30th Oct.~3rd Nov. 2017
“Collisionless Boltzmann (Vlasov) Equation and Modeling of Self-Gravitating Systems and Plasmas”
@ CIRM, Marseille

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Perturbative description of Vlasov-Poisson system in cosmology :

Approaching and going beyond shell-crossing

Atsushi Taruya
(Yukawa Institute for Theoretical Physics, Kyoto Univ.)

In collaboration with
Stéphane Colombi (Institut d’Astrophysique de Paris)
Shohei Saga (Yukawa Institute for Theoretical Physics)

Plan of talk

Perturbative description of 'shell-crossing' and beyond in nonlinear regime of cosmic structure formation

Refs. AT & Colombi, MNRAS 470, 4858 ('17), Saga, Colombi & AT (in prep.)

- Introduction & motivation
- Beyond shell-crossing in 1D
- Approaching shell-crossing in 3D
- Summary

Large-scale structure of the Universe

Matter inhomogeneity over Giga parsec scales

=

1000 Mpc =3*10^9 light years

is dominated by hypothetical invisible matter (i.e., *cold dark matter*)

It has evolved from tiny fluctuations (most likely seeded by inflation) under influence of cosmic expansion and gravity

- Provide a wealth of cosmological information
- Key observable in the era of precision cosmology

Macroscopically,

self-gravitating collisionless system in an expanding universe

Cosmological Vlasov-Poisson system

Vlasov-Poisson system in a cosmological background:

$$\left[\frac{\partial}{\partial t}+\frac{\boldsymbol{p}}{ma^2}\frac{\partial}{\partial \boldsymbol{x}}-m\frac{\partial\Phi}{\partial \boldsymbol{x}}\frac{\partial}{\partial \boldsymbol{p}}\right]f(\boldsymbol{x},\boldsymbol{p})=0,$$

$$\underbrace{\nabla^2\Phi(\boldsymbol{x})}_{\text{Newton potential}}=4\pi\,G\,a^2\left[\frac{m}{a^3}\int d^3\boldsymbol{p}\,f(\boldsymbol{x},\boldsymbol{p})-\rho_{\rm m}\right]$$

a(t) : scale factor of the Universe

Cold initial flow (or single-stream flow):

Dirac's delta function

$$f(\boldsymbol{x},\boldsymbol{p})=\overline{n}\,a^3\,\{1+\underbrace{\delta_{\rm m}(\boldsymbol{x})}_{\text{Mass density field}}\}\,\delta_{\rm D}[\boldsymbol{p}-m\,a\,\underbrace{\boldsymbol{v}(\boldsymbol{x})}_{\text{Velocity field}}]$$

System at an early phase is reduced to pressureless fluid system

Cosmic fluid and perturbation theory

Assuming single-stream flow,

cosmological Vlasov-Poisson system is reduced to fluid system

$$\frac{\partial \delta_{\rm m}}{\partial t}+\frac{1}{a}\nabla\left[(1+\delta_{\rm m})\boldsymbol{v}\right]=0,$$

$$\frac{\partial \boldsymbol{v}}{\partial t}+\frac{1}{a}(\boldsymbol{v}\cdot\nabla)\,\boldsymbol{v}=-\frac{1}{a}\frac{\partial\Phi}{\partial\boldsymbol{x}},$$

$$\frac{1}{a^2}\,\nabla^2\Phi=4\pi\,G\,\rho_{\rm m}\,\delta_{\rm m}.$$

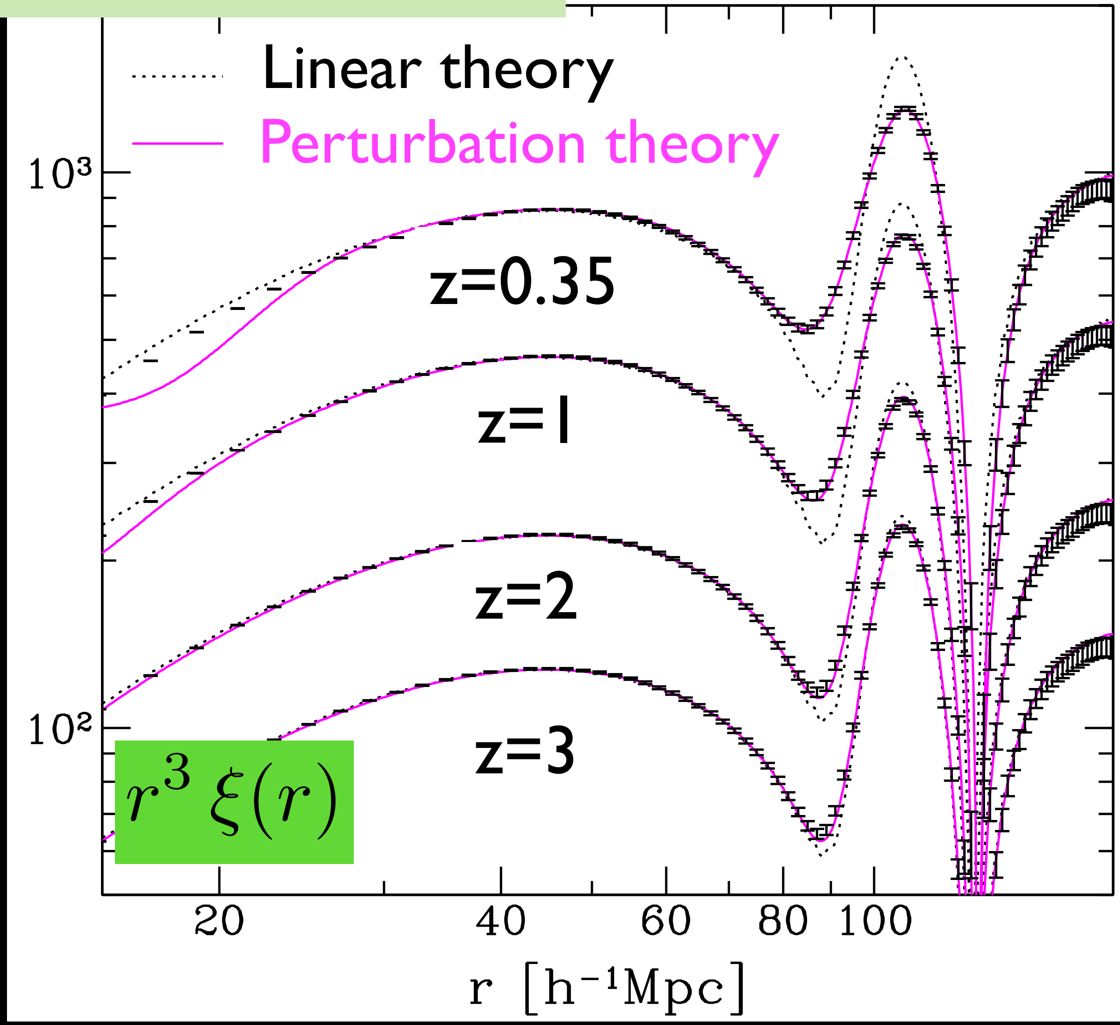
Perturbative expansion: $|\delta|\ll 1$

$$\delta=\delta^{(1)}+\delta^{(2)}+\delta^{(3)}+\cdots$$

Juszkiewicz ('81), Vishniac ('83), Goroff et al. ('86), Suto & Sasaki ('91), Makino, Sasaki & Suto ('92), Jain & Bertschinger ('94), ...

