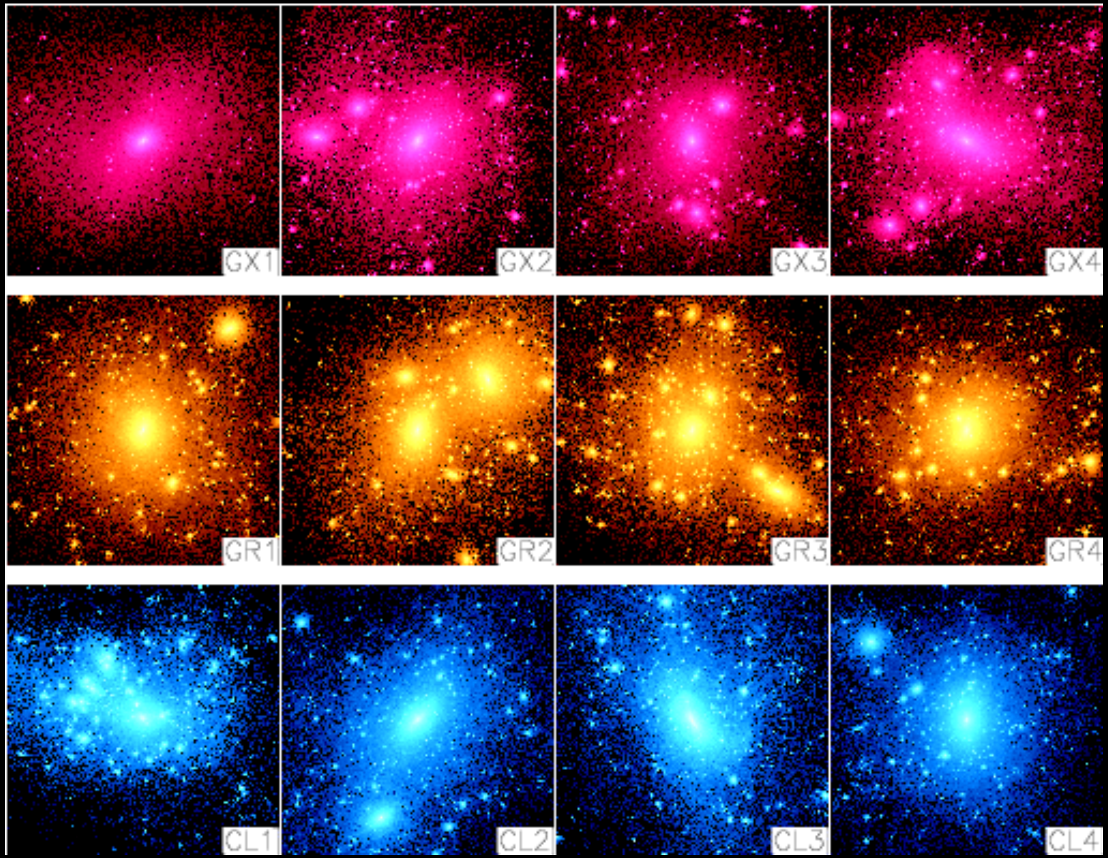
Non-sphericity of collisionless gravitating systems in the universe



dark matter halos from N-body simulations (Jing & Suto 2000)

Yasushi Suto

Department of Physics and RESCEU (Research Center for the early Universe) The University of Tokyo

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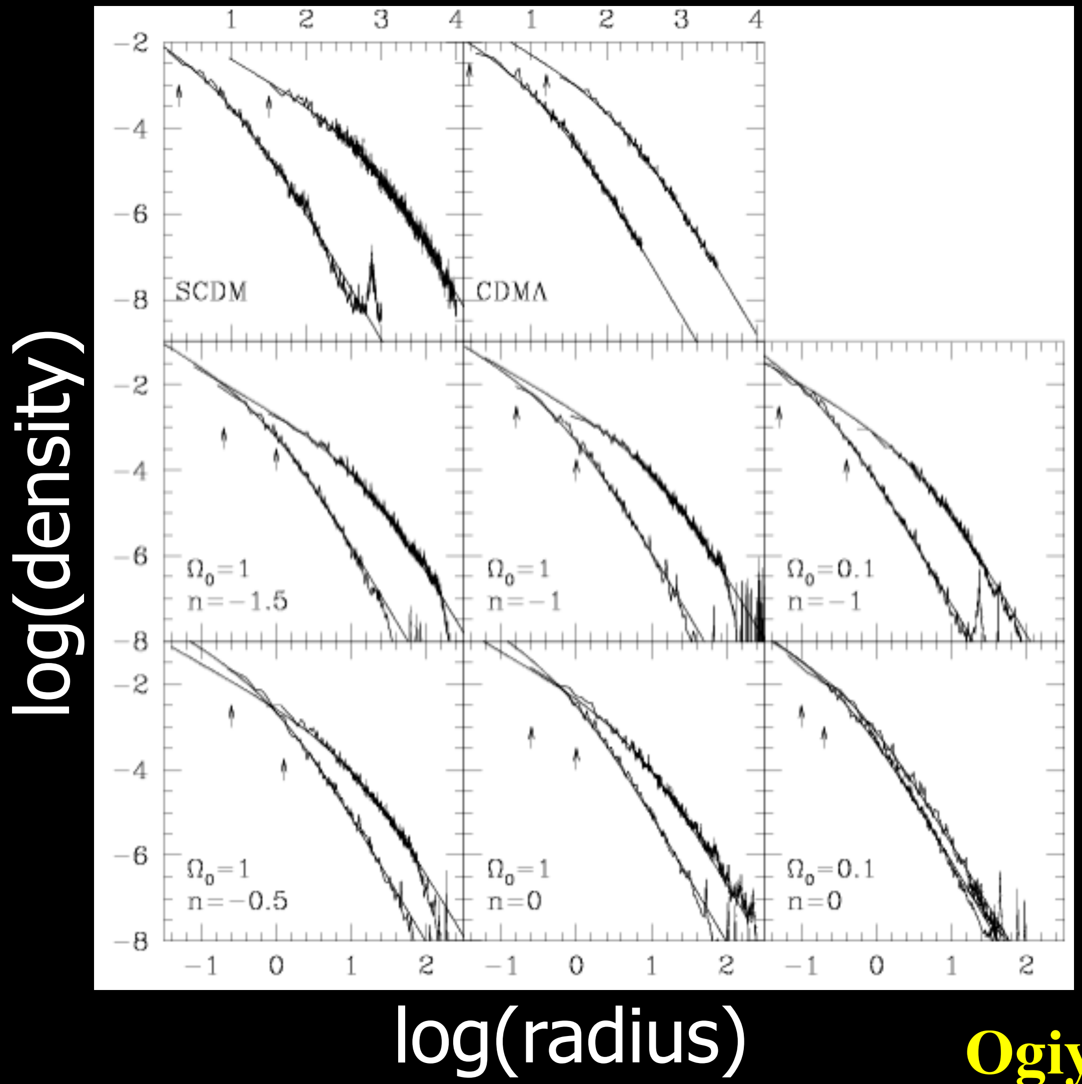
Shape and profile of **dark matter halos** (= collisionless self-gravitating systems in the universe)

- Theoretical question: *what is the final state of cosmological self-gravitating system (if any) ?*
  - Forget initial conditions and exhibit universality ?
  - Or initial memory is imprinted somewhere ?
- Practical importance: *testing cosmology and nature of dark matter against observations*
  - Gravitational weak/strong lensing
  - Optical/X-ray/radio observations of clusters of galaxies
  - Signature of dark matter decay/annihilation

Validity and limitation of spherical dust collapse model of dark matter halos

Universality of *spherically-averaged* density profiles: insensitive to initial conditions



- **NFW profile**
  - Spherically-averaged density profiles of collisionless dark matter halos

$$\rho(r) = \frac{\delta_c \rho_{crit}}{(r/r_s)(1 + r/r_s)^2}$$

Navarro, Frenk & White (1997); see also Fukushige & Makino (1997)

Ogiya's Talk on Friday !

