TILING AND RECURRENCE)

CIRM, LUMINY, 4-8 DECEMBER, 2017

Abstracts

Michael Baake

Autocorrelation and diffraction via renormalisation
Part II: Extensions and generalisations

Abstract: In this follow-up contribution, we will consider extensions to inflation tilings in higher dimensions, where the crucial aspect lies in the separation of the geometry of the space and that of the inflation structure itself. This works well for FLC tilings, but can also be generalised to non-FLC structures. This will be explained along examples from the class of block substitutions.

Artemi Berlinkov

Singular substitutions of constant length

Abstract: We consider primitive aperiodic substitutions of constant length q and prove that, in order to have a Lebesgue component in the spectrum of the associated dynamical system, it is necessary that one of the eigenvalues of the substitution matrix equals $\sqrt{q}$ in absolute value.
Joint work with Boris Solomyak.

Valérie Berthé

Dimension groups and recurrence for tree subshifts

Abstract: Dimension groups are invariants of orbital equivalence. We show in this lecture how to compute the dimension group of tree subshifts. Tree subshifts are defined in terms of extension graphs that describe the left and right extensions of factors of their languages: the extension graphs are trees. This class of subshifts includes classical families such as Sturmian, Arnoux-Rauzy subshifts, or else, codings of interval exchanges. We rely on return word properties for tree subshifts: every finite word in the language of a tree word admits exactly d return words, where d is the cardinality of the alphabet.
This is joint work with P. Cecchi, F. Dolce, F. Durand, J. Leroy, D. Perrin, S. Petite.

Adnene Besbes

Thermodynamic formalism On
Aperiodic Linearly Repetitive Tilings

Abstract: We show the convergence of thermodynamic quantities (pressure, mean energy and entropy) defined on the lattice of vertices of aperiodic linearly repetitive tilings. To show such convergence we establish two sorts of ergodic theorems: additive and subadditive theorems.

Alexander I. Bufetov

Hölder estimates for the spectrum of substitution systems and translation flows

Abstract: Uniform Hölder estimates are obtained for the spectral measure of suspension flows over substitution systems as well as for translation flows in genus two. The proof of the Hölder property relies on a variant of the Erdős-Kahane argument. Joint work with Boris Solomyak.

Danilo Antonio Caprio

Dynamics of stochastic Bratteli diagrams

Abstract: Given a stationary ordered Bratteli diagram B, with incidence d x d matrix, we define the stochastic Vershik map associated to B. This gives a Markov chain whose states are all non-negative integer numbers.We prove that the spectrum of transition operator associated to this Markov chain is a fractal set connected to the class of Julia sets on dimension d.
This is a joint work with Ali Messaoudi and Glauco Valle.

Julien Cassaigne

A family of infinite words with complexity 2n+1 associated with a bidimensional continued fraction algorithm

Abstract: After reviewing how Sturmian words and their s-adic expansions are linked with the usual additive continued fraction algorithm, we introduce a new continued fraction algorithm in two (projective) dimensions, that is designed to produce infinite words of complexity 2n+1. We study some of its properties and show that it is conjugated to an existing continued fraction algorithm, the Selmer algorithm.
Joint work with Sébastien Labbé and Julien Leroy.

Thierry Coulbois

Tree substitutions and Rauzy fractals

Abstract: For me, coming from the automorphisms of free groups community, a substitution is rather a free group automorphism (when invertible), the limit object is a lamination rather than the attracting shift and, the dual geometric representation is a real tree rather than a Rauzy fractal (under Pisot hypothesis). I will discuss these different objects.

In our joint work, we describe the real tree by a combinatorial tree substitution and we draw this tree inside the Rauzy fractal or inside the iterated images of the dual substitution. Finally, using the contour of the tree substitution we get an interval exchange transformation which covers the initial substitution.

The talk will present many images and possibly some computations with Sage.
Joint work with Milton Minervino.

Karma Dajani

Algebraic sums and products of univoque bases

Abstract: Given $x \in (0,1]$, let $\mathcal{U}(x)$ be the set of bases $\beta \in (1,2]$ for which there exists a unique sequence (d_i) of zeros and ones such that $x = \sum_{i=1}^{\infty} d_i/\beta^i$. In 2014, Lü, Tan and Wu proved that $\mathcal{U}(x)$ is a Lebesgue null set of full Hausdorff dimension. In this talk, we will show that the algebraic sum $\mathcal{U}(x) + \lambda\mathcal{U}(x)$, and the product $\mathcal{U}(x) \cdot \mathcal{U}(x)^\lambda$ contain an interval for all $x \in (0,1]$ and $\lambda \neq 0$. As an application we show that the same phenomenon occurs for the set of non-matching parameters associated with the family of symmetric binary expansions studied recently by the first speaker and C. Kalle.
This is joint work with V. Komornik, D. Kong and W. Li.