+ resummation technique

Correlation function



AT, Bernardeau, Nishimichi & Codis ('12)

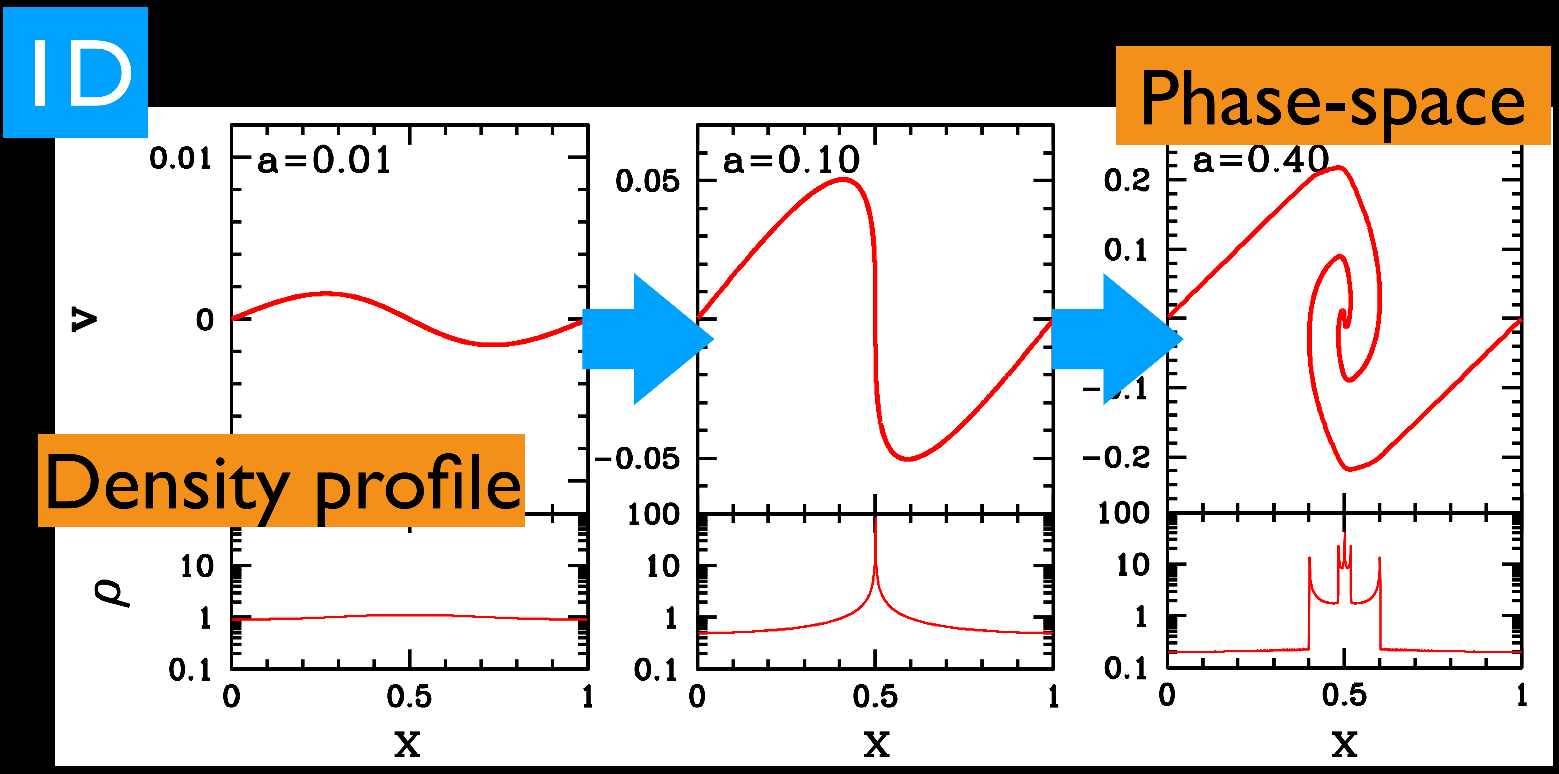
Shell-crossing: inevitable feature

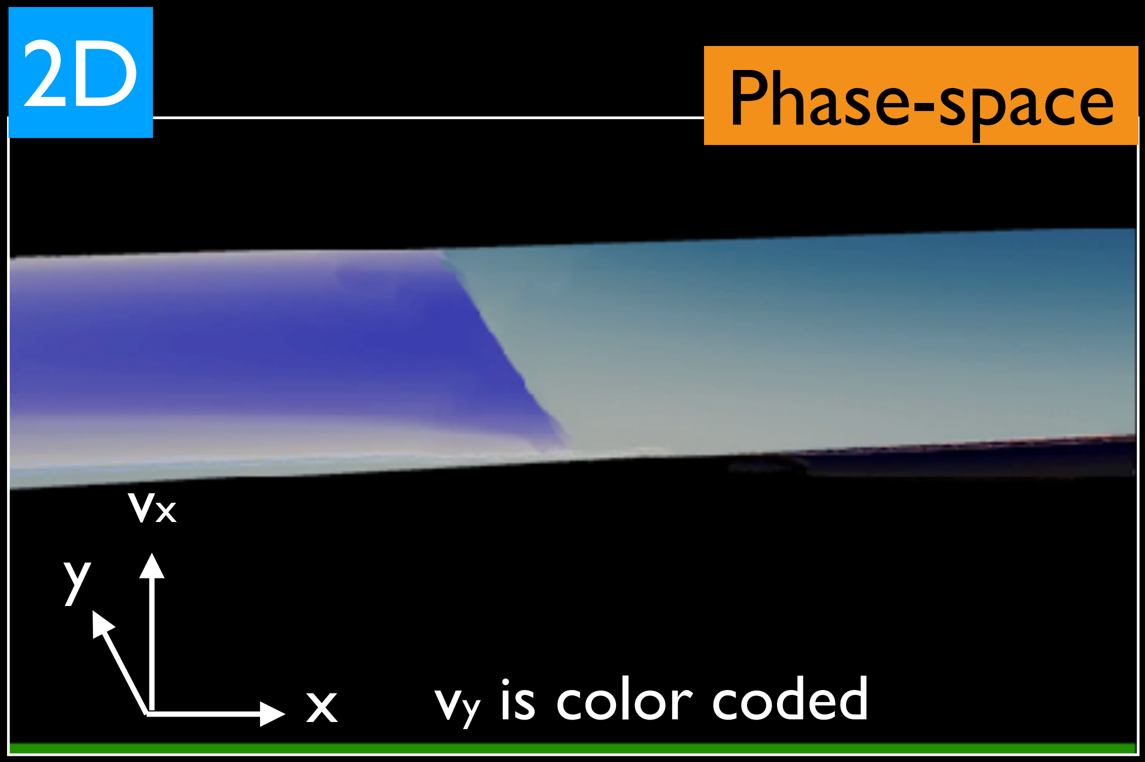
Fluid description of matter flow, however, breaks down at late time, followed by

