NEW ADVANCES IN SYMBOLIC DYNAMICS

MINI COURS

Bryna Kra. *Symbolic systems of low complexity*

This series of lectures will focus on shift systems of zero entropy, particularly on the behaviors that arise when placing restrictions on the complexity and contrasting this behavior with that of general shifts. Starting with the simplest case of linear complexity, we discuss various properties of the system, such as the automorphism group and the number of invariant measures. We then discuss how these properties change as the growth rate becomes quadratic, a higher order polynomial, or higher growth, studying both individual automorphisms and properties of the group of all automorphisms.

Ronnie Pavlov. *Entropy and mixing for multidimensional shifts of finite type*

I will speak about multidimensional shifts of finite type and their Gibbs measures. In particular, I will present results about computability of topological entropy/pressure for SFTs and measure-theoretic entropy/pressure of Gibbs measures. I'll focus on various mixing hypotheses, both topological and measure-theoretic, which imply different rates of computability for these objects, and give applications to various systems, including the hard square and Potts models.

EXPOSÉS

Shigeki Akiyama. *On corona limits*

Given a tiling, we study the limit shape of successive coronas of the patches. It is shown that corona limit exists if and only if all directional speeds exist in all directions. If the directional speed are uniform in the sense of location, the corona limit is convex and symmetric with respect to the origin. Finally we see that if the tiling is periodic, all the conditions are satisfied and we see that the corona limit is convex symmetric polyhedron.

Jean-Paul Allouche. *Mock-characters*

We introduce and study a family of functions we call the mock characters. They are completely multiplicative, non-periodic, automatic functions. Examples are Kronecker symbols in the non-periodic case (which does not seem to have been studied). Mock characters satisfy a number of interesting properties, and of all completely multiplicative arithmetic functions seem to come as close as possible to being Dirichlet characters.

Valérie Berthé. *From dual substitutions to Rauzy fractals*

We first recall several applications of the formalism of dual substitutions introduced by P. Arnoux and S. Ito. We then discuss recent developments in the context of S-adic dynamics with applications to pure discrete spectrum and Markov partitions, obtained in collaboration with P. Arnoux, M. Minervino, W. Steiner and J. Thuswaldner.

Anne Bertrand Mathis.

Srecko Brleck. *Almost everything I learned from Sebastien Ferenczi*

The study of palindromes and their generalizations in a word has gained a lot of interest in the last 20 years, motivated by applications in physics, biology, discrete geometry, to name only a few. Using Sebastien Ferenczi as an example, we illustrate the computation of its palindromic complexity and its relation with the usual factor complexity, via an identity attributed to Brlek and Reutenauer involving also the palindromic defect. Periodic infinite words as well as the family of words with language closed by reversal also satisfy the identity. The identity remains valid when palindromic is replaced by σ-palindromic, and we also discuss some other patterns. TBA Sébastien.

Yann Bugeaud. *On the decimal expansion of e*

We show that the block complexity of the decimal expansion of e, viewed as an infinite word over the alphabet $\{0, 1, \ldots, 9\}$, is not too small. This is a joint work with Dong Han Kim.

Jérôme Buzzi. *The Bowen property of some factor maps coding nonuniformly hyperbolic dynamics*

In a joint work with Mike Boyle, we introduced a property of factor maps first used in R. Bowen's classical analysis of Markov partitions. We will explain how it can be used to generalize some classical techniques of symbolic dynamics and improve Sarig's symbolic extensions of surface diffeomorphisms, not only on with respect to measures but also (work in progress) to get lower bounds on the number of periodic points.

Albert Fathi.

Alby Fisher. *Infinite measures for adic transformations*

Extending theorems of Bezuglyi, Kwiatkowski, Medynets and Solomyak, we classify the invariant Borel measures for adic transformations of finite rank which are finite on the path space of some sub-Bratteli diagram. Key ingredients of the proof include an appropriate definition of distinguished eigenvector sequence, a nonstationary Frobenius–Victory theorem, and the notions of adic tower and canonical cover : the measure may be locally infinite on the original space but is always locally finite on the cover space. An application is given to nested circle rotations, where our necessary and sufficient condition for the measure to be infinite is expressed in terms of continued fractions.

(Joint work with Marina Talet)

Hiroshi Fujisaki. *The topological entropy and correlational properties of the discretized Markov β-transformations and their applications*

We have previously defined the discretized Markov transformations and the full-length sequences based on such transformations. We have also defined the topological entropy of the discretized Markov transformations.

In this reserch, we first obtain the topological entropy of the discretized golden mean transformation. We also generalize this result and give the topological entropy of the discretized Markov β-transformations with the alphabet $\Sigma = \{0, 1, \cdots, k-1\}$ and the set $\mathcal{F} = \{(k-1)c, \cdots, (k-1)(k-1)\}$ $(1 \leq c \leq k-1)$ of $(k-c)$ forbidden blocks.

