

Mini-courses

Olivier Benoist

Compactified moduli of canonically polarized varieties

Abstract: The moduli space of stable curves of Deligne and Mumford is a compactification of the moduli space of smooth curves of genus ≥ 2 parametrizing certain nodal curves. It has proven to be a powerful tool to study algebraic curves. Analogues in higher dimension have been constructed by Kollár, Shepherd-Barron and Alexeev in dimension 2, by Viehweg when restricting to smooth varieties, and the general case has been settled very recently (notably thanks to the development of the minimal model program, and to the work of Kollár). We will give an introduction to the construction of these moduli spaces, with an emphasis on the class of singularities that appears in this context: semi-log-canonical singularities.

References:

Kovács, Young person's guide to moduli of higher-dimensional varieties.
Kollár, Singularities of the minimal model program
Kollár, Families of varieties of general type, in preparation, available at
<https://web.math.princeton.edu/~kollar/book/modbook20170720.pdf>

Jean Fasel

Cohomological classification of vector bundles over smooth affine varieties

Abstract: In this series of lectures, we are going to explain how recent results from Morel, Schlichting and Asok-Hoyois-Wendt allow in good cases to classify vector bundles on smooth affine varieties over a (sufficiently nice) field using cohomological methods. Instead of giving complete proofs, we will rather focus on the general philosophy and provide a toolbox for this classification. Time permitting, we will perform a few computations in the case of smooth affine complex threefolds as an illustration of the techniques.

References:

We will rely on the primer written by Antieau and Elmanto (arXiv:1605.00929), as well as on the paper of Asok-Hoyois-Wendt (arXiv:1507:08020).

Frank Loray

Transversely projective foliations

Abstract: A transversely projective foliation is a codimension one foliation whose transversals locally identify with the projective line in a natural way. We will first explain the classical definition, in the smooth case, starting from the notion of projective structures on curves, and end with the setting of singular holomorphic foliations. We will explain the link with flat meromorphic connections of rank two. We will end with a structure theorem for both connections and foliations, mostly relying on a result of K. Corlette et C. Simpson.

This is a joint work with J. V. Pereira and F. Touzet.

Reference:

F. Loray, J. V. Pereira and F. Touzet : Representations of quasi-projective groups, flat connections and transversely projective foliations. J.c. polytech. Math. 3 (2016) 263-308. free access : http://jep.cedram.org/jep-bin/fitem?id=JEP_2016_3_263_0

Alessandra Sarti

Automorphisms of Hyperkähler manifolds

Abstract: In the 80's Beauville generalized several foundational results of Nikulin on automorphism groups of K3 surfaces to hyperkähler manifolds. Since then the study of automorphism groups of hyperkähler manifolds and in particular of hyperkähler fourfolds got very much attention. I will present some classification results for automorphisms on hyperkähler fourfolds that are deformation equivalent to the Hilbert scheme of two points on a K3 surface and describe some explicit examples. I will give particular attention to double EPW sextics, that admit in a natural way a non-symplectic involution. Time permitting I will show how the rich geometry of double EPW sextics has an important connection to a classical question of U. Morin (1930).

References:

Boissière, Samuel; Camere, Chiara; Sarti, Alessandra. Classification of automorphisms on a deformation family of hyper-Kähler four-folds by p-elementary lattices. Kyoto J. Math. 56 (2016), no. 3, 465-499.

Boissière, Samuel; Cattaneo, Andrea; Nieper-Wisskirchen, Marc; Sarti, Alessandra; The automorphism group of the Hilbert scheme of two points on a generic projective K3 surface. K3 surfaces and their moduli, 1-15, Progr. Math., 315, Birkhäuser/Springer, [Cham], 2016.

O'Grady, Kieran G. Pairwise incident planes and hyperkähler four-folds. (English summary) A celebration of algebraic geometry, 553-566, Clay Math. Proc., 18, Amer. Math. Soc., Providence, RI, 2013.

Donten-Bury, Maria; van Geemen, Bert; Kapustka, Grzegorz; Kapustka, Michał; Wiśniewski, Jarosław A. A very special EPW sextic and two IHS fourfolds. (English summary) Geom. Topol. 21 (2017), no. 2, 1179-1230.

Carlos Simpson

Large Geometry of compactifications of representation varieties

Abstract: The goal will be to understand the geometrical structures present in the compactifications of moduli varieties of representations, Higgs bundles and vector bundles with connection, over curves. The primordial example is that of \mathbb{P}^1 minus 4 points, where the character variety M_{Betti} is an affine cubic surface whose configuration at infinity is a triangle of lines. We will see how to compare this structure with the structures of the Hitchin moduli space and the moduli space of vector bundles with connection. We'll also talk about WKB asymptotics of systems of ODE's and the relationship with harmonic mappings to buildings, leading to the Gaiotto-Moore-Neitzke spectral networks. Several active current research directions in the study of these structures will be presented.

Talks

Federico Lo Bianco

Une application de l'intégration p-adique à la dynamique d'un automorphisme préservant une fibration

Abstract: Une partie de la preuve de l'alternative de Tits pour les sous-groupes finiment engendrés de $\mathrm{PGL}_n(\mathbb{C})$ consiste à injecter un tel groupe Γ dans $\mathrm{PGL}(\mathbb{Q}_p)$ (pour un p bien choisi) et en tirer des informations sur les éléments de Γ . Dans cet exposé je vais rappeler ce résultat et expliquer comment le même genre d'idée, à l'aide d'éléments d'intégration p-adique, peut servir à décrire la dynamique d'un automorphisme (ou, plus généralement, d'une transformation birationnelle) préservant une fibration. Ceci s'applique en particulier au cas des variétés symplectiques irréductibles (ou hyperkähleriennes).

