On putting experimental data into the Vlasov-Poisson equations

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Outline

- Externally excited ion waves.
  - Calculation of terms and moments of the kinetic equation.
- Incoherent detection and identification of waves.
  - Use of the phase-space cross-correlation matrix.
- Electron wave-particle interaction with Alfven waves.

Electrostatic waves at ion frequencies

$$E = -\nabla\Phi$$

$$n_e \approx n_i$$ Quasi-neutral $\lambda >> \lambda_d$

$$\frac{\tilde{n}_{e1}}{n_{e0}} \approx e^{\frac{e\Phi}{\kappa T_e}}$$ Low frequency $\omega << \omega_{pi}$

$$\frac{\partial f}{\partial t} + \mathrm{v}\frac{\partial f}{\partial z} + \frac{e}{m}E\frac{\partial f}{\partial \mathrm{v}} = 0$$

Small amplitude

$$E = E_1$$

$$f = f_0 + f_1$$

Fluid Dispersion relation

$$\frac{\omega_{pi}^2}{\omega^2} k^2 = \frac{1}{\lambda_{de}^2}$$

$$\omega = kC_s$$

Kinetic Dispersion Relation

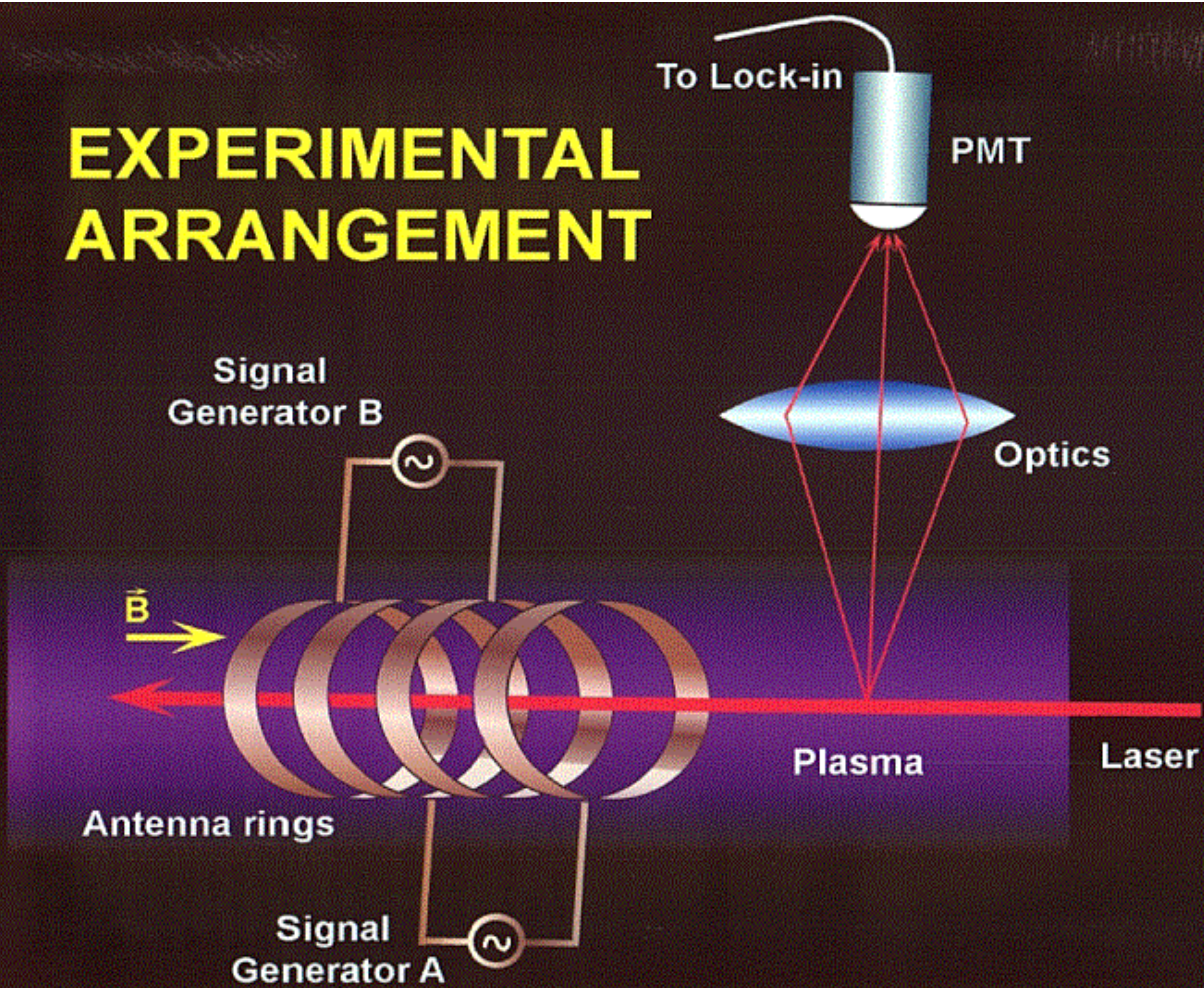
$$1 - C_s^2 \frac{1}{n_0} G\left(\frac{\partial f}{\partial v_{\mathrm{P}}}\right) = 0$$

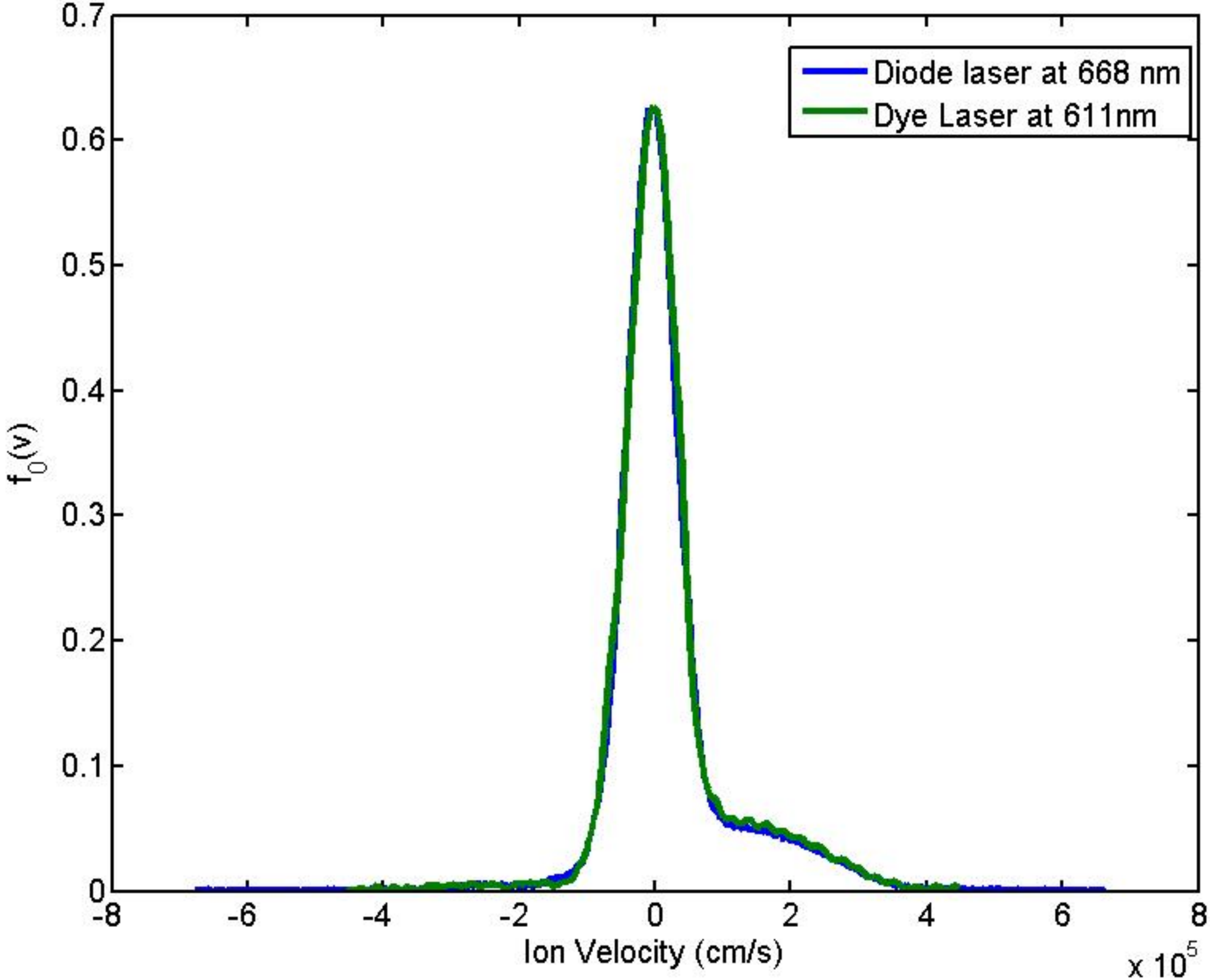
$$G(h(v))_u = p\int_{-\infty}^{\infty} \frac{h(v)}{v-u}\,dv + i\pi h(u)$$

$$G(f(v))\downarrow u \to n\downarrow 0 \;\; Z(u/v\downarrow t\;)$$

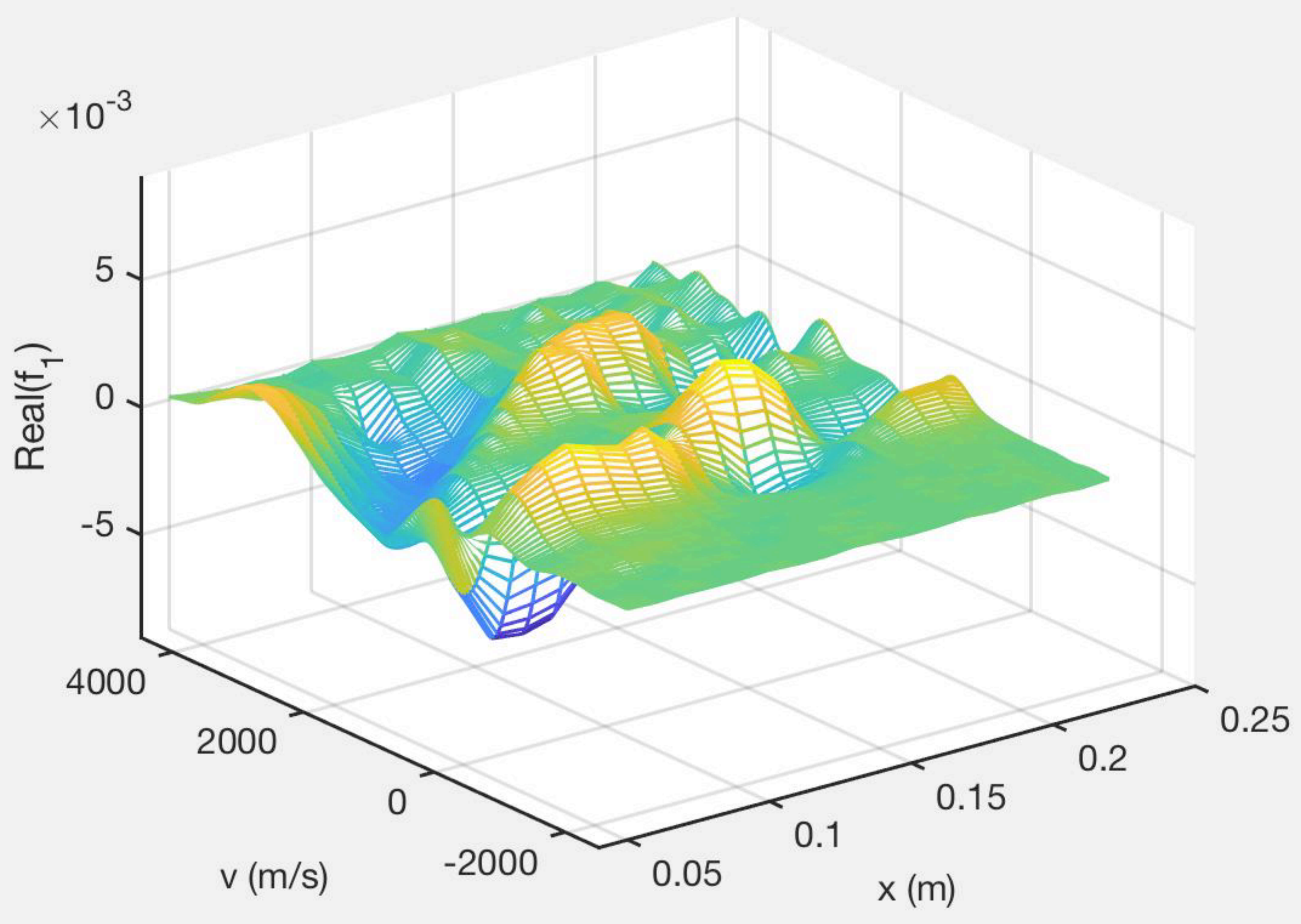
(For the case of a maxwellian *f(v)*)

