Collisional Dissipation of Fine Velocity Structures in Weakly Collisional Plasmas

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Collisionless Boltzmann (Vlasov) equation and modeling of self-gravitating systems and plasmas
CIRM, Marseille Oct 30-Nov 3, 2017

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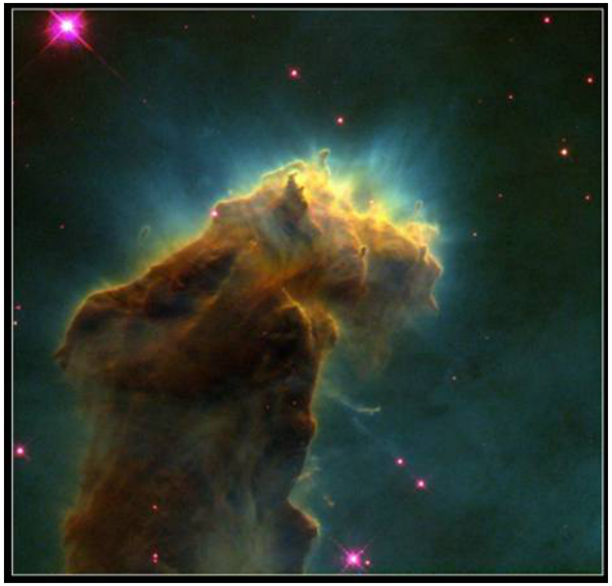
Outline

- **Introduction**
    - The heating and dissipation problem in weakly collisional plasmas, as the solar wind
- **Beyond the Vlasov approach…**

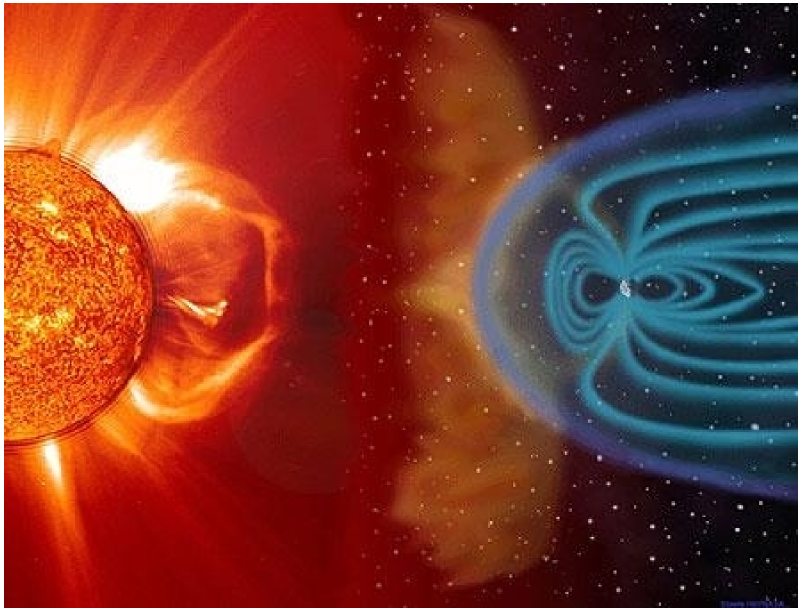
  … Can collisions be enhanced by the presence of strong non-Maxwellian features?

  The importance of considering collisions in a weakly collisional plasma
    - Collisional dissipation of fine structures in velocity space
    - Nonlinear vs. Linear collisional operators
    - Collisions and other dynamical processes
- **Conclusions**

