**Collisionless Boltzmann (Vlasov) equation and modeling of self-gravitating systems and plasmas**
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# Hidden drifts & turbulence evolution

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# *Outline*

***Plasma turbulence described by Vlasov eq. for the distribution functions***

$$\partial_t f^\alpha + \mathbf{v}\cdot\nabla f^\alpha + v_z \partial_z f^\alpha = 0,$$

$$\mathbf{B} = B\mathbf{e}_z, \quad \phi = -\frac{\phi_e}{B}, \quad \mathbf{v} = \nabla\phi\times\mathbf{e}_\mathbf{z}$$

***Fluid turbulence described by 2D Euler eq. for the vorticity***

$$\partial_t \omega + \mathbf{v}\cdot\nabla\omega = 0,$$

$$\mathbf{v} = \nabla\phi\times\mathbf{e}_\mathbf{z}$$

***Different nonlinearities:***

$$\delta n_i(\mathbf{x}, z, t) = \delta n_e(\mathbf{x}, z, t)$$

$$\omega(\mathbf{x}, t) = -\Delta\phi(\mathbf{x}, t)$$

***The same equations for the characteristics:***

$$\frac{dx_1}{dt} = \partial_2\phi(\mathbf{x}, z, t)$$

$$\frac{dx_2}{dt} = -\partial_1\phi(\mathbf{x}, z, t)$$

$$\frac{dz}{dt} = v_z$$

Motion of ion guiding centers

$$\frac{dx_1}{dt} = \partial_2\phi(\mathbf{x}, t)$$

$$\frac{dx_2}{dt} = -\partial_1\phi(\mathbf{x}, t)$$

Motion of vorticity elements

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***Hidden drifts***

***are found in these test particle trajectories***

## *Outline*

***Plasma turbulence described by Vlasov eq. for the distribution functions***

$$\partial_t f^{\alpha} + \mathbf{v}\cdot\nabla f^{\alpha}$$

$$\mathbf{B} = B\mathbf{e}_z, \qquad \mathbf{v} =$$

***Fluid turbulence described by 2D Euler eq. for the vorticity***

$$\mathbf{v}\cdot\nabla\omega = 0,$$

***The effects of hidden drifts on turbulence evolution***

***Different nonlinearities:***

$$\delta n_i(\mathbf{x}, z, t) = \delta n_e(\mathbf{x}, z, t)$$

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*Content*

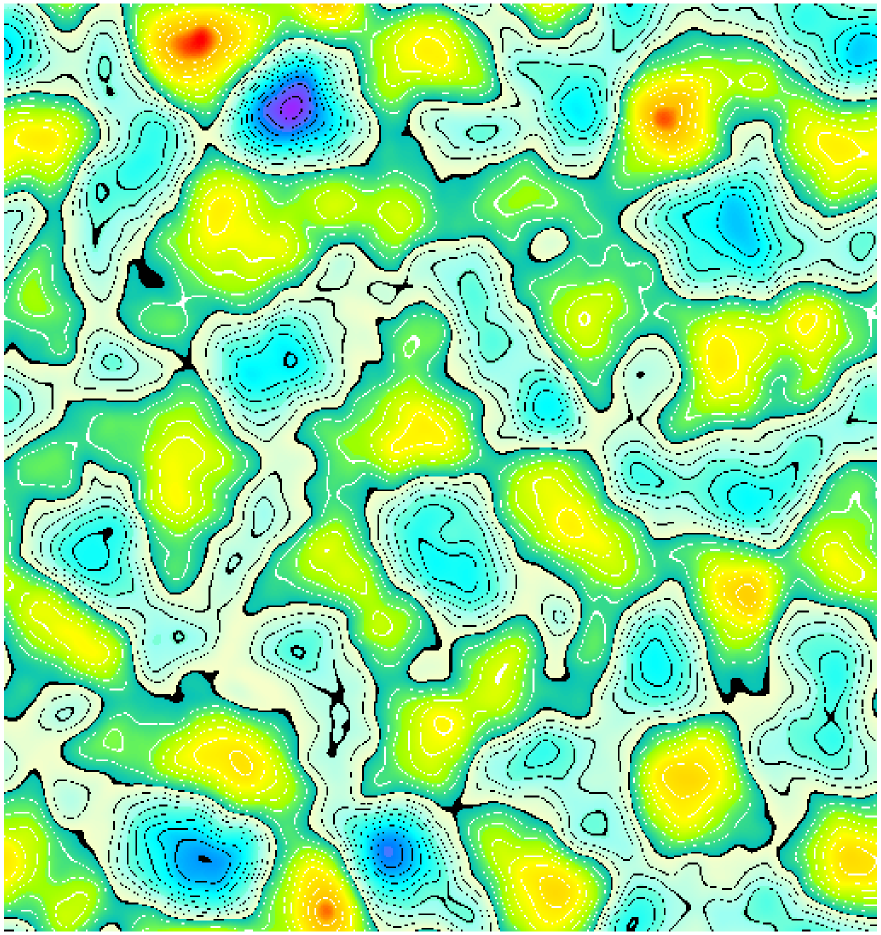
- ***1. Trajectory trapping or eddying in stochastic potential fields; random and quasi-coherent aspects in the statistics of trajectories***
- ***2. The statistical approach***
- ***3. Hidden drifts***
- ***4. Effects of the hidden drifts on plasma turbulence***
- ***5. Effects of hidden drifts on fluid turbulence***
- ***6. Conclusions***

# ➢ 1. Trajectory trapping or eddying in stochastic potential fields

The stochastic velocity is represented by a potential in 2d incompressible case

$$\phi(\vec{x},t)$$

$$\vec{v}(\vec{x},t) = \left( \frac{\partial}{\partial x_2}, -\frac{\partial}{\partial x_1} \right) \phi(\vec{x},t)$$



***Hamiltonian system of equations***

$$\frac{dx_i}{dt} = \varepsilon_{ij} \partial_j \phi(\vec{x},t)$$

- For static potential with $\tau \to \infty$
  the trajectories are closed and periodic
(situated on the contour lines of φ)

- For slow time variation of the potential, the trajectories are almost closed for long time intervals

***The Eulerian correlations (EC) of the potential and of the velocity:***

$$E(\vec{x},t) \equiv \left\langle \varphi(0,0)\,\varphi(\vec{x},t) \right\rangle = \Phi^2 f\left( \frac{\vec{x}}{\lambda}, \frac{t}{\tau} \right)$$

$$f(0,0) = 1, \quad f \to 0, \left|\vec{x}\right| >> \lambda, t >> \tau$$

$$E_{ij}(\vec{x},t) \equiv \left\langle v_i(0,0)\, v_j(\vec{x},t) \right\rangle,$$

$$E_{11} = -\partial_2 \partial_2 E, \; E_{22} = -\partial_1 \partial_1 E \qquad V = \frac{\Phi}{\lambda}$$

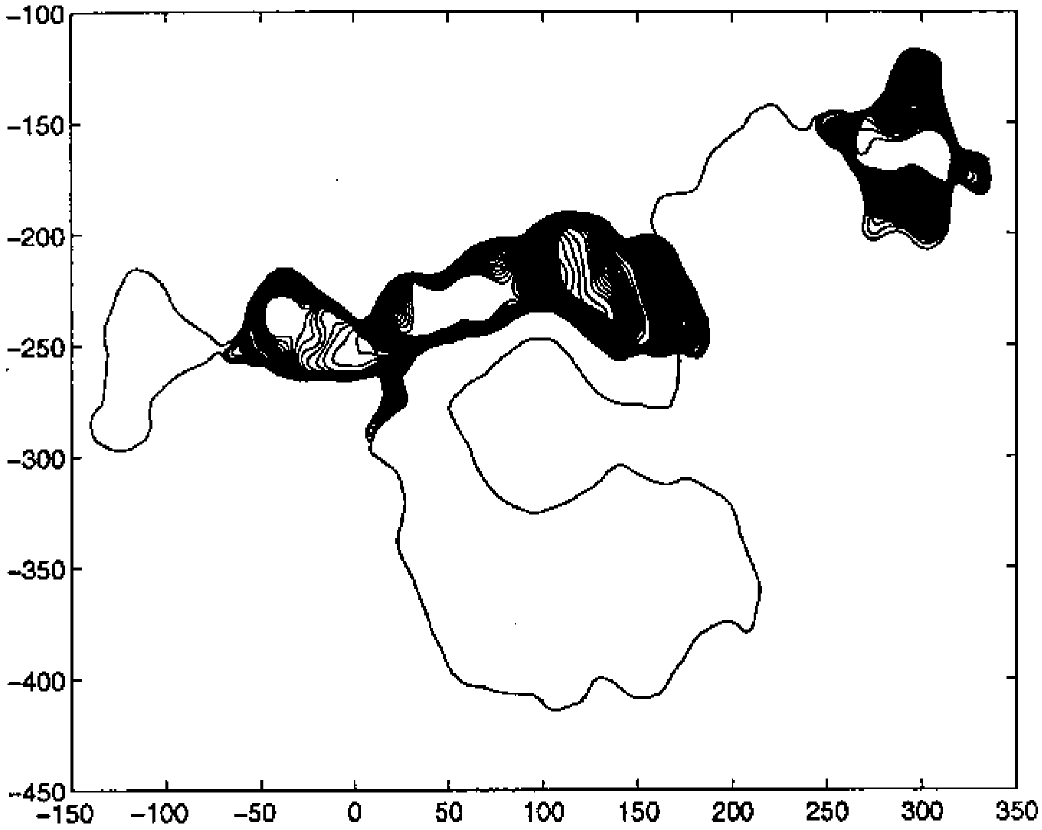
- **V** is the amplitude
- **λ** is the correlation length
- **τ** is the correlation time