Spherical dust collapse (SDC)

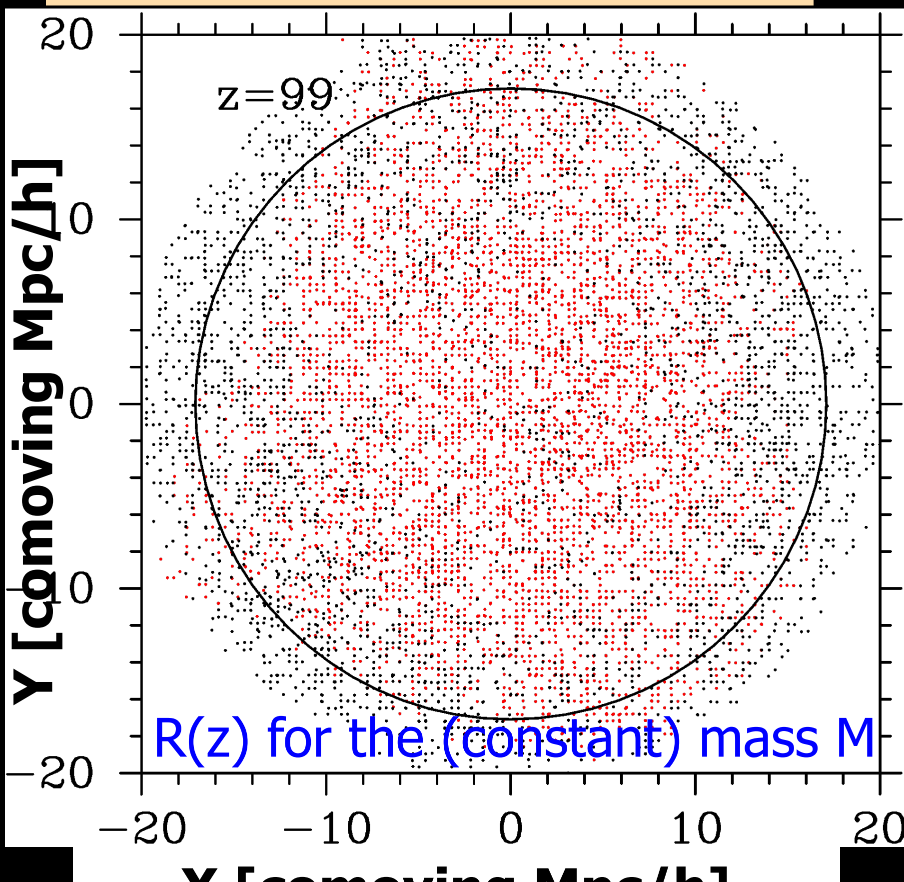
- An analytic solution to a spherical dynamics
  - A simple but widely-used approximation
  - e.g., dark matter halo abundance vs. cluster mass and temperature functions to determine cosmological parameters
- Attempts for improvement
  - shell crossing (e.g., Bertschinger 1985)
  - non-sphericity (e.g., Jing + YS 2002)
  - velocity dispersions (Suto, Kitayama, Osato, Sasaki + YS 2016a, PASJ 68, 14)

Comparison of the SDC model predictions against N-body simulation

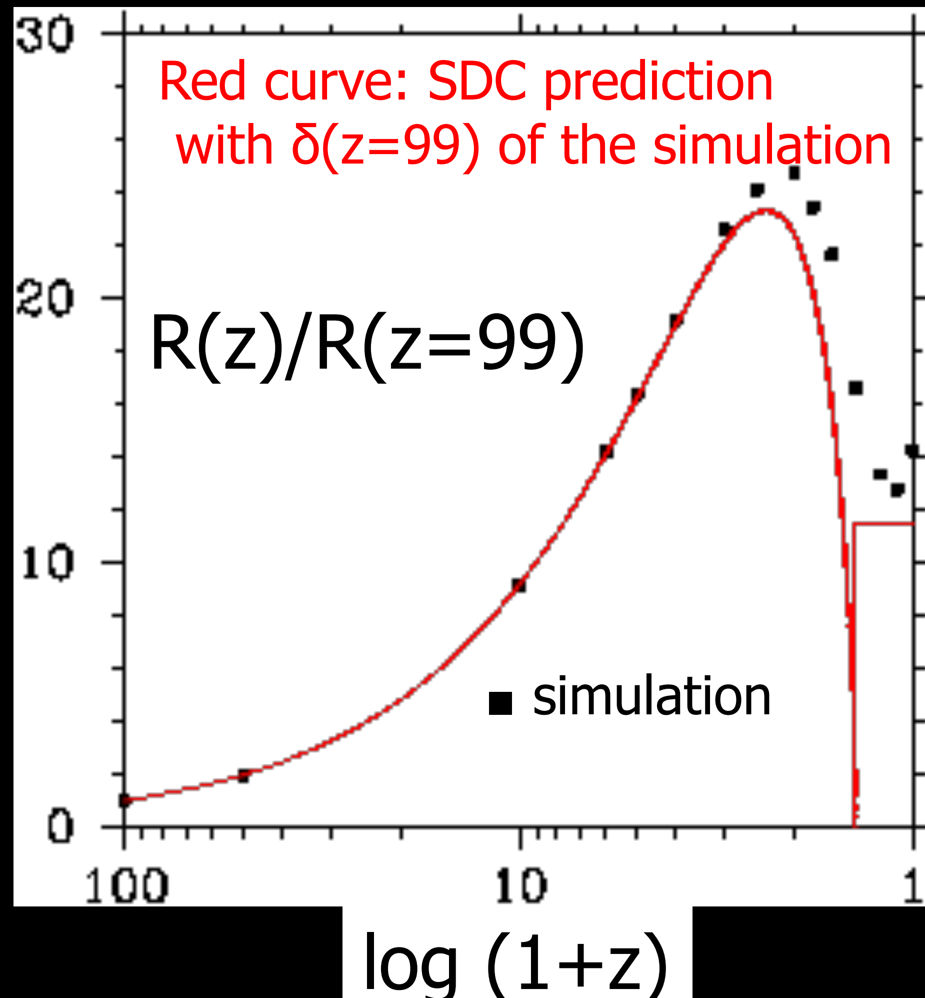
- Dark matter only simulations with GADGET-2
  - ΛCDM with WMAP9 cosmological parameters
  - $N=1024^3$ in $(360h^{-1}\ \mathrm{Mpc})^3$
  - $m=3.4 \times 10^9\ M_\odot$
- Self-gravitating systems identified at $z=0$
  - compute the spherical mass M and radius R of spherical overdensity of $\Delta=\rho/\rho_m=355.4$
  - Identifies the center-of-mass of the $z=0$ halo particles at z, and compute the radius $R(z)$ enclosing the mass M at $0<z<z_{initial} = 99$

