Optimal Contracting with Unobservable Managerial Hedging

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Motivation: Relative Performance Evaluation (RPE)

Holmstrom (82): market shocks, which are not affected by managers, should be removed from managerial compensations.

RPE: Investors filter the market return from the firm's output to compensate managers.

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- exert effort α with cost $h(\alpha) = \frac{1}{2}\alpha^2$

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$$X = I + M, \quad I \sim N(\alpha, \sigma_I), M \sim N(0, \sigma_M), M, N \text{ indep.}$$

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Principal (Investor)

- Risk neutral
- observe X and M, not α
- Pay Agent via the following linear contract

$$\xi = a + bX + cM,$$

where a, b, c are constants.

The optimal contract

Agent's optimization problem:

$$\max_{\alpha} \mathbb{E}\Big[-\frac{1}{\gamma} \exp\Big(-\gamma\big(\xi - h(\alpha)\big)\Big)\Big].$$

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Therefore, $c^* = -b^*$, RPE $\xi = a + b^*(X - M)$ is the best!

Empirical tests of RPE

Homlstrom (82) assumes that Managers do not hedge.

- Managers sell stocks to diversify Ofek-Yermack (00), trades financial derivatives Bettis-Bizjak-Lemmon (01)
- Cvitanić-Henderson-Lazrak (14): observable hedging

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Results are mixed:

- Negative: Antle-Smith (86), Barro-Barro (90), Jensen-Murphy (90), Janakiraman-Lambert-Larcker (92), Aggrwal and Samwick (99)...
- Positive: Gong-Li-Shin (11), Albuquerque-De Franco-Verdi (13)
- Jenter-Kanaan (15): CEOs are more likely to be fired when the peers/market perform badly.

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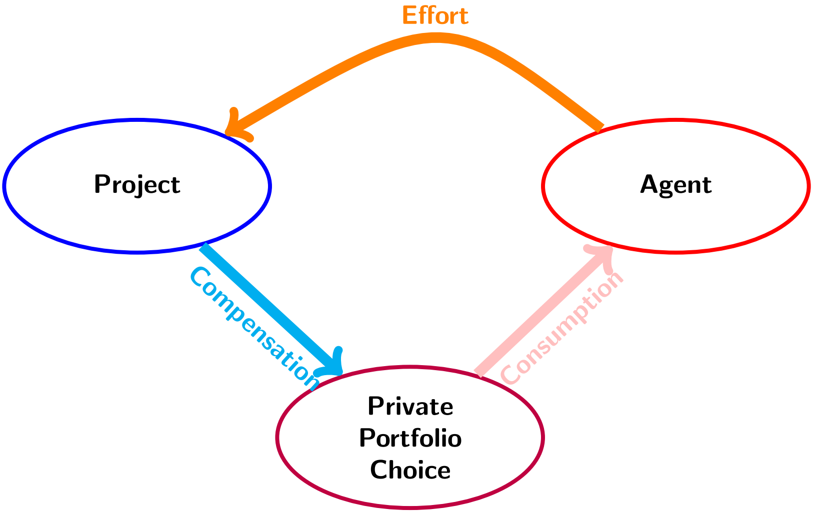
Messages from the empirical literatue:

- Difficult to provide incentive when managers hedge
- The market return is not completely filtered out from compensation
- The relationship between CEO turnover and RPE is puzzling

Unobservable Managerial Hedging



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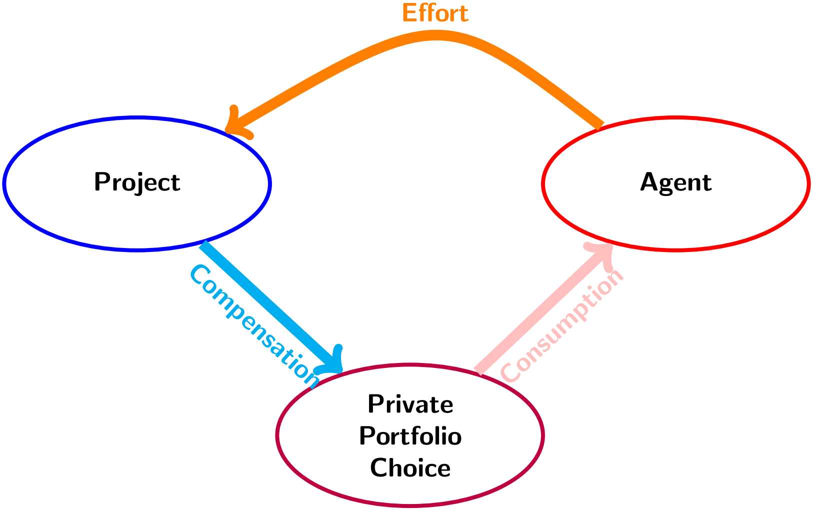

(Moral hazard I): hidden effort
(Moral harzard II): unobservable saving & hedging: may offset incentives

Economic contributions

Our model imposes limited liability restriction for contract compensation.