EXPERIMENTAL ARRANGEMENT

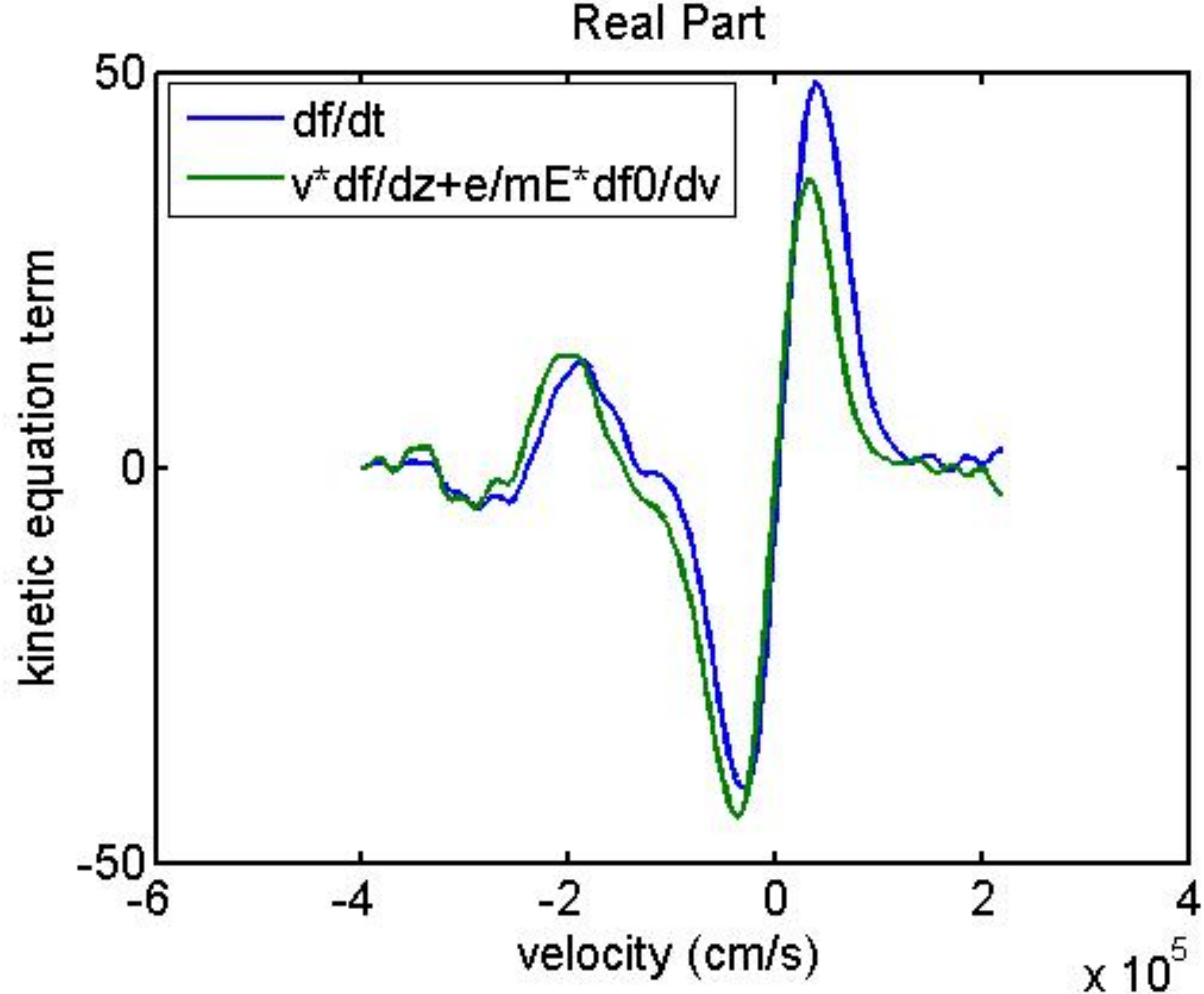




0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$f_0(v)$
-8
-6
-4
-2
0
2
4
6
8
Ion Velocity (cm/s)
x 10^5
Diode laser at 668 nm
Dye Laser at 611nm



×10^-3
5
0
-5
Real(f_1)
4000
2000
0
-2000
v (m/s)
0.05
0.1
0.15
0.2
0.25
x (m)



Real Part
df/dt
v*df/dz+e/mE*df0/dv
kinetic equation term
50
0
-50
-6
-4
-2
0
2
4
velocity (cm/s)
x 10^5

Generalized Hilbert Transform

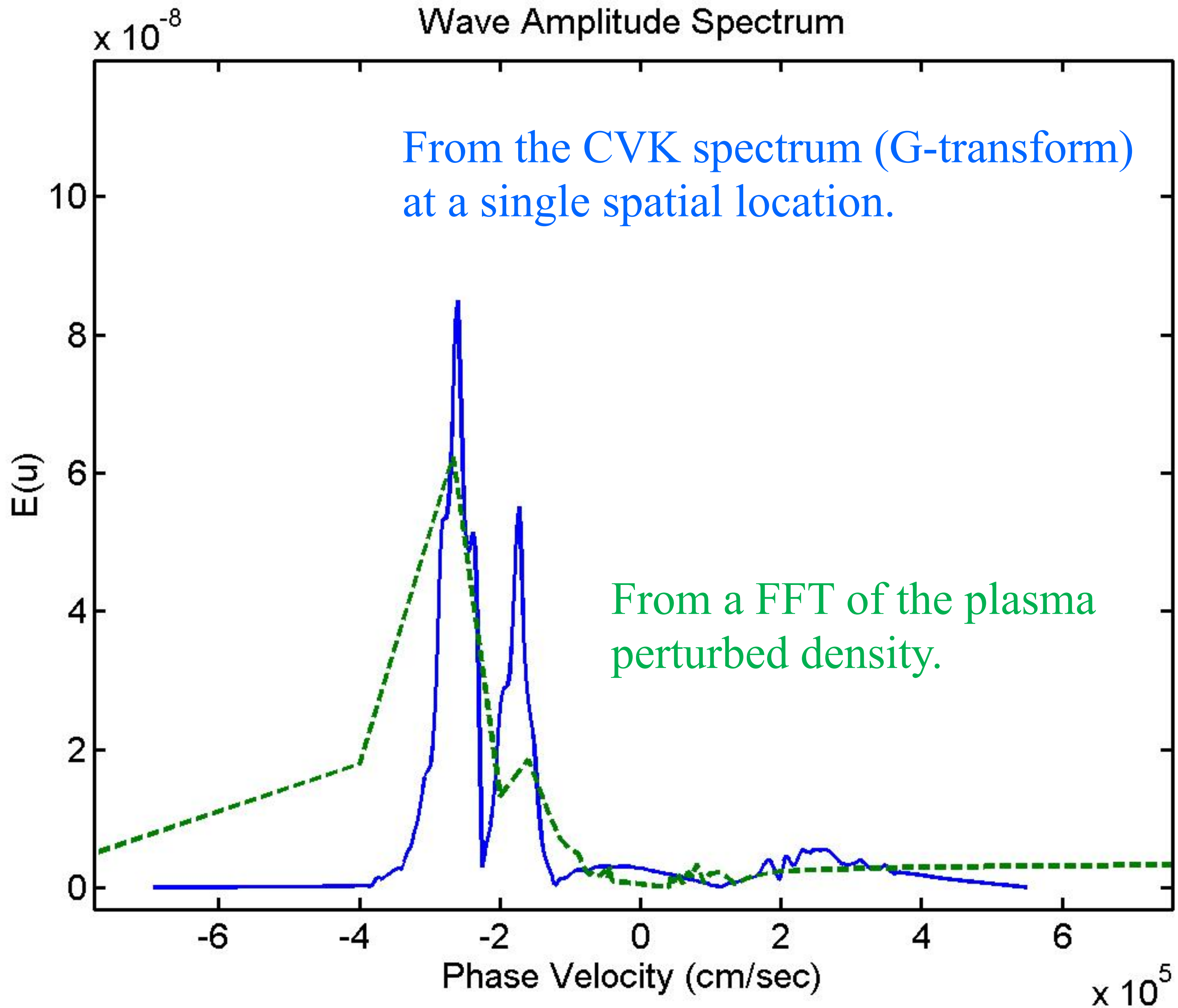
$$\mathcal{G}\left[f(\mathrm{v})\right]_u \equiv g = \alpha(u)\bar{f} + \beta(u) f(u)$$

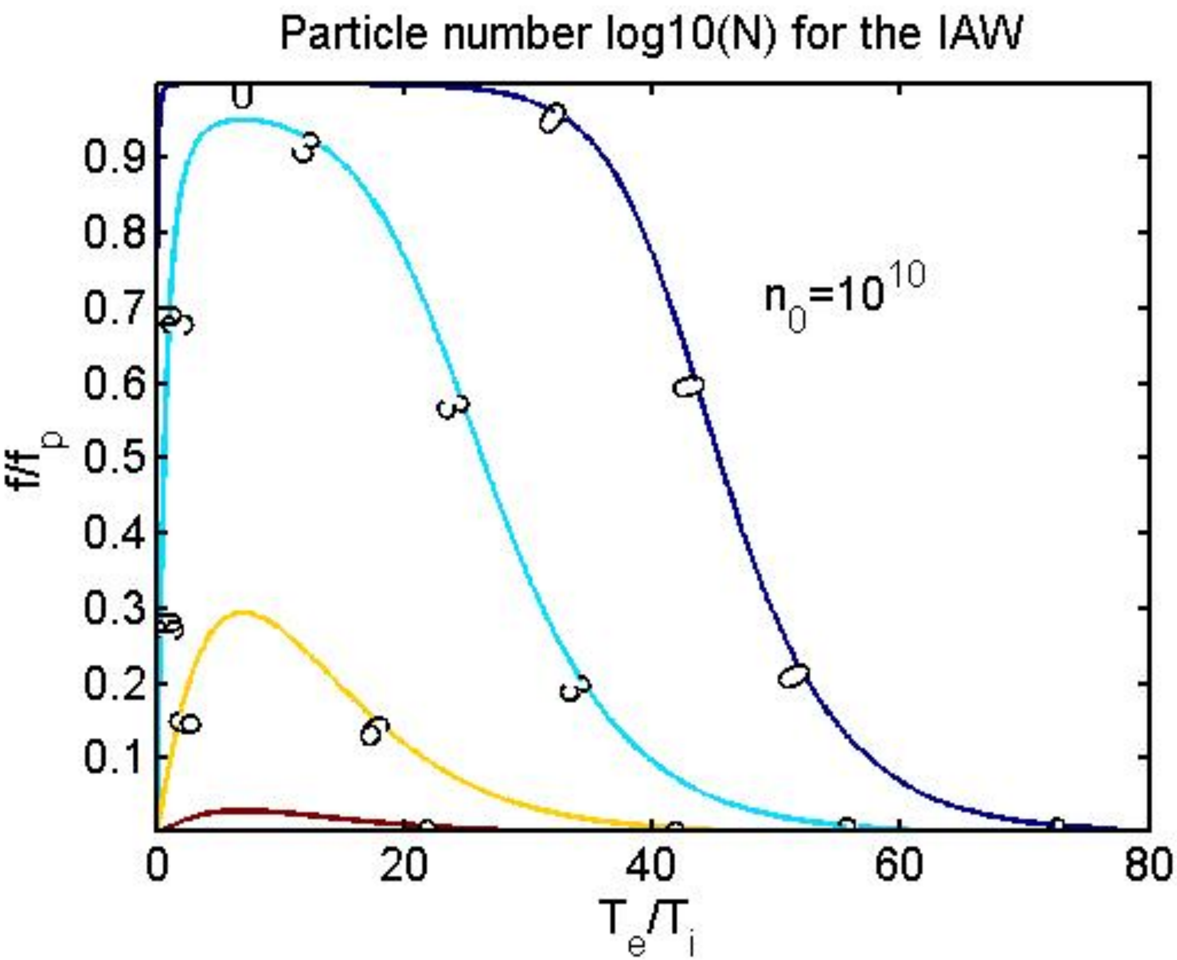
$$\mathcal{G}\left[g\right] = f = -\frac{\alpha}{\alpha^2+\beta^2}\bar{g} + \frac{\beta}{\alpha^2+\beta^2} g$$