The most massive halo with $M=1.66\times10^{15}\ M_\odot$

Red: FOF particles at z=0
Black: non-FOF particles

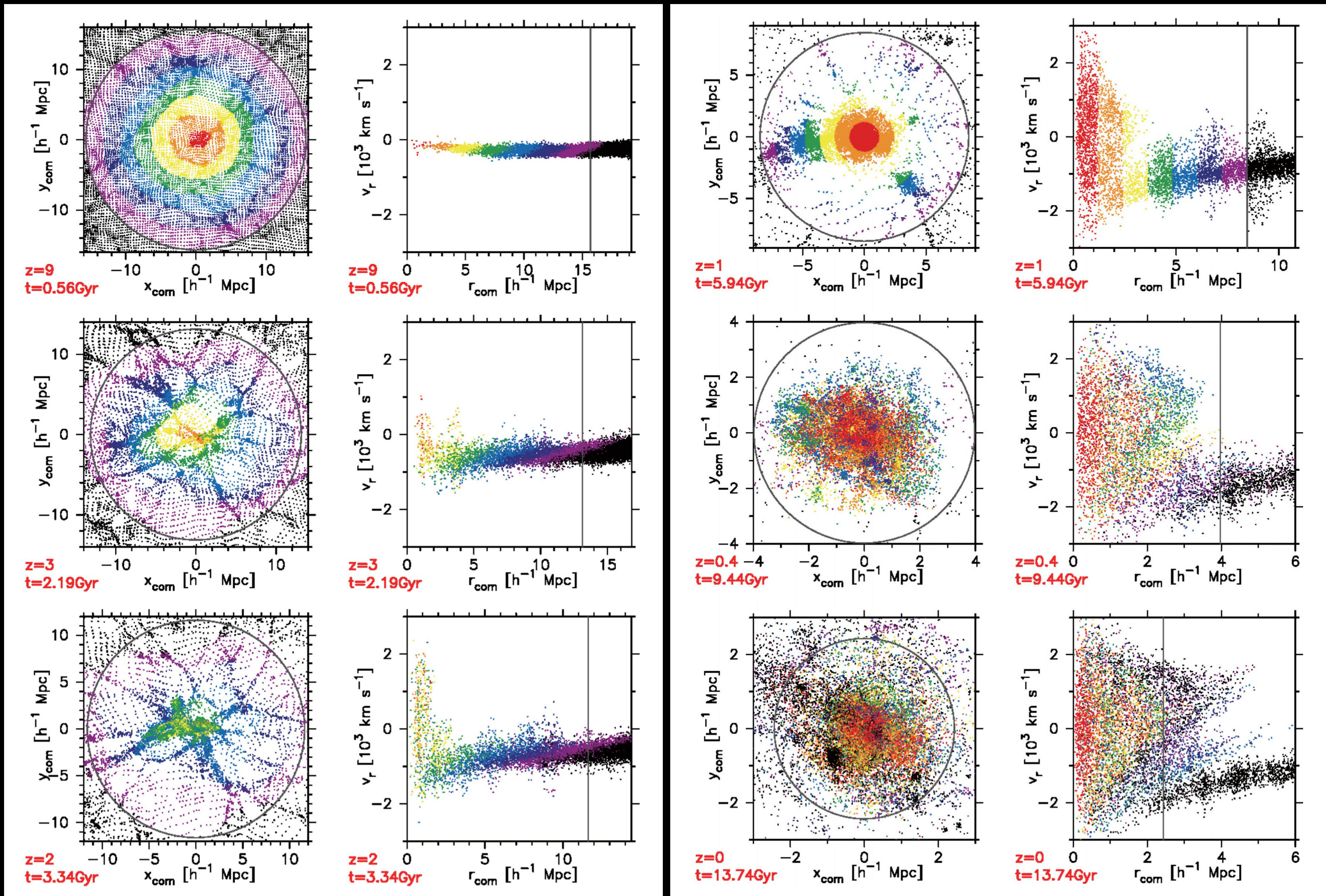


Sampled particles in a halo



Suto et al. (2016a)

Evolution in real and phase spaces



Effect of velocity dispersions

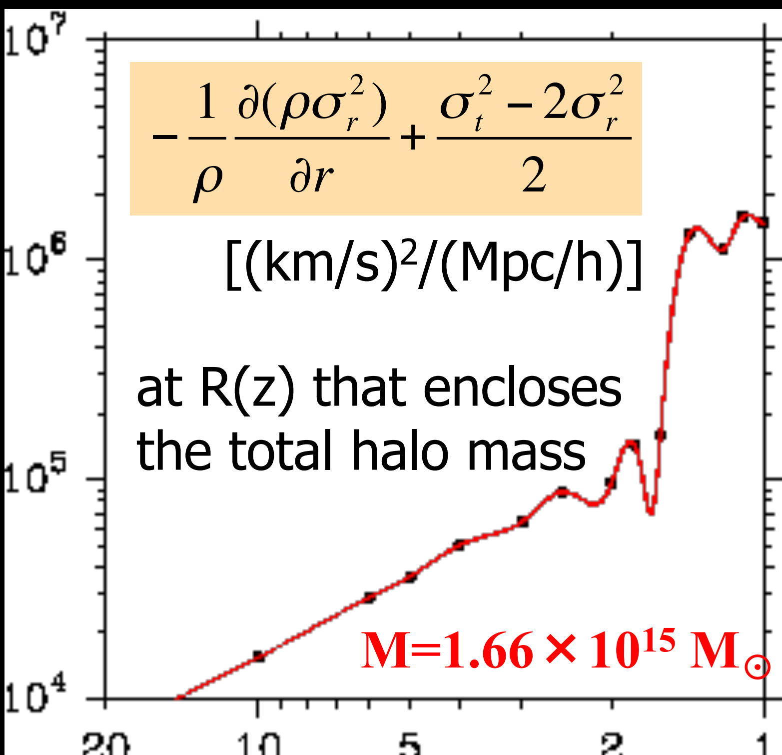
- Jeans equation for spherical collisionless system
  - radial velocity dispersion σ_r^2
  - tangential velocity dispersion σ_t^2

$$\frac{Dv_r}{Dt} = -\frac{1}{\rho}\frac{\partial(\rho\sigma_r^2)}{\partial r} + \frac{\sigma_t^2 - 2\sigma_r^2}{2} - \frac{GM}{r^2}$$

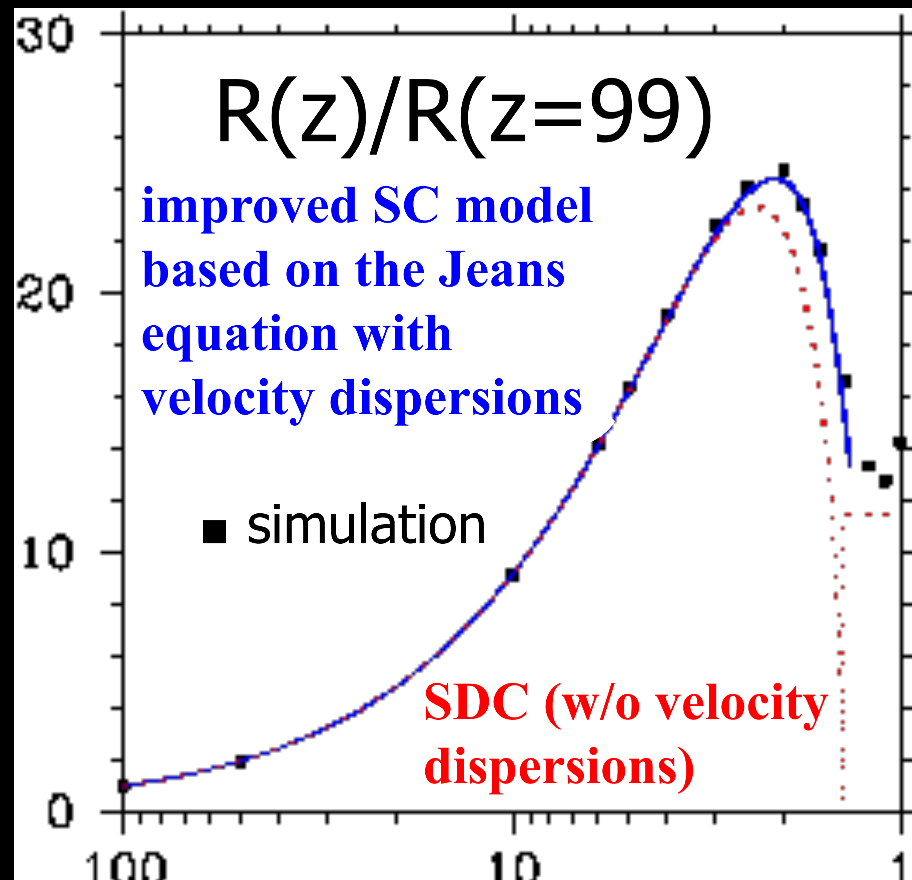
- SDC assumes an initially top-hat (homogeneous) sphere
  - neglects small-scale inhomogeneities, shell-crossing before turn-around, and thus no σ_r^2 or σ_t^2
- Larger $t_{\text{turn-around}}$ and R_{virial} than predicted by SDC

SDC improved with velocity dispersions

- Evaluate the velocity dispersions from simulation data and solve the Jean equation
- Better agreement!



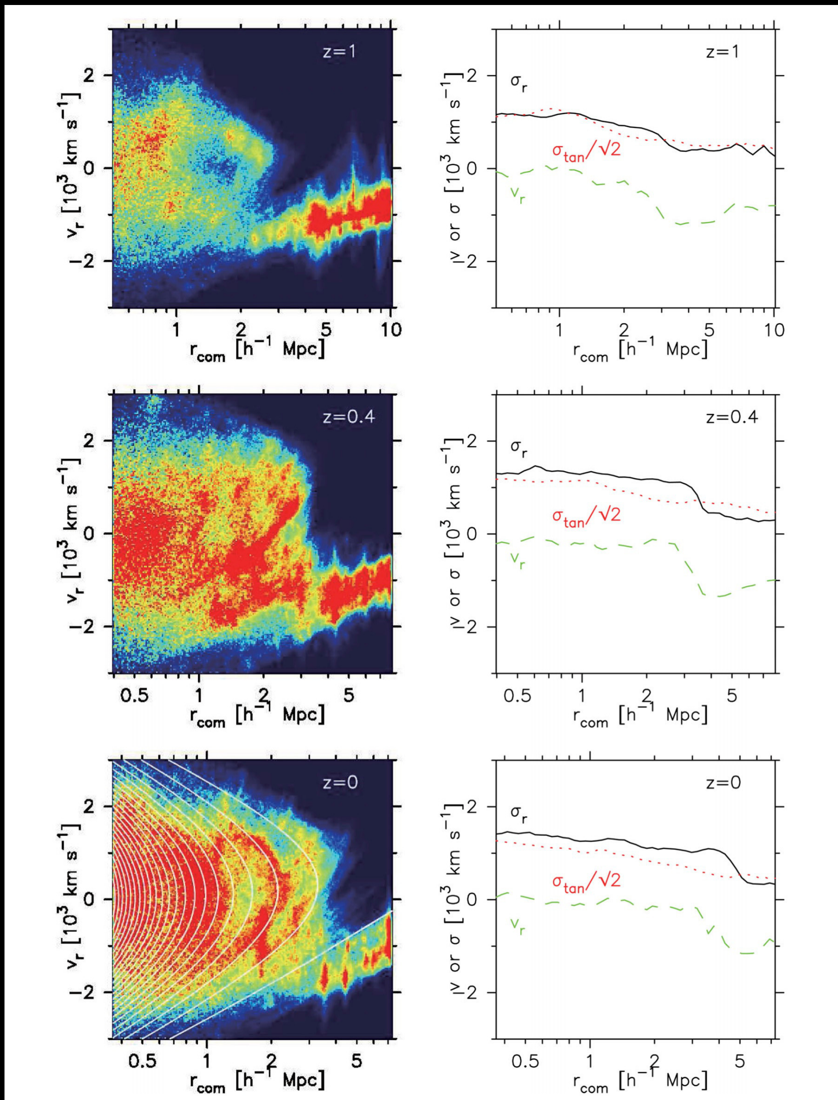
log(1+z)

