Quantum trajectories and feedback in a photon box

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Cavity Quantum Electrodynamics group

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Modeling and Control of Open Quantum Systems
Marseille, 16 - 20 April, 2018

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Projects:

QND measurement and non-local states

Cavity QED with slow atoms of atomic fountain

Atomic chip and Rydberg atom simulator

Rydberg atoms and quantum metrology

Simplest quantum systems

Better understanding of intimate quantum mechanisms

Measurement, entanglement, non-locality, “paradoxes”, …

Exploration of the quantum/classical boundary

Understanding decoherence of mesoscopic quantum superpositions

Applications to quantum information processing

Use quantum weirdness to process and transmit information

- Quantum cryptography
- Quantum simulators
- Quantum computers

Applications to quantum metrology

Use quantum coherence and enhanced non-local correlations to go beyond classical measurement precision limits

Nobel prize 2012

Nobelprize.org

The Official Web Site of the Nobel Prize

The Nobel Prize in Physics 2012

Serge Haroche, David J. Wineland

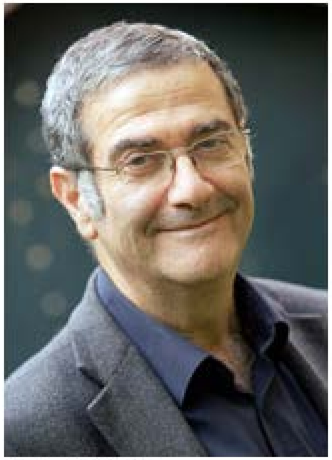
The Nobel Prize in Physics 2012

Serge Haroche

David J. Wineland

Controlling individual photons with atoms

"Photon box"



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Serge Haroche

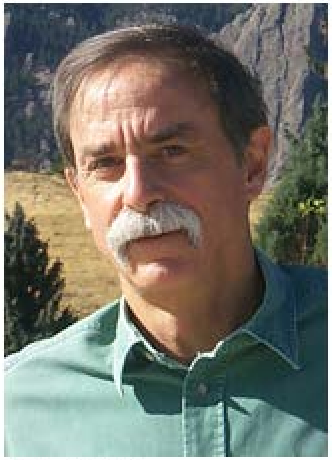
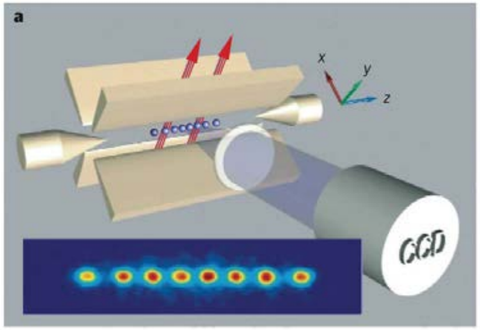


Photo: © NIST

David J. Wineland

Controlling individual atoms (ions) with photons (lasers)

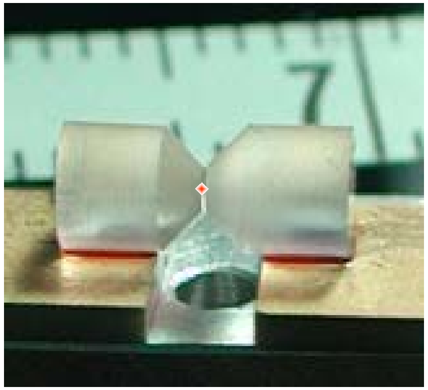


Linear ion trap

The Nobel Prize in Physics 2012 was awarded jointly to Serge Haroche and David J. Wineland *"for ground-breaking experimental methods that enable measuring and manipulation of individual quantum systems"*

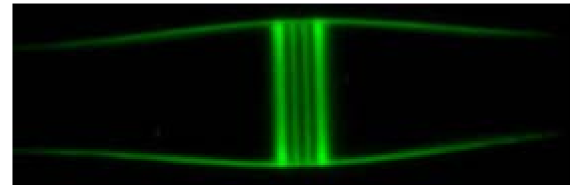
Cavity Quantum ElectroDynamics

Optical cavity QED



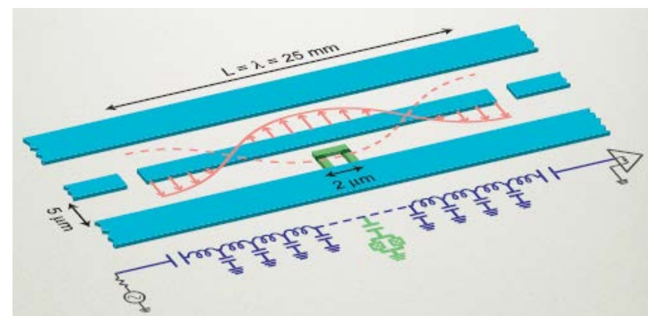
Cold atoms in optical cavities

Fiber-based cavity QED



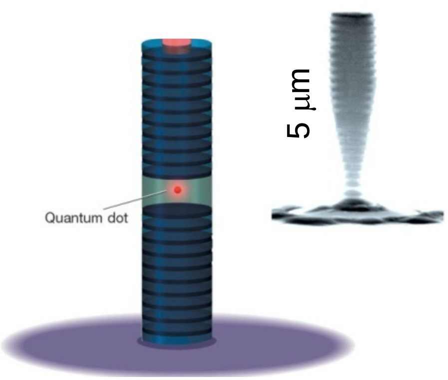
Laser-trapped atoms coupled to a bottle resonator

Circuit QED

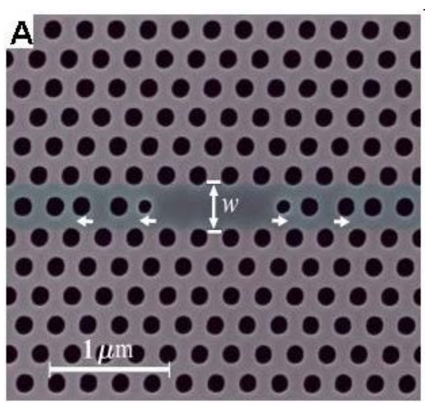


Superconducting qubits coupled to a strip-line cavity

Cavity QED in solids



Quantum dots



Photonic crystals

and many others …

Photon box

high-quality microwave superconducting cavity

- Resonance @ ν_c = 51 GHz
- Lifetime of photons T_c= 130 ms

- billion bounces on the mirrors

- a light travel distance of 40 000 km
(full turn around the Earth)

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  - Experimental setup : Photon box
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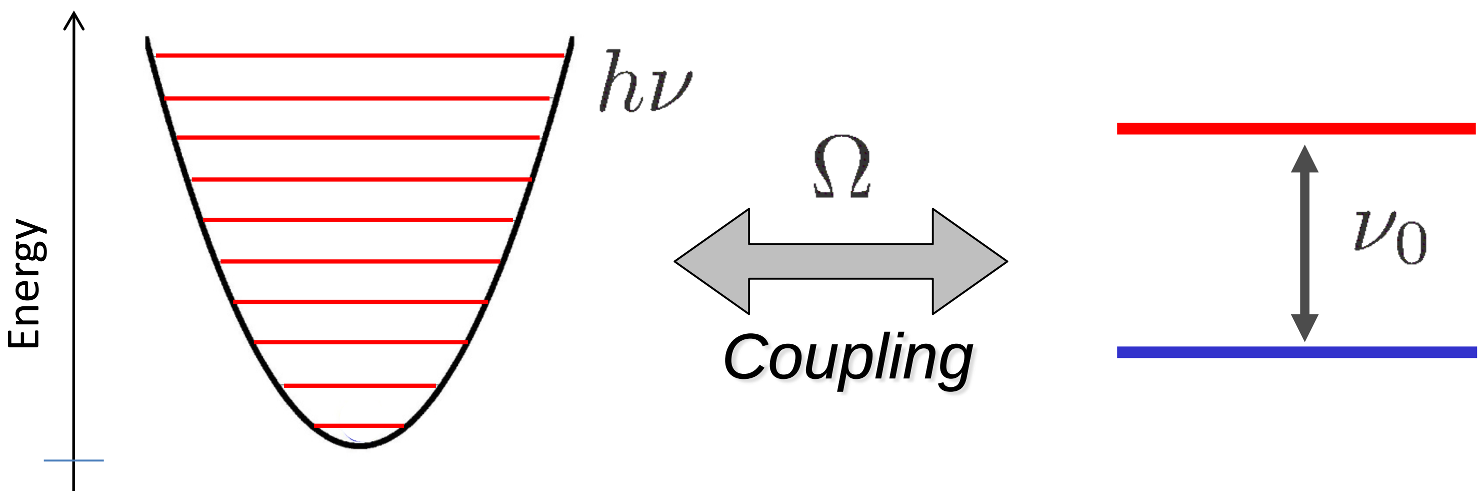
  - Past quantum states

- Quantum control
  - Quantum feedback stabilizing photon-number states
  - Adaptive QND measurement

Simplest quantum models

Spring: harmonic oscillator	*Spin ½: two-level atom*



- **Atomic** motion in a trap
- Vibrational levels of **molecules**
- Vibration of a **mechanical oscillator**
- Single mode of **light**…

- Electronic or nuclear **spins**
- **Atomic** states
- Light **polarization**
- **Artificial** “atoms”…

- Elementary block of quantum information processing: **qubit**

Harmonic oscillator

Hamiltonian of a harmonic oscillator given by momentum and position operators

$$H_x = \frac{P^2}{2m} + \frac{m\omega^2 X^2}{2} \qquad [X, P] = i\hbar$$

commutation relation

Dimensionless position and momentum

$$H_x = \hbar\omega \left[P_0^2 + X_0^2\right] \qquad X_0 = \sqrt{\frac{m\omega}{2\hbar}}X \qquad P_0 = \frac{1}{\sqrt{2\hbar m\omega}}P$$

Operator of a normalized complex amplitude: $a = X_0 + iP_0$

$$H_x = \hbar\omega \left[a^\dagger a + 1/2\right] \qquad \left[a, a^\dagger\right] = \mathbb{1}$$

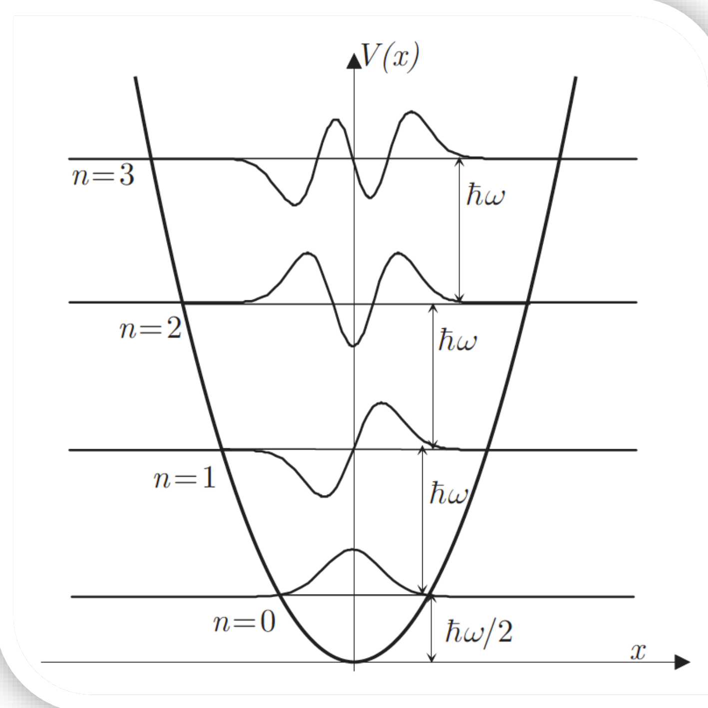
Photon-number operator and energy eigenstates

$$N = a^\dagger a \qquad N\left|n\right\rangle = n\left|n\right\rangle$$

n ∈ N photon number

Annihilation and creation operators

$$a\left|n\right\rangle = \sqrt{n}\left|n-1\right\rangle \qquad a^\dagger\left|n\right\rangle = \sqrt{n+1}\left|n+1\right\rangle$$

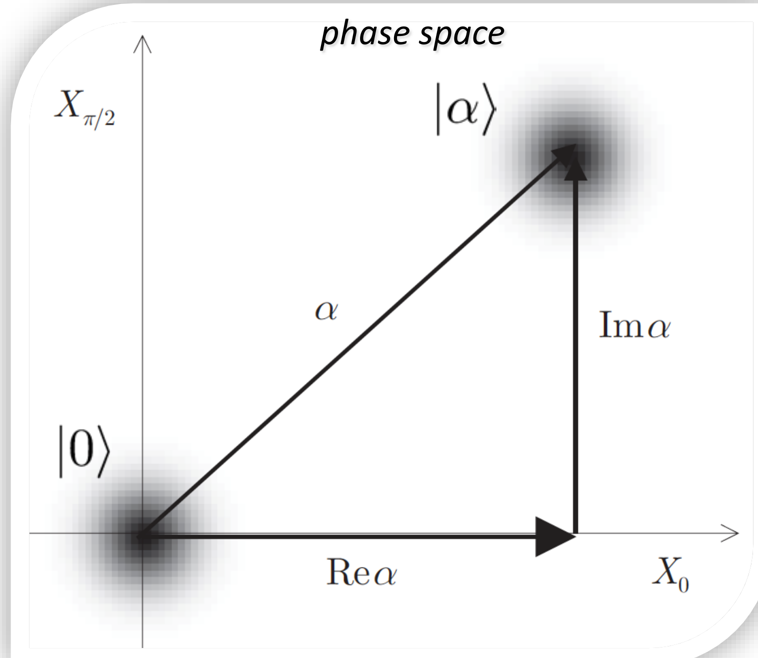


Field displacement and coherent states

Coupled to (driven by) a resonant external force:
field displacement

$$D(\alpha) = e^{(\alpha a^\dagger - \alpha^* a)}$$

α : complex drive strength



Coherent (classical) states

$$|\alpha\rangle = D(\alpha)\,|0\rangle$$

applied to vacuum

Eigenstate of annihilation operator

$$a|\alpha\rangle = \alpha|\alpha\rangle$$

complex amplitude operator

Coherent state in a photon-number basis

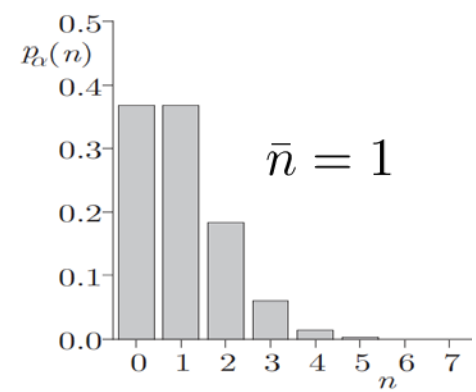
$$|\alpha\rangle = \sum_n c_n\,|n\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}}\,|n\rangle$$

Poisson distribution of photon numbers

$$p_\alpha(n) = e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!}$$

$$\overline{n} = |\alpha|^2$$

mean photon number

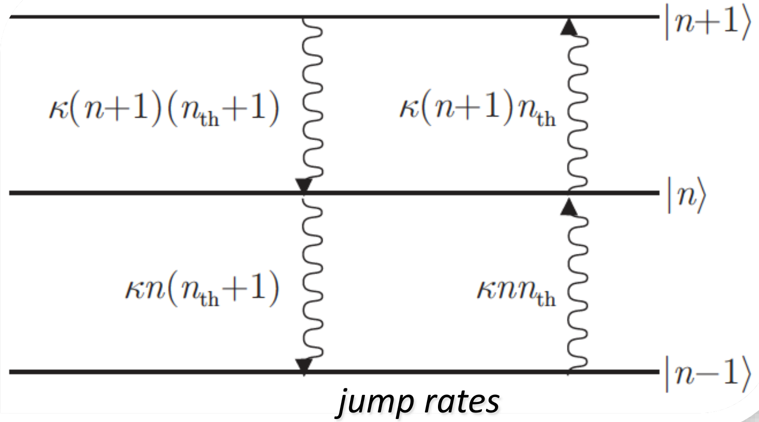


Cavity relaxation

Losses due to cavity relaxation (damping)

energy decay rate
and cavity lifetime

$$\kappa = T_c^{-1}$$



Excitation due to thermal field

thermal photon number

$$n_{\rm th} = \frac{1}{e^{\hbar\omega_c/k_b T} - 1}$$

Photon number probabilities

$$\frac{dp(n)}{dt} = \kappa(1+n_{\rm th})(n+1)p(n+1) + \kappa n_{\rm th} n p(n-1) - [\kappa(1+n_{\rm th})n + \kappa n_{\rm th}(n+1)]p(n)$$

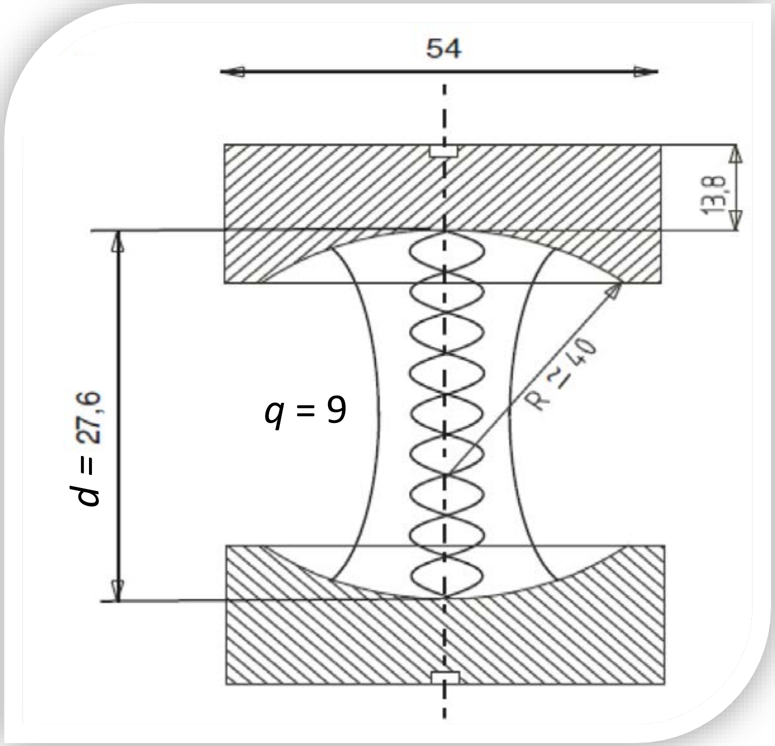
jump down from (n+1) *jump up from (n-1)* *jump away from n*

Master equation

density matrix evolution

$$\frac{d\rho}{dt} = -\frac{\kappa}{2}(1+n_{\rm th})(a^\dagger a\rho + \rho a a^\dagger - 2a\rho a^\dagger) - \frac{\kappa}{2}n_{\rm th}(aa^\dagger\rho + \rho a^\dagger a - 2a^\dagger\rho a),$$

Fabry-Perot interferometer



microwave cavity with standing-wave mode structure

Mode spectrum $$\nu_q = \frac{c}{2d}\left(q + \frac{1}{\pi}\arccos\left(1 - \frac{d}{R}\right)\right)$$

TEM$_{900}$ mode $$\nu = 51\,\mathrm{GHz} \qquad \lambda = 5.9\,\mathrm{mm}$$

Compact waist (mode width) $$w_0 = \left(\frac{\lambda}{2\pi}\sqrt{d(2R-d)}\right)^{1/2} \approx \lambda$$

Small mode volume

$$V = \int |f(r)|^2 \mathrm{d}^3 r = \frac{\pi {w_0}^2 d}{4} \approx 3.8\lambda^3$$

Large vacuum field fluctuation

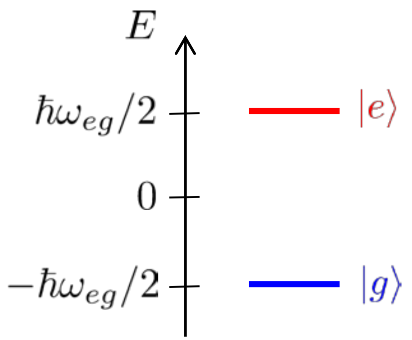
$$\mathcal{E}_0 = \sqrt{\frac{\hbar\omega_0}{2\epsilon_0 V}} \approx 1{,}58 \cdot 10^{-3}\,\mathrm{V/m}$$

High quality factor $$Q = \frac{\nu}{\Delta\nu} = 2\pi \cdot \nu \cdot T_{\mathrm{cav}} \approx 3 \cdot 10^{10}$$

Two-level atom

Hamiltonian

$$H_a = \frac{\hbar\omega_{eg}}{2}\sigma_Z \qquad \sigma_Z = |e\rangle\langle e| - |g\rangle\langle g|$$

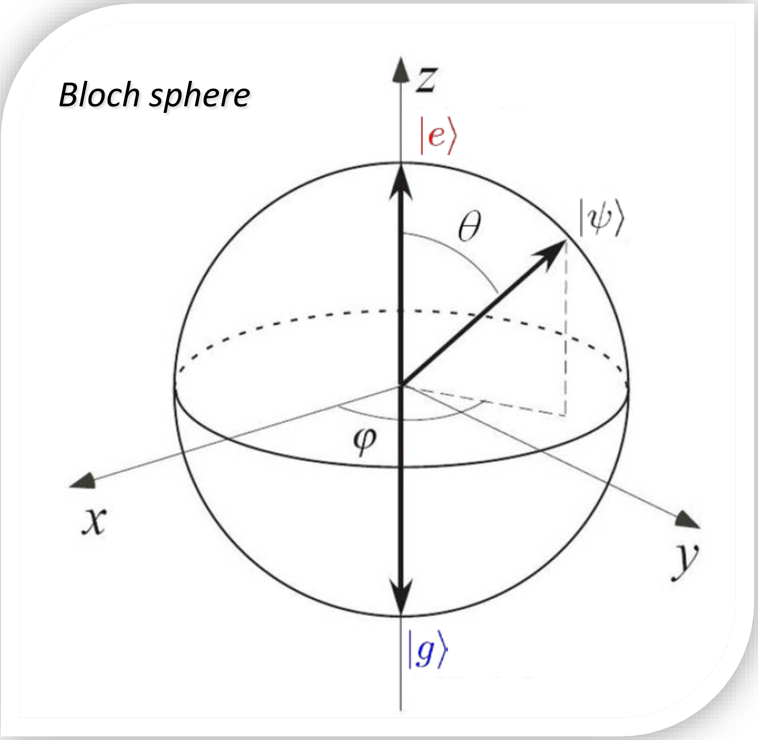


Arbitrary state as a superposition

$$|\psi\rangle = \cos\frac{\theta}{2}|e\rangle + e^{i\varphi}\sin\frac{\theta}{2}|g\rangle$$

Bloch sphere representation

states on the sphere = pure
states inside the sphere = mixed
state at the origin = equal mixture of two orthogonal states
opposite vectors = orthogonal states



Rabi oscillation

Coupling to a resonant classical light

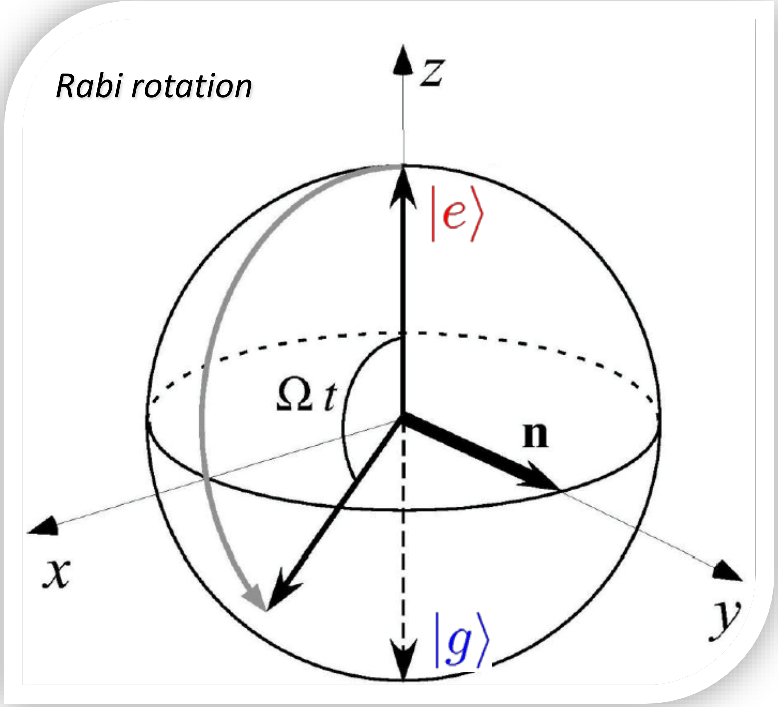
$$H_r = i\hbar\frac{\Omega_r}{2}[\sigma_- - \sigma_+]$$

raising and lowering operators:

$$\sigma_+ = |e\rangle\langle g|$$
$$\sigma_- = |g\rangle\langle e|$$

light-atom coupling:
(Rabi frequency)

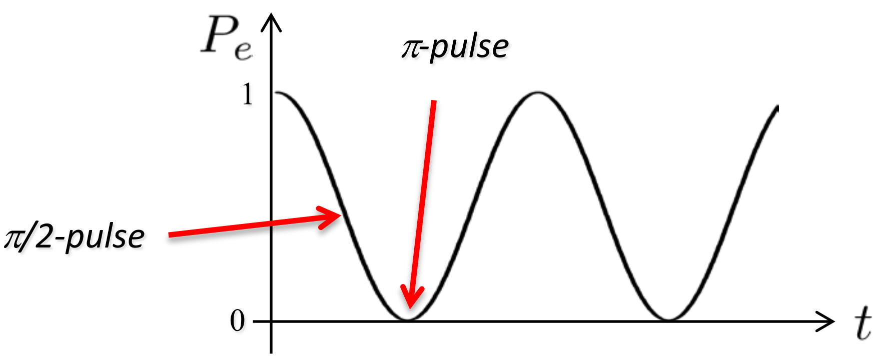
$$\Omega_r = \frac{2d}{\hbar}\mathcal{E}_r \boldsymbol{\epsilon}_a^* \cdot \boldsymbol{\epsilon}_r$$



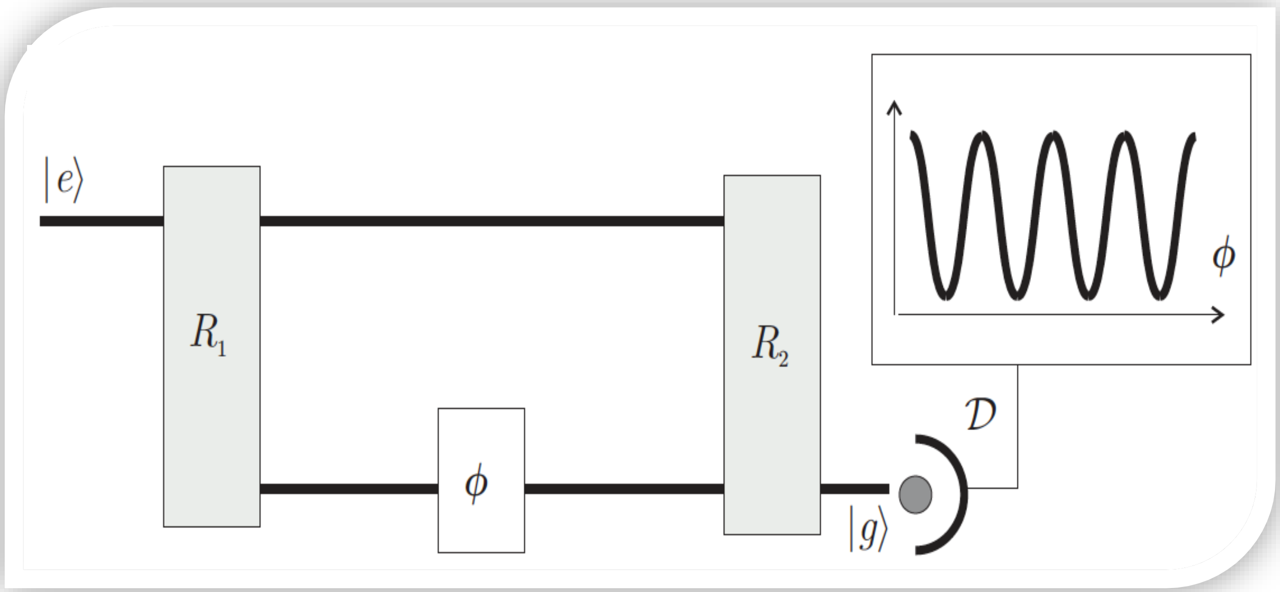
Rabi oscillations (if started in $|e\rangle$)

$$|\Psi\rangle = \sin(\Omega_r t/2)|g\rangle + \cos(\Omega_r t/2)|e\rangle$$

$$P_e = \cos^2(\Omega_r t/2)$$



Two π/2-pulses in Ramsey zones R_1 and R_2



atomic analog to the optical Mach-Zehnder interferometer: Ramsey zones = beam splitters