***The time of flight:***

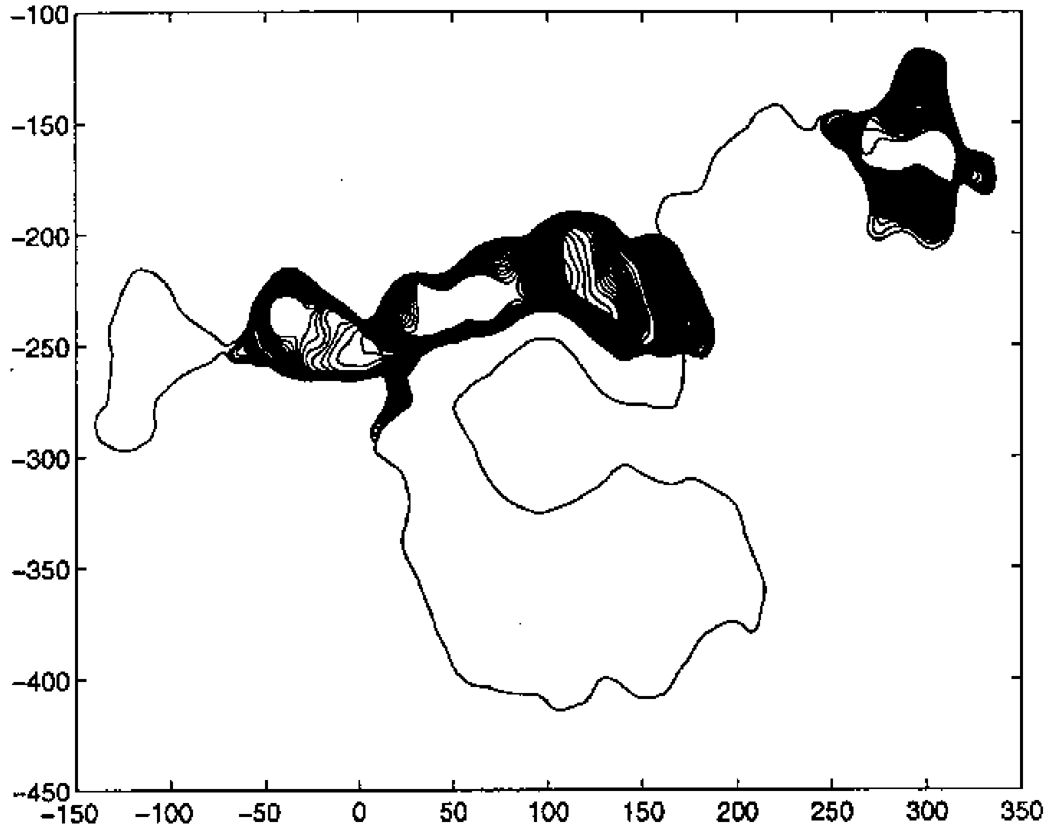
$$\boxed{\tau_{fl} = \frac{\lambda}{V} = \frac{\lambda^2}{\Phi}}$$

**The Kubo number :**

$$\boxed{K = \frac{\tau}{\tau_{fl}} = \frac{V\tau}{\lambda}}$$

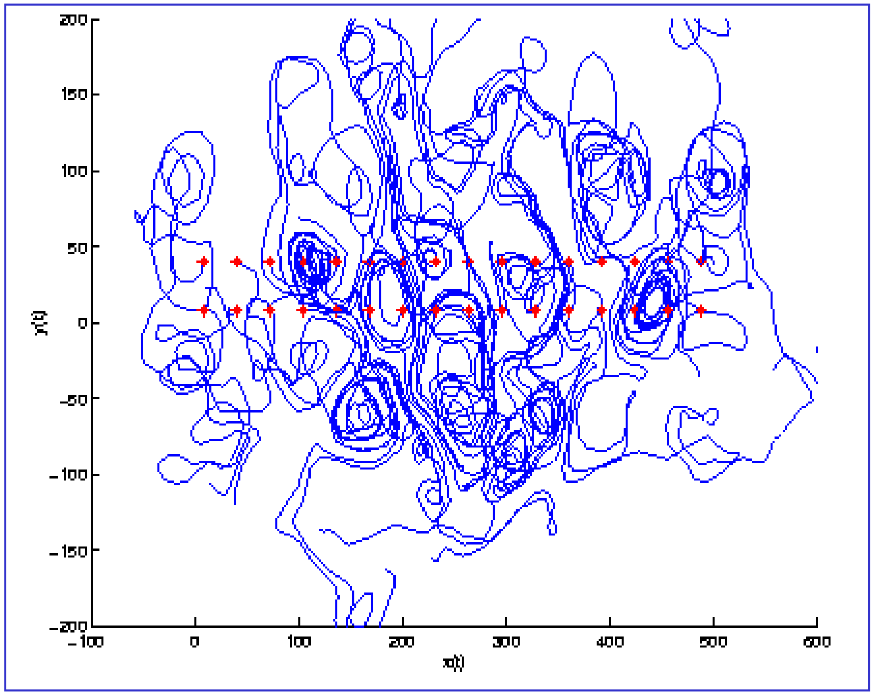



***A typical trajectory in 2-d turbulence***
***(incompressible fluids, magnetized plasmas)***
***is a stochastic sequence of trapping events and long jumps***

