McKean FBSDE applied to the management of microgrid

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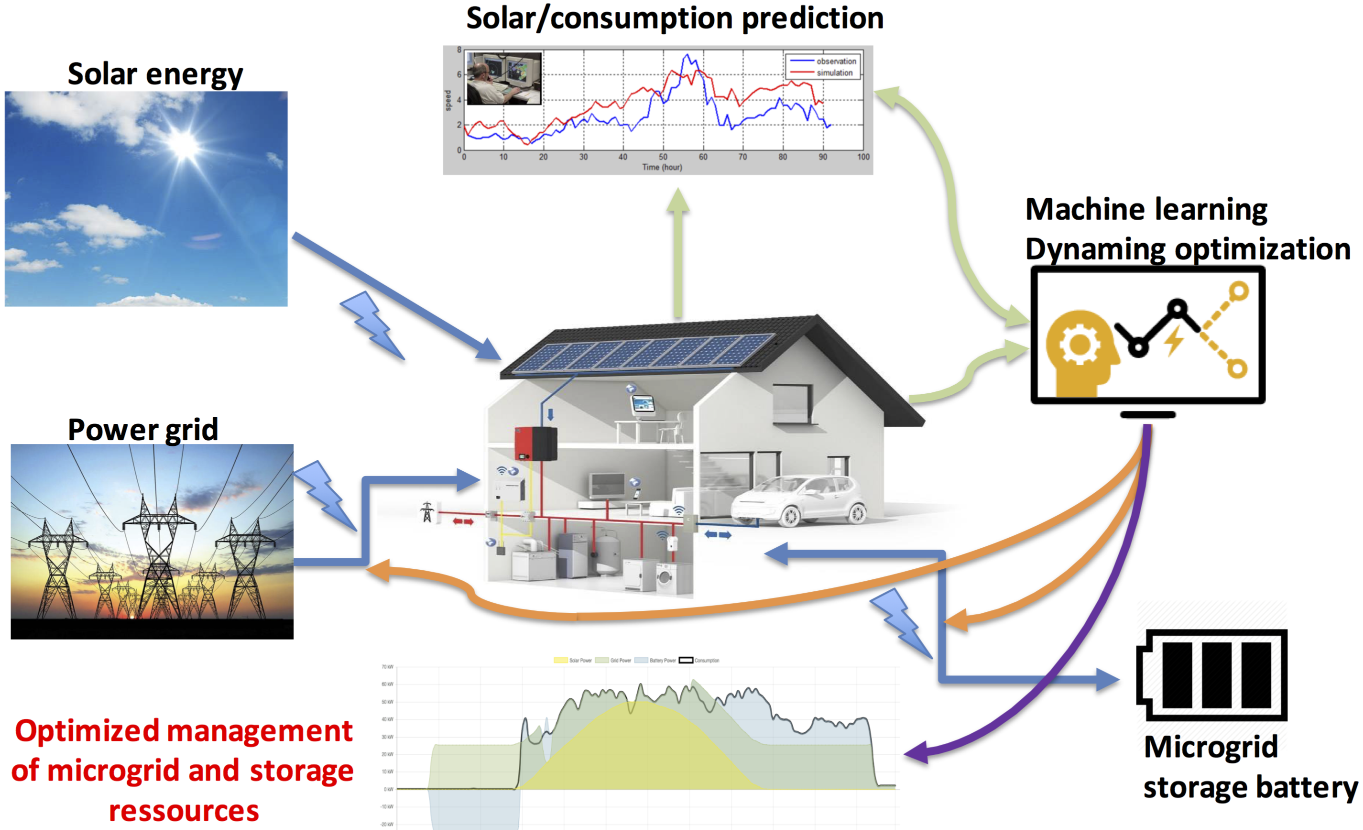
Solar modeling with Jordi Badosa and Daeyoung Kim (TREND-X).