Morrison G transform

$$\varepsilon = i\alpha + \beta$$

Appropriate dielectric function ε


Wave Amplitude Spectrum
x 10^-8
From the CVK spectrum (G-transform)
at a single spatial location.
From a FFT of the plasma
perturbed density.
E(u)
10
8
6
4
2
0
-6
-4
-2
0
2
4
6
Phase Velocity (cm/sec)
x 10^5



Number of resonant particles:

$$N = f(\omega/k)\,\lambda^3\,(\delta v)$$

$$\delta v \sim 2 C_s * \epsilon_I (\omega/k)$$

So this approach is problematic for Te>>Ti

Second order perturbations

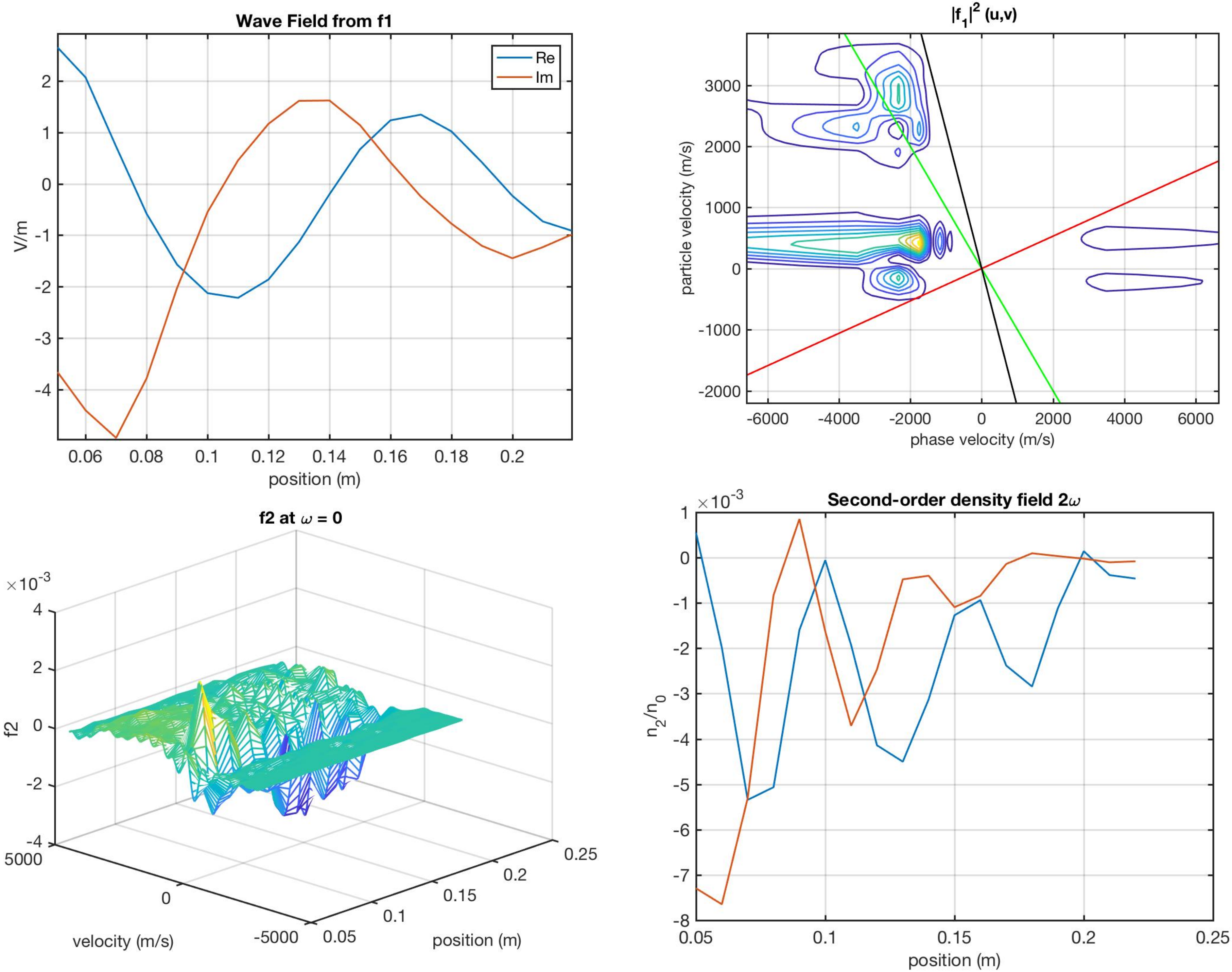
$$f_2(\omega=0)=f(\omega=0,\ with\ waves)-f_0$$

$$f_2(2\omega)$$

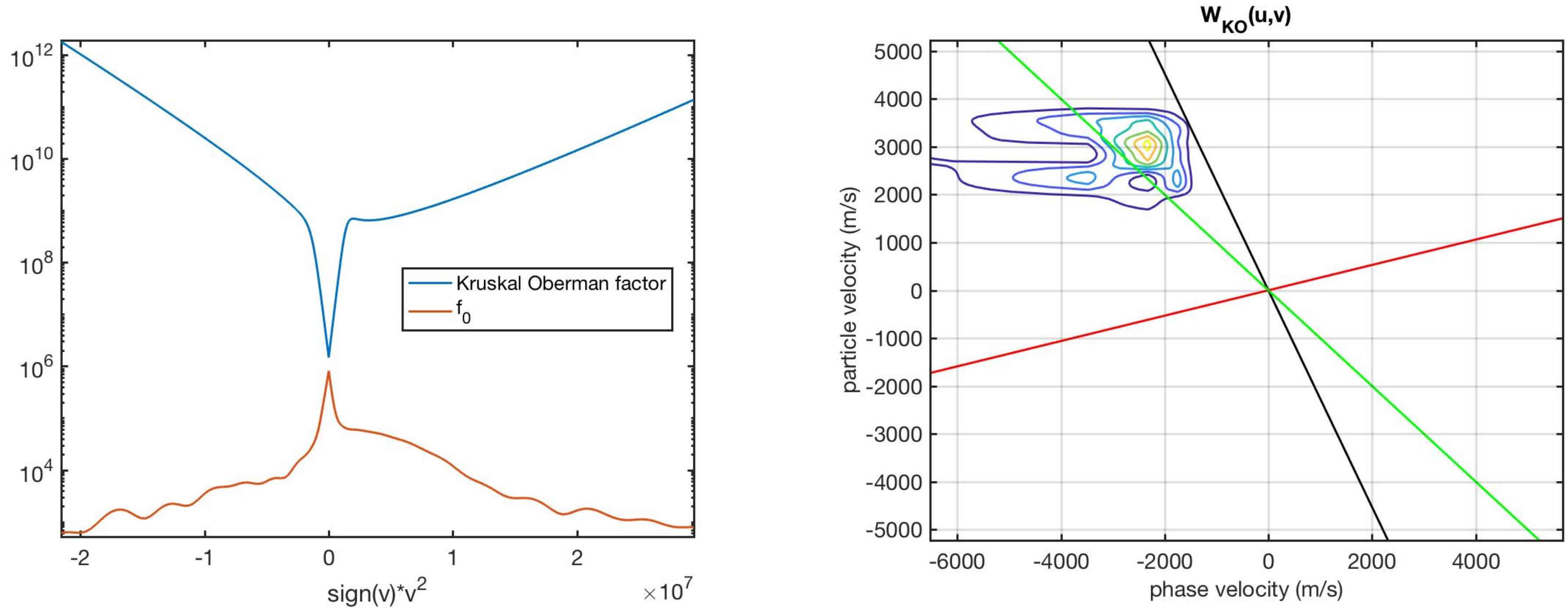
Energy density:

$$W_{KO}=\frac{1}{2}\iint \left(-\frac{mv}{\partial f_0/\partial v}\right)|\delta f|^2\,dz\,dv$$

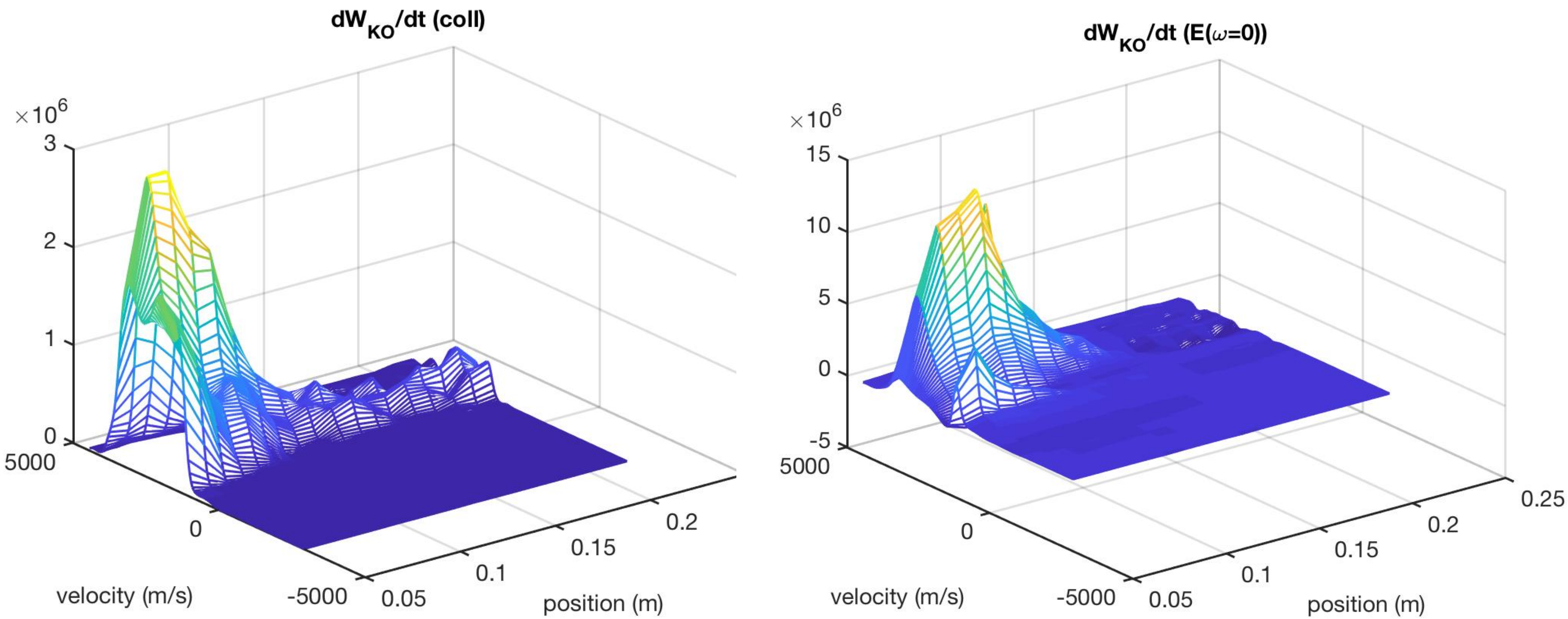
$$\frac{\partial}{\partial t}W_{KO}=\frac{e}{2}\iint \frac{\partial}{\partial v}\left(\frac{v}{\partial f_0/\partial v}\right)E|\delta f|^2\,dz\,dv$$