Final state

$$|\Psi\rangle = \frac{1}{2}\left[(1-e^{i\phi})\,|e\rangle + (1+e^{i\phi})\,|g\rangle\right]$$

Probability of finding the atom in $|e\rangle$ and $|g\rangle$

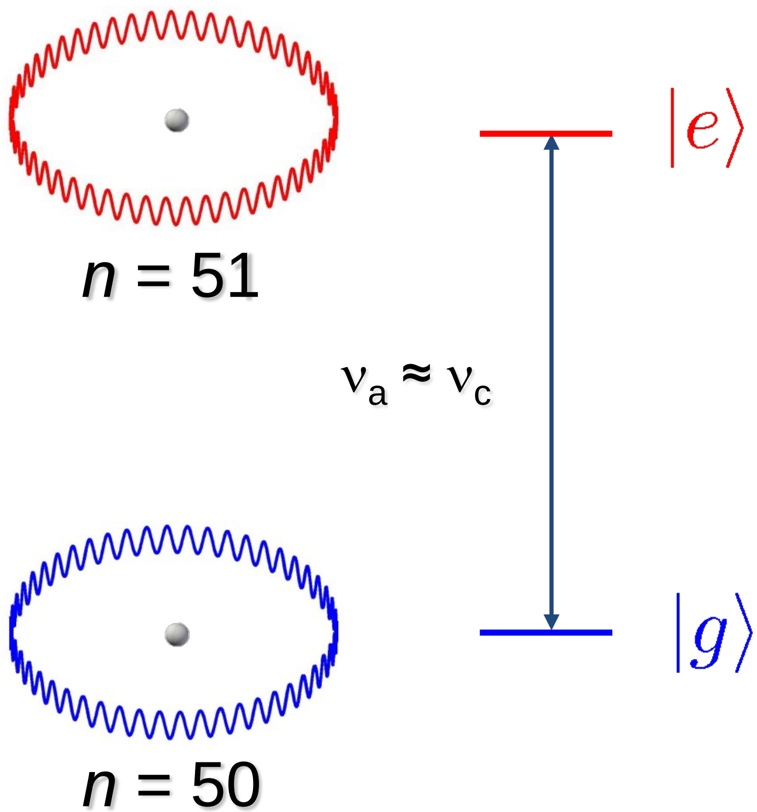
can be used for metrology (atomic clocks) and photon-number counting

$$P_{e,g|e} = (1 \mp \cos\phi)/2$$

Ramsey fringes

Circular Rydberg atoms

Rydberg atoms: large principle quantum number *n*
circular states: maximal *l* = |*m*| = *n* -1



Useful properties:

- two-level system ⇒ "simple" to treat and manipulate
- big electric dipole ⇐ huge sensitive "micro" antenna

$$(r \approx 1/4\,\mu m,\ d \approx 1800\,ea_0)$$

- long lifetime ⇒ more time for read-out

$$(T_a \approx\ 30\,\mathrm{ms})$$

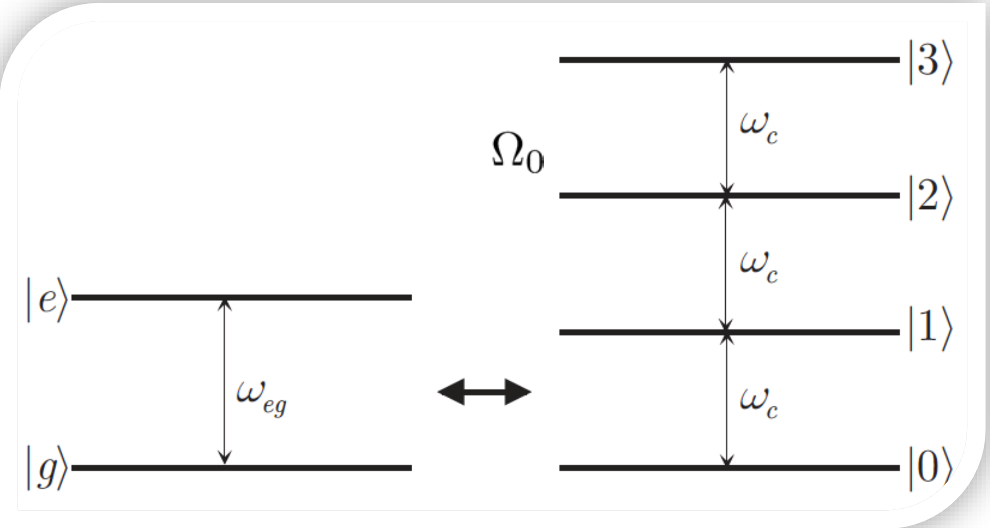
- easily tunable via Stark effect ⇐ small vertical electric field of several V/cm
- easy state detection ⇐ ionization electric field different for two states

Atom-cavity interaction

Atom-field Hamiltonian in rotating wave approximation

$$H_{ac} = i\hbar\frac{\Omega_0}{2}(a^\dagger\sigma_- - a\sigma_+)$$

$$\Omega_0 = 2\frac{d\mathcal{E}_0\boldsymbol{\epsilon}_a^*\cdot\boldsymbol{\epsilon}_c}{\hbar} \approx 2\pi\cdot 51\mathrm{kHz}$$



Jaynes-Cummings model

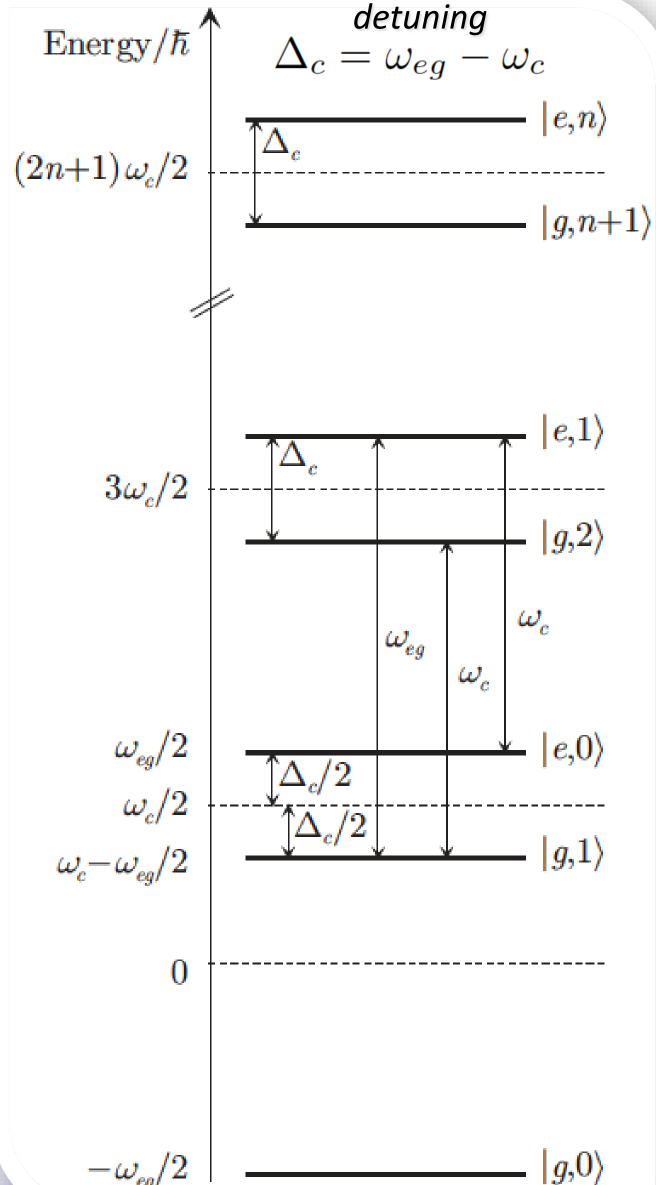
$$H = H_a + H_c + H_{ac} = \hbar\omega_c(a^\dagger a + 1/2) + \frac{\hbar\omega_{eg}}{2}\sigma_Z + i\hbar\frac{\Omega_0}{2}(a^\dagger\sigma_- - a\sigma_+)$$

Strong coupling regime – loss rates smaller than coupling

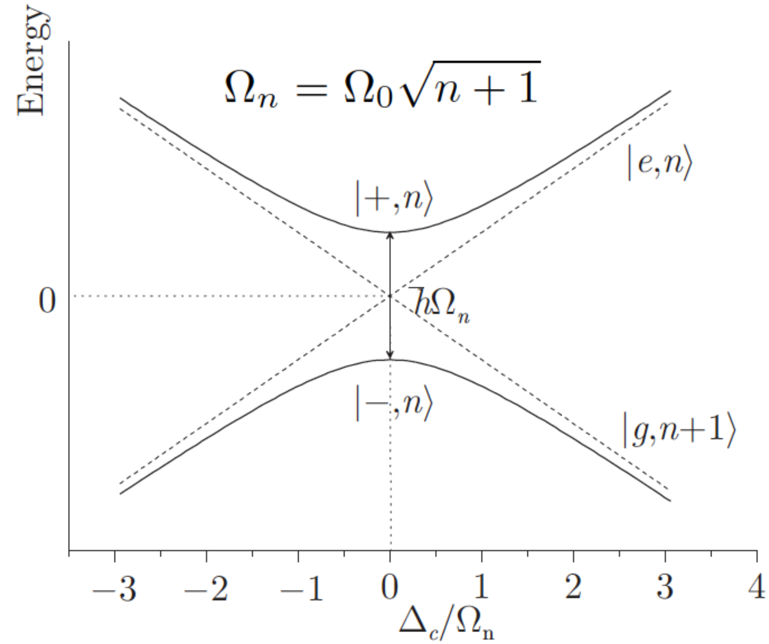
$$\Omega_0/2\pi \approx 50\,\mathrm{kHz} \quad \gg \quad (\kappa \approx 10\,\mathrm{Hz}, \quad \gamma \approx 35\,\mathrm{Hz})$$

$$T_{\mathrm{Rabi}} \approx 20\,\mu\mathrm{s} \quad \ll \quad (T_{\mathrm{c}} \approx 100\,\mathrm{ms}, \quad T_{\mathrm{a}} \approx 30\,\mathrm{ms})$$

Dressed states



Restriction to the n^{th} doublet (n excitation quanta)



$$E_n^{\pm} = (n + 1/2)\,\hbar\omega_c \pm \frac{\hbar}{2}\sqrt{\Delta_c^2 + \Omega_n^2}$$

Dressed states – eigenstates of coupled atom-cavity system

$$\begin{aligned} |+, n\rangle &= \cos(\theta_n/2)\,|e, n\rangle + i\sin(\theta_n/2)\,|g, n+1\rangle \\ |-, n\rangle &= \sin(\theta_n/2)\,|e, n\rangle - i\cos(\theta_n/2)\,|g, n+1\rangle \end{aligned}$$

mixing angle $\tan\theta_n = \Omega_n/\Delta_c$

Resonant and dispersive

Resonant regime: $\Delta_c = 0$

Consider initial state of *n* photons and atom in $|e\rangle$: $\left|\widetilde{\Psi}_e(t)\right\rangle = \cos\frac{\Omega_n t}{2}\,|e,n\rangle + \sin\frac{\Omega_n t}{2}\,|g,n+1\rangle$

Rabi oscillations (photon exchange) at frequency $\mathbf{\Omega}_n$

Dispersive regime: $|\Delta_c| > \Omega_0$

Energy eigenvalues $$E_n^{\pm} = (n+1/2)\,\hbar\omega_c \pm \hbar\left(\frac{\Delta_c}{2} + \frac{\Omega_n^2}{4\Delta_c}\right)$$

Quantized light shifts of uncoupled levels $$\Delta_{e,n} = \hbar(n+1)s_0\ ; \qquad \Delta_{g,n} = -\hbar n s_0 \qquad s_0 = \frac{\Omega_0^2}{4\Delta_c}$$

Two atomic levels are shifted in opposite directions, proportional to the photon number *n*

$$\delta\omega_{eg} = (2n+1)s_0$$

light shift (→ $2n$) *vacuum Lamb shift* (→ 1)

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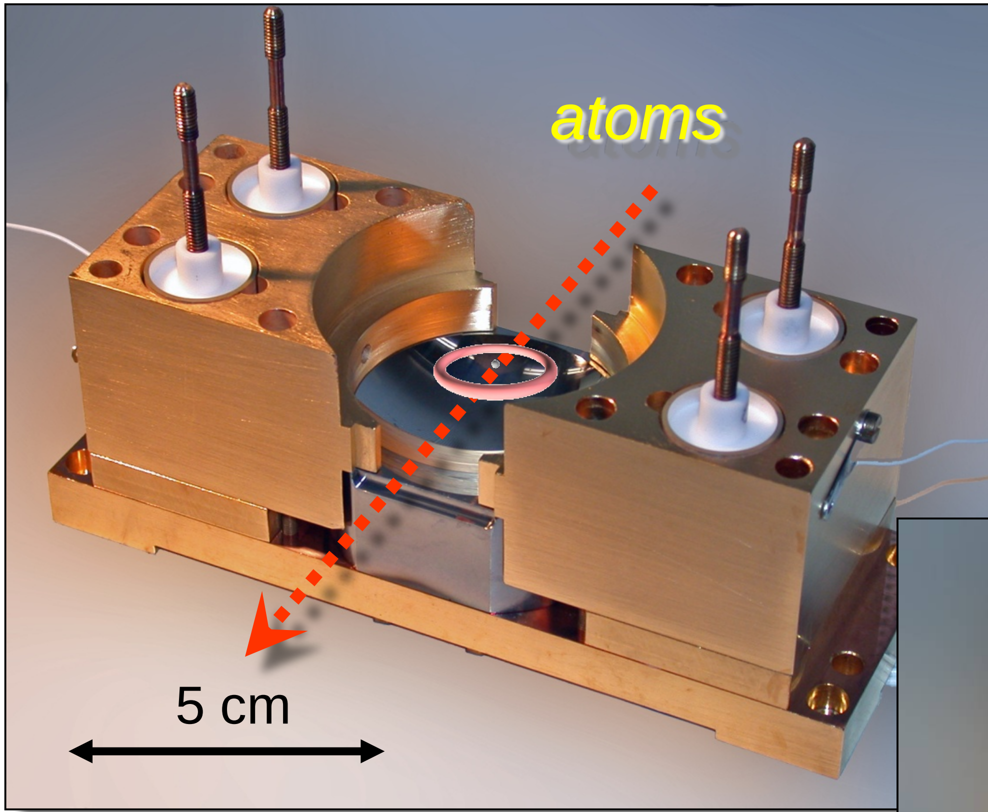
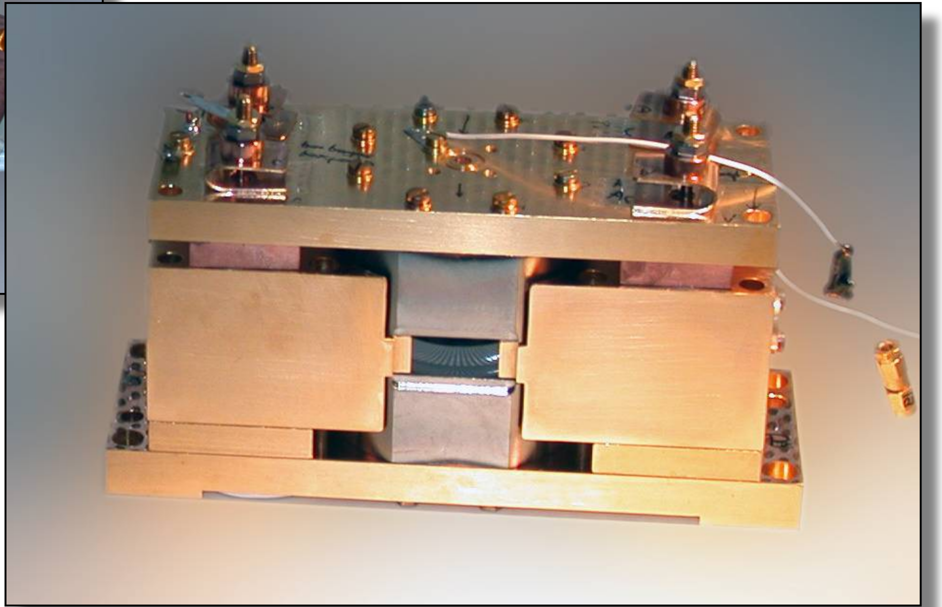
Cavity assembly