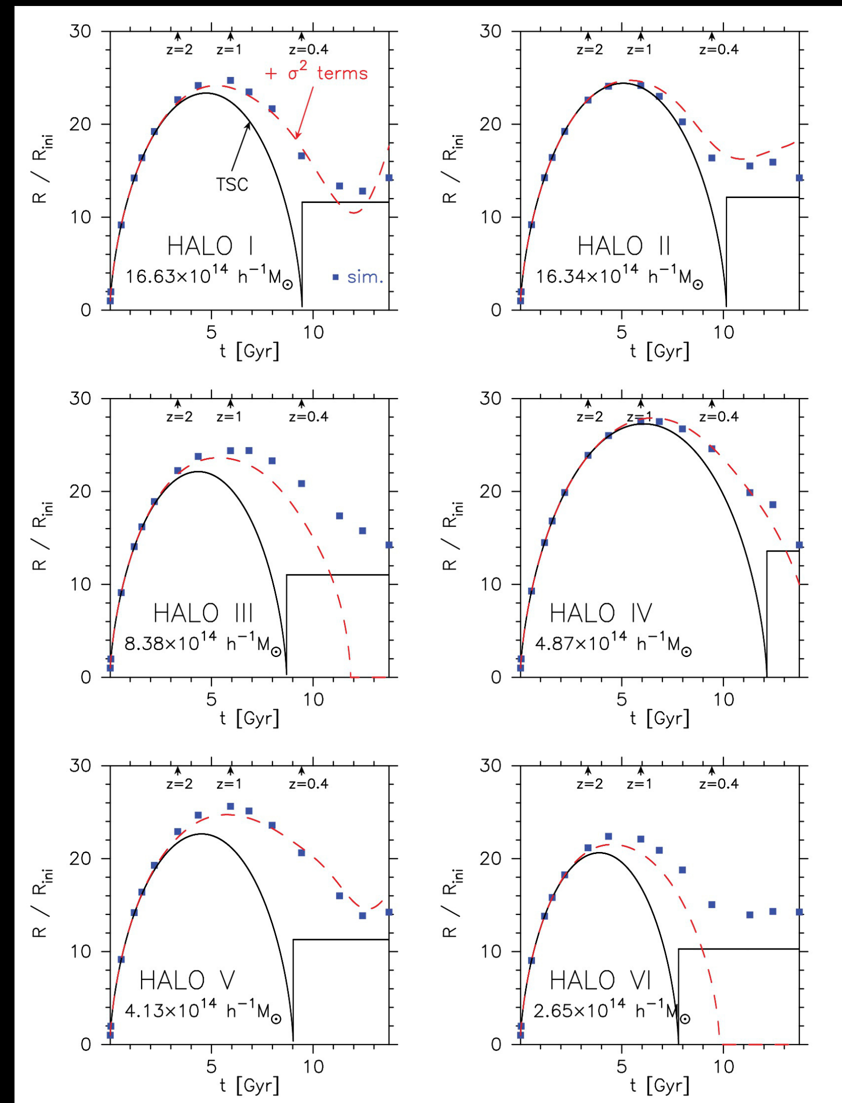


log(1+z)

Suto et al. (2016a)

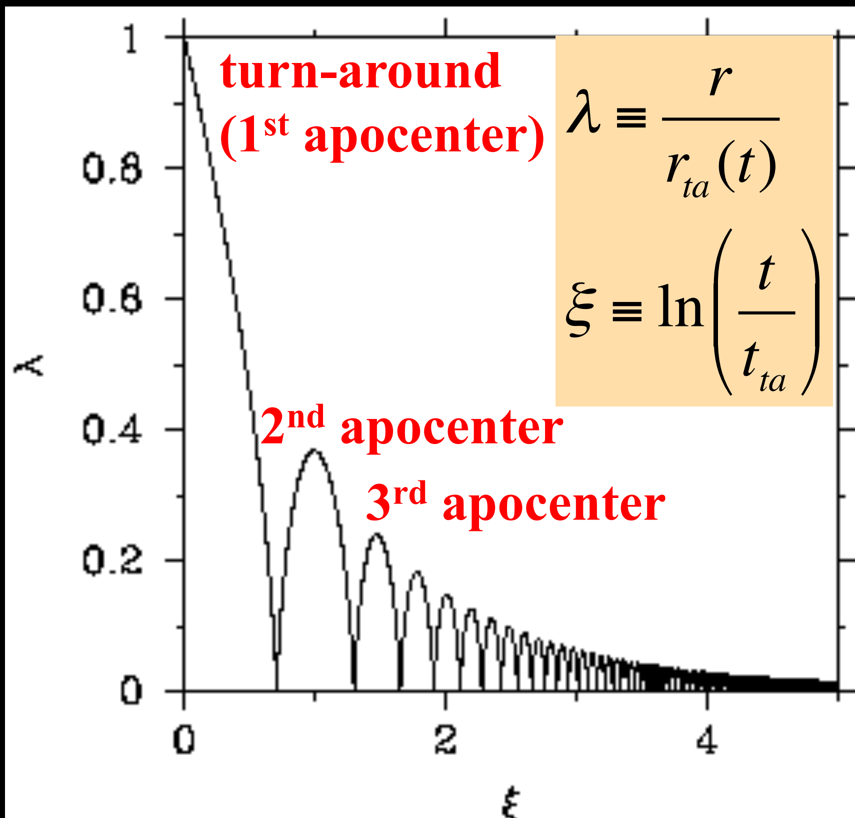
Spherical collapse with velocity dispersion

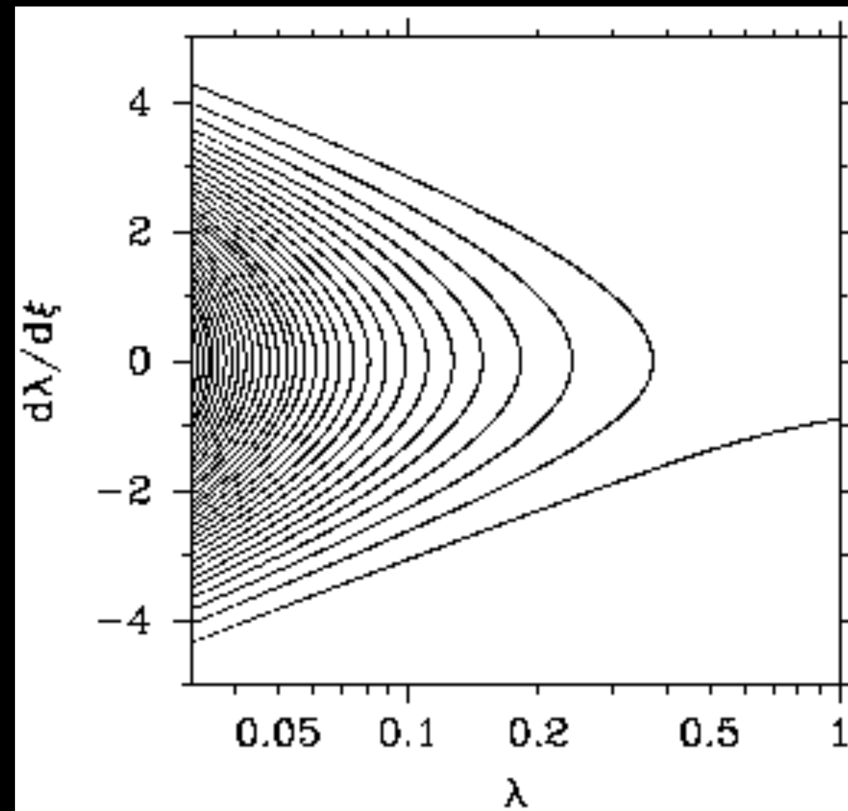




Suto, Kitayama, Osato, Sasaki + YS 2016a, PASJ 68, 14

Bertschinger's self-similar solution





- Self-similar shell crossing of collisionless particles: spherical secondary infall

