Random Matrices and Determinantal Process

February 27 - March 3, 2017

Gernot Akemann: Universality in products of two coupled random matrices: Finite rank perturbations

The singular values of products of independent Gaussian random matrices form a determinantal point process. In the scaling limit of large matrices at the origin the Meijer G-kernel is found. It generalises the Bessel-kernel and its universality is not yet fully understood. We investigate the product of two matrices that are coupled, and where the matrix elements are correlated. Depending on the coupling, we find back the kernel of one or two independent random matrices at the origin, including finite rank perturbations, thus indicating some degree of universality. In a third limit a kernel interpolating between the two is found.
This is joint work with T. Checinski, D.-Z. Liu and E. Strahov.

Jinho Baik: Fluctuations of the free energy of spherical Sherrington-Kirkpatrick model

Spherical Sherrington-Kirkpatrick (SSK) model is an example of disordered systems, called spin glasses. The free energy of 2-spin SSK, which is a random variable, is a finite temperature version of the largest eigenvalue of a random symmetric matrix. We show that the fluctuations of the free energy converge to the GOE Tracy-Widom distribution for the temperatures below a critical value. In the temperatures above the critical value, the limiting distribution is Gaussian. We also consider a SSK plus ferromagnetic Hamiltonian and discuss a relation with spiked random matrices.
This is a joint work with Ji Oon Li.

Gerard Ben Arous: Complexity of high dimensional random landscapes: a phase transition

I will report on recent work on the role of random matrix thery for the understanding the complexity of smooth random functions of many variables and in particular for energy landscapes of spin glasses and of loss functions for deep learning algorithms.

Marco Bertola: The Kontsevich matrix integral and Painlevé hierarchy; rigorous asymptotics and universality at the soft edges of the spectrum in random matrix theory

The Kontsevich integral is a matrix integral (aka "Matrix Airy function") whose logarithm, in the appropriate formal limit, generates the intersection numbers on $\mathcal{M}_{g,n}$. In the same formal limit it is also a particular tau function of the KdV hierarchy; truncation of the times yields thus tau functions of the first Painlevé hierarchy. This, however is a purely formal manipulation that pays no attention to issues of convergence. The talk will try to address two issues: Issue 1: how to make an analytic sense of the convergence of the Kontsevich integral to a tau function for a member of the Painlevé I hierarchy? Which particular solution(s) does it converge to? Where (for which range of the parameters)? Issue 2: it is known that (in fact for any β) the correlation functions of K points in the GUE_β ensemble of size N are dual to the correlation functions of N points in the $GUE_{4/\beta}$ of size K. For $\beta = 2$ they are self-dual. Consider $\beta = 2$: this duality is lost if the matrix model is not Gaussian; however we show that the duality resurfaces in the scaling limit near the edge (soft and hard) of the spectrum. In particular we want to show that the correlation functions of K points near the edge of the spectrum converge to the Kontsevich integral of size K as $N \to \infty$. This line of reasoning was used by Okounkov in the GUE_2 for his "edge of the spectrum model". This is based on joint work with Mattia Cafasso (Angers). Time permitting I will discuss a work-in-progress with G. Ruzza (SISSA) extending these results to the generating function of open intersection numbers.

Philippe Biane: Random Matrix Theory and Representation Theory

Pavel Bleher: Random Matrices and Exact Solution of the Six-Vertex Model with Half-Turn Boundary Conditions

We obtain asymptotic formulas for the partition function of the six-vertex model with half-turn boundary conditions in each of the phase regions. The proof is based on the Izergin–Korepin–Kuperberg determinantal formula for the partition function, its reduction to orthogonal polynomials, and on an asymptotic analysis of the orthogonal polynomials under consideration in the framework of the Riemann–Hilbert approach.
This is a joint work with Karl Liechty.

Tom Claeys: Near-extreme eigenvalues of random matrices and systems of coupled Painlevé II equations

For a wide class of Hermitian random matrices, the limit distribution of the eigenvalues close to the largest one is governed by the Airy kernel determinantal point process. In such ensembles, the limit distribution of the k-th largest eigenvalue can be expressed either in terms of a Fredholm determinant or in terms of the Tracy-Widom formulas involving solutions of the Painlevé II equation. Limit distributions for quantities involving two or more near-extreme eigenvalues, such as the gap between the k-th and the l-th largest eigenvalue, are given in terms of Fredholm determinants with several discontinuities. I will show that these Fredholm determinants have simple expressions in terms of solutions of systems of coupled Painlevé II equations, which are traveling wave reductions of the vector non-linear Schrödinger equation. The talk will be based on joint work in progress with Antoine Doeraene.

Andrey Dymov: A functional limit theorem for the sine-process

It is well-known that a large class of determinantal processes including the sine-process satisfies the Central Limit Theorem. For many dynamical systems satisfying the CLT the Donsker Invariance Principle also takes place. The latter states that, in some appropriate sense, trajectories of the system can be approximated by trajectories of the Brownian motion. I will present results of my joint work with A. Bufetov, where we prove a functional limit theorem for the sine-process, which turns out to be very different from the Donsker Invariance Principle. We show that the anti-derivative of our process can be approximated by the sum of a linear Gaussian process and small independent Gaussian fluctuations whose covariance matrix we compute explicitly.