Contents

1 Modeling the micro-grid

1.1 Context

Recent deep transformations in the mechanisms of energy purchase / sale, distribution / consumption: Renewable Energies, storage, aggregation, home automation, setting up of microgrids…



1.2 The system

✓ **Context:** management of smart building

- equipped with solar panels
- connected to a "public" grid providing electricity
- equipped with a battery

✓ **State variables:**

- power from the grid ($\mathtt{P}_{\mathtt{grid}}$)
- State of Charge of the battery ($\mathtt{SOC}$)
- weather variables (irradiance, humidity, outside temperature)
- inside temperature
- building consumption (HVAC, appliances, lighting. . .)

✓ **Uncertainty:** building consumption, PV production

✓ **Controls:** HVAC, lighting, **battery**

✓ **Battery:** plays the role of a buffer, absorbing both

- the unpredictable part of the consumption
- the intermittency of the PV production

➠ Different economic models for the use of battery are possible. . .

Economic criterion 1.

Prosumer.

Store PV electricity and sell it when electricity prices are at the highest

Economic criterion 2.

Uncertainty reduction of the demand on the grid load.

- ☺ Grid manager: better sizing of energy-production units
- ☺ Consumer: contract with lower electricity price

➠ Our choice...

1.3 Ingredients for designing the micro-grid management

Goal: How to minimize the variability of the grid load?

1. **Consider an optimization criterion**

   For instance: over $T = 1$ day horizon,

$$\min_{\mathbf{control}_t} \int_0^{\mathbf{T}} \left(\kappa \, \mathbb{V}\mathrm{ar}\left[\mathtt{P}_{\mathtt{grid}}(\mathbf{t}) \right] + \mu \, \mathbb{E}\left[\mathtt{P}^{\mathbf{2}}_{\mathtt{bat}}(\mathbf{t}) \right] + \nu \, \mathbb{E}\left[\left(\mathtt{SOC}(\mathbf{t}) - \frac{\mathbf{1}}{\mathbf{2}} \right)^{\mathbf{2}} \right] \right) \mathrm{d}\mathbf{t}$$

   ✓ Compromise between
   - variability of $\mathtt{P}_{\mathtt{grid}}$ averaged over the day
   - and large charge/discharge of the battery (aging effect)

   ✓ Ideally: $\mathtt{SOC}(0) = \mathtt{SOC}(T) = \frac{1}{2}$. Relaxation of the constraint by adding a penalization with parameter $\nu \to +\infty$.

   ✓ In the case of linear dynamics, see **[Yong, SICON 2013]**

   ✓ **Optimal stochastic control problem of McKean type** (involving the distribution of the State Variables and **of the Control**), see **[Carmona-Delarue, AoP 2015, etc]**

