The Complexity of the Dark Matter Sheet

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MPA Garching

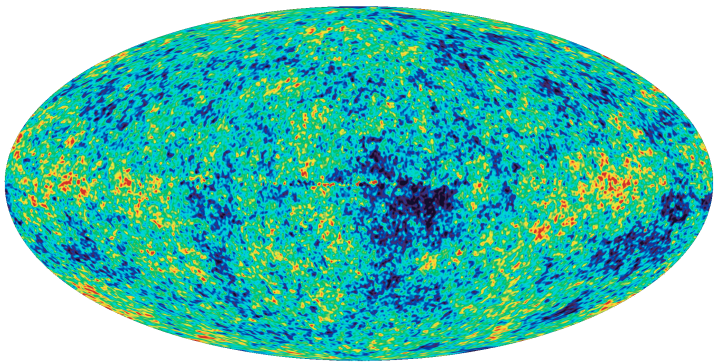
With Simon White, Oliver Hahn and Raul Angulo

November 3rd, 2017

Structure Formation

Our universe at $z \sim 1000$:

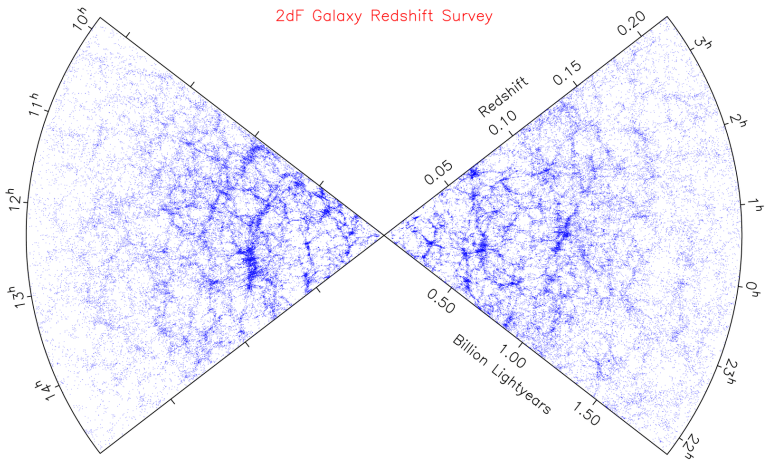
- Extremely homogeneous
- Small Density perturbations $\delta \ll 1$
- Mathematically Simple



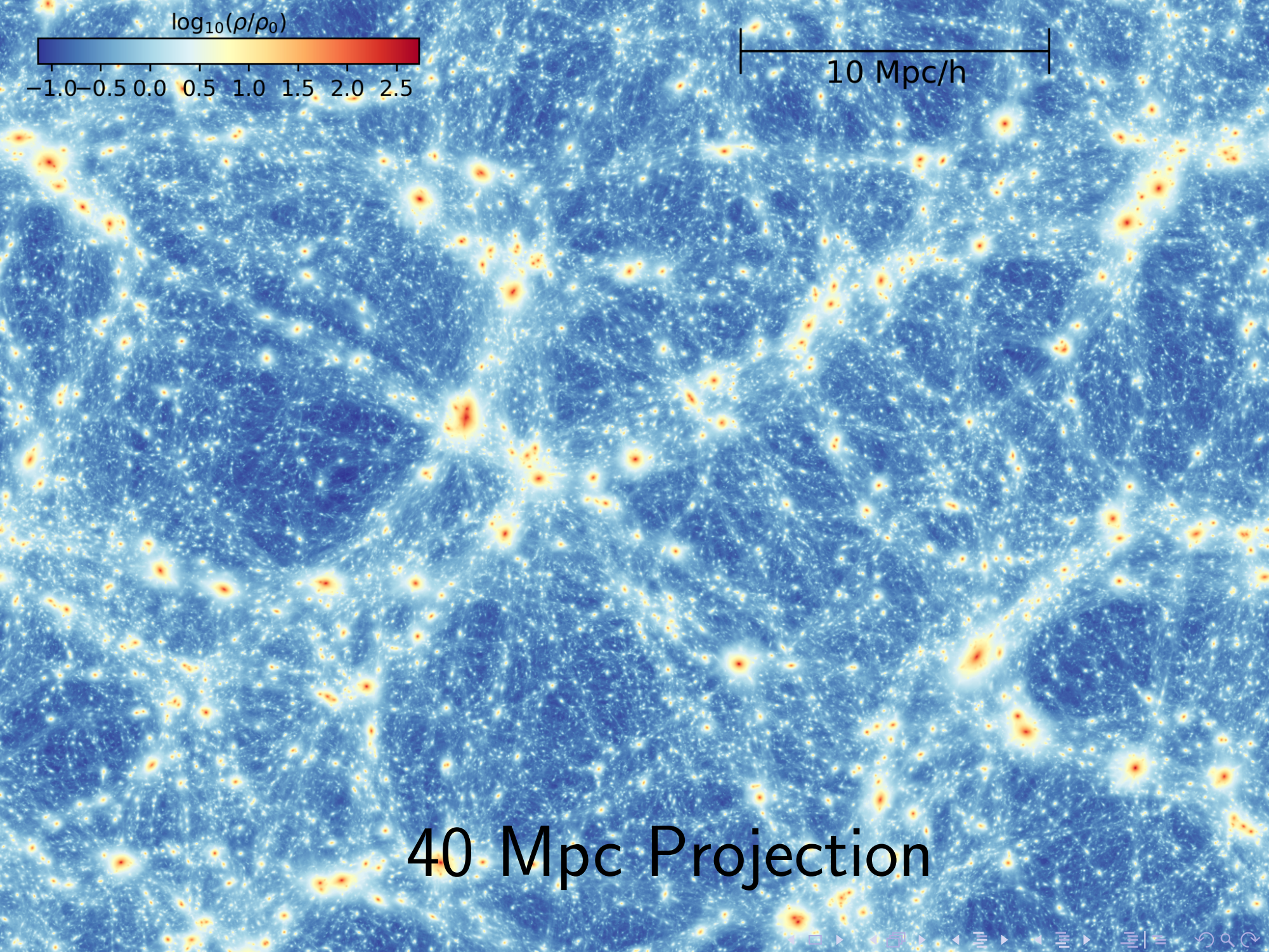
WMAP (2015)

Our universe today ($z = 0$):

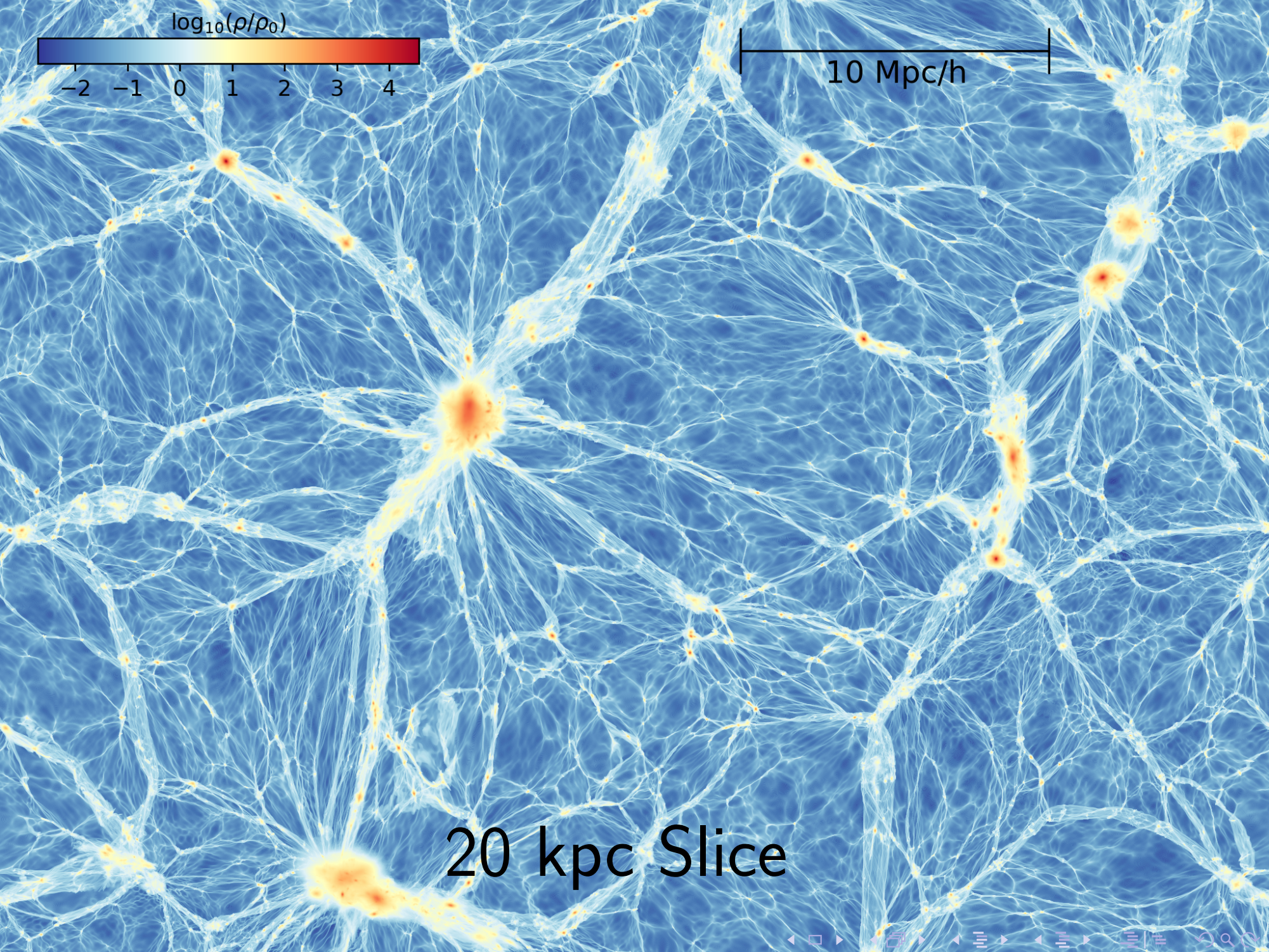
- Very inhomogeneous $\delta \gg 1$
- Cosmic Web, Haloes, Galaxies
- Very Complex

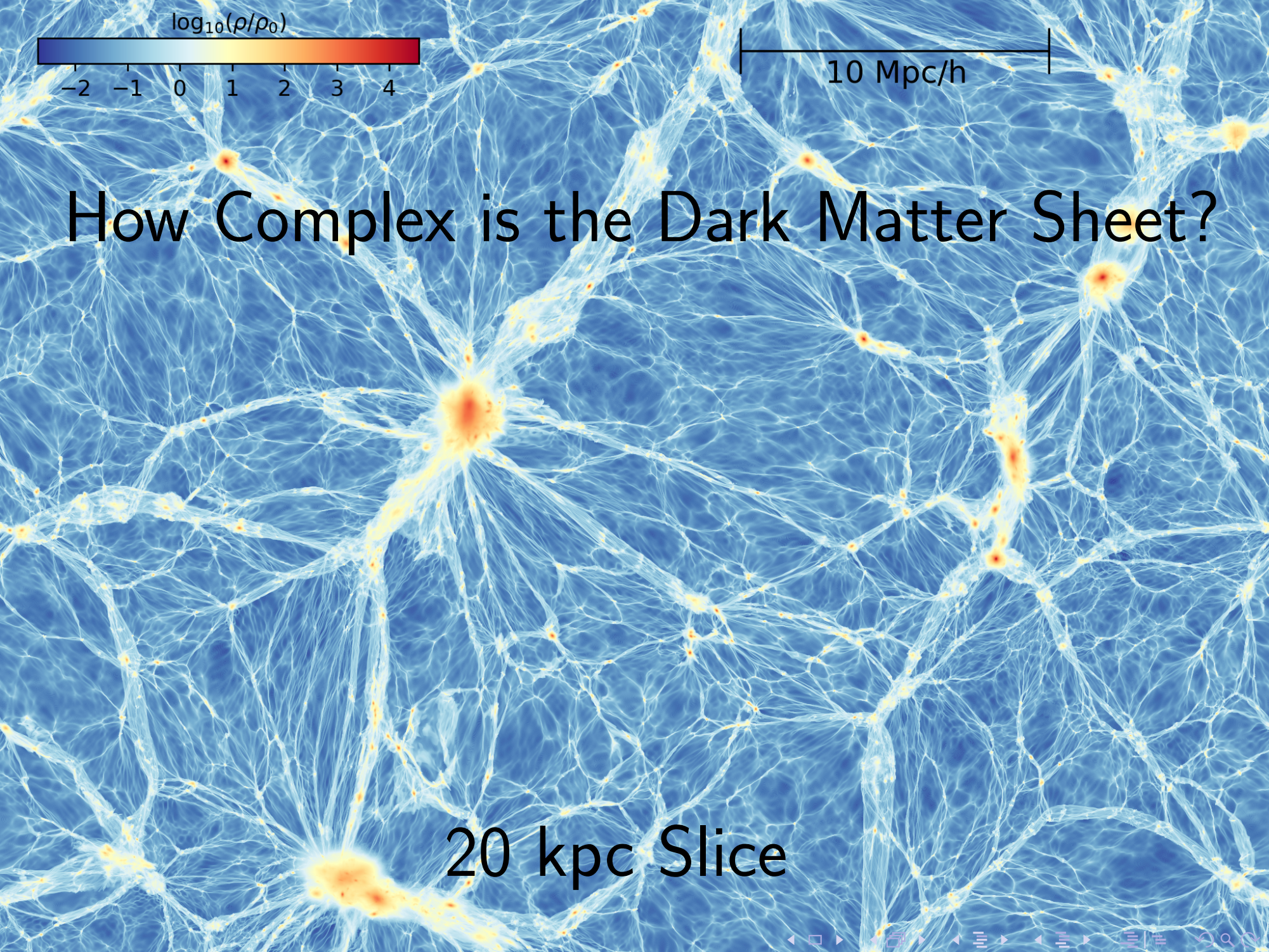


2DFGRS (2017)



40 Mpc Projection

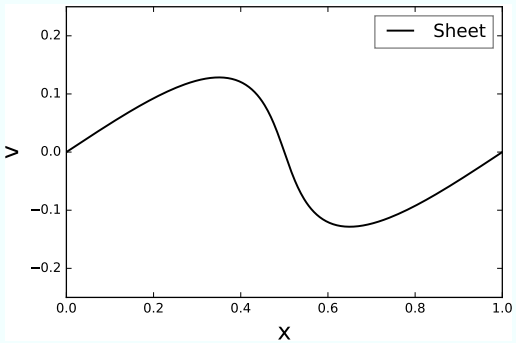

$\log_{10}(\rho/\rho_0)$
−2 −1 0 1 2 3 4
10 Mpc/h
20 kpc Slice

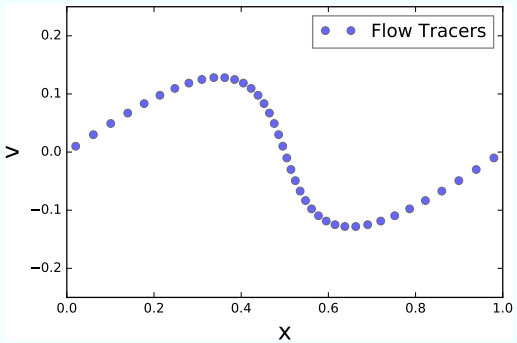

$\log_{10}(\rho/\rho_0)$
−2 −1 0 1 2 3 4
10 Mpc/h
How Complex is the Dark Matter Sheet?
20 kpc Slice

The Dark Matter Sheet in Phase Space

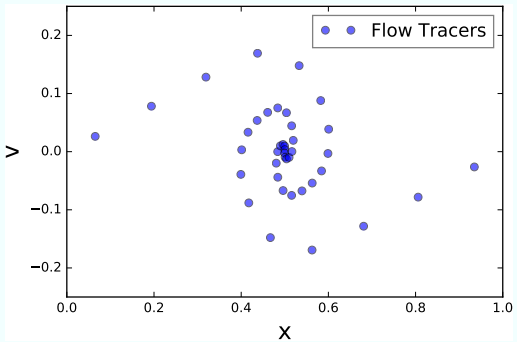
Shandarin et al. (2012), Abel et al. (2012), Hahn & Angulo (2016), Sousbie & Colombi (2016)

Discretize $\Rightarrow$





$\Downarrow$ Evolve

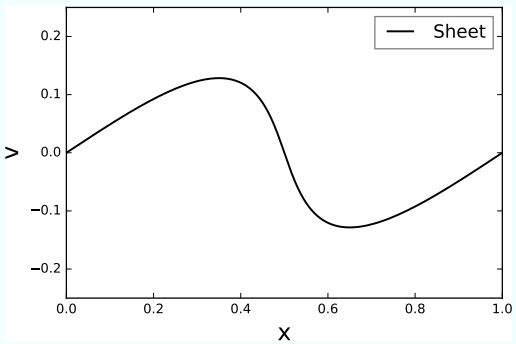


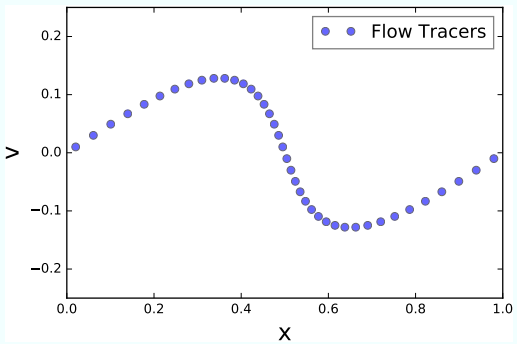
$\Leftarrow$?

The Dark Matter Sheet in Phase Space

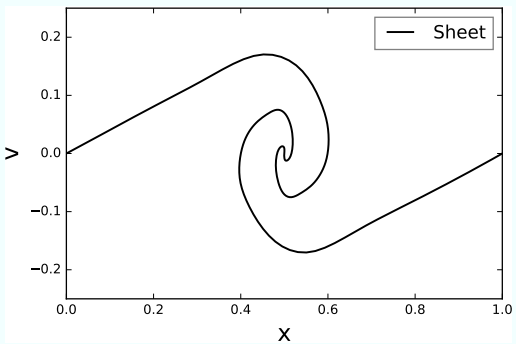
Shandarin et al. (2012), Abel et al. (2012), Hahn & Angulo (2016), Sousbie & Colombi (2016)

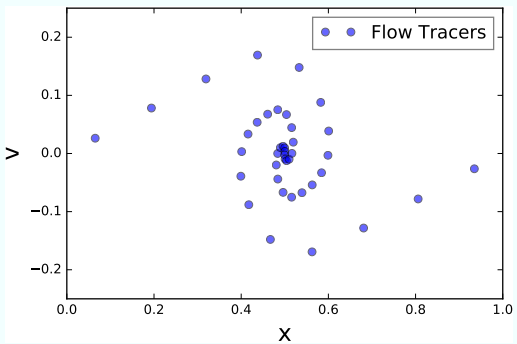
Discretize ⇒





⇓ Evolve

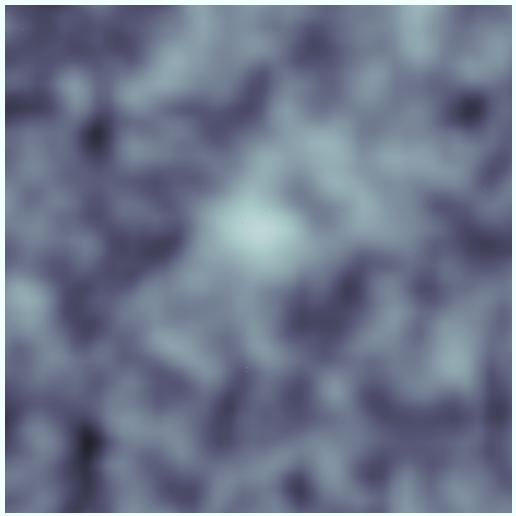


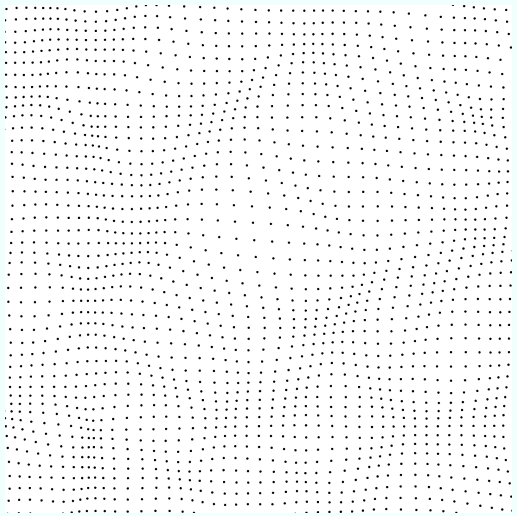


⇐ Interpolate

Discretize $\Rightarrow$

$z \sim 100$

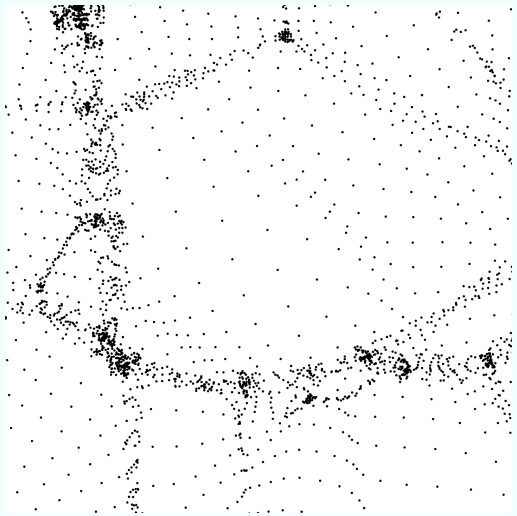




$\Downarrow$ Evolve

$z = 0$

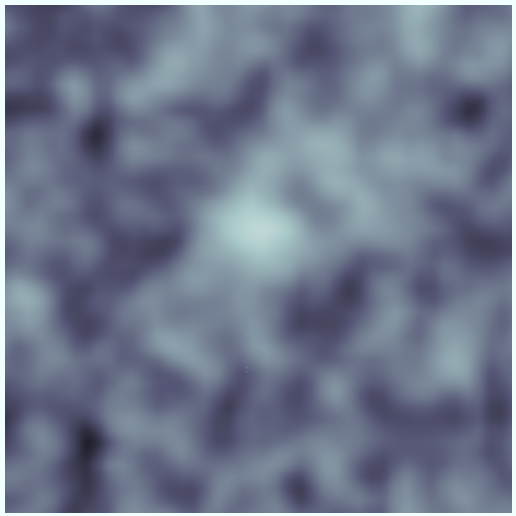
?