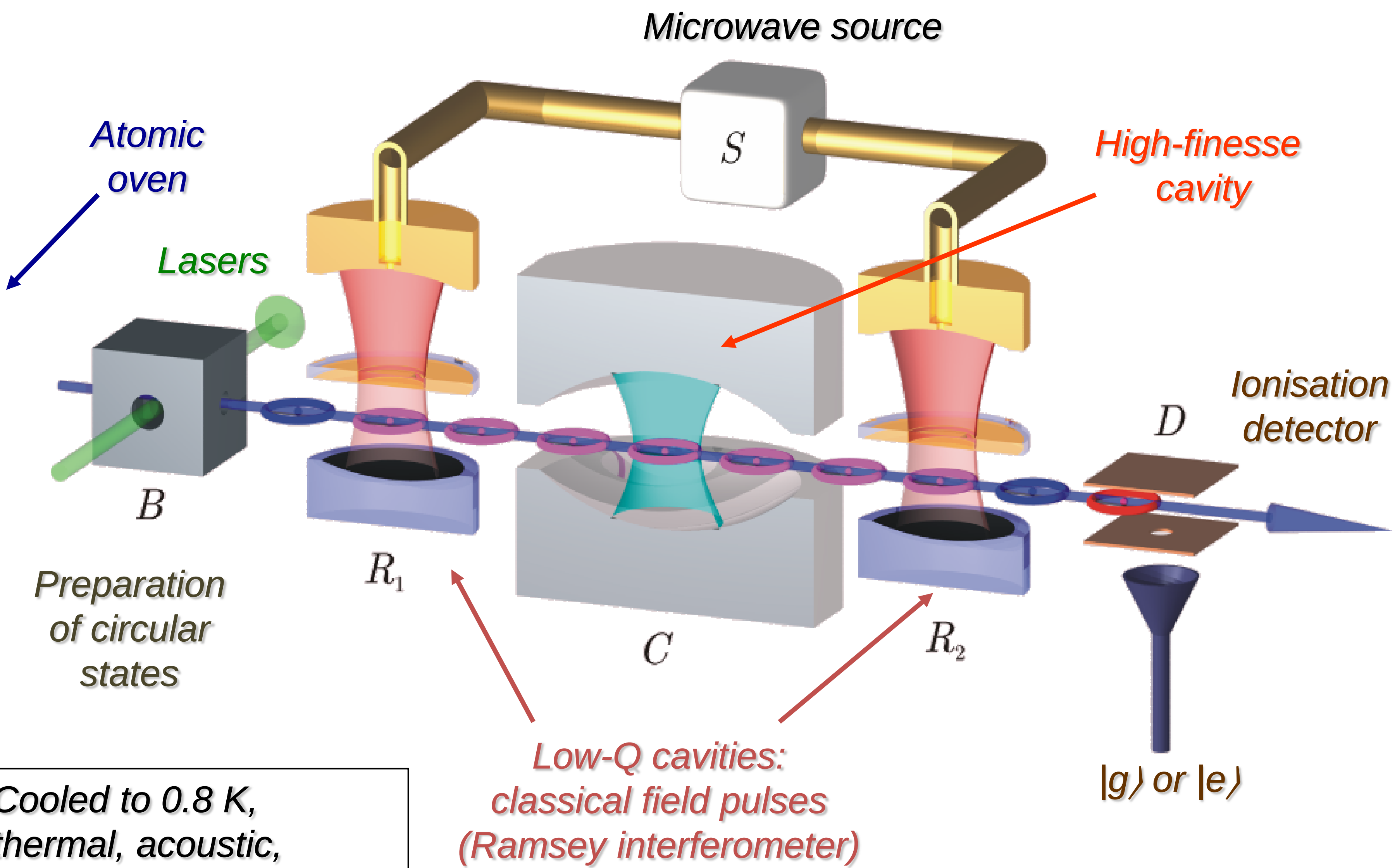


⇐ *one mirror-and-piezo mount*

assembled cavity block ⇒

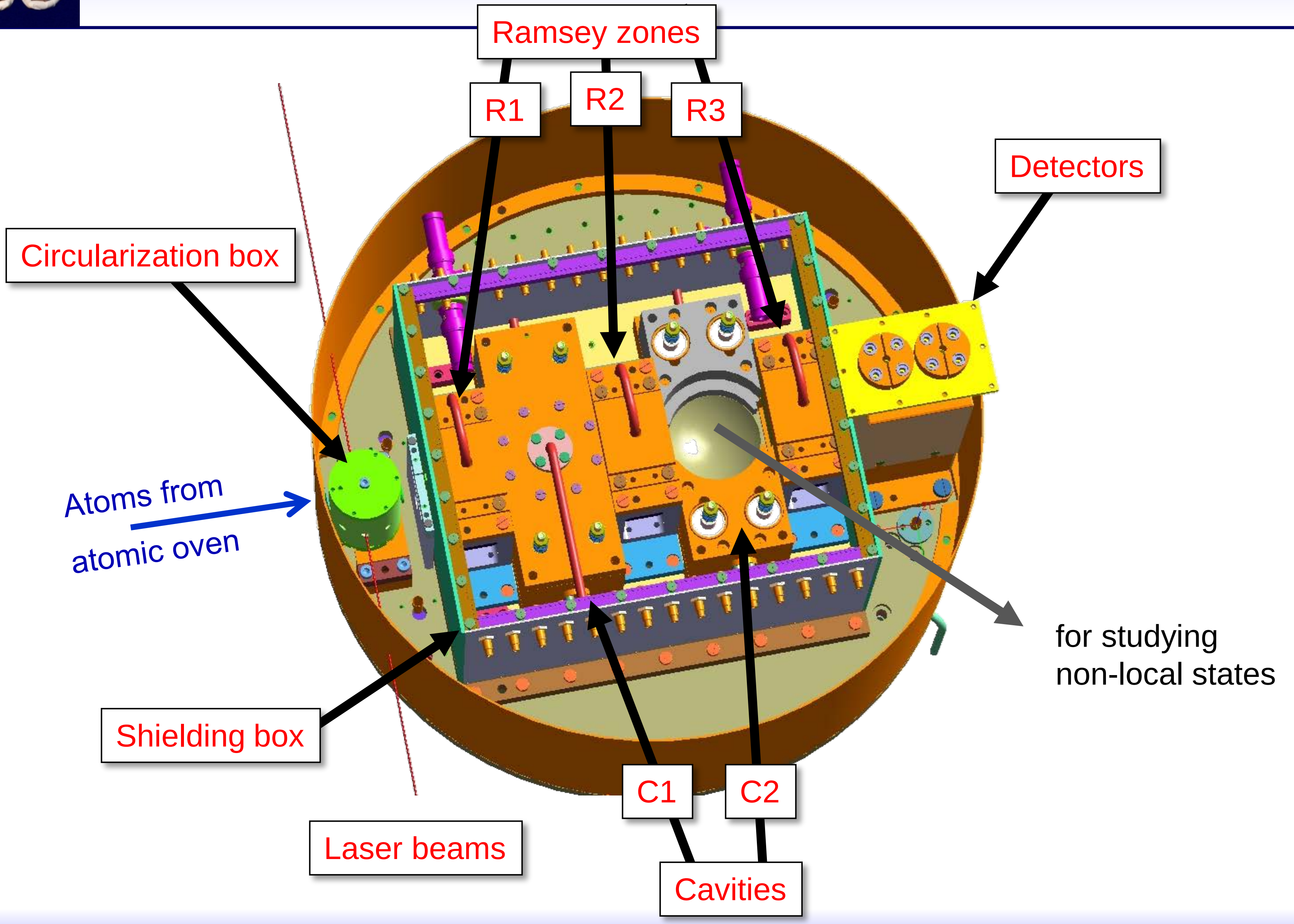


Meeting atoms with photons

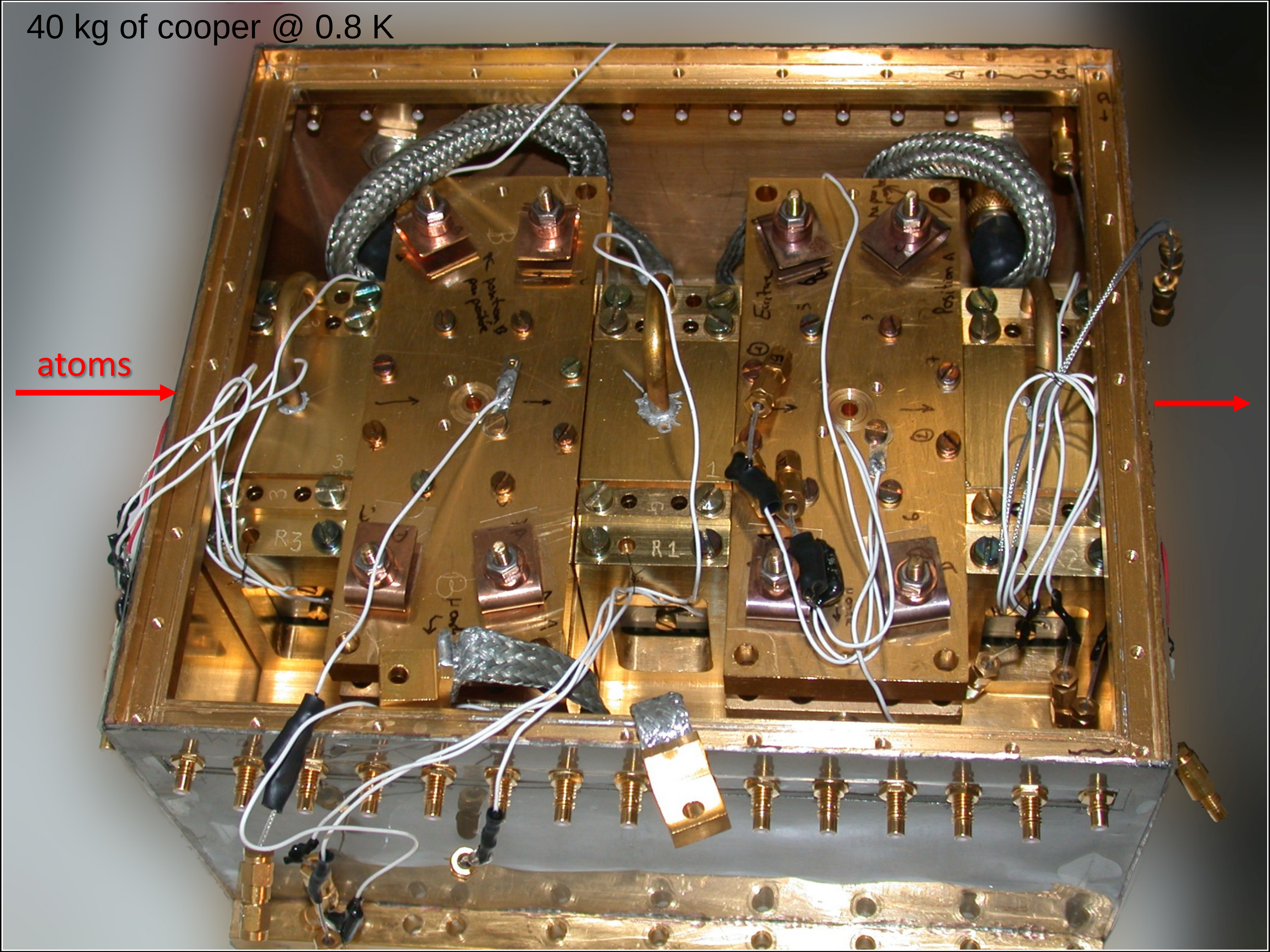


Cooled to 0.8 K, thermal, acoustic, magnetic & electric isolation

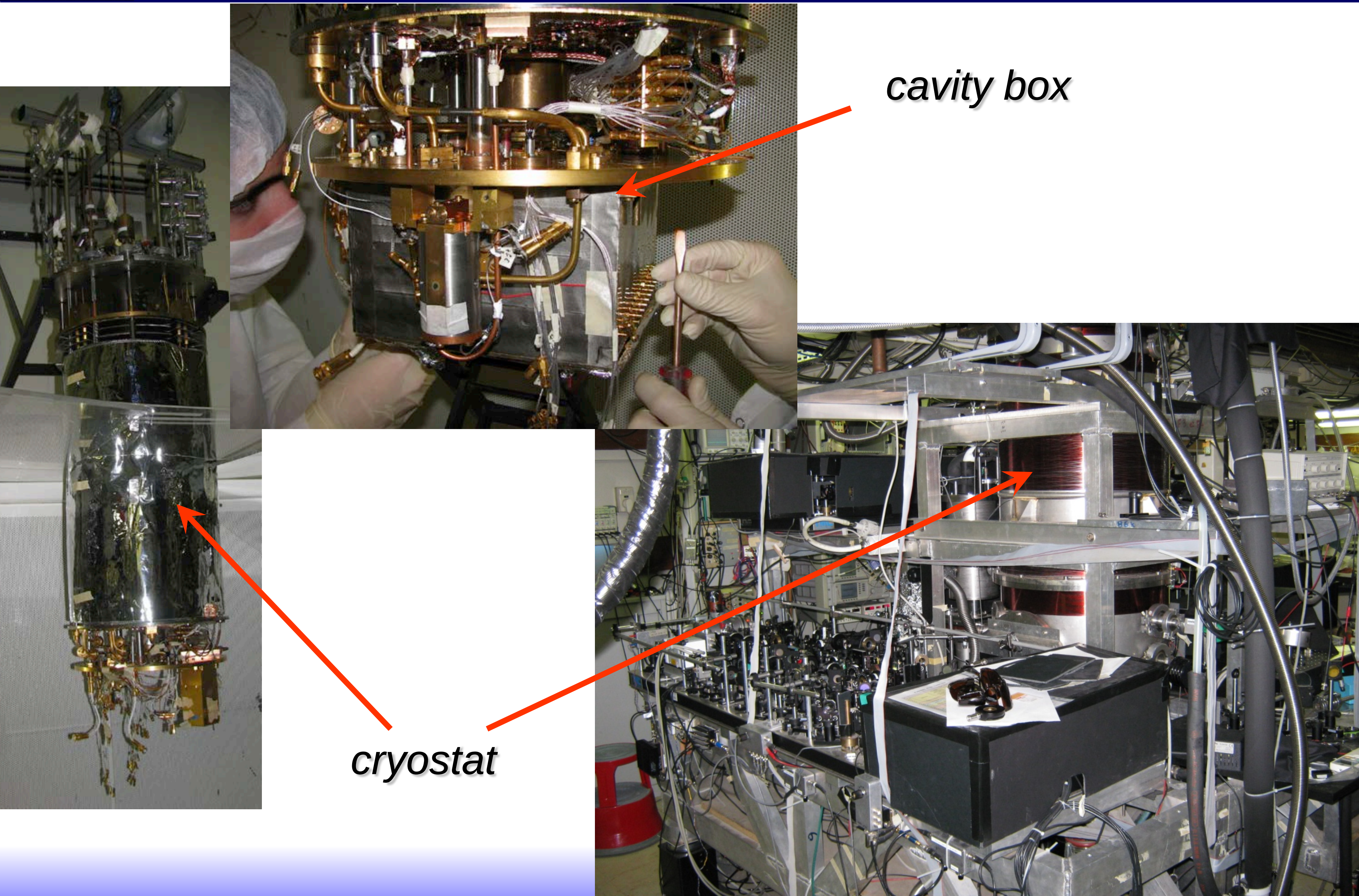
More realistic drawing



40 kg of cooper @ 0.8 K



In the lab



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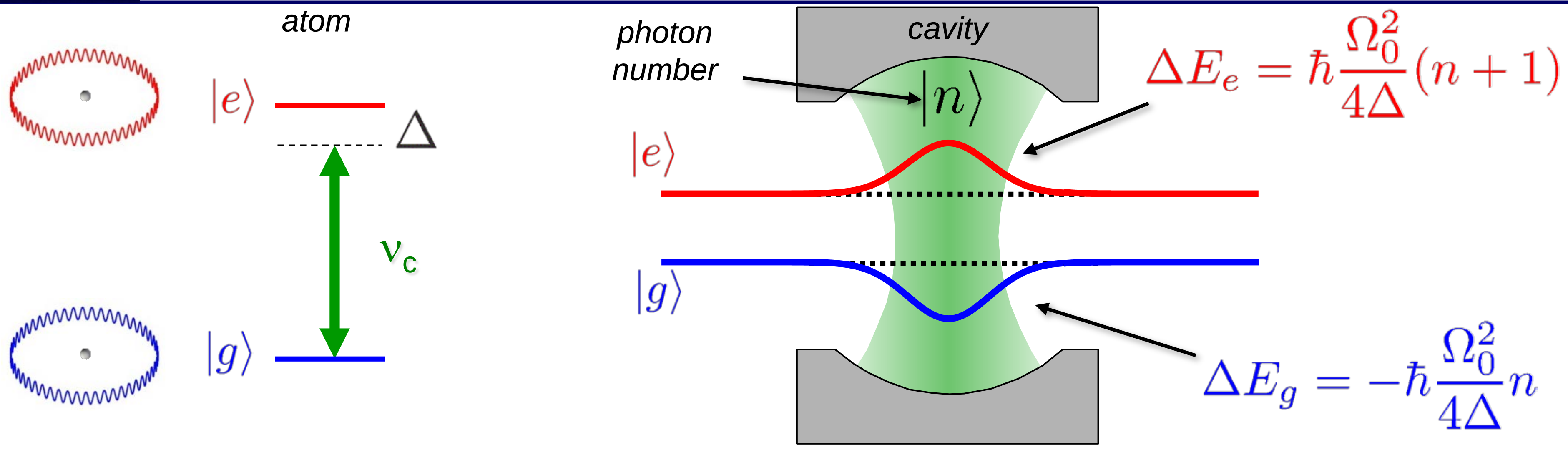
Flying atom and cavity mode

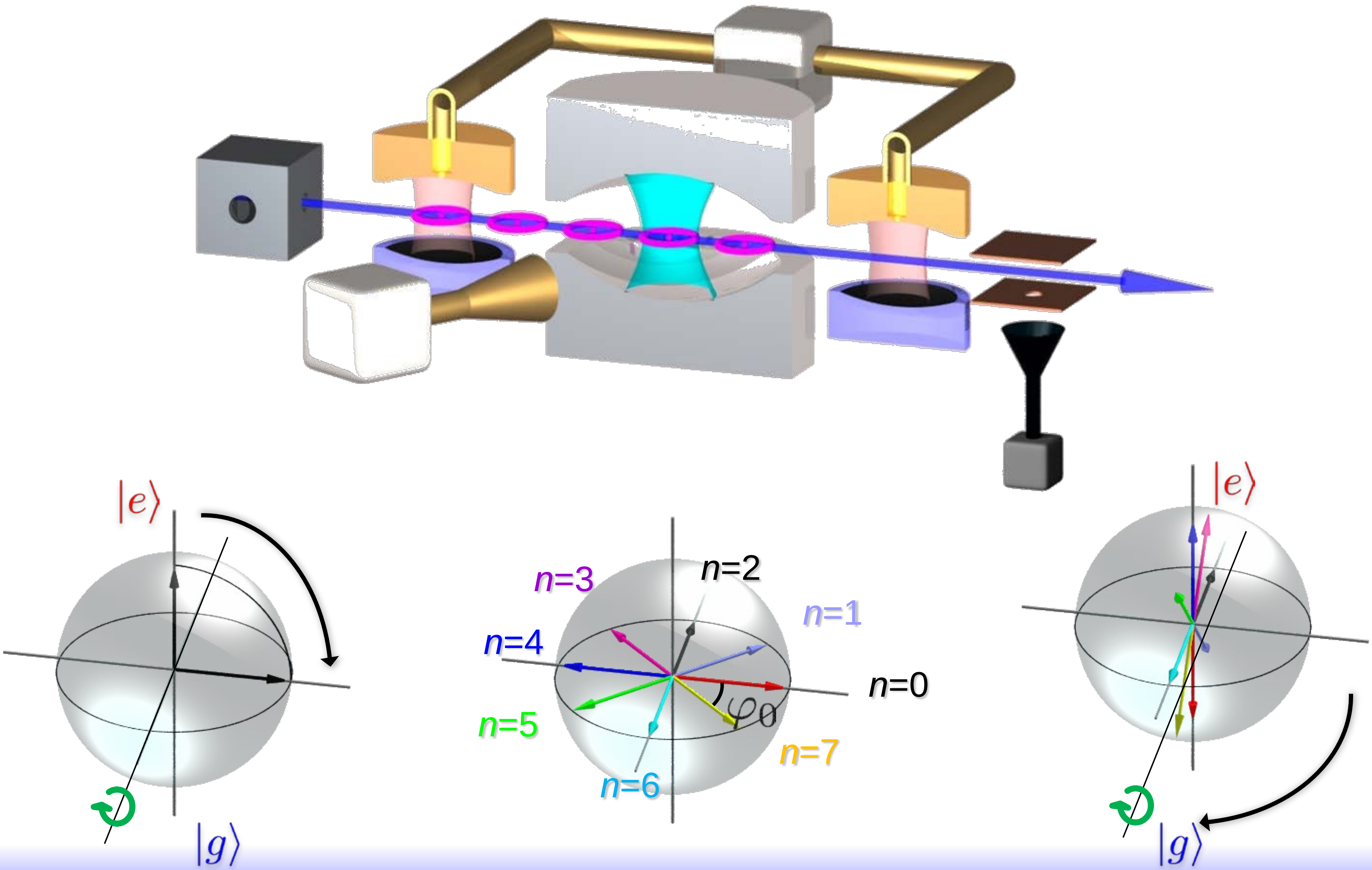


- ➔ Energy conservation + adiabatic coupling $\Rightarrow$ the field (*and thus* n) is preserved
- ➔ Atom-field interaction modifies ν_{at} proportional to n
- ➔ Due to frequency change, atomic dipole acquires a phase

$$\varphi(n) = (n + 1/2)\varphi_0$$

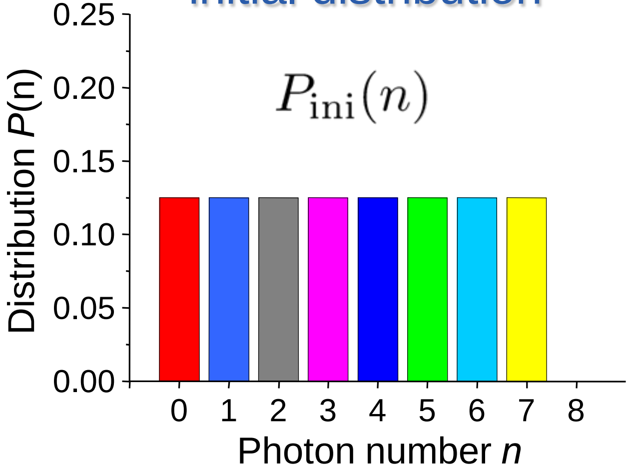
$$\varphi_0 = \frac{\Omega_0^2}{2\Delta} t_{\mathrm{int}}$$

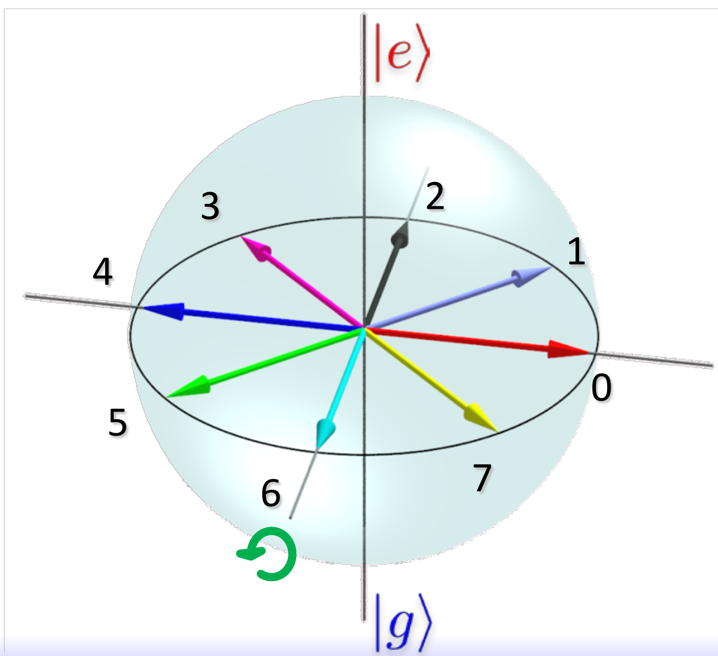
phase shift per photon


$|e\rangle$
$|g\rangle$
n=3
n=2
n=1
n=4
n=0
φ_0
n=5
n=7
n=6
$|e\rangle$
$|g\rangle$

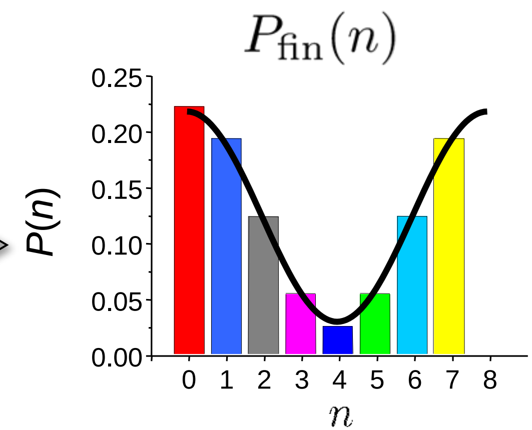
Photon-number measurement

initial distribution

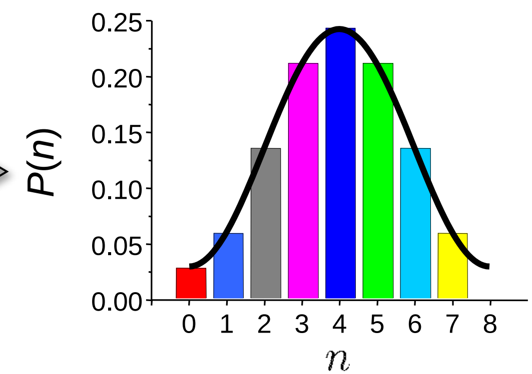




atomic detection modifies photon-number distribution



atom in $|e\rangle$



atom in $|g\rangle$

Photon-number measurement

Bayesian inference
and measurement at time t_i :

final *initial*

$$P(n, t_i^+) \propto P(a_i|\phi_{ri}, n) P(n, t_i^-)$$

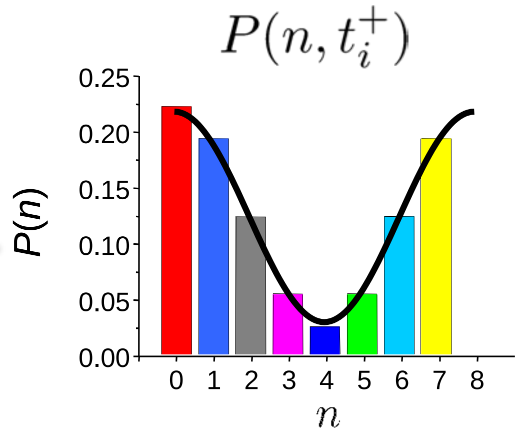
atomic state $a_i \in \{g, e\}$

with conditional probability :

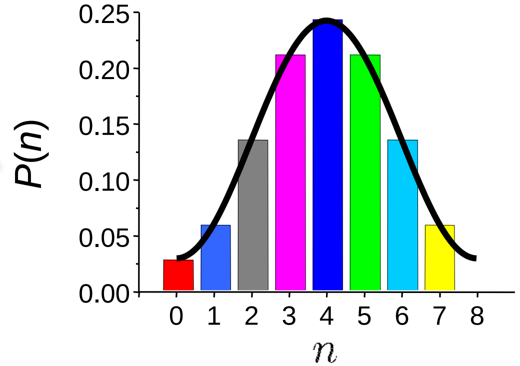
$$P(g|\phi_r, n) = \cos^2[\varphi_0(n + 1/2) + \phi_r]$$

dephasing per photon *Ramsey phase*

atomic detection modifies
photon-number distribution

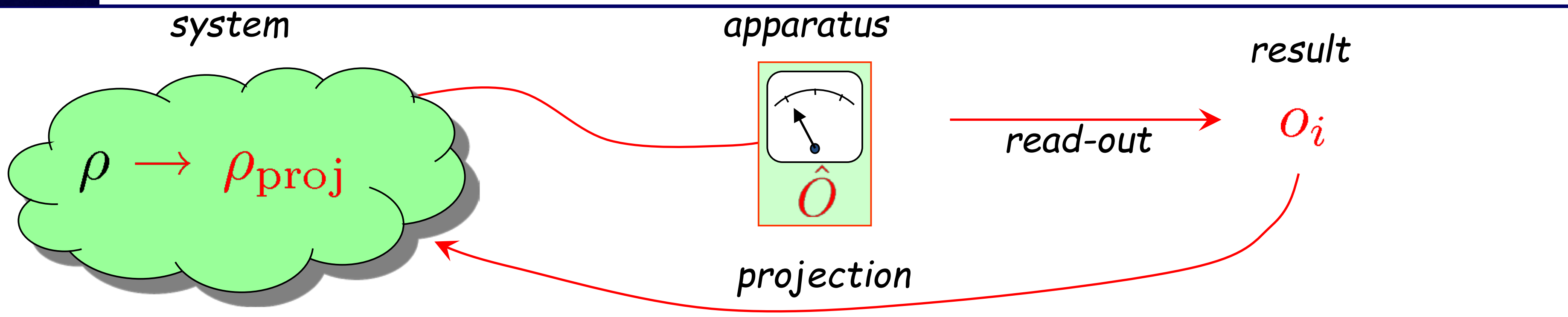


atom in |*e*⟩



atom in |*g*⟩

Projective measurement



Measured observable is described by a Hermitian operator O. Set of projectors $\{P_i\}$ onto the sub-space of possible results $\{o_i\}$ of O have the following properties:

$$O = \sum_i o_i P_i \qquad \sum_i P_i = I \qquad P_i = P_i^2 = P_i^\dagger$$

Rule 1: Measurement result is **random** with probability

$$p_i = \mathrm{Tr}(\rho P_i)$$

Rule 2: After result i the system is **projected** onto the state

$$\rho_{\rm proj} = \frac{P_i \rho P_i}{p_i}$$

Repeated projective (ideal) measurements give the same result

Positive operator valued measure (*POVM*)

Every generalized measurement can be associated to a set of (not necessarily Hermitian) operators $\{M_i\}$ with ***i*** running through all possible measurement results.

POVMs $E_i = M_i^\dagger M_i$ satisfy the completeness relation:

$$\sum_i M_i^\dagger M_i = I$$

(if $M_i = M_i^2$, then POVM reduces to a projective measurement)

Rule 1: Applied to a system in ρ the measurement gives the result ***i*** with probability of

$$p_i = \mathrm{Tr}(\rho\, M_i^\dagger M_i)$$

Rule 2: The measurement projects the system onto the state

$$\rho_{\mathrm{proj}} = \frac{M_i\, \rho\, M_i^\dagger}{p_i}$$

Opposite to projective, POVM measurement can be non-repeatable !

Also called weak measurement.

Weak measurement

initial state

$$(j = e, g) \quad \rho_{\mathrm{proj}} = \frac{M_j \, \rho \, M_j^\dagger}{\mathrm{Tr}(M_j \, \rho \, M_j^\dagger)}$$

state after projection

measurement direction

$$M_e = \sin\left(\frac{\varphi_0(N + 1/2) + \phi_r}{2}\right)$$

photon number operator

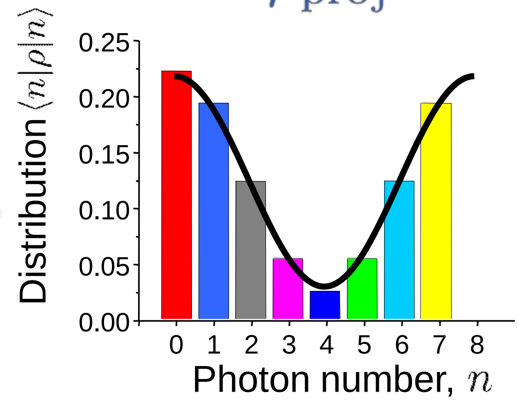
Two (Kraus) operators corresponding to two possible outcomes

dephasing per photon

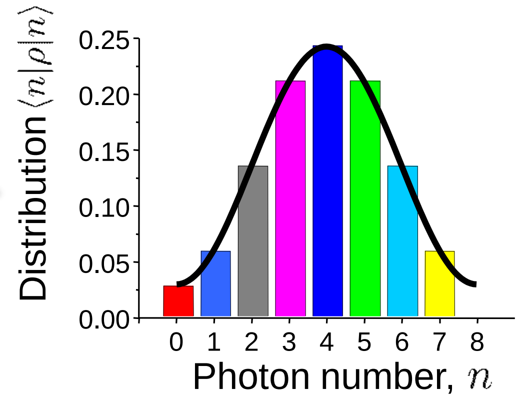
$$M_g = \cos\left(\frac{\varphi_0(N + 1/2) + \phi_r}{2}\right)$$

atomic detection modifies quantum state

ρ_{proj}

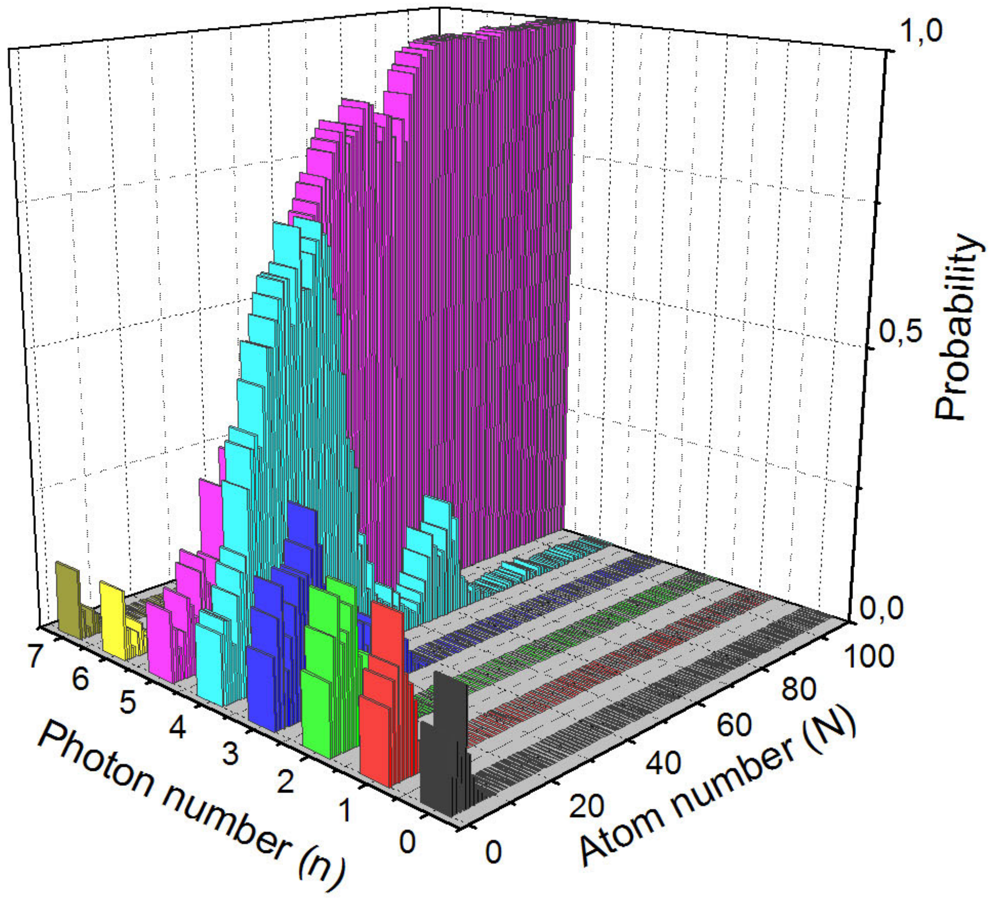


atom in $|e\rangle$



atom in $|g\rangle$

From weak to projective measurement



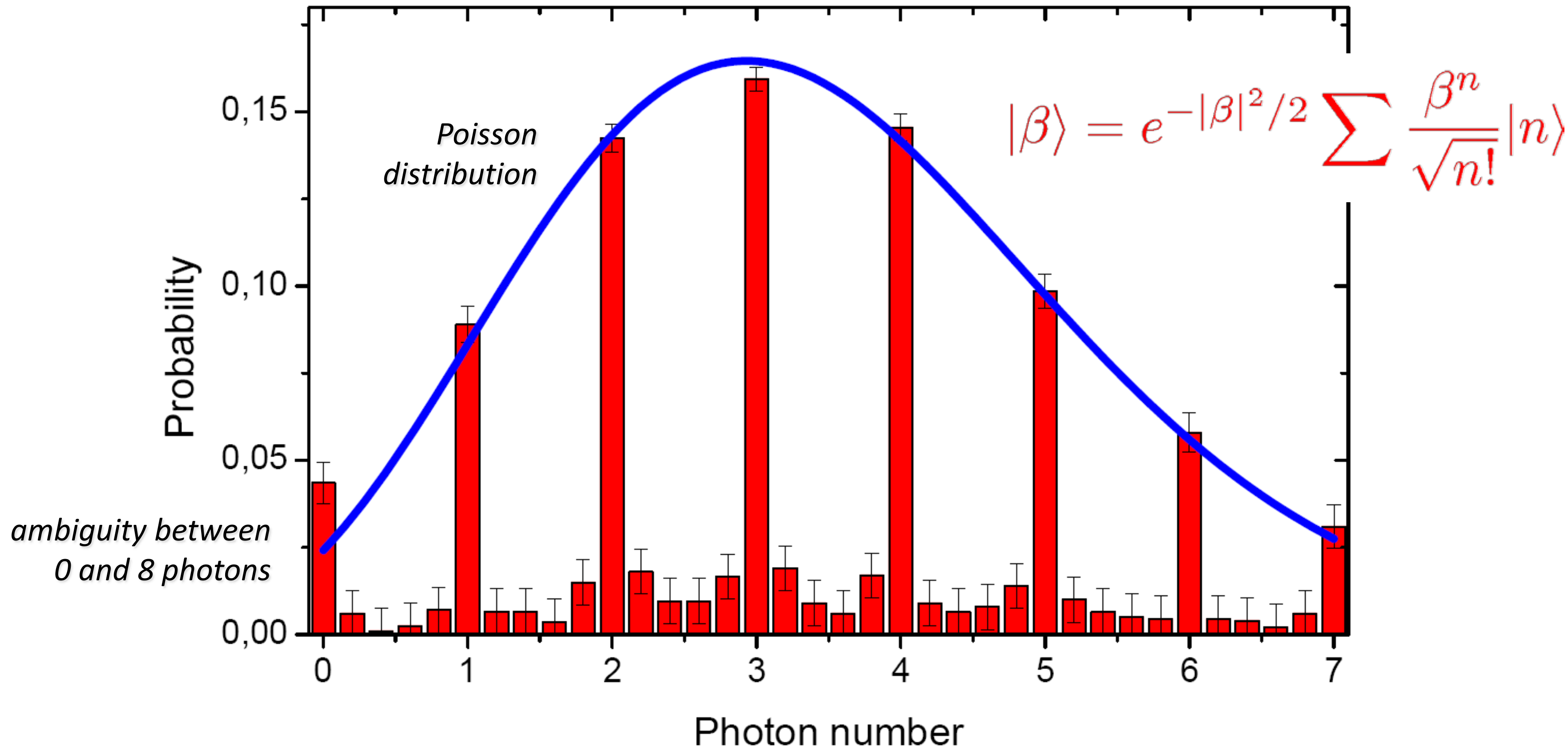
• Evolution of photon number distribution while detecting many atoms in a single sequence with 4 alternating detection directions