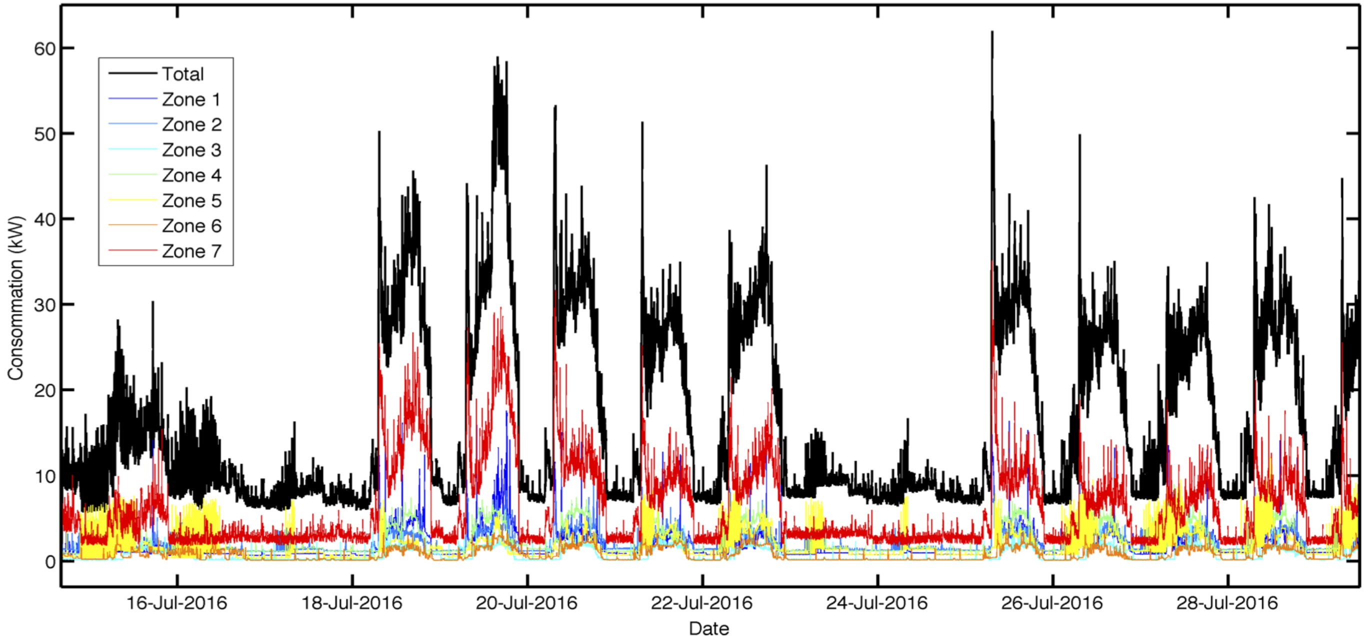
2. **Building consumption:**

$$\mathtt{P}_{\mathtt{cons}}(t) = \mathtt{P}_{\mathtt{HVAC}}(t) + \mathtt{P}_{\mathtt{Appliance}}(t) + \mathtt{P}_{\mathtt{Lighting}}(t).$$

✓ Lighting: automatic mode (working building), depends on the season and the hour of the day. Negatively correlated to the irradiance.

✓ HVAC: automatic mode to maintain a inside temperature within a range (e.g. $[19^o\text{C} - 20^o\text{C}]$). Correlated to the weather conditions.

✓ Usually modeled with **mean-reverting process with jumps** (when switch off-on devices). See e.g. **[Aswani, Master, Taneja, Culler, Tomlin, IEEE 2011]**.



3. **Power balance:**

$$\mathtt{P_{cons}}(t) = \mathtt{P_{bat}}(t) + \mathtt{P_{sun}}(t) + \mathtt{P_{grid}}(t)$$

with $\mathtt{P_{bat}} \gtrless 0$, $\mathtt{P_{sun}} \geq 0$, $\mathtt{P_{grid}} \geq 0$ (no selling of extra production).

✓ $\mathtt{P_{sun}}$: depend on irradiance (see later), humidity, temperature, PV panel...

✓ $\mathtt{P_{bat}}$: depend on the controller u_t

- $\mathtt{SOC}$: the State Of Charge variable.
- Power delivered by the battery:

$$\mathtt{P_{bat}}(t) = \phi^{\mathtt{bat}}(u_t, \mathtt{SOC}(t)).$$

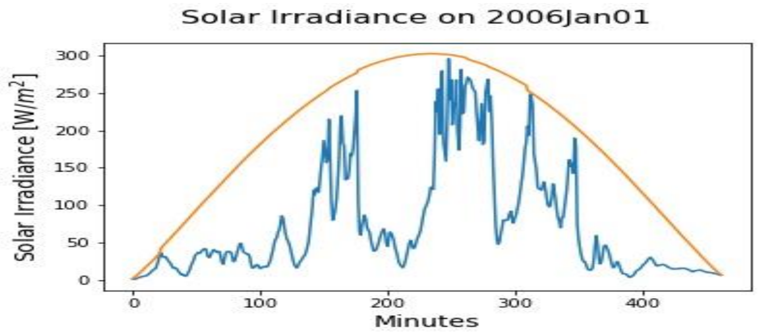
  - u and $\mathtt{P_{bat}}$ have the same signs
  - if $\mathtt{SOC}(t) = 0$ and $u_t > 0$, no extra discharge ($\mathtt{P_{bat}}(t) = 0$). And vice-versa.
- Evolution of SOC:

$$\frac{\mathrm{d}\mathtt{SOC}^u(t)}{\mathrm{d}t} = \phi^{\mathtt{SOC}}(u_t, \mathtt{SOC}^u(t)).$$

