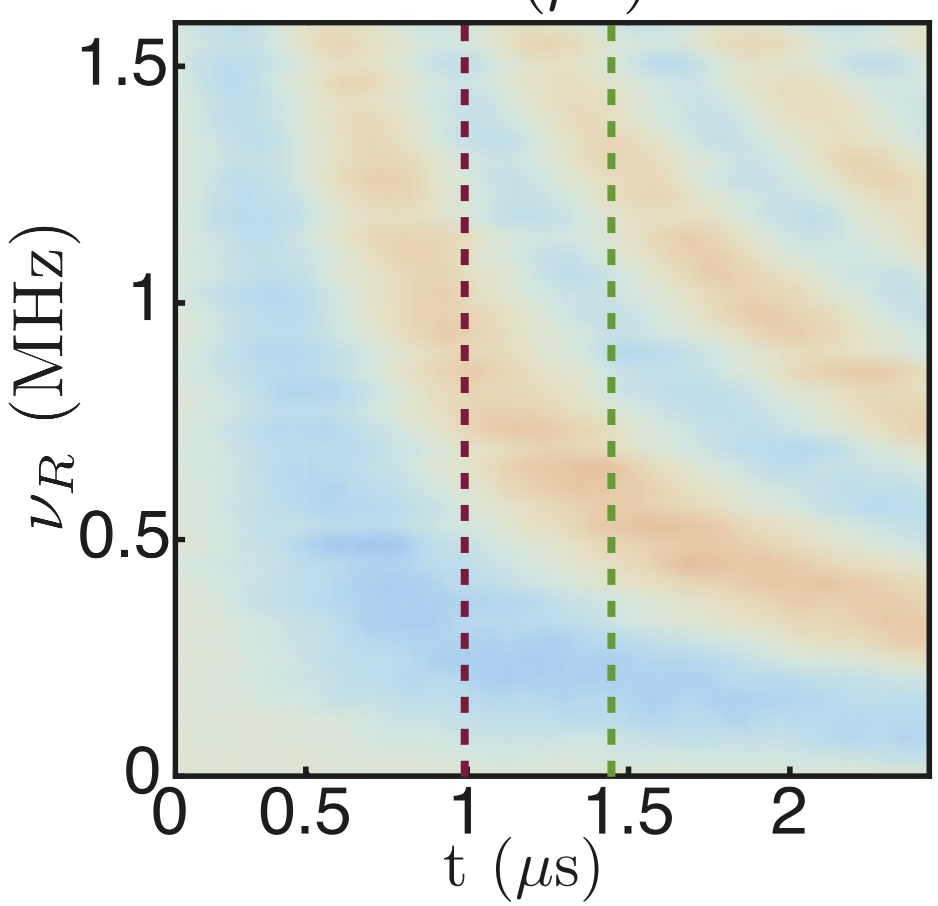


Comparison with generalized theory



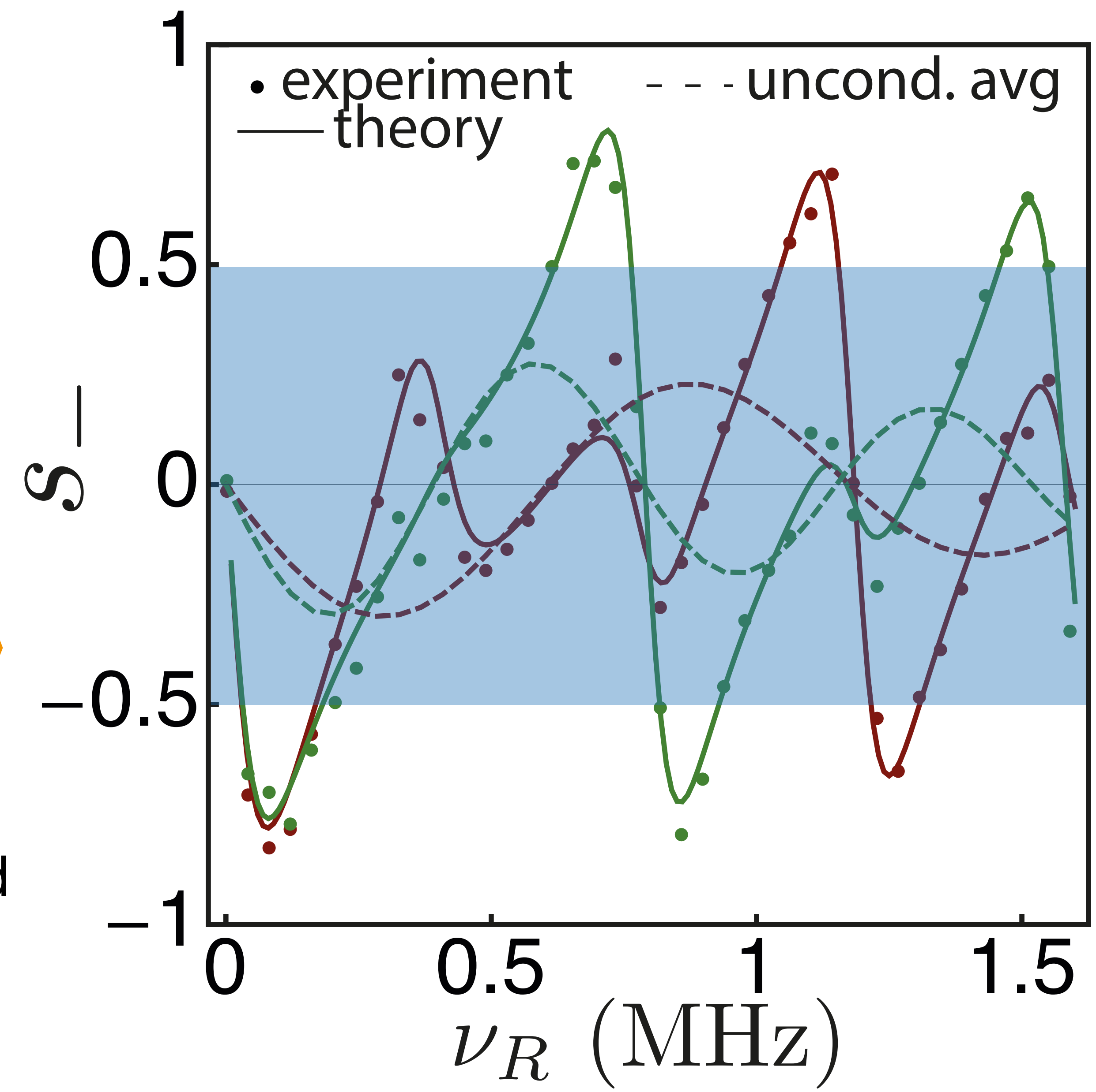
qubit starts in $|e\rangle$ and ends in $|g\rangle$