• Progressive collapse of the field state vector during information acquisition into a random photon-number state

(initial coherent field with 3.7 photons)

Many repeated **weak** measurements result in the ideal **projective** measurement of the photon number

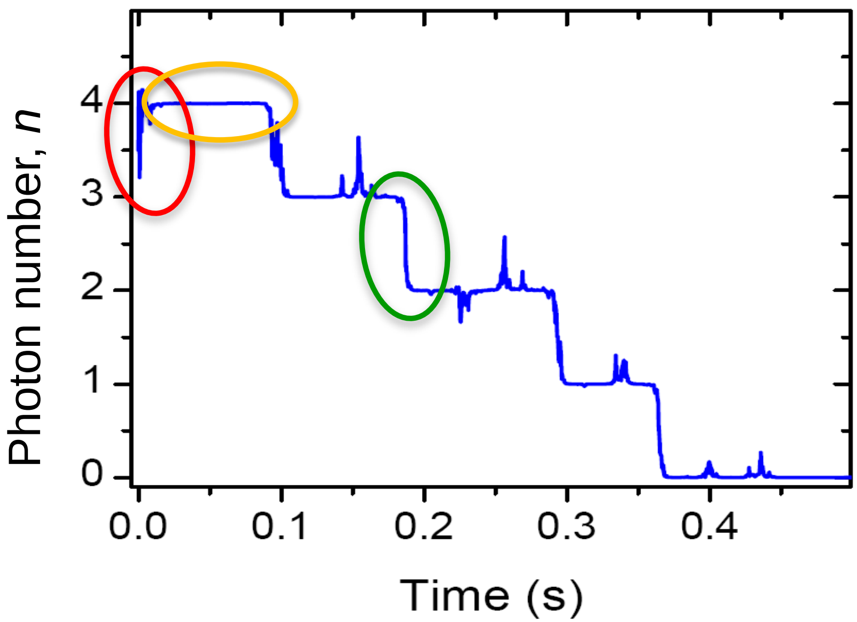
Photon number statistics



Statistics of final photon number
reveals the initial coherent field of 3.7 photons

Quantum jumps of light

Real-time observation of the quantum field evolution



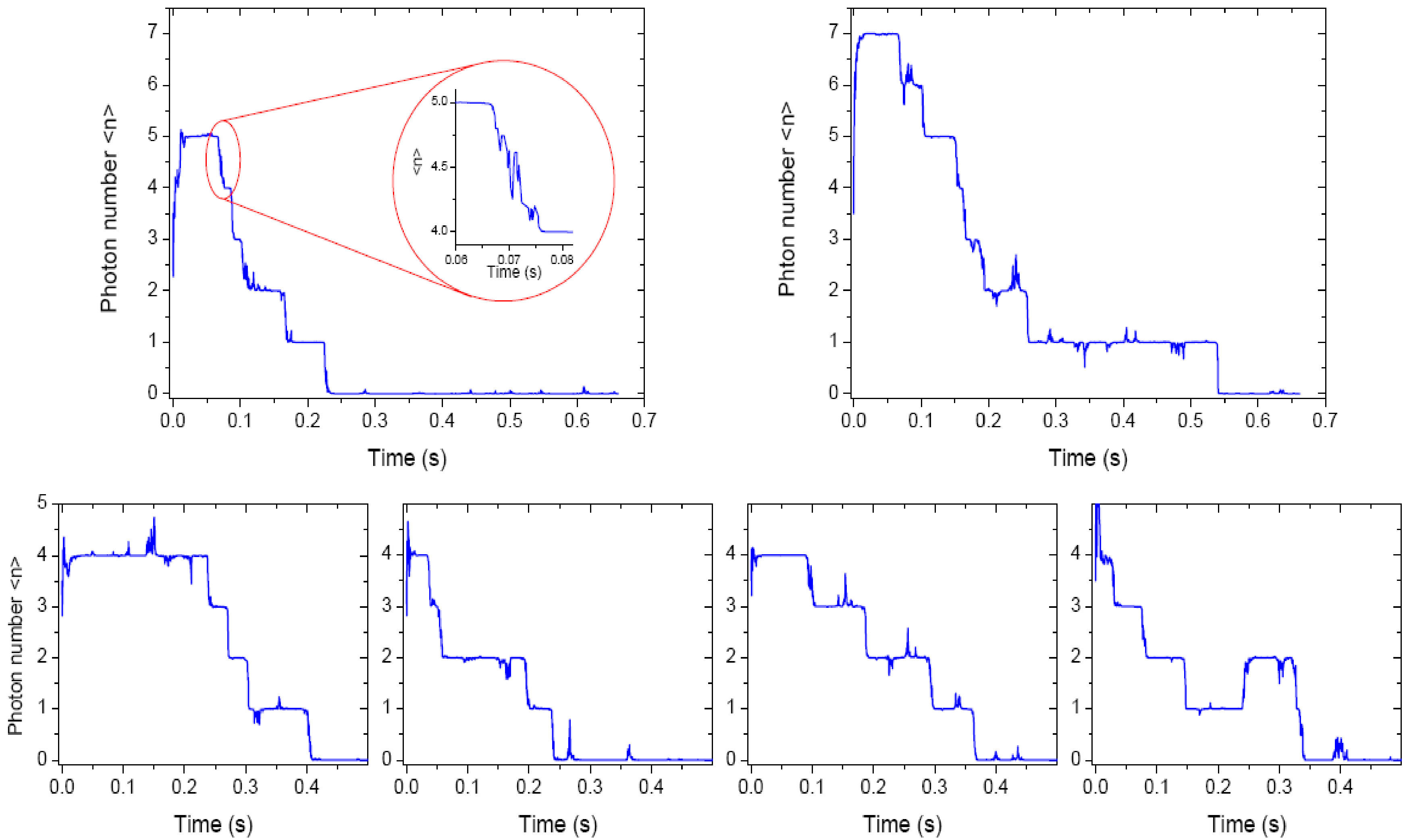
Random projection onto one of n values, but with an *a priori* known probability

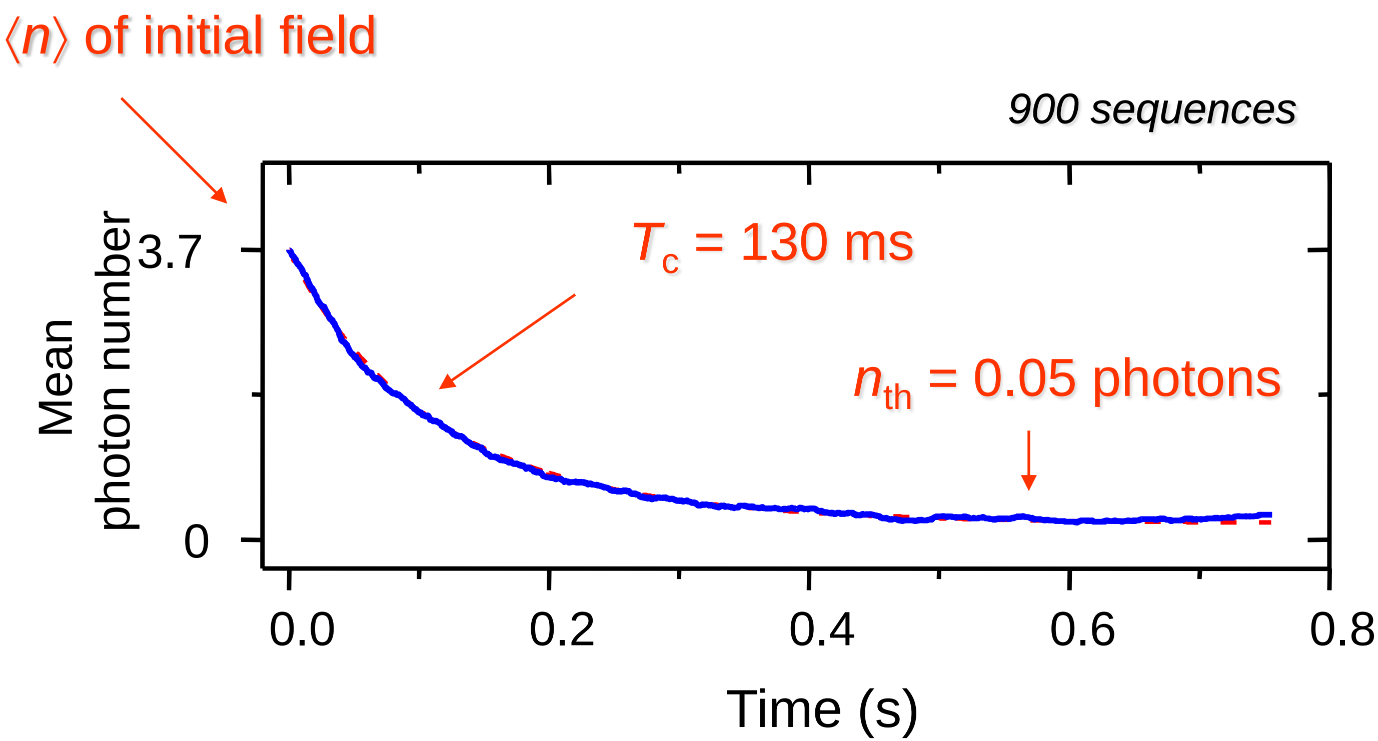
Repeatability of QND measurement

Quantum jumps between discrete values of n: damping of the field caught in the act

A vivid illustration of quantum measurement postulates

Gallery of trajectories





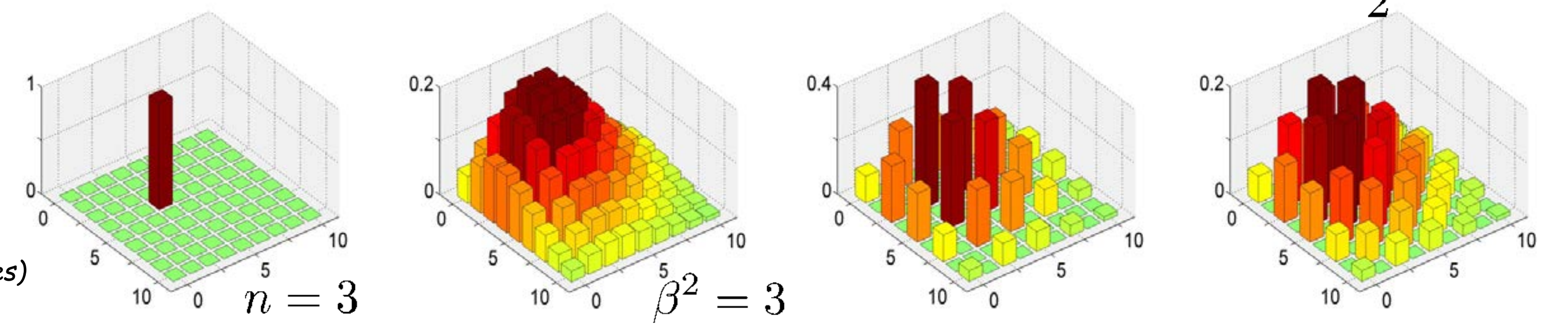
Smooth exponential decay of a harmonic oscillator:

A direct illustration of the difference between individual quantum realizations and ensemble average

Cavity field states

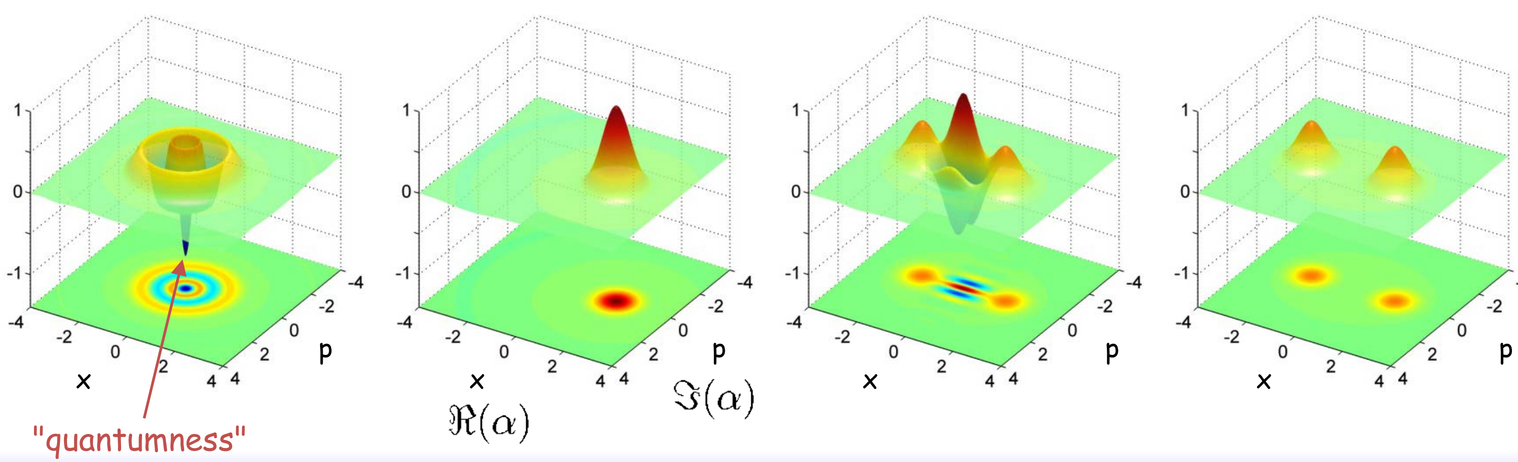
	Photon-number state	Coherent state	"Schrödinger's cat" state	Statistical mixture
Wave function $\vert\Psi\rangle$	$\vert n\rangle$	$\vert\beta\rangle = \frac{1}{\mathcal{N}} \sum_n \frac{\beta^n}{\sqrt{n!}} \vert n\rangle$	$\frac{\vert\beta\rangle + \vert-\beta\rangle}{\sqrt{2}}$	
Density matrix $\hat{\rho}$	$\vert n\rangle\langle n\vert$	$\vert\beta\rangle\langle\beta\vert$	$\vert\Psi\rangle\langle\Psi\vert$	$\frac{\vert\beta\rangle\langle\beta\vert + \vert-\beta\rangle\langle-\beta\vert}{2}$

(for 3 photon states)



Wigner function $W(\alpha)$

(phase space representation)



Wigner function

$$W(\alpha = x + ip) = \frac{2}{\pi}\mathrm{Tr}\left[\hat{\rho}\,\hat{D}(\alpha)(-1)^{\hat{N}}\hat{D}(-\alpha)\right]$$

State displaced by complex amplitude -α

Parity measurement

$$\hat{P}|n\rangle = (-1)^{\hat{N}}|n\rangle = \begin{cases} +|n\rangle & \text{if } n \text{ even} \\ -|n\rangle & \text{if } n \text{ odd} \end{cases}$$

Direct Wigner function measurement recipe:

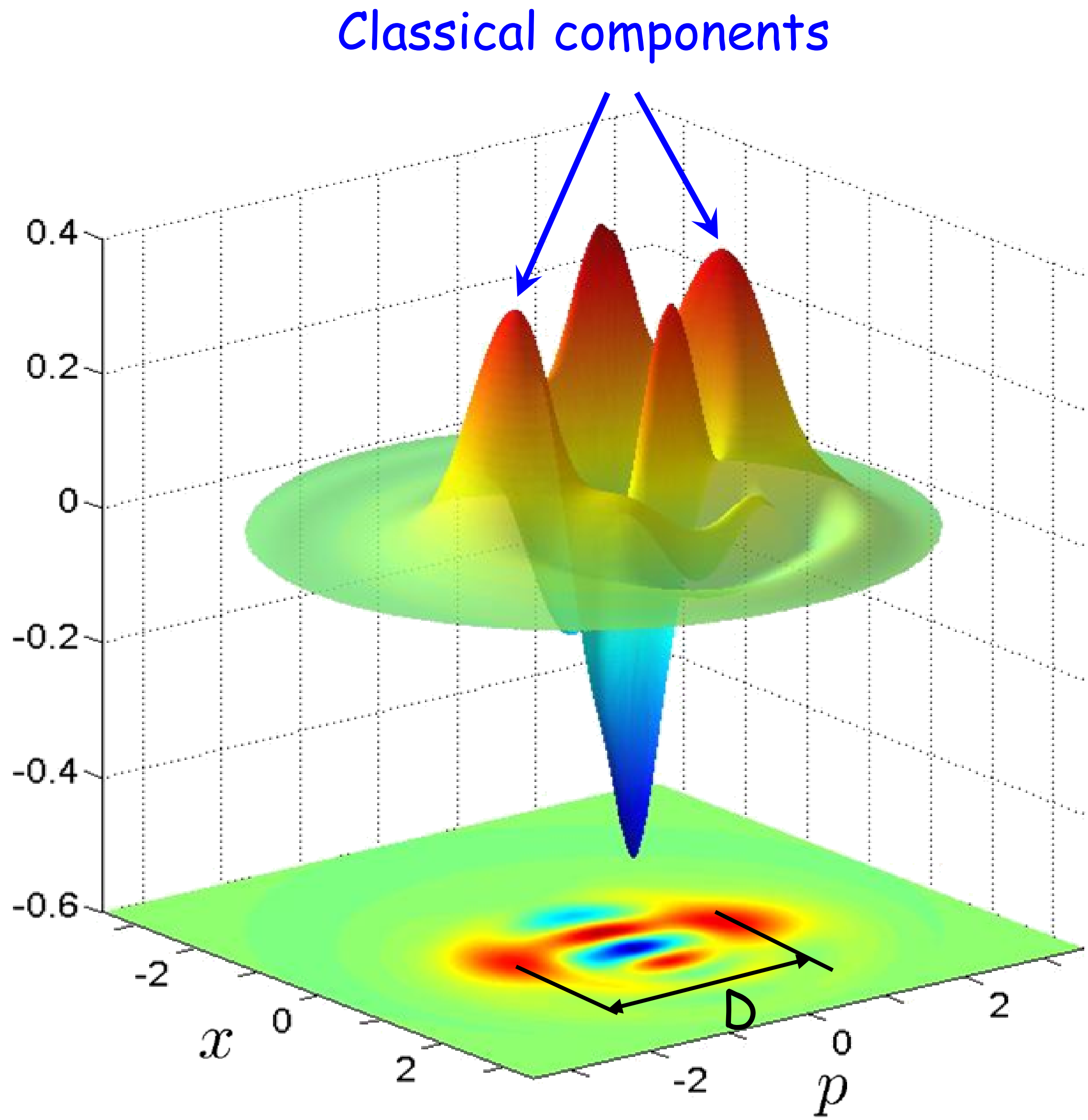
1. Inject a coherent field (–α).
2. Measure repeatedly photon number parity: its average value yields W(α).
3. Resume for different α values

Schrödinger's cat state of light

2 photons in each classical component (*amplitude of the initial coherent field*)

cat size $D^2 \approx 7$ photons

coherent components are completely separated ($D > 1$)

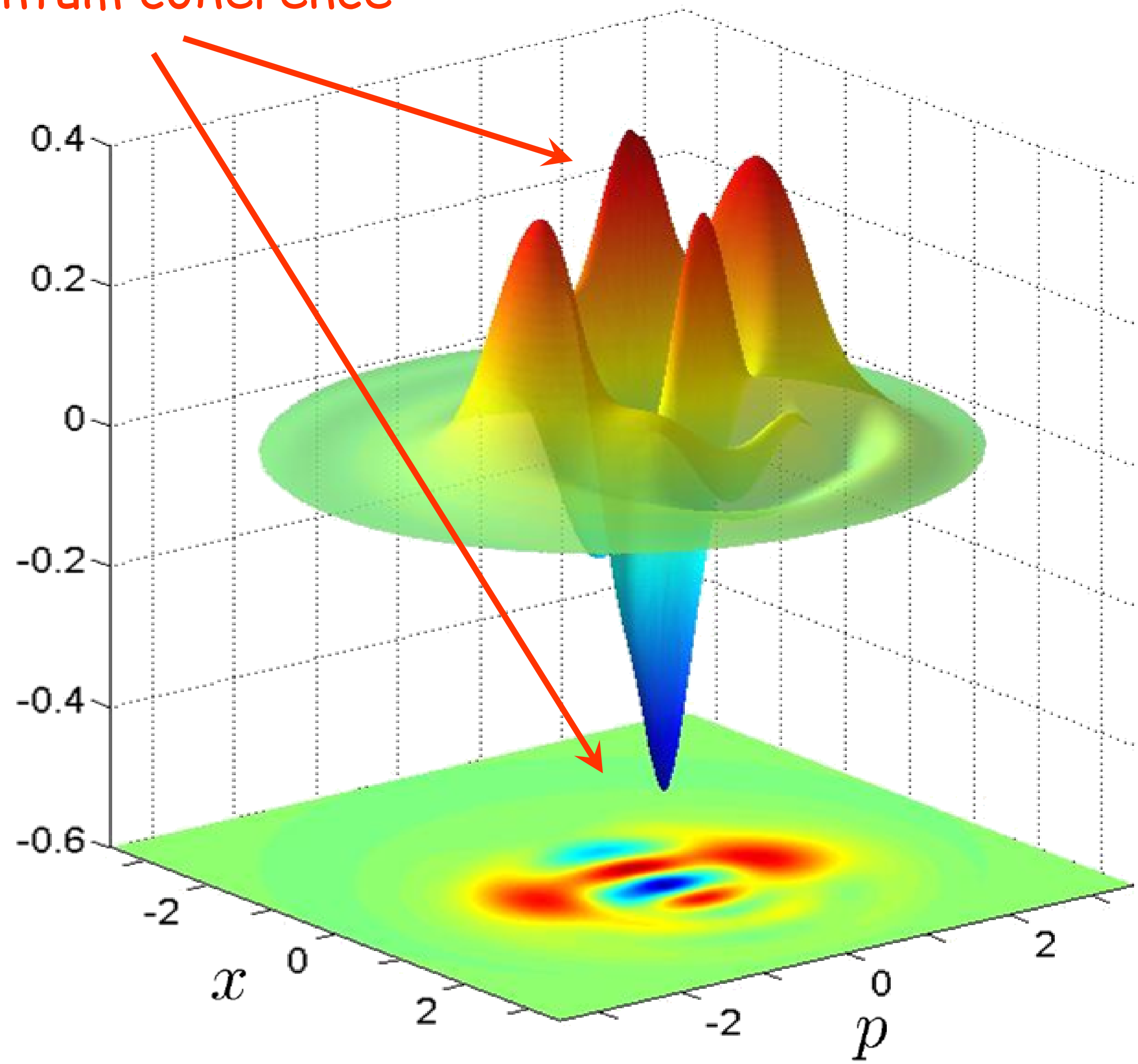


Schrödinger's cat state of light

quantum superposition
of two classical fields
(interference fringes)

quantum signature of
the prepared state
*(negative values of
Wigner function)*

Quantum coherence

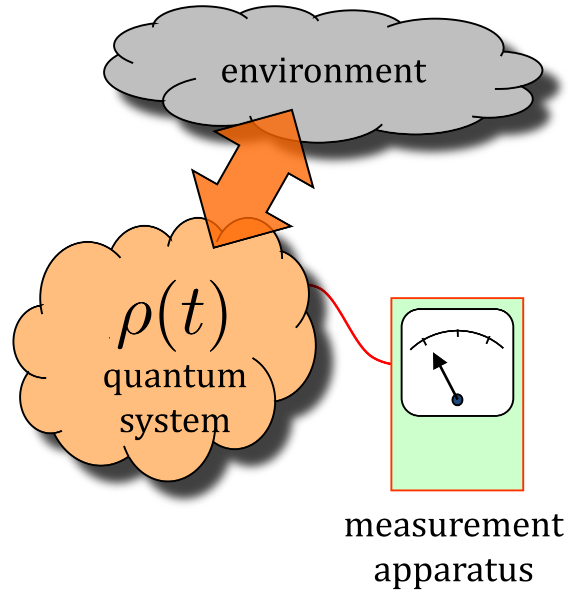


Outline

- Quantum non-demolition measurement
  - Basics of atom-cavity interaction : a spring and a spin
  - Experimental setup: Photon box
  - QND photon-number measurement
  - Past quantum states
- Quantum control
  - Quantum feedback stabilizing photon-number states
  - Adaptive QND measurement

Quantum state

State of a system defines all its properties (results of any measurement).

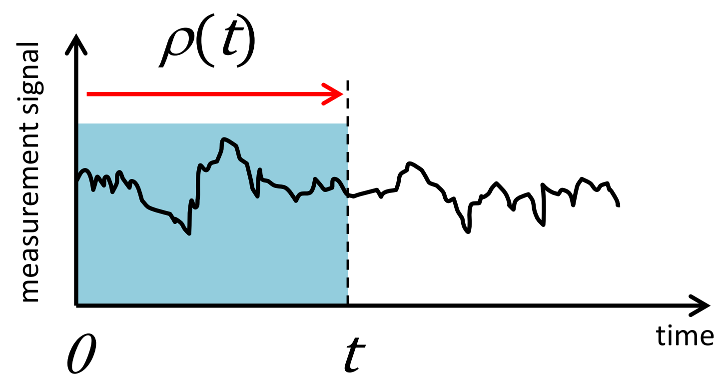


Estimation of the density matrix $\rho(t)$ from:

- initial system's state
- coherent evolution (*Hamiltonian*)
- coupling to environment (*decoherence*)
- measurement results (*measurement back-action*)

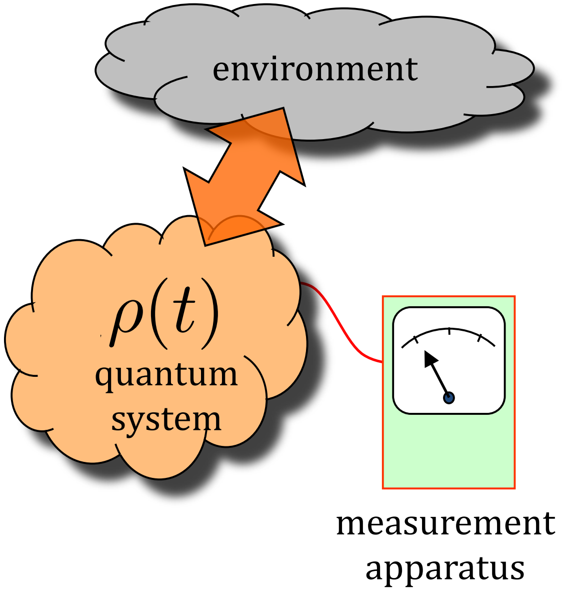
The probability of a measurement outcome n defined by Ω_n:

$$P^f(n,t) = \mathrm{Tr}\left[\hat{\Omega}_n^\dagger \hat{\Omega}_n \rho(t)\right]$$



density matrix

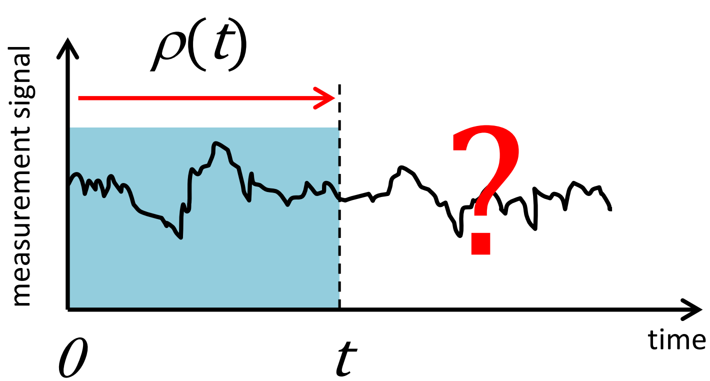
Our knowledge on the measured system is subject to:
- technical noise (*e.g.* non-ideal detectors)
- fundamental noise (*e.g.* measurement back-action)



Ways out:
- improve technology (*e.g.* better detectors)
- optimize measurement settings (*e.g.* chose appropriate measurement basis)

- more efficiently use all available data:
  use data measured after time t
  in order to get better estimate
  of the state at t.

$$P^f(n,t) = \mathrm{Tr}\left[\hat{\Omega}_n^\dagger \hat{\Omega}_n \rho(t)\right] \; ?$$



density matrix

Past quantum state

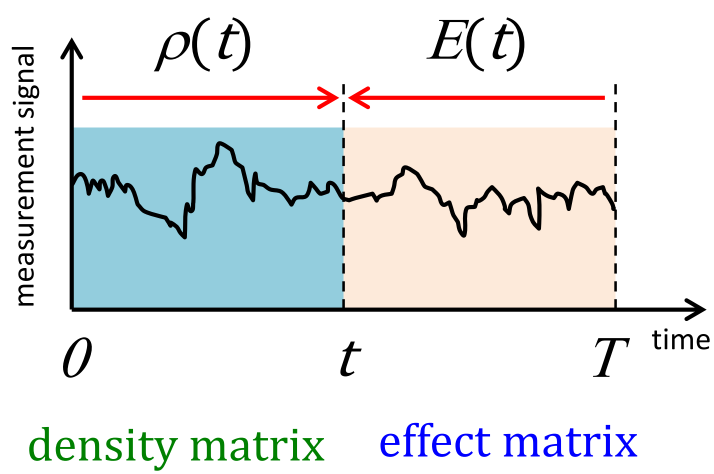
Complete the density matrix $\rho(t)$ with an effect matrix $E(t)$, calculated similarly to $\rho(t)$, but on data obtained after time t in a backward time direction.