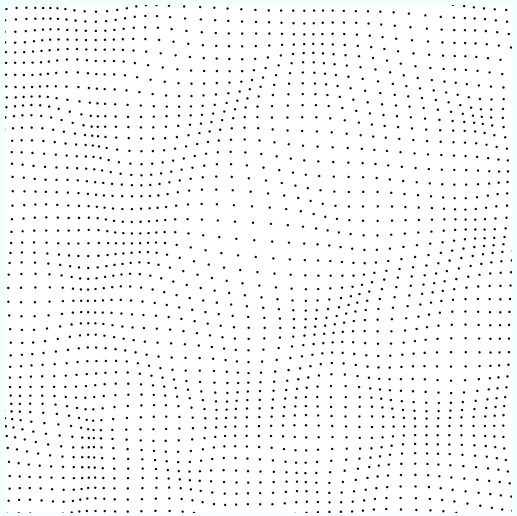


$\Leftarrow$?

Discretize $\Rightarrow$

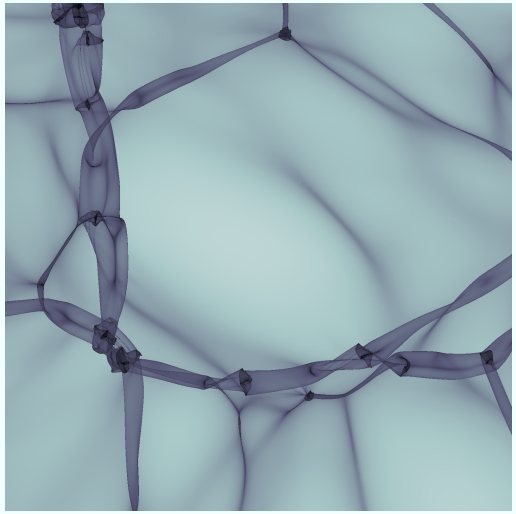
$z \sim 100$

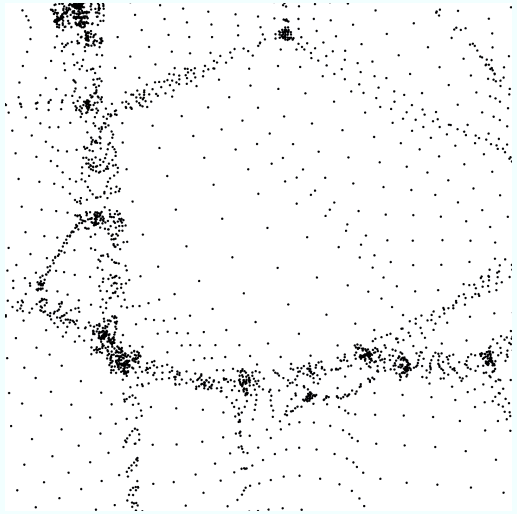




$\Downarrow$ Evolve

$z = 0$





$\Leftarrow$ Interpolate (in Phase Space)

The Dark Matter Sheet in Phasespace

Why is this useful?

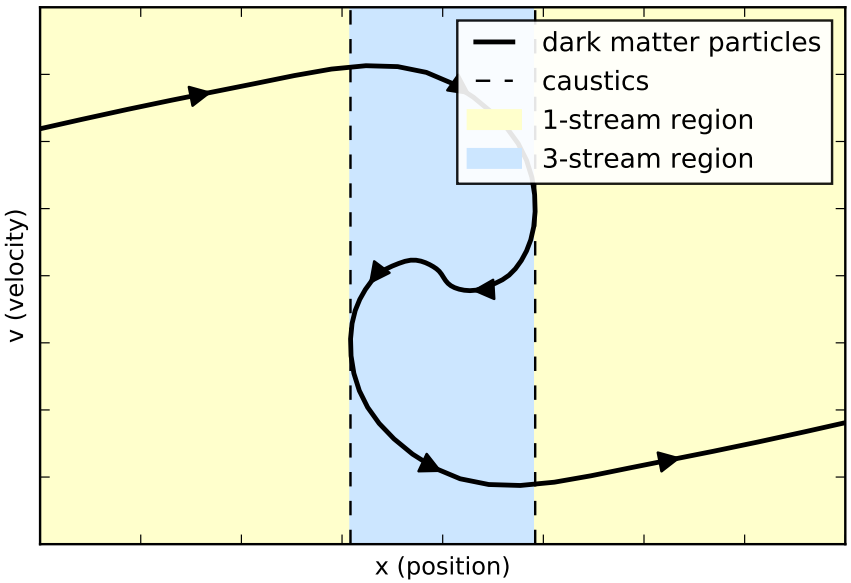
- Recover continuum ($N \to \infty$) from finite number of particles
- Avoid Fragmentation in Warm Dark Matter Simulations (Oliver Hahn's talk)
- Full phase space information
- (Pretty pictures!)

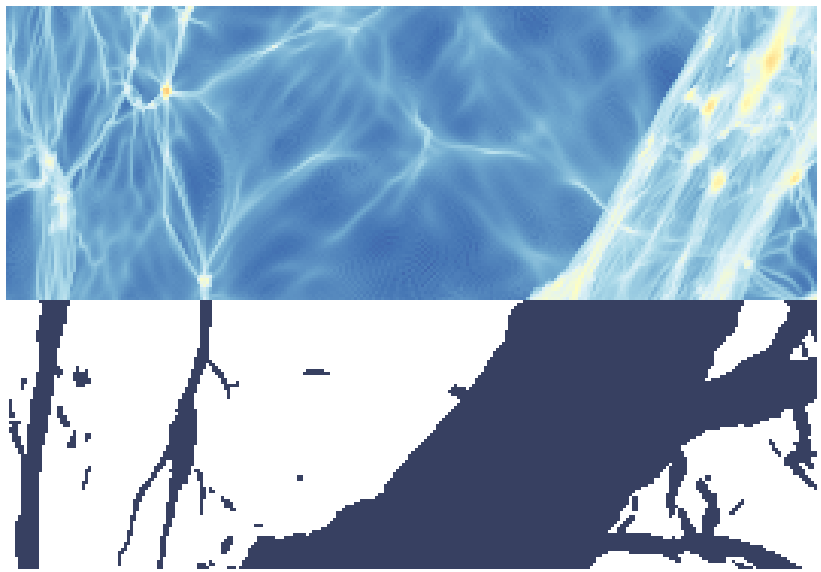
How complex is the dark matter sheet?

(a) Where can we recover the continuum limit $N \to \infty$ by interpolation?

(b) What to do where we can't?

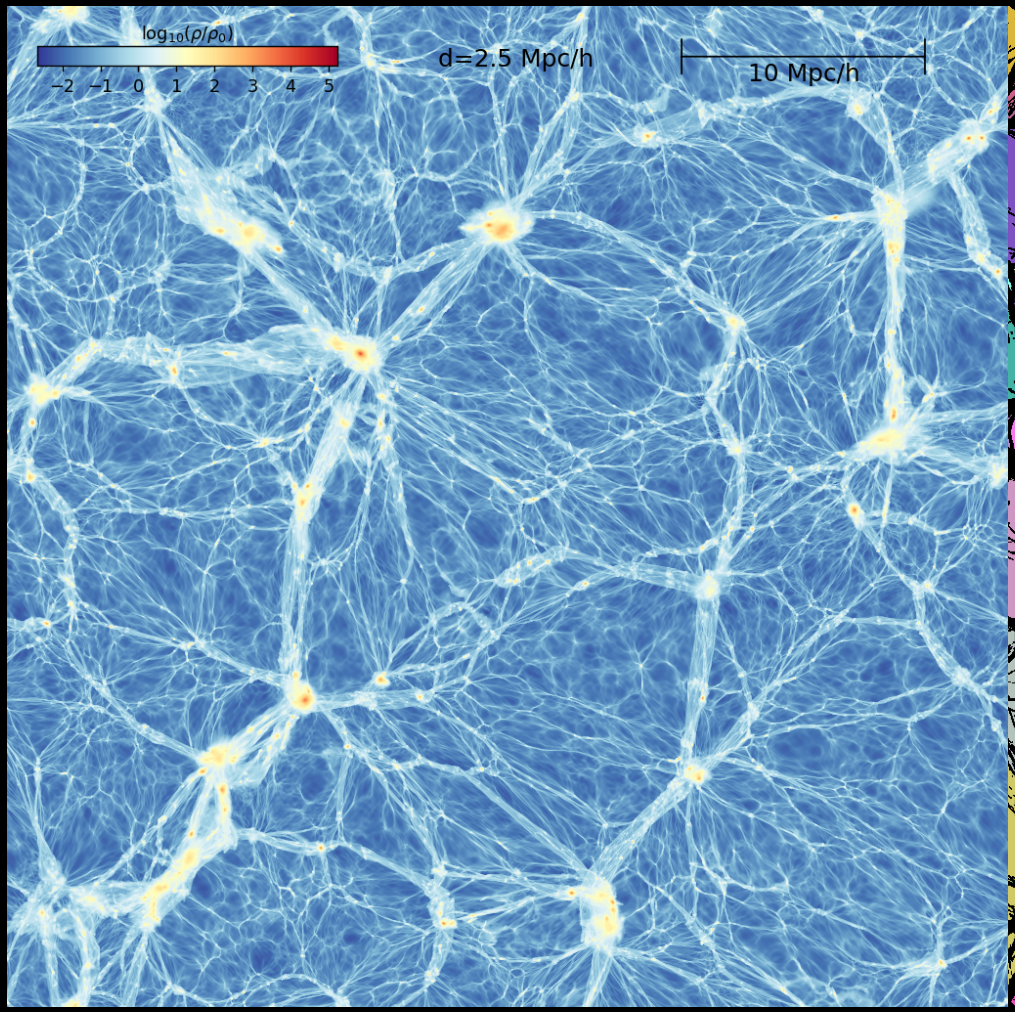
Single-Stream Regions

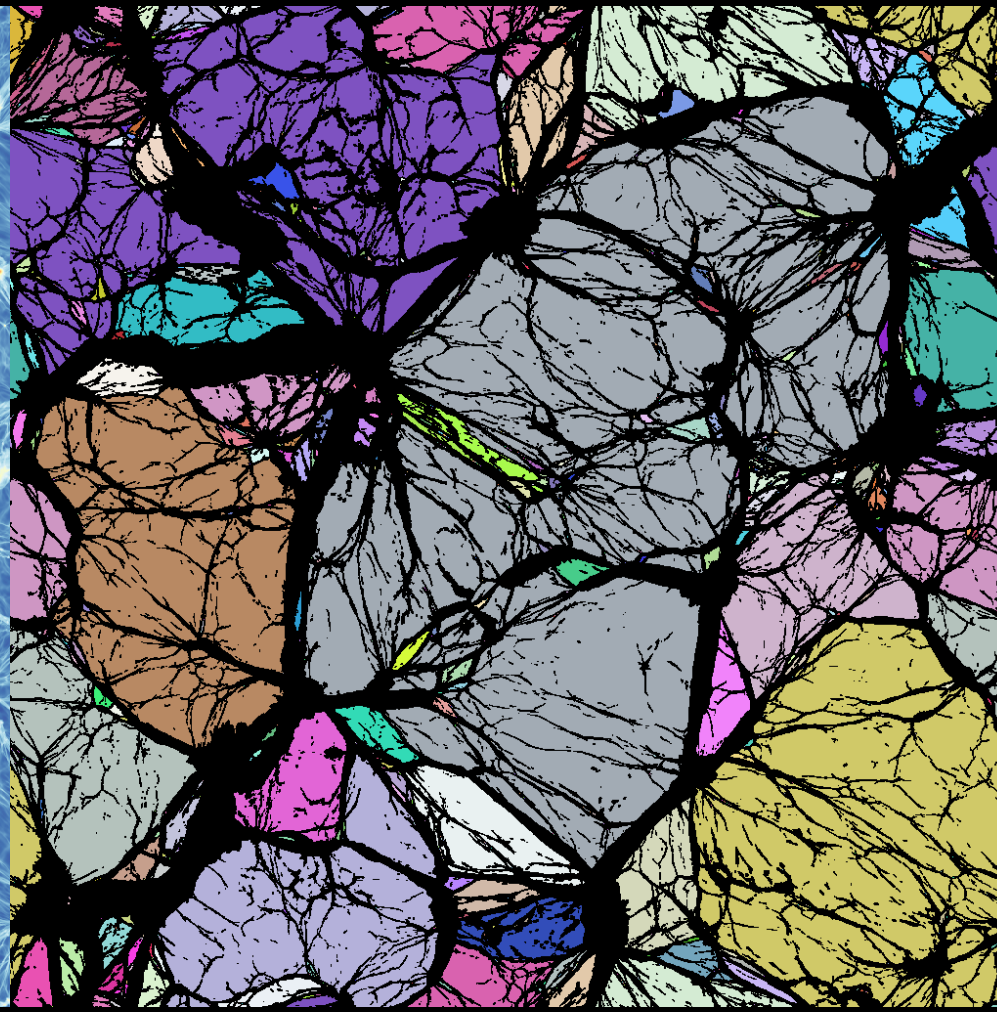




Single-Stream regions contain particles with $\left|det \frac{d\vec{x}}{d\vec{q}}\right| < 0$

Single-Stream Regions





Stücker, Busch & White (2017)

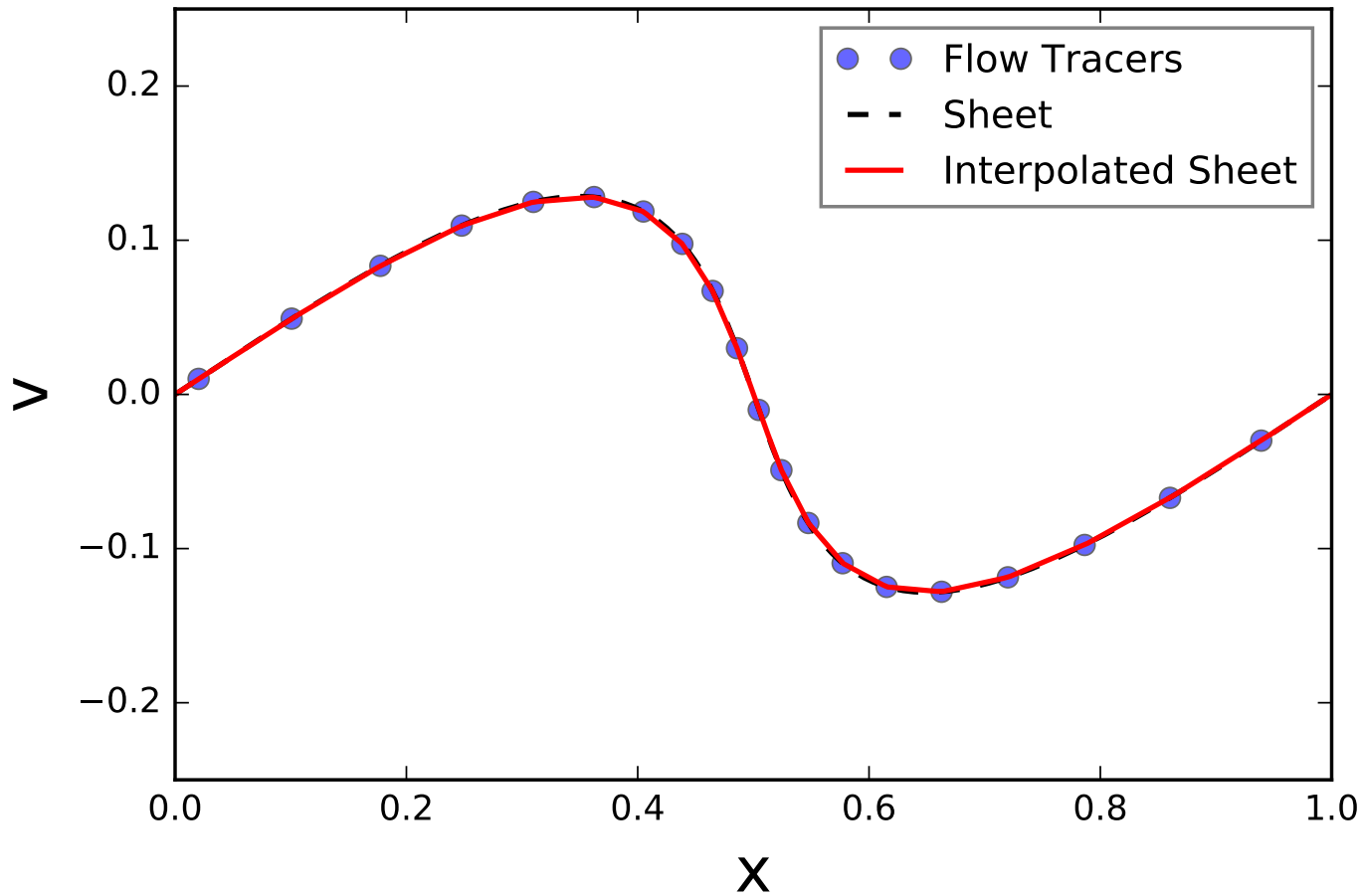
Single-Stream Regions are simple

- Can be well reconstructed with moderate number of particles
- They make up $\sim 75\%$ of the volume
- Can reasonably well be described by simple models, like e.g. the Zeldovich Approximation or a triaxial collapse model

The median density of the Universe, Stücker, Busch & White (2017)

What are the Limitations of the Interpolation?

The Interpolation breaks down when the Sheet becomes too complex



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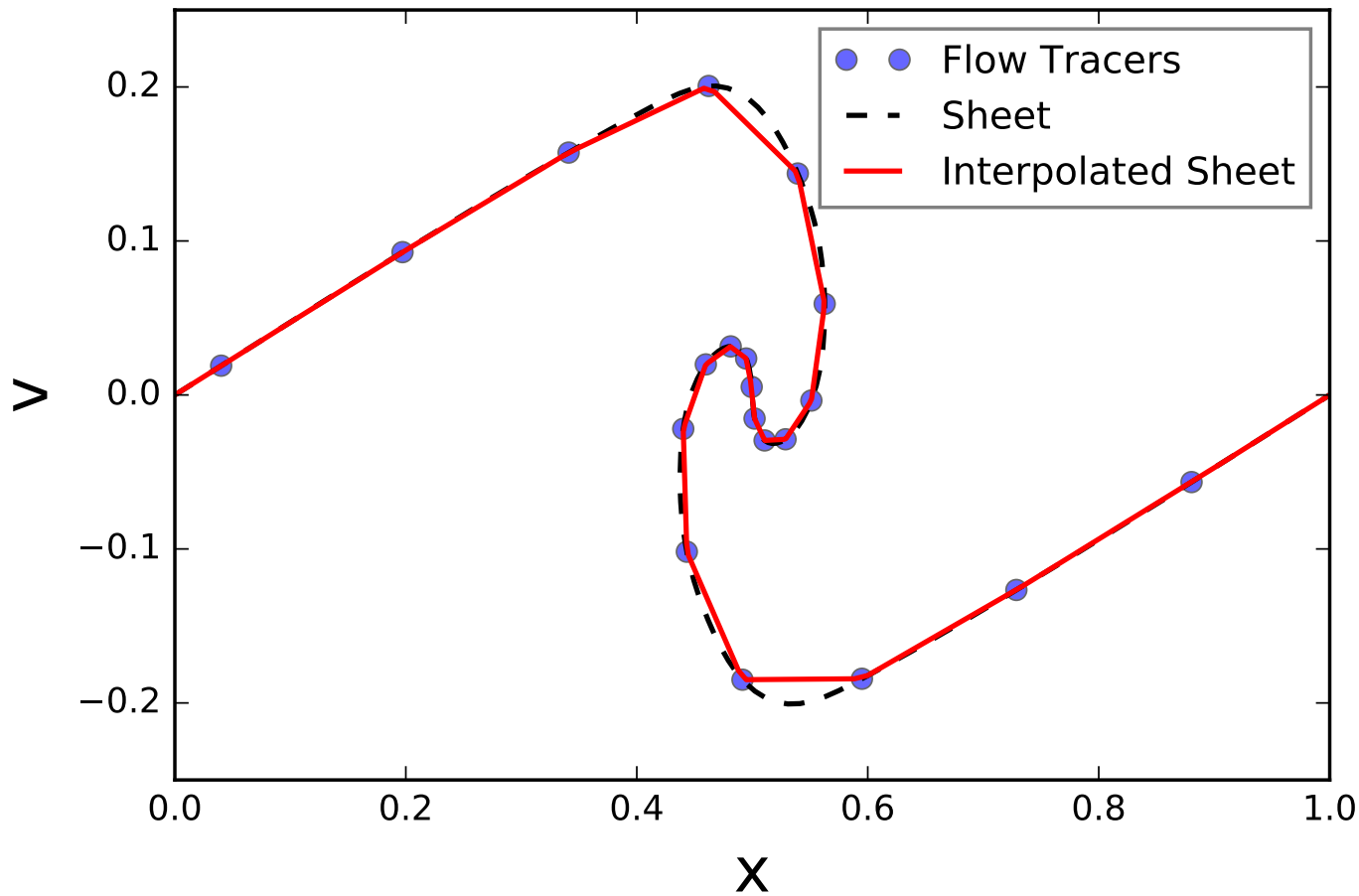