Maurice Duits: Fluctuations of linear statistics for biorthogonal ensembles

In this talk a recurrence matrix approach to the study of fluctuations of linear statistics for biorthogonal ensembles will be discussed. This approach leads to general criteria for Central Limit Theorems and establishes the universality of GFF correlations for a variety of determinantal point processes. After a review of the subject and an introduction to the approach, special emphasis will be on determinantal point processes on the unit circle and relative Szegö asymptotics for Toeplitz determinants. The talk is based on several works, including joint works with Jonathan Breuer and Rostyslav Kozhan.

Yan V Fyodorov: Exponential number of equilibria and depinning threshold for a directed polymer in a random potential

Using the Kac-Rice approach, we show that the mean number of all possible equilibria of an elastic line (directed polymer), confined in an harmonic well and submitted to a quenched random Gaussian potential grows exponentially with its length L. The growth rate is found to be directly related to the fluctuations of the Lyapunov exponent of an associated Anderson localization problem of a 1-d Schroedinger equation in a random potential. We will discuss analytical results for this rate for strong and weak confinement and apply our results for describing the depinning threshold in presence of an applied force.
The talk is based on joint results with Pierre Le Doussal and Christophe Texier.

Vadim Gorin: Height fluctuations through Schur generating functions

For a large number of stochastic systems of $2d$ statistical mechanics and asymptotic representation theory the smoothed global fluctuations remain finite as the size of the system grows. Their asymptotic is governed by (typically log-correlated) Gaussian fields. I will present a new approach for studying such fluctuations through Schur generating functions of the underlying measures. The approach produces in a unified way the asymptotic theorems for many systems, including random domino and lozenge tilings, non-intersecting random walks, decompositions of tensor products, quantum random walks on irreducible representations.

Adrien Hardy: Determinantal processes in higher dimension and Monte Carlo

In Monte Carlo methods, namely numerical intergration algorithms using random variables, the error made in the approximation is associated with the fluctuations size of the linear statistics in the underling random variables. Using determinantal point processes (DPPs) in this context seems promising: in several class of models, DPPs present much smaller fluctuations than independent random variables, due to the repulsion between the variables. This approach naturally asks for natural classes of DPPs in any dimensions. With this goal in mind, we introduce a class of DPPs associated with multivariate orthogonal polynomials and prove a central limit theorem for the linear statistics. As a consequence, we obtain Monte Carlo estimators where the error term decays faster than in the usual Monte Carlo algorithms. This is a joint work with Rémi Bardenet.

Kurt Johansson: The Airy point process in the two-periodic Aztec diamond

The two-periodic Aztec diamond is a dimer or random tiling model with three phases, solid, liquid and gas. The dimers form a determinantal point process with a somewhat complicated but explicit correlation kernel. I will discuss in some detail how the Airy point process can be found at the liquid-gas boundary by looking at suitable averages of height function differences. The argument is a rather complicated analysis using the cumulant approach and subtle cancellations.
Joint work with Vincent Beffara and Sunil Chhita.

Jon Keating: Extreme value statistics: from random matrices to number theory

I will review some recent developments concerning the extreme value statistics of the characteristic polynomials of random matrices and of the Riemann zeta function, focusing in particular on connections with the extreme value statistics of log-correlated Gaussian fields.

Konstantin Khanin: On point fields related to Airy processes

We shall discuss point fields associated with Airy processes in connection with the problem of KPZ universality. We shall also introduce a new renormalisation procedure for these fields.

Antti Knowles: Spectral radii of sparse random matrices

Abstract: We establish bounds on the spectral radii for a large class of sparse random matrices, which includes the adjacency matrices of inhomogeneous Erdős-Rényi graphs. For the Erdős-Rényi graph $G(n, d/n)$, our results imply that the smallest and second-largest eigenvalues of the adjacency matrix converge to the edges of the support of the asymptotic eigenvalue distribution provided that $d \gg \log n$. This establishes a crossover in the behaviour of the extremal eigenvalues around $d \sim \log n$. Our results also apply to non-Hermitian sparse random matrices, corresponding to adjacency matrices of directed graphs. Joint work with Florent Benaych-Georges and Charles Bordenave.

Mylène Maida: Concentration for Coulomb gases and Coulomb transport inequalities

This is based on a joint work with Djalil Chafaï (Dauphine) and Adrien hardy (Lille). We study the non-asymptotic behavior of Coulomb gases in dimension two and more. Such gases are modeled by an exchangeable Boltzmann-Gibbs measure with a singular two-body interaction. We obtain concentration of measure inequalities for the empirical distribution of such gases around their equilibrium measure, with respect to bounded Lipschitz and Wasserstein distances. Our approach is remarkably simple and bypasses the use of renormalized energy. It relies on new inequalities between probability metrics, including Coulomb transport inequalities which will be presented in the talk.