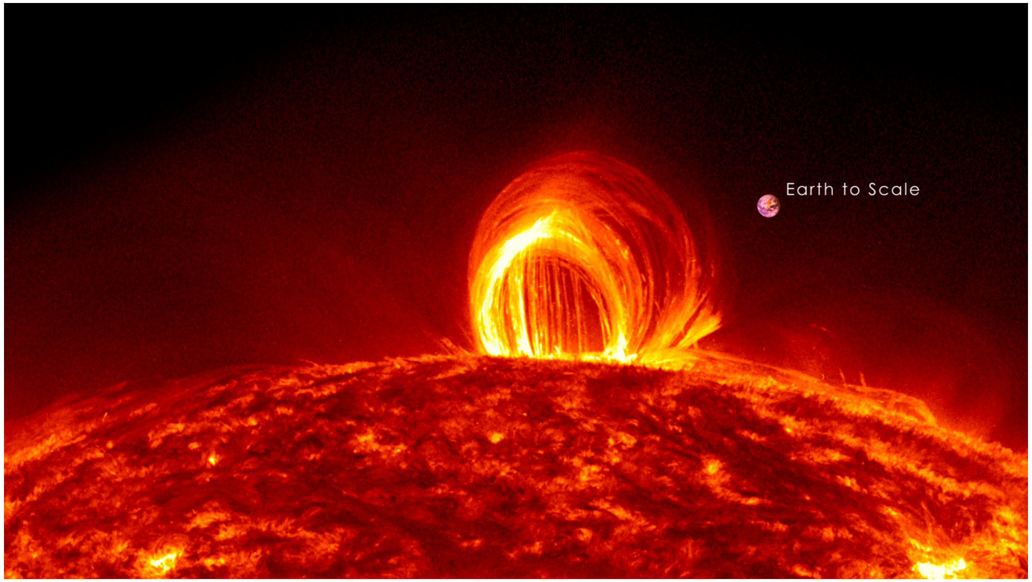
Plasmas across the Universe



M16 eagle nebula. Hubble (NASA, 2/11/95)

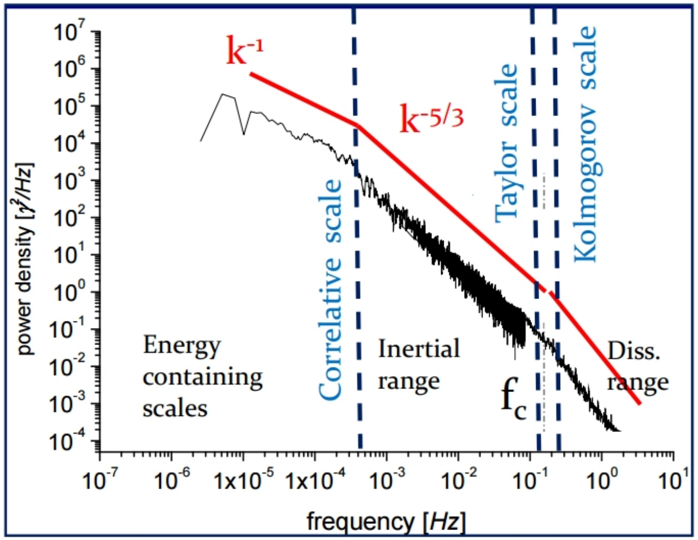


Northern light. ISS (NASA, 19/02/12)



Solar flare. SDO (NASA, 19/07/12)