David Damanik
The Fibonacci Trace Map

Abstract: In this talk we explain how the Fibonacci trace map arises from the Fibonacci substitution and leads to a unified framework in which a variety of models can be studied. We discuss the associated foliations, hyperbolic sets, stable and unstable manifolds, and how the intersections of the stable manifolds with the model-dependent curve of initial conditions allow one to translate dynamical into spectral results.

Hiromi Ei
Tilings associated to the nearest integer complex continued fractions over imaginary quadratic fields

Abstract: The following facts are known: in the case of imaginary quadratic fields, the unique factorization theorem holds only for $\mathbb{Q}(\sqrt{-d})$ with $d = 1, 2, 3, 7,$
$11, 19, 43, 67$ and 163, and the Euclidean algorithm does not work for $d = 19, 43, 67$ and 163. So it is natural to focus on the nearest integer complex continued fractions over $\mathbb{Q}(\sqrt{-d})$ for $d = 1, 2, 3, 7$ and 11. As natural extension domains of maps of these nearest integer complex continued fractions, we obtain tilings with finite prototiles which have fractal boundaries. I will explain about some cases.
In collaboration with Hitoshi Nakada and Rie Natsui

Thomas Fernique
Local rules for planar tilings

Abstract: The cut and project method is one of the prominent method to define quasiperiodic tilings. In order to model quasicrystals, where energetic interactions are only short-range, it is important to know which of these tilings can be characterized by local configurations (in dynamical terms: which of these tiling spaces are of finite type or sofic). In this talk we shall review known results, in particular those obtained these last years with Nicolas Bedaride and Mathieu Sablik.

Uwe Grimm
Autocorrelation and diffraction via renormalisation
Part I: Concepts and examples in one dimension

Abstract: This talk will give an introduction to autocorrelation and diffraction of substitution-based structures, concentrating on the one-dimensional case. Examples with pure point, absolutely continuous and singular continuous diffraction will be discussed. Particular emphasis will be put to the use of exact renormalisation equations for the autocorrelation which follow from the substitution structure, and how these can be exploited to determine the nature of the diffraction measure.

Pierre Guillon

Deterministic and expansive directions in 2D subshifts

Abstract: Many tilings or 2D subshifts have the property that some half-plane *codes* for the whole plane, i.e. any two configurations that coincide in the half-plane are actually equal. The directions of such half-planes are called *symbolically deterministic*. This refines the notion of *expansive direction*. We prove some properties of this set of directions, sketch a proof of realization of some sets by self-simulating partial cellular automata which are close to S-adic systems, and survey some links to group shifts, automorphism groups, Nivat conjecture, tilings by one tile?

Alan Haynes

Bounded remainder sets for rotations on p-adic solenoids

Abstract: Bounded remainder sets for a dynamical system are sets for which the Birkhoff averages of return times differ from the expected values by at most a constant amount. These sets are rare and important objects which have been studied for over 100 years. In the last few years there have been a number of results which culminated in explicit constructions of bounded remainder sets for toral rotations in any dimension, of all possible allowable volumes. In this talk we are going to explain these results, and then explain how to generalize them to give explicit constructions of bounded remainder sets for rotations in p-adic solenoids. Our method of proof will make use of a natural dynamical encoding of patterns in non-Archimedean cut and project sets.