In view of basic properties of the normalized cross- and auto-correlation functions for the de Bruijn sequences that can be regarded as the full-length sequences based on the discretized dyadic transformation, we obtain correlational properties of the full-length sequences based on the discretized golden mean transformation. We generalize this result and give the correlational properties of the discretized Markov β-transformations with the alphabet $\Sigma = \{0, 1, \cdots, k-1\}$ and the set $\mathcal{F} = \{(k-1)(k-1)\}$ of forbidden blocks $(k \geq 2)$. We also apply the generalized result to evaluate the auto-correlation function for the optimum binary spreading sequences of Markov chains based on discretized β-transformations.

Guilhem Gamard. *Coverable words and languages*

This talk deals with right-infinite words. A factor q is said to be a cover of an infinite word w if each position of w belongs to an occurrence of q. An infinite word might have several, or even infinitely many covers. First, we will survey recent results about coverability, most notably that periodic and standard Sturmian words can be characterized in terms of covers.

Then, we will review ongoing work to generalize these results to bi-infinite words, and see how the bi-infinite case is surprisingly different from the right-infinite case. Finally we will give a few open questions and suggest developments to define coverable languages instead of coverable words.

Arek Goetz.

Benjamin Hellouin. *Computing the entropy of mixing tiling spaces*

The entropy of a language is a measure of its complexity and a well-studied dynamical invariant. I consider two related questions : for a given class of languages, can this parameter be computed, and what values can it take ?

In 1D tiling spaces (subshifts) of finite type, we have known how to compute the entropy for 30 years, and the method gives an algebraic characterisation of possible values. In higher dimension, a surprise came in 2007 : not only is the entropy not computable in general, but any upper-semi-computable real number appears as entropy - a weak computational condition. Since then new works have shown that entropy becomes computable again with aditionnal mixing hypotheses. We do not know yet where the border between computable and uncomputable lies.

In this talk, I will explore the case of general subshifts (not of finite type) in any dimension, hoping to shed some light on the finite type case. I relate the computational difficulty of computing the entropy to the difficulty of deciding if a word belongs to the language. I exhibit a threshold in the mixing rate where the difficulty of the problem jumps suddenly, the very phenomenon that is expected in the finite type case.

This is a joint work with Silvère Gangloff and Cristobal Rojas.

Pascal Hubert. *Exemple d'Arnoux-Yoccoz, fractal de Rauzy, problème de Novikov : brins d'une guirlande éternelle*

Teturo Kamae. *Pattern recognition and complexity*

Olena Karpel. *Ergodic invariant measures for finite rank Bratteli diagrams*

The main goal of this talk is to give an explicit description of the set of all invariant probability measures on a Bratteli diagram $B = (V, E)$ of finite rank k. This set is a simplex $\Delta(B)$ with l vertices, where l is some number between 1 and k. The vertices of $\Delta(B)$ correspond to the ergodic invariant probability measures on B. We determine the vertices of $\Delta(B)$ in terms of the incidence matrices of B. This is a joint work with S. Bezuglyi and J. Kwiatkowski.

Michal Kupsa. *On the partitions with Sturmian-like refinements*

In the dynamics of a rotation of the unit circle by an irrational angle $\alpha \in (0,1)$, we study the evolution of partitions whose elements are not intervals, but unions of intervals. We show that if the end-points of these intervals belong to the past trajectory of the point 0, then the refinements of the partition eventually coincide with the refinements of the standard "Sturmian" partition, which consists of two intervals $[0, 1-\alpha)$ and $[1-\alpha, 1)$. It means that even though we start with the partition that divides the circle into disconnected sets, its dynamical refinements eventually consist of intervals, i.e. connected sets. We reformulate this result into the language of symbolic dynamics as a "finite level" injectivity of sliding-block codes from Sturmian subshifts. This is a joint work with Stěpán Starosta.

Martha Łącka. *The Weyl pseudometric and the Krieger Theorem*

During the talk we will present some consequences of the convergence with respect to the Weyl pseudometric in dynamical systems generated by an amenable group actions. This will lead us to the alternative proof of the Krieger theorem, which says that for any number between 0 and $\log k$ one can find a Toeplitz shift over a k-letter alphabet with entropy equal to this number. The talk is based on a joint work with Marta Pietrzyk.

Ali Messaoudi. *Dynamics of of adding machines*

In this lecture, we will study dynamical properties of infinite matrices associated to countable Markov chains. In particular, we will prove that a large class of these matrices are frequently hypercyclic and Devaney chaotic.

Aleksey Minabutdinov. *Limiting curves for a class of self-similar adic transformations*

We prove the existence of limiting curves resulting from deviations of partial sums in the ergodic theorem in the case of polynomial adic systems (generalized odometers) and cylindrical functions. These systems are related to self-similar graphs. For a general ergodic measure-preserving transformation and a summable function we give a necessary condition for a limiting curve to exist. Our work generalizes results by É. Janvresse, T. de la Rue and Y. Velenik and answers several questions from their work.