The plasma heating problem

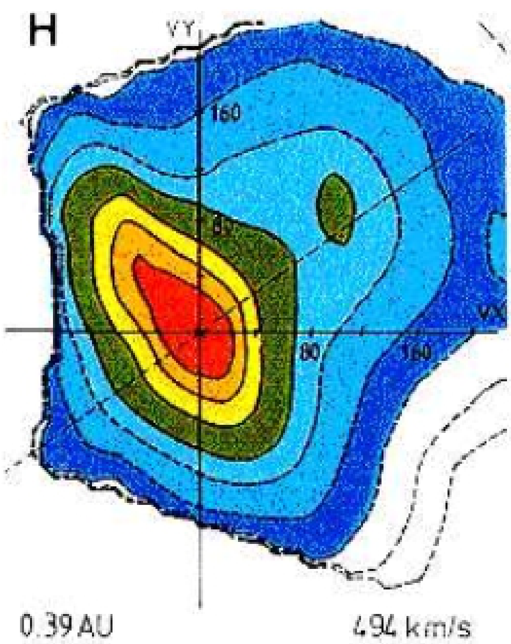


Bruno & Carbone, LRSP 2013

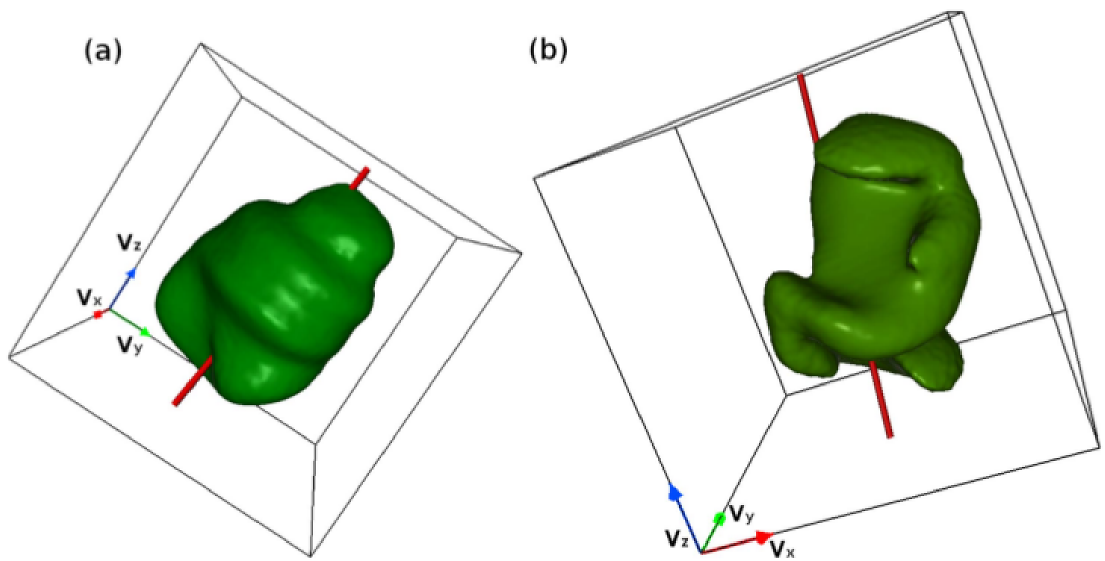
Turbulence

Kinetic physics

When the turbulent energy is transferred to small scales, **a kinetic approach is mandatory to describe the plasma dynamics!**