Shell crossing

e.g., Shandarin & Zel'dovich ('89)

- Singular density field
- Development of multi-stream flow after shell-crossing



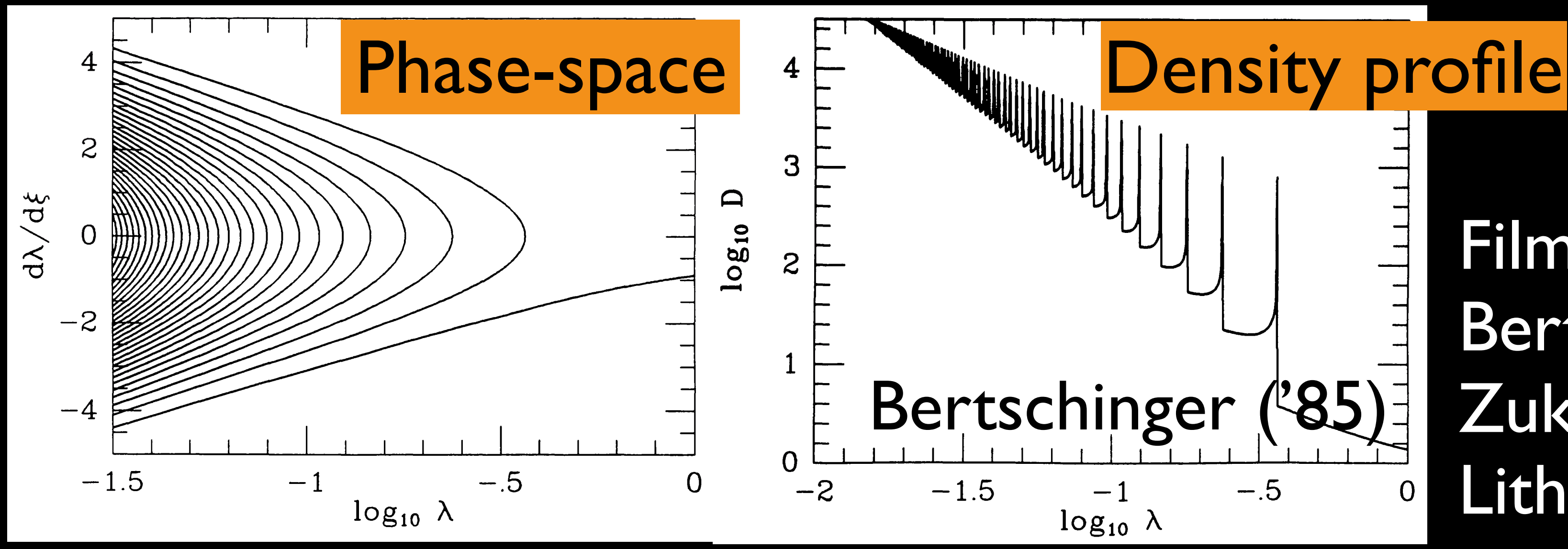


http://www.vlasix.org/index.php?n=Main.ColDICE

Analytic description of shell-crossing

How well one can analytically describe shell-crossing and beyond ?

Self-similar solution under a certain symmetry (spherical, planar, …)



Filmore & Goldreich ('84);
Bertschinger ('85); Ryden ('93);
Zukin & Bertschinger ('10);
Lithwick & Dalal ('11); Alard ('13)

In this talk,

Perturbative description of formation of shell-crossing and beyond

Key Lagrangian description of matter flow:

$$\boldsymbol{x}(\boldsymbol{q},t)=\boldsymbol{q}+\boldsymbol{\Psi}(\boldsymbol{q},t), \qquad \boldsymbol{v}(\boldsymbol{q},\,t)=\frac{d\boldsymbol{\Psi}(\boldsymbol{q},\,t)}{dt}$$

$\boldsymbol{q}$: Lagrangian coordinate $\quad \boldsymbol{\Psi}$: displacement field

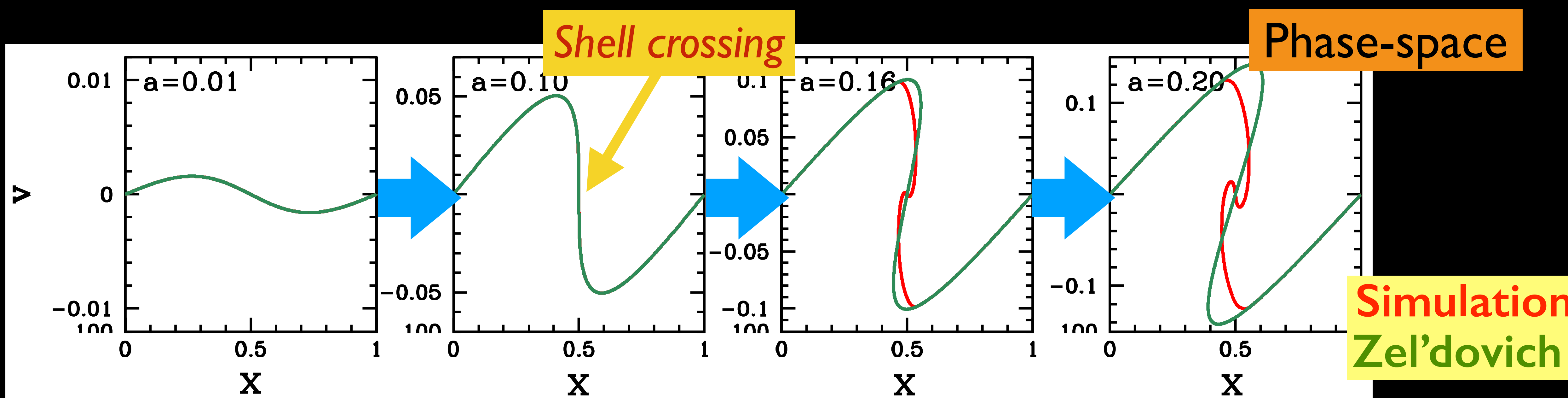
Zel'dovich solution

Zel'dovich ('70), Novikov ('69)
Shandarin & Zel'dovich ('89)

Exact single-stream solution in 1D:

$$x(q;\tau) = q + \psi(q)\, D_+(\tau) \qquad \mathrm{v}(q;\tau) = \psi(q)\, \frac{dD_+(\tau)}{d\tau}$$