Thierry Monteil. *On a conjecture of Sebastien*

As a motivating question for the week, we will discuss a conjecture of Sebastien about the codings of 2-dimensional rotations.

Kyewon Park. *Z^2-subshifts and their directional complexities*

Topological entropy dimension was introduced to measure the complexities of entropy zero systems. In particular it classifies the systems of subexponential growth rate. We define the directional entropy dimension together with entropy dimension to study the properties of Z^2-actions. We present stirictly ergodic Z^2-subshifts of positive entropy dimension with diverse directional complexities.

Martine Queffelec. *Sébastien's first steps*

Sébastien Férenczi started his research with the "ergodic theory" team of Paris VI, in the beginning of the eighties. I shall give a survey of his first results, centered on low complexity systems, finite rank systems and connections between them. Those results contributed efficiently to future developments in symbolic dynamics.

Filipp Rukhovich. *Outer billiards : aperiodic points outside regular polygons*

For any convex figure, outer billiard map T can be defined as following. Let A be a figure and x be a point outside it. There are two tangent lines to A containing x ; choose one of them, say "right" ; let y be point of tangency. Then, $T(x)$ is a point so that y is the middle of segment connecting x and $T(x)$.

In case when A is a polygon, the set of points outside A can be divided into three sets : 1) set of points with "finite" points, i.e. points so that $T(T(...(T(x))...))$ is not defined for some iteration of T ; 2) set of periodic points $(T(T(T(...(T(x))...))) = x$ for some number of iterations ; 3) set of aperiodic points.

It can be easily proved that outer billiards outside triangle, square and regular hexagon do not have aperiodic points. In 1993, S.Tabachnikov proved that outer billiard outside regular pentagon has such points, and set of aperiodic points has fractal structure. We shall discuss a computer proof of the existence of aperiodic points for outer billards outside regular octagon and dodecagon.

Tom Schmidt. *α-Deformations for an infinite class of continued fractions*

In 1981, H. Nakada defined his α-continued fractions, $T_\alpha : [(\alpha - 1), \alpha] \to [(\alpha - 1), \alpha)$ for $\alpha \in [0, 1]$ such that $\alpha = 1$ is the Gauss (continued fractions) map, $x \mapsto 1/x - \lfloor 1/x \rfloor$. This family has been studied by numerous authors ; with Arnoux, we showed that each T_α is a factor of the Poincare return map to a cross-section for the geodesic flow on $T^1(\text{PSL}_2(\mathbb{Z}))$, the unit tangent bundle of the modular surface.

With Calta and Kraaikamp, we define and study α-type maps $T_{n,\alpha}$ for each of a countable number of (Fuchsian triangle) groups (thus playing the role of $\mathrm{PSL}_2(\mathbb{Z})$) ; these groups are defined over number fields (whose degree is unbounded). We show that the key property of orbit synchronization of the endpoints of the interval of definition holds for each n on a set of α of full measure, and identify cross-sections for these $T_{n,\alpha}$.

Asaki Saito. *Continued fraction algorithms and Lagrange's theorem in* $\mathbb{Q}_p$

We give several continued fraction algorithms, each of which generates a periodic expansion for every quadratic element of $\mathbb{Q}_p$ over $\mathbb{Q}$ and generates a finite expansion for every rational number. This is a joint work with Junichi Tamura and Shin-ichi Yasutomi.

Jun-Ichi Tamura. *Rauzy fractals in the p-adic world*

We introduce p-adic substitutions, i.e., substitutions over $\mathbb{C}_p$-powered symbols. Here, $\mathbb{C}_p$ is the closure of $\mathbb{R}_p$ with respect to the p-adic topology, $\mathbb{R}_p$ is the the algebraic closure of $\mathbb{Q}_p$, which is the closure of the rational number field $\mathbb{Q}$ with respect to the p-adic topology. We proved a norm theorem which completely describes the distribution of p-adic absolute value of the zeros of a given polynomial in $\mathbb{C}_p[x]$ in terms of the coefficients. As an application of the norm theorem, we can show some convergence theorems for certain p-adic substitutions.

Under convergence results, we can define the Rauzy fractal for a p-adic substitution in the vector space $\mathbb{C}_p^d$. Making visualization, we show some examples of Rauzy fractals.

Ren Yi. *The Triple Lattice PETs*

Polytope exchange transformations (PETs) are higher dimensional generalizations of interval exchange transformations (IETs) which have been well-studied for more than 40 years. A general method of constructing PETs based on multigraphs was described by R. Schwartz in 2013. The triple lattice PETs are examples of multigraph PETs. In this talk, I will show that there exists a renormalization scheme of the triple lattice PETs in the interval (0,1). I will discuss the properties of the limit set with respect to the golden ratio. Pictures and computer simulations will be presented in the talk.