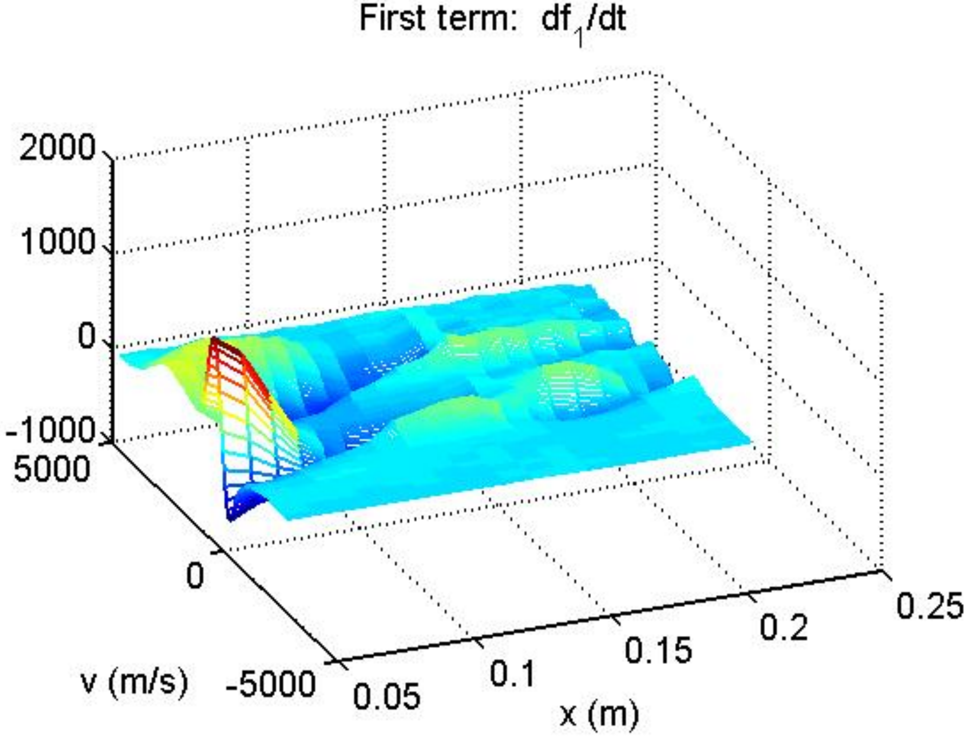
Wave Field from f1
Re
Im
V/m
position (m)
|f1|2 (u,v)
particle velocity (m/s)
phase velocity (m/s)
f2 at ω = 0
×10-3
f2
velocity (m/s)
position (m)
Second-order density field 2ω
×10-3
n2/n0
position (m)

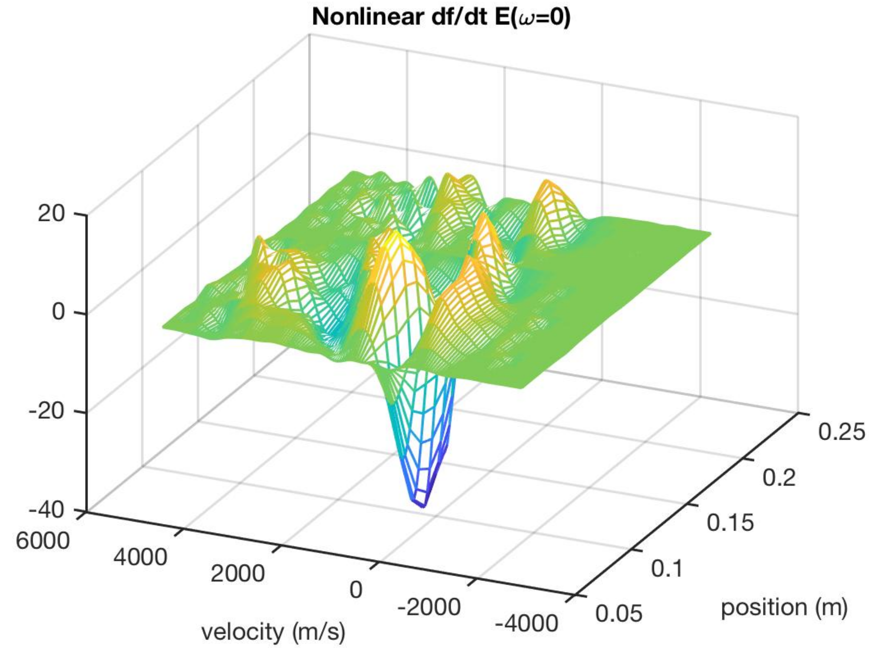


Kruskal Oberman factor
f_0
sign(v)*v^2
$\times 10^7$
$W_{KO}(u,v)$
particle velocity (m/s)
phase velocity (m/s)

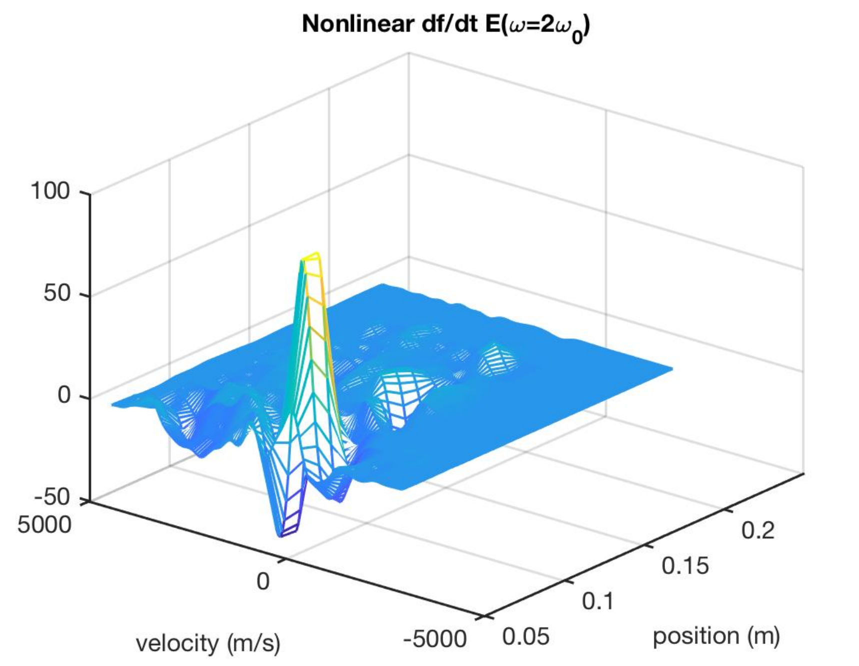


dW_KO/dt (coll)
×10^6
3
2
1
0
5000
0
-5000
velocity (m/s)
0.05
0.1
0.15
0.2
position (m)
dW_KO/dt (E(ω=0))
×10^6
15
10
5
0
-5
5000
0
-5000
velocity (m/s)
0.05
0.1
0.15
0.2
0.25
position (m)





$$-e/m\, E\partial f/\partial v$$



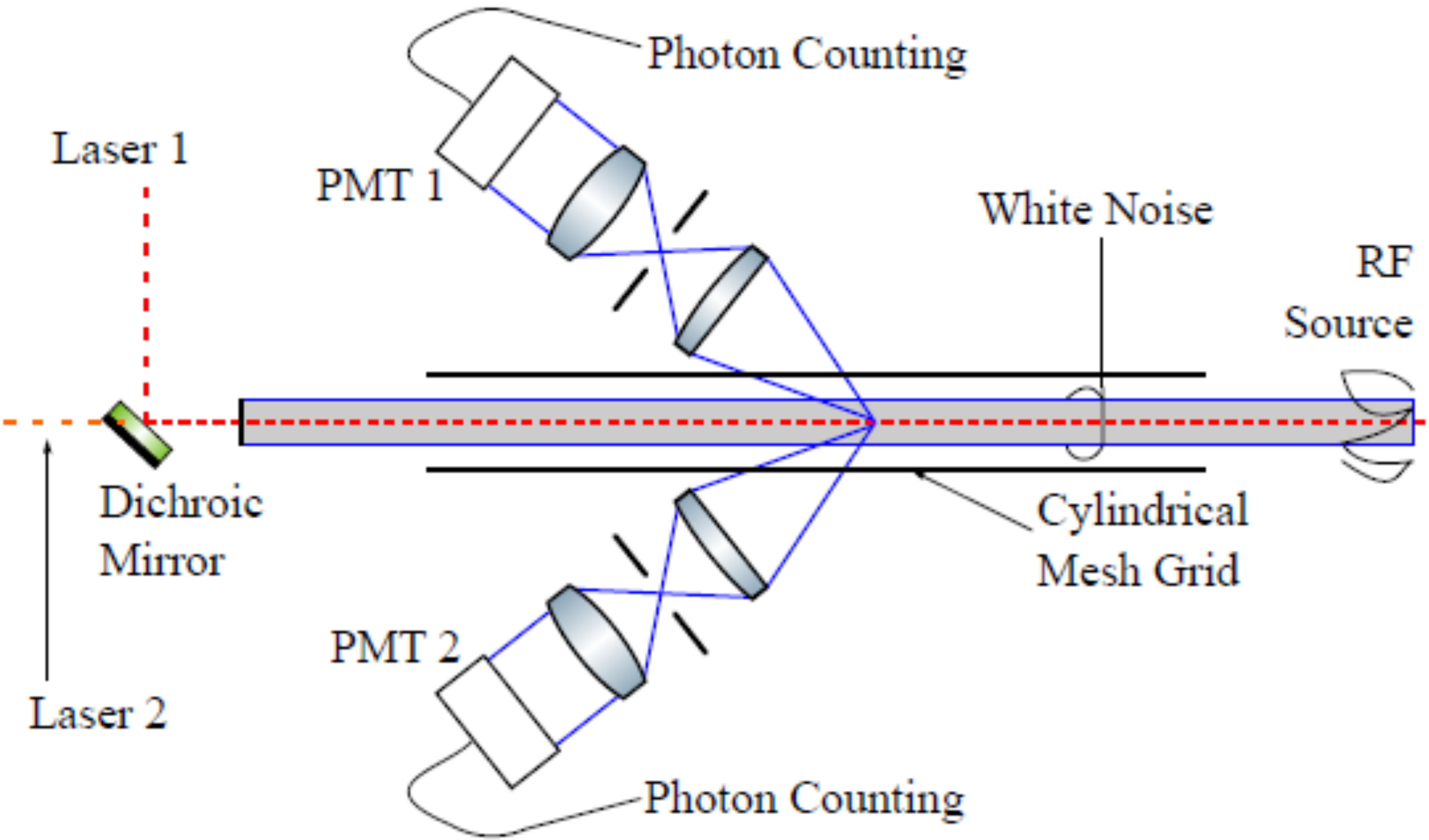
Phase-space correlation function : C

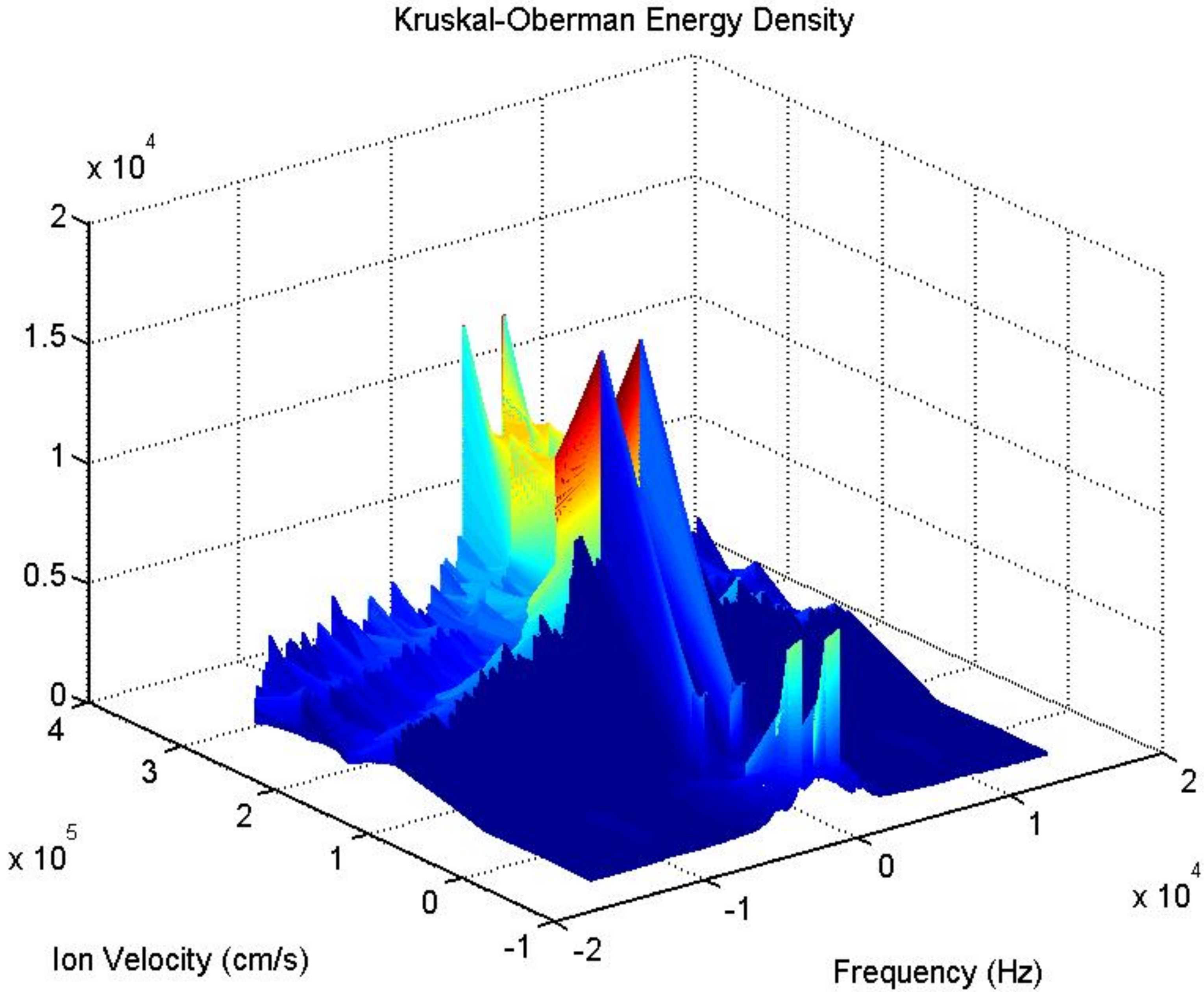
$$C(\vec{x}_1, \vec{x}_2, \vec{v}_1, \vec{v}_2; \tau) = \langle \delta f(\vec{x}_1, \vec{v}_1, t) \delta f(\vec{x}_2, \vec{v}_2, t - \tau) \rangle_t$$