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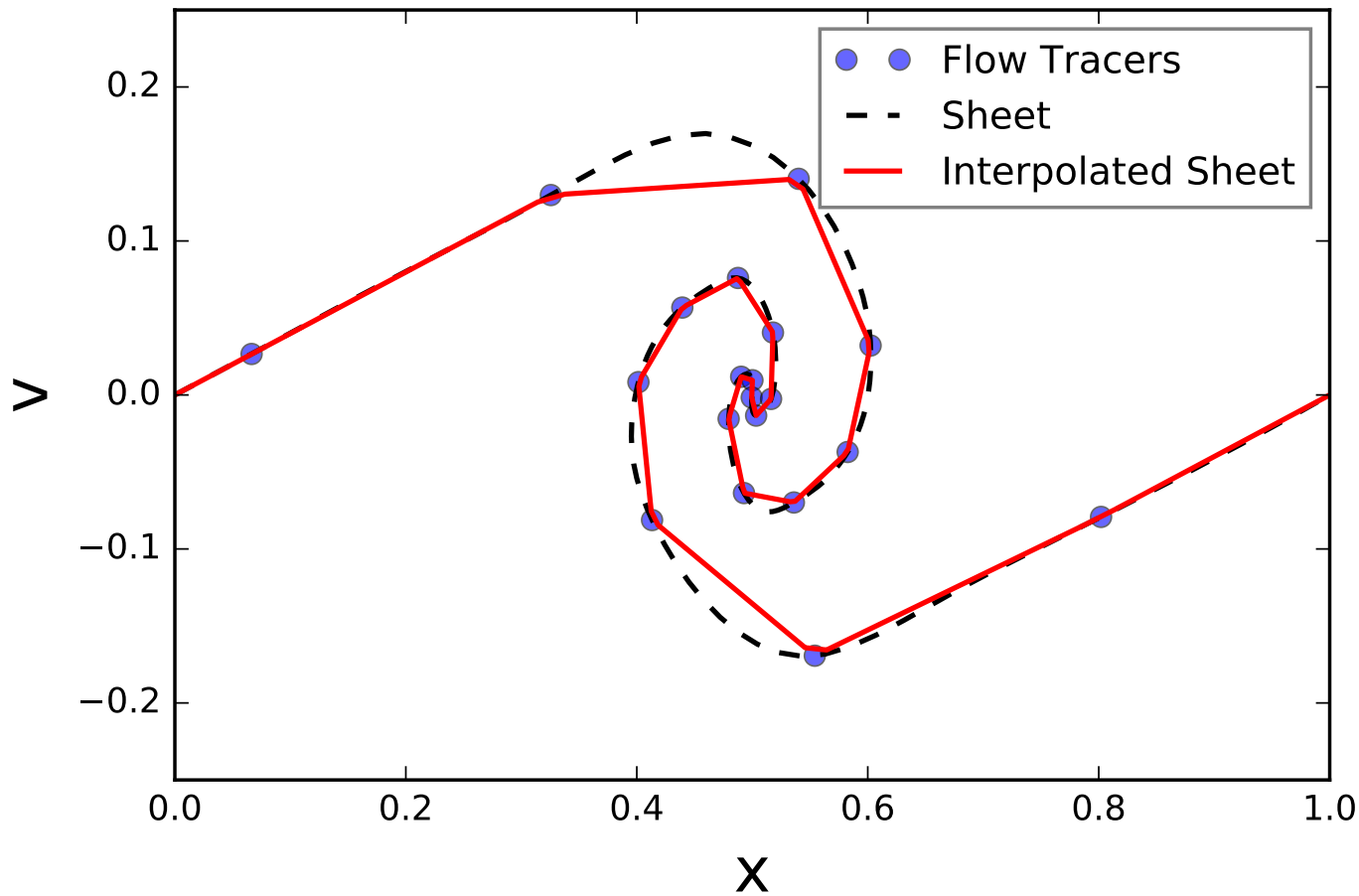

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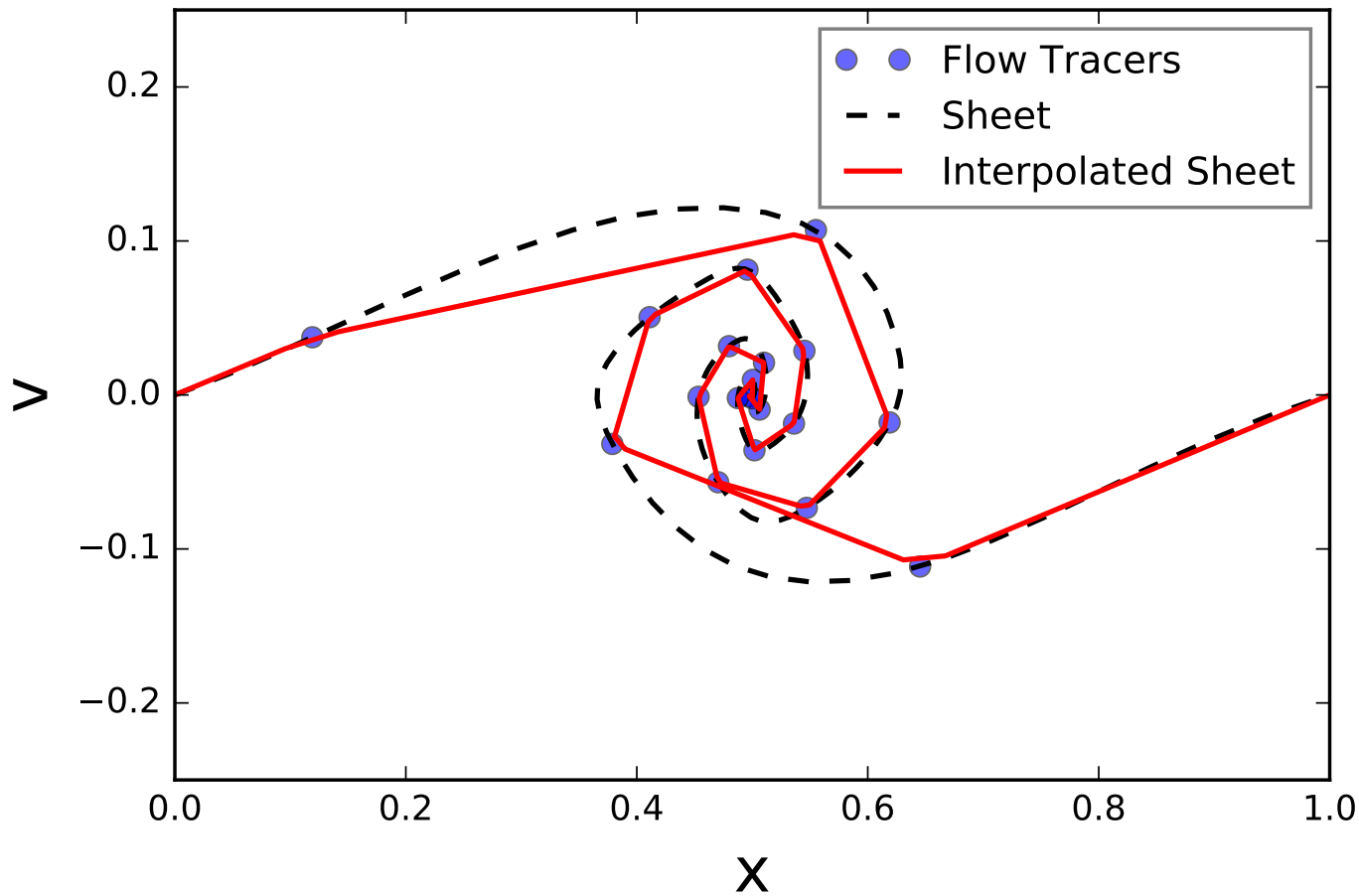


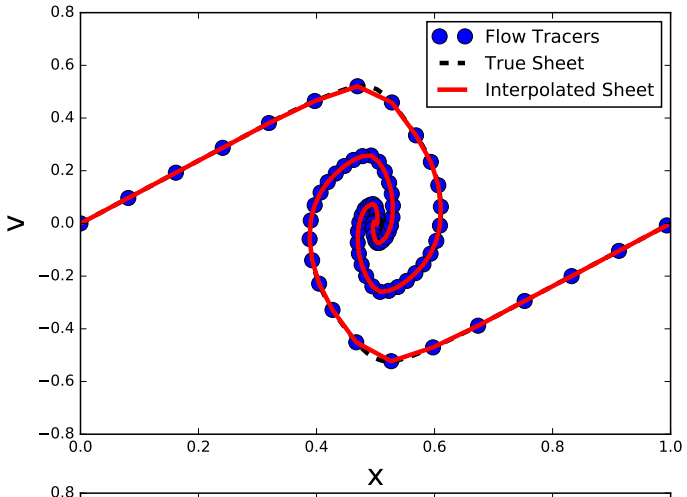
How can we test for this?