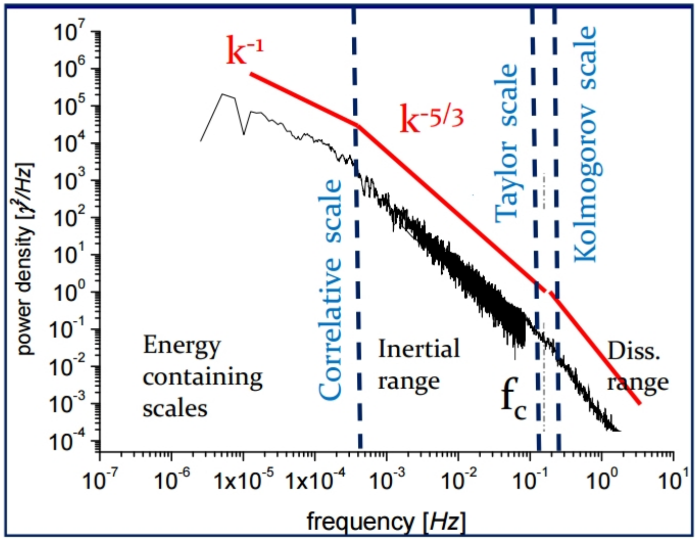


Marsch et al., JGR 1982

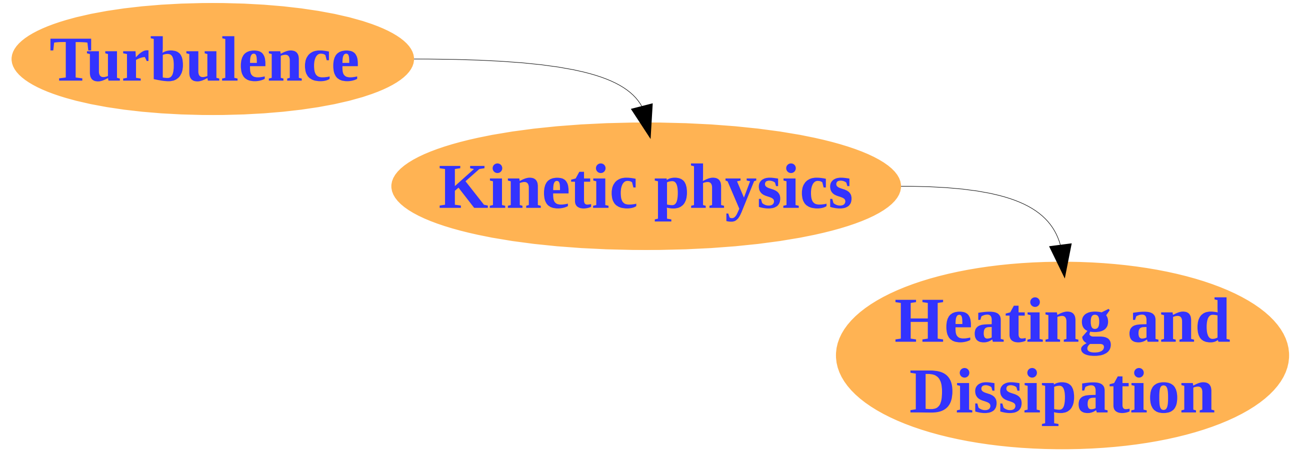


Valentini et al., NJP 2016

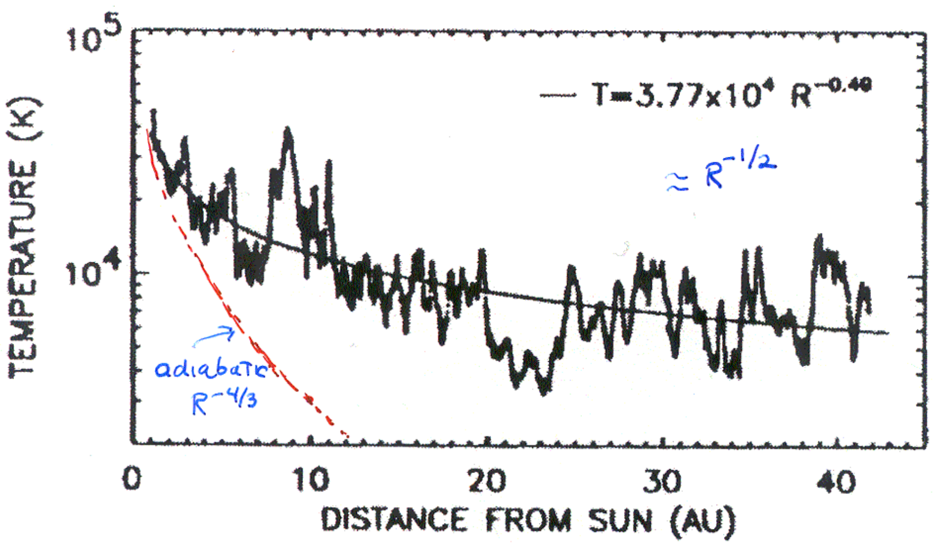
The plasma heating problem



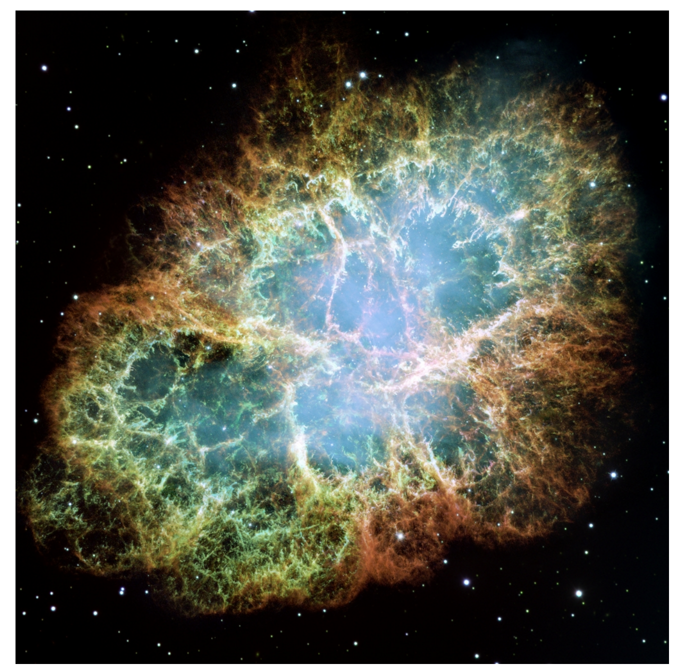
Bruno & Carbone, LRSP 2013



A fundamental yet unanswered question is how plasmas are heated, in particular dilute plasmas which reach temperatures of million degrees despite ***collisions between particles are weak***



Richardson et al., GRL 1995



Crab Nebula NASA/Hubble

Although several heating processes, often based on the ***collisionless assumption***, have been proposed, a clear answer about which mechanism is dominant is still missing.