From now on, the quantum state of the system is defined by a pair:

$$\Xi(t) = \{\rho(t), E(t)\}$$

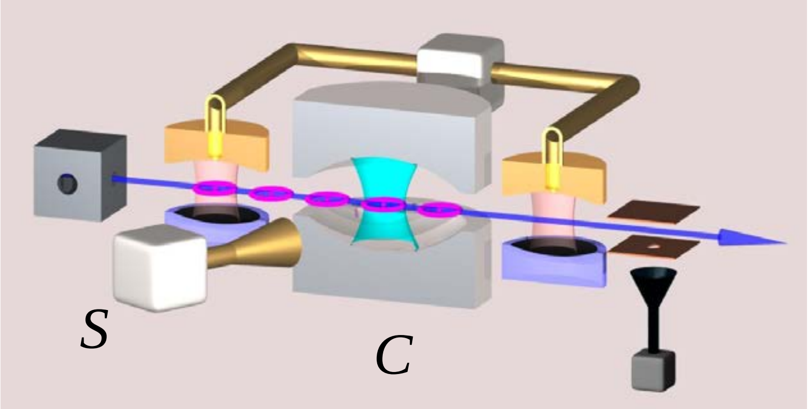
The improved estimate of the measurement outcome:

$$P^{fb}(n,t) = \frac{\mathrm{Tr}\left[\hat{\Omega}_n \rho(t) \hat{\Omega}_n^\dagger E(t)\right]}{\Sigma_m \mathrm{Tr}\left[\hat{\Omega}_m \rho(t) \hat{\Omega}_m^\dagger E(t)\right]}$$



S. Gammelmark *et al.*, PRL **111**, 160401 (2013)

Field evolution measurement

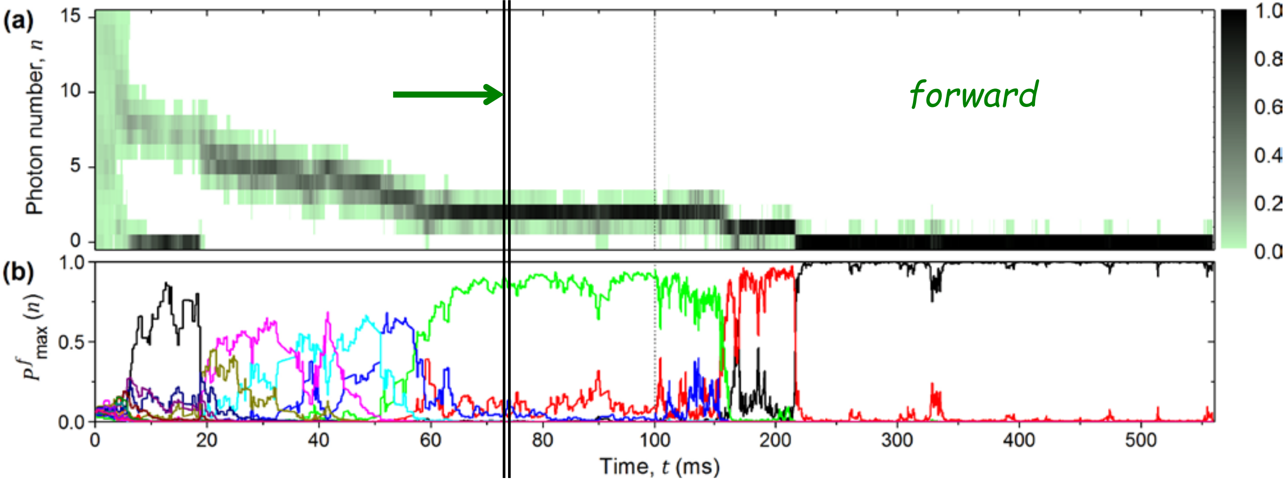


- Prepare a coherent field in the cavity:
  Poisson photon-number distribution
  on average 6 photons

- Send a long sequence of QND probes
  dephasing per photon $\pi/4$, *i.e.* distinguish from 0 to 7 photons

- Each detection projects the field into a new state:
  - take into account detection result
  - take into account decoherence from the last detection ($\sim$0.2 ms)

- Goal: estimate evolution of the photon-number distribution on a single quantum trajectory



- Broad initial distribution
- 8-photon measurement periodicity
- Some noise

- Properties similar to forward analysis

- Narrower distributions (smaller uncertainty on *n*)
- No periodicity problems
- Reduced noise

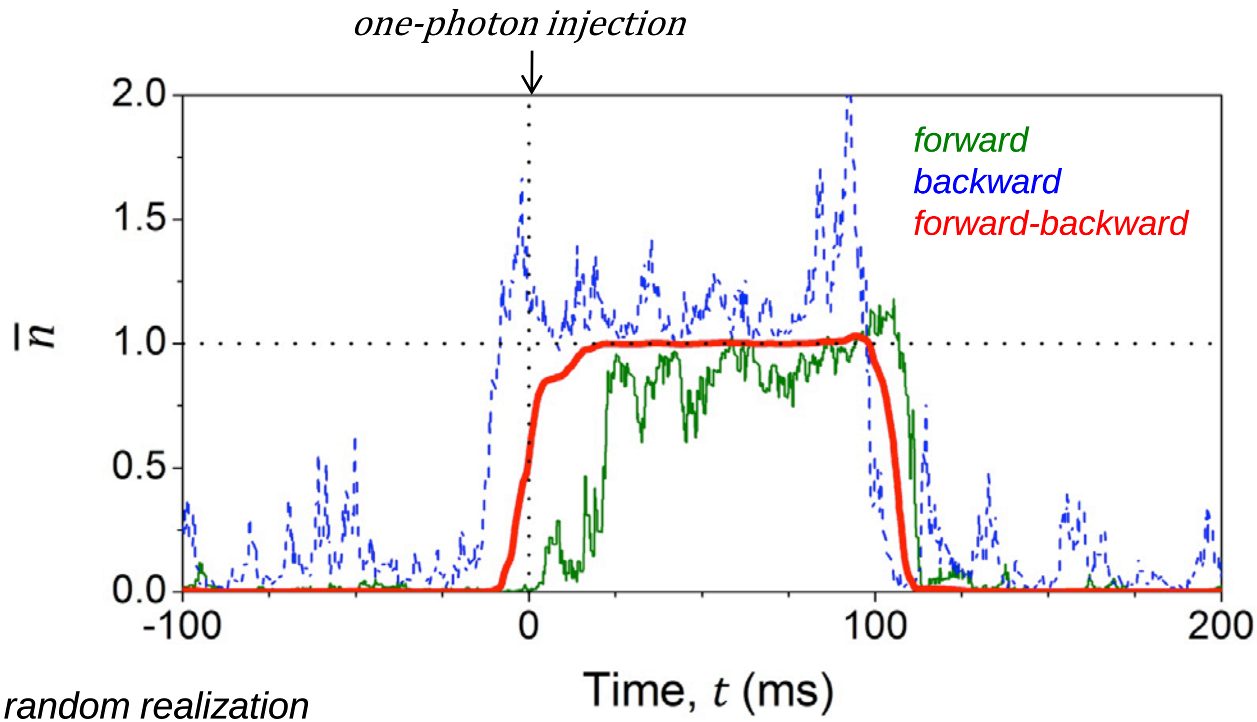
$$P^{fb}(n,t) \propto P^f(n,t)\, P^b(n,t)$$

Quantum jump detection

Simple measurement protocol:

- use empty cavity (*no photons*);
- inject a *single photon* with a resonant atom at time t = 0;
- try to detect this photon with dispersive QND probes sent before and after t.

- delayed detection, noise
- advanced detection, noise
- detection on time, (almost) no noise



one random realization

Quantum jump detection

Simple measurement protocol:

- use empty cavity (*no photons*);
- inject a *single photon* with a resonant atom at time $t = 0$;
- try to detect this photon with dispersive QND probes sent before and after t.

- delayed detection, noise
- advanced detection, noise
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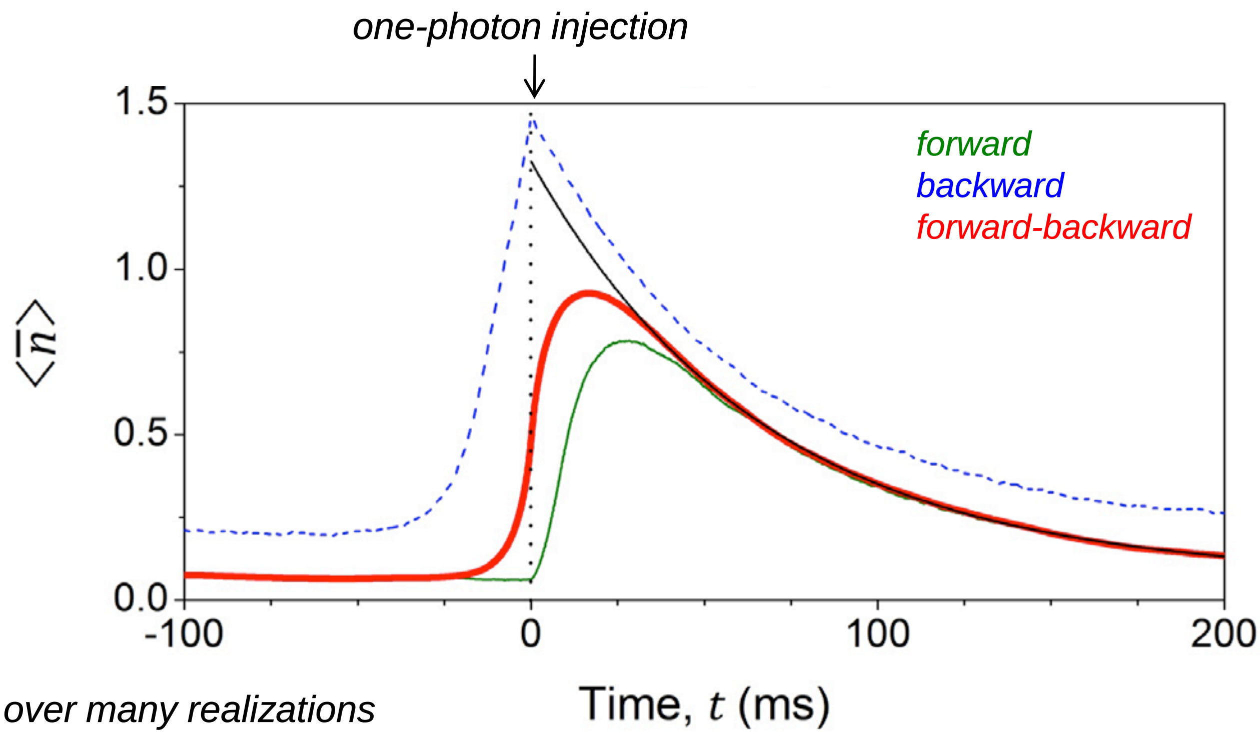
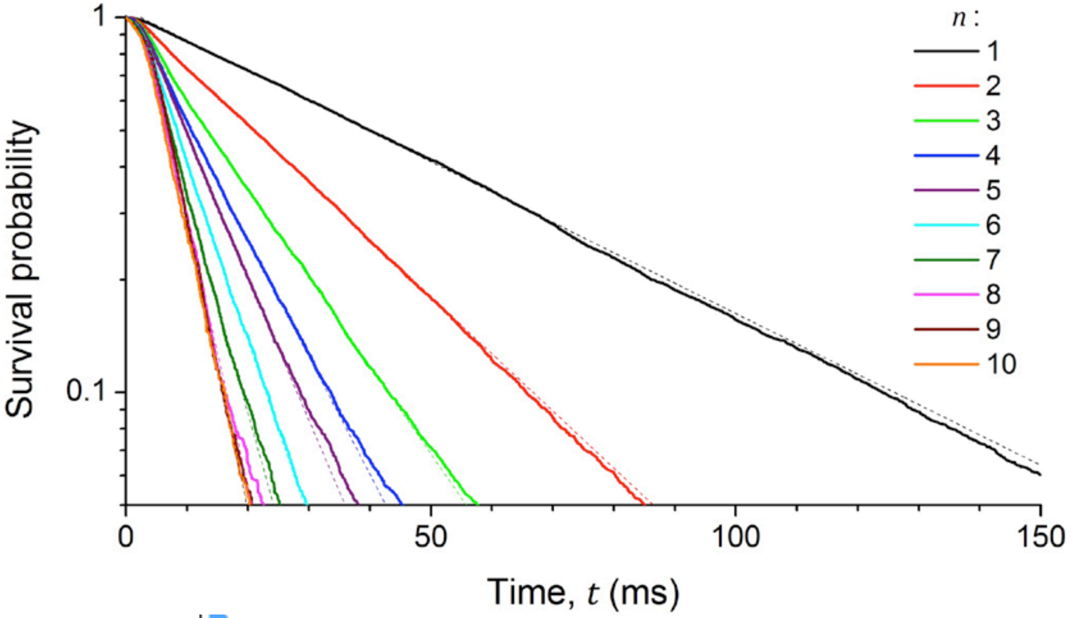


average over many realizations

Photon-number lifetimes

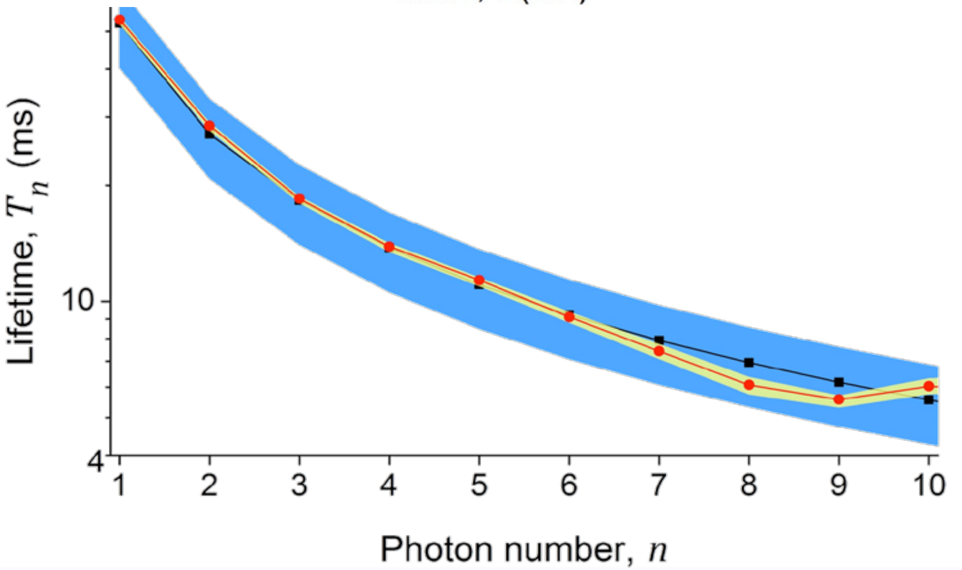
Do these "jumps" make sense?

Statistics of individual quantum jumps from initial coherent field of 6 photons



Lifetime of different *n*-states

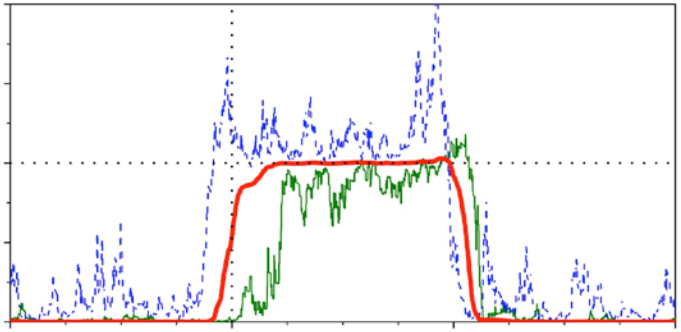
$$T_n = \frac{T_c}{n(1+n_b)+n_b(n+1)}$$



Summary on past quantum state

Approaches similar to the *past quantum state* method:

- *forward-backward algorithm*
- *quantum state smoothing*
- *…*



T. Rybarczyk *et al.,* PRA **91**, 062116 (2015)

At no additional "experimental" cost:

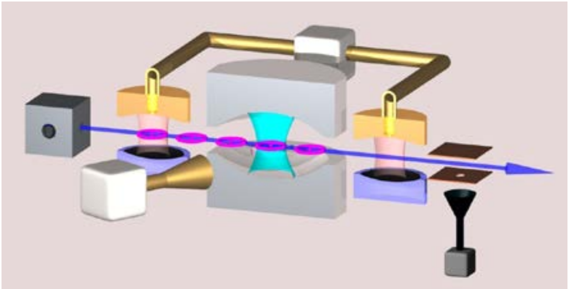
- we (almost) *get rid of noise* and obtain purer quantum properties
- we overcome some *measurement limitations* (like fundamental periodicity of interferometric measurement) and *access* previously hardly accessible states
- we *improve detection* of quantum state changes

Applications ?

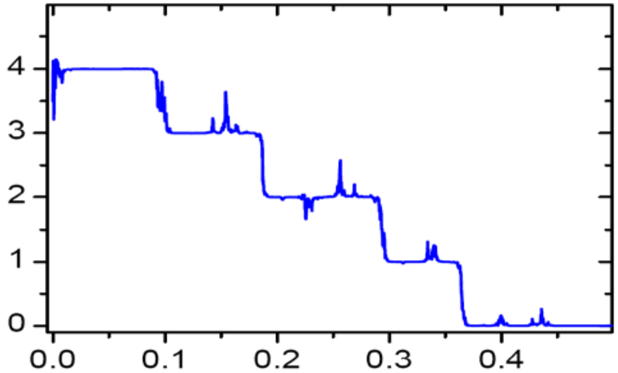
- to get more precise property evolution, *e.g.* for parameter estimation and metrology
- to learn about the property "in the past", *e.g.* for state reconstruction and post-selecting data
- …

Summary I: Photon-number measurement

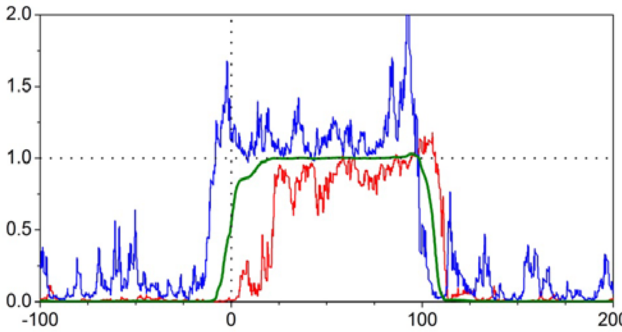
- **Photon box**
  high-quality cavity storing photons for long time and single circular Rydberg atoms with control on the atom-cavity interaction



- **QND photon-number measurement**
  dispersive atom-cavity interaction and photon-number dependent phase shift; individual weak measurements lead to the ideal projective one



- **Past quantum state**
  use results of measurements performed both before and after some moment in order to better estimate of a real quantum trajectory



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- Quantum non-demolition measurement
    - Basics of atom-cavity interaction : a spring and a spin
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    - Quantum feedback stabilizing photon number states
    - Adaptive QND measurement

Challenges in experimenting with quantum systems

Preparation of individual quantum objects and systems

Individual atoms, ions, photons, nuclear or electron spins, quantum dots, superconducting circuits, ...

Manipulation of individual quantum systems in a well-controlled way

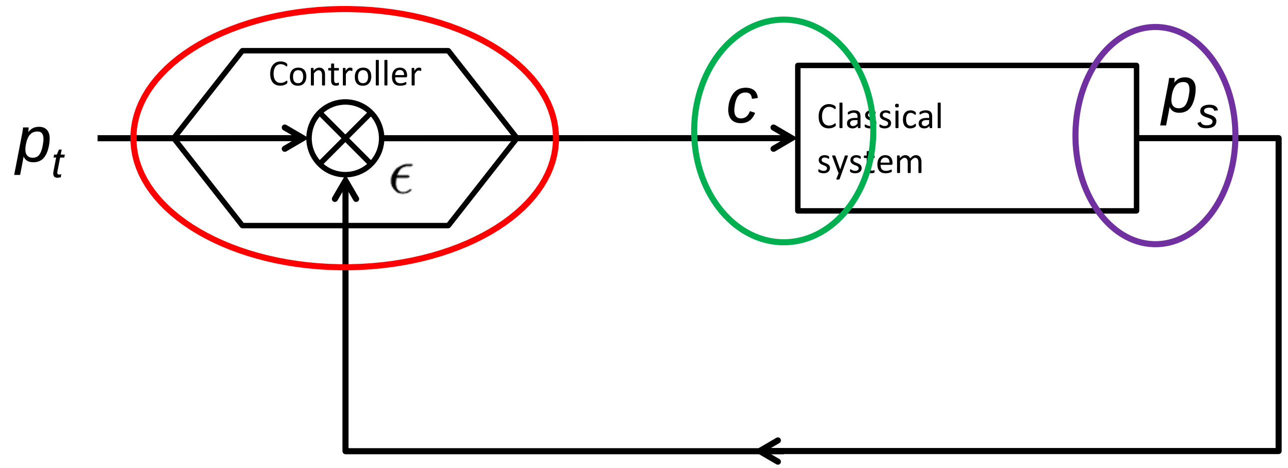
Sophisticated measurements at the quantum limit

Isolation from the environment

Vacuum, electric/magnetic/light traps for particles, cryogenic environment, low noise equipment, ...

If the isolation is not ideal, use active control on the system to maintain its state, *i.e.* implement quantum feedback loop

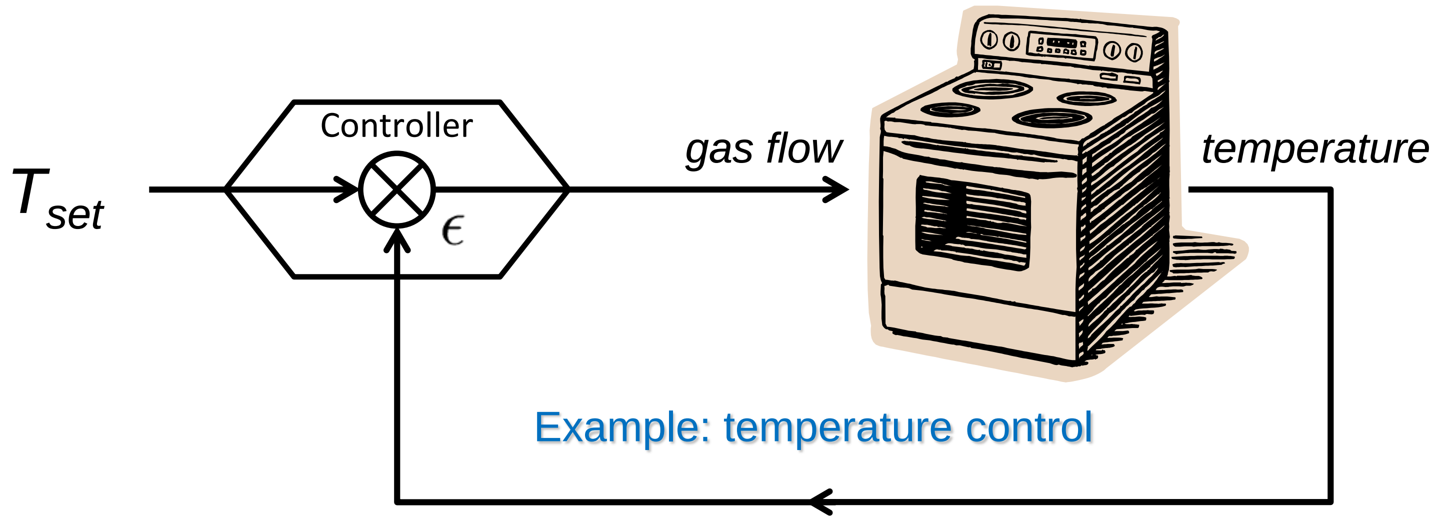
Use of **feedback loops**



- **Sensor:** measures the current state of the system
- **Controller:** compares to the set-point and chooses feedback control
- **Actuator:** acts on the system to bring it closer to the target

Control of a classical system

Use of **feedback loops**



- **Sensor:** measures the current state of the system
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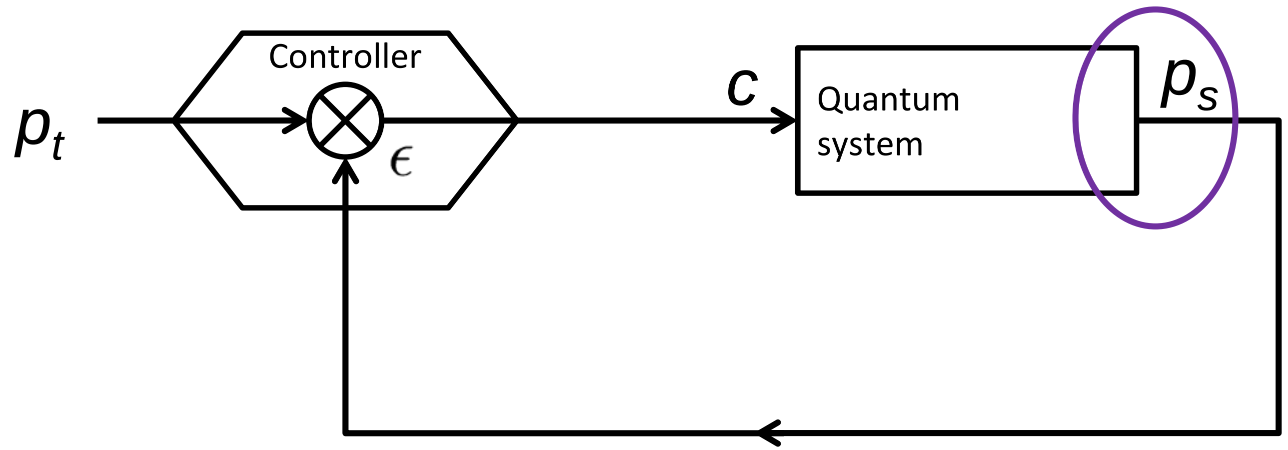
Use of **feedback loops** ?