$D_+(\tau)$: linear growth factor $\qquad \psi(q)$: arbitrary function of q



Beyond shell-crossing, solution is no longer exact

⟶ exploiting new treatment with *post-collapse PT in 1D*

In 2D & 3D, it is no more exact even at single-stream regime

⟶ improved description by *higher-order Lagrangian PT in 3D*

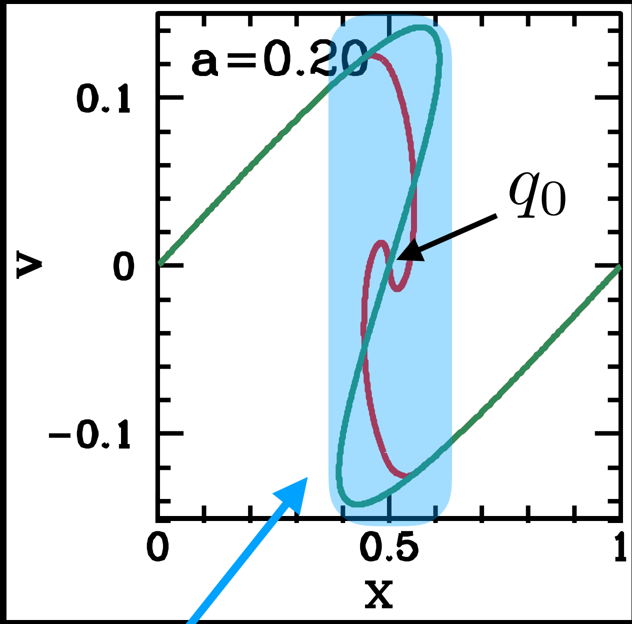
Beyond shell-crossing in 1D

AT & Colombi, MNRAS 470, 4858('17)
see also, Colombi, ibid. 446, 2902 ('15)

New formalism

Post-collapse perturbation theory (PT)

Computing back-reaction to the Zel'dovich flow:



1. Expand the displacement field around *shell-crossing* *(Lagrangian) point*, q_0:

$$x(q;\tau) \simeq A(q_0;\tau) - B(q_0;\tau)(q-q_0) + C(q_0;\tau)(q-q_0)^3$$

2. Compute force $F(x(q;\tau)) = -\nabla_x \Phi(x(q;\tau))$ at multi-valued region

Newton potential

$$\Delta \mathrm{v}(Q;\tau,\,\tau_\mathrm{q}) = \int_{\tau_\mathrm{q}}^{\tau} d\tau'\,F(x(Q,\tau')), \quad \Delta x(Q;\tau,\,\tau_\mathrm{q}) = \int_{\tau_\mathrm{q}}^{\tau} d\tau'\,\Delta \mathrm{v}(Q;\tau',\tau_\mathrm{q})$$

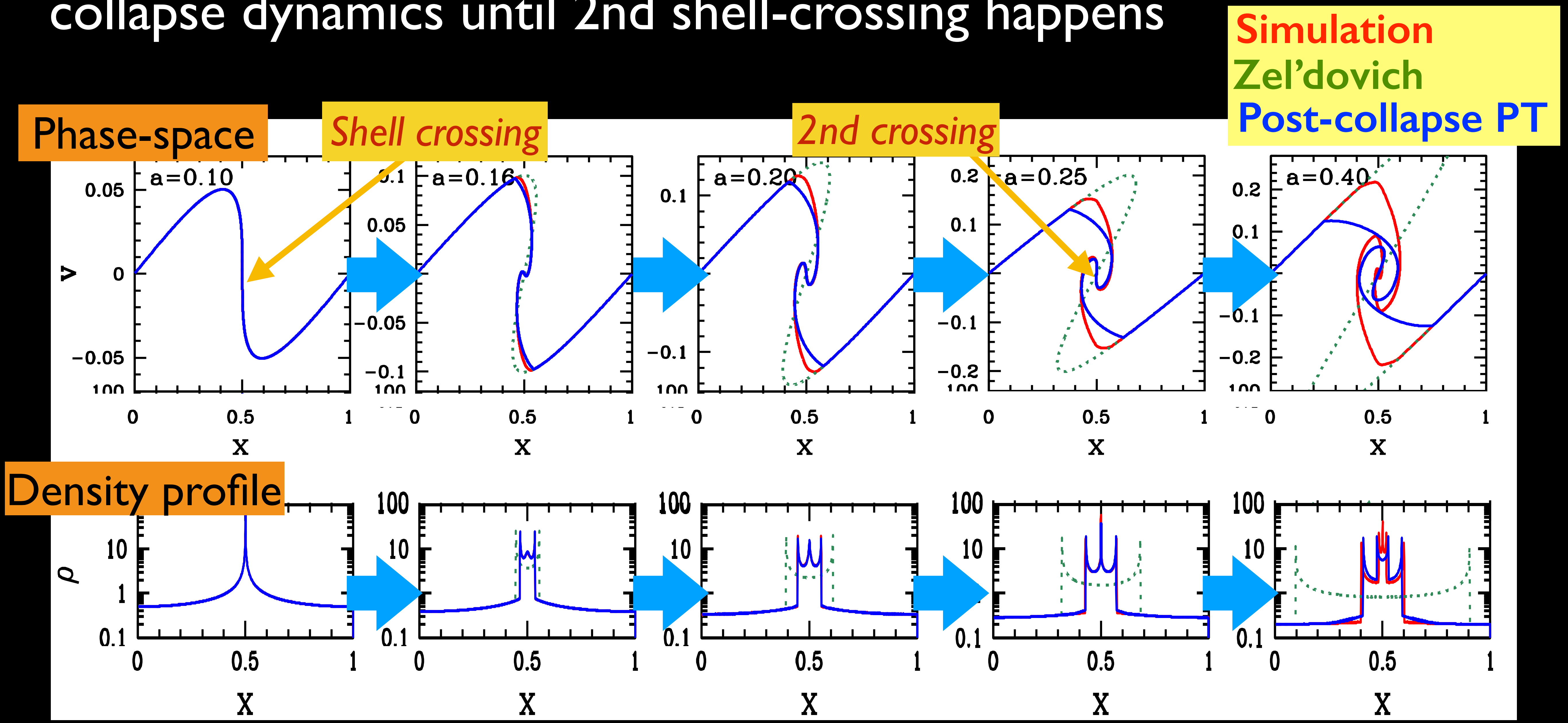
Backreaction

...... *polynomial function of Q=q-q₀ up to 7th order*

Post-collapse PT: single cluster

Post-collapse PT basically can capture the post-collapse dynamics until 2nd shell-crossing happens

AT & Colombi ('17)