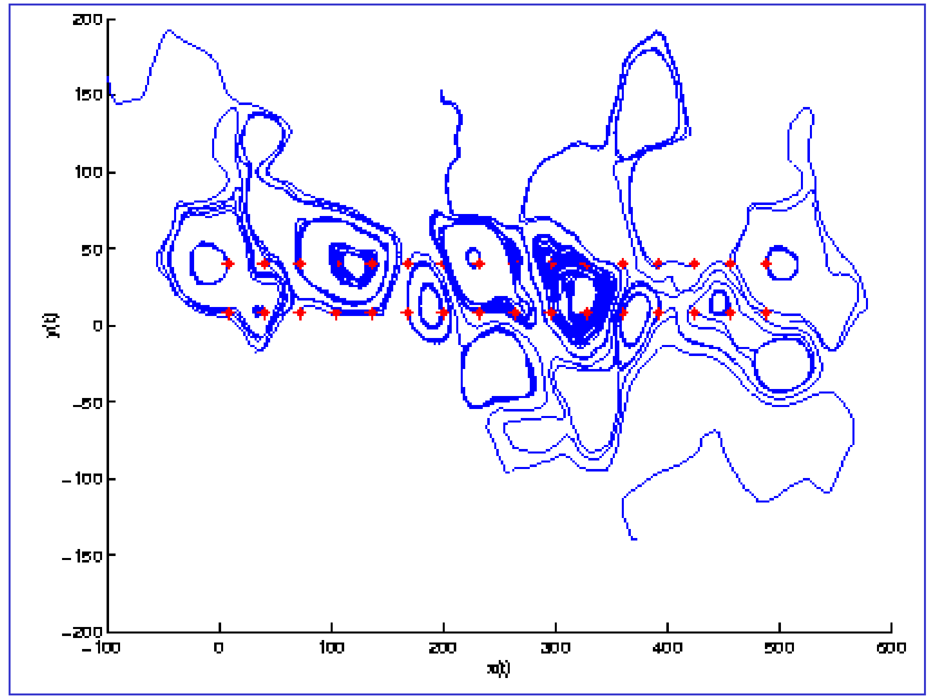



*Trapping is quasi-coherent motion.*

*The long jumps are random.*




$K = 1$

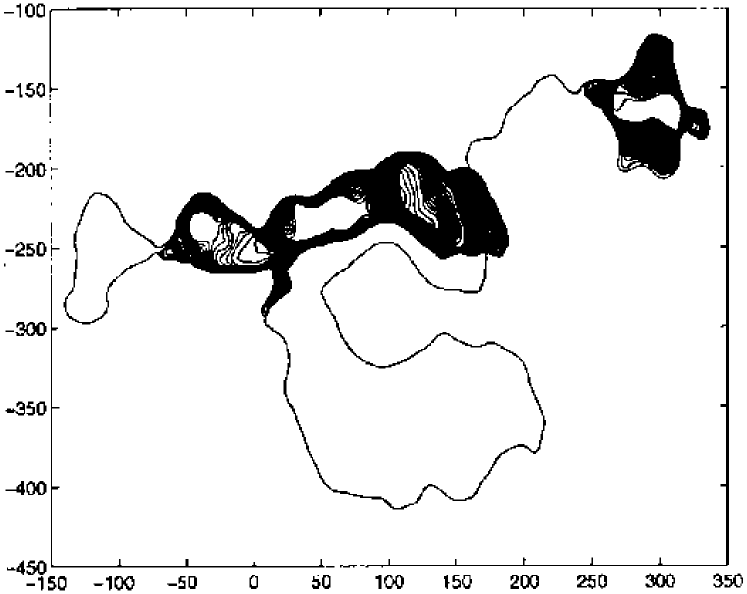



$K = 10$

Numerical simulations show that

- Trapping appears at $K \approx 1$ and the fraction of trapped trajectories increases with K
- Trapping is generic. It affects a part of the trajectories at small time and, in a along time, each trajectory is trapped during a fraction of this time.
- The trapping events are multi-size
- ***Trapping determines the coherent aspects in particle motion***

## ➢ 2. The statistical approach



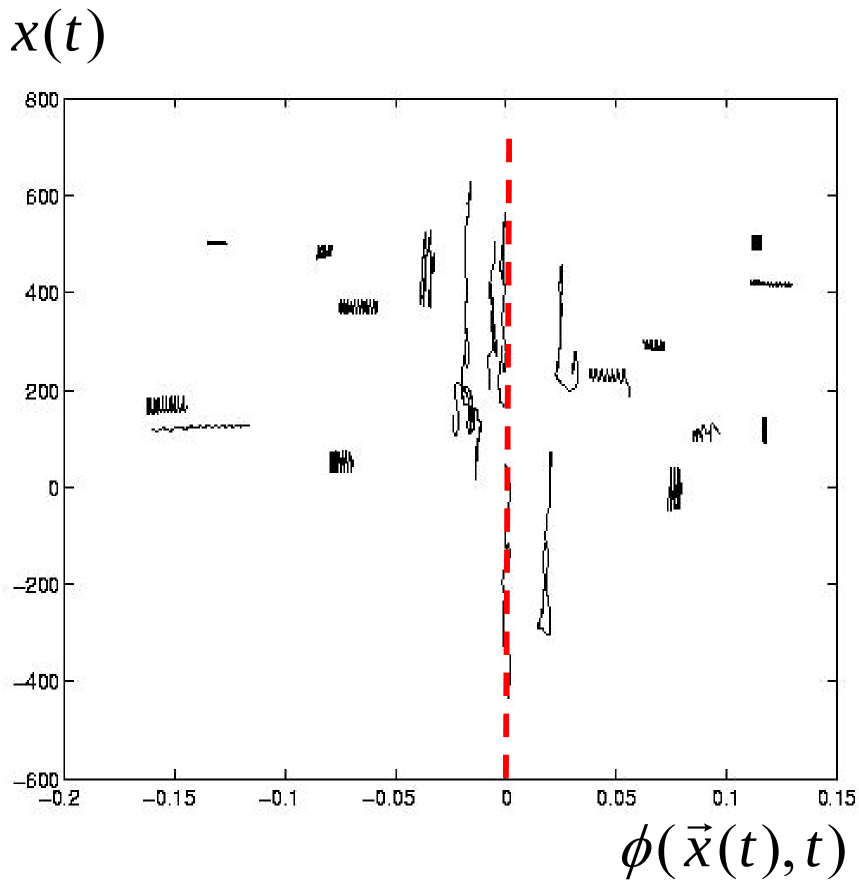

***Trapping*** is due to the ***Hamiltonian motion*** determined by 2-dimensional, zero-divergence velocity fields. The potential (stream function) is invariant for time independent fields.

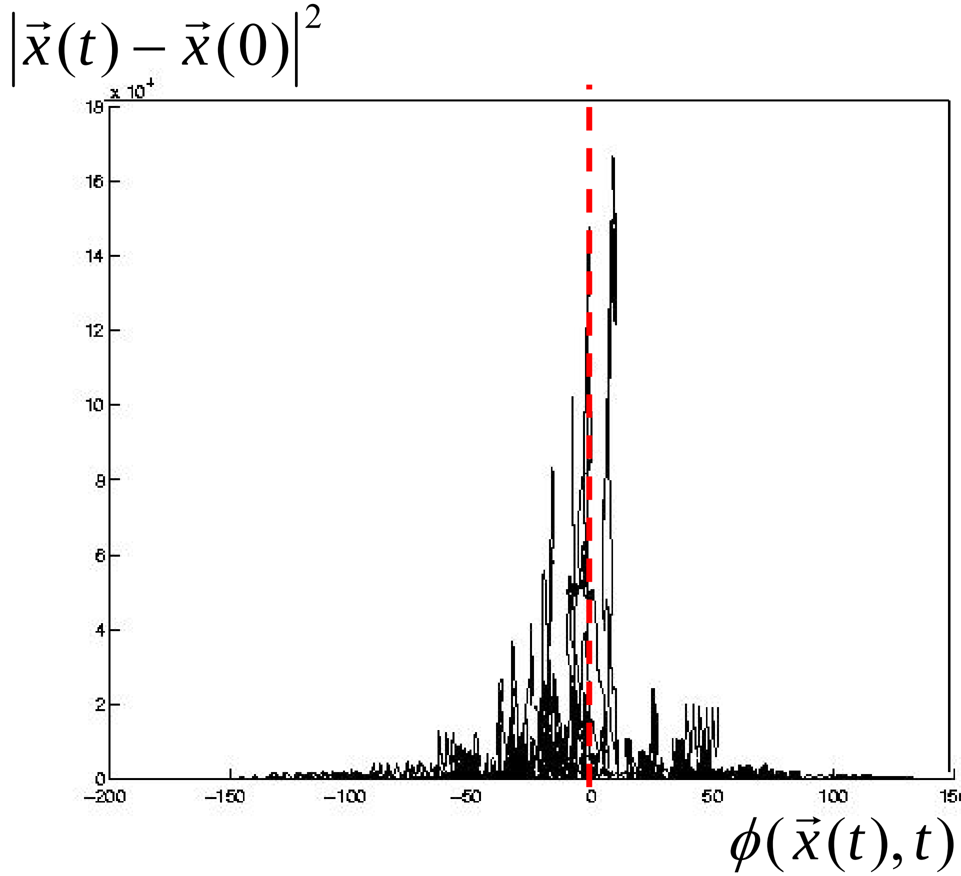
***Main requirements for the theoretical methods:***

- To be in agreement with the statistical constraints imposed by the invariance of the potential
  (Corrsin approximation, DIA, renormalization group technique are not)

- To describe both the random and the quasi-coherent characteristics of the trajectories

- long jumps appear when the trajectory is at $\phi(\vec{x}(t),t) \cong 0$
- trapping appears on the contour lines with large $\left|\phi(\vec{x}(t),t)\right|$




The segments of trajectories that correspond to similar values of the potential are not much different

## ***Semi-analytical statistical methods***

### ❑ ***The decorrelation trajectory method (DTM)***

M.Vlad, F. Spineanu, J. H. Misguich, R. Balescu, “Diffusion with intrinsic trapping in 2-d incompressible stochastic velocity fields”, **Physical Review E 58** (1998) 7359

### ❑ ***The nested subensemble method (NSM)***

M. Vlad, F. Spineanu, “Trajectory structures and transport”, **Physical Review E 70** (2004) 056304(14))

DTM and NSM are based on a set of simple trajectories determined from the Eulerian correlation EC of the stochastic potential,
***the decorrelation trajectories***

***The main idea*** of this approach is to determine the Lagrangian averages not on the whole set of trajectories but *to group together trajectories that are similar*, to average on them and then to perform averages of these averages.