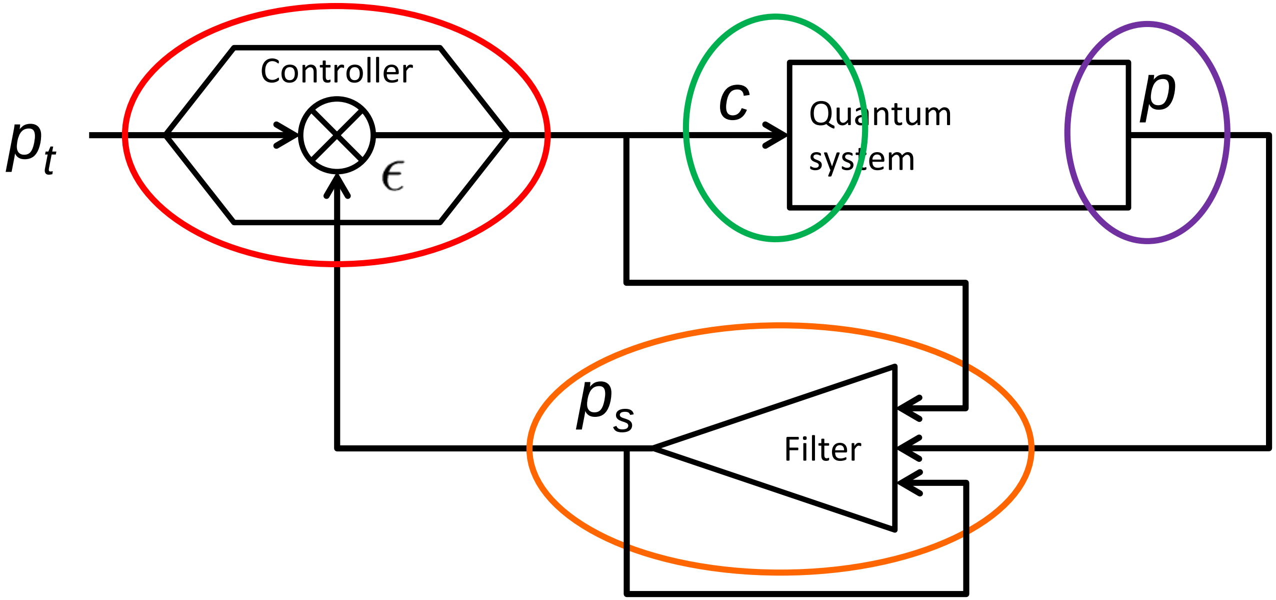
- **Sensor:** measures the current state of the system …

The measurement modifies

the state of the quantum system to be controlled !

Control of a quantum system

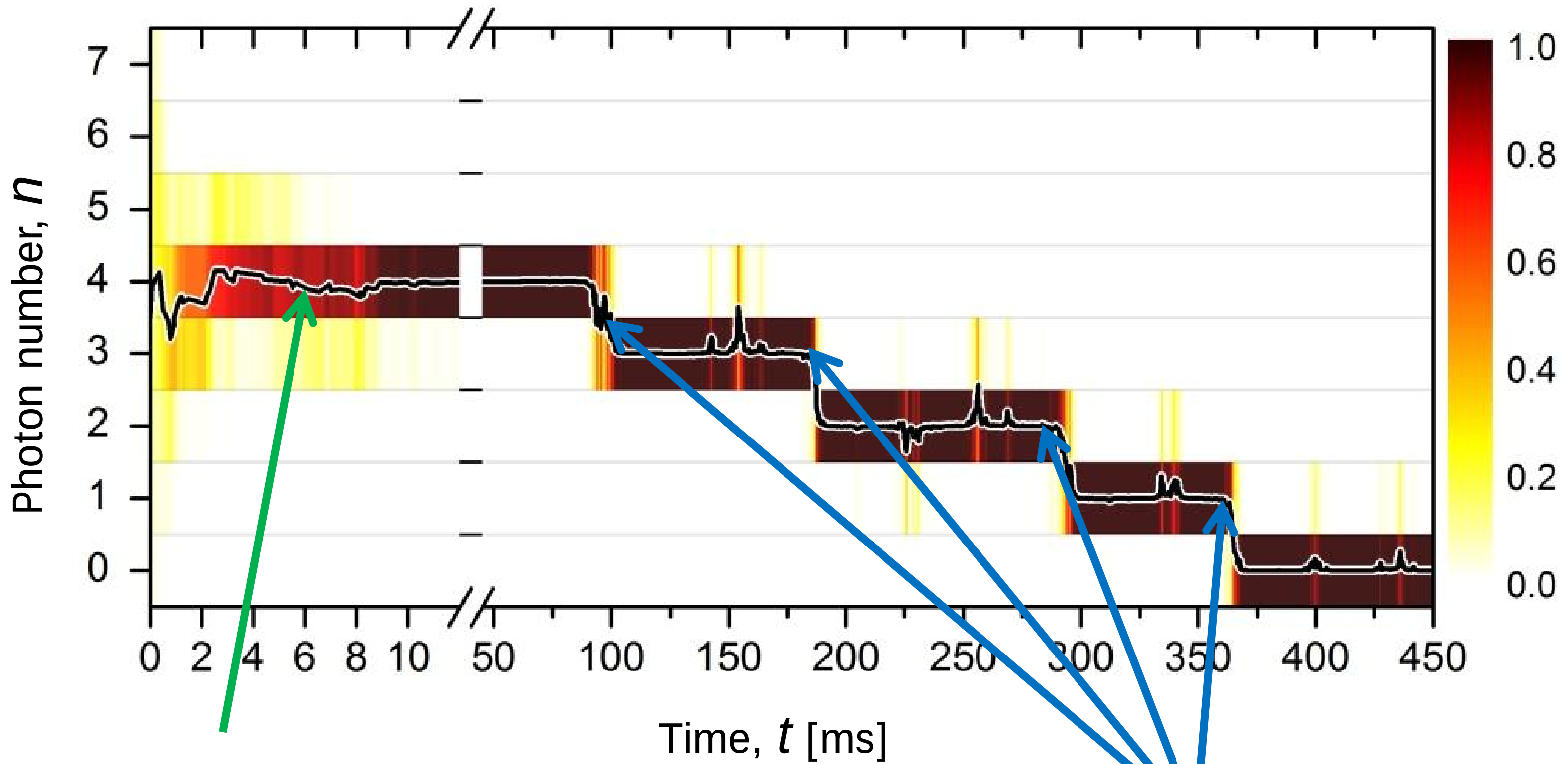
Use of **feedback loops**



- **Sensor:** measures the current state of the system
- **Filter:** estimates system's state using all available knowledge
- **Controller:** compares to the set-point and chooses feedback control
- **Actuator:** acts on the system to bring it closer to the target

Quantum jumps and Relaxation

Real-time repetitive non-destructive photon number measurement



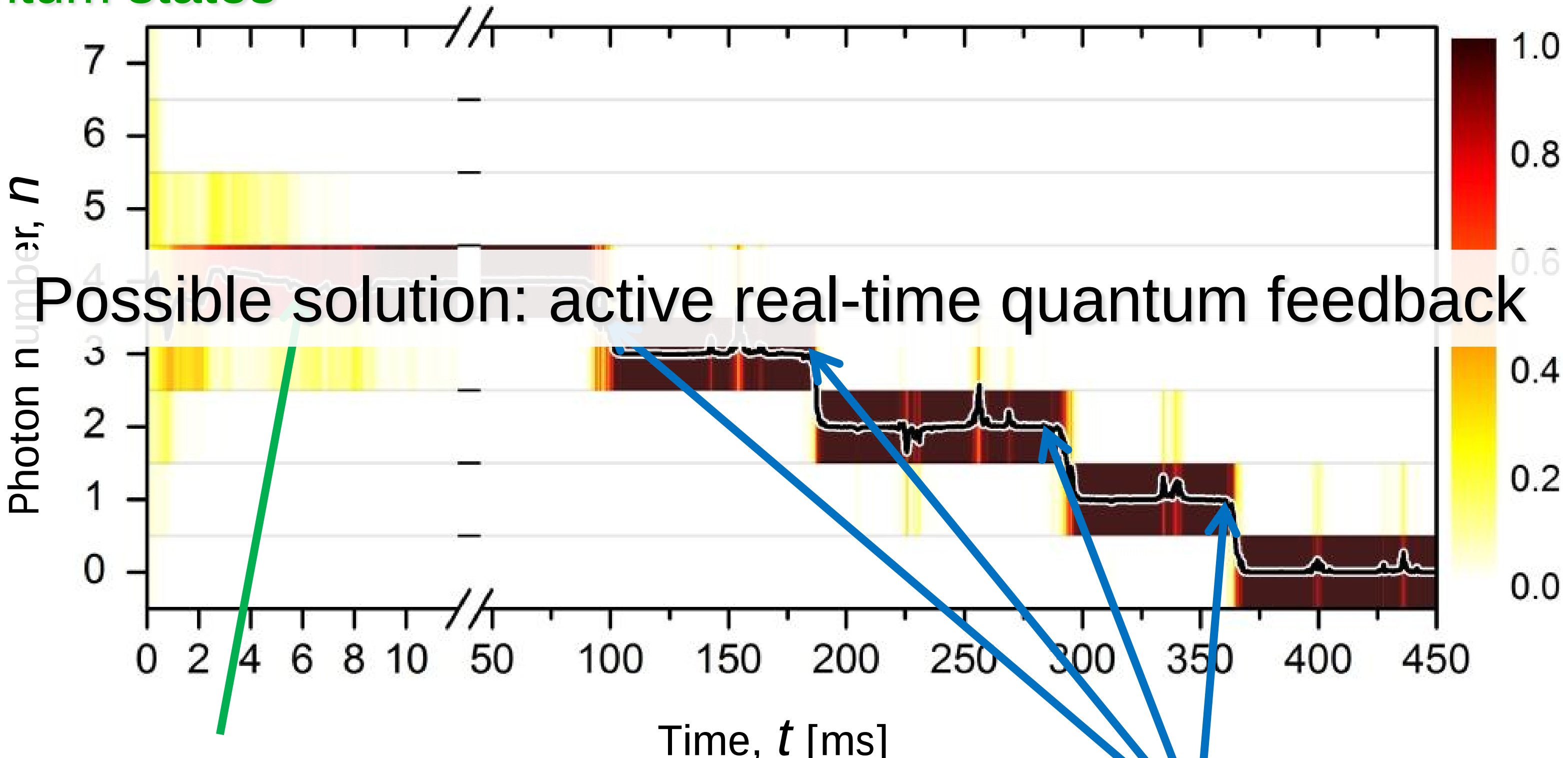
Projection onto a **random** photon-number state $|n\rangle$

Cascade of **quantum jumps** due to **decoherence**

Quantum jumps and Relaxation

Robust and deterministic preparation of quantum states

Protection against decoherence



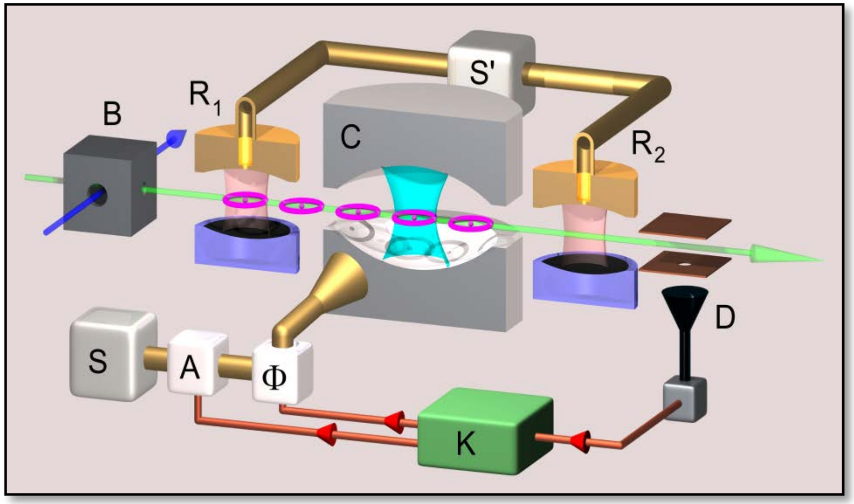
Possible solution: active real-time quantum feedback

Projection onto a **random** photon-number state $|n\rangle$

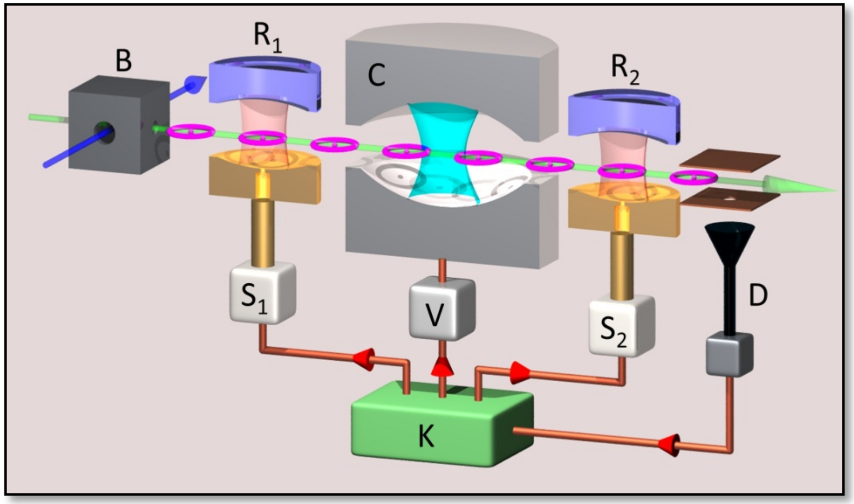
Cascade of **quantum jumps** due to **decoherence**

Quantum feedback loops

Classical control

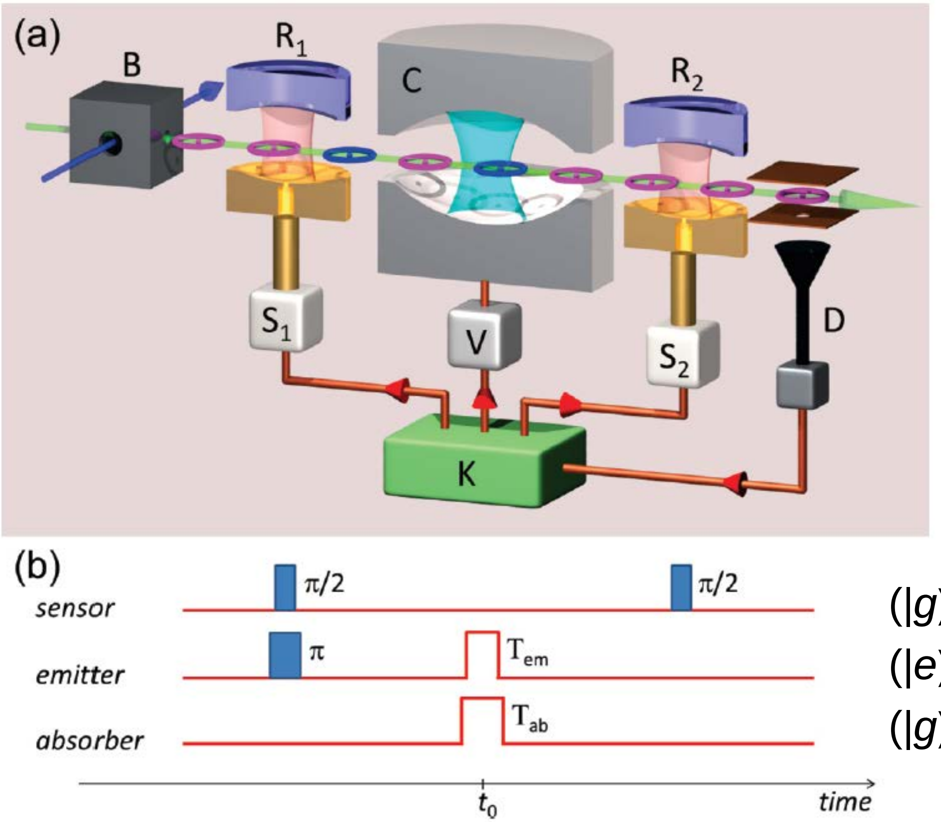


Quantum control



- **System**: light trapped in a cavity
- **Set point**: fixed number of photons
- **Sensor**: single atoms performing weak measurements of photon number
- **Filter/controller:** state estimation and optimal feedback action
- **Actuator classical:** injection of coherent fields with controlled amplitude
- **Actuator quantum:** injection or subtraction of a photon by resonant atoms

Quantum feedback with quantum actuators



Actuators: resonant atoms

Challenges:
- switch between different interactions in real time
- accurately calibrate resonant interaction
- properly take it into account all relevant experimental imperfections
- ...

Controller chooses between:

$(\|g\rangle+\|e\rangle$, dispersive)	$\Leftarrow$	no correction required
$(\|e\rangle$, resonant)	$\Leftarrow$	too few photons
$(\|g\rangle$, resonant)	$\Leftarrow$	too many photons

Atomic state is set by the microwave injection S_1 into R_1

Interaction type is set by the Stark shift controlled by potential V

Filter: estimation of a field's state

- In each feedback loop *k*, the filter performs real-time estimation ρ_k of the state of the field based on:
  - actual field's state ρ_{k-1} estimated at the end of the previous loop,
  - actual measurement result on sensor and actuator atoms,
  - last feedback action,
  - free state evolution (relaxation) during the loop duration.

- Initially, the field in the cavity is in the state of density matrix ρ_0 prepared by us (e.g. vacuum or coherent)

Filter I: measurement sensors

For every detected atom: outcome $\mu \in \{e,g\}$

$$\rho_{\rm proj} = \mathbb{M}_\mu^{sn} \rho = \frac{M_\mu \rho M_\mu^\dagger}{\mathrm{Tr}(M_\mu \rho M_\mu^\dagger)}$$

« Ideal » description:

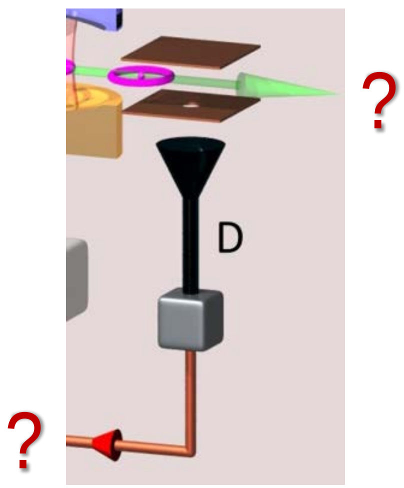
does not take into account the **imperfections** of the experimental setup !

Filter I: measurement sensors

Difficulty : imperfections of the experimental setup

- Detection efficiency ε:

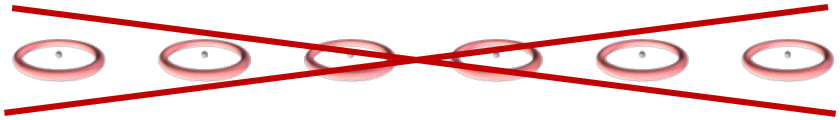
  atom can miss detection (65% chance)



- Detection errors η_g and η_e:

  state can be wrongly attributed (12% chance)

- Poissonian atom statistics:



  random number of atoms per sample (0, 1 or even 2), so

nobody knows how many atoms have passed through the cavity

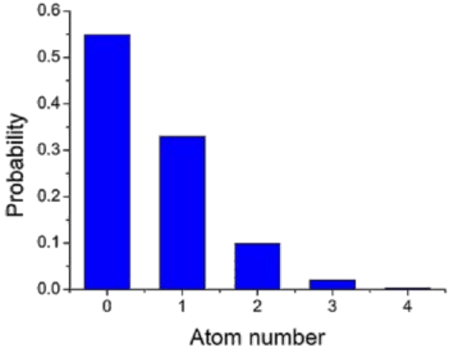
Filter I: measurement sensors

Poisson distribution P_a of atom number in atomic samples

To be considered:

- mean atom number $n_a = 0.6$
- include possible 0- and 2-atom events
- suppose negligible probability of more than 2 atoms
- consider a proper normalization for the Kraus operators

$$P_a(n) = e^{-n_a} \frac{n^{n_a}}{n!}$$



New set of Kraus operators:

$$\begin{aligned} L_0 &= \sqrt{P_a(0)} I \\ L_g &= \sqrt{P_a(1)} M_g \\ L_e &= \sqrt{P_a(1)} M_e \\ L_{gg} &= \sqrt{P_a(2)} M_g^2 \\ L_{ge} &= \sqrt{2\, P_a(2)} M_g M_e \\ L_{ee} &= \sqrt{P_a(2)} M_e^2 \end{aligned}$$

Filter I: measurement sensors

Detection efficiency ε and detection errors η_g and η_e

Statistical mixture of all possible states:

for every detection outcome $\mu \in \{\varnothing, g, e, gg, ge, ee\}$

$$\mathbb{M}^{sn}_{\mu'}\rho = \frac{\sum_\mu P(\mu'|\mu)\,\mathbb{L}^{sn}_\mu \rho}{\mathrm{Tr}\left(\sum_\mu P(\mu'|\mu)\,\mathbb{L}^{sn}_\mu \rho\right)}$$

our detection result (→ μ')

result of ideal detection (→ μ)

$$\mathbb{L}^{sn}_\mu \rho \equiv L_\mu \rho L^\dagger_\mu$$

(for the sake of simplicity)

probabilities are given by the stochastic matrix :

$J \setminus j$	$\varnothing$	g	e	gg	ee	ge
$\varnothing$	1	$1-\varepsilon$	$1-\varepsilon$	$(1-\varepsilon)^2$	$(1-\varepsilon)^2$	$(1-\varepsilon)^2$
g	0	$\varepsilon(1-\eta_g)$	$\varepsilon\eta_e$	$2\varepsilon(1-\varepsilon)(1-\eta_g)$	$2\varepsilon(1-\varepsilon)\eta_e$	$\varepsilon(1-\varepsilon)(1-\eta_g+\eta_e)$
e	0	$\varepsilon\eta_g$	$\varepsilon(1-\eta_e)$	$2\varepsilon(1-\varepsilon)\eta_g$	$2\varepsilon(1-\varepsilon)(1-\eta_e)$	$\varepsilon(1-\varepsilon)(1-\eta_e+\eta_g)$
gg	0	0	0	$\varepsilon^2(1-\eta_g)^2$	$\varepsilon^2\eta_e^2$	$\varepsilon^2\eta_e(1-\eta_g)$
ge	0	0	0	$2\varepsilon^2\eta_g(1-\eta_g)$	$2\varepsilon^2\eta_e(1-\eta_e)$	$\varepsilon^2((1-\eta_g)(1-\eta_e)+\eta_g\eta_e)$
ee	0	0	0	$\varepsilon^2\eta_g^2$	$\varepsilon^2(1-\eta_e)^2$	$\varepsilon^2\eta_g(1-\eta_e)$

Ideal Rabi oscillation

tow types of atoms: ν = {*absorber, emitter*}

state transformation
$$\mathbb{M}_\mu^\nu \rho \equiv \frac{R_\mu^\nu \rho R_\mu^{\nu\,\dagger}}{\mathrm{Tr}\left(R_\mu^\nu \rho R_\mu^{\nu\,\dagger}\right)}$$

detection results

$$R_e^{em} = \sum_n \cos\frac{\Omega_n t}{2} \left|n\right\rangle \left\langle n\right| ,$$ *emitter failed*

$$R_g^{em} = \sum_n \sin\frac{\Omega_n t}{2} \left|n+1\right\rangle \left\langle n\right| ,$$ *emitter succeeded*

$$R_e^{ab} = \sum_n \sin\frac{\Omega_n t}{2} \left|n\right\rangle \left\langle n+1\right| ,$$ *absorber succeeded*

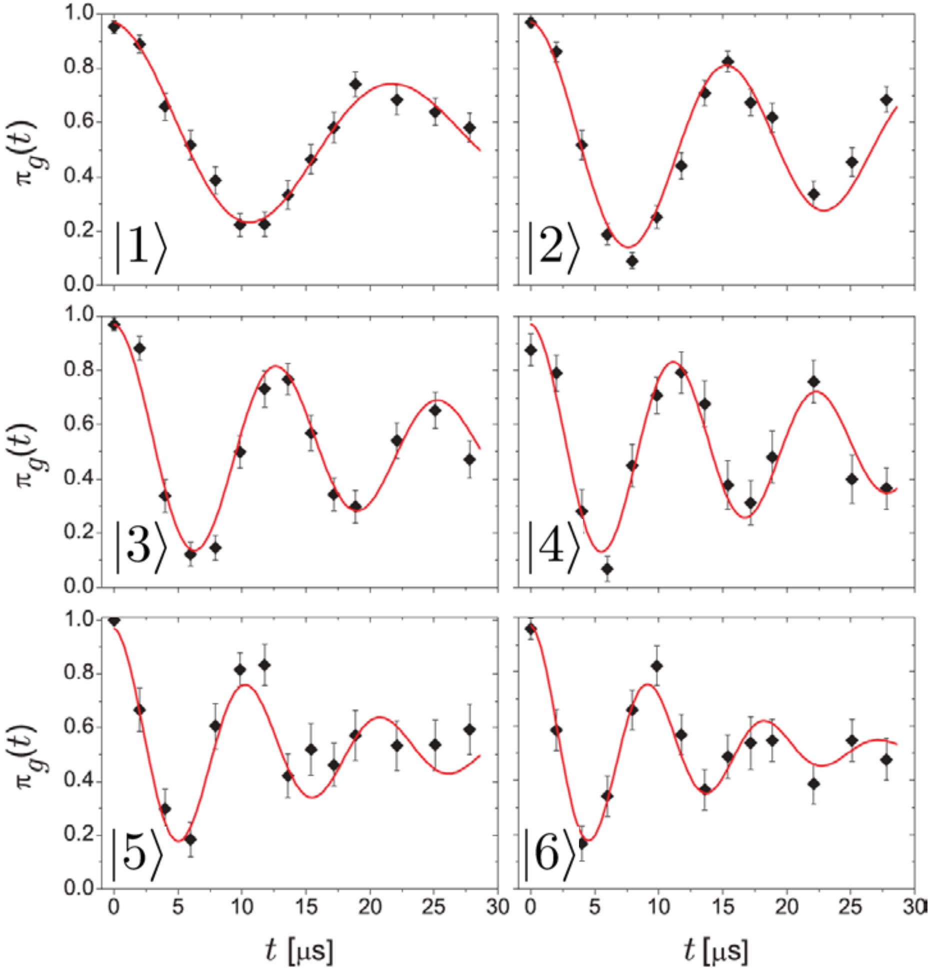
$$R_g^{ab} = \sum_n \cos\frac{\Omega_n t}{2} \left|n+1\right\rangle \left\langle n+1\right| + \left|0\right\rangle \left\langle 0\right|$$ *absorber failed*

Imperfections

Dispersion in Rabi oscillations

- get Rabi frequency $\boldsymbol{\Omega}_n$ and damping $\boldsymbol{\tau}_n$ from calibration in different photon-number states
- here, consider only populations, but not coherence of the field
- modify Kraus operators:

$$\left[\mathbb{R}_e^{em}\rho\right]_{nn} = \tfrac{1}{2}[1 + e^{-t/\tau_n}\cos(\Omega_n t)]\rho_{nn},$$
$$\left[\mathbb{R}_g^{em}\rho\right]_{nn} = \tfrac{1}{2}[1 - e^{-t/\tau_{n-1}}\cos(\Omega_{n-1} t)]\rho_{n-1,n-1},$$
$$\left[\mathbb{R}_e^{ab}\rho\right]_{nn} = \tfrac{1}{2}[1 - e^{-t/\tau_n}\cos(\Omega_n t)]\rho_{n+1,n+1},$$
$$\left[\mathbb{R}_g^{ab}\rho\right]_{nn} = \tfrac{1}{2}[1 + e^{-t/\tau_{n-1}}\cos(\Omega_{n-1} t)]\rho_{nn},$$



Detection imperfections

- taken into account similar to the sensor atoms

Coupling to environment results in a sudden loss/capture of photons

Characteristic time: lifetime of a photon $T_{\mathrm{c}} = 65\,\mathrm{ms}$

Loop duration: interval between atoms $T_{\mathrm{a}} = 82\,\mu\mathrm{s}$ $(\xi = T_{\mathrm{a}}/T_{\mathrm{c}} \ll 1)$

Thermal field: $n_{\mathrm{th}} = 0.05$

Result of the quantum master equation:

$$\mathbb{T}\rho = J_0 \rho J_0{}^\dagger + J_\downarrow \rho J_\downarrow{}^\dagger + J_\uparrow \rho J_\uparrow{}^\dagger$$

no change: $J_0 = (1 - \xi n_{\mathrm{th}}/2)\, I - \xi(1/2 + n_{\mathrm{th}})\, a^\dagger a,$

loss: $J_\downarrow = \sqrt{\xi(1 + n_{\mathrm{th}})}\, a,$

jump operators

capture: $J_\uparrow = \sqrt{\xi n_{\mathrm{th}}}\, a^\dagger,$

Filter III: field relaxation

Coupling to environment results in a sudden loss/capture of photons

Characteristic time: lifetime of a photon $T_\mathrm{c} = 65\,\mathrm{ms}$

Loop duration: interval between atoms $T_\mathrm{a} = 82\,\mu\mathrm{s}$ $(\xi = T_\mathrm{a}/T_\mathrm{c} \ll 1)$