Collisions in ‘collisionless’ plasmas

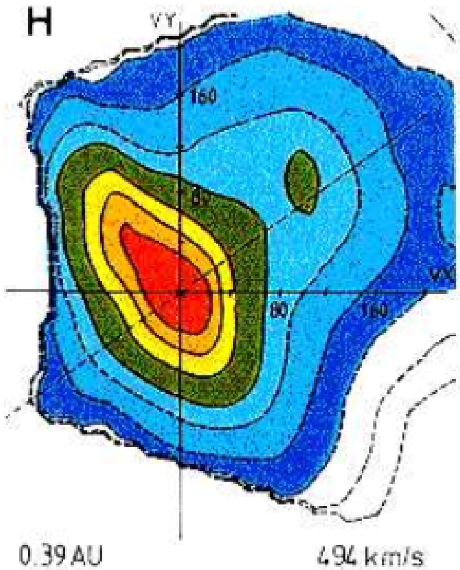
Despite inter-particle collisions are neglected, they are **the most robust physical effect to introduce irreversibility** in the system, therefore to describe heating and dissipation in the general thermodynamic sense.

QUASI-MAXWELLIAN APPROACH

Spitzer, 1956,
Hernandez & Marsch, JGR 1985

Frequency	Slow wind	Fast wind
proton cyclotron	~ 0.1 Hz	~ 0.1 Hz
electron cyclotron	$\sim 2 \times 10^2$ Hz	$\sim 2 \times 10^2$ Hz
plasma	$\sim 2 \times 10^5$ Hz	$\sim 1 \times 10^5$ Hz
proton-proton collision	$\sim 2 \times 10^{-6}$ Hz	$\sim 1 \times 10^{-7}$ Hz

… but …
is the SW quasi-Maxwellian!?
NO!



Marsch et al., JGR 1982

Can collisionality, since it depends on gradients in velocity space, be enhanced by the presence of fine velocity phase structures?

How to model collisions in a weakly collisional plasma?

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}} + \frac{\mathbf{F}}{m}\cdot\frac{\partial}{\partial \mathbf{v}}\right] f(\mathbf{r},\mathbf{v},t) = \left.\frac{\partial f}{\partial t}\right|_{coll}$$

+ Maxwell

… due to the huge computational cost of the Landau collisional operator, we focused on the **collisional relaxation of a spatially homogeneous force-free plasma ...**

Landau operator

$$\frac{\partial f(\mathbf{v})}{\partial t} = \pi\left(\frac{3}{2}\right)^{\frac{3}{2}}\frac{\partial}{\partial v_i}\int d^3v'\, U_{ij}(\mathbf{u})\left[f(\mathbf{v}')\frac{\partial f(\mathbf{v})}{\partial v_j} - f(\mathbf{v})\frac{\partial f(\mathbf{v}')}{\partial v'_j}\right]$$

$$U_{ij}(\mathbf{u}) = \frac{\delta_{ij}u^2 - u_i u_j}{u^3}$$

u = v – v'

N^9 for a 3D-3V grid!!
N^6 for a 3V grid!!

Normalization quantities: λ_D, $T=1/\upsilon_{SH}$, v_{th}

$$\nu_{SH} \simeq 8 \times (0.714\pi n e^4 \ln\Lambda)/(m^{0.5}(3k_B T)^{3/2})$$

Pezzi, Valentini and Veltri, PRL 2016

Numerical methods:
6th order FD scheme for velocity derivatives;
4th order AB scheme for temporal evolution.

Plateau vs Anisotropies

$$f_1(\mathbf{v}) = C_1 f_{M,T_\perp}(v_x) f_{M,T_\perp}(v_y) f_{p,T_\parallel}(v_z)$$