- Similar trajectories are obtained by imposing supplementary initial conditions besides the necessary one.
- Particularly important initial conditions are provided by the ***conserved quantities*** (the potential in this case). The value of the initial potential determines *the average path of the trajectories*.
- **Subensembles S** of realizations of the stochastic potential and ***conditional averages*** are used in order to impose the initial conditions
- ***The decorrelation trajectories (DT)*** are obtained from the conditional average potential by an equation with the same structure as the equation for trajectories

$$\boxed{\frac{d\vec{X}}{dt} = \nabla\Phi^{S}(\vec{X})\times\vec{e}_z, \quad \Phi^{S}(\vec{x},t) = \left\langle \varphi(\vec{x},t)\right\rangle_S = v_i^0 E_{i\varphi} + \varphi^0 E}$$

- The ***DT's*** are determined from the EC of the stochastic fields and depend on the set of initial conditions
- The ***DT's*** are smooth, simple trajectories much different from particle trajectories.
- Each ***DT*** has an associated probability (weighting factor).

$$\boxed{\vec{X}(t;\varphi^0, v_i^0)}$$

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• M. Vlad, F. Spineanu, J.H. Misguich, R. Balescu, “Collisional effects on diffusion scaling laws in electrostatic turbulence”, **Physical Review E 61** (2000) 3023-3032.
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• M. Vlad, F. Spineanu, J.H. Misguich, R. Balescu, “Reply to ‘Comment on Diffusion in biased turbulence’”, **Physical Review E 66** (2002) 038302.
• M. Vlad, F. Spineanu, J.H. Misguich, R. Balescu, “Electrostatic turbulence with finite parallel correlation length and radial diffusion”, **Nuclear Fusion 42** (2002), 157-164.
•M. Vlad, F. Spineanu, J.H. Misguich, R. Balescu, “Magnetic line trapping and effective transport in stochastic magnetic fields”, **Physical Review E 67** (2003) 026406.
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•M. Vlad, F. Spineanu, “Larmor radius effects on impurity transport in turbulent plasmas”, **Plasma Physics and Controlled Fusion 47** (2005) 281-294.
•M. Vlad, F. Spineanu, S.-I. Itoh, M. Yagi, K. Itoh , “Turbulent transport of the ions with large Larmor radii”, **Plasma Physics and Controlled Fusion 47** (2005) 1015-1029.
•M. Vlad, F. Spineanu, S. Benkadda, “Impurity pinch from a ratchet process”, **Physical Reviews Letters 96** (2006) 085001.
• M. Vlad, F. Spineanu, S. Benkadda, “*Collision and plasma rotation effects on ratchet pinch*”, **Physics of Plasmas 15** (2008) 032306.

•M. Vlad, F. Spineanu, S. Benkadda, "*Turbulent pinch in non-homogeneous confining magnetic field*", **Plasma Physics Controlled Fusion 50** (2008) 065007.
• M. Vlad, F. Spineanu, "*Vortical structures of trajectories and transport of tracers advected by turbulent fluids"*, **Geophysical and Astrophysical Fluids Dynamics 103** (2009) 143.
•M. Vlad, *„Ion stochastic trapping and drift turbulence evolution"*, **Physical Review E 87** 053105 (2013).
•M. Vlad, F. Spineanu, *„Test particle study of ion transport in drift type turbulence"*, **Physics of Plasmas 20** 122304 (2013).
•M. Vlad, F. Spineanu, *„Diffusion and stochastic island generation in the magnetic field line random walk"*, **Astrophysical Journal 791** (2014) 56-64.
•M. Vlad, F. Spineanu, *Trajectory statistics and turbulence evolution*, **Chaos, Solitons and Fractals 81** (2015) 463-472
•M. Vlad, F. Spineanu, *Electron heat transport multi-scale turbulence*, **Physics of Plasmas 22** (2015) 112305
•M. Vlad et al *Nonlinear effects in particle transport in stochastic magnetic fields,* **Astrophysical Jornal** 815 (2015) 11
•M. Vlad, F. Spineanu, *Direct effects of the resonant magnetic perturbation on turbulent transport,* **Nuclear Fusion 56** (2016) 092003.
•Croitoru A. M., Palade D. I., Vlad M., Spineanu F., *Turbulent transport of alpha particles in tokamak plasmas,* **Nuclear Fusion 56** (2017) 036019
• Vlad M, Spineanu F, *Randon and quasi-coherent aspects in particle motion and their effects on transport and turbulence evolution*, Focus on Turbulence in Astrophysical and Laboratory Plasmas, **New Journal of Physics 19** (2017) 025014

## ***Examples of other people work with the DTM method***

M. Neuer, K. Spatschek, Phys. Rev. E 74 (2006) 036401
T. Hauff, F. Jenko, Physics of Plasmas 13 (2006) 102309
Petrisor I., Negrea M., Weyssow B., Physica Scripta 75 (2007) 1.
Negrea M., Petrisor I., Balescu R., Physical Review E 70 (2004) 046409.
Balescu R., Physical Review E 68 (2003) 046409.
Balescu R., Petrisor I., Negrea M., Plasma Phys. Control. Fusion 47 (2005) 2145.

***Still many other applications and developments to be done***

The averages of Lagrangian quantities are estimated by weighted averages on the decorrelation trajectories. The DTM is used fot transport studies while more complicated quantities are obtained with the NSM, which can provide more statistical information.

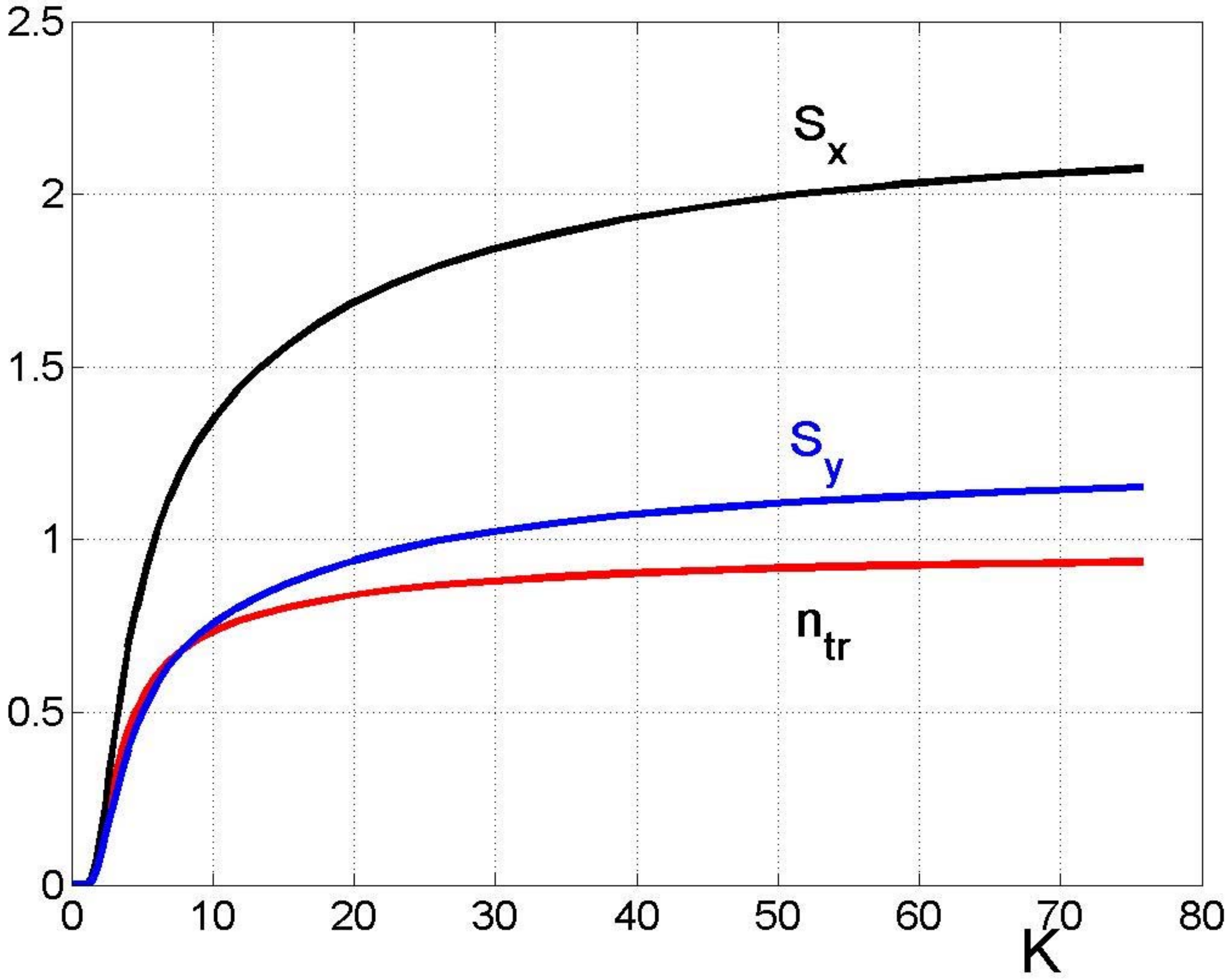
• The probability of the displacements
• the statistical characteristics of the distance at a time t between initialy neighbour trajectories