Even after 2nd crossing,

it still gives a reasonable description for density profiles

Post-collapse PT: random initial condition

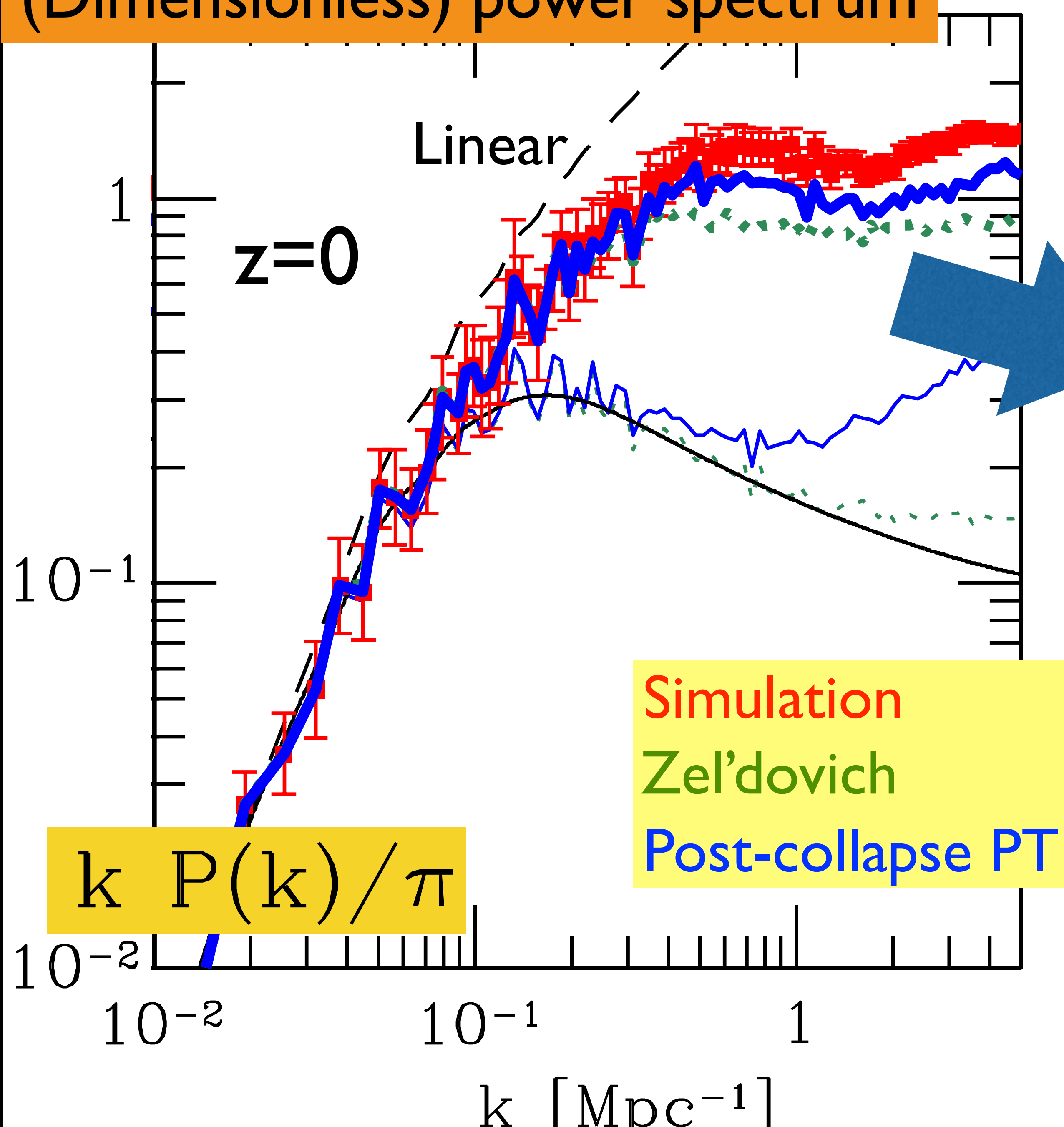
AT & Colombi ('17)

Initial condition

$$P_{\rm 1D}(k)=\frac{k^2}{2\pi}\,P_{\rm 3D}(k)$$

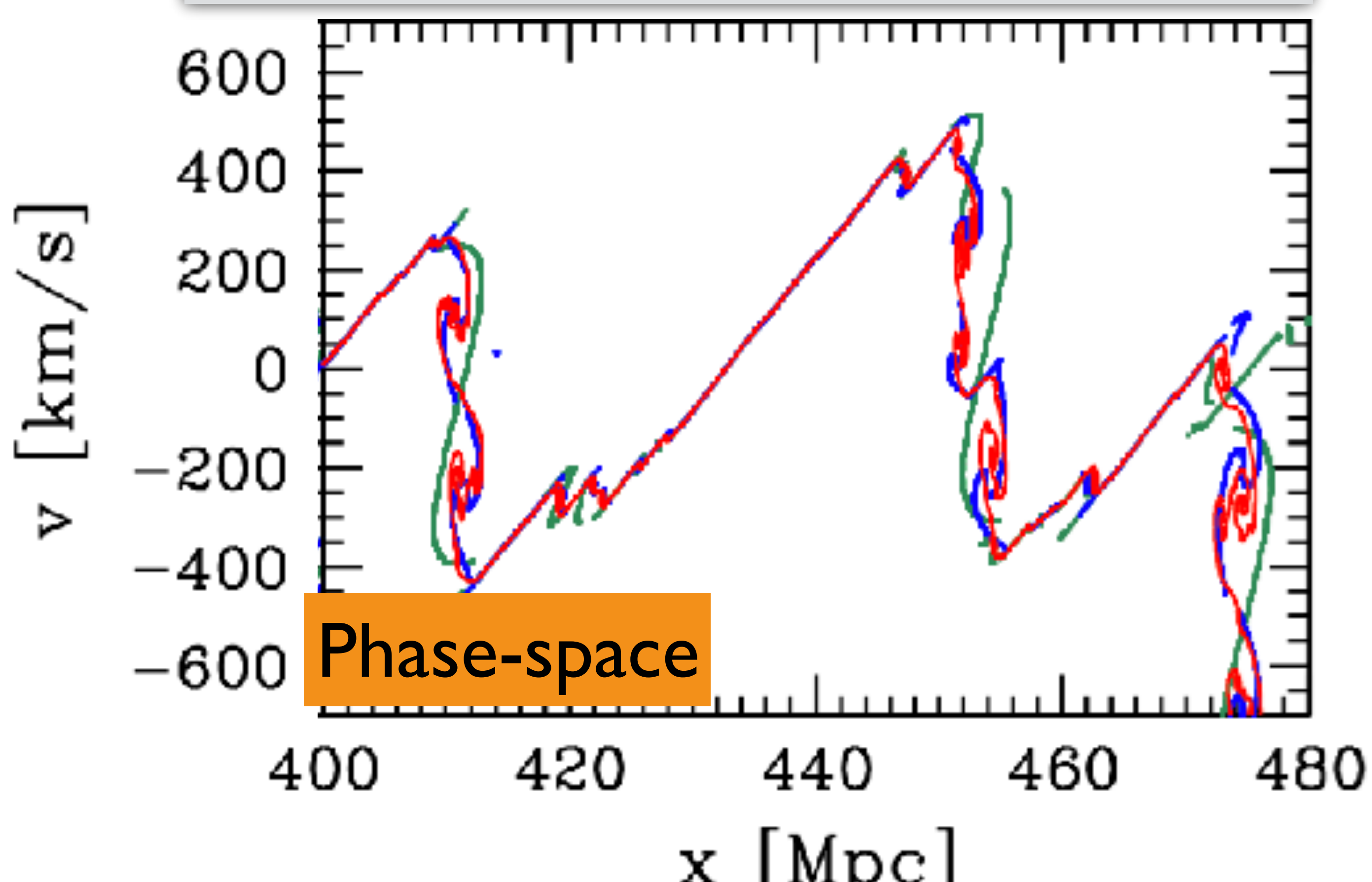
Planck ΛCDM

(Dimensionless) power spectrum



Adaptive smoothing

applied to initial density peaks
(with filter scales determined
by first-barrier crossing)