Vladimiro Benedetti

Orbital degeneracy loci

Abstract: I will present a joint work with Sara Angela Filippini, Laurent Manivel and Fabio Tanturri (arXiv: 1704.01436). We introduce a new class of varieties, called orbital degeneracy loci. The idea is to use any orbit closure in a representation of an algebraic group to generalise the classical construction of degeneracy loci of morphisms between vector bundles, and of zero loci as well. After giving the definition of an orbital degeneracy locus, I will explain how to control the canonical bundle of these varieties: under some Gorenstein condition on the orbit closure, it is possible to construct examples of varieties with trivial canonical bundle or of Fano type. Finally, if time will permit, I will give some explicit examples of such degeneracy loci, which allow to construct many Calabi-Yau varieties of dimension three and four, and some new Fano fourfolds.

Yohan Brunebarbe

Hyperbolicity of varieties supporting a variation of Hodge structure

Abstract: Complex algebraic varieties supporting a non-trivial variation of Hodge structure tend to have a hyperbolic behavior, at least in the directions in which the period map is immersive. This applies in particular to many moduli spaces, including moduli spaces of polarized abelian varieties, K3 surfaces and more generally varieties with trivial canonical bundle. We will present a new approach to this phenomenon using singular metrics, which both simplify the former approaches and extend their range of applicability. In particular, whereas the preceding approaches use difficult results about degenerations of Hodge structure, the proof that we will explain uses only the special curvature properties of period domains.

This is joint work with Benoît Cadorel.

Junyan Cao

Un théorème de décomposition des variétés à fibré anticanonique nef

Abstract: Soit X une variété projective lisse à fibré anticanonique nef. En étudiant l'application d'Albanese et l'application MRC, on montre que son revêtement universel \tilde{X} se décompose comme $\tilde{X} = \mathbb{C}^n \times Y \times Z$, où Y est une variété projective de $c_1(Y) = 0$ et Z est une variété rationnellement connexe.

Il s'agit d'un travail en commun avec Andreas Höring.

Sara Angela Filippini

Stability data, irregular connections and tropical curves

Abstract: I will outline the construction of isomonodromic families of irregular meromorphic connections on \mathbb{P}^1 with values in the derivations of a class of infinite-dimensional Poisson algebras, and describe two of their scaling limits. In the “onformal limi” we recover a version of the connections introduced by Bridgeland and Toledano-Laredo, while in the “large complex structure limit” the connections relate to tropical curves in the plane and, through work of Gross, Pandharipande and Siebert, to tropical/GW invariants.

This is joint work with M. Garcia-Fernandez and J. Stoppa.

Victoria Hoskins

Group actions on quiver moduli spaces and applications

Abstract: We study two types of actions on King’s moduli spaces of quiver representations over a field k , and we decompose their fixed loci using group cohomology in order to give modular interpretations of the components. The first type of action arises by considering finite groups of quiver automorphisms. The second is the absolute Galois group of a perfect field k acting on the points of this quiver moduli space valued in an algebraic closure of k ; the fixed locus is the set of k -rational points, which we decompose using the Brauer group of k , and we describe the rational points as quiver representations over central division algebras over k . Over the field of complex numbers, we describe the symplectic and holomorphic geometry of these fixed loci in hyperkaehler quiver varieties using the language of branes. Over the reals, the rational points of these quiver moduli spaces come from either real or quaternionic quiver representations, and we compute the Poincaré polynomials of both components.

This is joint work with Florent Schaffhauser.

Arvid Perego

Kählerianité des espaces de modules de faisceaux stables sur les surfaces K3 non-projectives

Abstract: Un espace de module M de faisceaux H –stables sur une surface K3 S est une variété holomorphiquement symplectique, qui s’avère être compacte et connexe dès qu’on choisit de façon appropriée le caractère de Chern des faisceaux et la polarisation H . Dans un travail récent en collaboration avec M. Toma on a montré que le même choix nous permet de conclure que M est aussi équivalent par déformation à un schéma de Hilbert de points sur une surface K3 projective, et que si on suppose en plus l’existence d’une métrique kählerienne sur M , alors il est une variété hyperkählerienne irréductible (fait déjà connu lorsque la surface S est projective). L’existence d’une telle métrique kählerienne est conjecturée, et démontrée dans certains cas particuliers. Dans cet exposé on montrera qu’une métrique kählerienne existe sur M si et seulement si le deuxième nombre de Betti de M est égal à la somme des nombres de Hodge $h^{2,0}(M)$, $h^{1,1}(M)$ et $h^{0,2}(M)$.

Xavier Roulleau

Irrationality of cubic threefolds by their reduction mod 3

Abstract: A smooth cubic threefold X is unirational: there exists a dominant rational map $f : \mathbb{P}^3 \rightarrow X$. Clemens and Griffiths proved that X (over the complex field) is irrational, i.e. the degree of such f is always > 1 . That was the first counter-example to the Roth Conjecture. The difficult part of their proof was to show that the intermediate Jacobian of X is the Jacobian of a curve. In this talk we will prove that result anew for a generic cubic, by elementary methods: reduction mod p and point counting.

It is a joint work with Dimitri Markouchevitch.