***Trapping determines:***

- ***non-Gaussian distribution of displacements***
- ***Long time correlation of the Lagrangian velocity***
- ***Micro-confinement effects***
- ***high degree of coherence*** (very strong clump effect of the trapped traj.)
- ***quasi-coherent structures of trajectories***
- ***Quasi-coherent flows*** (due to space variation of the confining magnetic field or of the amplitude of the turbulence)

***The characteristics of the quasi-coherent structures***
(non-isotropic, time-dependent potential):

- ***the fraction of trapped trajectories*** $n_{tr}$
- ***the average sizes of the trajectory structures*** $S_i$




$n_{tr}\,,\; S_x\,,\; S_y$ are increasing functions of the Kubo number K

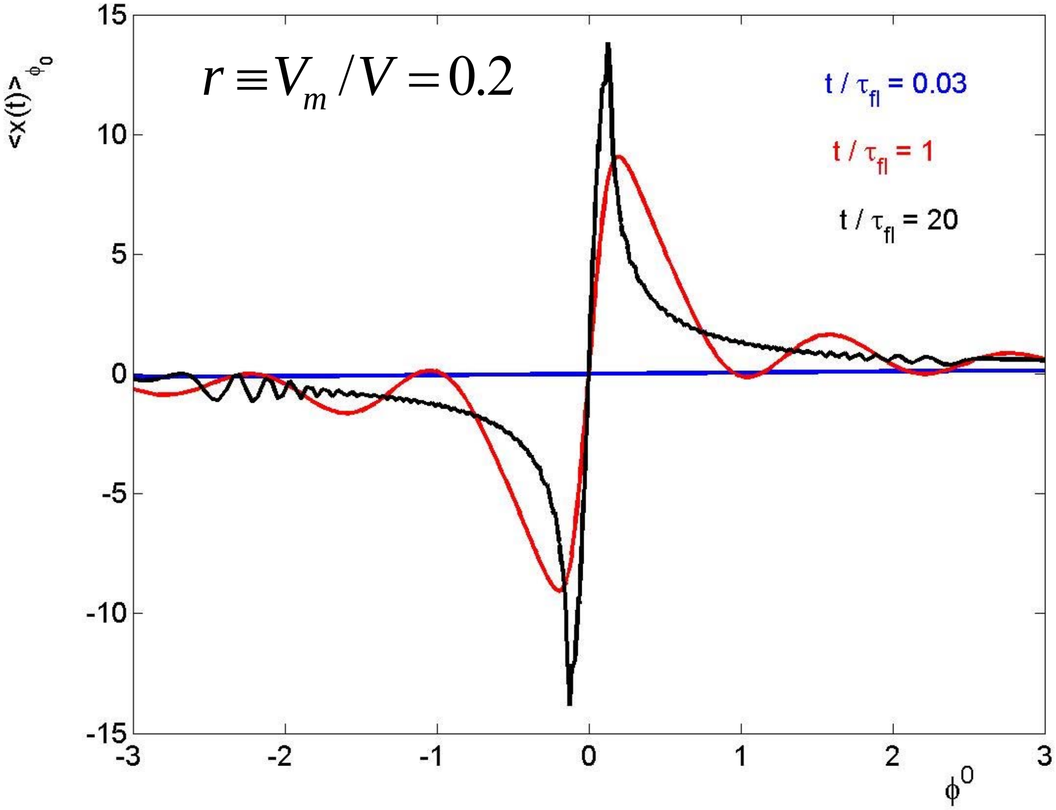
The struncture are absent at small K. They exist for $K \approx 1$.

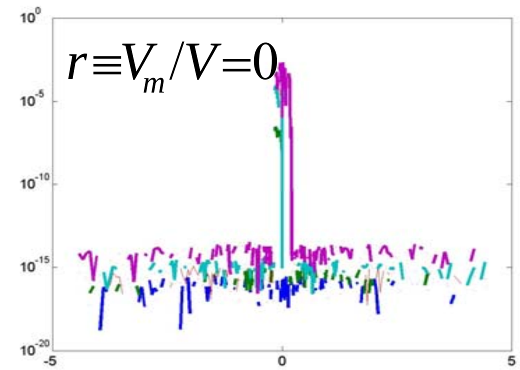
$$K = \frac{\tau}{\tau_{fl}} = \frac{V\tau}{\lambda}$$

## ➢ 3. The hidden drifts

- Hidden drifts (HDs) are a more subtle manifestation of the coherence of the stochastic trajectories
- HDs are ordered displacements that average to zero and do not drive flows
- ***HDs appear in the presence of an average velocity and they are perpendicular to this velocity***

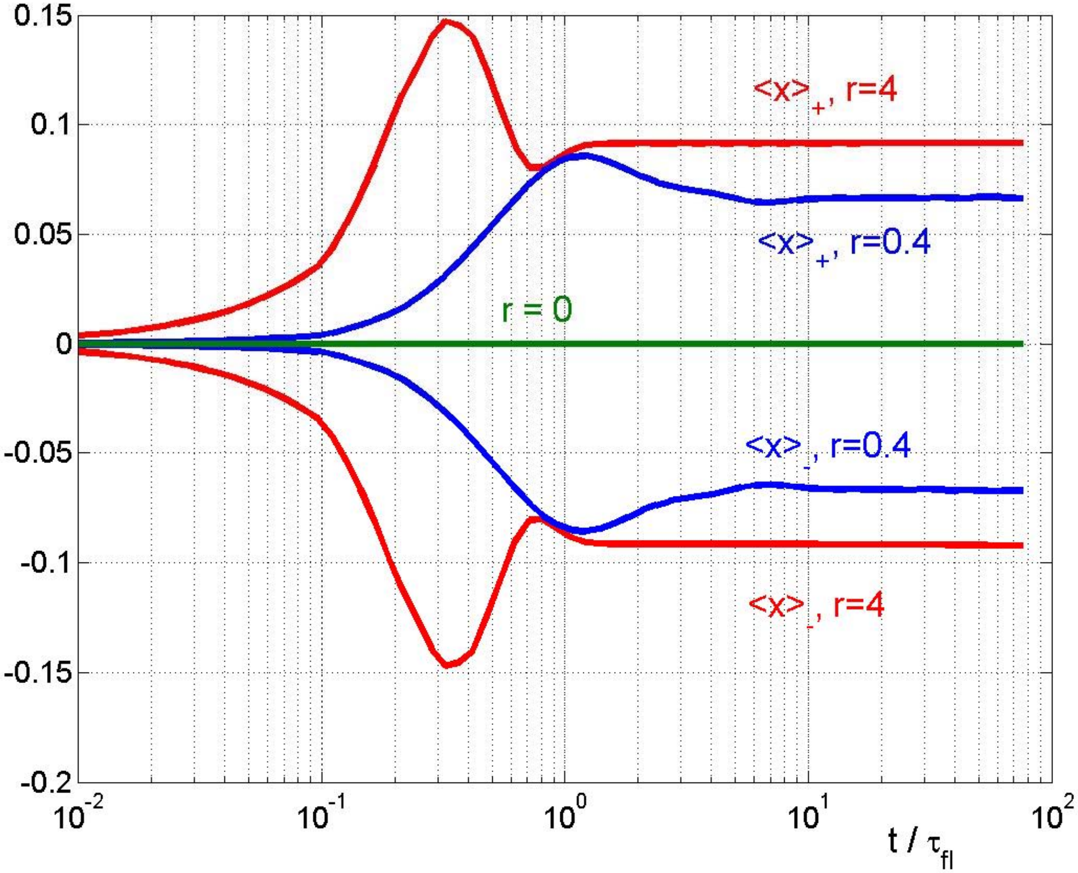
$$V_m \mathbf{e}_y$$




$$\langle x(t) \rangle_{\phi_0}$$

The displacement conditioned by the potential is an even function of $\phi_0$




$$\langle x(t)\rangle_+ = \int_0^{\infty} d\phi_0 \langle x(t)\rangle_{\phi_0} = \int_0^{\infty} d\phi_0 X(t;\phi_0)$$

$$\langle x(t)\rangle_- = \int_{-\infty}^{0} d\phi_0 \langle x(t)\rangle_{\phi_0} = \int_{-\infty}^{0} d\phi_0 X(t;\phi_0)$$

The average perpendicular displacements at saturation for the positive and negative initial potential

***The velocities of the hidden drifts:***

$$V_{h+} = \frac{\langle x(\tau_c)\rangle_+}{\tau_c}, \quad V_{h-} = \frac{\langle x(\tau_c)\rangle_-}{\tau_c} = -V_{h+}$$

## ➢ 4. Effect of the hidden drifts on plasma turbulence evolution

- Analytical study of test modes in turbulent plasmas that takes into account trajectory trapping
- *The frequency and the growth rate are determined as function of the statistical characteristics of the background turbulence.*