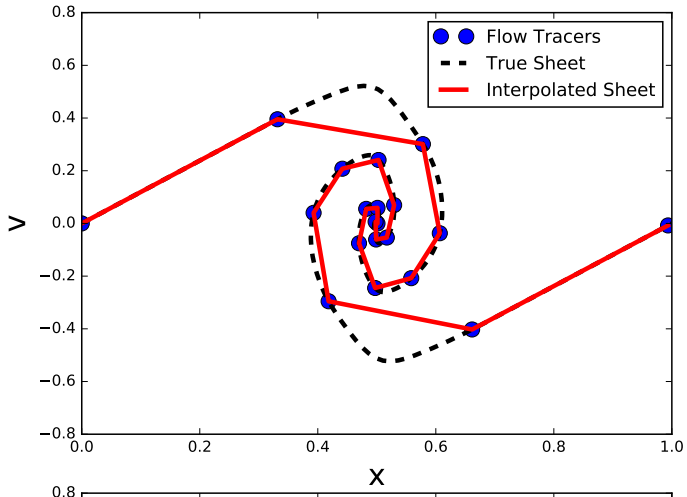
Well reconstructed

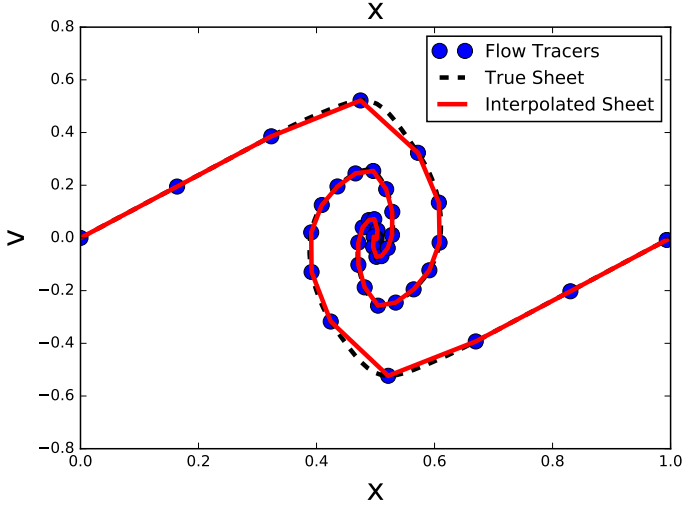
Poorly reconstructed

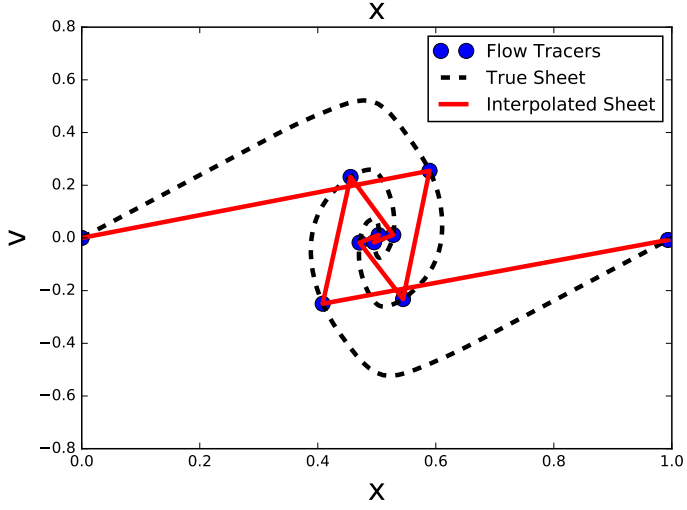
All particles

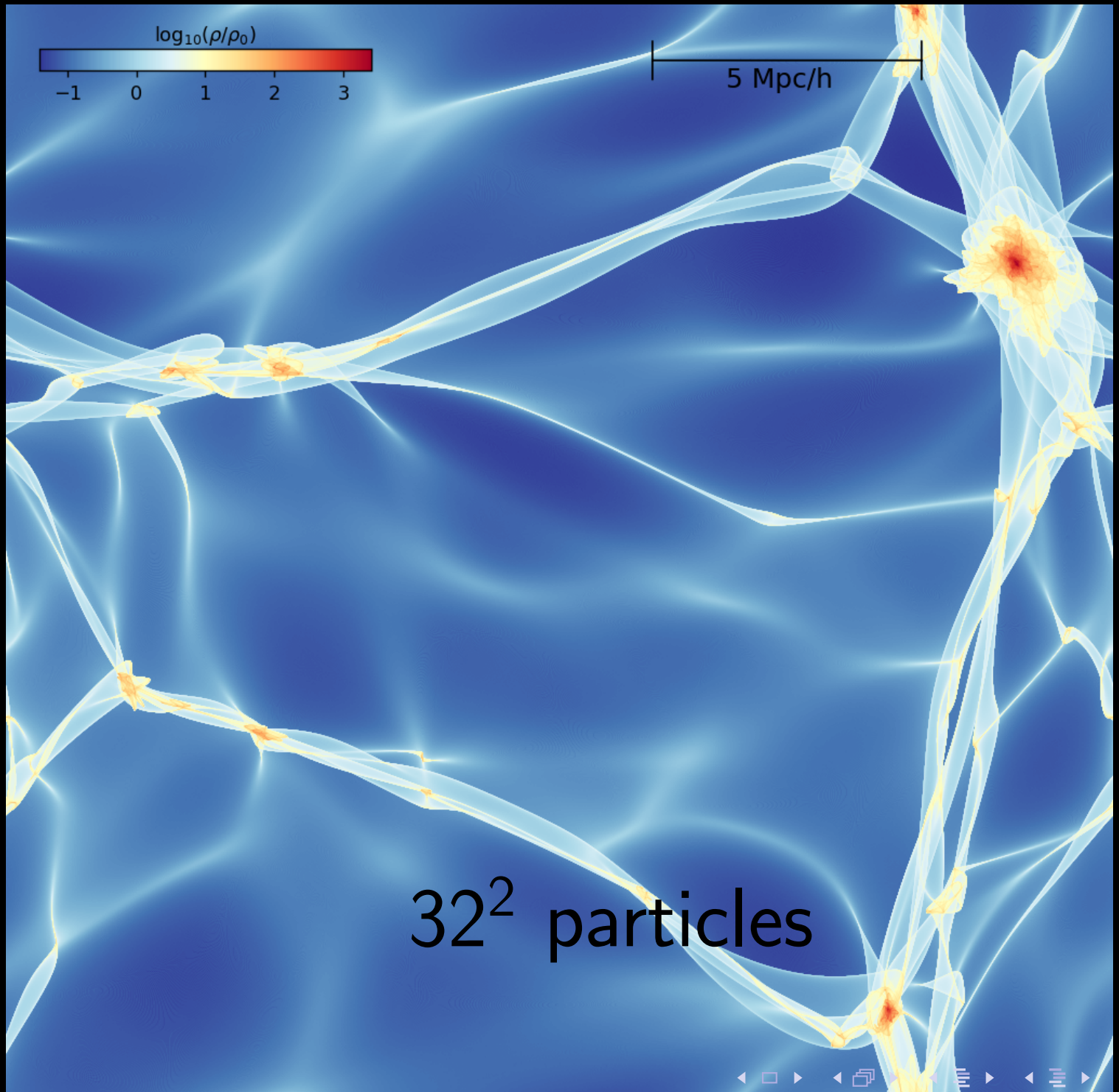
Every second particle

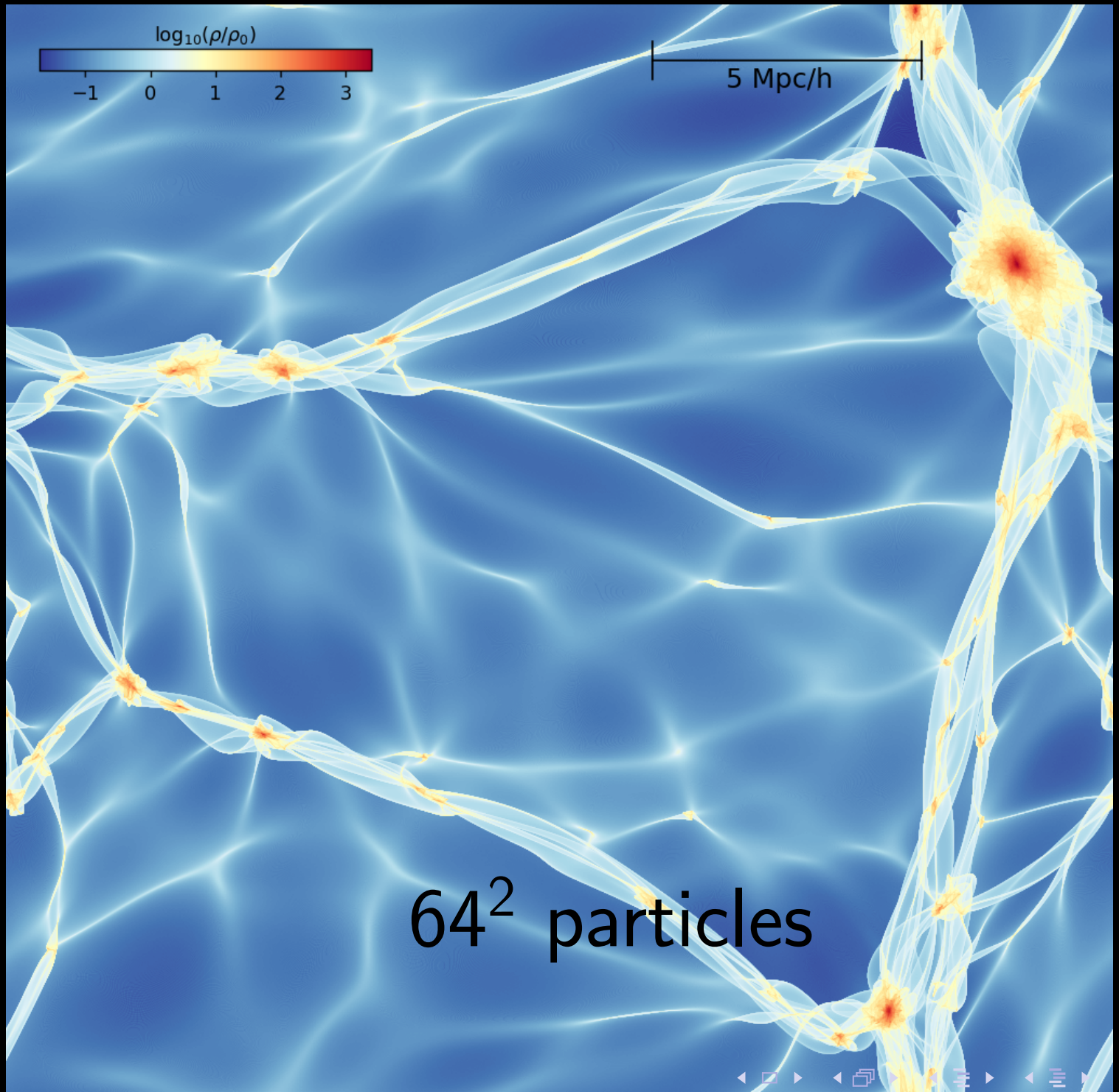


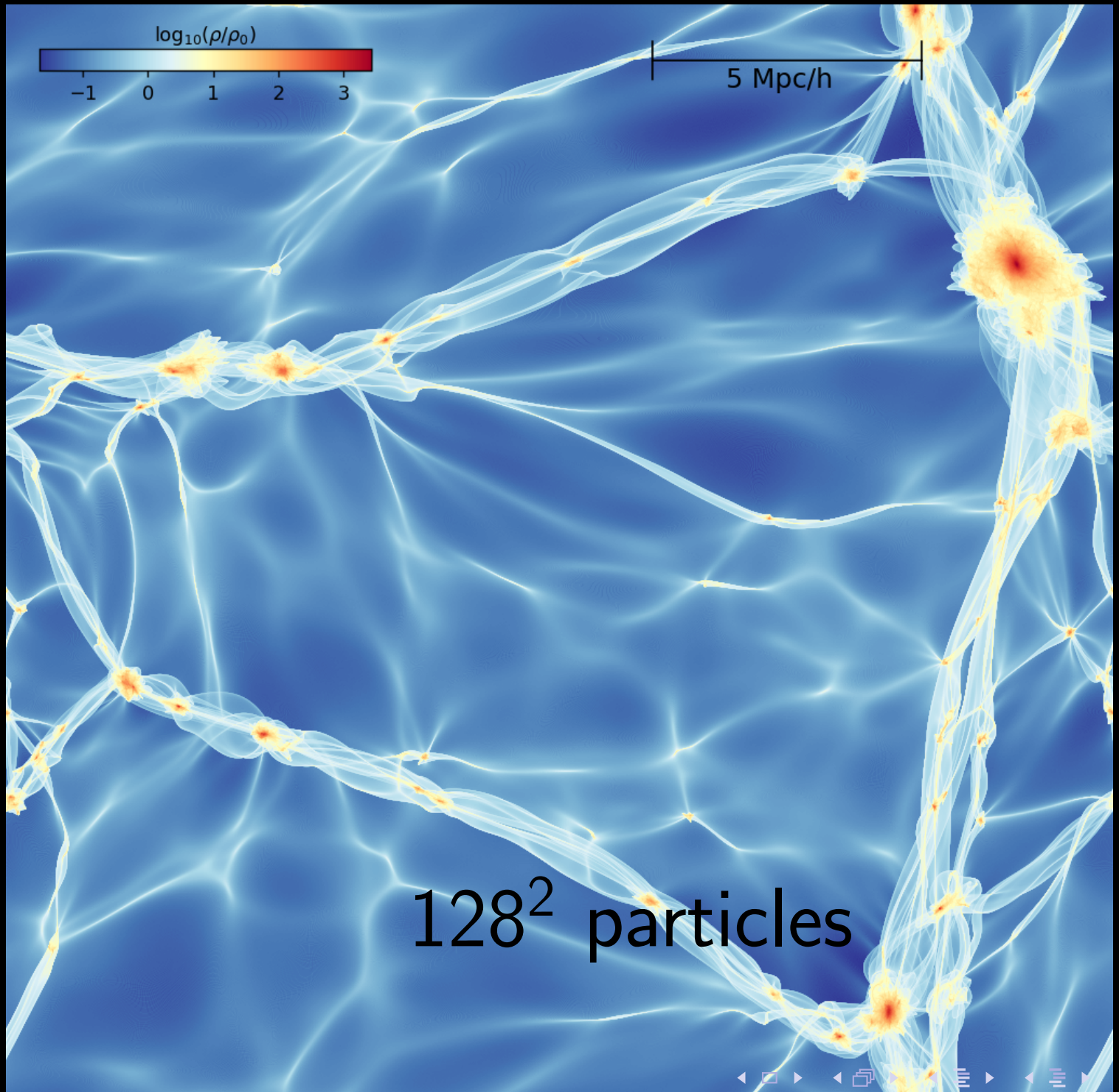


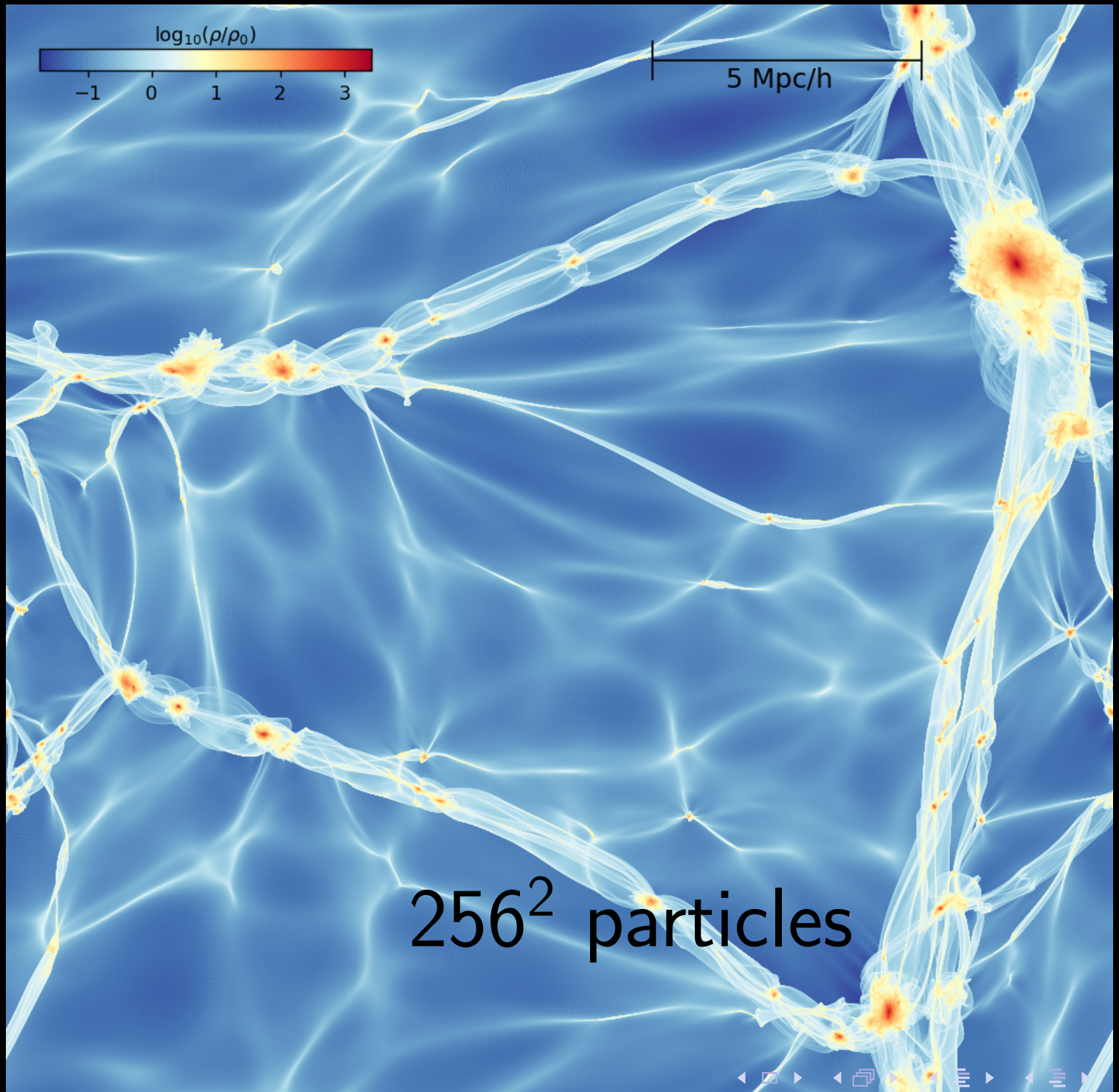


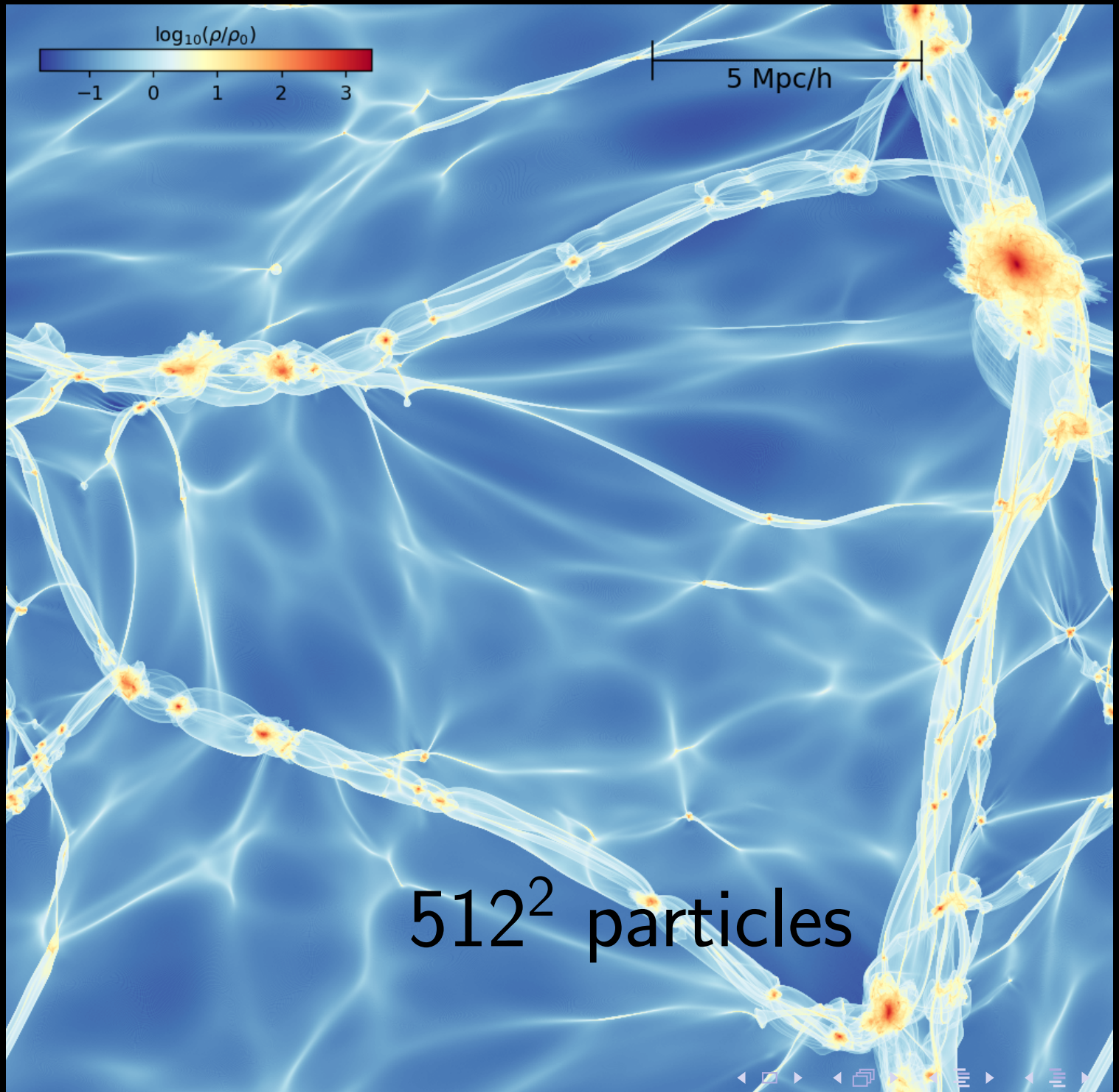


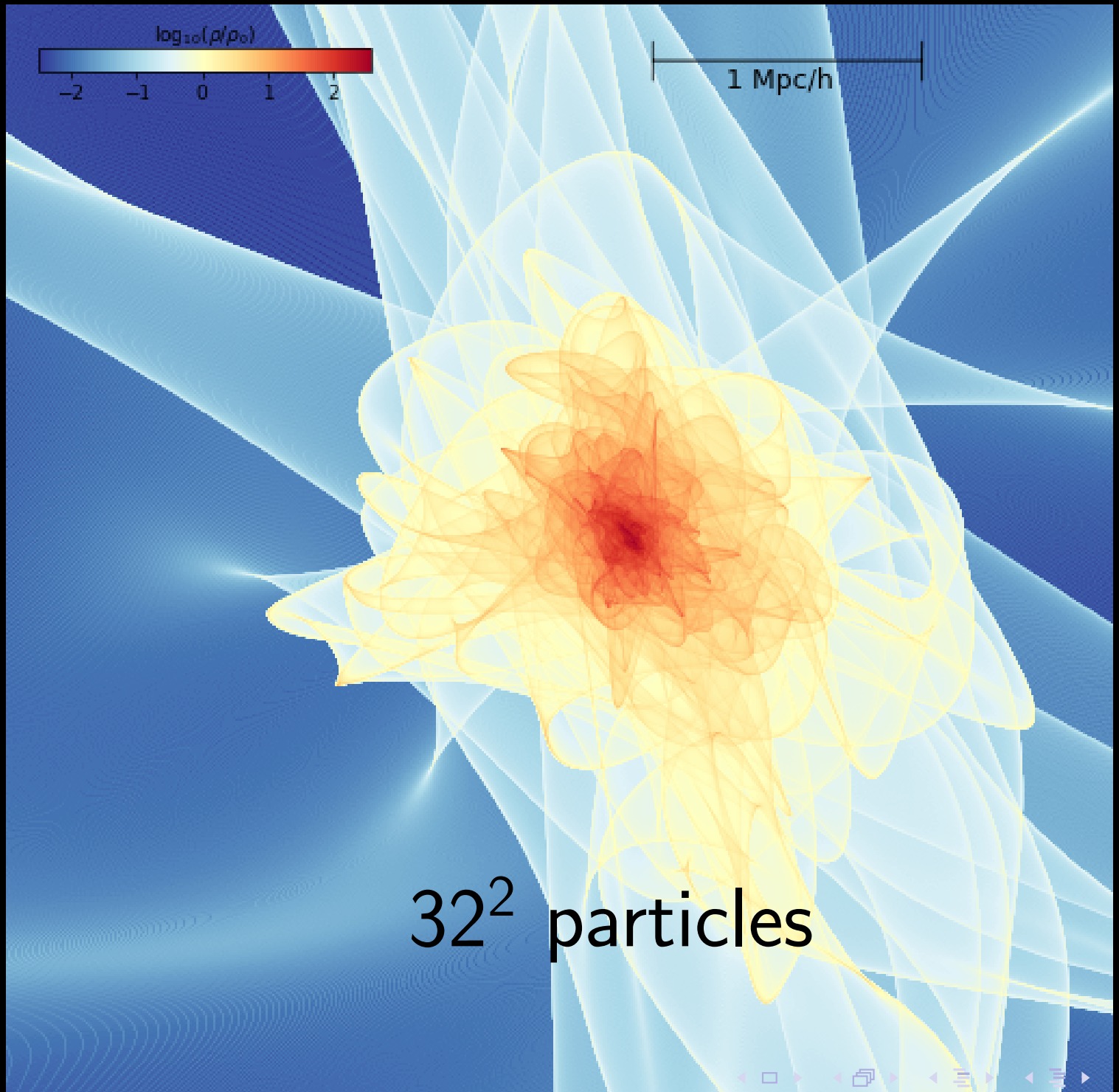

$\log_{10}(\rho/\rho_0)$
−1
0
1
2
3
5 Mpc/h
32^2 particles

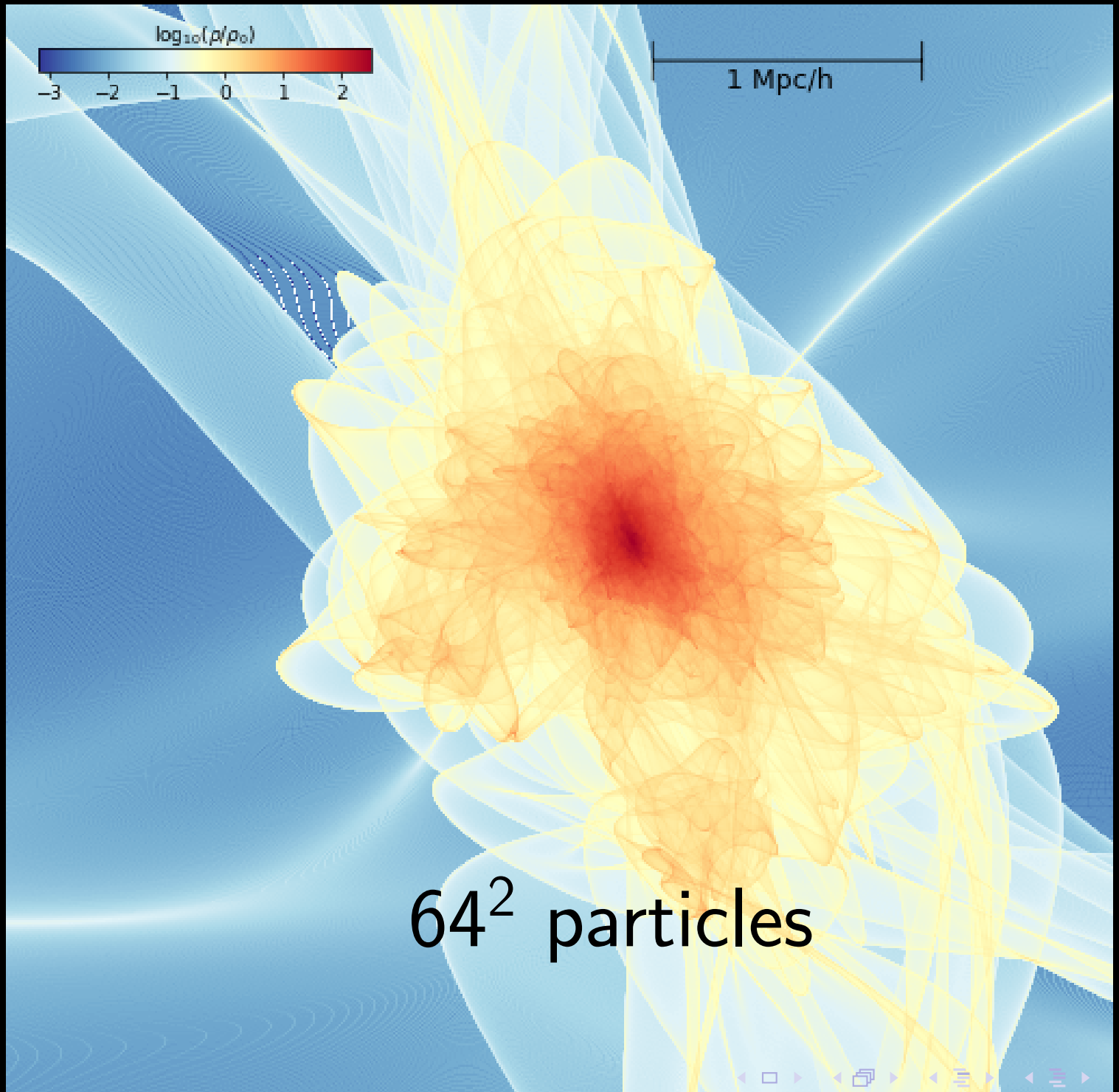

$\log_{10}(\rho/\rho_0)$
−1
0
1
2
3
5 Mpc/h
64^2 particles

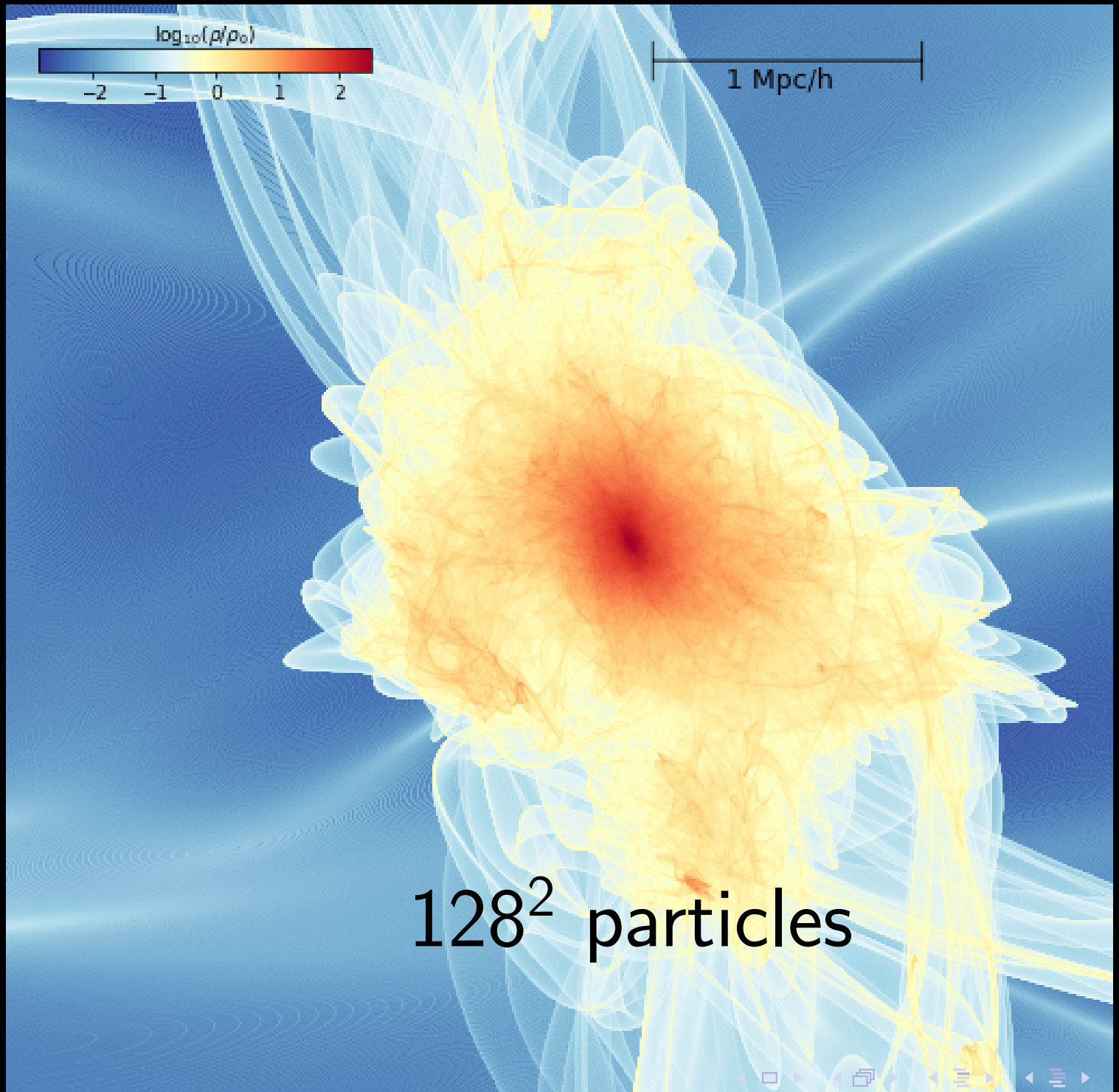

$\log_{10}(\rho/\rho_0)$
−1
0
1
2
3
5 Mpc/h
128^2 particles


$\log_{10}(\rho/\rho_0)$
−1
0
1
2
3
5 Mpc/h
256^2 particles


$\log_{10}(\rho/\rho_0)$
−1
0
1
2
3
5 Mpc/h
512^2 particles


$\log_{10}(\rho/\rho_0)$
−2
−1
0
1
2
1 Mpc/h
32^2 particles


$\log_{10}(\rho/\rho_0)$
−3 −2 −1 0 1 2
1 Mpc/h
64^2 particles


$\log_{10}(\rho/\rho_0)$
−2
−1
0
1
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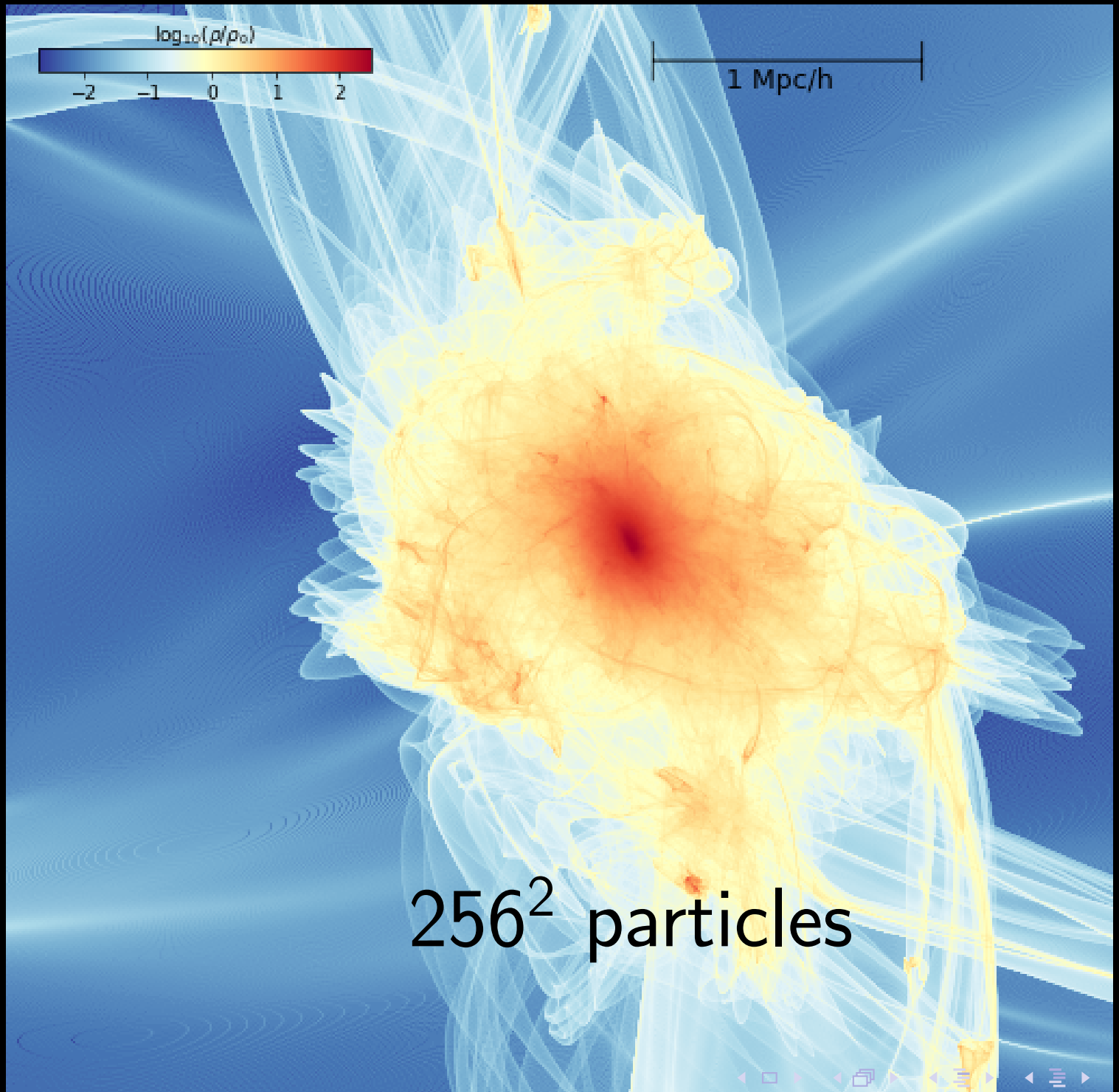
1 Mpc/h
128^2 particles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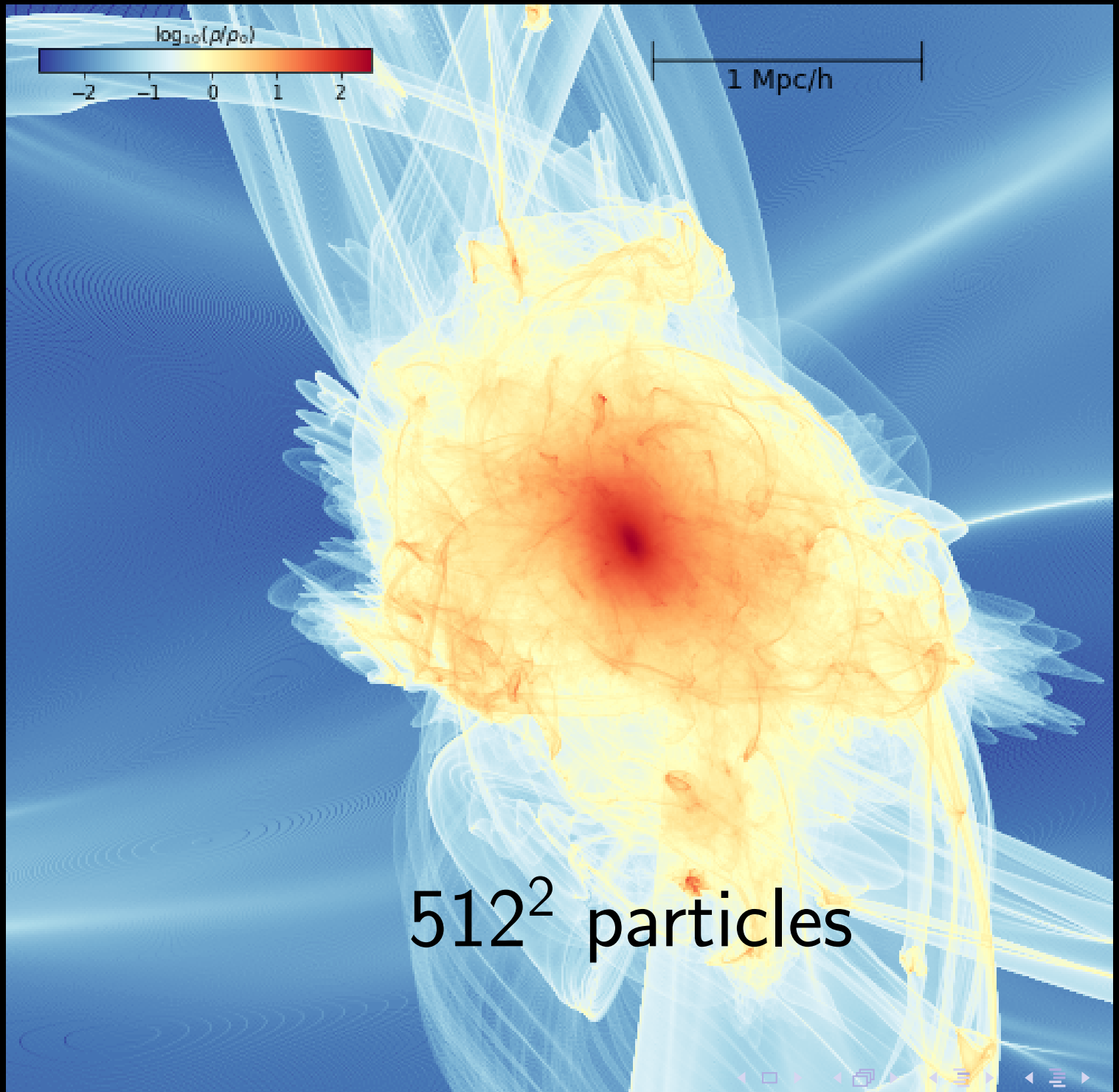
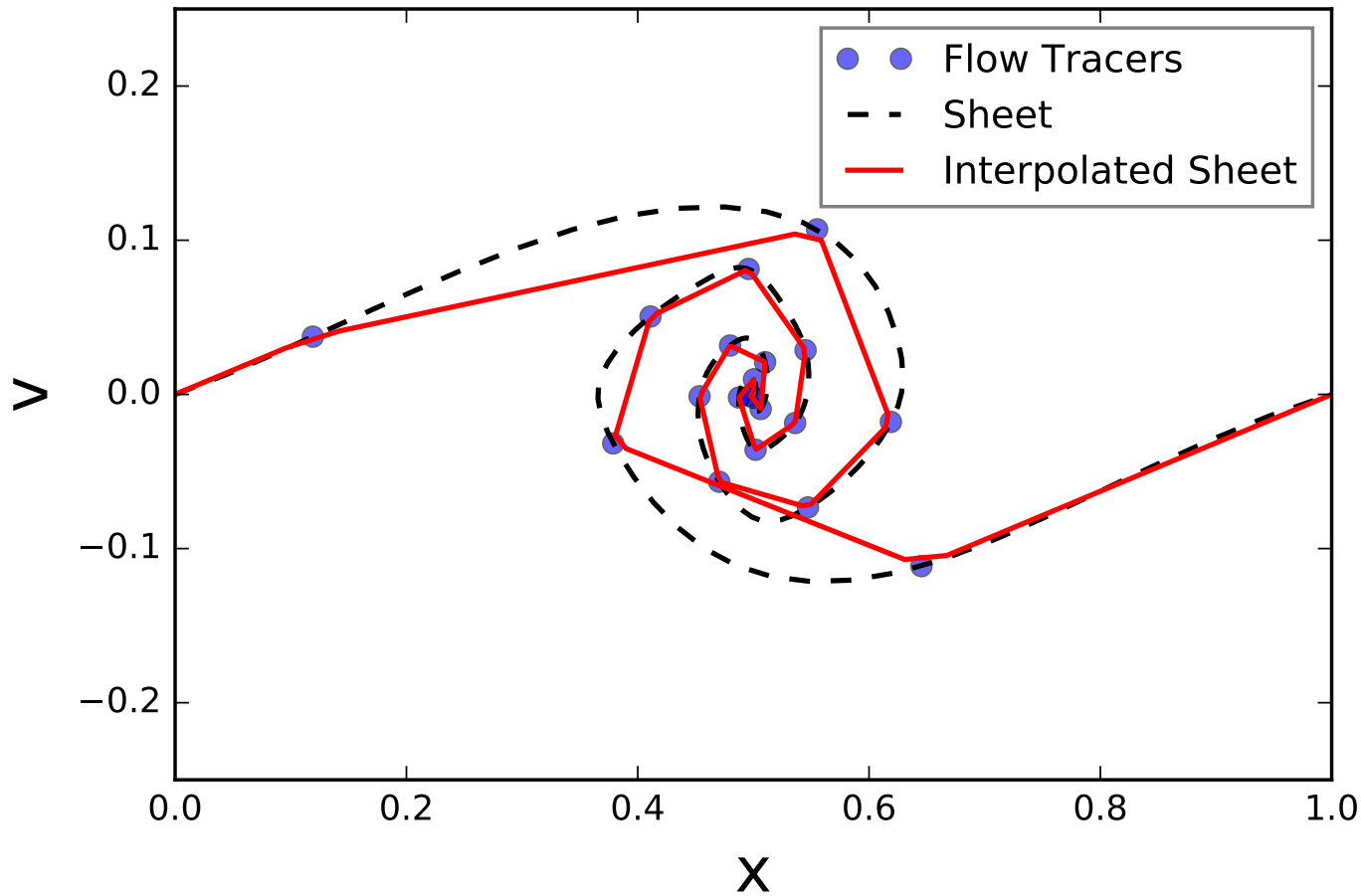

$\log_{10}(\rho/\rho_0)$
−2
−1
0
1
2
1 Mpc/h