$$C(x_1, x_1, v_1, v_2; \tau) = C(x_1, x_1, v_2, v_1; -\tau)$$

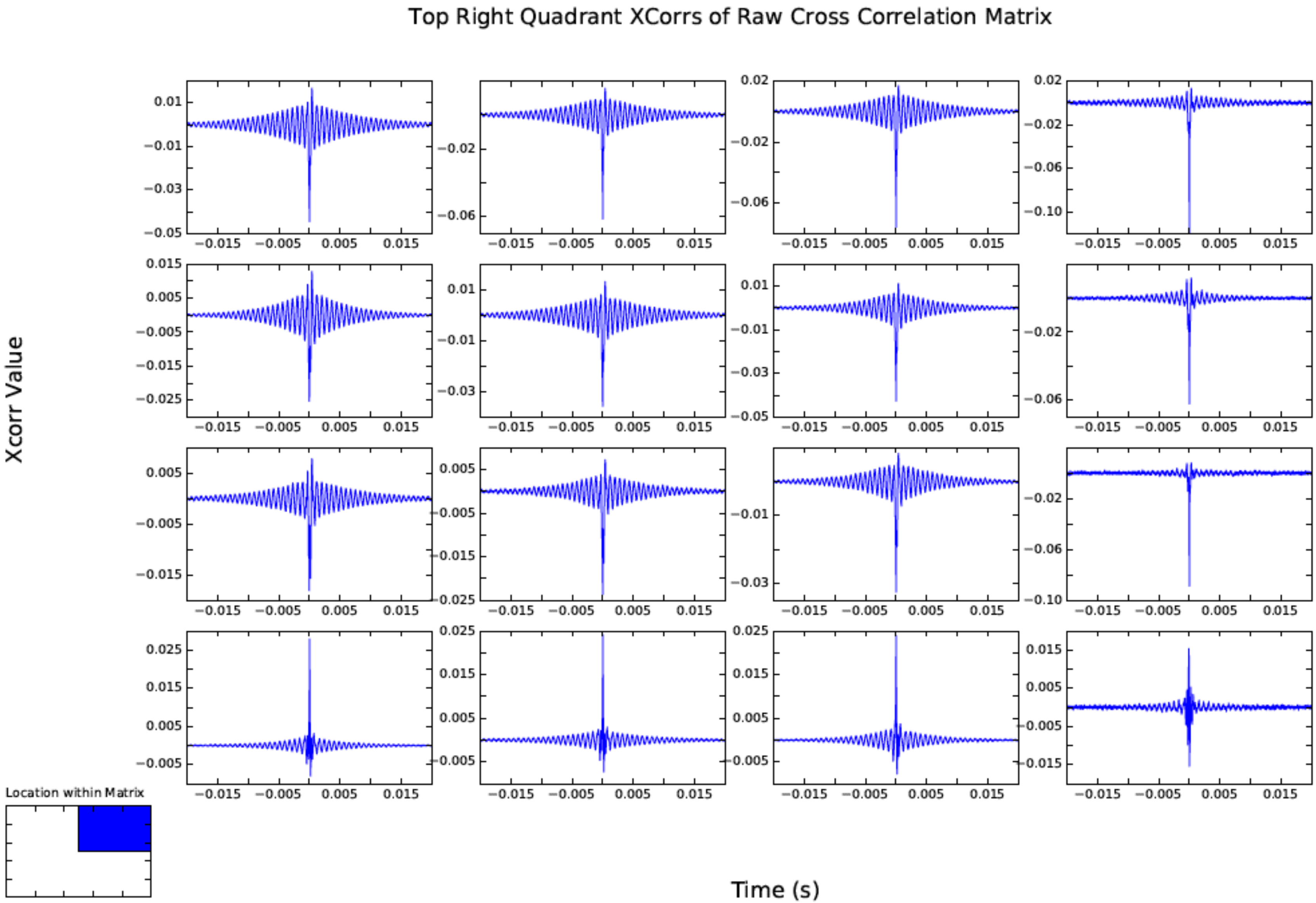
For a set of velocities $\varpi = \varpi_i$ the matrix $C(\omega, \varpi_i, \varpi_j)$ is Hermitian

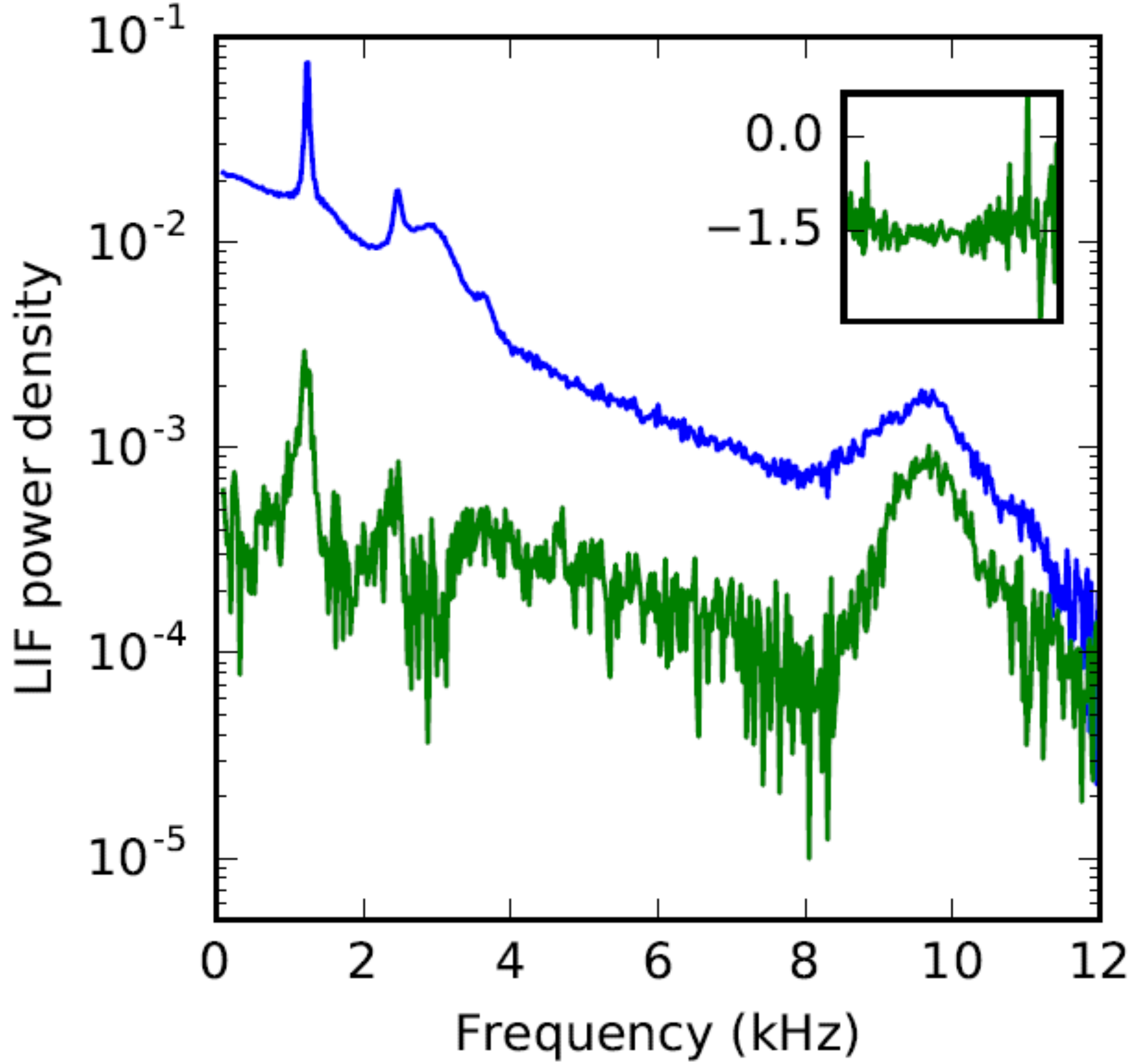
$$C(x_1, x_1, v_1, v_2; \omega) = C(x_1, x_1, v_2, v_1; \omega)^{*}$$


Photon Counting
Laser 1
PMT 1
White Noise
RF
Source
Dichroic
Mirror
Cylindrical
Mesh Grid
PMT 2
Laser 2
Photon Counting

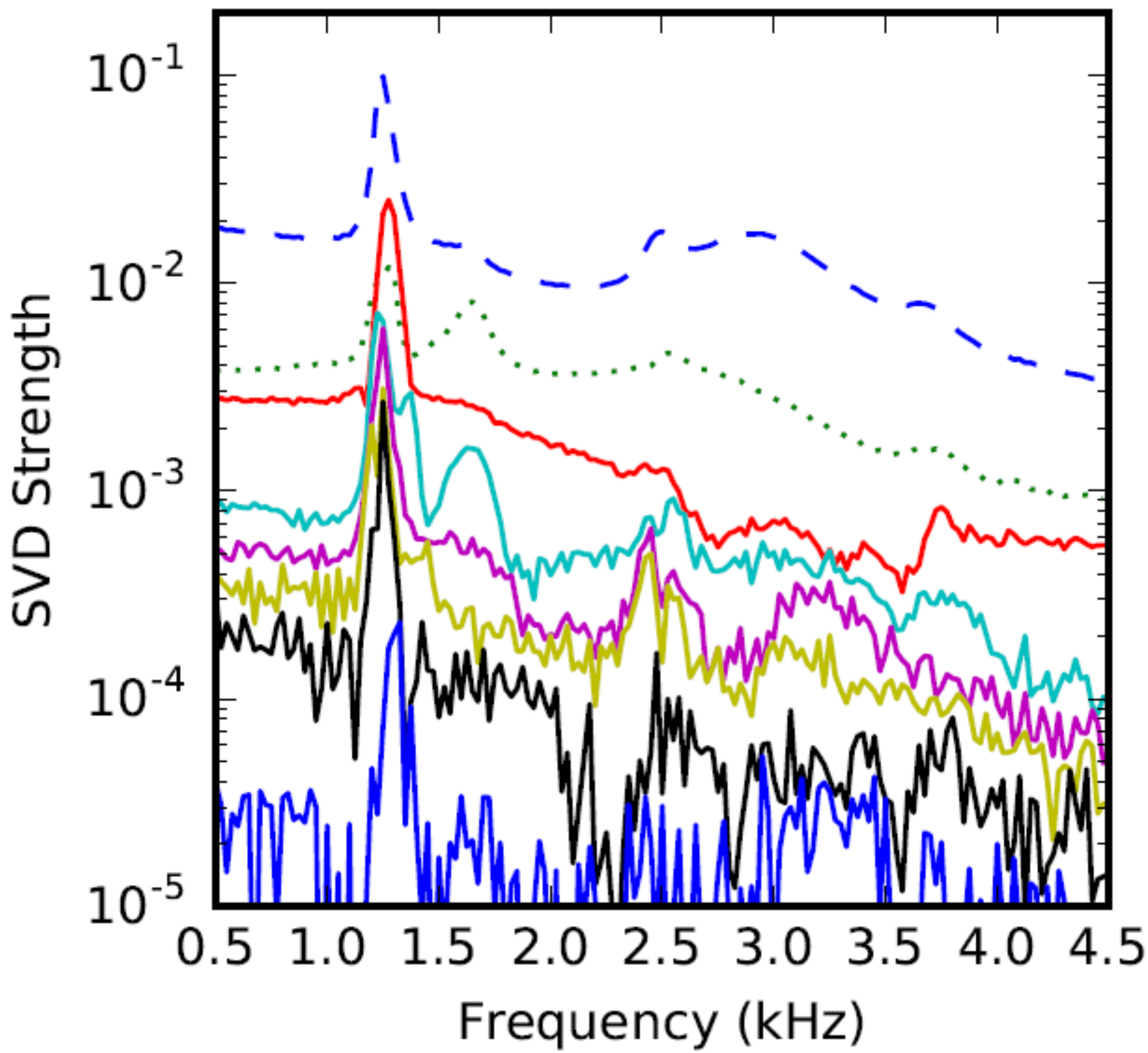


Kruskal-Oberman Energy Density
x 10^4
2
1.5
1
0.5
0
4
3
2
1
0
-1
x 10^5
Ion Velocity (cm/s)
-2
-1
0
1
2
x 10^4
Frequency (Hz)

