Uncountably Many Ergodic Maximizing Measures for Dense Continuous Functions

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Ergodic Optimization

X compact metric space
$T : X \to X$ continuous map

The aim of ergodic optimization is to describe maximizing measures for a given continuous function ϕ.

Definition 1

$\mu \in \mathcal{M}(X, T)$ is a ϕ-maximizing measure if

$$\sup_{\nu \in \mathcal{M}(X,T)} \int \phi \, d\nu = \int \phi \, d\mu.$$

where $\mathcal{M}(X, T)$ denotes the space of all invariant Borel probability measures

Motivation to study maximizing measures

· Birkhoff's ergodic theorem enables us to interpret maximization of time average as that of space average.
For all continuous function ϕ on X we have

$$\sup_{x\in X} \limsup_{n\to\infty} \frac{1}{n}\sum_{k=0}^{n-1} \phi \circ T^k(x) = \max_{\nu\in\mathcal{M}(X,T)} \int \phi \, d\nu$$

· Maximizing measures appear as the zero temperature limit of equilibrium measures.

$\{\mu_\beta\}$ a sequence of equilibrium measures of $\beta\phi$

$$\frac{1}{\beta}\mathcal{P}(\beta\phi) = \sup\left\{\frac{1}{\beta}h(\mu) + \int \phi \, d\mu : \mu \in \mathcal{M}(X,T)\right\}$$
$$\to \sup\left\{\int \phi \, d\mu : \mu \in \mathcal{M}(X,T)\right\} \quad \text{as } \beta \to \infty$$

Main Theorems

$C(X)$ the space of continuous functions on X with the supremum norm.
$\mathcal{M}_e(X,T)$ the set of all ergodic Borel probability measures on X

Theorem 2 (Jenkinson)

There exists a residual subset $\mathcal{R}$ of $C(X)$ such that for every $\phi \in \mathcal{R}$ there exists a unique ϕ-maximizing measure

Theorem A

Assume T is not uniquely ergodic and $\mathcal{M}_e(X,T)$ is arcwise-connected. Then there exists a dense subset $\mathcal{D}$ of $C(X)$ such that for every $\phi \in \mathcal{D}$ there exist uncountably many ergodic maximizing measures.