512^2 particles

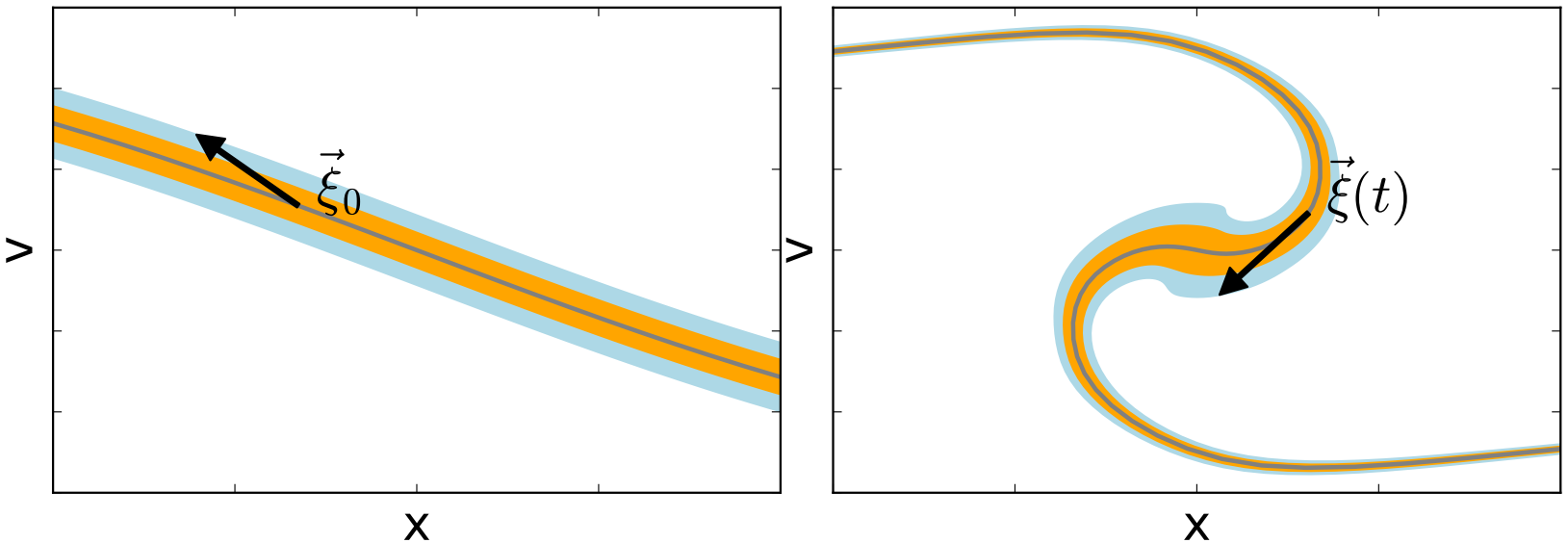
How to quantify this?

Compare derivatives $d\vec{x}/d\vec{q}$ of the interpolated sheet with the real ones



The Geodesic Deviation Equation

How does a small displacement in initial phase space evolve?



$$\vec{\xi}(t) := \begin{pmatrix} \Delta\vec{x} \\ \Delta\vec{v} \end{pmatrix}$$
$$=: D(t)\vec{\xi}_0$$

Vogelsberger et al. (2008), Vogelsberger & White (2011)

The Geodesic Deviation Equation

$$\begin{pmatrix} \Delta\vec{x}(t) \\ \Delta\vec{v}(t) \end{pmatrix} = D(t) \begin{pmatrix} \Delta\vec{x}_0 \\ \Delta\vec{v}_0 \end{pmatrix}$$

$$D = \begin{pmatrix} D_{xq} & D_{vp} \\ D_{vq} & D_{vp} \end{pmatrix}$$

Evolution equations of the distortion tensor

$$\dot{D}_{xq} = a^{-2} D_{vq}$$

$$\dot{D}_{vq} = a^{-1} T D_{xq}$$

$$T_{ij} = -\frac{\partial^2 \phi}{\partial x_i \partial x_j}$$

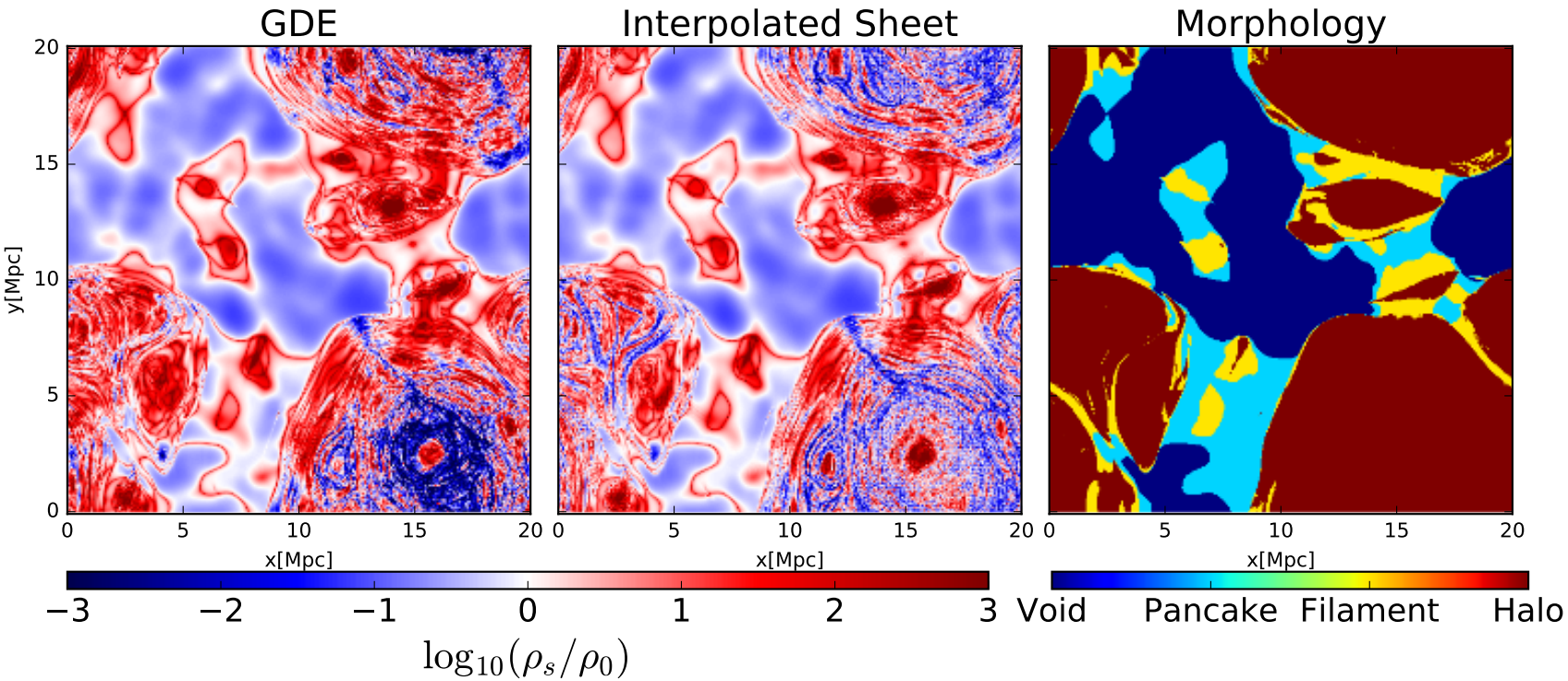
Vogelsberger et al. (2008), Vogelsberger & White (2011)

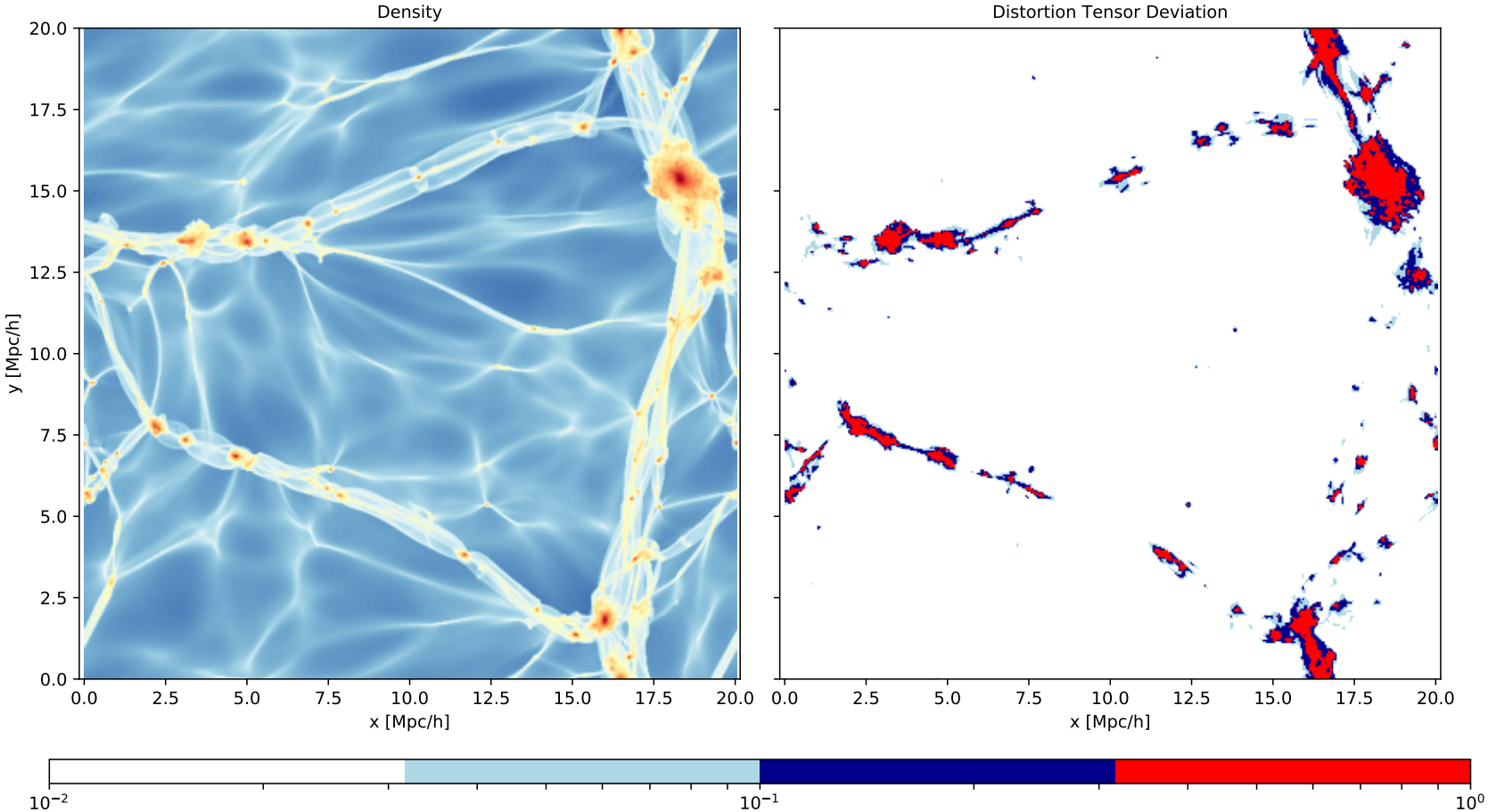
Stream Densities in Lagrangian Space

$$\rho_s = \left| det \frac{d\vec{x}}{d\vec{q}} \right|^{-1}$$

$\vec{x}$: Eulerian space position

$\vec{q}$: Lagrangian coordinates

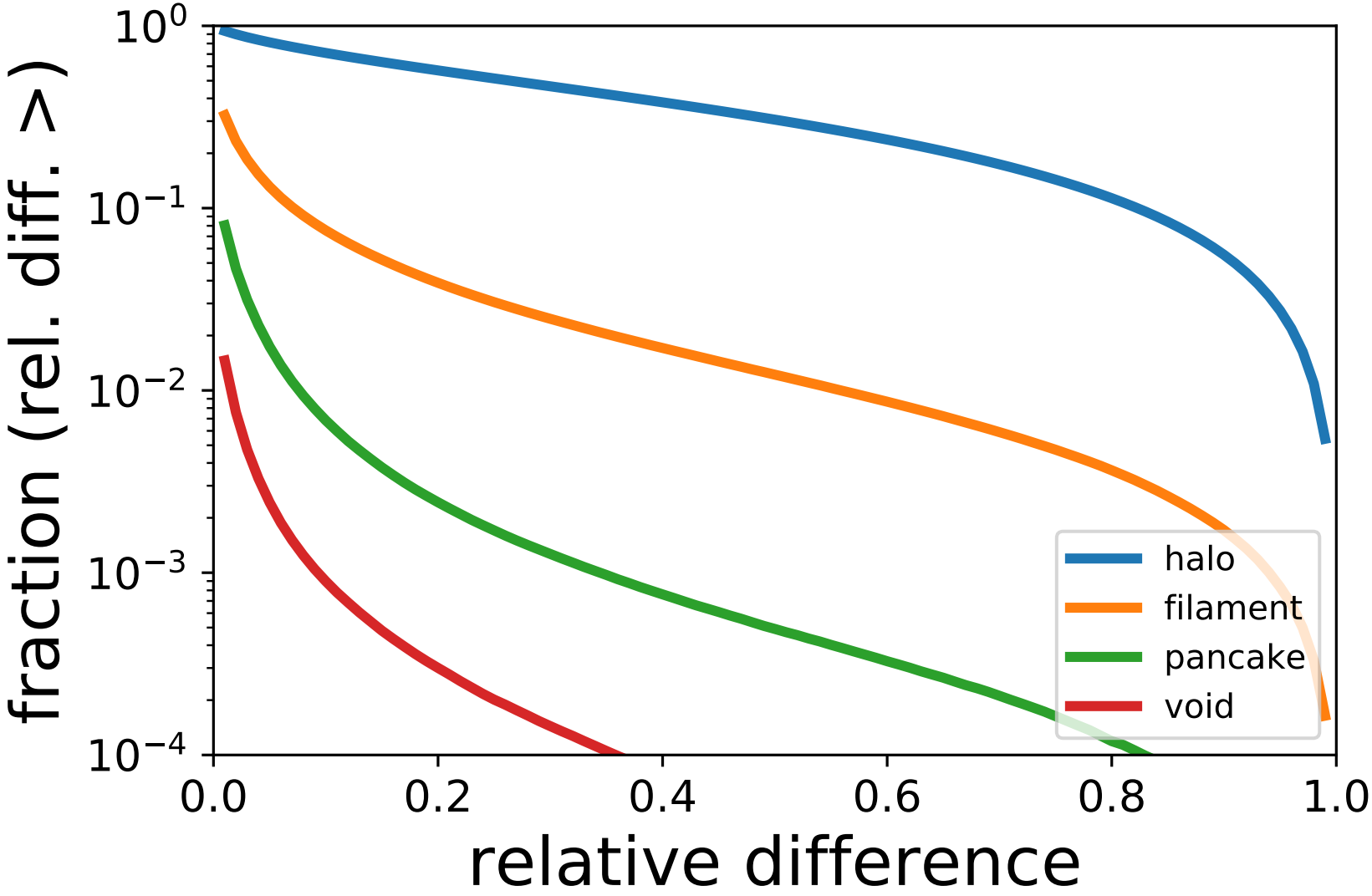




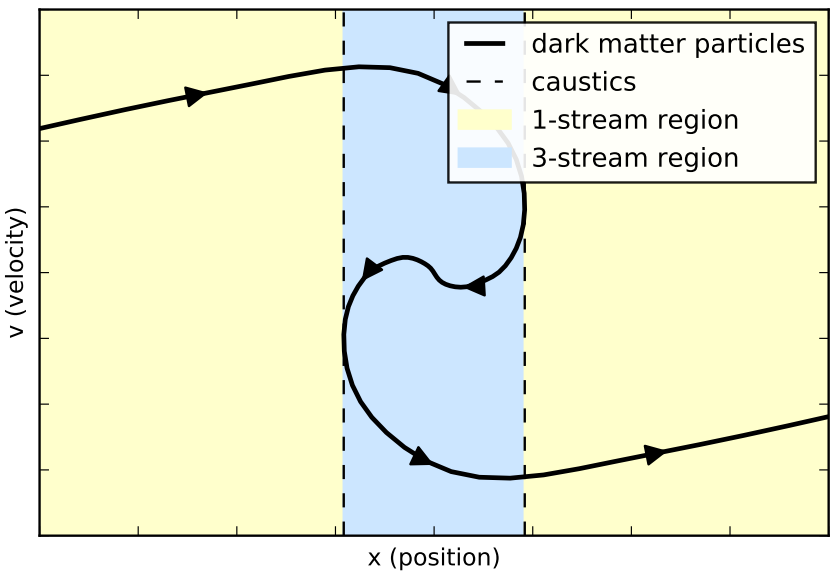
Relative Difference between Distortion Tensors

