Homogeneous Spaces, Diophantine Approximation and Stationary Measures

February 6 - 10, 2017

MINI-COURSES

Yves Benoist: Dense subgroups in simple groups.

In this series of lectures, we will focus on simple Lie groups, their dense subgroups and the convolution powers of their measures. In particular, we will dicuss the following two questions. Let G be a Lie group. Is every Borel measurable subgroup of G with maximal Hausdorff dimension equal to the group G? Is the convolution of sufficiently many compactly supported continuous functions on G always continuously differentiable? Even though the answer to these questions is no when G is abelian, the answer is yes when G is simple. This is a joint work with N. de Saxce. First, I will explain the history of these two questions and their interaction. Then, I will relate these questions to spectral gap properties. Finally, I will discuss these spectral gap properties."

Anish Ghosh: Dynamics on homogeneous spaces and Diophantine approximation.

I will discuss approaches to several problems concerning values of linear and quadratic forms using the ergodic theory of group actions on the space of unimodular lattices, and more generally, on homogeneous spaces of semisimple Lie groups.

François Maucourant & Barbara Schapira: Dynamics on quotients of SL(2,C) by discrete subgroups.

We will discuss old and recent results on topological and measurable dynamics of diagonal and unipotent flows on frame bundles and unit tangent bundles over hyperbolic manifolds. The first lectures will be a good introduction to the subject for young researchers.

TALKS

Jayadev Athreya: Variance estimates on spaces of lattices.

We discuss estimates for the variance of the Siegel (or theta) transform on various subsets on the space of lattices, starting with classical work of Rogers, recent work of Kelmer, and finally our recent joint work with Y. Konstantoulas on symplectic lattices.

Yann Bugeaud: Exponents of Diophantine approximation.

For an irrational real number x, let w(x) denote the supremum of the real numbers w such that there are arbitrarily large integers q with $||qx|| < q^{-w}$, where $||\ .\ ||$ denotes the distance to the nearest integer. Let n be a positive integer. This definition can be extended to approximation to x by algebraic numbers of degree at most n, to small values of integer polynomials of degree at most n evaluated at x, or to simultaneous approximation to $x, x^2, ?, x^n$ by rational numbers with the same denominator. Moreover, we may as well introduce uniform exponents of approximation by replacing the requirement 'there are arbitrarily large integers' by 'for all sufficiently large integers' in the above definition. Thus, we have introduced six families of Diophantine exponents. We survey the relations between them and try to determine the sets of values taken by these exponents on the set of real numbers. We formulate several open questions and mention some recent advances.

Nicolas De Saxcé: Approximation diophantienne sur les variétés.

Pour retrouver des résultats de Kleinbock-Margulis et al. sur l'extrémalité des sous-variétés de matrices, et pour répondre à certaines autres conjectures, nous proposerons une nouvelle approche, basée sur le théorème du sous-espace.

Dmitry Kleinbock: Shrinking targets on homogeneous spaces and improving Dirichlet's Theorem.

Optimal results on the improvements to Dirichlet's Theorem are obtained in the one-dimensional case. For simultaneous approximation the problem is open. I will describe reduction of the problem to dynamics both in one-dimensional case (via continued fractions) and for higher dimensions (via diagonal flows on the space of lattices. If time allows I'll mention an inhomogeneous version which is easier than the homogeneous one. Joint work with Nick Wadleigh.

Frédéric Paulin: Counting and equidistribution of integral representations by quadratic norm forms in positive characteristic.

In this talk, we will prove the projective equidistribution of integral representations by quadratic norm forms in positive characteristic, with error terms, and deduce asymptotic counting results of these representations. We use the ergodic theory of lattice actions on Bruhat-Tits trees, and in particular the exponential decay of correlation of the geodesic flow on trees for Hölder variables coming from symbolic dynamics techniques.

Anke Pohl: Dynamical approaches to automorphic functions and resonances, and reduction theories for indefinite quadratic forms.

We report on the current status of a program to develop dynamical approaches to automorphic functions and resonances, and its relation to reduction theories for indefinite quadratic forms.

Uri Shapira: Generalizing Benoist-Quint to homogeneous spaces of non-lattice type.

In this talk I will discuss a joint work with Oliver Sargent in which we attempt to generalize the Benoist-Quint classification of stationary measures to the homogeneous $SL(3,R)-spaceX(2,3)$ of rank-2 discrete subgroups in R^3 identified up to homothety. I will discuss both positive and more surprisingly negative results. On the one hand we show that when the acting measure is Zariski dense in $SL(3,R)$ then there is a unique stationary measure. On the other hand, we show that when the acting measure is $SO(2,1)(Z)$ there are boundary measures hidden in $X(2,3)$ in which integrality manifest itself in a mysterious way. In the talk I will try to emphasis the features of the homogeneous space $X(2,3)$ which allow for such unexpected behaviour (namely, non-discrete stabilizers).

Barak Weiss: Random walks on homogeneous spaces and diophantine approximation on fractals.

We extend results of Y. Benoist and J.-F. Quint concerning random walks on homogeneous spaces of simple Lie groups to the case where the measure defining the random walk generates a semigroup which is not necessarily Zariski dense, but satisfies some expansion properties for the adjoint action. Using these dynamical results, we study Diophantine properties of typical points on some self-similar fractals in $\mathbb{R}^d$. As examples, we show that for any self-similar fractal $\mathbb{K} \subset \mathbb{R}^d$ satisfying the open set condition (for instance any translate or dilate of Cantor's middle thirds set or of a Koch snowflake), almost every point with respect to the natural measure on $\mathbb{K}$ is not badly approximable. Furthermore, almost every point on the fractal is of generic type, which means (in the one-dimensional case) that its continued fraction expansion contains all finite words with the frequencies predicted by the Gauss measure. Joint work with David Simmons.