Approaching shell-crossing in 3D

Improved description of shell-crossing by

Poster by Shohei Saga

higher-order Lagrangian perturbation theory (LPT)

Lagrangian displacement field

$$\boldsymbol{\Psi}(\boldsymbol{q},t) = \boldsymbol{\Psi}^{(1)}(\boldsymbol{q},t) + \boldsymbol{\Psi}^{(2)}(\boldsymbol{q},t) + \boldsymbol{\Psi}^{(3)}(\boldsymbol{q},t) + \cdots$$

Eq. of motion for displacement field *assuming single-stream flow*

Longitudinal:

$$\left(\hat{\mathcal{T}} - 4\pi\, G\,\overline{\rho}_{\rm m}\right)\Psi_{k,k} = -\epsilon_{ijk}\epsilon_{ipq}\,\Psi_{j,p}\left(\hat{\mathcal{T}} - 2\pi\, G\,\overline{\rho}_{\rm m}\right)\psi_{k,q}$$
$$- \frac{1}{2}\epsilon_{ijk}\epsilon_{pqr}\,\Psi_{i,p}\Psi_{j,q}\left(\hat{\mathcal{T}} - \frac{4\pi\, G}{3}\,\overline{\rho}_{\rm m}\right)\Psi_{k,r}$$

Transverse:

$$\epsilon_{ijk}\,\hat{\mathcal{T}}\,\Psi_{j,k} = -\epsilon_{ijk}\,\Psi_{p,j}\hat{\mathcal{T}}\Psi_{p,k}$$

$$\hat{\mathcal{T}}f(t) \equiv \ddot{f}(t) + 2H\dot{f}(t)$$

Moutarde et al. ('91); Bouchet et al. ('92); Buchert ('92); Buchert & Ehlers ('93); Bouchet et al. ('95), …, Matsubara ('15), Rampf & Frisch ('17)

Approaching shell-crossing in 3D

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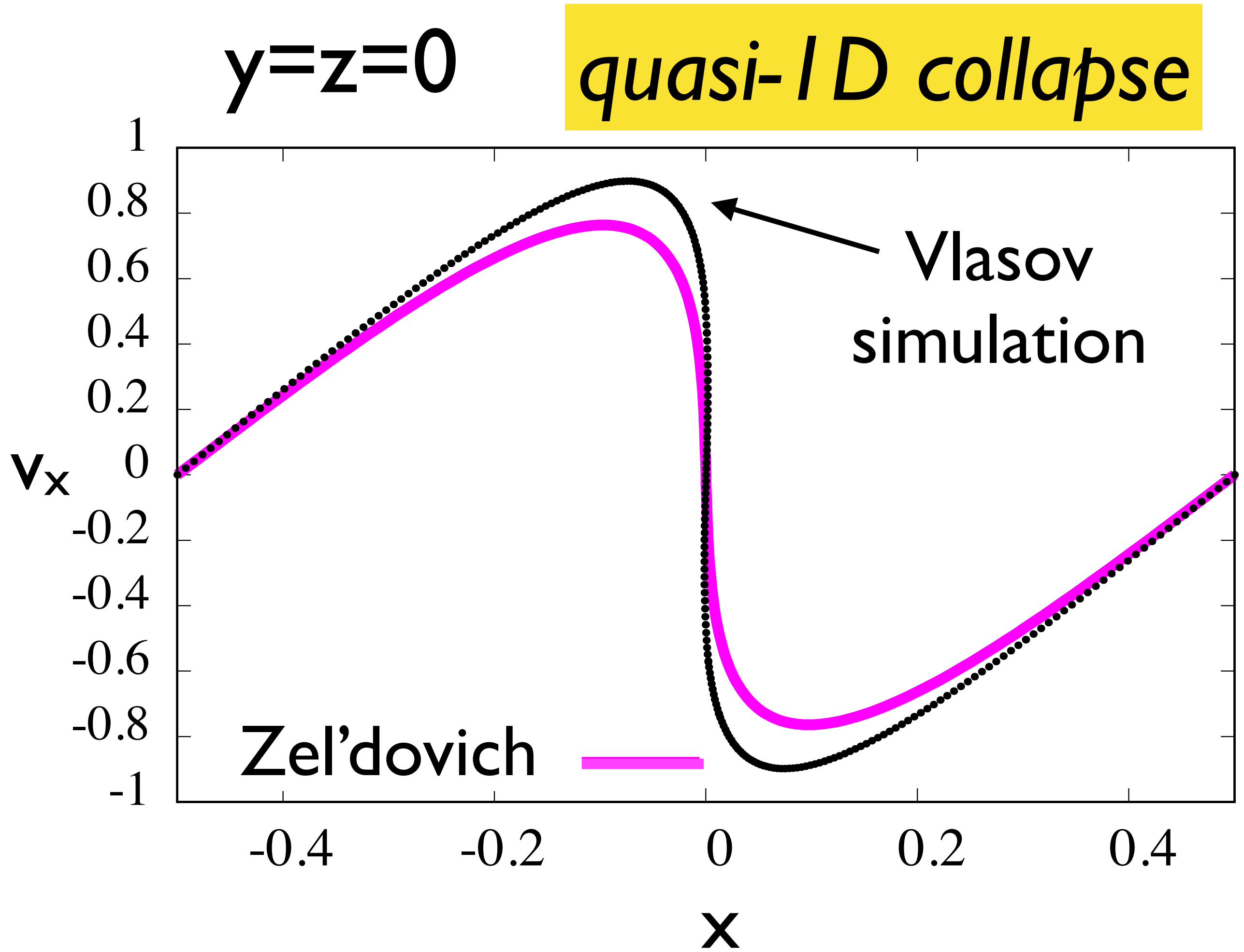