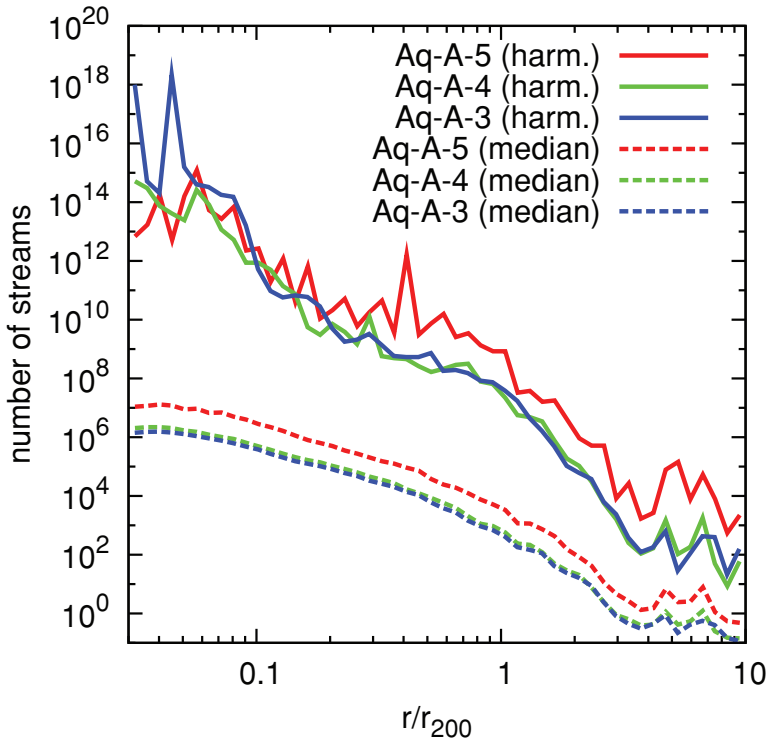
$$\text{rel. diff.} := 1 - \frac{\sum A_{ij} B_{ij}}{||A||_2 ||B||_2}$$

...


fraction (rel. diff. > λ)
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0.0
0.2
0.4
0.6
0.8
1.0
relative difference
halo
filament
pancake
void

Stream-Number in Halos





Vogelsberger & White (2011)

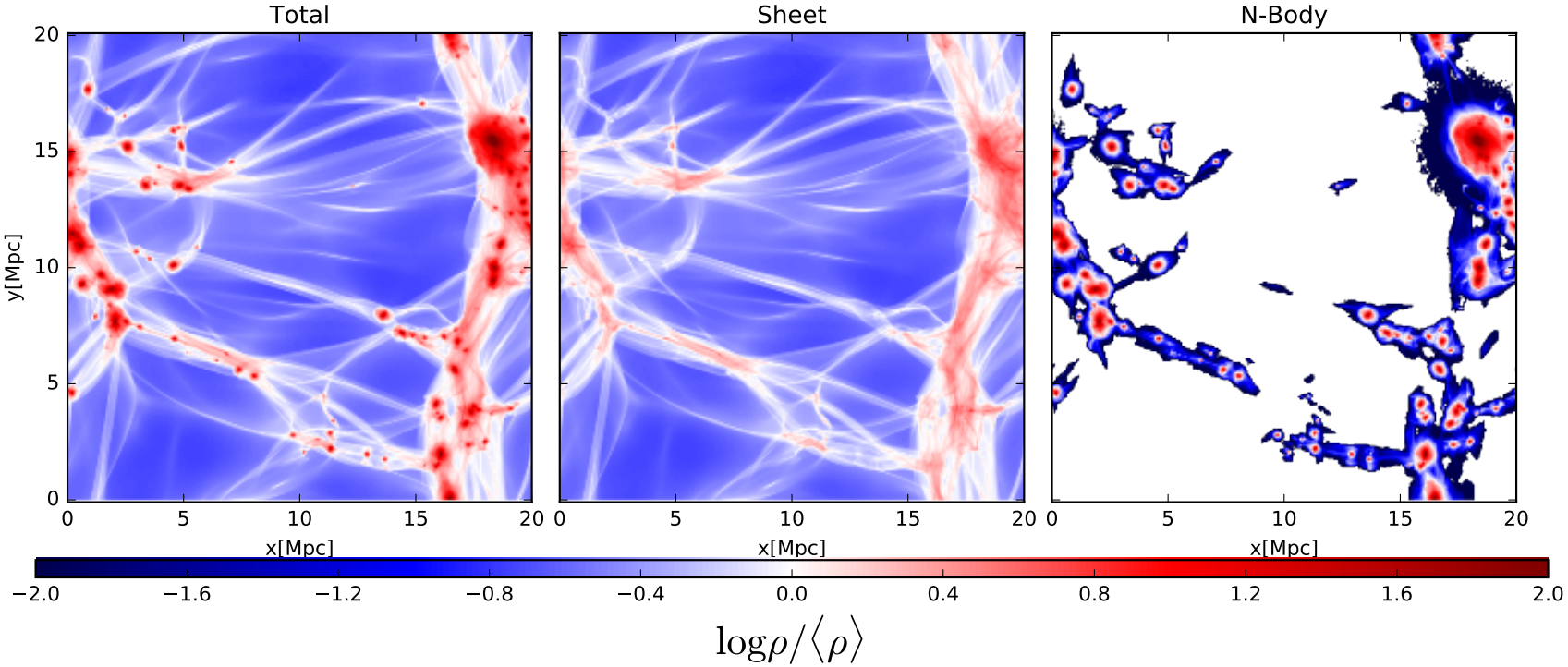
How can we use the Dark Matter Sheet in Simulations?

	Sheet	N-Body
Haloes	too complex	OK
$\sim$ Everywhere else	almost perfect	fragments

$\rightarrow$ Combine Sheet- and N-Body simulations

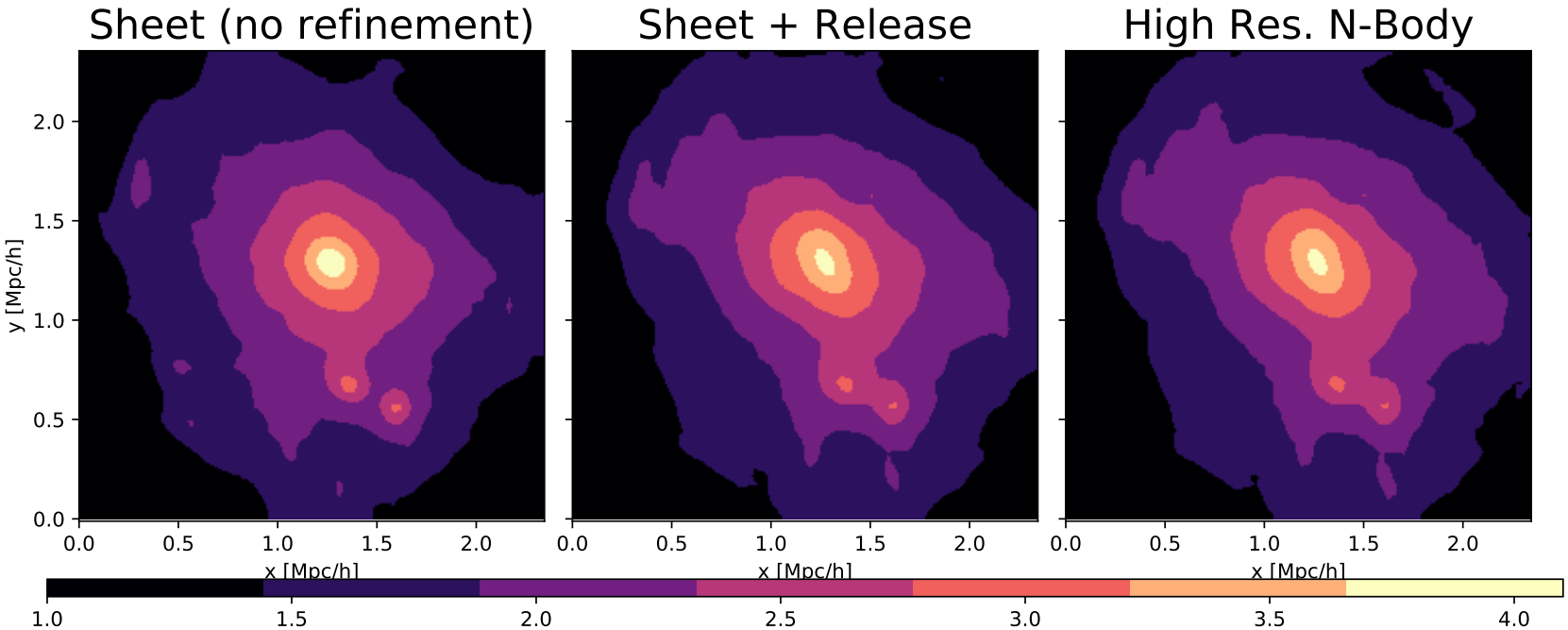
- Use sheet in the beginning everywhere
- Trace GDE for all particles
- Switch to N-Body where the derivatives disagree

Combined Sheet + N-Body + GDE Simulations



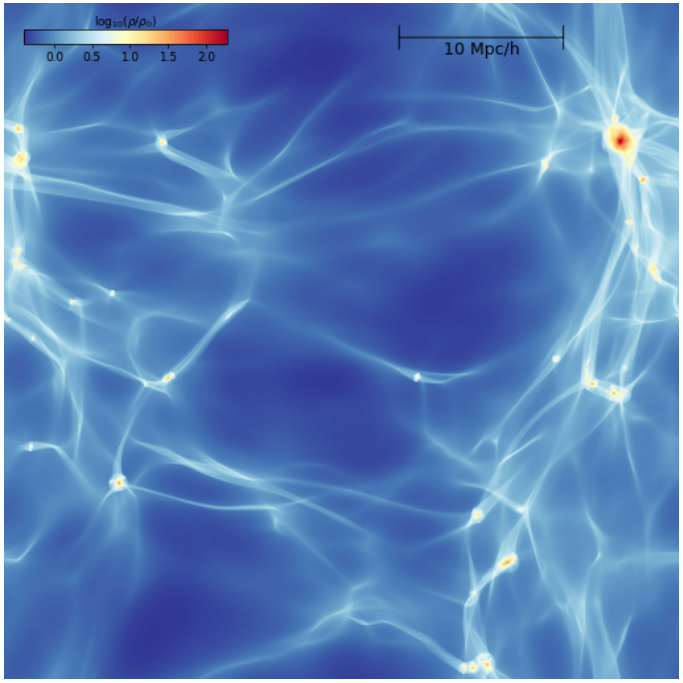
(preliminary)

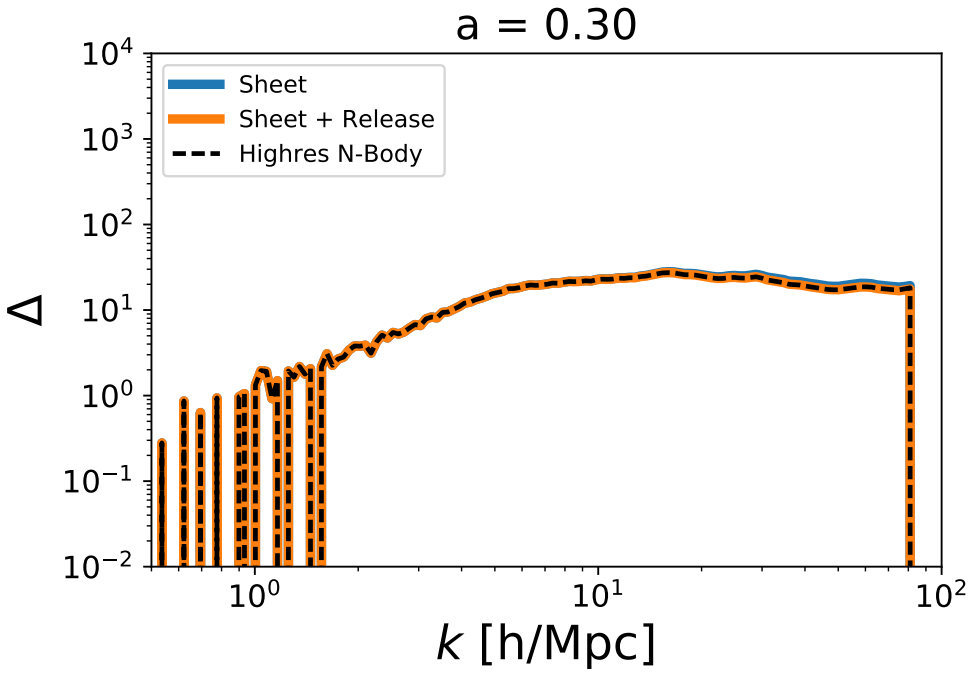
Combined Sheet + N-Body + GDE Simulations



(preliminary)

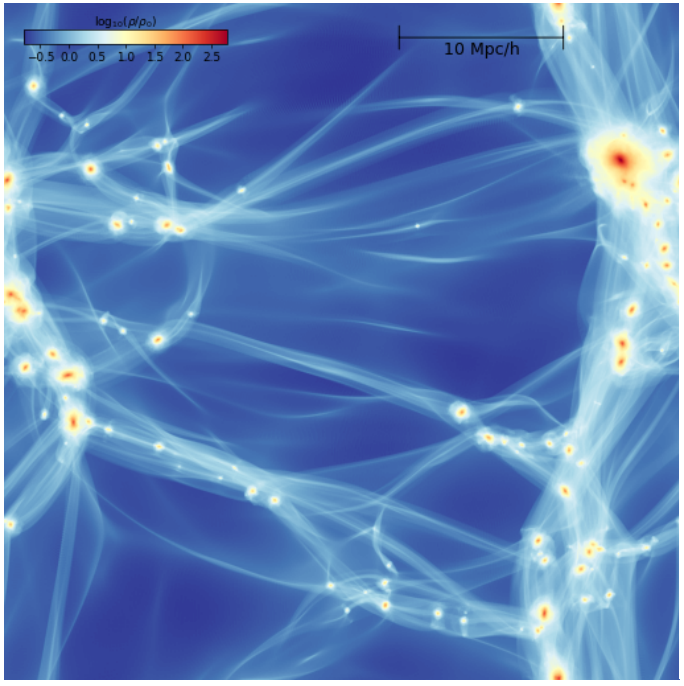
Combined Sheet + N-Body + GDE Simulations

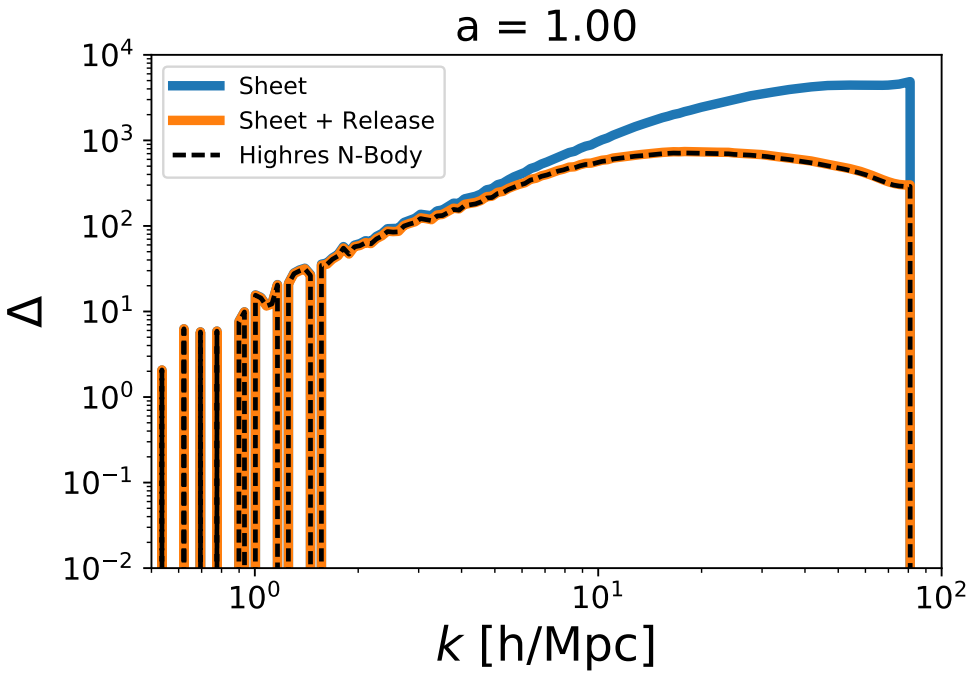




(preliminary)

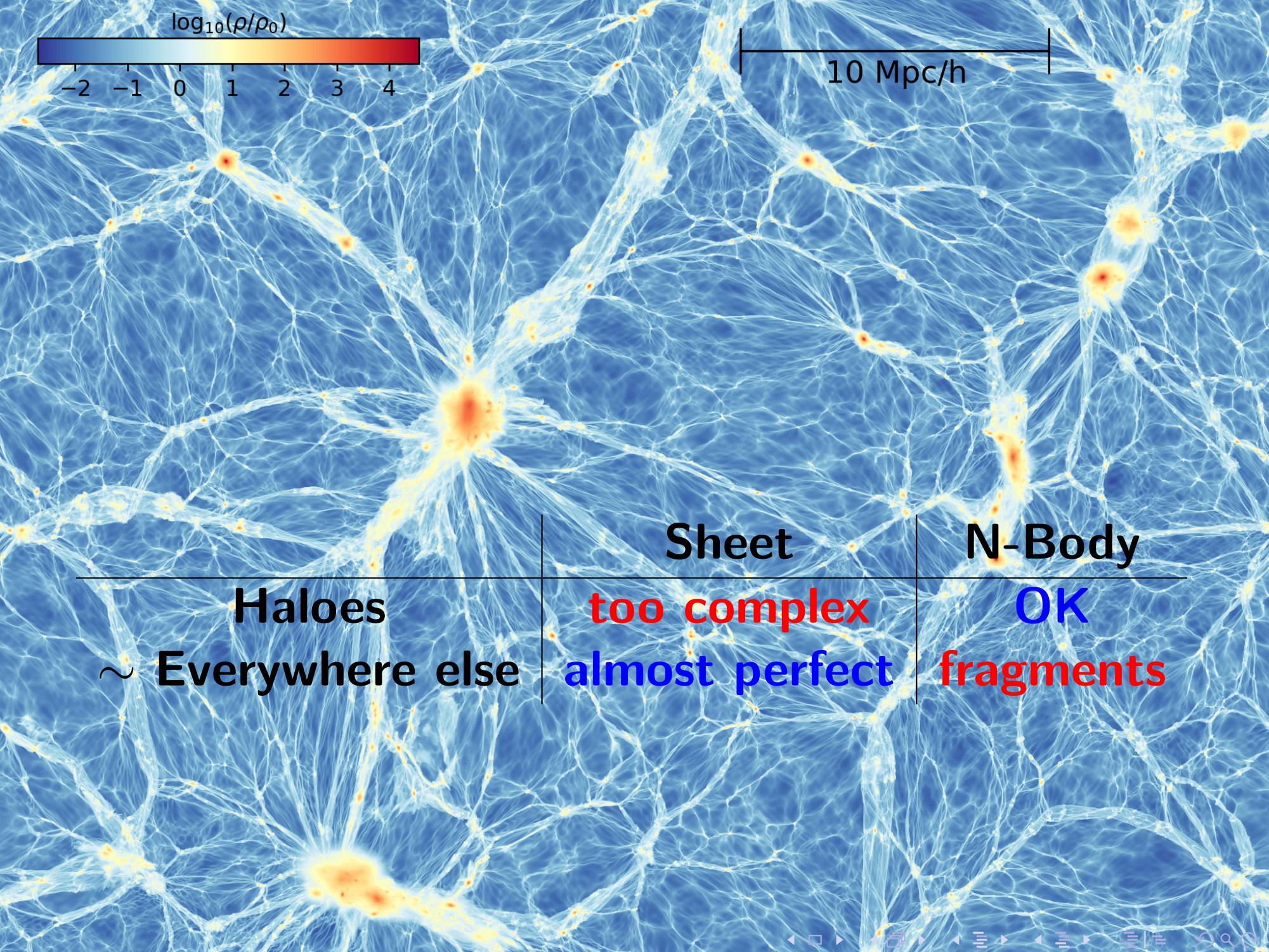
Combined Sheet + N-Body + GDE Simulations





(preliminary)

currently working on force resolution



	Sheet	N-Body
Haloes	too complex	OK
∼ Everywhere else	almost perfect	fragments

Abel T., Hahn O., Kaehler R., 2012, Monthly Notices of the Royal Astronomical Society, 427, 61