Thermal field: $n_\mathrm{th} = 0.05$

If field coherence is not important, considered only *P*(*n*,*t*) evolution

$$\frac{dP^f(n,t)}{dt} = \sum_m K_{n,m} P^f(m,t)$$

no change: $K_{n,n} = -\kappa[(1+n_b)n + n_b(n+1)]$

loss: $K_{n,n+1} = \kappa(1+n_b)(n+1)$ *relaxation matrix*

capture: $K_{n,n-1} = \kappa n_b n$

Filter IV: putting all together

Quantum state estimated in a feedback loop *k*

not-yet-detected atoms
(s=3, flying between cavity and detector)

atom type

initial state

$$\rho_k = \prod_{i=k+1}^{k+s} (\mathbb{T}\,\mathbb{N}^{\nu_i}) \prod_{i=1}^{k} \left(\mathbb{T}\,\mathbb{M}_{\mu_i'}^{\nu_i}\right) \rho_0$$

detection result

Challenge : perform these calculation fast

Use different approximations, operator properties, precalculated stuff, mathematical tricks, etc

Controller: feedback action

Controller's task is to find the actuator action minimizing the distance between the actual *p*(*n*) and the target state n_t

Distance to target:

mean *variance*

$$d(n_{\mathrm{t}}, p(n)) = \sum_i (i - n_{\mathrm{t}})^2 p(i) = (\bar{n} - n_{\mathrm{t}})^2 + \Delta n^2$$

Distance matrix

$$\mathfrak{D}'_{n_{\mathrm{t}}} = \sum_i (i - n_{\mathrm{t}})^2 \left| i \right\rangle \left\langle i \right| \qquad d(n_{\mathrm{t}}, p(n)) = \mathrm{Tr}\left[\mathfrak{D}'_{n_{\mathrm{t}}} \rho\right]$$

Distance to be minimized:

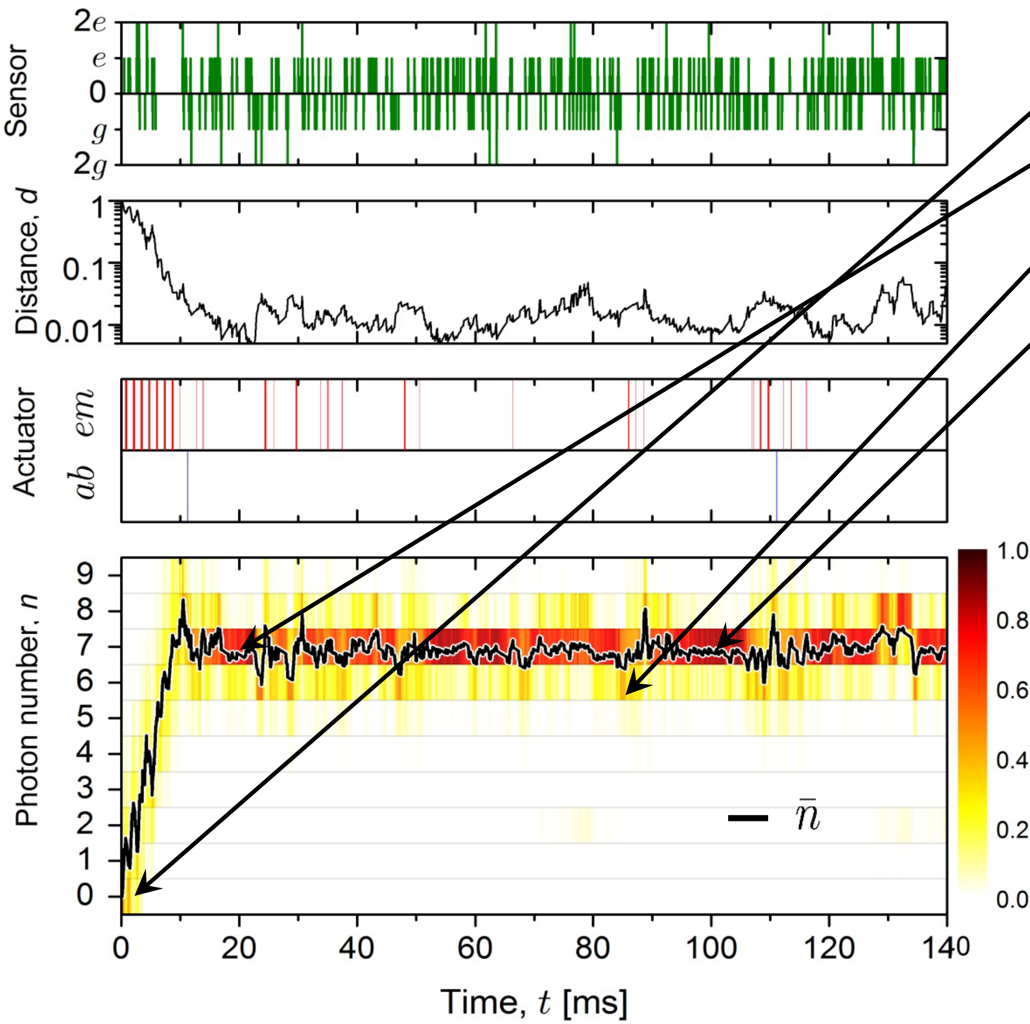
$$d^{\{\nu_{k+8}, \nu_{k+7}, \nu_{k+6}\}} = \mathrm{Tr}\left[\mathfrak{D}'_{n_{\mathrm{t}}} \prod_{i=k+6}^{k+8} \left(\mathbb{T}\, \mathbb{N}^{\nu_i}\right) \rho_k\right]$$

decides for the next 3 atoms in fly minimizing distance

$$\approx \mathrm{Tr}\left[\mathfrak{D}'_{n_{\mathrm{t}}} (\mathbb{T})^3 \prod_{i=k+6}^{k+8} \mathbb{N}^{\nu_i} \rho_k\right]$$

Feedback loop in action

Feedback stabilization of Fock state |7⟩



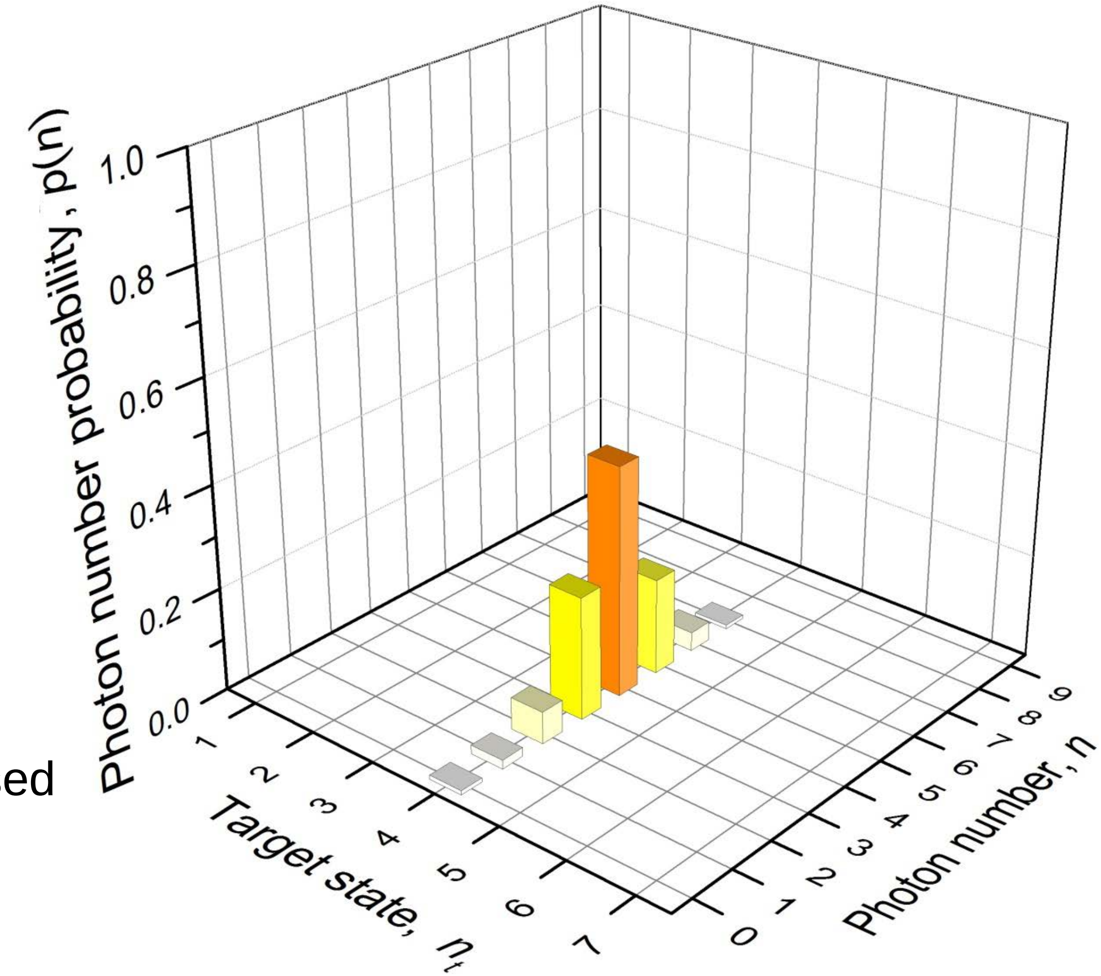
- Vacuum state |0⟩
- Initial preparation of |7⟩
- Detection of a quantum jump to |6⟩
- Correction and restoration of |7⟩
- and so on …

Deterministic preparation of fragile photon-number states (demonstrated up to n = 7) and their stabilization against decoherence

Note: Lifetime of state |7⟩ is only 9 ms

Stationary regime:
ensemble average of many trajectories has a steady *P*(*n*) distribution

- Each trajectory is terminated at about $2T_c$
- An independent QND measurement is performed.
- Field state is reconstructed based on the measurement results of 4000 individual trajectories.



Stationary regime:
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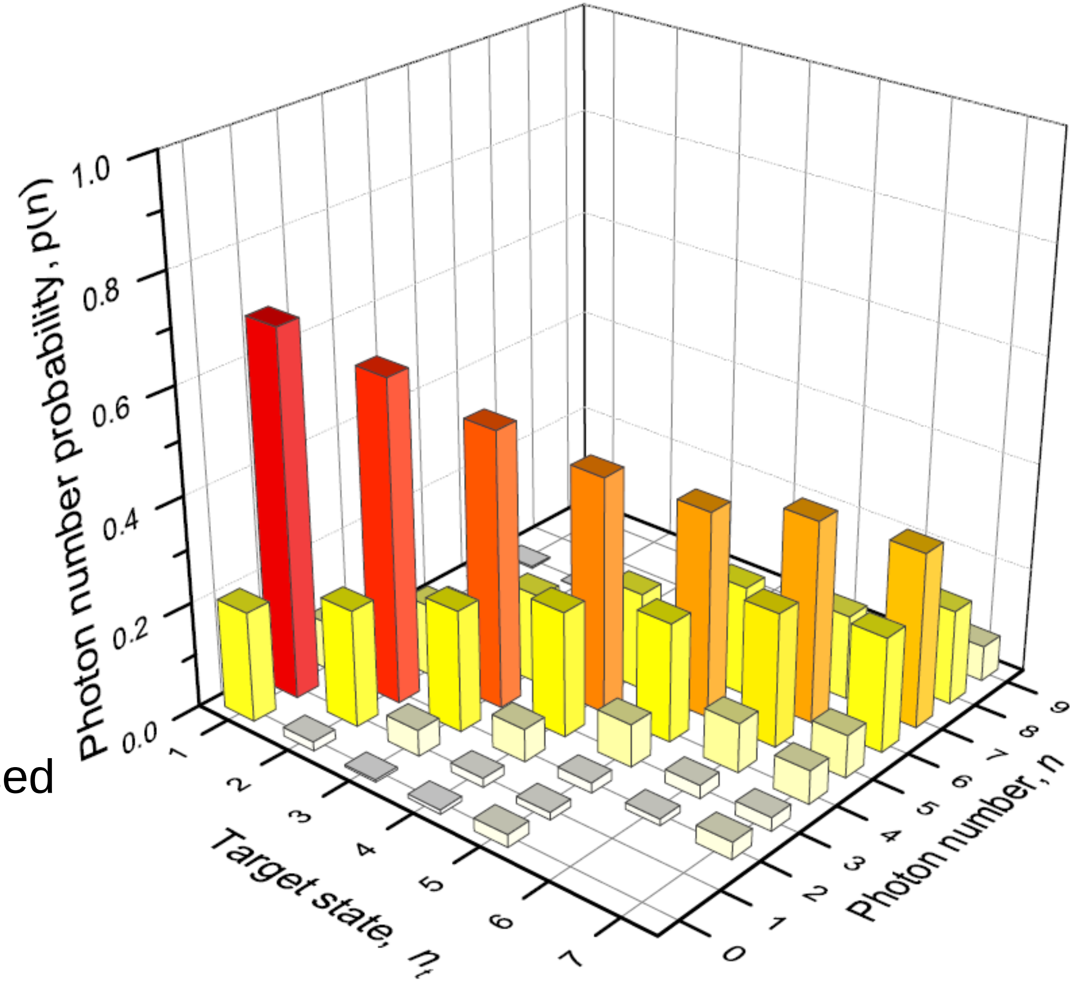
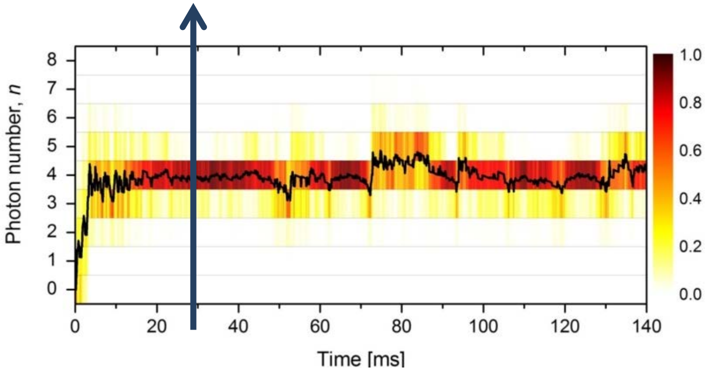
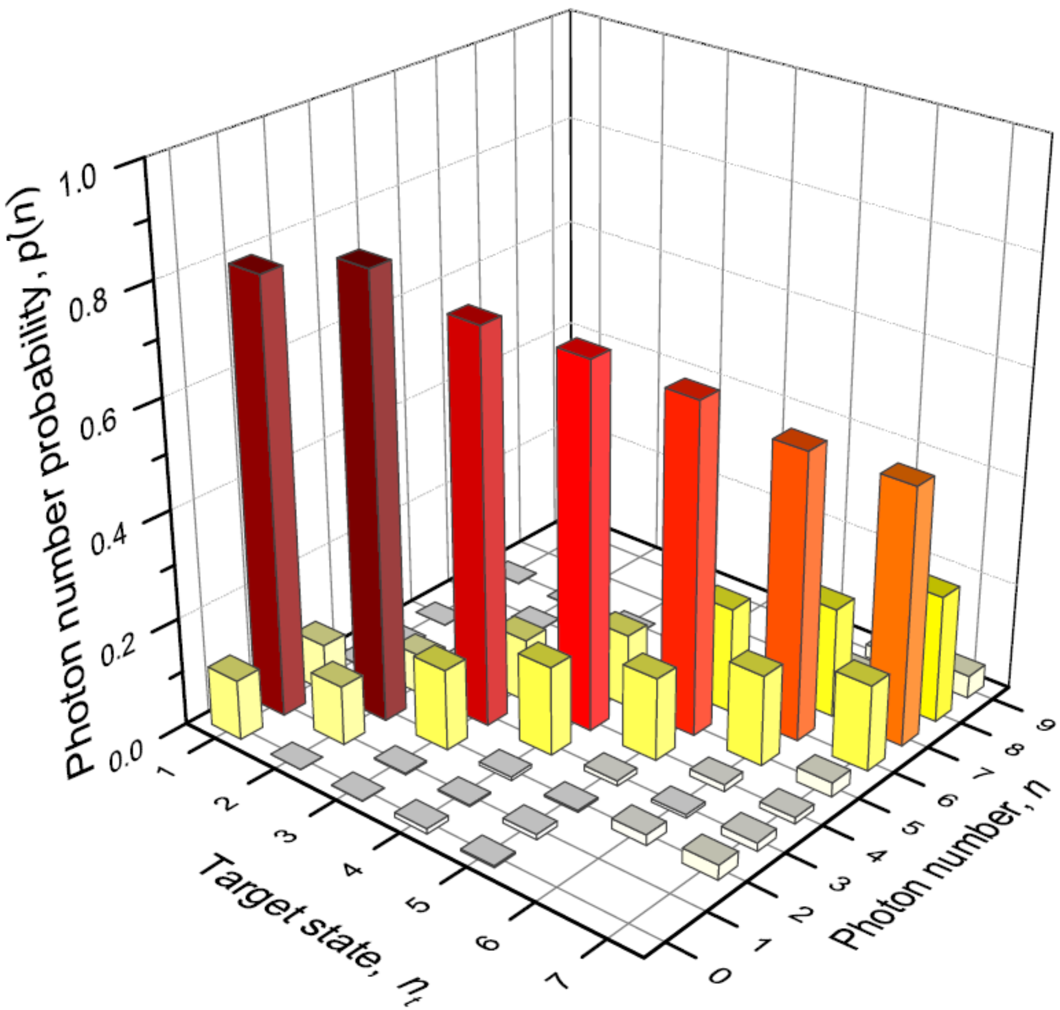




- Use knowledge of the controller:

  interrupt feedback loop
  as soon as *e.g.* $P(n_t) > 0.8$

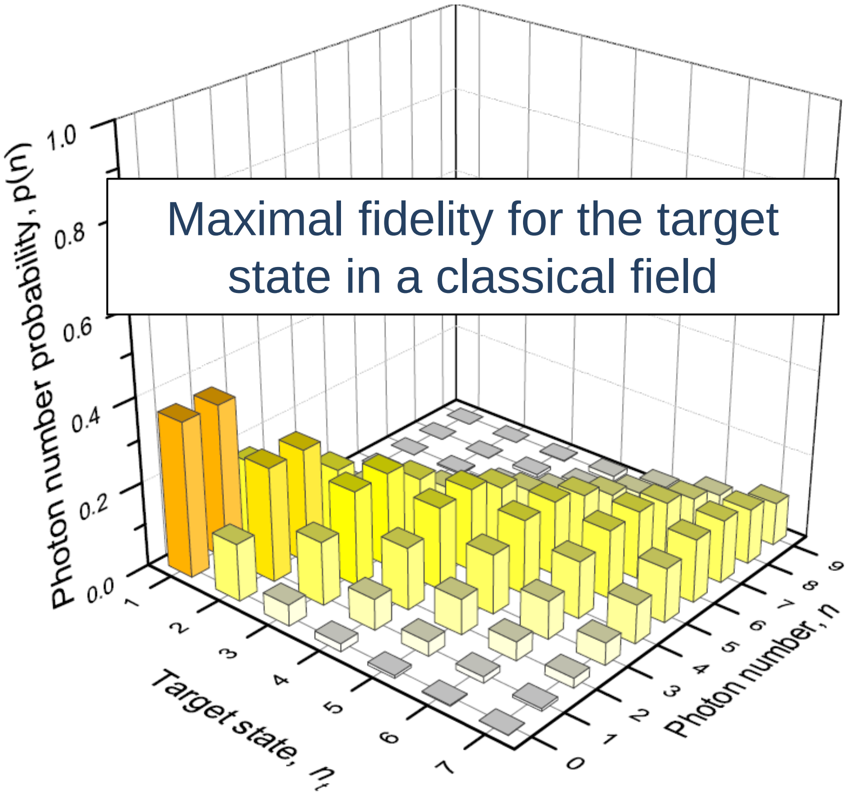
- State purity is improved



Photon-number distribution

Coherent field $|\alpha\rangle$

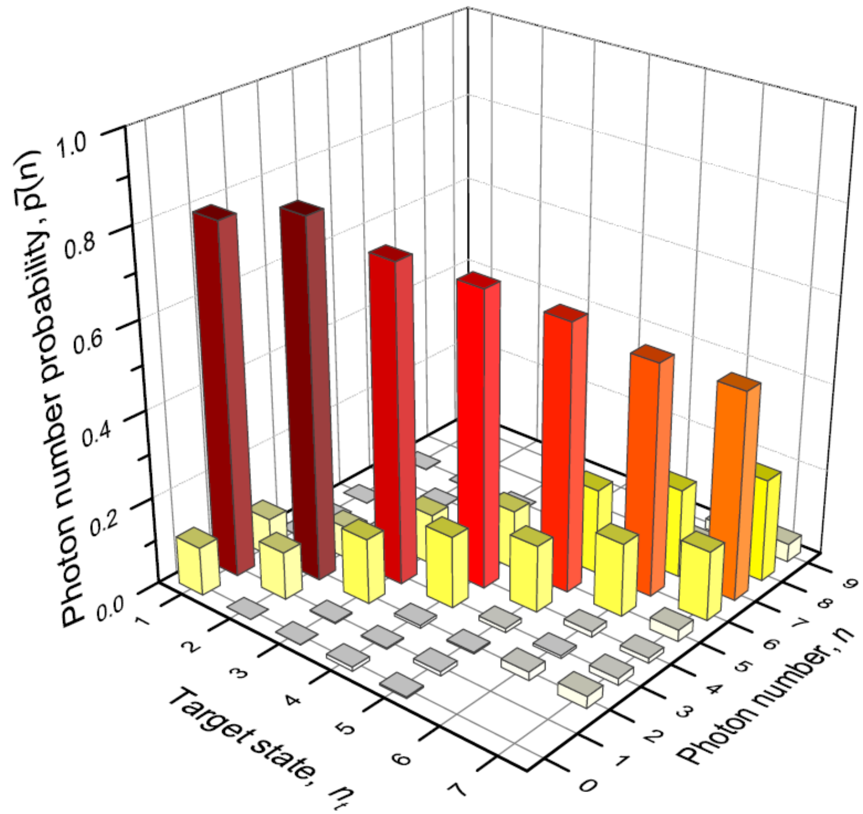
$$|\alpha|^2 = n_t$$



Maximal fidelity for the target state in a classical field

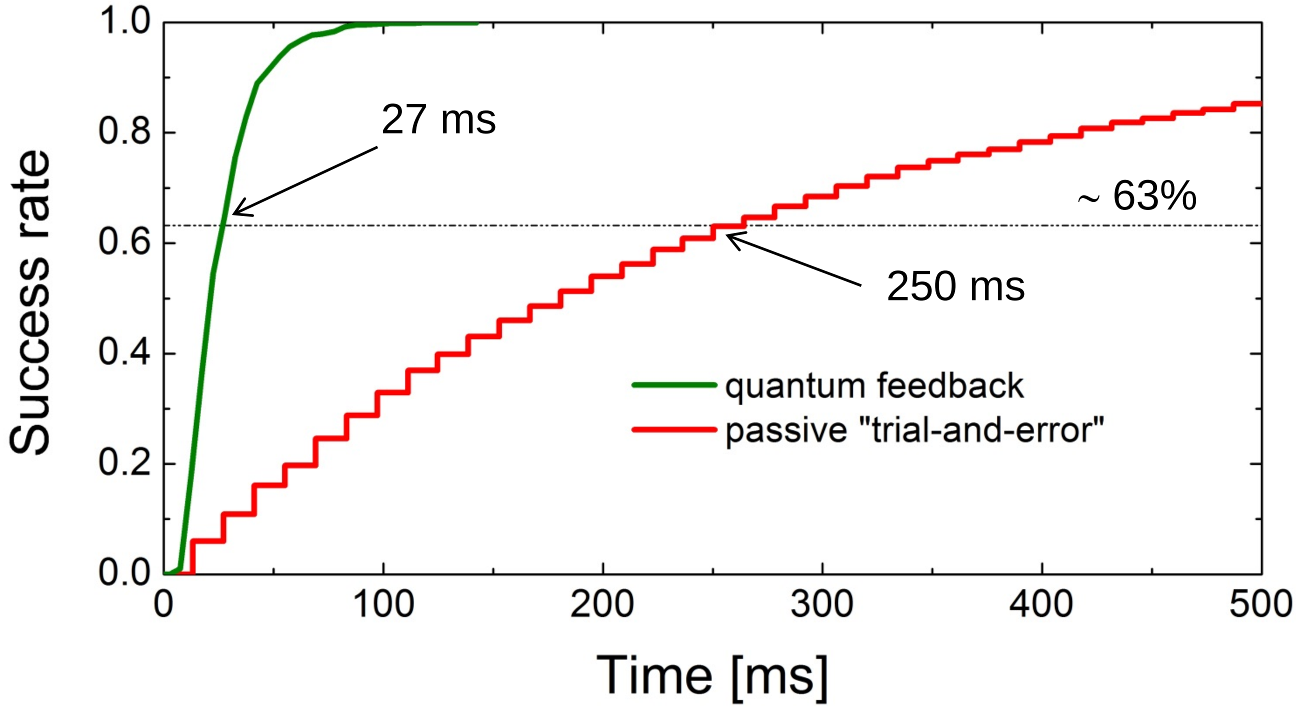
(Poisson distribution)

Feedback preparation



(squeezed distribution)

Preparation speed-up



$n_t = 3$

Active quantum feedback:

- prepare *vacuum* field
- activate feedback
- wait until $P(n_t)>0.8$

Passive "trial-and-error":

- prepare *coherent* field
- measure photon number
- if $P(n_t)<0.8$, start from the beginning

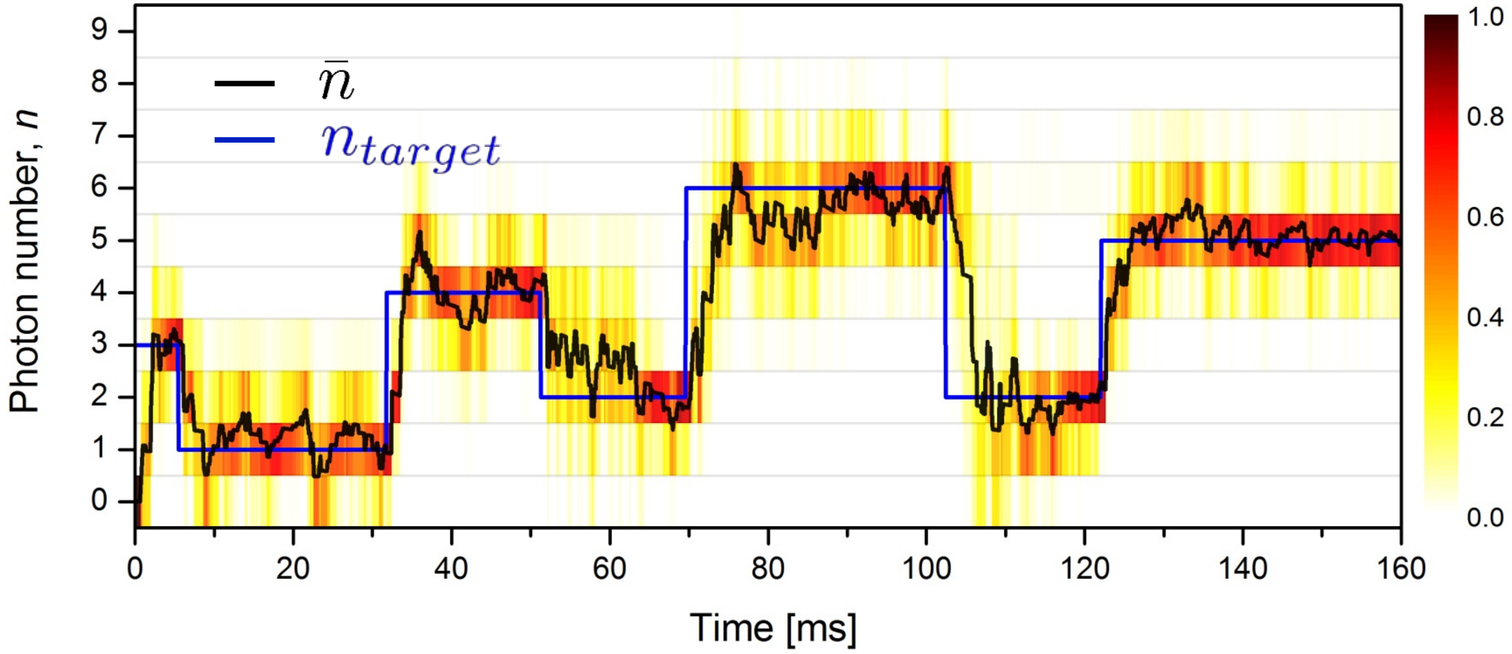
Active preparation is 9 times faster than passive for $n_t = 3$.