Plateau v_z + Temp. Anisotropy

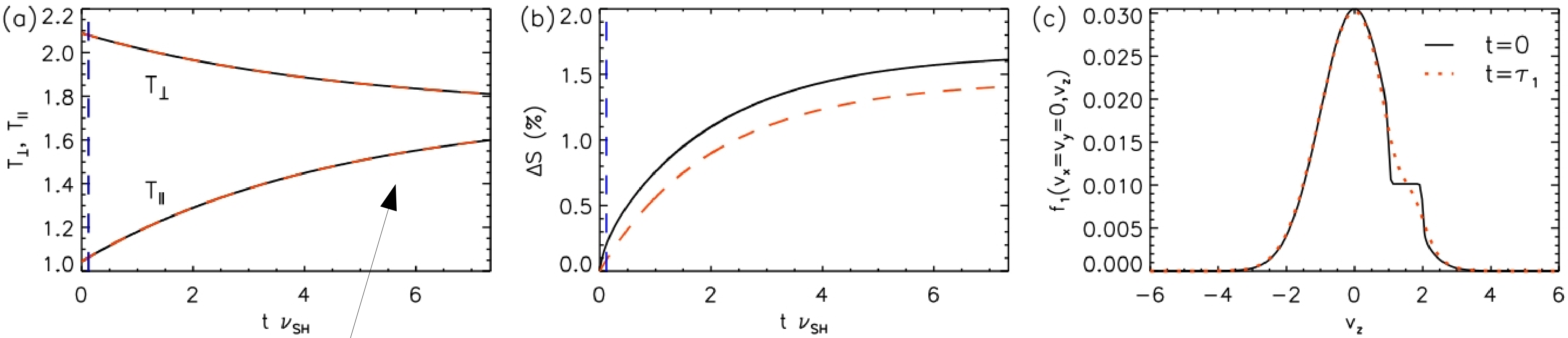
$$f_2(\mathbf{v}) = C_1 f_{M,T_\perp}(v_x) f_{M,T_\perp}(v_y) f_{M,T_\parallel}(v_z)$$

NO Plateau v_z + Temp. Anisotropy

$$N_{v_x} = N_{v_y} = 51$$

$$N_{v_z} = 1601$$

$$A = T_\perp / T_\parallel = 2$$



The temperature evolution does not “feel” the presence of the small scales structure (plateau)

Plateau vs Anisotropies

$f_1(\mathbf{v}) = C_1 f_{M,T_\perp}(v_x) f_{M,T_\perp}(v_y) f_{p,T_\parallel}(v_z)$ Plateau v_z + Temp. Anisotropy

$f_2(\mathbf{v}) = C_1 f_{M,T_\perp}(v_x) f_{M,T_\perp}(v_y) f_{M,T_\parallel}(v_z)$ NO Plateau v_z + Temp. Anisotropy

$$N_{v_x} = N_{v_y} = 51$$
$$N_{v_z} = 1601$$
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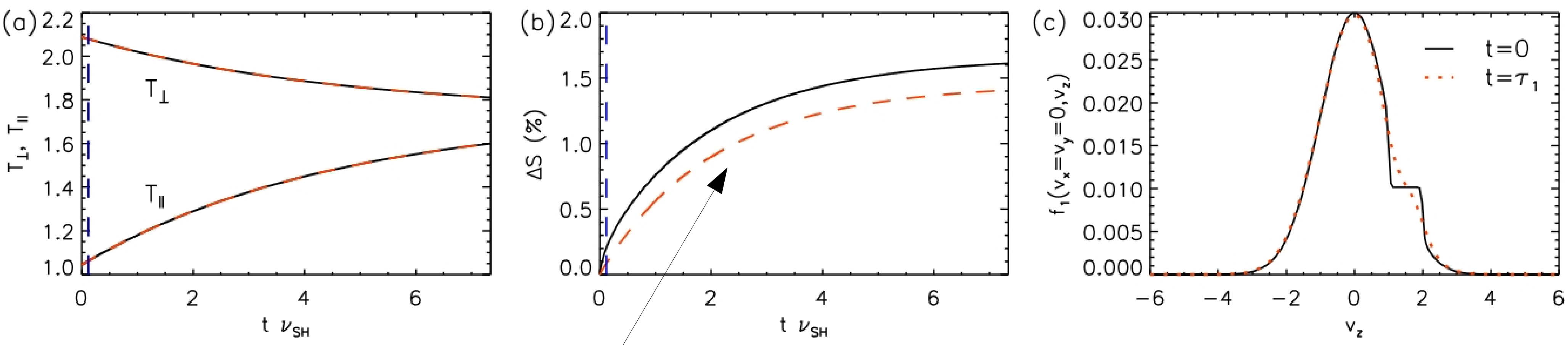