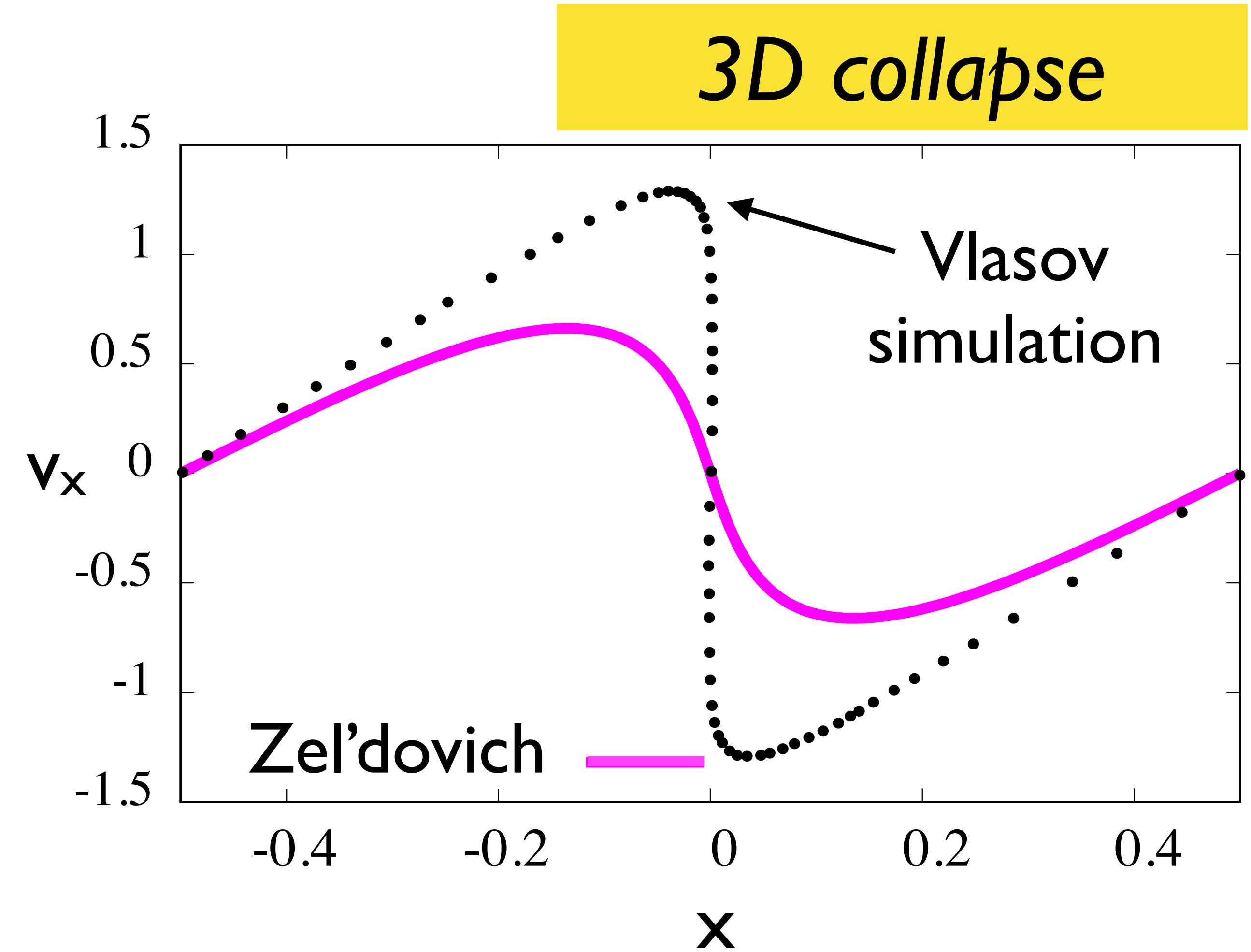

Simulation: ColDICE (Sousbie & Colombi '16)

sine-wave initial condition

(Moutarde et al. '91)

Approaching shell-crossing in 3D

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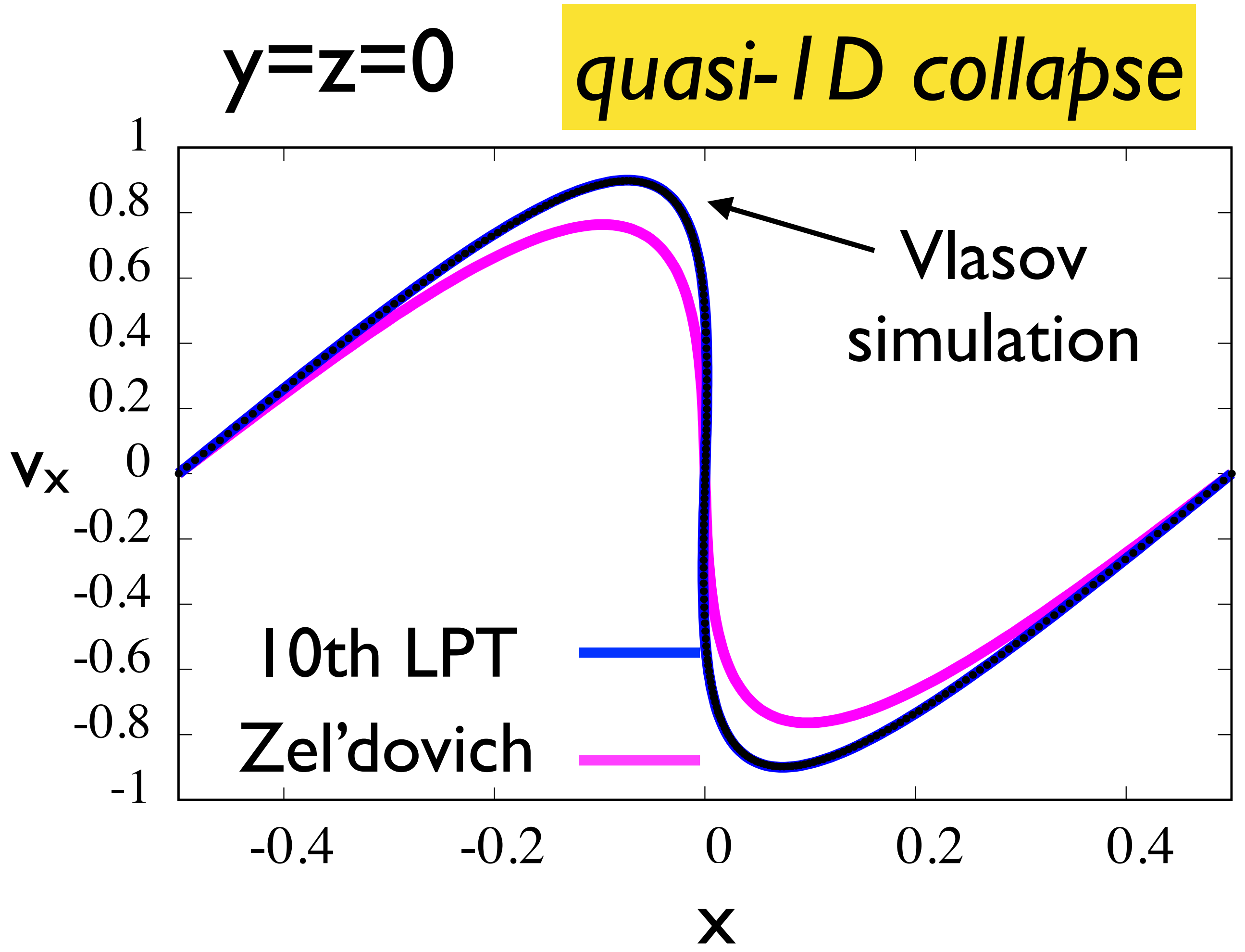
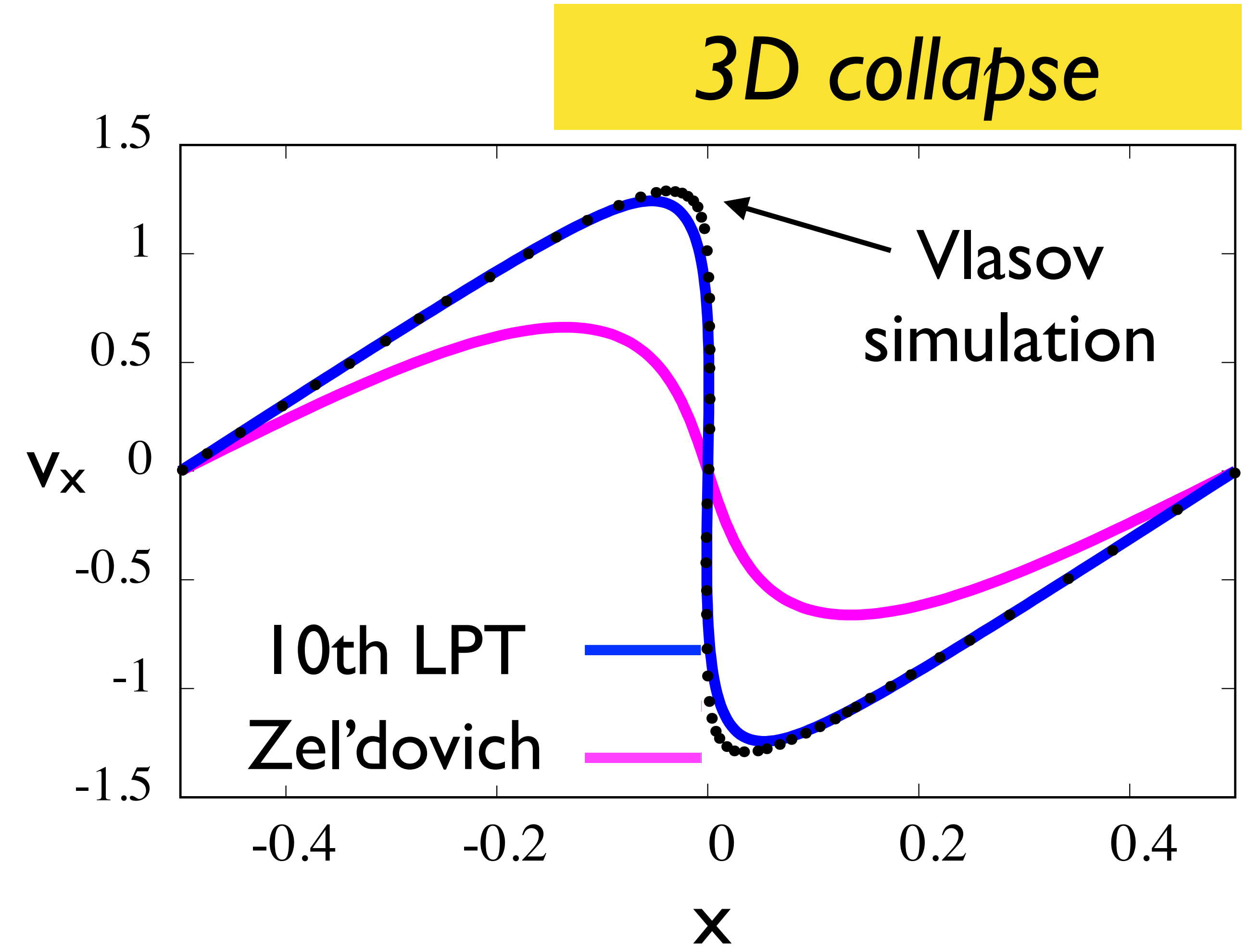
higher-order Lagrangian perturbation theory (LPT)