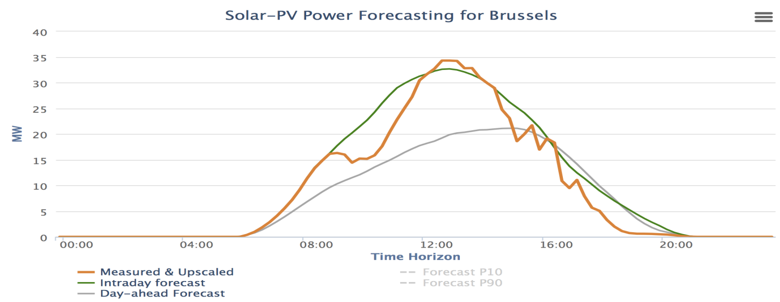
- Rough approximation: linear dynamics

4. Irradiance

a) Fluctuations of irradiance depend much on the size of the area

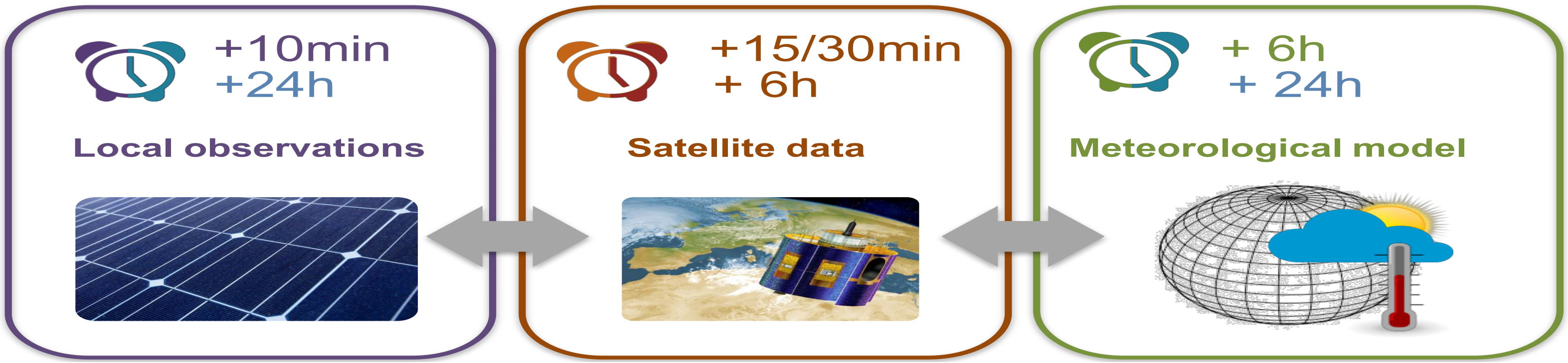


Small site



Large site

b) Long term forecast are especially difficult



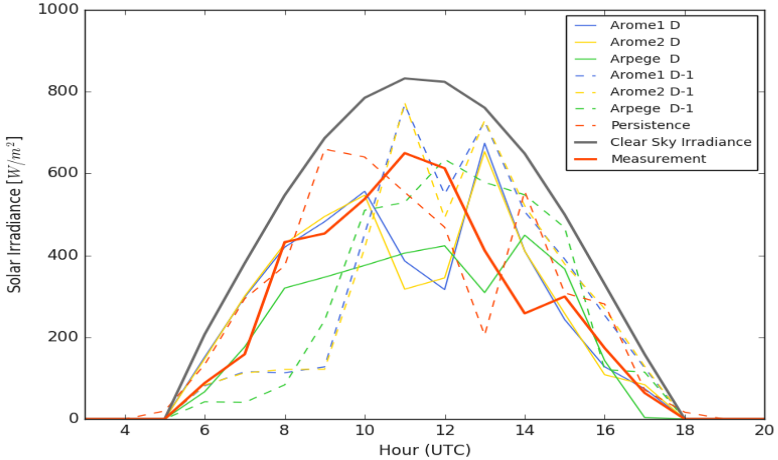
Here we mainly need forecast other several hours ➠ data from MeteoFrance.

c) We need probabilistic forecast (and not only a pointwise forecast)

How to design a stochastic model?