Multi-exponential fit

$$\Delta S(t) = \sum_{i=1}^{K} \Delta S_i (1 - e^{-t/\tau_i})$$

$\tau_1 = 0.14 \nu_{SH}^{-1}$ → $\Delta S_1 / \Delta S_{tot} = 25\%$

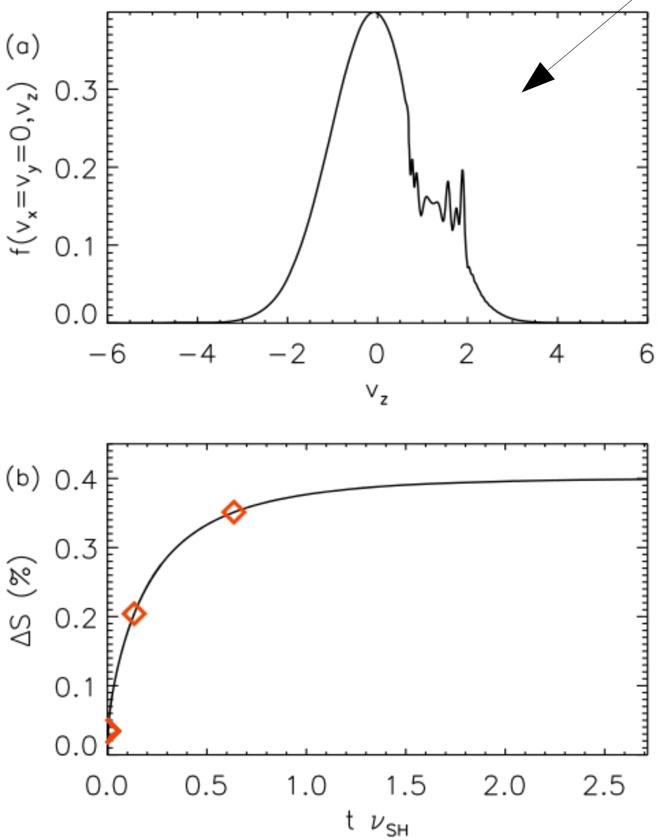
$\tau_2 = 2.03 \nu_{SH}^{-1}$ → $\Delta S_2 / \Delta S_{tot} = 75\%$

The plateau is dissipated faster than global anisotropy and the entropy evolution shows a net entropy growth due to its presence!

Existence of several characteristic times

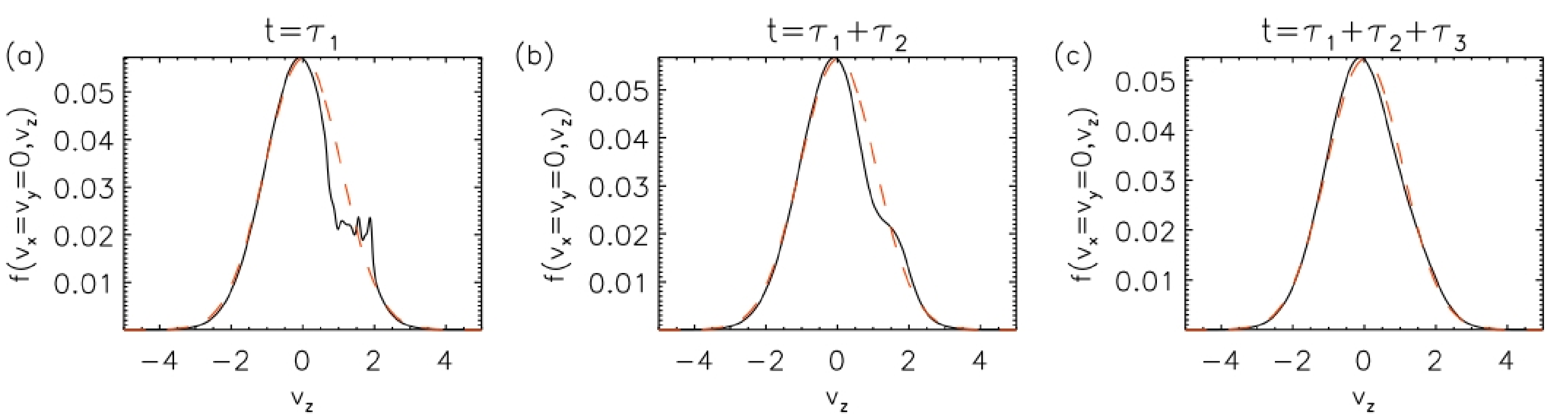
$$f(v_x, v_y, v_z) = f_{M,T_e}(v_x) f_{M,T_e}(v_y) \hat{f}_e(v_z)$$

Complex resonant region (typical of electrostatic waves) on v_z + NO temp. anisotropy



- $\tau_1 = 3.5 \cdot 10^{-3} \nu_{SH}^{-1} \rightarrow \Delta S_1 / \Delta S_{tot} = 13\%$ Spikes dissipation
- $\tau_2 = 1.3 \cdot 10^{-1} \nu_{SH}^{-1} \rightarrow \Delta S_2 / \Delta S_{tot} = 42\%$ Plateau smoothing
- $\tau_3 = 4.9 \cdot 10^{-1} \nu_{SH}^{-1} \rightarrow \Delta S_3 / \Delta S_{tot} = 40\%$ Final approach to equilibrium Maxwellian

… characteristic times associated with fine velocity structures gets smaller as finer scales are present in the VDF ...