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Sousbie T., Colombi S., 2016, Journal of Computational Physics, 321, 644

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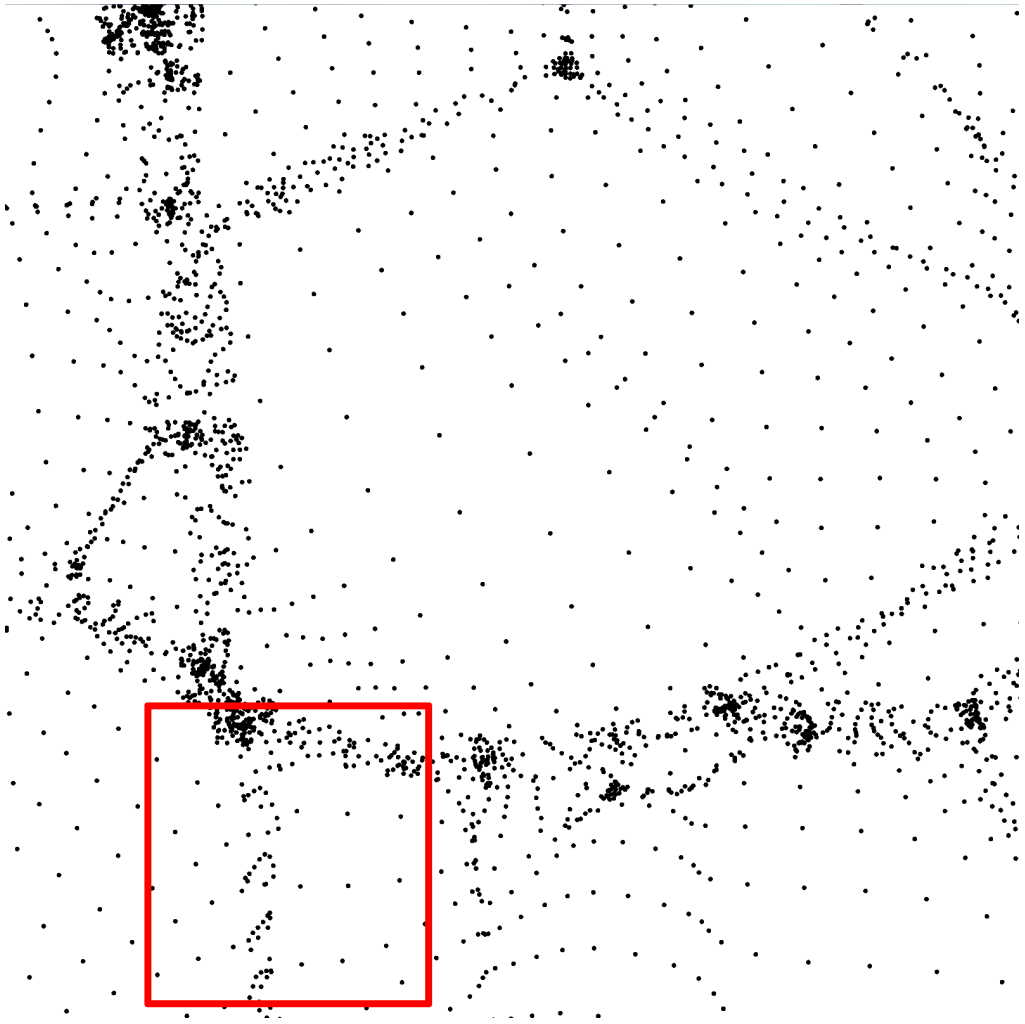
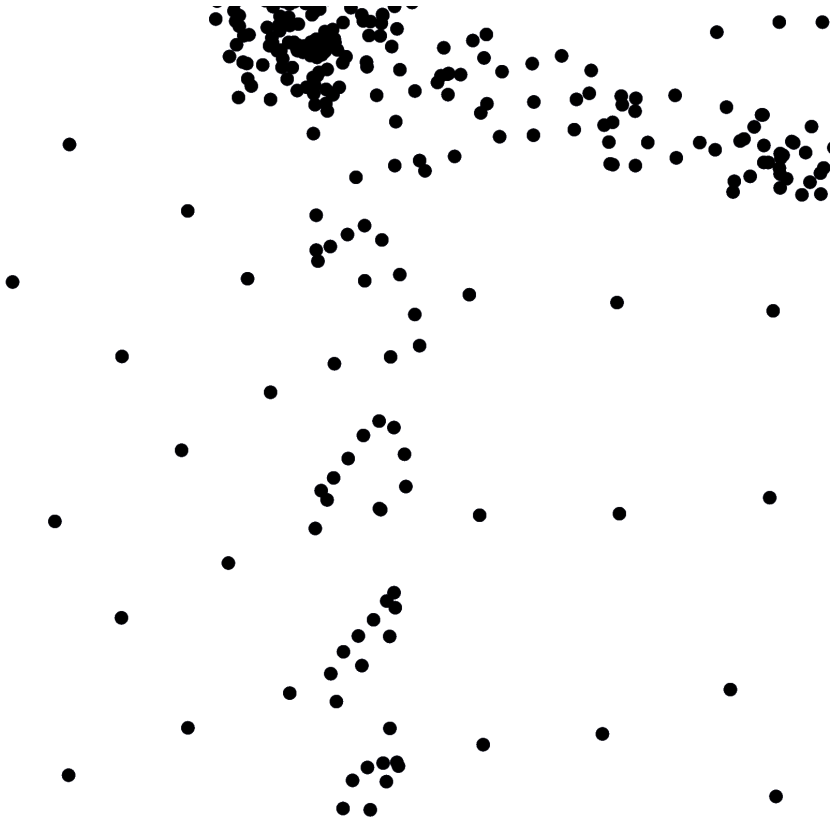
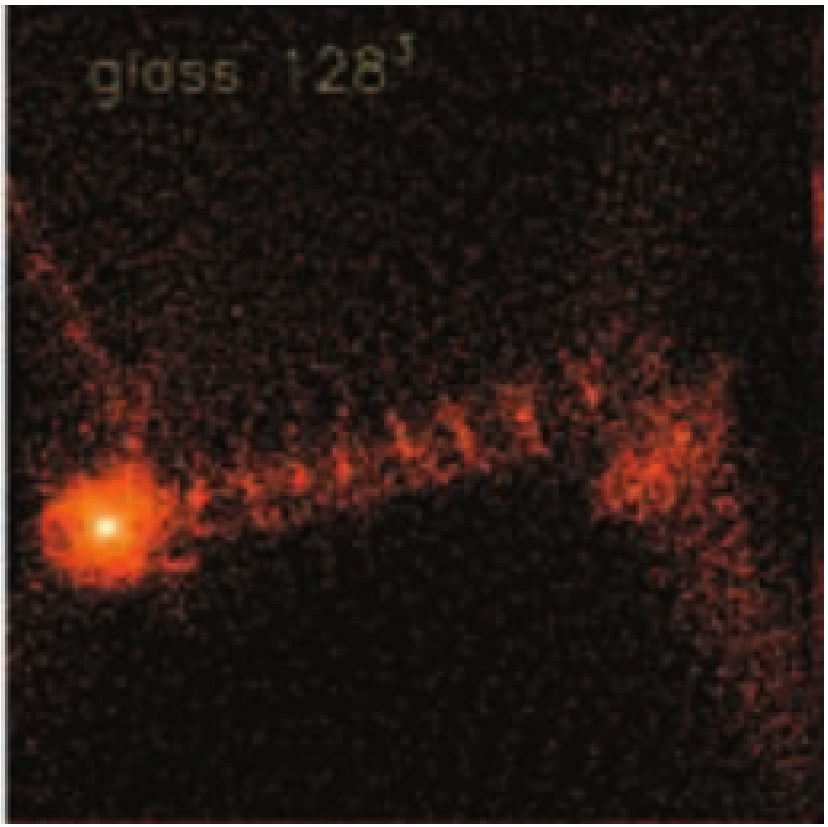
Wang J., White S. D. M., 2007, Monthly Notices of the Royal

Discreteness Effects & Artificial Haloes



Discreteness Effects & Artificial Haloes

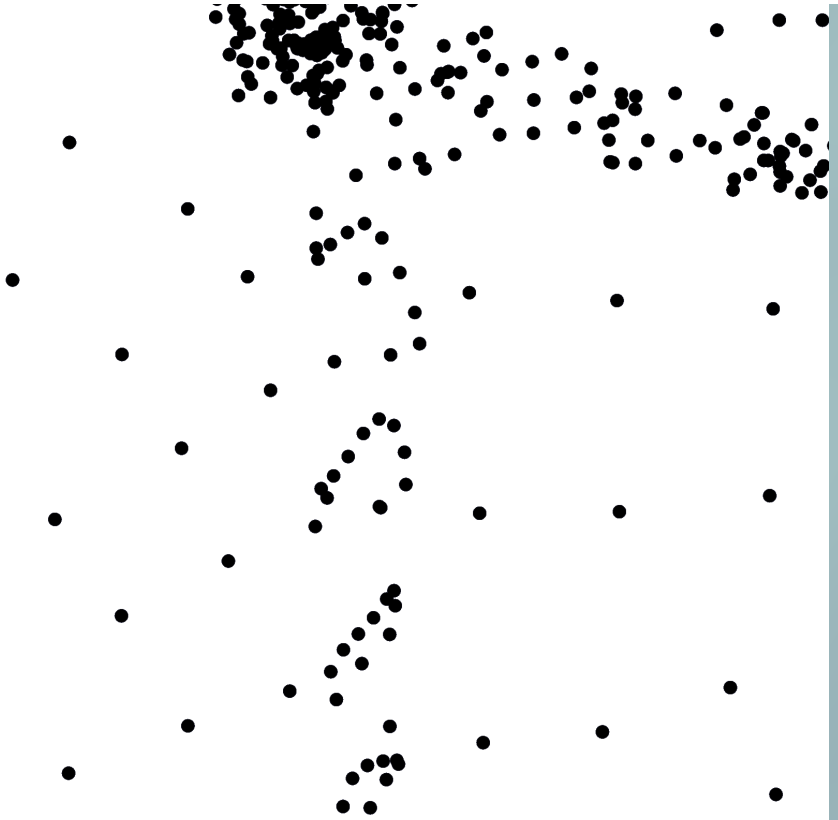


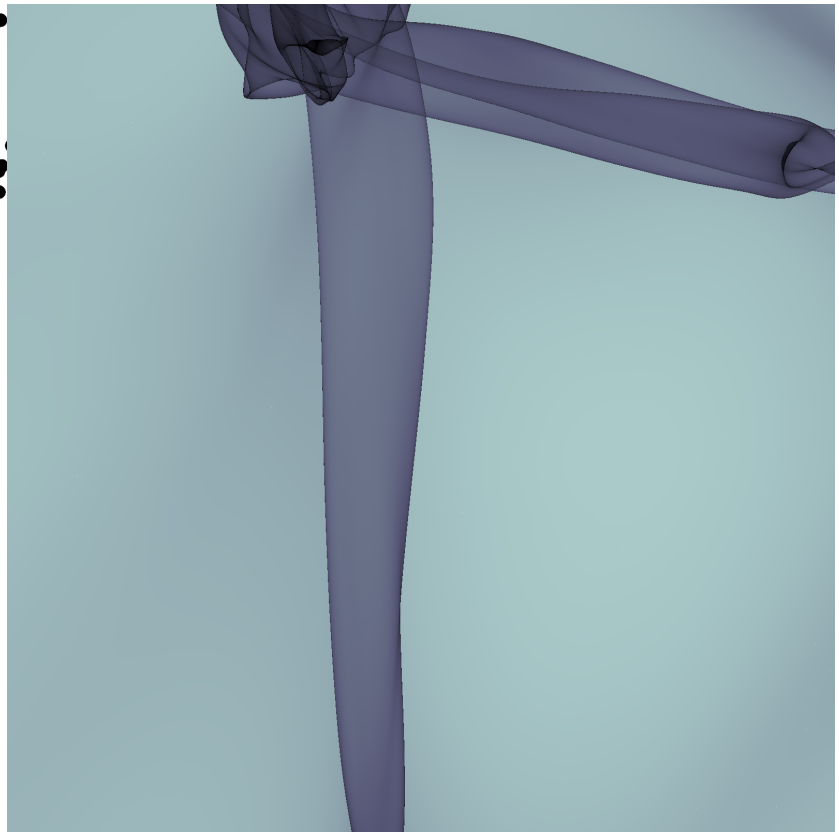


Wang & White (2007)

How to get rid of discreteness effects?

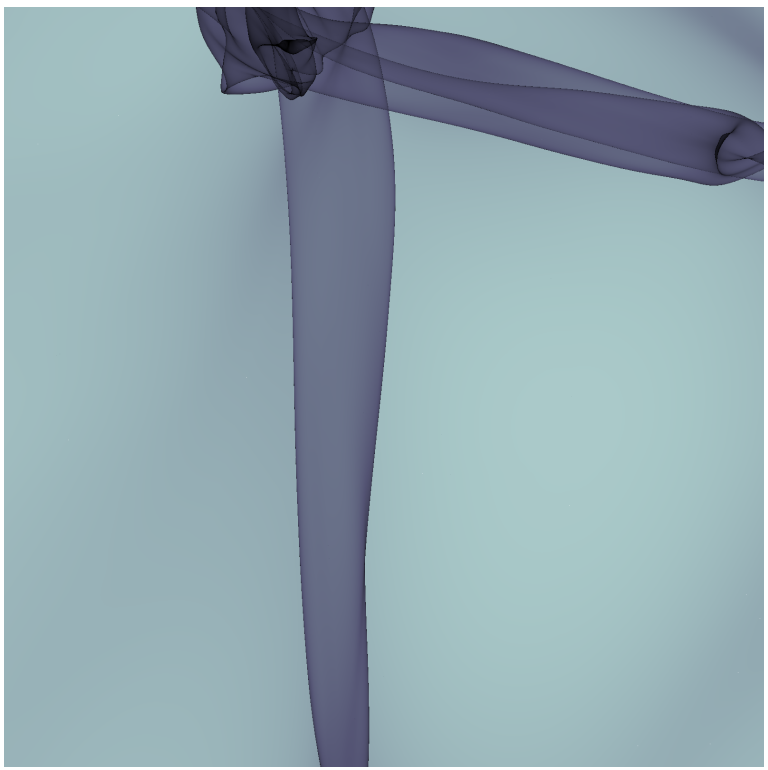
Discretization vs. Continuum

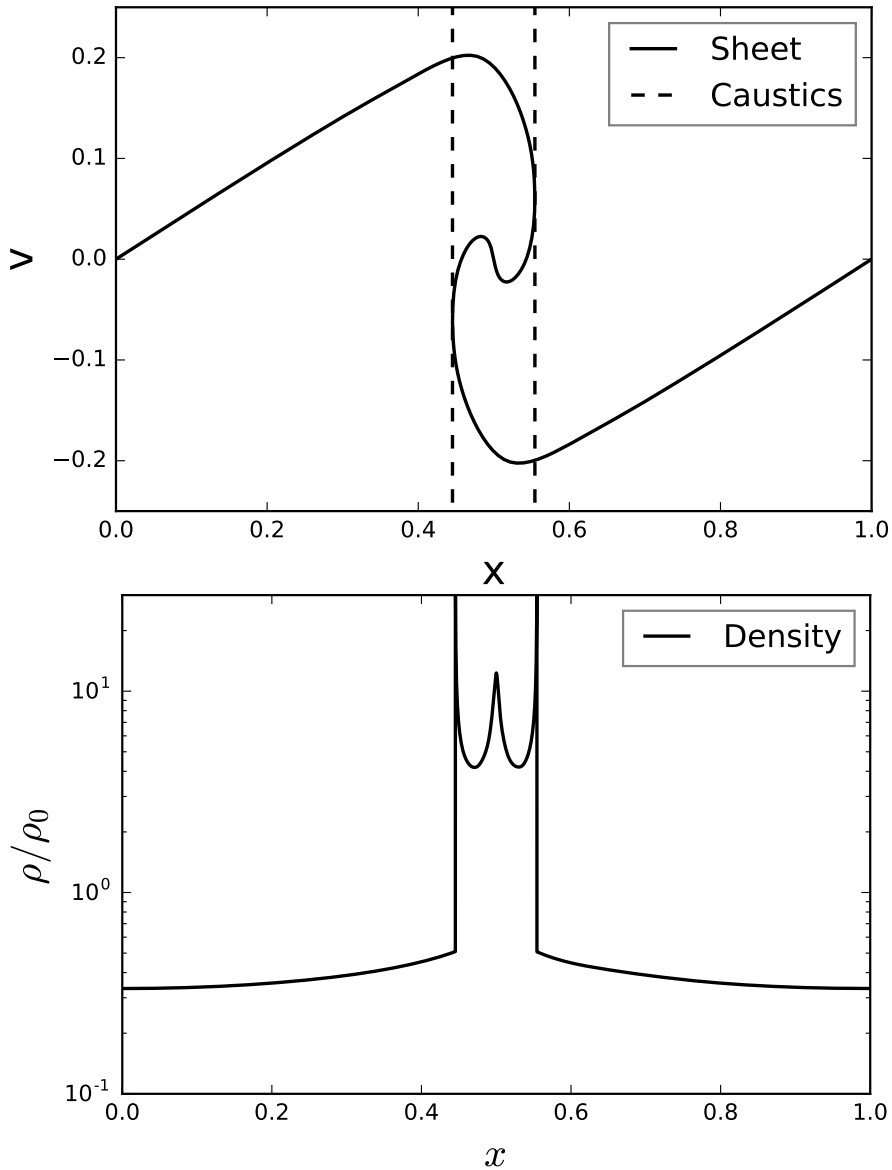




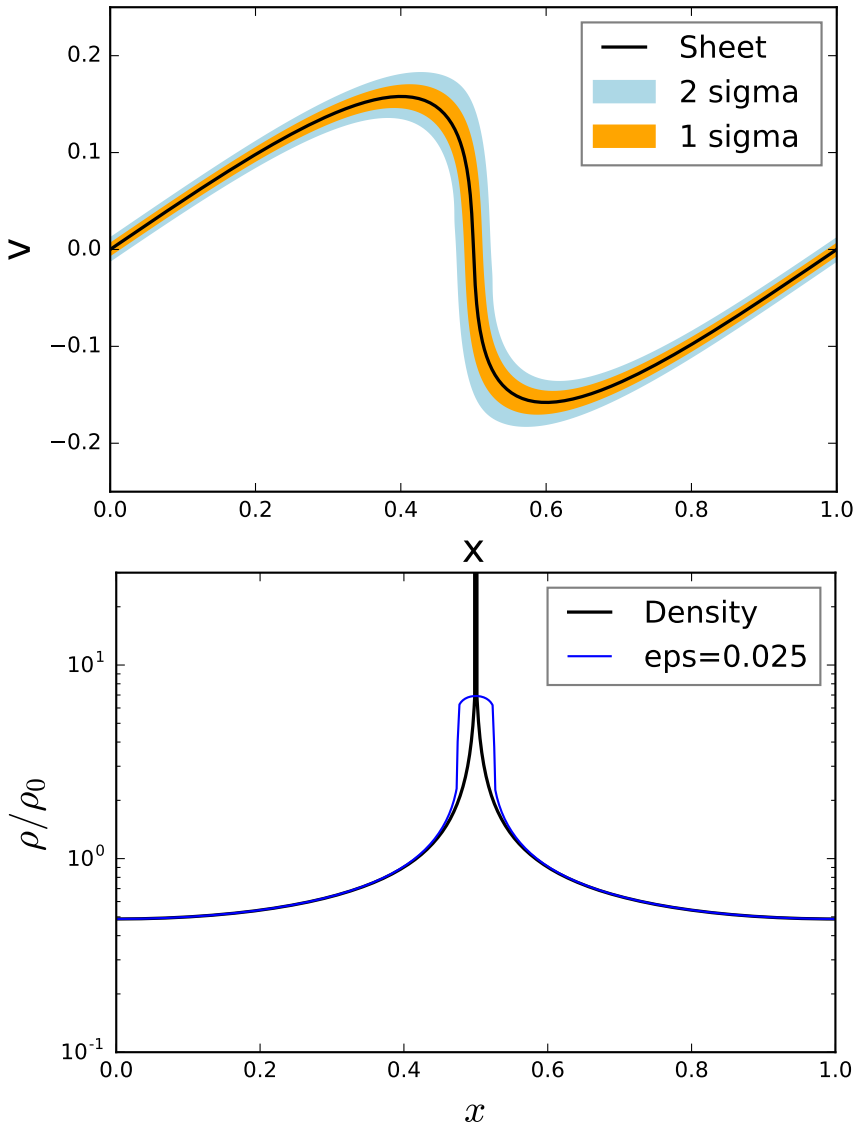
Use this density estimate in simulations to get rid of artificial fragmentation.
(Angulo et al., 2013; Hahn & Angulo, 2016; Sousbie & Colombi, 2016)