✓ No stationarity property in weather variables

✓ We shall take advantage of day-ahead forecasts (performed on day D-1)



Different MeteoFrance forecasts (AROME, ARPEGE).
Horizons: Day=D, Day before=D-1.

✓ We shall account for the maximal irradiance (Clear Sky Model).

✓ Clear Sky Index: $X_t = \dfrac{I(t)}{I^{\texttt{clear sky model}}(t)} \in [0,1]$

✓ Expected CSI: $x_t^{\texttt{forecast}} := \dfrac{I^{\texttt{forecast}}(t)}{I^{\texttt{clear sky model}}(t)} \in [0,1].$

✓ SDE model (like Fisher-Wright or Jacobi process):

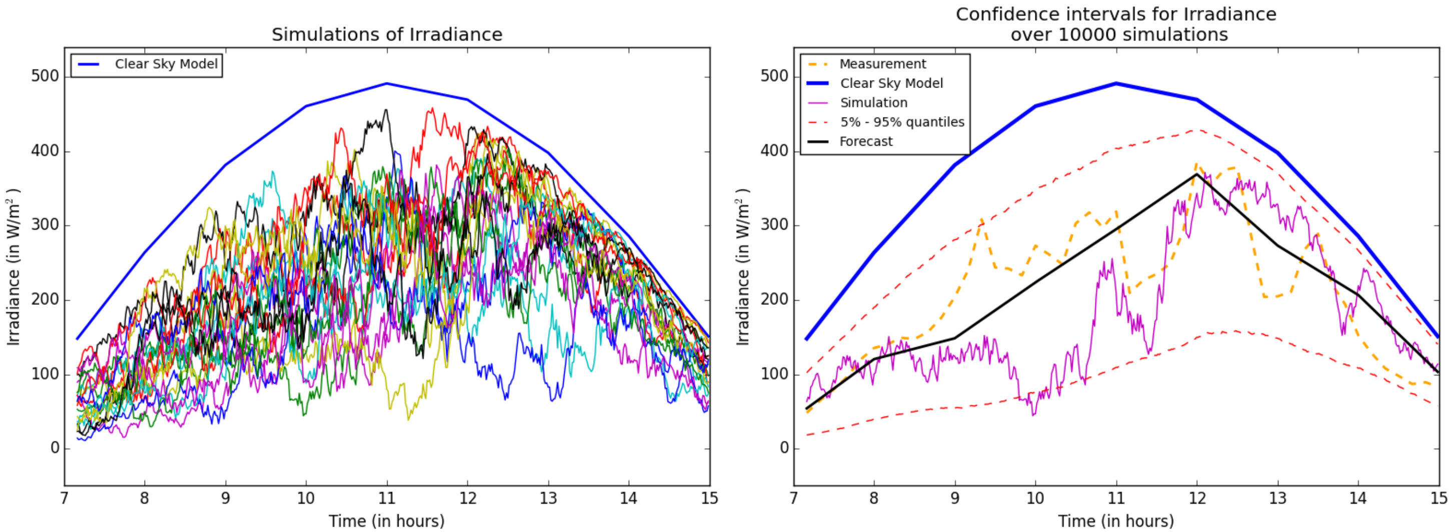
$$\mathrm{d}X_t = -a(X_t - x_t^{\texttt{forecast}})\mathrm{d}t + \sigma X_t^{\alpha}(1 - X_t)^{\beta}\mathrm{d}W_t$$

with $\alpha, \beta \in [1/2, 1]$.