Moreover, this difference rapidly increases with n_t.

Sequential preparation of quantum states

- An pre-set sequence of targets is chosen
- As soon as fidelity of the current target reaches 80%, the target is changed.
- Illustration of adaptive measurement

Arbitrary sequence {3,1,4,2,6,2,5}



Summary on Quantum feedback

Proposal and simulations:

I. Dotsenko, M. Mirrahimi, M. Brune, J.-M. Raimond, S. Haroche, & P. Rouchon,
« *Quantum feedback by discrete QND measurements:*
towards on-demand generation of photon-number states »,
Phys. Rev. A 80, 013805 (2009)

coherent injection

Stability analysis:

M. Mirrahimi, I. Dotsenko & P. Rouchon, **IEEE Conference on Decision and Control ,1451-1456 (2010)**
A. Somaraju *et al.*, **Proceedings of the American Control Conference, 5084-5089 (2012)**
...

coherent injection

Experiment:

C. Sayrin *et al.*,
« *Real-time quantum feedback prepares and stabilizes photon number states* »,
Nature 477, 73-77 (2011)

coherent injection

X. Zhou *et al.*,
« *Field Locked to a Fock State by Quantum Feedback with Single Photon Corrections* »,
Phys. Rev. Lett. 108, 243602 (2012)

resonant atoms

B. Peaudecerf *et al.*,
« *Quantum feedback experiments stabilizing Fock states of light in a cavity* »,
Phys. Rev. A 87, 042320 (2013)

review of both methods

Outline

- Quantum non-demolition measurement
  - Basics of atom-cavity interaction : a spring and a spin
  - Experimental setup: Photon box
  - QND photon number measurement
  - Past quantum states
- Quantum control
  - Quantum feedback stabilizing photon number states
  - Adaptive QND measurement

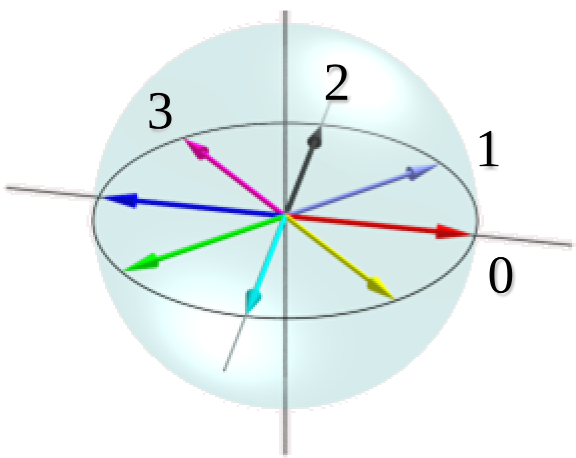
Adaptive QND measurement

So far:

for complete QND measurement we use a sequence of sensors with 4 alternative Ramsey phases (*i.e.* detection direction set by R_2) in order to be equally sensitive to all photon number from 0 to 7.

Idea:

for each sensor choose a detection direction which has a maximum chance to give the most new information on the field in order not to waste time on less useful measurements



$$\varphi_r \in \{\varphi_{r0}, \varphi_{r1}, \varphi_{r2}, \varphi_{r3}\}$$

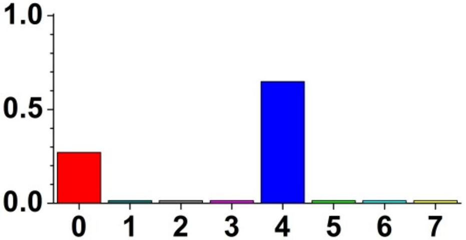
We choose the measurement which can still "surprise" us

B. Peaudecerf *et al.,* PRL **112**, 080401 (2014)

Optimal phase choice

Example:

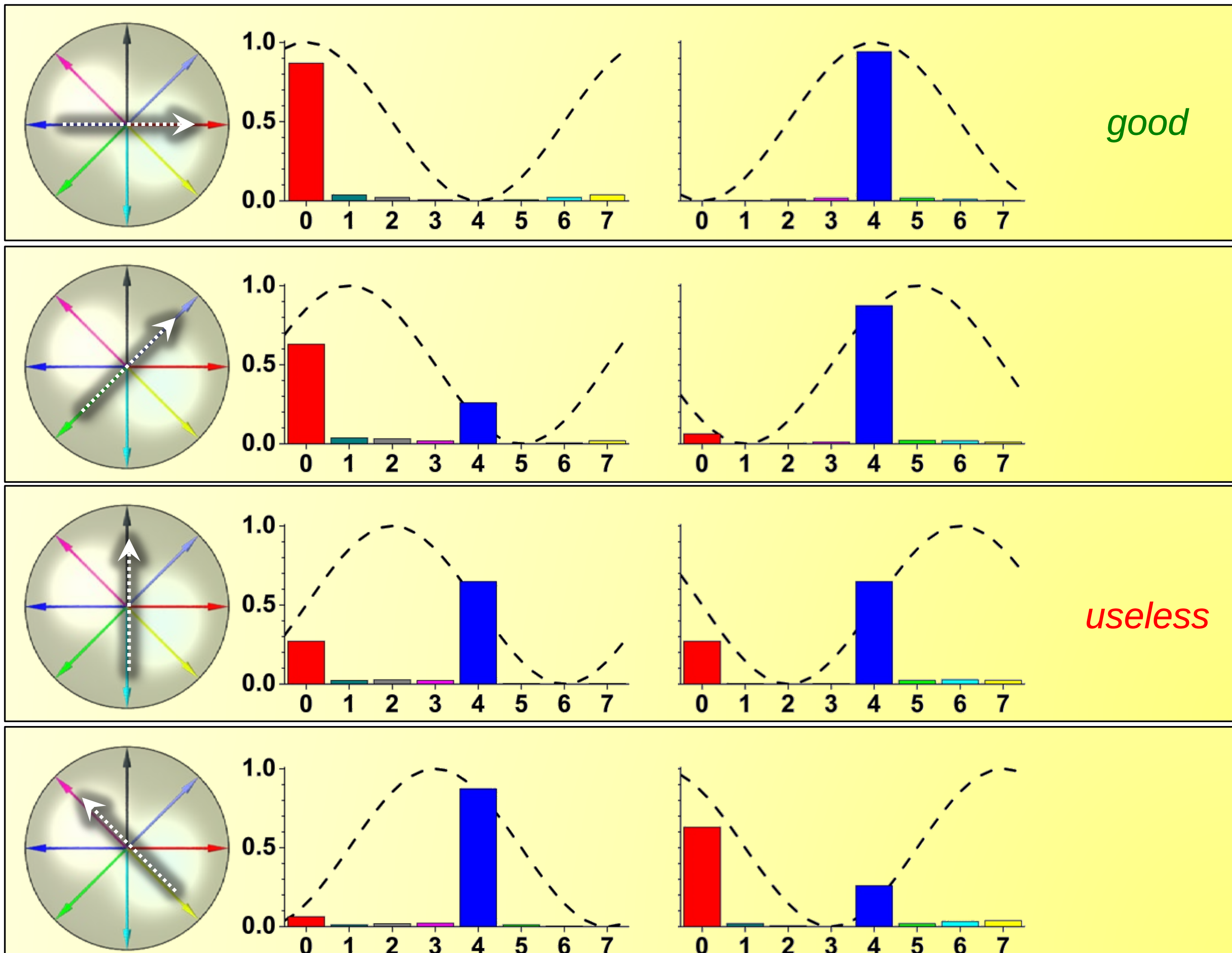
distribution $P_k(n)$ after k detections:



Four available measurements

Expected photon number distribution $P_{k+1}(n)$

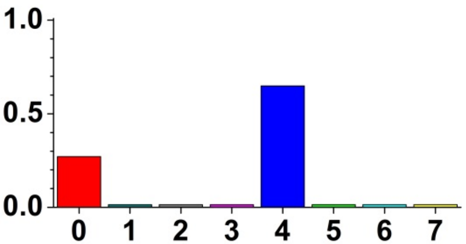
$|e\rangle$ $|g\rangle$



Entropy as information measure

Example:

distribution $P_k(n)$
after *k* detections:

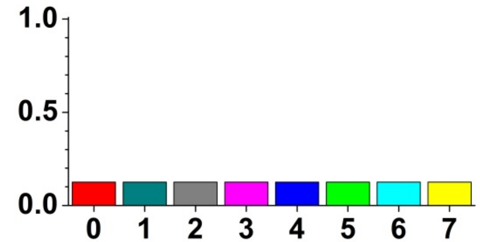


Shannon entropy as a measure of knowledge $P(n)$ on photon number:

$$S = -\sum_n P(n)\ln P(n)$$

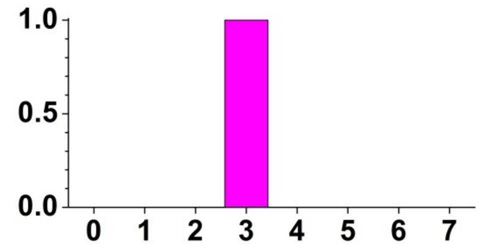
No information on photon number (uniform distribution):

$$S = S_{\max} = -\ln(1/N) = 2.1$$



Full information on a photon number (single peak distribution):

$$S = S_{\min} = 0$$



Smaller entropy = more knowledge

Optimal phase choice

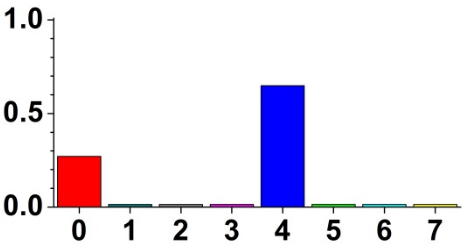
Available measurements

Expected photon number distribution $P_{k+1}(n)$

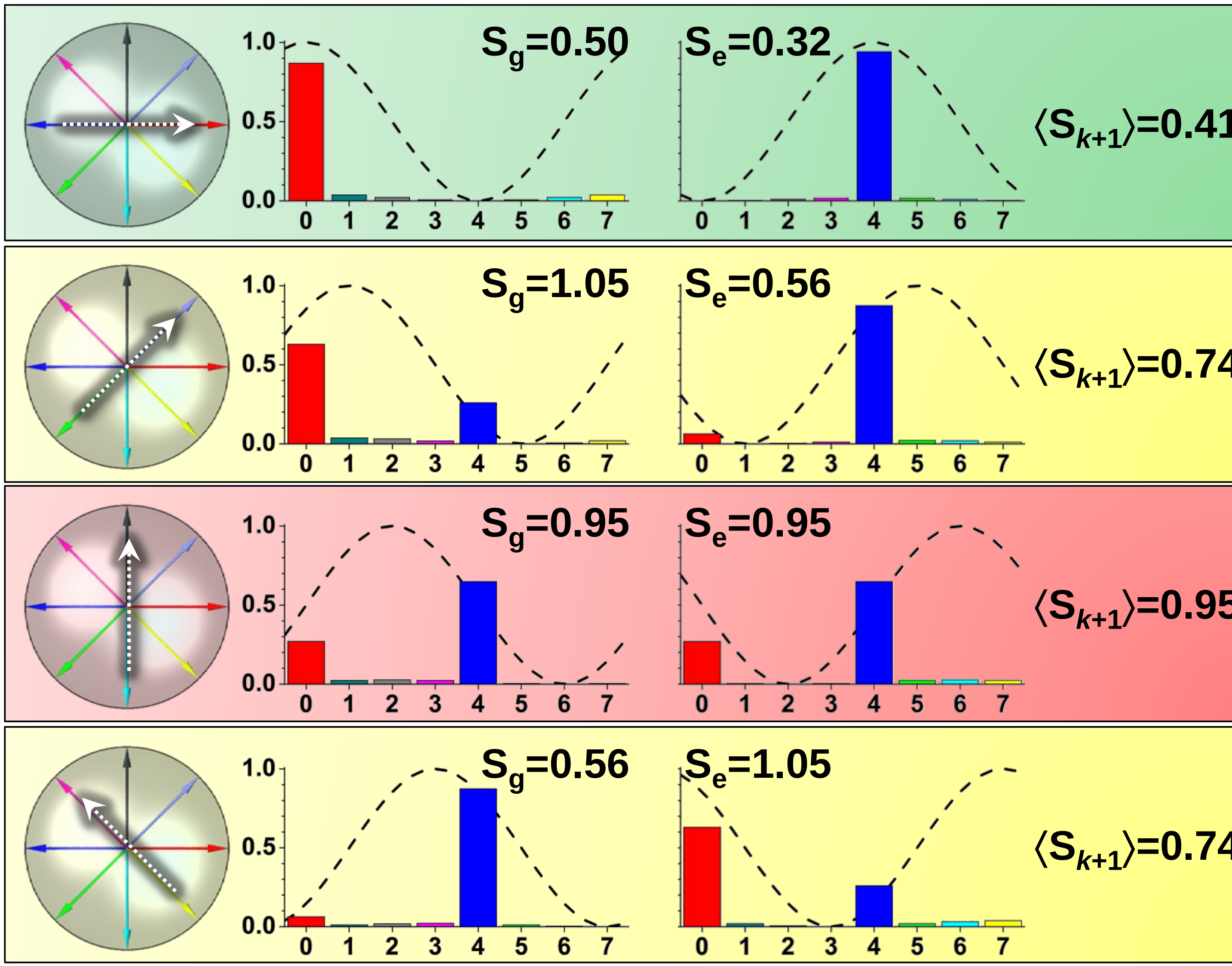
$|e\rangle$ $|g\rangle$

Example:

distribution $P_k(n)$ after k detections:



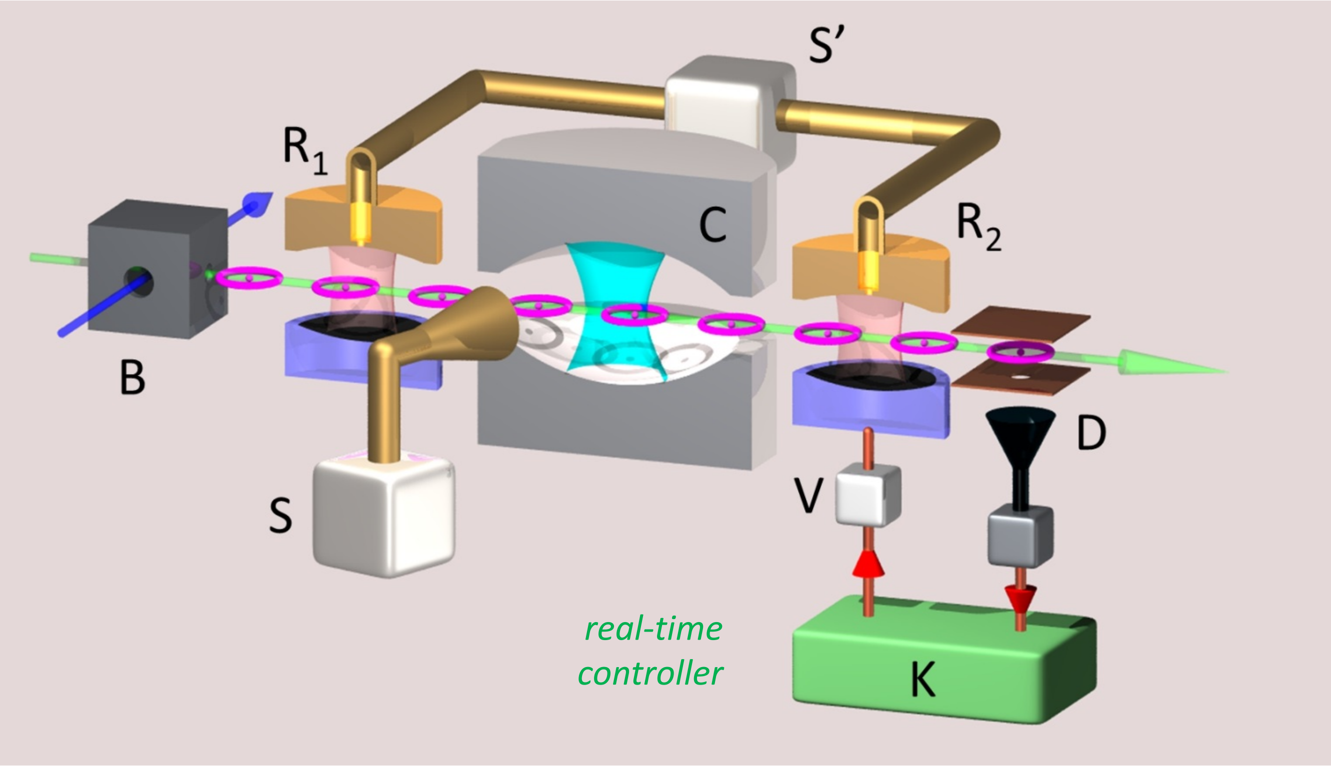
$S_k = 0.98$



Adaptive QND measurement

Algorithm: for each sensor choose a Ramsey phase which has a maximum chance to minimize entropy

$$S = -\sum_n P(n)\ln P(n)$$



Experimental sequence

We want to test phase adaptation in a context free from relaxation :

Initialization	Adaptive measurement	Verification
projection onto a random $\|n_i\rangle$	measurement of $\|n\rangle$ (25 ms)	independent measurement of $\|n_f\rangle$

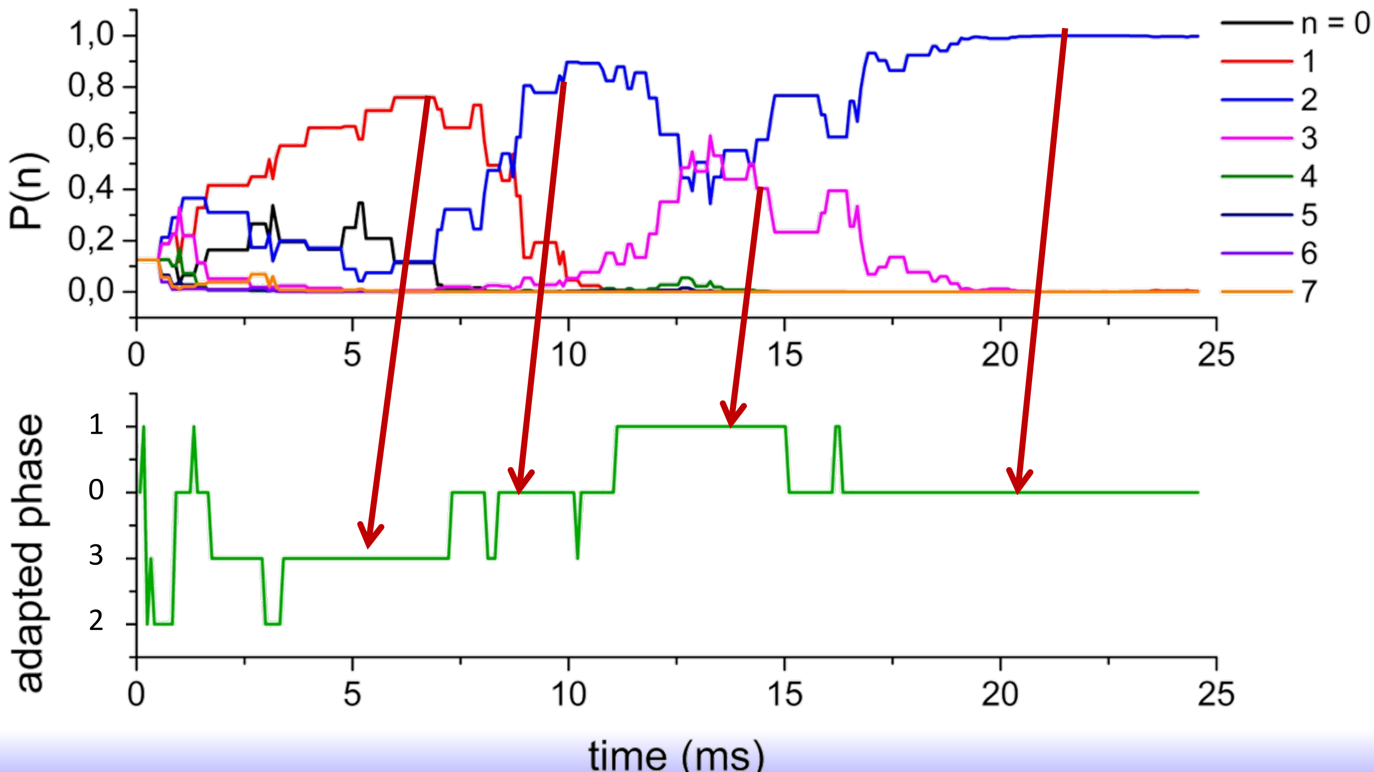
Select trajectories with **no relaxation :**
$n_i = n_f$

Experimental sequence

We want to test phase adaptation in a context free from relaxation :

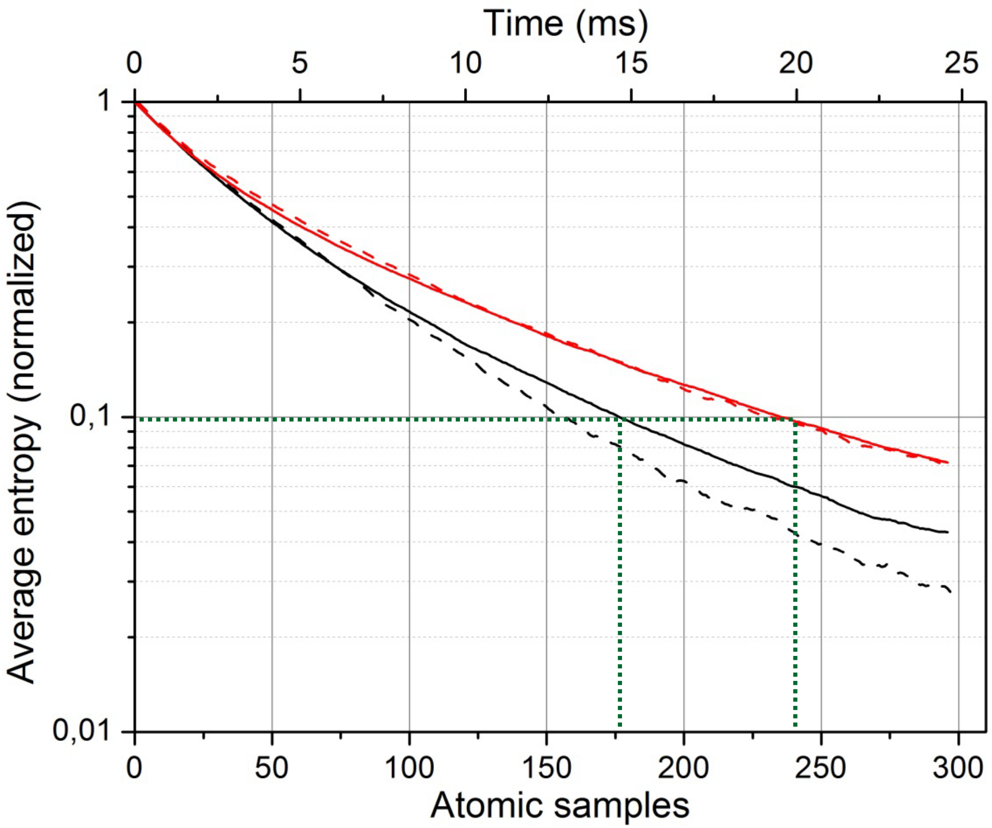
Phase, aligned orthogonally to a maximally probable *n* *(so-called mid-fringe setting, better distinguishability from its neighbors),* is chosen more often.

Controller's measurement choice follows *P*(*n*)



measurement of state |n=2⟩

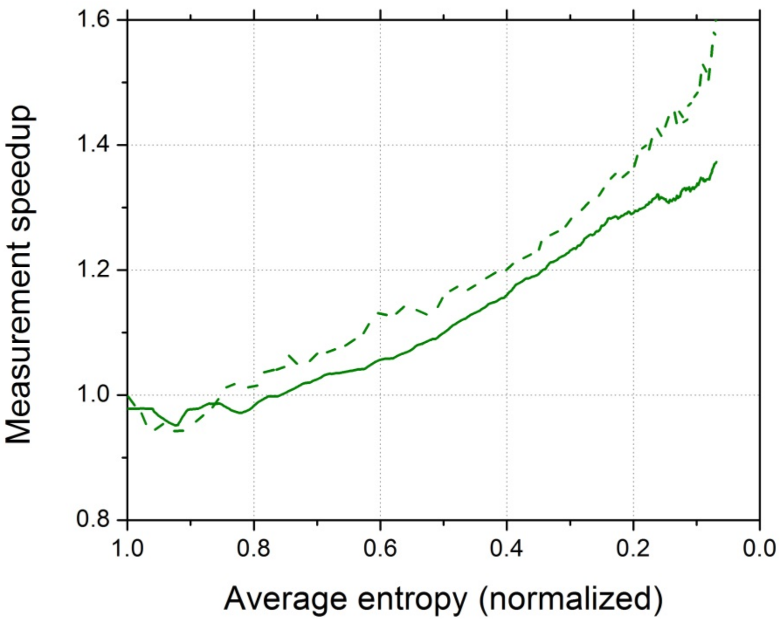
Speed-up of information acquisition



passively alternated

actively adapted

(dashed lines: simulations)

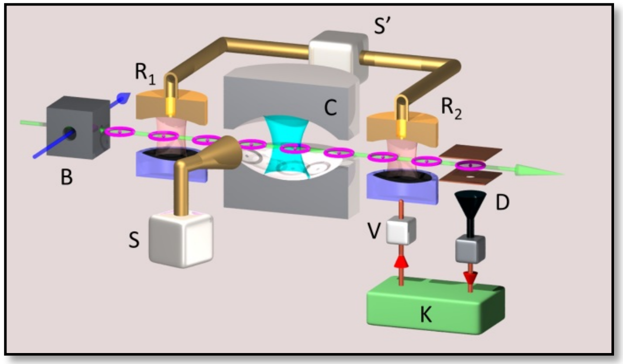


Faster entropy reduction
if phase is **adapted**

Summary II: Quantum control

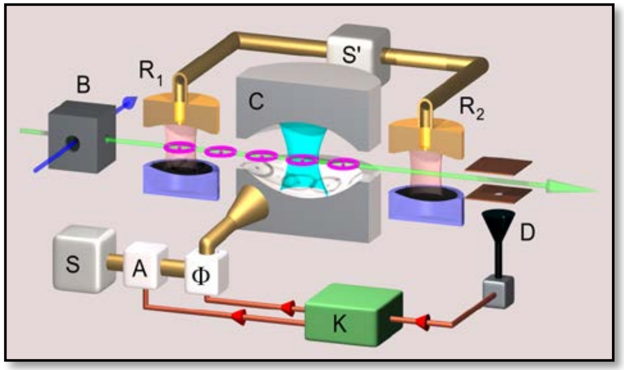
➢ Adaptive QND measurement

choosing in real time a Ramsey phase
for speeding-up information acquisition
(for faster state reduction/projection)



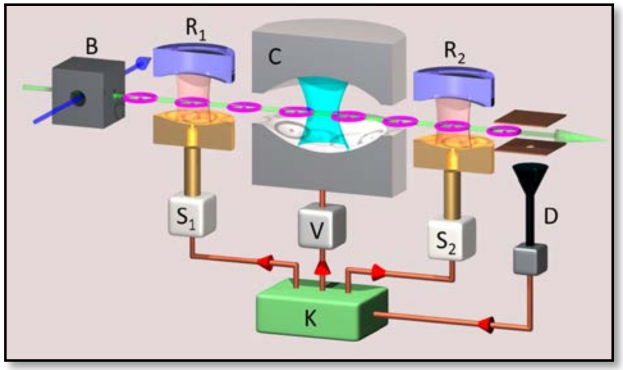
➢ Quantum feedback with coherent injection

preparation and stabilization of
photon-number states (up to 4)



➢ Quantum feedback with resonant atoms

faster feedback, thus higher
photon-number states (up to 7 so far)




Thank you for your attention!