Bertschinger 1985, ApJS, 58, 39

Beyond the spherical model: ellipsoidal collapse and phenomenological triaxial model

Dark matter halos are not spherical

galaxies

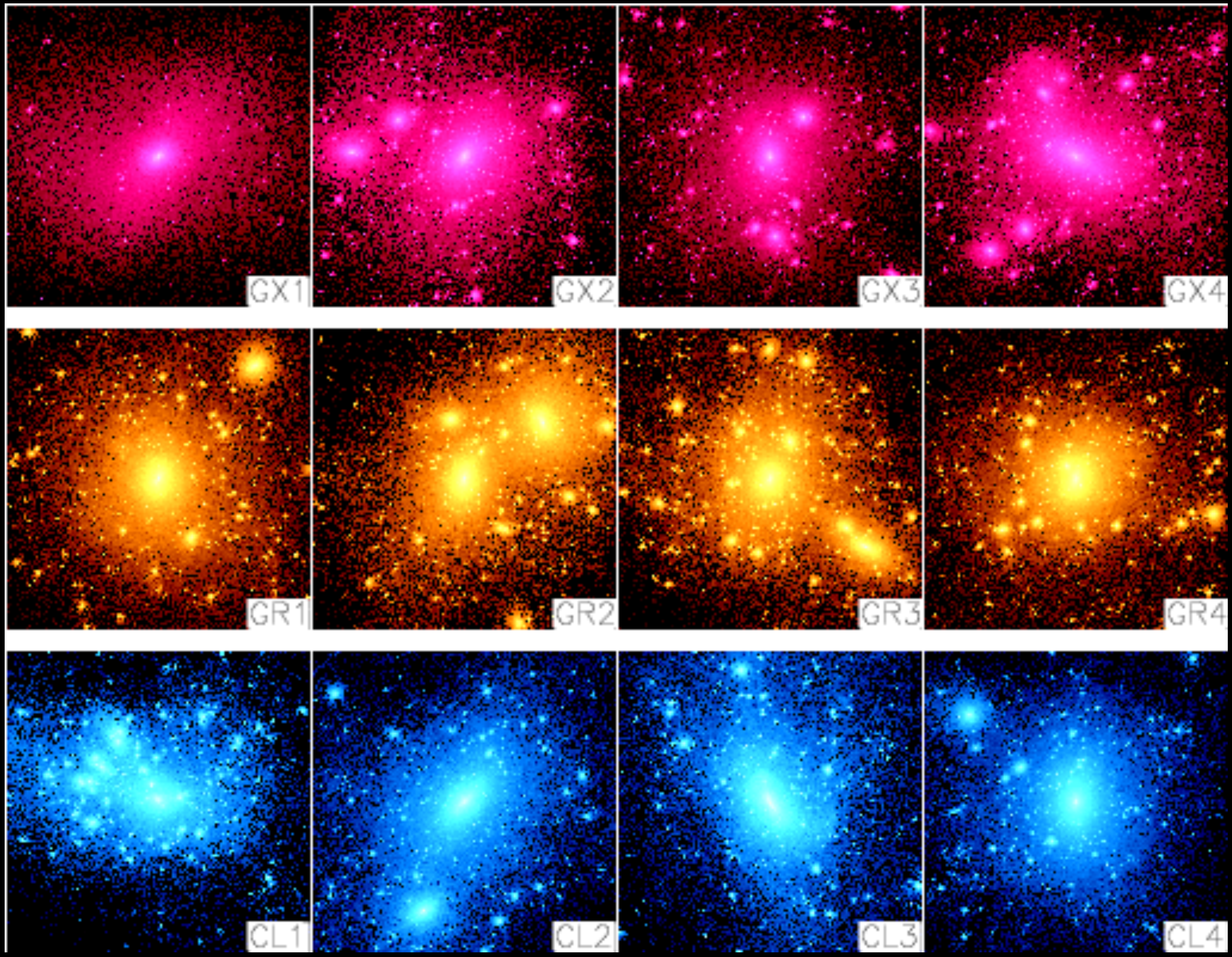
~ $5 \times 10^{12} M_{sun}$

groups

~ $5 \times 10^{13} M_{sun}$

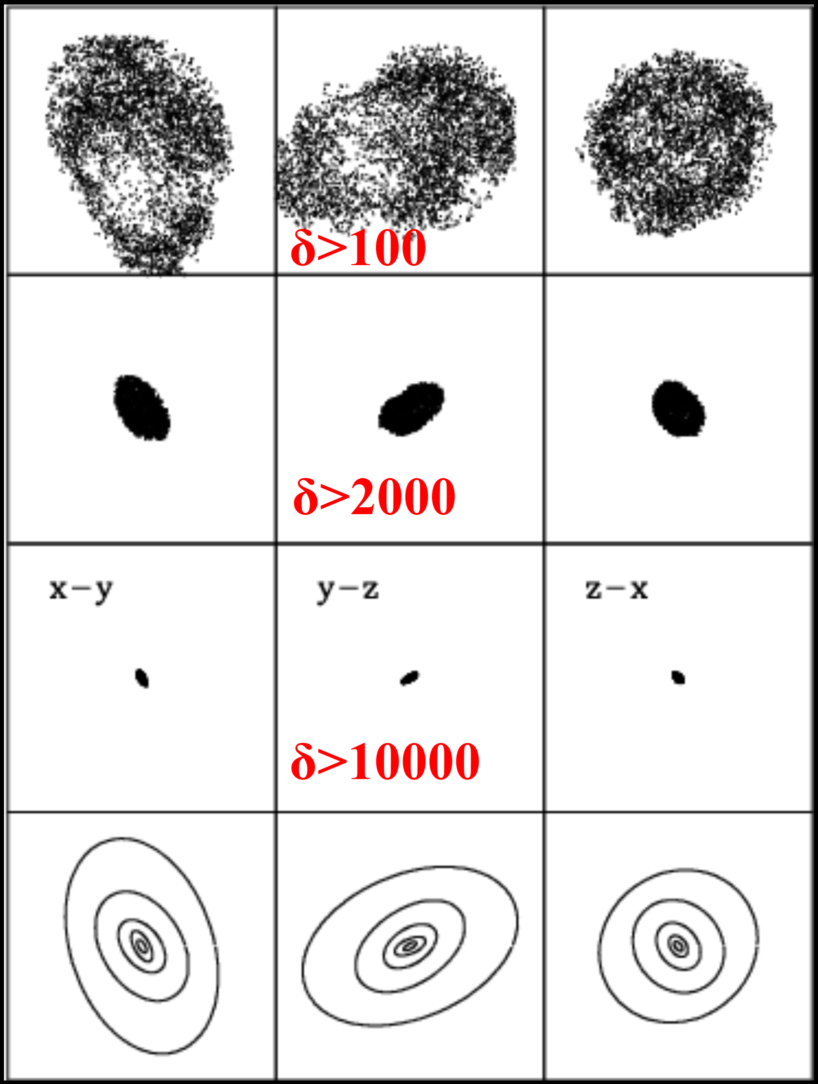
clusters

~ $3 \times 10^{14} M_{sun}$



Jing & Suto ApJL 529(2000) L69

Beyond spherical modelling: phenomenological triaxial fit



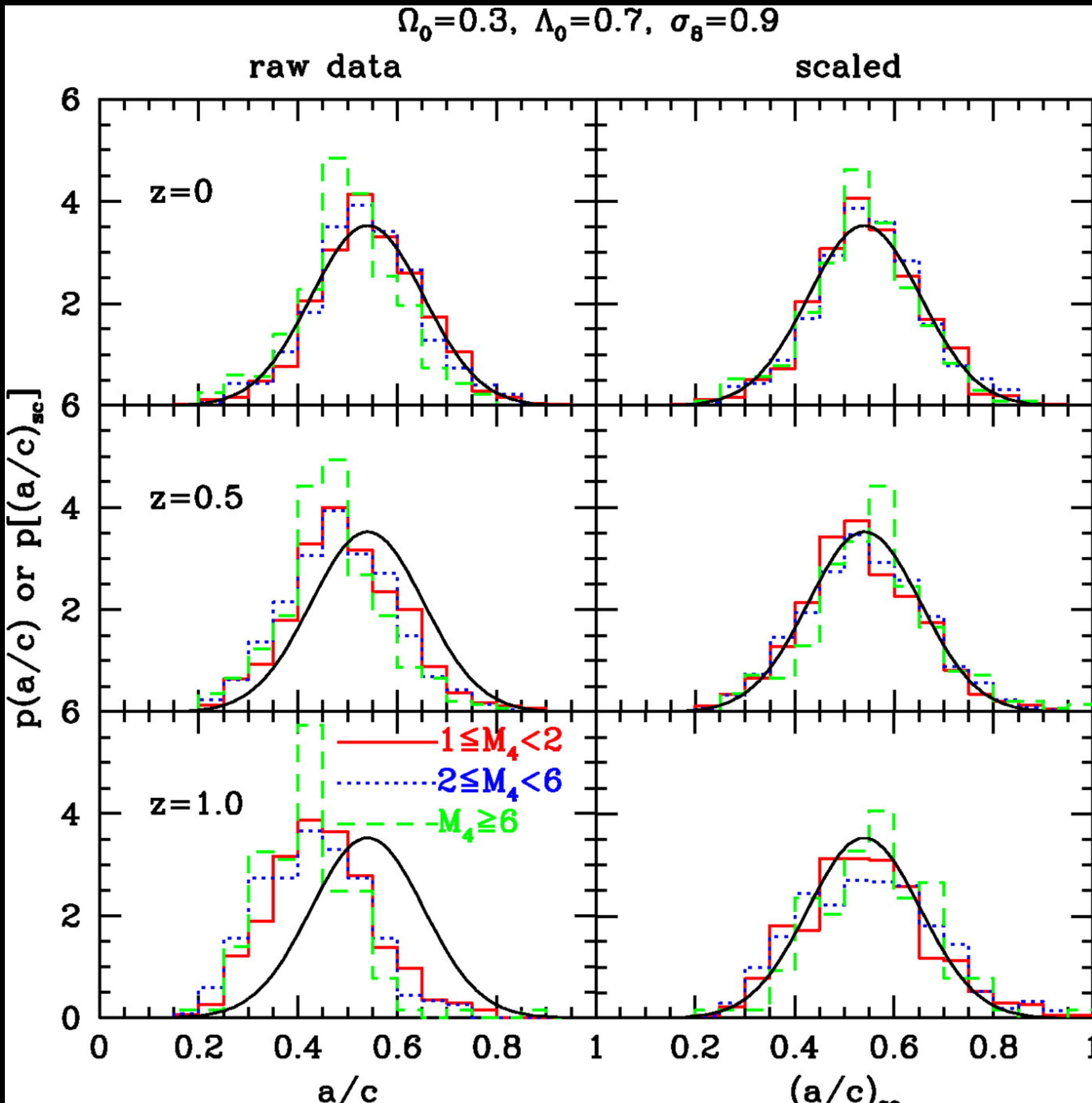
$$\rho(R) = \frac{\delta_c \rho_{crit}}{(R/R_s)^{\alpha}(1+R/R_s)^{3-\alpha}}$$

$$R^2(\rho) = \frac{x^2}{a^2(\rho)} + \frac{y^2}{b^2(\rho)} + \frac{z^2}{c^2(\rho)}$$

Jing & Suto ApJ 574 (2002) 538

- While it is widely applied for many cosmological problems, it is very simplified
  - Concentric & self-similar (axis ratio is independent of radius)

Probability density function of axis ratios



Jing & Suto (2002)

Scaled axis ratio

$$\tilde{r}_{ac} = \left(\frac{a}{c}\right)_{scaled} = \left(\frac{a}{c}\right)\left(\frac{M_{vir}}{M_{nonlinear}(z)}\right)^{0.07\Omega(z)^{0.7}}$$

PDF of the scaled axis ratio

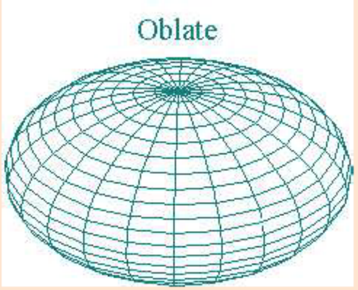
$$p(\tilde{r}_{ac}) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(\tilde{r}_{ac}-0.54)^2}{2\sigma^2}\right)$$

$$\sigma = 0.113$$

- Higher z for a given mass, less spherical