No negative compensation!

- Inefficient liquidation
- Risk-neutral principal is endogenously risk averse
- Principal shares market risk with agent
- Market contract sensitivity can be positive near liquidation boundary

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Main contributions:

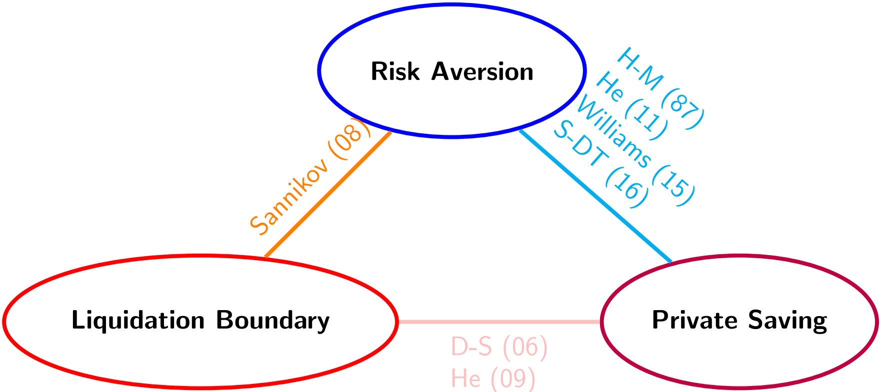
- Contract sensitivities are state dependent

$$\text{Compensation} \sim dY_t = Z(Y_t)\ \text{output}\ + U(Y_t)\ \text{market},$$

  where Y is called Agent's contract value.
- When Y is close to liquidation boundary, $U(Y)$ can be positive

$$\text{market} \downarrow \quad \Rightarrow \quad Y \downarrow \quad \Rightarrow \quad \text{Liquidation probability} \uparrow.$$

"Impossible Trinity in Contracting"



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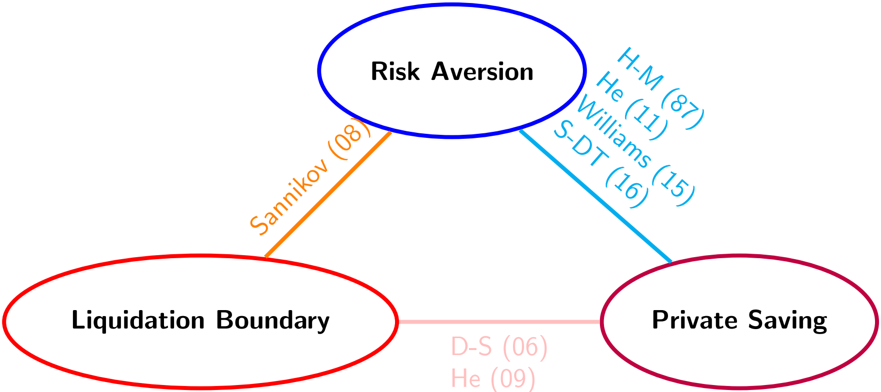

- Use agent's certainty equivalence as the state variable
- Principal's problem is stochastic control with regular + singular controls

Model

A risk-free bond with rate r

A market portfolio with return process

$$dR_t = m dt + \sigma dB_t.$$

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The output process of the project

$$dX_t = (\mu + A_t)dt + \rho\psi dB_t + \sqrt{1-\rho^2}\psi dB_t^{\perp},$$

X and R are observable to the principal continuously.

Agent's problem

Agent's private wealth process

$$dS_t = rS_t dt + \pi_t (m - r)dt + \pi_t \sigma dB_t + dI_t - h(A_t)dt - c_t dt,$$

- π: monetary value invested in the market
- I: cumulative compensation, nondecreasing (limited liability)
- $h(A) = \frac{\kappa}{2}A^2 + bA$: monetary cost for agent's effort A
- c: private consumption rate
- Admissibility: transversality condition: $\lim_{T\to\infty} \mathbb{E}[e^{-\bar{\delta}T} e^{-r\gamma S_T}] = 0$.