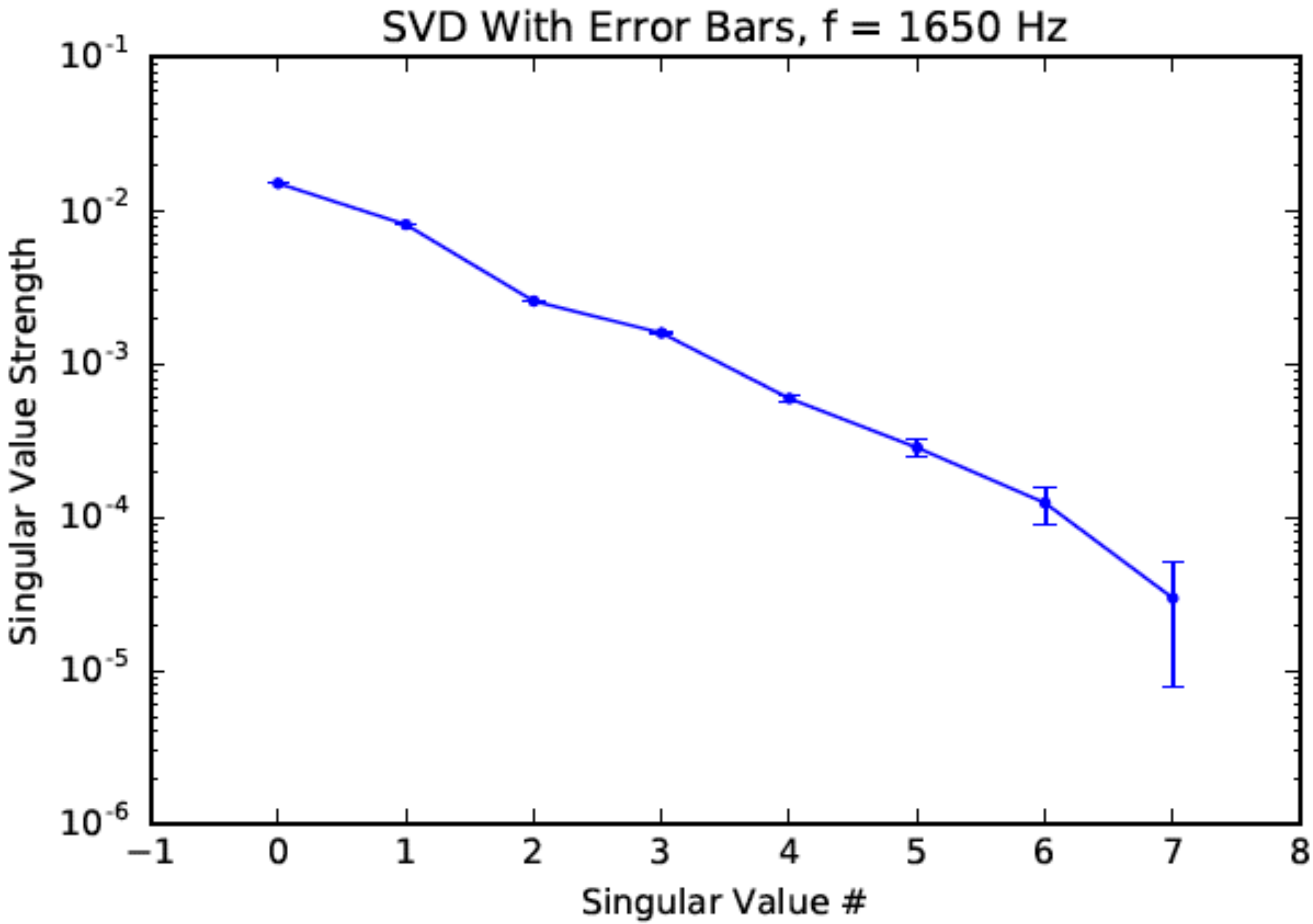

Top Right Quadrant XCorrs of Raw Cross Correlation Matrix
Xcorr Value
Time (s)
Location within Matrix

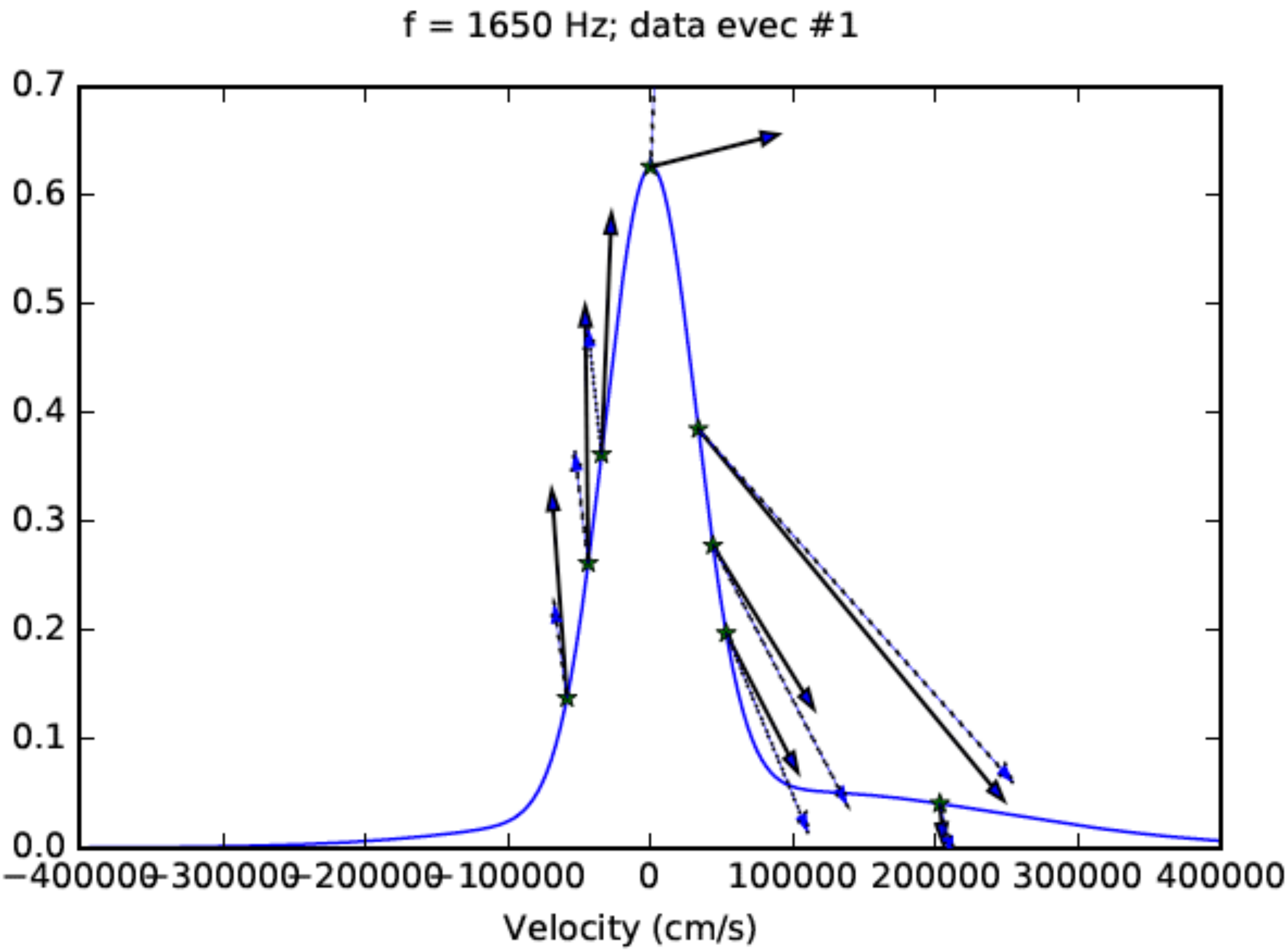


LIF power density
Frequency (kHz)
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
2
4
6
8
10
12
0.0
−1.5

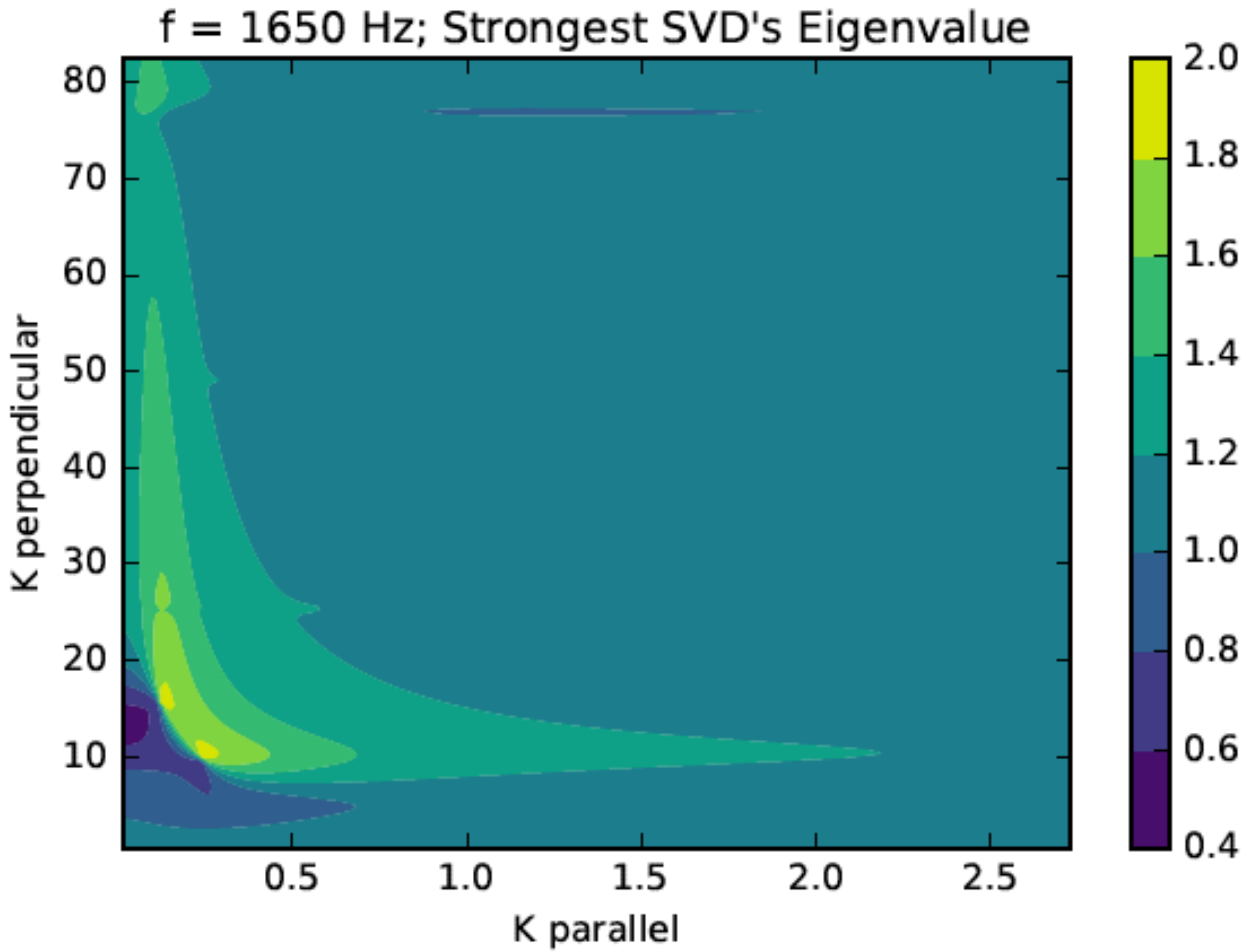


SVD Strength
Frequency (kHz)
10^-1
10^-2
10^-3
10^-4
10^-5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5