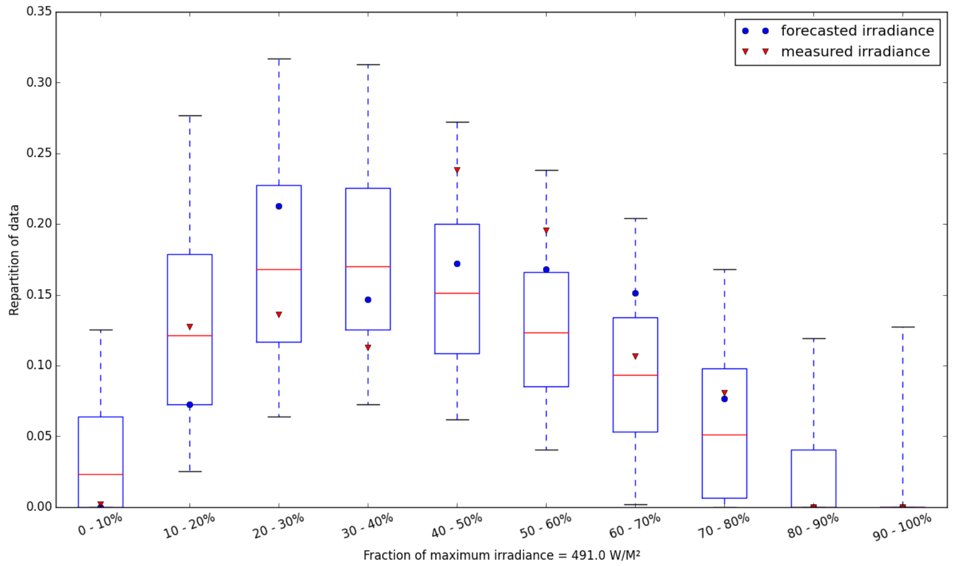
✓ Parameter estimations:

- $\mathbf{a} \approx \mathbf{0.75h^{-1}}$: estimated from autocorrelogram
- $(\alpha, \beta) \approx (\mathbf{0.8}, \mathbf{0.7})$
- the **volatility** σ is adjusted everyday as a function of the averaged increments of the $D-1$-forecast

Results for a day with mitigated weather (October, 24th, 2015)



Results for the probabilistic forecast



Distribution of the irradiance over the day

2 Optimal stochastic control of McKean type

Accounting on the distribution of the system (through its **moments**): set

$$X_t^u = \begin{pmatrix} \mathtt{SOC}^u(t) \\ \mathtt{P}_{\mathtt{load}}(t) \end{pmatrix} \text{ and}$$

$$\mathcal{J}(\mathbf{u}) = \mathbb{E}\left[\int_0^T l\left(u_t, X_t^u, , \mathbb{E}\left[g(t, u_t, X_t^u)\right]\right) \mathrm{d}t + \psi(X_T^u)\right] \longrightarrow \min_u .$$

References of such a problem (without the distribution on the control): **[Yong 2013]**, **[Carmona, Delarue, Lachapelle, 2013]**, **[Carmona, Delarue, 2015]** ...

Our strategy of analysis, using Pontryagin principle:

1. necessary conditions by Gateaux differentiability ➠ McKean Forward Backward SDE
2. well-posedness of the McKean FBSDE
3. sufficient conditions under convexity conditions

2.1 Necessary conditions

Theorem (Gâteaux derivatives). Let $u \in \mathbb{H}^2$ and set $g_t^u := g(t, u_t, X_t^u)$. Assume smooth coefficients, define the FBSDE (Y, M)

$$\begin{cases} -\mathrm{d}Y_t = \Big[\phi_{\mathsf{SOC}}^{\mathsf{SOC}}(t, u_t, \mathsf{SOC}^u(t))Y_t + l_{\mathsf{SOC}}(t, u_t, X_t^u, g_t^u) \\ \qquad\qquad + \mathbb{E}\left[l_g(t, u_t, X_t^u, g_t^u)\right] g_{\mathsf{SOC}}(t, u_t, X_t^u)\Big] \mathrm{d}t - \mathrm{d}M_t, \\ Y_T = \psi_{\mathsf{SOC}}(X_T^u) \end{cases}$$

and assume that it has a square integrable solution (Y, M). Then, for any $v \in \mathbb{H}^2$,