- Drift turbulence in a plasma slab, constant and uniform magnetic field
- The instability is produced by the combined effect of the non-adiabatic response of the electrons and of the finite Larmor radius (FLR) of the ions determined here by the polarization drift $\mathbf{u}_p = \frac{m_i}{eB^2}\partial_t \mathbf{E}_\perp.$

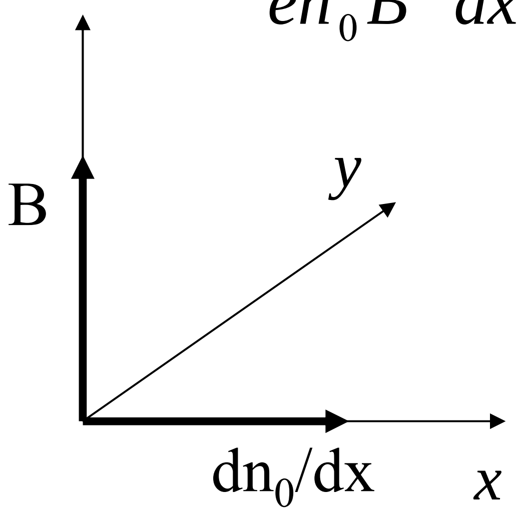
$$V_{*e} = \frac{T}{en_0 B}\frac{dn_0}{dx}$$

The limit of zero Larmor radius of the ions:

- *stable drift waves*

- *potentials that move with the diamagnetic velocity*

$$\phi_b(\mathbf{x} - V_{*e}\,\mathbf{e}_y\, t)$$



- We consider a turbulent plasma $\phi_b(\vec{x}, z, t)$
- Small perturbation $\delta\phi,\ \phi = \phi_b + \delta\phi,\ \delta\phi << \phi_b$
- The collisionless Vlasov equation $\partial_t f^{\alpha} + \mathbf{v}\cdot\nabla f^{\alpha} + v_z \partial_z f^{\alpha} = 0,$

- electrons – same response as in quiescent plasma

$$\delta n^e = n_0(x)\frac{e\delta\phi}{T_e}\left(1 + i\sqrt{\frac{\pi}{2}}\frac{\omega - k_y V_{*e}}{|k_z| v_{Te}}\right)$$

- ion response

$$f_0^i = n_0(x) F_M^i \exp\left(\frac{e\phi_b(\mathbf{x} - \mathbf{V}_{*e} t)}{T_e}\right)$$ the short time approximate equilibrium

$$f^i = f_0^i + h$$

- The linearized Vlasov equation in the perturbation of the potential:

$$(\partial_t h + \mathbf{v}\cdot\nabla h) + h\nabla\cdot\mathbf{u}_p = -i f_0^{\,i}\frac{e\delta\phi}{T_e}\left(k_y V_{*e} - \omega\rho_s^2 k_\perp^2\right)$$

- The formal solution for ion density perturbation obtained with the method of the characteristics

$$h(\mathbf{x}, v, t) = -n_0(x) F_M^i \frac{e\delta\phi}{T_e}(k_y V_{*e} - \omega\rho_s^2 k_\perp^2)\overline{\Pi}^i$$

The propagator :
$$\overline{\Pi}^i = i\int_{-\infty}^{t} d\tau\, M(\tau)\, \exp[-i\omega(\tau - t)]$$

$$M(\tau) \equiv \left\langle \exp\left[\frac{e\phi_b\left(\mathbf{x}(\tau) - \mathbf{V}_{*\mathbf{e}}\tau\right)}{T_e} + i\mathbf{k}\cdot\left(\mathbf{x}(\tau) - \mathbf{x}\right) - \int_\tau^t d\tau'\, \nabla\cdot\mathbf{u}_p\left(\mathbf{x}(\tau')\right)\right]\right\rangle.$$

The average on the trajectories: $\dfrac{d\mathbf{x}(\tau)}{d\tau} = -\dfrac{\nabla\phi_b(\mathbf{x} - \mathbf{V}_{*e}t)\times\mathbf{e}_z}{B}$,
integrated backwards in time with the condition at $\tau = t$ $\mathbf{x}(t) = \mathbf{x}$

-The dispersion relation for test modes in turbulent plasma
(the same as in quiescent plasma, except for the function $M(\tau)$ that appears in the propagator )

$$-(k_y V_{*e} - \omega\rho_s^2 k_\perp^2)\overline{\Pi}^i = 1 + i\sqrt{\frac{\pi}{2}}\frac{\omega - k_y V_{*e}}{|k_z| v_{Te}}.$$

Thus, all the effects of the background turbulence are contained in the function:

$$\boxed{M(\tau) \equiv \left\langle \exp\left[ \frac{e\phi_b\left(\mathbf{x}(\tau) - \mathbf{V}_{*\mathbf{e}}\tau\right)}{T_e} + i\mathbf{k}\cdot\left(\mathbf{x}(\tau) - \mathbf{x}\right) - \int_\tau^t d\tau' \, \nabla\cdot\mathbf{u}_p\left(\mathbf{x}(\tau')\right) \right] \right\rangle.}$$

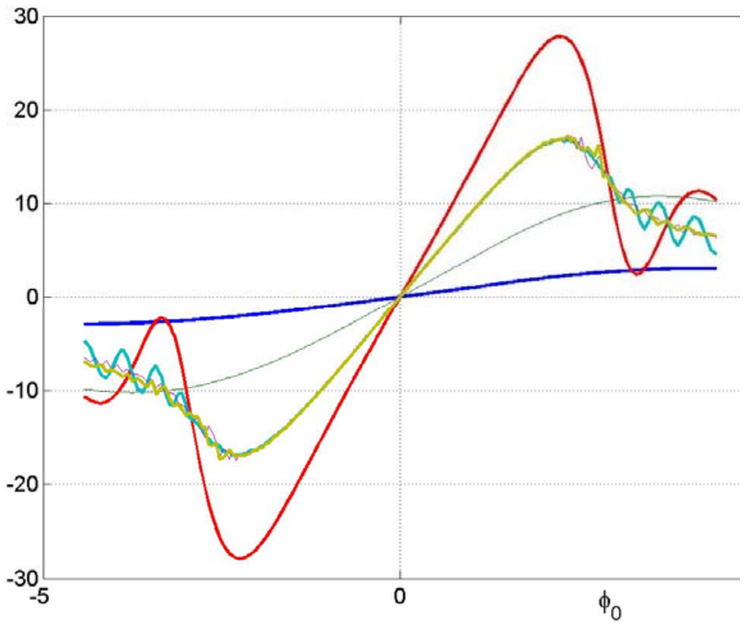
- Implicite dependence on the ion trajectories through the average
- Explicite dependence on the turbulence potential and polarization drift

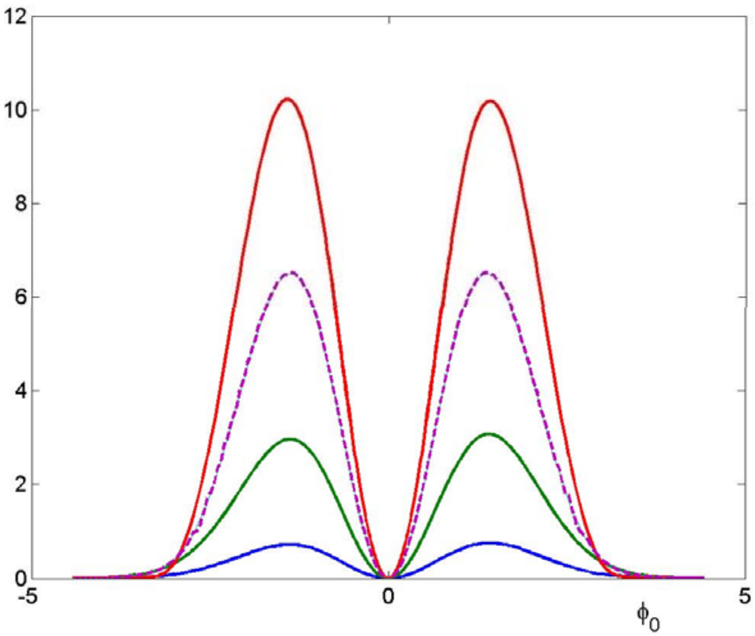
- ❑ In the case of quiescent plasmas $\mathrm{M} = 1$
- ❑ In turbulent plasmas, M contains the correlation potential-displacement, which is determined by ***the hidden drifts***

$$\left\langle \phi_0 \, x(t) \right\rangle = \int d\phi_0 \, \phi_0 \left\langle x(t) \right\rangle_{\phi_0} P(\phi_0)$$

$\left\langle x(t) \right\rangle_{\phi_0}$

$r = V_m / V = 4$



## *The hidden drifts determine in the propagator an average velocity*

$$\boxed{V_{hd} = \frac{e}{T_e} \frac{\langle \phi_b \, x(\tau_c) \rangle}{\tau_c}}$$

$$\overline{\Pi}^i = -\frac{F}{\omega + k_x V_{hd} + i k_i^2 D_i}, \qquad F \equiv \exp\left( -\frac{1}{2} k_i^2 S_i \right)$$

$$\omega = \frac{F k_y V_{*e} - k_x V_{hd}}{1 + F\,(k_x^2 + k_y^2)}$$

$$\gamma = \sqrt{\frac{\pi}{2}} \frac{F}{|k_z| v_{T_e}} \frac{\left(k_y V_{*e} + k_x V_{hd}\right)\left(k_y V_{*e} + k_x V_{hd}\,(k_x^2 + k_y^2)\right)}{\left(1 + F\,(k_x^2 + k_y^2)\right)^3} - k_i^2 D_i$$

- The hidden drift velocity modifies both frequency and growth rate of the drift modes
- The background turbulence also determines the factor *F*, which is due to the trajectory structures.