SVD With Error Bars, f = 1650 Hz
Singular Value Strength
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
−1
0
1
2
3
4
5
6
7
8
Singular Value #



f = 1650 Hz; data evec #1
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
−400000
−300000
−200000
−100000
0
100000
200000
300000
400000
Velocity (cm/s)



f = 1650 Hz; Strongest SVD's Eigenvalue
K perpendicular
K parallel
80
70
60
50
40
30
20
10
0.5
1.0
1.5
2.0
2.5
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4

Alfven wave-particle interaction with electrons.

$$-i\omega f+v\partial f/\partial z+i\omega/(k_x\delta)^2\ [(V_A\,\partial/\partial z/\omega)^2+1]\int^{}v f\,dv\,1/n$$

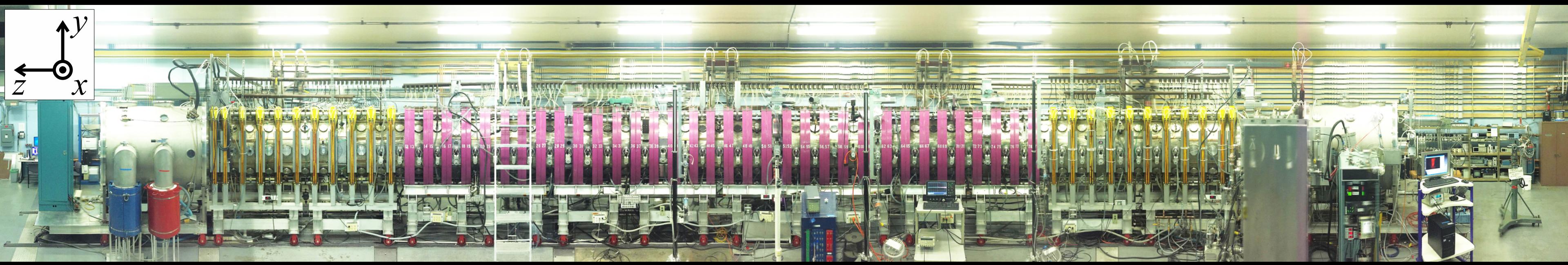
This gives the dispersion relation for the perturbed electron current:

$$u=\omega/k$$

$$\epsilon=1+(V_A^2-u^2)/v_t^2\ \ 1/k_x^2\,\delta^2\ \ (1+\zeta Z(\varsigma))$$

$$\varsigma=u/\sqrt{2}\,v_t$$

The LAPD generates a linear magnetized plasma.

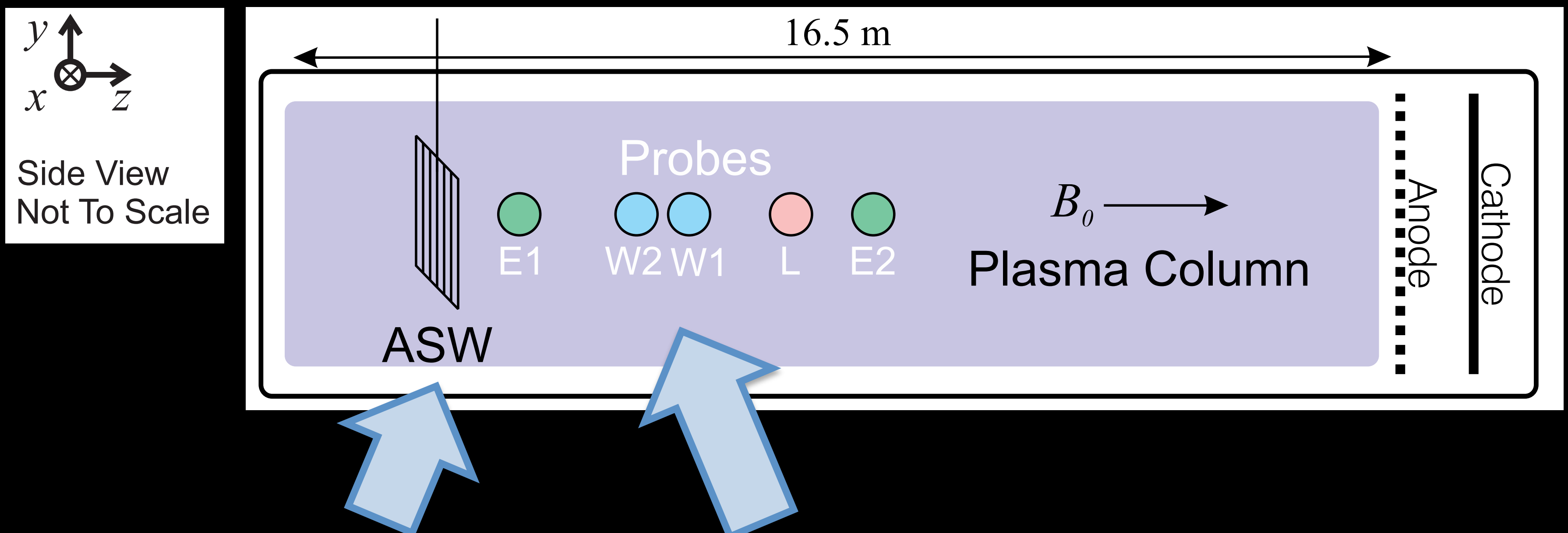


- Length: 16.6 m
- Diameter: 50 cm
- Duration: 10 ms
- Repetition rate: 1 Hz

- Fill gas: H_2
- $B_0 = 1800$ G

- $n_e = 10^{12}$ cm^{-3}
- $T_e = 2$ eV
- $v_A = 3.4 \times 10^8$ cm/s
- $\beta = 3.4 \times 10^{-5} < m_e/m_i$
- $v_{te}/v_A = 0.18$ (can produce inertial Alfvén waves)

LAPD setup for generating Alfvén waves and measure $g_e(v_\parallel)$.



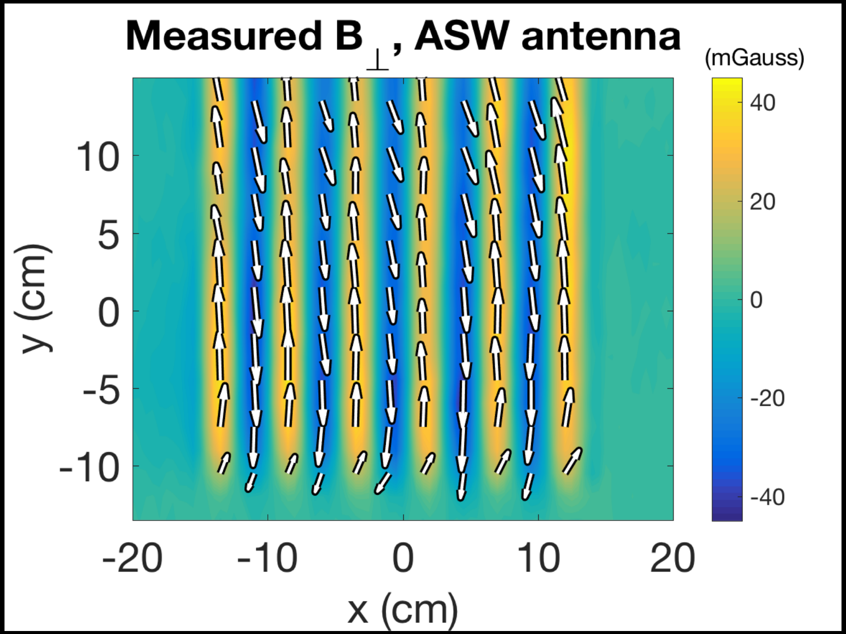
ASW antenna launches Alfvén waves.

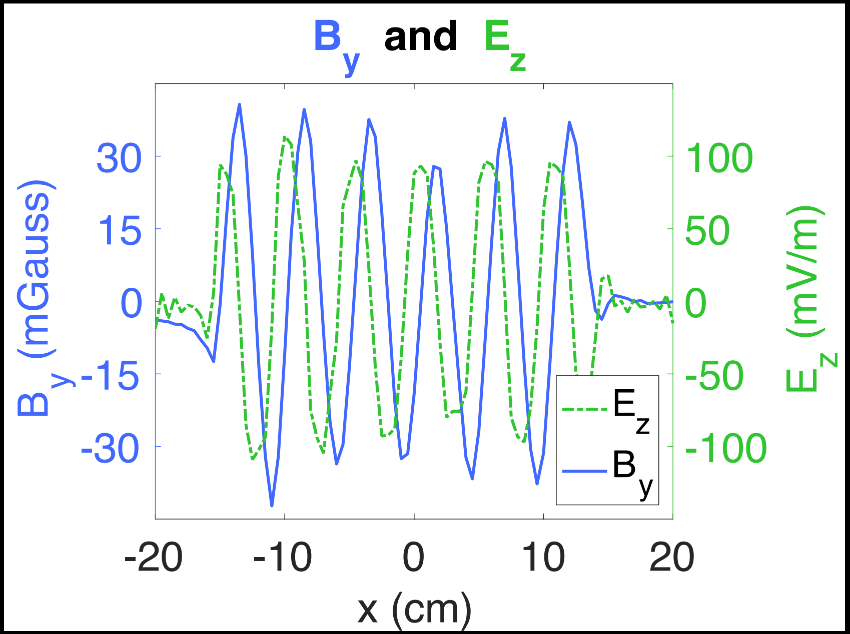
Whistler probes (W1, W2) record $g_e(v_\parallel)$.

Note: In the LAPD, $\hat{z}$ is the parallel direction.

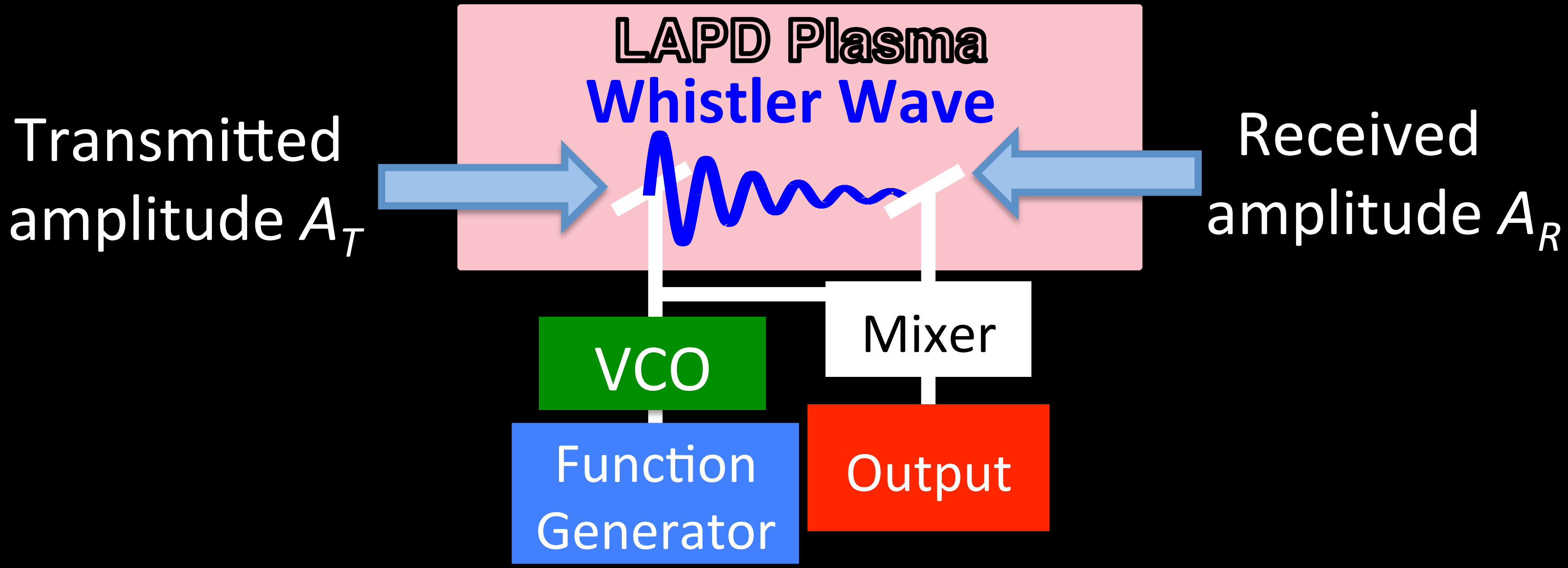
E_z cannot be measured directly.