- More massive at a given z, very slightly less spherical

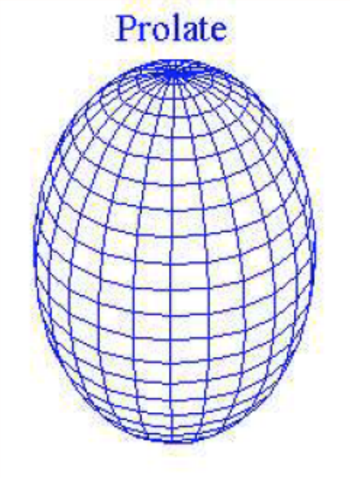
Triaxial fitting parameters for halo shape

- $a \simeq b \gg c$: Oblate object (disk-shaped) $\rightarrow$ pancake



0<p<e

- $b \simeq c \ll a$: Prolate object (cigar-shaped) $\rightarrow$ filament



0>p>-e

G.Rossi (2011)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad (a > b > c)$$

$$c = R(1 - \lambda_1)$$

$$b = R(1 - \lambda_2)$$

$$a = R(1 - \lambda_3)$$

$$\lambda_1 > \lambda_2 > \lambda_3 \qquad \delta = \lambda_1 + \lambda_2$$

- **Ellipticity**

$$e = \frac{\lambda_1 - \lambda_3}{2\delta}$$

- **Prolateness**

$$p = \frac{\lambda_1 + \lambda_3 - 2\lambda_2}{2\delta}$$

Ellipsoidal collapse model

- **Basic equations**

Axis length

$$\frac{d^2 A_k(t)}{dt^2} = \Omega_{\Lambda,0} H_0^2 A_k(t) - 4\pi G\bar{\rho}(t) A_k(t)\left[\frac{1+\delta(t)}{3} + \frac{b'_k\delta(t)}{2} + \lambda'_{\mathrm{ext},k}(t)\right]$$

Tidal force within the homogeneous ellipsoid

$$b'_k(t) = \prod_j A_j(t)\int_0^\infty \frac{d\tau}{[A_k^2(t)+\tau]\prod_j\sqrt{A_j^2(t)+\tau}} - \frac{2}{3}$$

External tidal force assuming linear growth

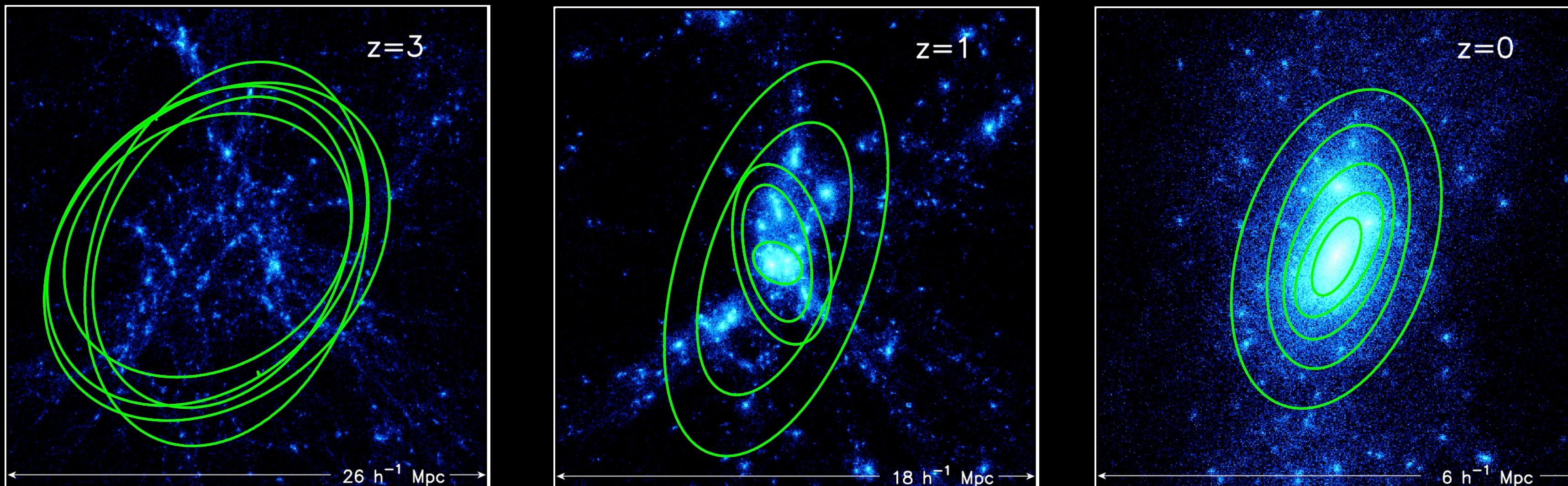
$$\lambda'_{\mathrm{ext},k}(t) = \frac{D(t)}{D(t_{\mathrm{ini}})}\left[\lambda_k(t_{\mathrm{ini}}) - \frac{\delta(t_{\mathrm{ini}})}{3}\right]$$