Simulation: ColDICE (Sousbie & Colombi '16)

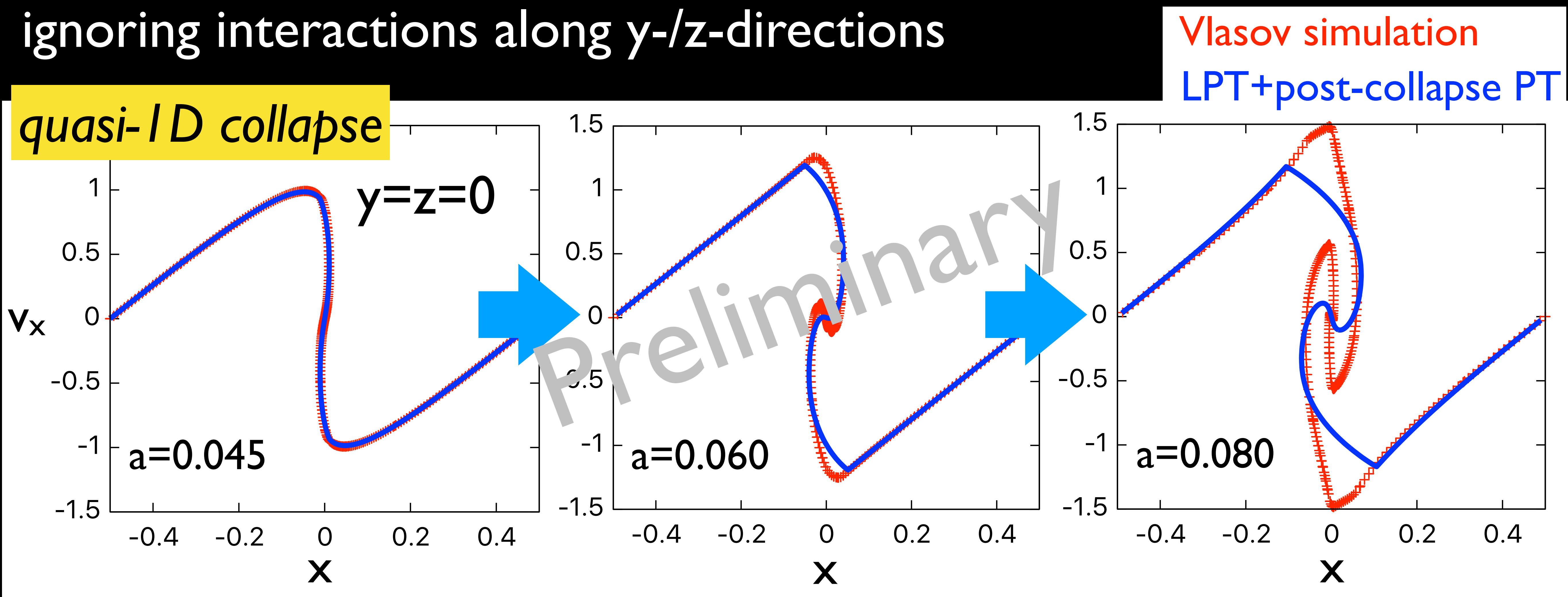
sine-wave initial condition (Moutarde et al. '91)

Beyond shell-crossing in 3D

(in progress)

Matching higher-order LPT with post-collapse PT (in 1D)

ignoring interactions along y-/z-directions



Multi-dimensional effect significantly alters multi-stream flow. Naive prediction with 1D post-collapse PT fails → need further progress

Summary

Analytical description of shell-crossing and beyond with perturbation theory (PT) calculations in cosmic structure formation

	Single-stream	*Multi-stream*
1D	Zel'dovich solution	$\boxed{\text{Post-collapse PT}}$
2D & 3D	$\boxed{\text{Higher-order Lagrangian PT}}$	*Work in progress*

- Combining with adaptive smoothing,
  Post-collapse PT gives an accurate statistical prediction of random density field
- Higher-order Lagrangian PT
  precisely matches results of the state-of-the-art Vlasov simulation

Extension to 3D case is still challenging, but worth for investigation