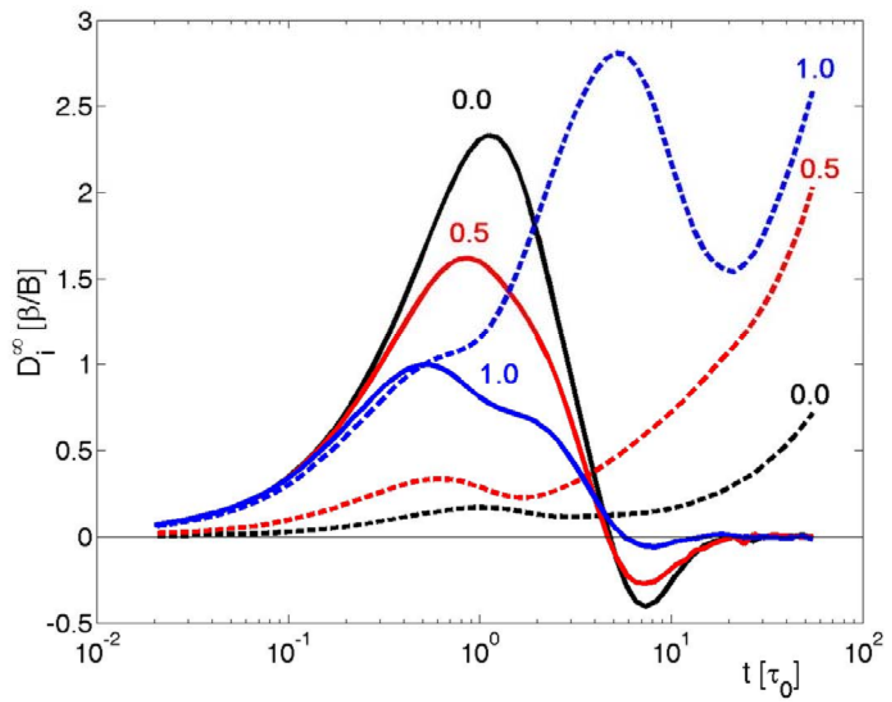
The hidden drifts determine unstable modes with $k_y \to 0$ ***(zonal flow modes)***

$$\gamma_{zf} = \sqrt{\frac{\pi}{2}} \frac{F}{|k_z| v_{T_e}} \frac{(k_x V_{hd})^2 k_x^2}{(1 + F k_x^2)^3} - k_i^2 D_i$$

These unstable zonal flow modes appear when the amplitude of the drift turbulence is large enough $V_{hd} \approx \Phi$

Zonal flow modes are very important because:

- ❑ they influence the saturation of the drift turbulence
- ❑ They determine the decrease of the diffusion coefficient along density gradient (confinement quality) $D_x$
- ❑ They strongly increase the perpendicular diffusion that has a damping effect on drift turbulence $D_y$



## ➢ 5. Effect of the hidden drifts on fluid turbulence evolution

***Ideal fluid turbulence described by 2D Euler equation for the vorticity***

$$\partial_t \omega + \mathbf{v} \cdot \nabla \omega = 0,$$

$$\mathbf{v} = \nabla \phi \times \mathbf{e}_z, \quad \omega(\mathbf{x}, t) = -\Delta \phi(\mathbf{x}, t)$$

***Process of vorticity separation according to its sign:***

The relaxation of turbulent initial states clearly show the evolution toward order, and it can lead to coherent states that consists of two large vortices of system size.

- The same Vlasov equation as in plasma case, with exactly the same statistical properties of the characteristics (including the ***hidden drifts***).
- The equation of motion of the vorticity elements are completely independent on the vorticity they carry.
- How occurs vorticity separation

❑ We show that the different nonlinearities of Euler and plasma turbulence equations (and the different nature of the advected quantities) determines ***completely different effects of the hidden drifts***

## *The Eulerian correlations (EC) of the potential and of the velocity:*

$$E_{\omega\phi}(\vec{x},t) \equiv \Delta E(\vec{x},t), \qquad E(\vec{x},t) \equiv \langle \varphi(0,0)\,\varphi(\vec{x},t)\rangle = \Phi^2 f\left(\frac{\vec{x}}{\lambda}, \frac{t}{\tau}\right)$$

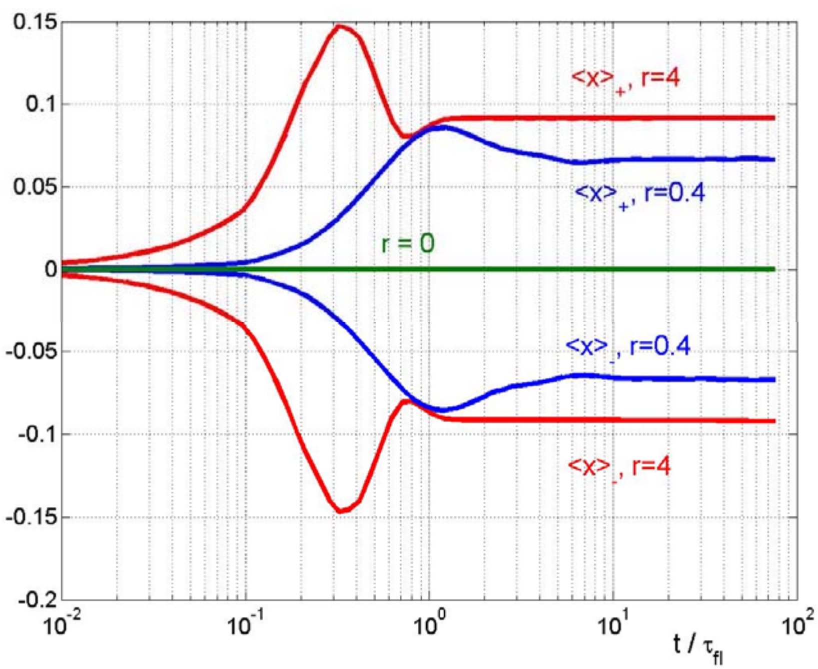
$E_{\omega\phi}(0,0) > 0$ correlation of the vorticity and stream function

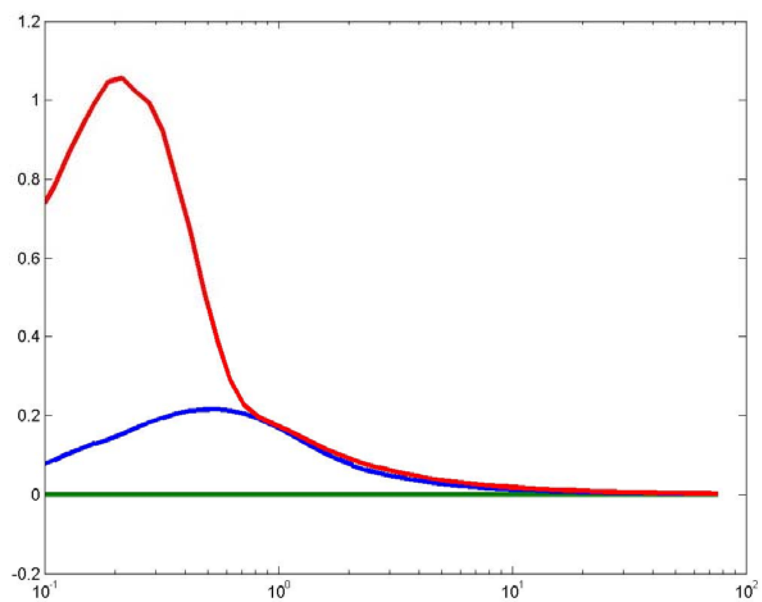
Where $\phi > 0$ also $\omega > 0$, and where $\phi < 0$ also $\omega < 0$

• ***Consider small scale turbulence in the presence of a large scale vortex represented by an average velocity*** $V_m \mathbf{e}_y$ → ***hidden drifts are generated***