$\overline{s_-}$



qubit starts in $|e\rangle$

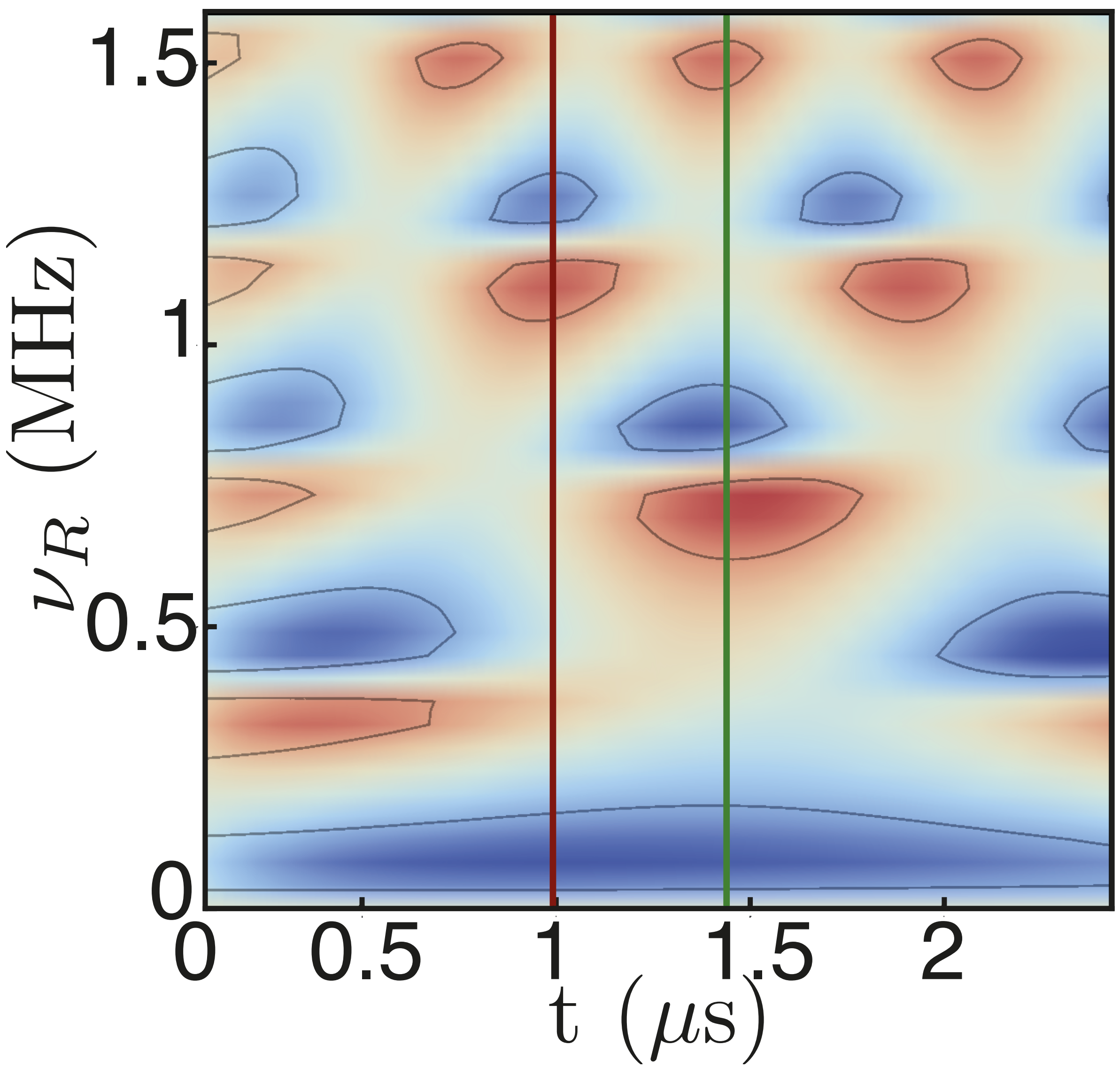
$\overline{s_-}$ post-selected

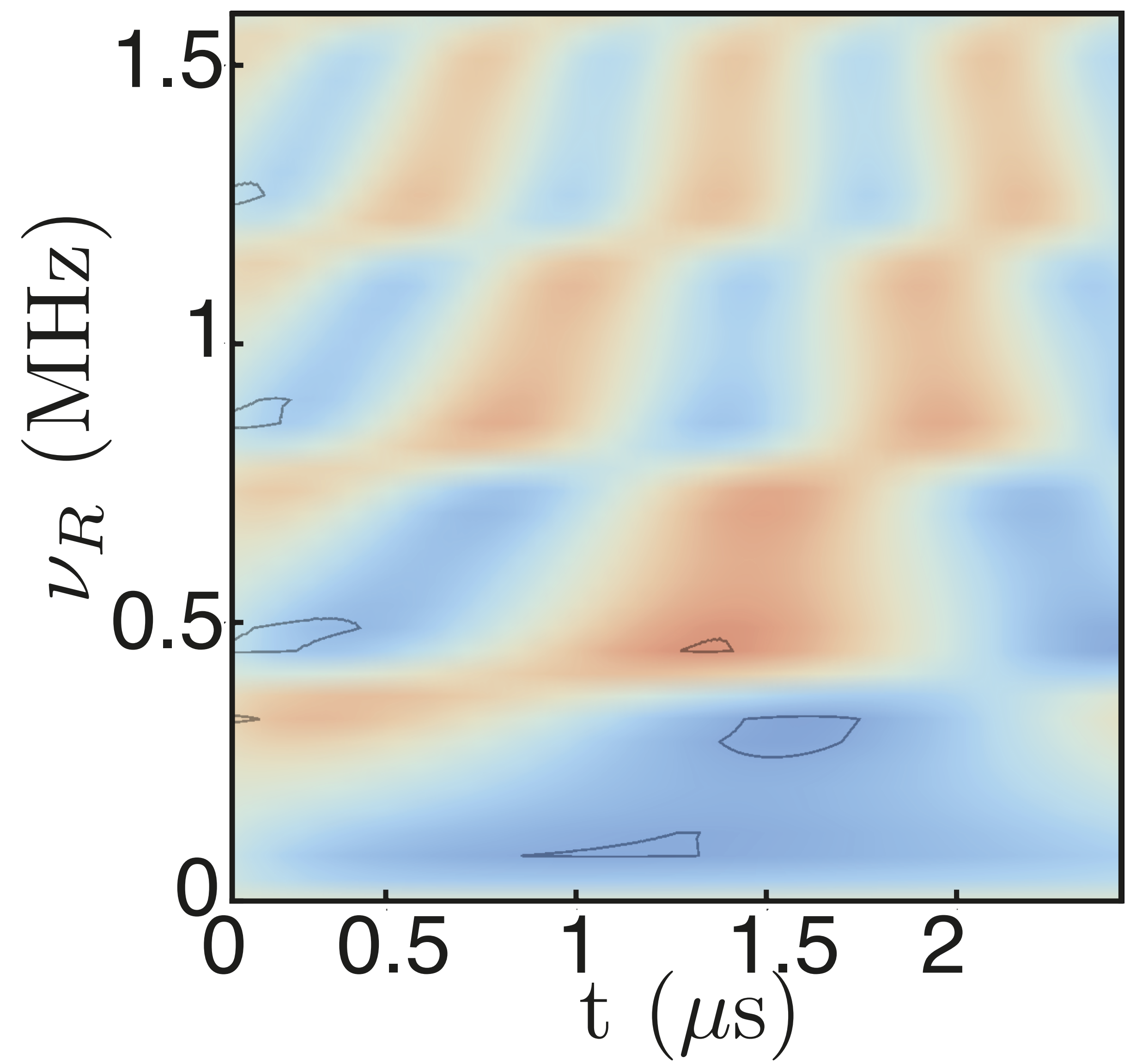


Steep slope => high sensitivity to parameters

Generalized master equation

$$\mathrm{Re}\langle\sigma_-(t)\rangle_w \neq \langle \mathrm{Re}\sigma_-(t)\rangle_w = \langle\sigma_X(t)/2\rangle_w$$