Steve Hurder

Wild solenoids and tilings

Abstract: A minimal action of a finitely generated group G on a Cantor set is said to be wild if it does not satisfy the strongly quasi-analytic (SQA) property. An action of an abelian group is always SQA, while there are examples of minimal free actions of higher rank arithmetic lattices on Cantor sets which are equicontinuous but do not satisfy the SQA property. In this talk, we will introduce the SQA property and derive some of its basic properties, including its algebraic interpretation for Cantor actions defined by group chains, and its meaning for tiling spaces. We discuss the SQA condition in relation to the problem of classifying Cantor actions up to return equivalence, and the study of weak solenoids up to homeomorphism, and construct some examples. The main results of this talk are described in the preprint arXiv:1702.03032

Dong Han Kim

On the higher-dimensional three-distance theorem

Abstract: For a given real number α, let us place the fractional parts of the points $0, \alpha, 2\alpha, \cdots, N\alpha$ on the unit circle. These points partition the unit circle into $N+1$ intervals having at most three lengths, one being the sum of the other two. This is the three distance theorem. We consider a two-dimensional version of the three distance theorem obtained by placing the points $n\alpha + m\beta$, for $0 \leqslant n, m \leqslant N$, on the unit circle. We provide examples of pairs of real numbers (α, β) for which there are finitely many lengths between successive points (with $1, \alpha, \beta$ rationally independent and not badly approximable), as well as examples for which there are infinitely many of them.
This is joint work with Valérie Berthé.

Henna Koivusalo
Cut and project sets, linear repetition of patterns, and the Littlewood conjecture

Abstract: Cut and project sets are, in many senses of the word, regular, but aperiodic point patterns obtained by projecting an irrational slice of the integer lattice to a subspace. A flexible formalism describes how to translate information on Diophantine approximation to regularity properties of cut and project sets. In this talk I explain recent development of the theory: how to quantify the relationship between Diophantine approximation and regularity properties of cut and project sets. In particular, I give an explicit characterization of linearly repetitive cut and project sets; show that existence of certain types of cut and project sets with a very high regularity is, in fact, equivalent to the Littlewood conjecture from Diophantine approximation being false; and explain some ideas for future research.
The talk is based on joined work with Alan Haynes and Jamie Walton.

Sébastien Labbé
On the dynamics of Jeandel-Rao aperiodic tilings

Abstract: We present a definition of Jeandel-Rao aperiodic tilings as the coding of a $\mathbb{Z}^2$-action on the torus. We conjecture that this is a complete characterization.

Paul Mercat
Yet another characterization of the Pisot conjecture

Abstract: In the way of Arnoux-Ito, we give a general geometric criterion for a subshift to be measurably conjugated to a domain exchange and to a translation on a torus. For a subshift coming from an unit Pisot irreducible substitution, we will see that it becomes a simple topological criterion. More precisely, we define a topology on $\mathbb{Z}^d$ for which the subshift has pure discrete spectrum if and only if there exists a domain of the domain exchange on the discrete line that has non-empty interior. We will see how we can compute exactly such interior using regular languages. This gives a way to decide the Pisot conjecture for any example of unit Pisot irreducible substitution.
Joint work with Shigeki Akiyama.

Yasushi Nagai
A generalization of local derivability and its consequences

Abstract: Local derivability and mutual local derivability (MLD) were defined by Baake, Schlottmann and Jarvis and are important concepts in aperiodic order. For example, a Penrose tiling is MLD with a tiling with Robinson triangles. In this talk we generalize local derivability and MLD and discuss the consequences. In particular, we show that several aspects of aperiodic order are understood in a simple manner by this generalization.

Wolfgang Steiner

Recognizability for sequences of morphisms

Abstract: We investigate different notions of recognizability for a free monoid morphism $\sigma : A^* \to B^*$. Full recognizability occurs when each (aperiodic) two-sided sequence over B admits at most one tiling with words $\sigma(a)$, $a \in A$. This is stronger than the classical notion of recognizability of a substitution σ, where the tiling must be compatible with the language of the substitution. We show that if A is a two-letter alphabet, or if the incidence matrix of σ has rank $|A|$, or if σ is permutative, then σ is fully recognizable. Next we investigate the classical notion of recognizability and improve earlier results of Mossé (1992) and Bezuglyi, Kwiatkowski and Medynets (2009), by showing that any substitution is recognizable for aperiodic points in its substitutive shift. Finally we define (eventual) recognizability for sequences of morphisms which define an S-adic shift. We prove that a sequence of morphisms on alphabets of bounded size, such that compositions of consecutive morphisms are growing on all letters, is eventually recognizable for aperiodic points. We provide examples of eventually recognizable, but not recognizable, sequences of morphisms, and sequences of morphisms which are not eventually recognizable. As an application, for a recognizable sequence of morphisms, we obtain an almost everywhere bijective correspondence between the S-adic shift it generates and the measurable Bratteli-Vershik dynamical system that it defines.
This is joint work with Valérie Berthé, Jörg Thuswaldner and Reem Yassawi.