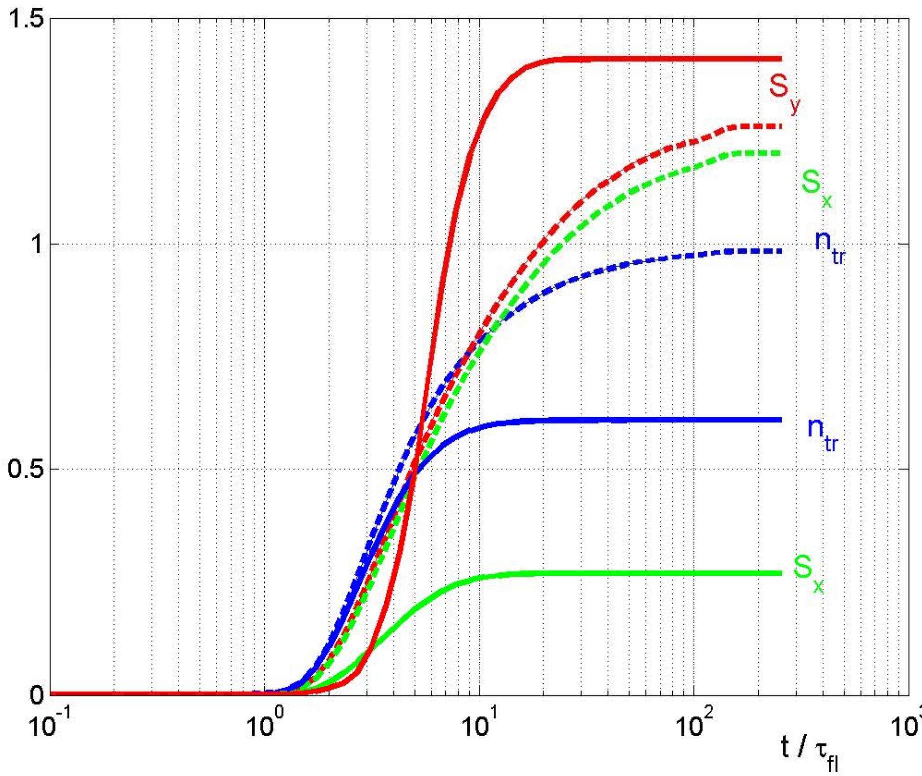
$$\langle x(t)\rangle_+ = \int_0^\infty d\phi_0 \langle x(t)\rangle_{\phi_0}, \quad \langle x(t)\rangle_- = \int_{-\infty}^0 d\phi_0 \langle x(t)\rangle_{\phi_0}$$

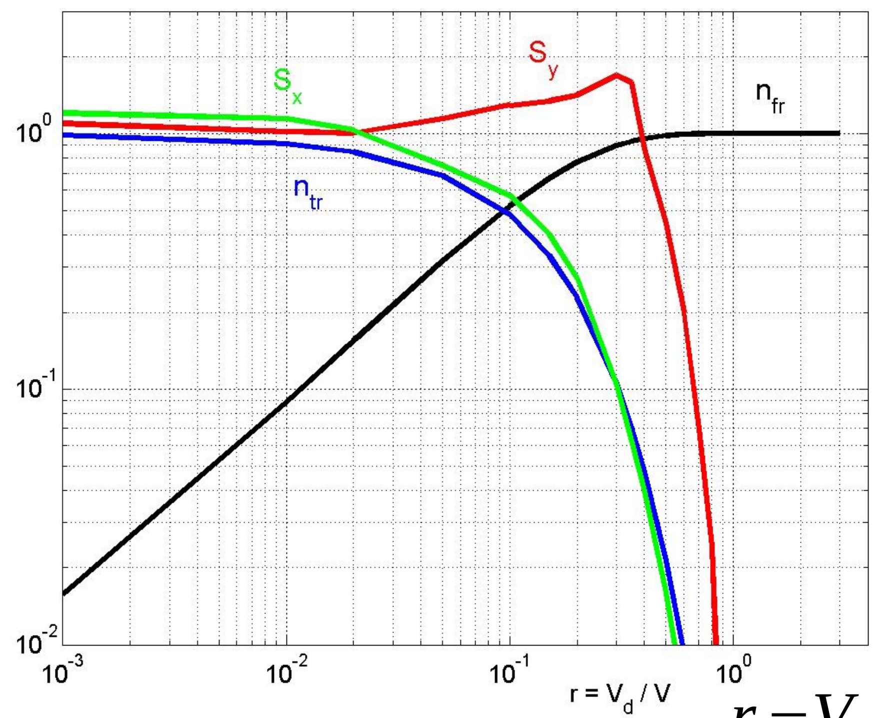
***Velocity difference between the hidden flows***



The velocity difference leads to vorticity separation, but only in the presence of trajectory structures

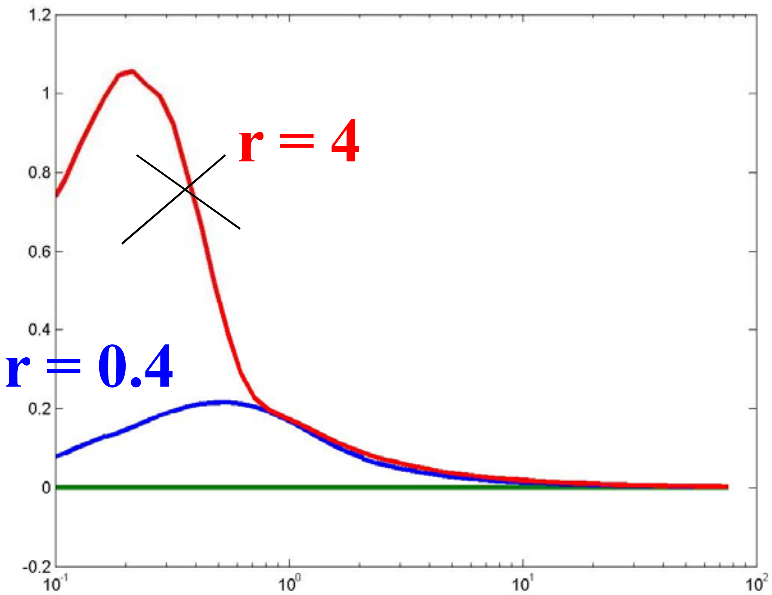


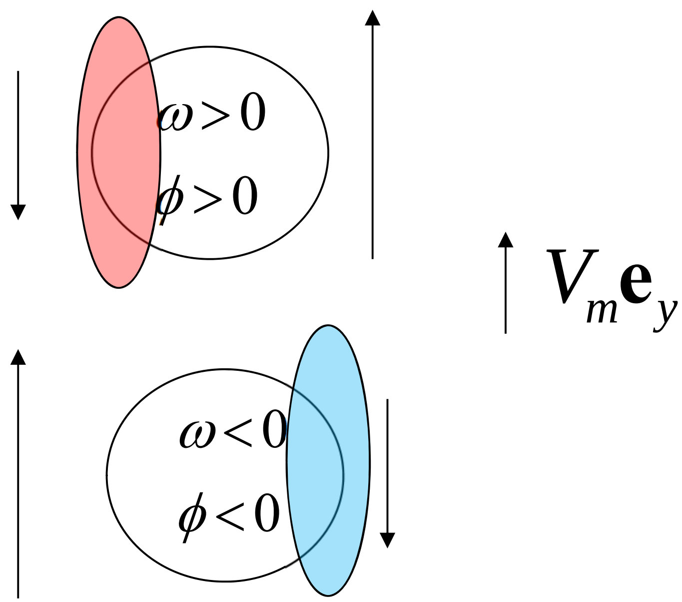

$$r = V_m / V$$

$V_m \mathbf{e}_y$ modifies the shapes of the structures (elongated along the average velocity), and eventually destroys all the structures when $r = V_m / V > 1$

***The velocity difference between the hidden flows leads to vorticity separation only in the presence of trajectory structures***




Positive vorticity concentration




Corresponds to a large scale negative vortex

Negative vorticity concentration

***The hidden drifts determine the attraction of the small scale vortices by a large vortex with the same sign***

# *Conclusion*

- *Hidden drifts (HDs) appear as ordered motion in 2D turbulence in the presence of an average velocity*
- *HDs have strong effect on both plasma and fluid turbulence, but by different mechanisms that are imposed by the specific nonlinearity*

*Plasma turbulence:*

- *HDs lead to an average velocity perpendicular to the average velocity by correlation with the background turbulence*
- *This velocity directed along the gradient of the density generates zonal flow modes;*
- This process of zonal flow generation appears from the quasilinear regime of the drift turbulence and adds to other physical mechanism (Reynolds stress, ion flows, etc.)

*Fluid turbulence:*

- *HDs contribute to the vorticity separation according to its sign*