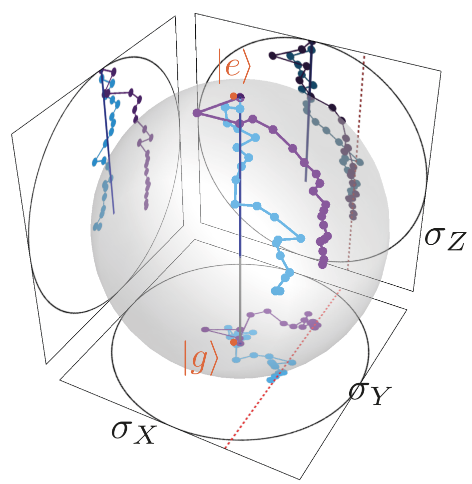


The post-selected average does not commute with « real part » operator

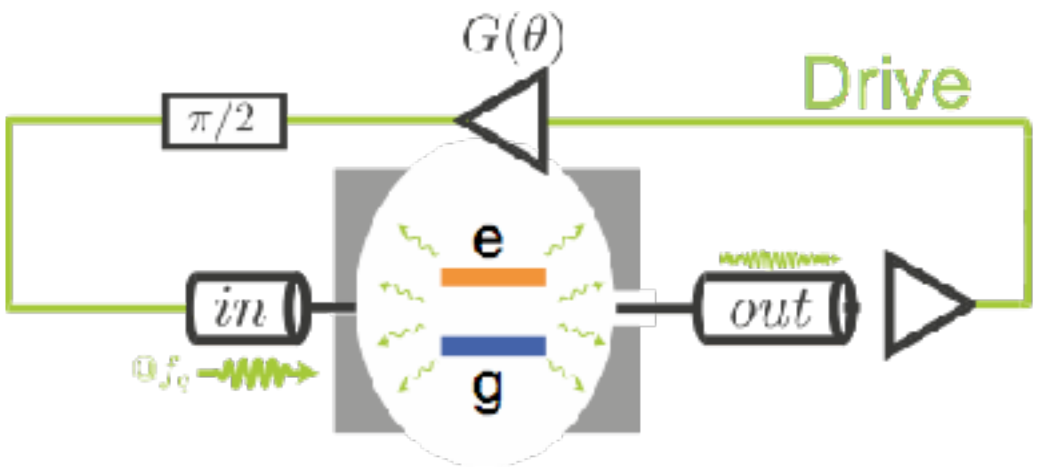
Conclusion

Superconducting circuits are a testbed for quantum measurement backaction

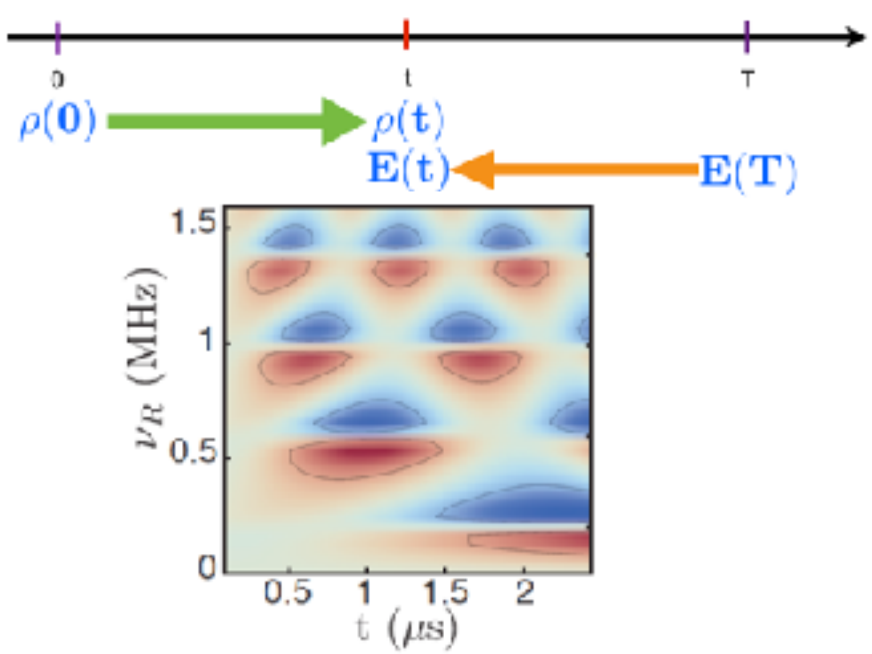
quantum trajectories



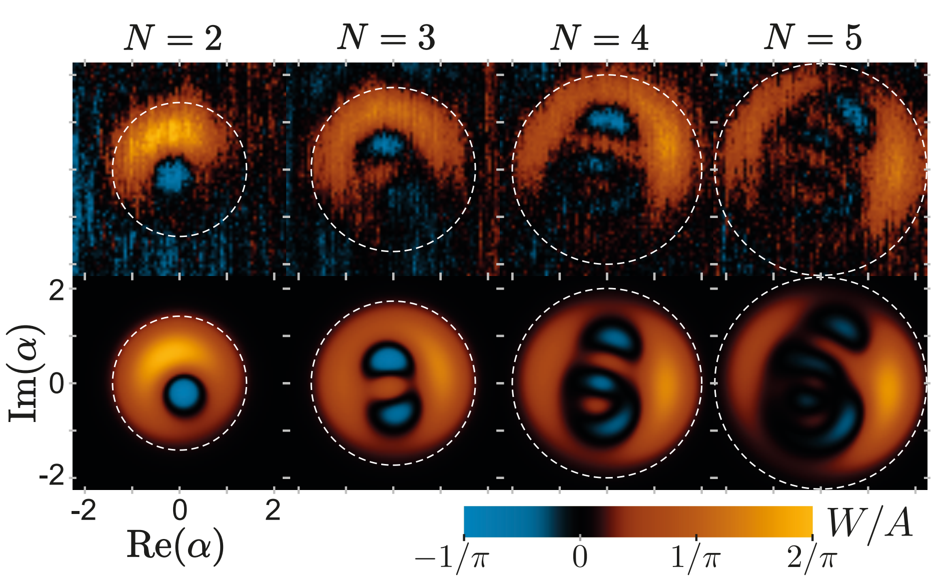
feedback



post-selection and quantum states



quantum Zeno dynamics



[Campagne-Ibarcq et al., PRL 2013]

[Bretheau et al., Science 2016]

also: autonomous feedback, quantum smoothing, parameter estimation, link with thermodynamics...

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Emmanuel Flurin (now at Saclay)

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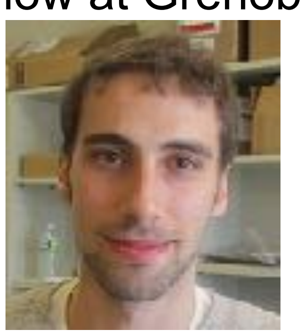

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