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A CARA agent with $u(c) = -\frac{1}{\gamma}e^{-\gamma c}$

Discounting rate $\bar{\delta}$

Agent's outside option:

$$\underline{V}_t = \text{ess sup}_{c,\pi} \mathbb{E}_t \left[\bar{\delta} \int_t^\infty e^{-\bar{\delta}(s-t)} u(c_s) ds \right],$$

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Agent's problem

$$u(\mathcal{G}_t) = \text{ess sup}_{A,\pi,c}\mathbb{E}_t\Big[\bar{\delta}\int_t^{\tau} e^{-\delta(s-t)}u(c_t)dt + e^{-\bar{\delta}(\tau-t)}u(rS_\tau - \ell)\Big],$$

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Define

$$\mathcal{G}_t = rS_t - \ell + rY_t.$$

Contract's additional value to the agent is Y, the Agent's contract value.

Principal's problem

Principal is risk neutral

Discounting rate δ

Principal's problem:

$$\sup_{I} \mathbb{E}\Big[\delta \int_0^{\tau} e^{-\delta t}\big((\mu + A^*)dt - dI_t\big) + e^{-\delta\tau}\phi\mu\Big].$$

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We choose Y as Principal's unique state variable.

Principal does not know Agent's private wealth S.

CARA utility assumption is essential.

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Goal: Find Principal's optimal contract I^*, Agent's optimal effort A^*.

Dynamics of Y

Suppose that the dynamics of Y follows

$$dY_t = dH_t + Z_t dX_t + U_t dR_t$$

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H can be determined by the Martingale Principal:
El Karoui-Rouge (00), Hu-Imkeller-Muller (05)

1. $e^{-\bar{\delta}t} u(\mathcal{G}_t) + \bar{\delta} \int_0^t e^{-\bar{\delta}s} u(c_s) ds$ is a supermartingale until τ for arbitrary strategy A, π, c;
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$$\begin{aligned} dY_t = & \Big[rY_t + \frac{r\gamma}{2} \psi^2 (1-\rho^2) Z_t^2 + h(A^*(Z_t)) + (m-r)\zeta_t \Big] dt - dI_t \\ & + \zeta_t \sigma dB_t + Z_t \sqrt{1-\rho^2} \psi dB_t^{\perp} \end{aligned}$$

- $\zeta = \frac{\rho\psi}{\sigma} Z + U$ is the agent's exposure to the market
- Agent's optimal portfolio is $\pi^* = \frac{m-r}{r\gamma\sigma^2} - \zeta$
- $\tau = \inf\{t \geq 0 : Y_t \leq 0\}$ is the liquidation time
- $A^*(Z) = \arg\min\{h(A) - ZA\}$

Principal's problem

Consider Y as Principal's unique state variable

$$W(y) = \sup_{I,Z,\zeta} \mathbb{E}\Big[\delta \int_0^\tau e^{-\delta t}\big((\mu + A^*(Z_t))dt - dI_t\big) + e^{-\delta\tau}\mu\Big],$$

where $\tau = \inf\{t \geq 0 \;:\; Y_t \leq 0\}$

- Z, ζ: regular control, $Z \in [\underline{Z}, \overline{Z}]$
- I: singular control

Variational inequality

$$\min\Big\{\delta W - \sup_{Z,\zeta}\Big\{\delta\big(\mu + A^*(Z)\big) + \big(ry + g(Z,\zeta)\big)W' + \tfrac{1}{2}\left[\sigma^2\zeta^2 + (1-\rho^2)\psi^2 Z^2\right]W''\Big\}, W' + \delta\Big\} = 0,$$

where

$$g(Z,\zeta) = \underbrace{\tfrac{r\gamma}{2}\psi^2(1-\rho^2)Z^2 + (m-r)\zeta}_{\text{Cost of hedging}} + \underbrace{h(A^*(Z))}_{\text{Cost of effort}}.$$

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A free boundary problem:

$$\delta W = \sup_{Z,\zeta}\Big\{\delta\big(\mu + A^*(Z)\big) + \big(ry + g(Z,\zeta)\big)W' + \tfrac{1}{2}\big[\sigma^2\zeta^2 + (1-\rho^2)\psi^2 Z^2\big]W''\Big\}$$