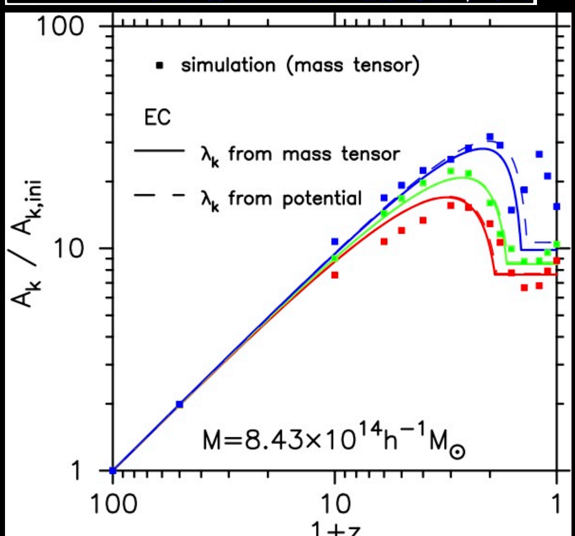
- **Initial condition at t_{ini}**

$$A_k(t_{\mathrm{ini}}) = a(t_{\mathrm{ini}})[1-\lambda_k(t_{\mathrm{ini}})]$$

$$\frac{dA_k(t_{\mathrm{ini}})}{dt} = H(t_{\mathrm{ini}})\left[A_k(t_{\mathrm{ini}}) - a(t_{\mathrm{ini}})\lambda_k(t_{\mathrm{ini}})\left.\frac{d\ln D}{d\ln a}\right|_{t=t_{\mathrm{ini}}}\right]$$

Evolution of non-sphericity: ellipsoidal collapse vs. N-body

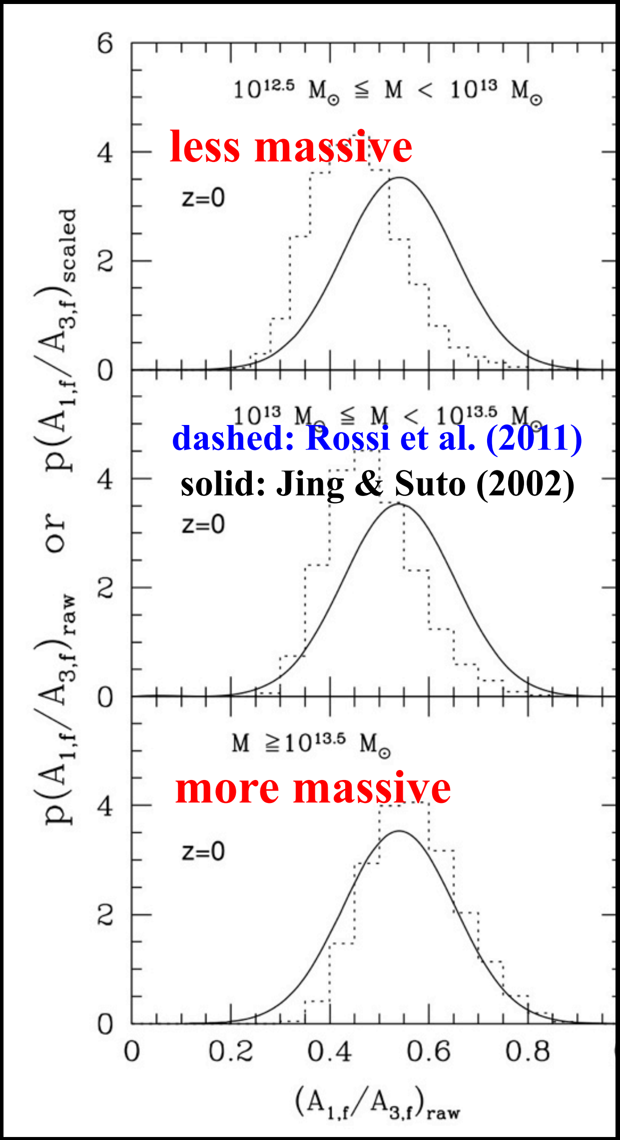




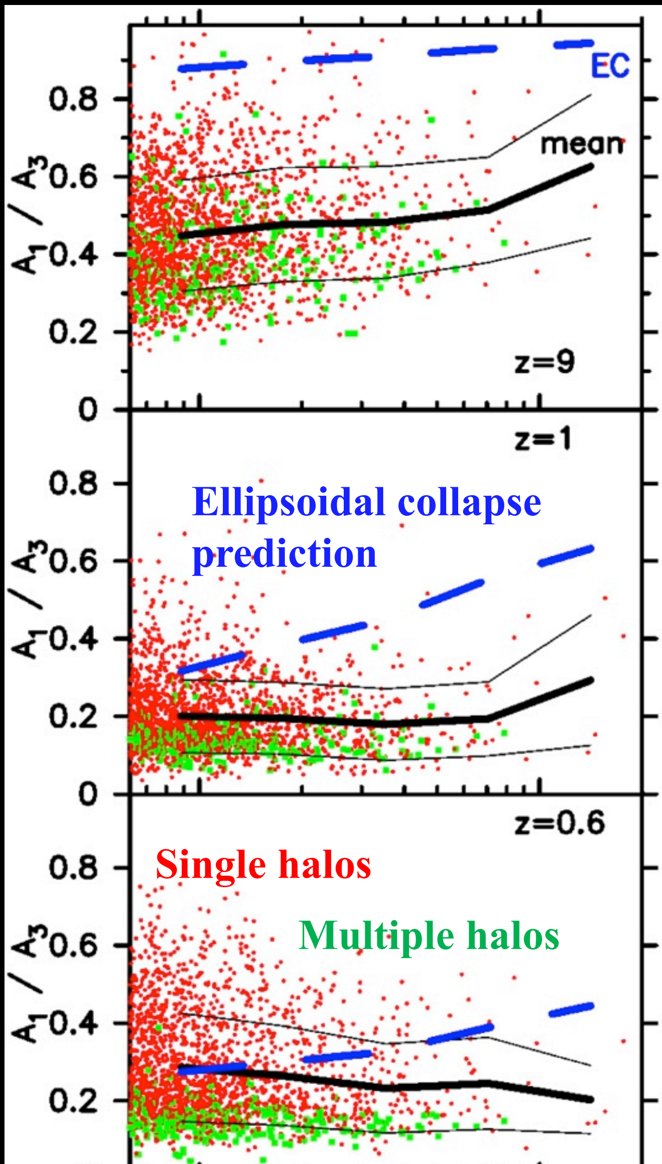
- Individual halo evolution is in reasonable, even if not good, agreement with ellipsoidal collapse *before virialization*
  - Suto et al. (2016b) PASJ, 68, 97

Does ellipsoidal collapse model improve the spherical collapse model ?