$$\partial_\varepsilon \mathcal{J}(\mathbf{u} + \epsilon \mathbf{v})|_{\varepsilon=\mathbf{0}} = \mathbb{E}\Bigg[\int_{\mathbf{0}}^{\mathbf{T}} \mathbf{v_t} \Big\{ \mathbf{l_u}(\mathbf{t}, \mathbf{u_t}, \mathbf{X_t^u}, \mathbf{g_t^u}) + \mathbb{E}\left[\mathbf{l_g}(\mathbf{t}, \mathbf{u_t}, \mathbf{X_t^u}, \mathbf{g_t^u})\right] \mathbf{g_u}(\mathbf{t}, \mathbf{u_t}, \mathbf{X_t^u}) \\ + \mathbf{Y_t} \phi_{\mathbf{u}}^{\mathsf{SOC}}(\mathbf{t}, \mathbf{u_t}, \mathbf{X_t^u}) \Big\} \mathrm{d}t\Bigg].$$

2.2 McKean FBSDE

Theorem (Existence, uniqueness). Under technical assumptions, the optimal control must fulfill

$$\mathbf{Y_t} = \mathbb{E}\left[\boldsymbol{\Psi}'(\mathbf{X_T^u}) + \int_{\mathbf{t}}^{\mathbf{T}} \mathbf{h_1}\left(\mathbf{Y_s}, \mathbf{u_s}, \mathbf{X_s^u}, \mathbb{E}\left[\mathbf{h_2}(\mathbf{u_s}, \mathbf{X_s^u})\right]\right) \mathrm{d}\mathbf{s} \mid \mathcal{F}_\mathbf{t}\right],$$

$$\mathbf{h_3}\left(\mathbf{Y_t}, \mathbf{u_t}, \mathbf{X_t^u}, \mathbb{E}\left[\mathbf{h_4}(\mathbf{u_t}, \mathbf{X_t^u}, \mathtt{P_{load}}(\mathbf{t}))\right]\right) = \mathbf{0},$$

for some h_1, h_2, h_3, h_4, and there exists a unique solution to the above system.

✓ For linear-quadratic problems, explicit solution through the solution of Ricatti equations.

✓ In general, resolution via regression Monte Carlo (like for BSDEs)

2.3 Sufficient conditions

Assume

1. The terminal cost ψ is convex in the first variable.
2. The mapping

$$\mathcal{H}:\begin{cases}[0,T]\times\mathbb{H}^2(\mathbb{R})\times\mathbb{S}^2(\mathbb{R})\times\mathbb{S}^2(\mathbb{R})\to\mathbb{R}\\ (t,u,X,Y)\mapsto\mathcal{H}(t,u,X,Y):=\mathbb{E}\left[l\left(t,u_t,\begin{pmatrix}X_t\\ \mathtt{P}_{\mathtt{load}}(t)\end{pmatrix},\mathbb{E}\left[g\left(t,u_t,\begin{pmatrix}X_t\\ \mathtt{P}_{\mathtt{load}}(t)\end{pmatrix}\right)\right]\right)\right]\\ \qquad\qquad +\mathbb{E}\left[Y_t\phi^{\mathtt{SOC}}(t,u_t,X_t)\right]\end{cases}$$

   is convex in (u,X) for any t and Y.