$$W'(\bar{y}) = -\delta, \quad W''(\bar{y}) = 0,$$

$$W(0) = \phi\mu.$$

Main result

Theorem

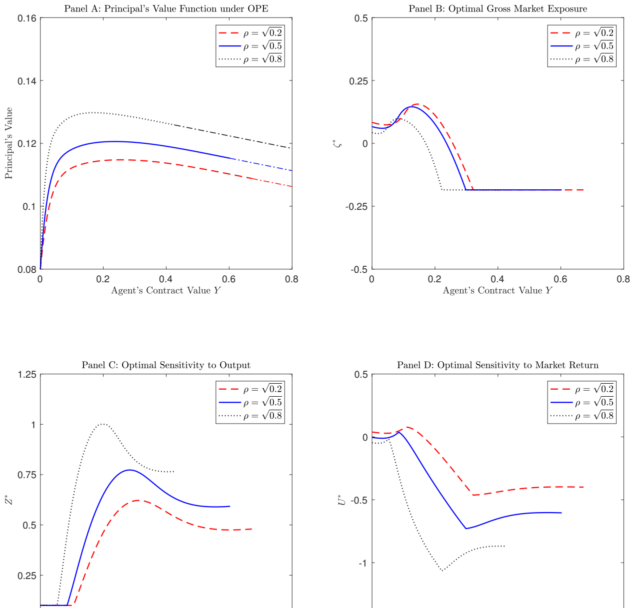
Assume that

- $r > \delta$ *(ensure* $\bar{y}$ *is finite),*
- $\underline{Z} > 0$ *(ensure the HJB is uniform elliptic).*

There is a unique solution $W \in C^2(0, \infty)$ *of the variational inequality. Moreover,*

- W *is strictly concave on* $(0, \bar{y})$*,*
- W *satisfies the free boundary problem,*
- *The optimal contract is a "local time" type, which reflects* Y *at* $\bar{y}$*.*

Risk sharing and incentive provision



Economic results

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2. The optimal exposure to the market is

$$\zeta^* = -\frac{m-r}{\sigma^2}\frac{W'(y)}{W''(y)}.$$

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- When Y is close to the liquidation boundary: $W' > 0 \implies \zeta^* > 0$
- When Y is close to the payment boundary: $W' < 0 \implies \zeta^* < 0$

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4.

$$dY_t = dH_t + Z_t dX_t + U_t dR_t.$$

   When Y is close to 0, positive U implies

$$dR_t < 0 \quad \implies \quad Y_t \text{ closer to } 0 \quad \implies \quad \mathbb{P}(\text{liquidation}) \uparrow$$

Conclusion

- A model with unobservable managerial hedging
- Market contract sensitivity is dynamic and can be positive; OuYang (05), Ozdenoren-Yuan (17)
- Risk aversion + private saving/investment + liquidation
- Positive market contract sensitivity implies more liquidation when market falls

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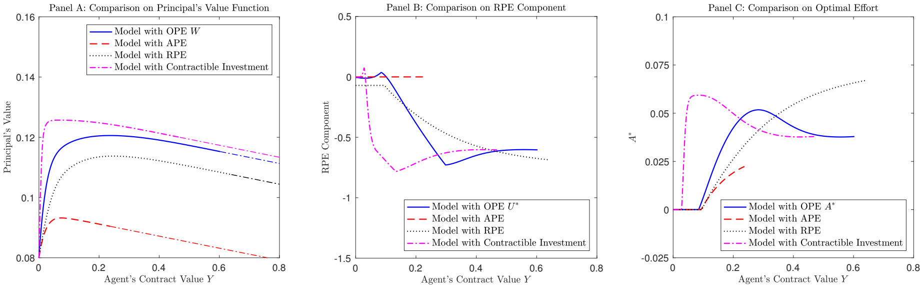
Thanks for your attention!

Model comparison

$$dY_t = dH_t + Z_t dX_t + U_t dR_t.$$

- APE: $U \equiv 0$
- RPE: $U = -\frac{\rho\psi}{\sigma} Z$
- OPE: U can be chosen freely
- Benchmark: observable hedging, unobservable effort



Proofs

$$\min\left\{\delta W - \sup_{Z,\zeta}\left\{\delta\big(\mu + A^*(Z)\big) + \big(ry + g(Z,\zeta)\big)W' + \tfrac{1}{2}\Sigma(Z,\zeta)W''\right\},\right.$$
$$\left. W' + \delta\right\} = 0$$

1. $\underline{W} \leq W \leq \overline{W}$, where $\overline{W}(y) = \mu - \delta y + \sup_{Z,\zeta}\{A^*(Z) - g(Z,\zeta)\}$ and $\underline{W}(y) = \phi\mu - \delta y$.
2. W is viscosity solution (DPP)
3. unique viscosity solution with linear growth (controls need to be bounded), hence W is continuous
4. $\tilde{W}(y) = \mu - ry + \sup_{Z,\zeta}\{A^*(Z) - g(Z,\zeta)\}$. The free boundary is before the intersection of $\underline{W}$ and $\tilde{W}$ ($r > \delta$)
5. W is concave

$$W'' = \inf_{Z,\zeta}\left\{\frac{\delta W - \delta\big[\mu + A^*(Z) - (ry + g(Z,\zeta))\big]W'}{\frac{1}{2}\Sigma(Z,\zeta)}\right\}$$

6. W is C^2 (uniformly elliptic) Strulovici-Szydlowski (2015), Pham (09)