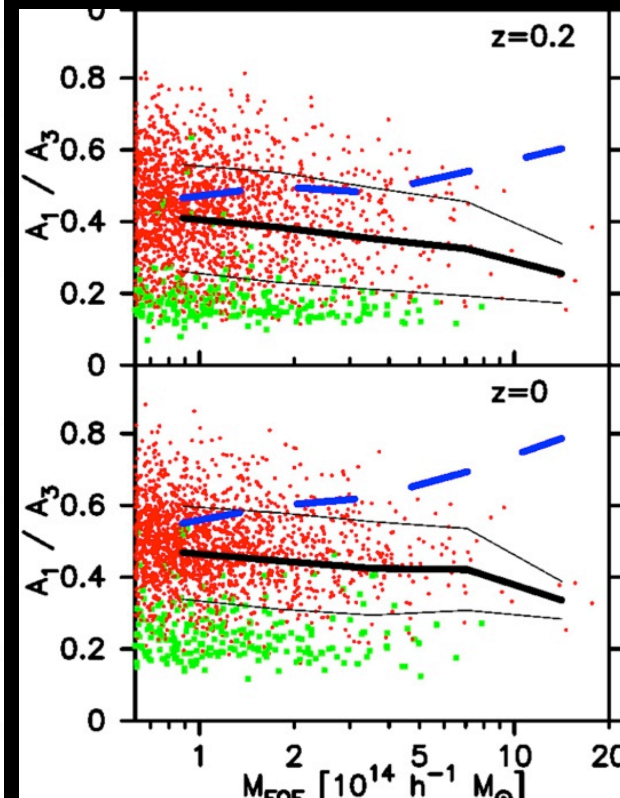
- Unfortunately no (not so much)
  - Ellipsoidal collapse model (Rossi, Sheth & Tormen 2011; dashed) predicts that more massive halos are more spherical
  - N-body simulations (Jing & Suto 2002; solid) indicate that non-sphericity is fairly insensitive to mass (more massive halos are slightly less spherical)



Axis ratio evolution of N-body halos

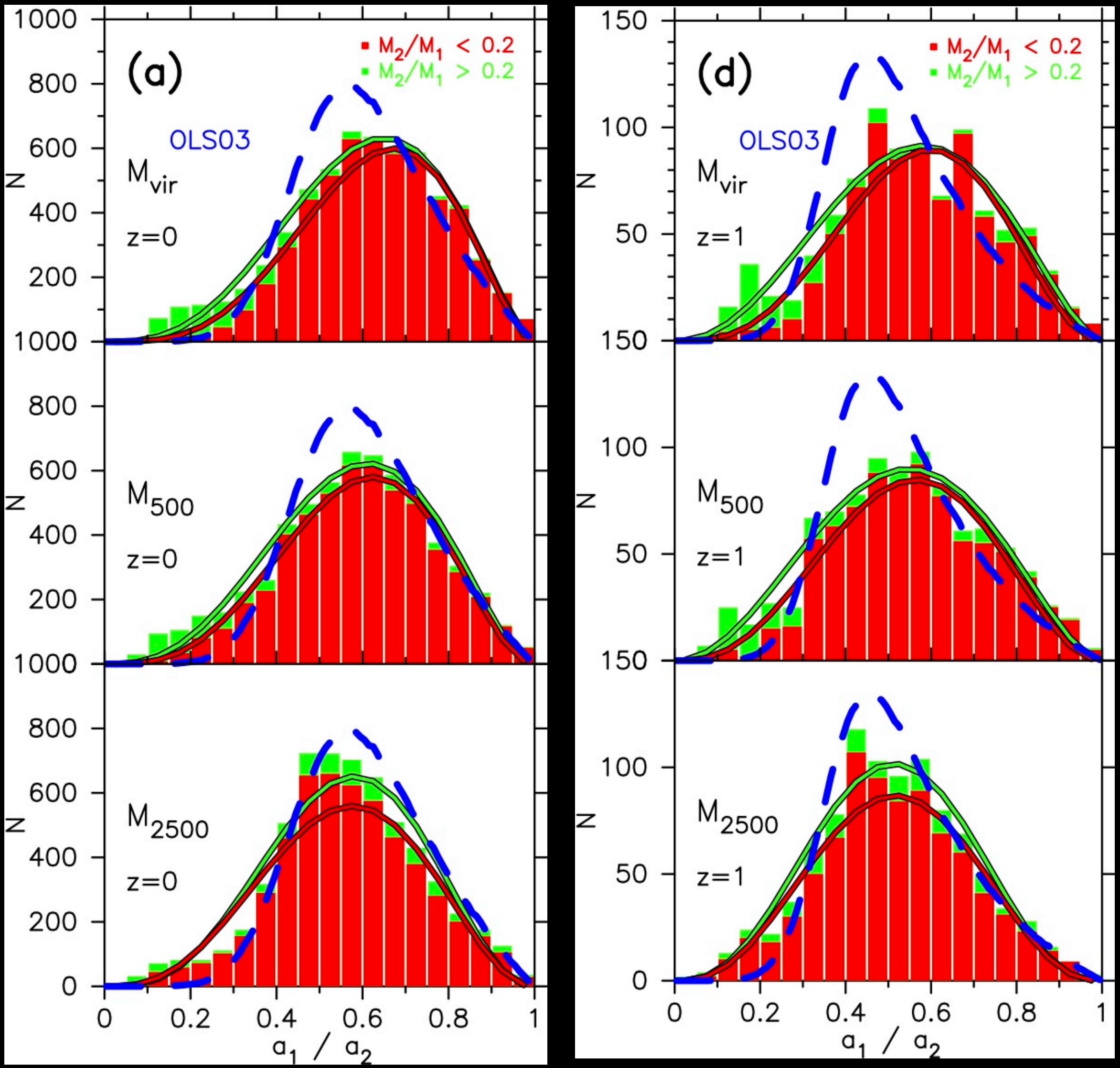


- Mass dependence
  - very slightly less spherical for larger mass, which is opposite to ellipsoidal collapse prediction



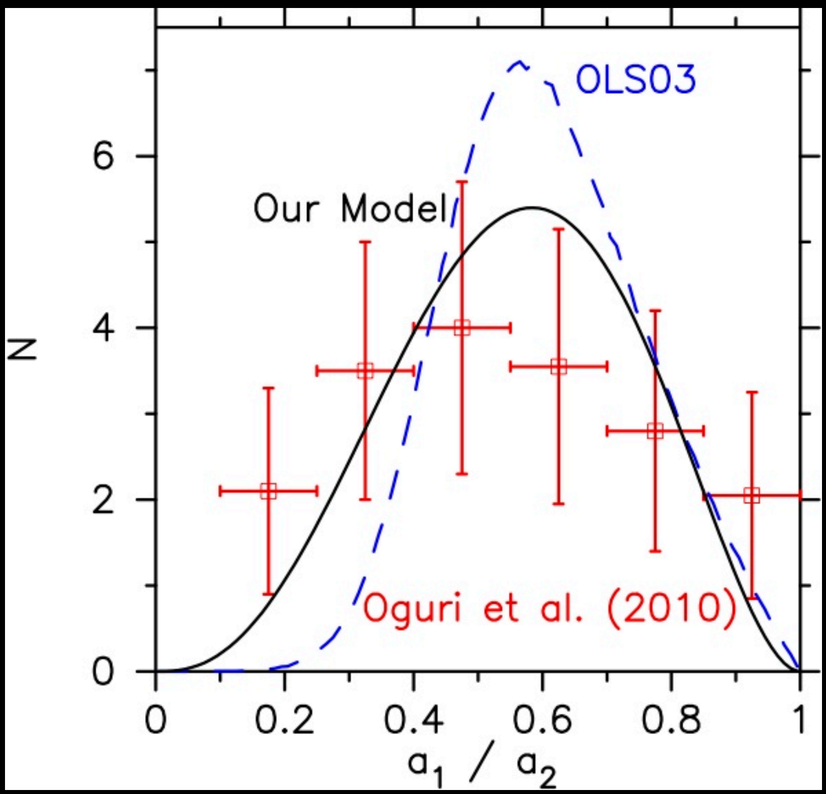
- Time dependence
  - Become less spherical until turn-around, and then more spherical

PDF of projected axis ratios



- insensitive to redshift
- Slightly less spherical towards inner region
- Very different from the self-similar projected model (Oguri, Lee & Suto 2003)
- Empirically fitted to β-distribution

Tentative comparison with observed axis ratio distribution from weak lensing



- Subaru Suprime-Cam weak-lensing map for 18 massive clusters (Oguri et al. 2010, MNRAS 405, 2215)
- Our result fits the observed data better than the OLS03 prediction
- Can be tested against future data from Subaru Hyper Supreme-Cam lensing survey

Summary

- Dark matter halos (collisionless self-gravitating systems) exhibit a certain universality
  - Seem to forget its initial condition during virialization (collisionless relaxation)
  - Including velocity dispersion improves the spherical collapse model
  - Ellipsoidal collapse model does not reproduce N-body results so well
- Phenomenological triaxial model to N-body results is useful for comparison with observations, e.g., constraining self-interacting dark matter