- $E_z/E_\perp \approx 0.003$
- Calculate E_z using Faraday's law.





Whistler mode damping is measured with two dipole antennas.

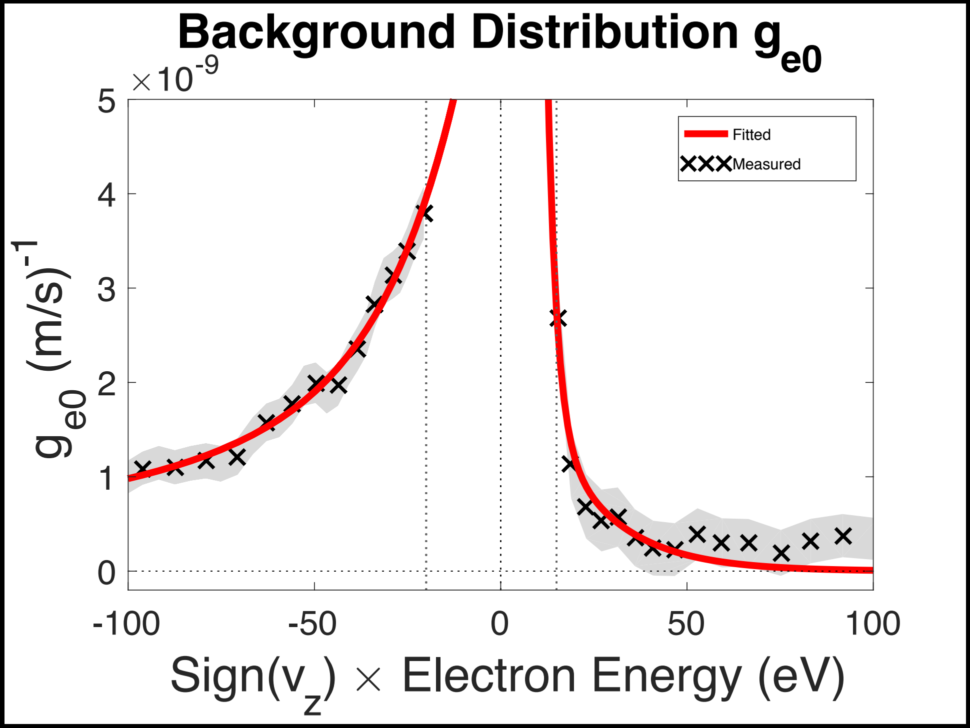


$$A_R = A_T e^{-(k_{wzi})z}$$

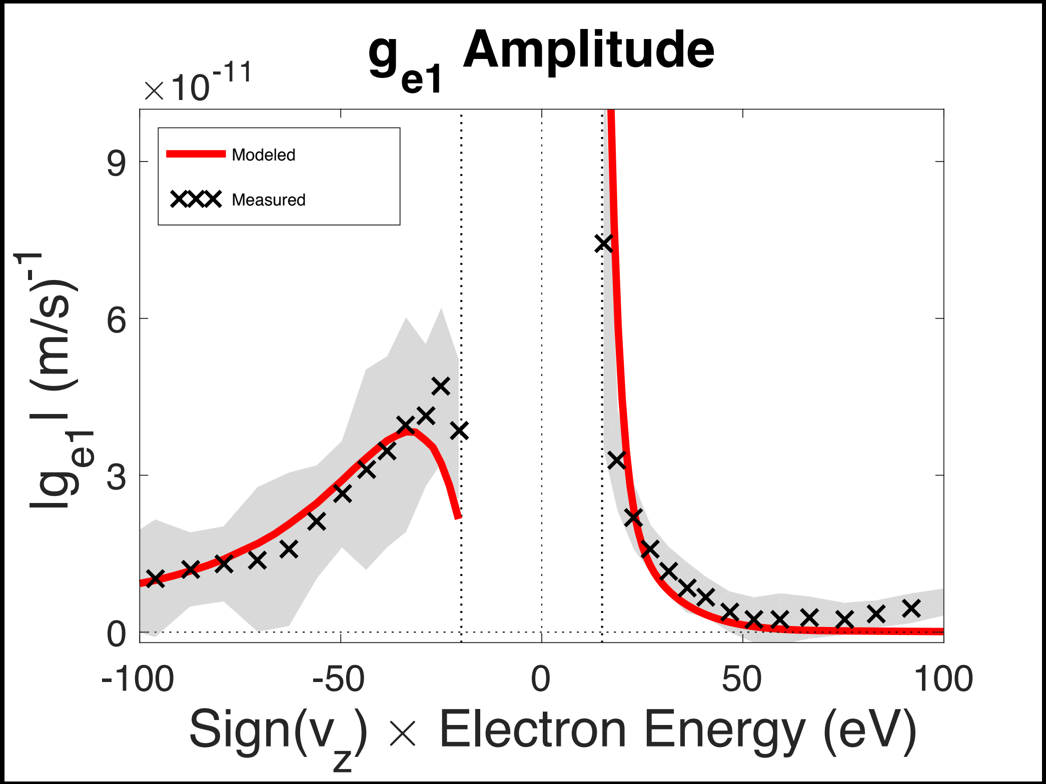
Measured background

g_{e0}

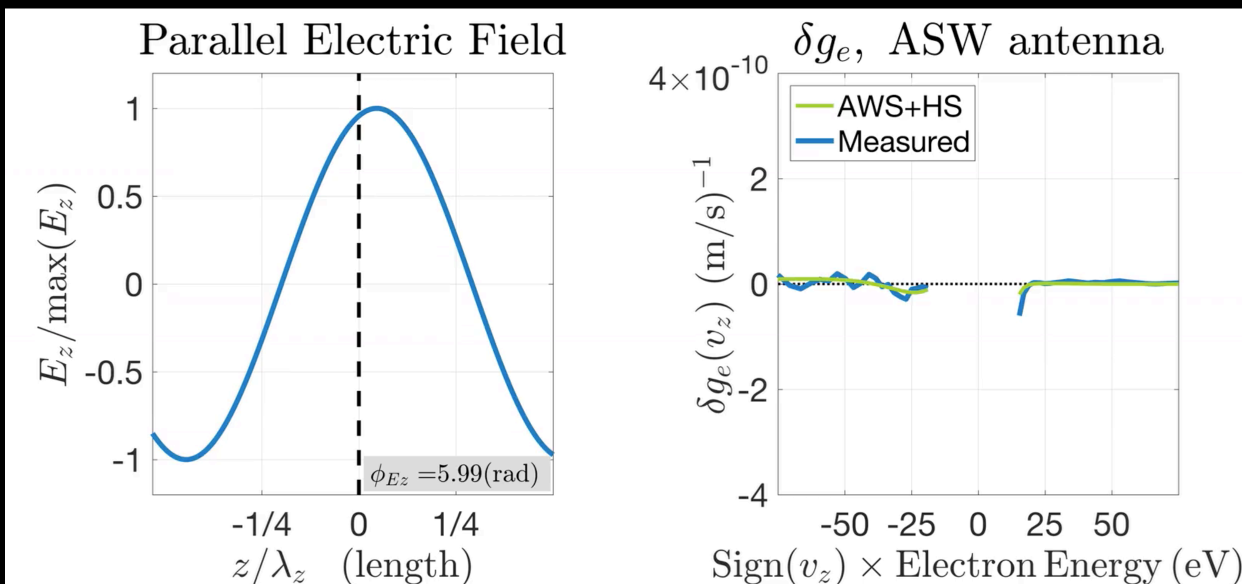
- Asymmetry from cathode source



Amplitude of modeled and measured oscillations agree.



$\delta g_e(v_z)$ is more Alfvénic during experiments with the inductive antenna.



Electrostatic antenna

- Density-like oscillations

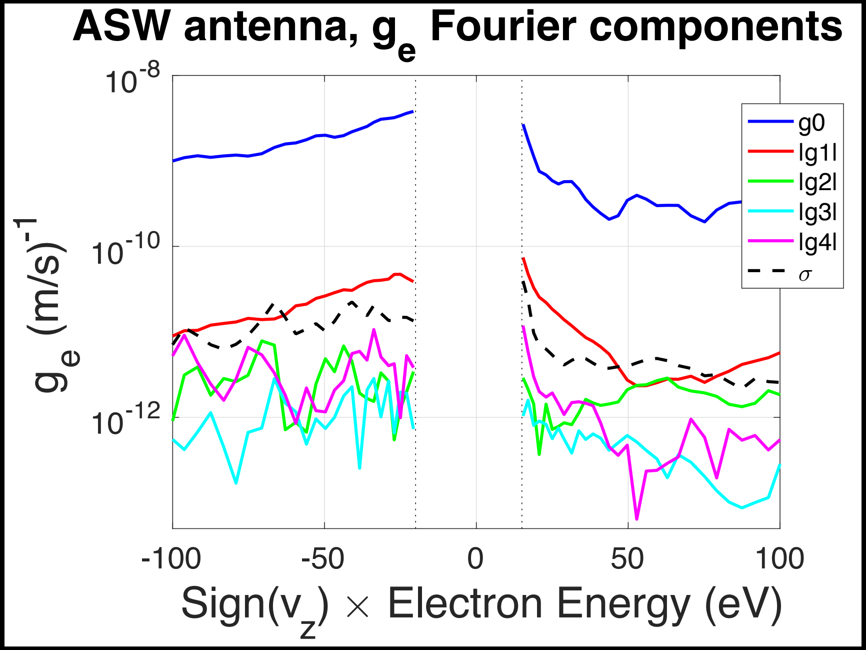
Inductive antenna

- Current-like oscillations

$g_e(v_z)$ harmonics suggest nonlinear wave-particle interaction.

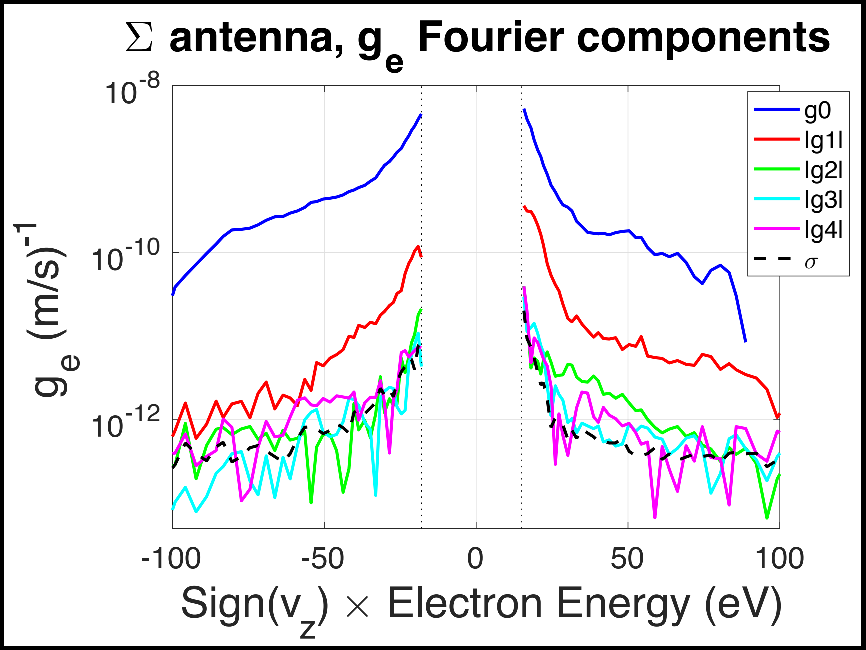
Electrostatic antenna

- No harmonics above noise



Inductive antenna

- Harmonics above noise



Conclusions

- Velocity sensitive diagnostics allow direct substitution of data into the Vlasov equation.
- This validation is a strict test of physical processe as well as of systematic effects in the diagnostic.
- The many electromechanical degrees of freedom of the Vlasov equation can be observed experimentally.

Acknowledgements

- Graduate students doing LIF: Sean Mattingly, Jorge Berumen, Feng Chu, and Ryan Hood.
- Gradute student on electron acceleration: Jim Schroeder.
- Collaborators on LAPD: Craig Kletzing, Greg Howes, Troy Carter, Seth Dorfman, Steve Vincena.
- NSF, DOE.