Streams and Caustics



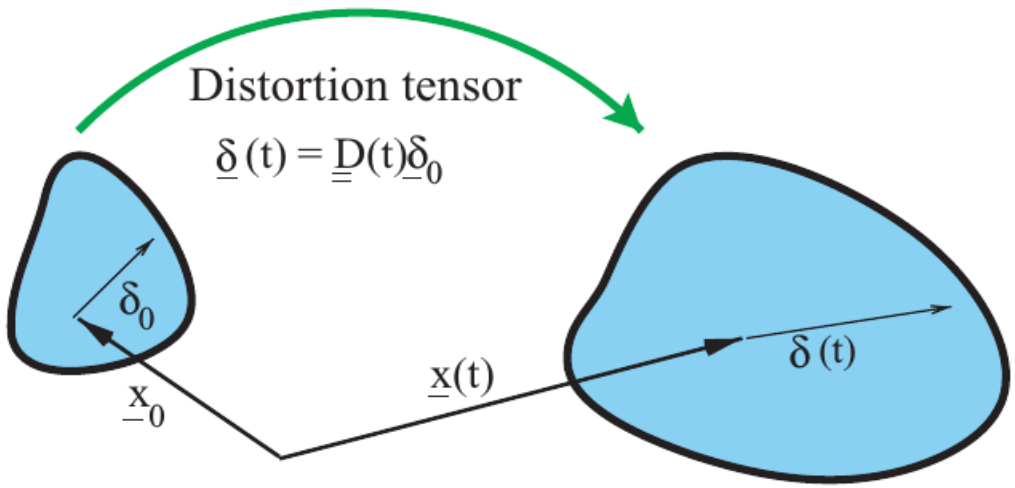


Caustics and Velocity Dispersion



The Geodesic Deviation Equation

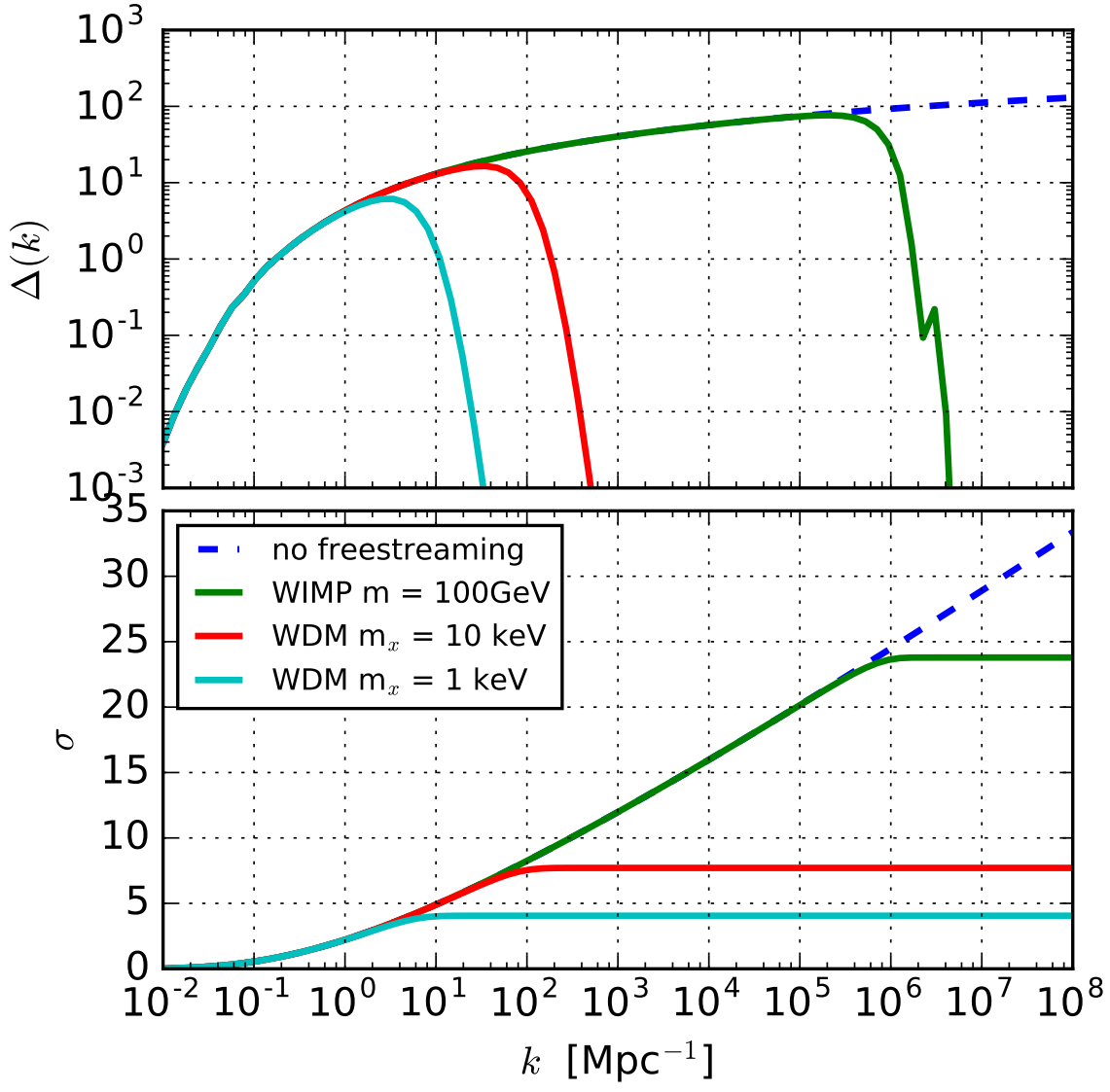
The distortion tensor is the Jacobian $d\vec{\xi}/d\vec{\xi}_0$ (in 6D phase space) where $\xi = (\vec{x}, \vec{v})$



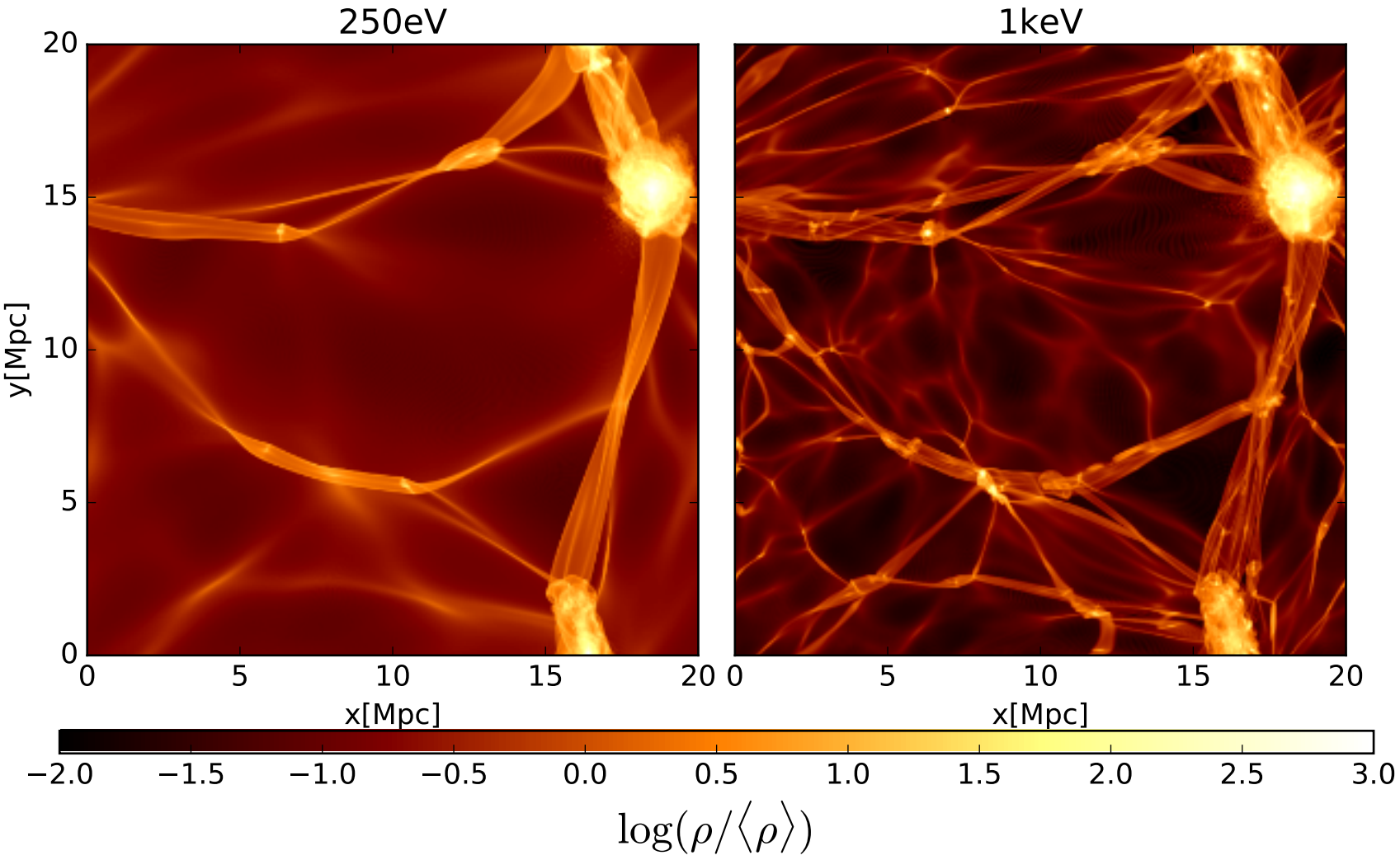
Vogelsberger et al. (2008)

Its evolution is given by the tidal forces (Geodesic deviation equation)

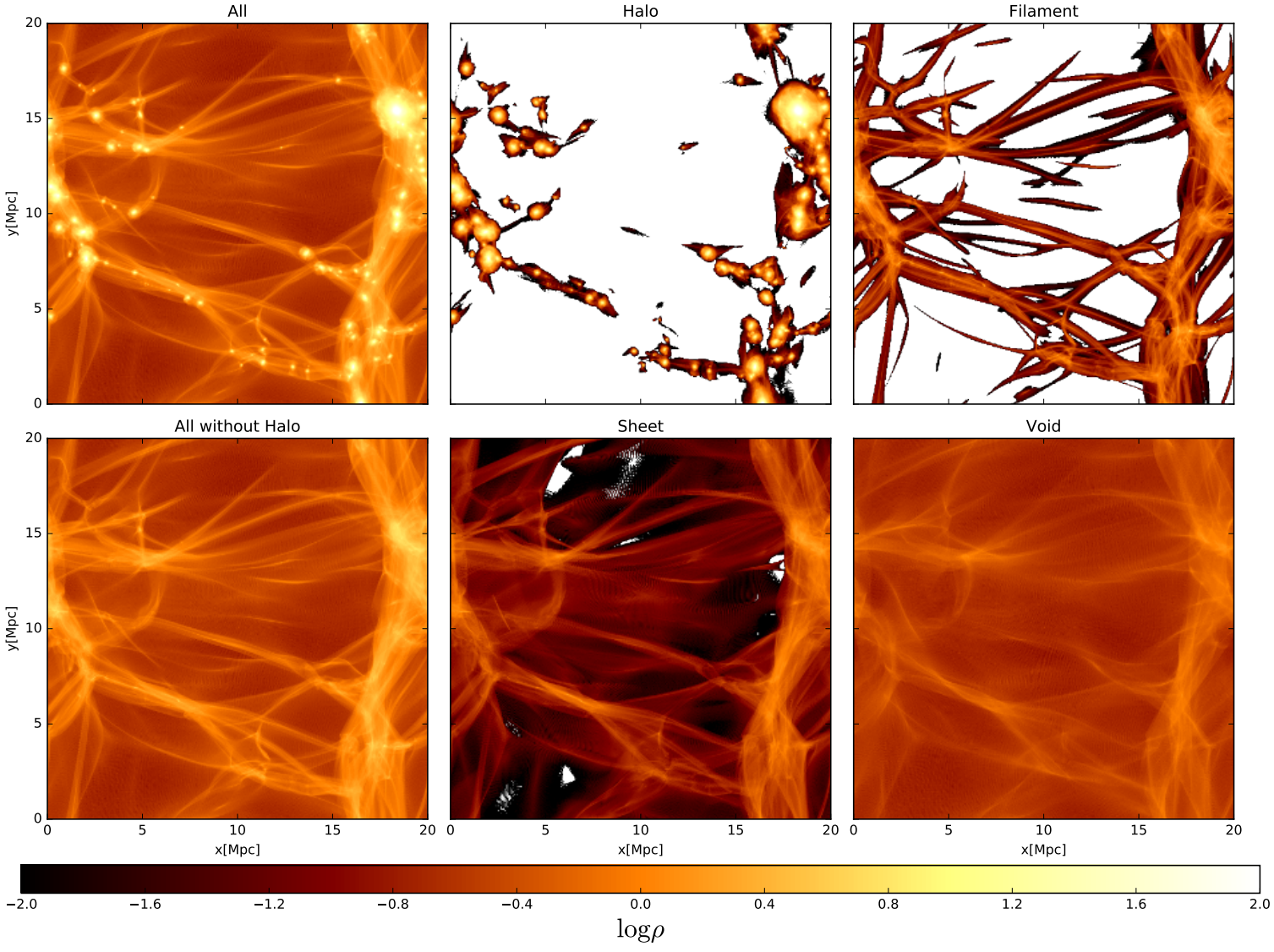
Power Spectrum



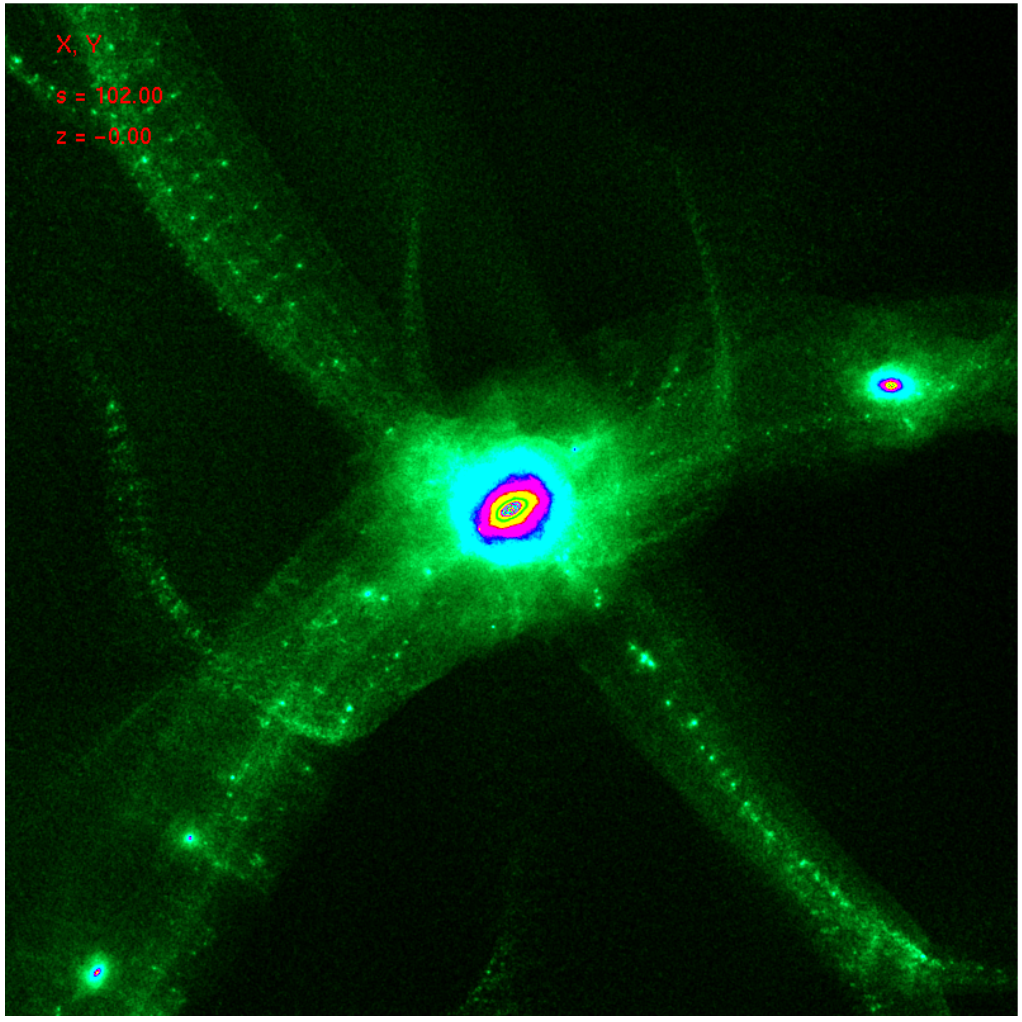
Warmth and Substructure



Structure Classification



Artificial Haloes



Liouvilles Theoreme in an expanding universe

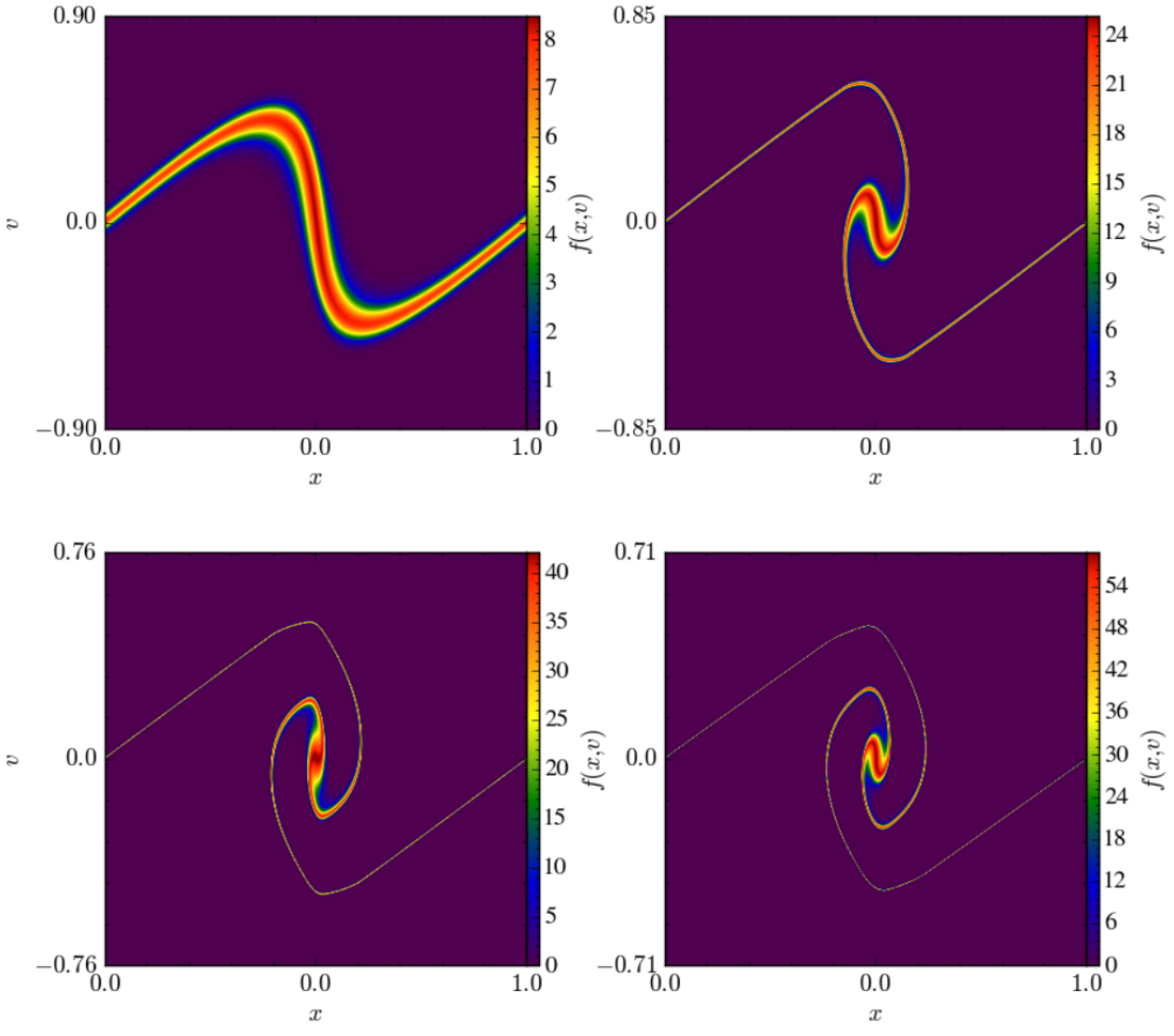
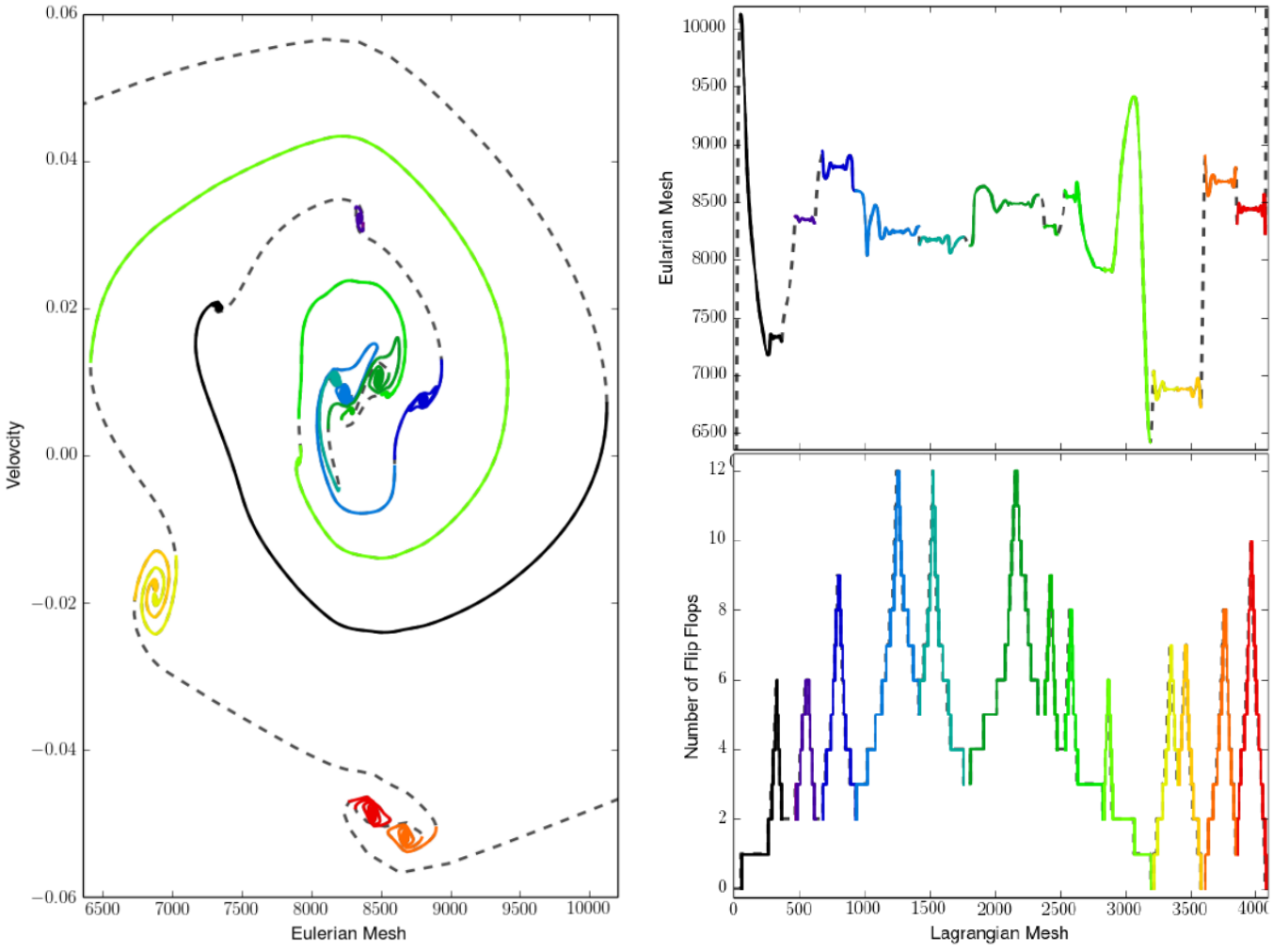


Figure from Myers et al. (2015)

Caustic Counts and Substructures



Shandarin & Medvedev (2014)