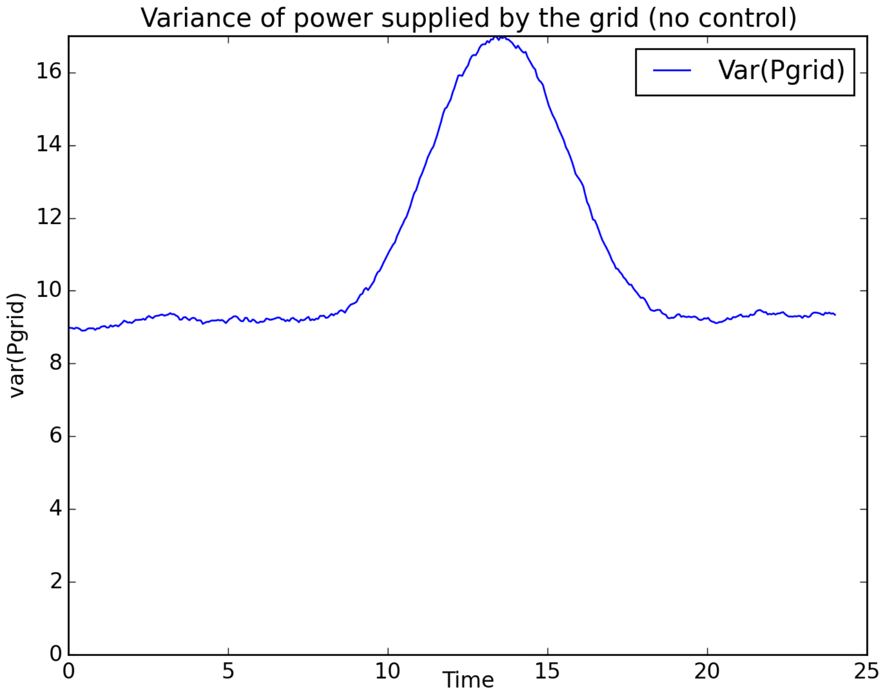
Hamiltonian in expectation and not pathwise.

Theorem. If (u,Y) is the solution of McKean FBSDE, then the control u is optimal.

All conditions are satisfied in the initial microgrid problem.

2.4 Numerical illustration: with or without battery command

Here we consider the Linear-Quadratic case (explicit solution).



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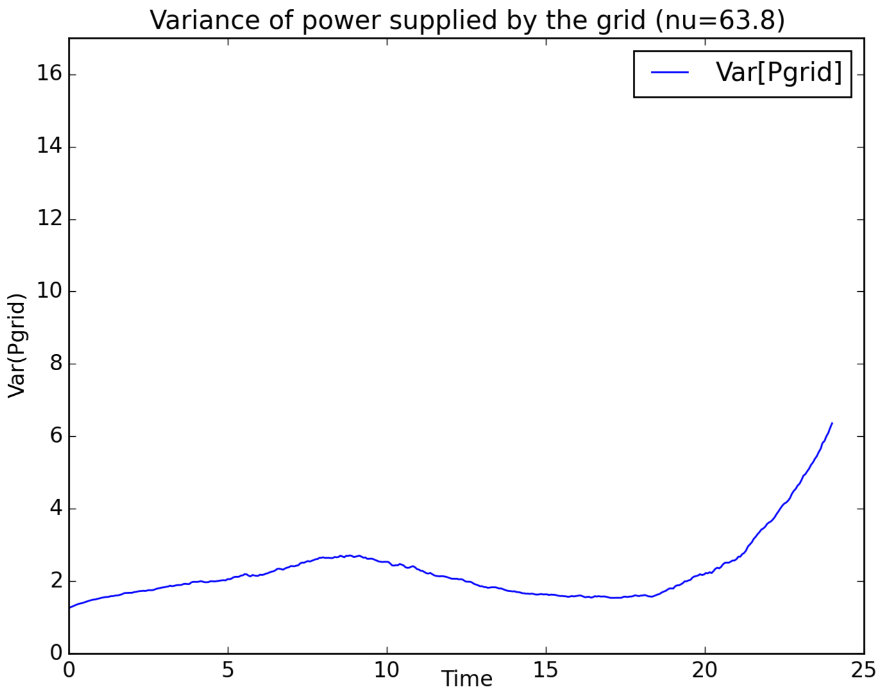

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Conclusion

- ✓ Modeling micro-grid management
- ✓ Optimization criterion: variability of $\mathtt{P}_{\mathtt{grid}}$
- ✓ New irradiance modeling: using SDE. Good probabilistic forecast.
- ✓ Optimal control: solution by a new type of Pontryagin principle, and McKean FBSDE.
- ✓ Perspectives:
  - numerical solution in the general case
  - tests on real situation.

Thank you for your attention!

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