Nikita Nekrasov: The Magnificent Four

In studying gauge theories in various dimensions one comes across various enumerative problems: counting of holomorphic curves in Calabi-Yau manifolds, intersection theory on moduli spaces of instantons, limit shapes for random Young diagrams in two and three dimensions. The latter problem is related to dimer models and the models of crystal melting. We shall report on the recent developments in four dimensions, the ultimate dimension for crystal melting coming from supersymmetric gauge theory. The string theory context of the problem is the counting the bound states of D0 branes in the presence of the D8 brane and a B-field. The random configurations are tilings of the 3-space by four types of squashed cubes. The similar problem of D0-D6 brane counting led to the partition function which was conjectured in 2004 to be given by the Witten index of 11d supergravity. The conjecture was proven in 2015 by A.Okounkov. I will present the conjecture on the partition function of the new model. It hints at the twelve-dimensional origin of the problem.

Hirofumi Osada: Dynamical universality for random matrices

Universality for random matrices is a topic which has been extensively studied by many researchers recently. In this talk, I present a dynamical counter part of it. That is, I prove that, if the distributions of N-particle systems converge to that of the limit point process strongly and the infinite-dimensional stochastic differential equation (ISDE) describing the limit stochastic dynamics has a unique solution, then the natural reversible dynamics of N-particle systems converge to the solution of the ISDE in the limit. I thus prove the universality of the Dyson interacting Brownian motion in the bulk, Airy interacting Brownian motion at the soft edge, and the Ginibre interacting Brownian motion in two dimensions. These ISDEs are universal as well as the associated point processes such as sine, Airy and Ginibre point processes. To prove this I prepare the general theory for the convergence of N-particle dynamics and the uniqueness for the quasi-regular Dirichlet forms describing the limit stochastic dynamics.

This talk is based on the joint works with Yosuke Kawamoto and Hideki Tanemura.

Yanqi Qiu: Palm equivalence and quasi-symmetries of determinantal point processes associated with Hilbert spaces of holomorphic functions

Two important examples of the determinantal point processes associated with the Hilbert spaces of holomorphic functions are the Ginibre point process and the set of zeros of the Gaussian Analytic Functions on the unit disk. In this talk, I will talk such class of determinantal point processes in greater generality. The main topics concerned are the equivalence of the reduced Palm measures and the quasi-invariance of these point processes under certain natural group action of the group of compactly supported diffeomorphisms of the phase space. This talk is based partly on the joint works with Alexander I. Bufetov and partly on a more recent joint work with Alexander I. Bufetov and Shilei Fan.

Roman Romanov: On reproducing kernel Hilbert spaces related to determinantal processes

We discuss the problem of characterization of reproducing kernel Hilbert spaces arising in the theory of determinantal processes. The goal is to find suitable generalizations of theorems on axiomatic characterization of de Branges spaces and on the existence of the underlying differential equation (inverse problem).

Alexander Shamov: Conditioned determinantal processes are determinantal

A determinantal point process governed by a Hermitian contraction kernel K on a measure space E remains determinantal when conditioned on its configuration on a subset $B \subset E$. Moreover, the conditional kernel can be chosen canonically in a way that is "local" in a non-commutative sense, i.e. invariant under "restriction" to closed subspaces $L^2(B) \subset P \subset L^2(E)$. Using the properties of the canonical conditional kernel we establish a conjecture of Lyons and Peres: if K is a projection then almost surely all functions in its image can be recovered by sampling at the points of the process.
Joint work with Alexander Bufetov and Yanqi Qiu, https://arxiv.org/abs/1612.06751

Tomoyuki Shirai: Determinantal point processes associated with reproducing kernel Hilbert spaces.

Determinantal point processes (DPPs) are associated with reproducing kernel Hilbert spaces. Some of the most important DPPs are the sine process and the Ginibre point process, which correspond to the Paley-Wiener space and Bargmann-Fock space, respectively. In the talk, we introduce two classes of DPPs as their generalizations and discuss their properties such as rigidity and absolute continuity/singularity.

Sasha Sodin: Summability of 1/N expansions

We shall discuss the problem of resummation for 1/N expansions of random matrix theory, and illustrate it by the recent results of Offer Kopelevitch.

Anatoly Vershik: The Absolute of random walks on the groups

Theory of random walk on the groups (mainly discrete) is very well developed area with many interesting new results. The central question is the asymptotic behaviour of the trajectories of Markov process and it boundary (so called Poisson-Furstenberg (PF) boundary and related questions). But there is some general and natural question about more wide boundary of the group G with generators S — which I called absolute $\mathcal{A}(G,S)$ which defined as a set of all Markov processes with the same COtrasition probabilities as a given random walk. The absolute could be nontrivial when PF boundary does. The absolute is tightly related to the set of positive eigenfunctions of Laplasian on the group but n general more complicate that PF-, Martin boundaries. In our paper with A.Malyutin we had found the absolute for free groups and for some nilpotent groups.

Balint Virag: Random matrices and canonical systems

Many models of random matrices, such as beta Hermite ensembles and circular beta ensembles can fit in the world of canonical systems, a theory worked out by de Branges and others in the sixties. I will discuss how this framework helps simplify many old results and yield several new ones.