Effect of smoothing on VDF

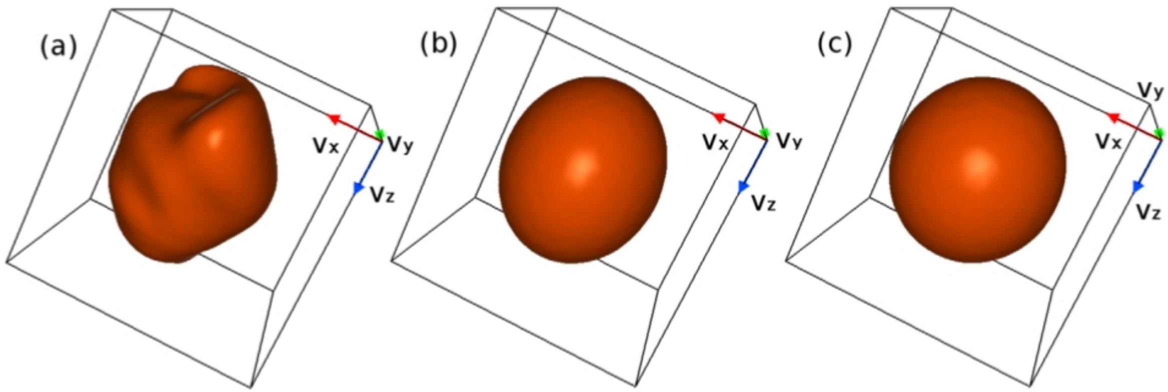


FIG. 4. Iso-surface plot of the initial VDFs $f_{sw}(\mathbf{v})$ (a), $\tilde{f}_{sw}(\mathbf{v})$ (b), and $\hat{f}_{sw}(\mathbf{v})$ (c), respectively

What happens if small scales structures are artificially smoothed out by some fitting procedure ? Let's focus on a solar-wind like, turbulent VDF!

NO fit

Tri-Maxwellian fit

Bi-Maxwellian fit

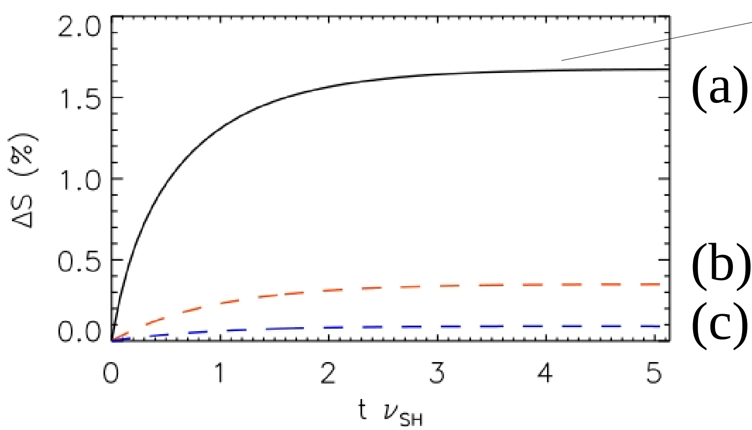


FIG. 5. Entropy growth for the initial VDFs $f_{sw}(\mathbf{v})$ (black line), $\tilde{f}_{sw}(\mathbf{v})$ (red dashed line), and $\hat{f}_{sw}(\mathbf{v})$ (blue dashed line), respectively.

$\tau_1 = 0.20\nu_{SH}^{-1}$ → $\Delta S_1/\Delta S_{tot} = 26\%$

$\tau_2 = 0.82\nu_{SH}^{-1}$ → $\Delta S_2/\Delta S_{tot} = 74\%$

The smaller characteristic times and the associated entropy growth are not recovered if small scales are artificially smoothed out

…high-resolution phase space measurements of the VDF are crucial to properly identify and take into account dissipative processes …

Nonlinearities in the collisional operator

Pezzi, JPP 2017

It is also interesting to extend the analysis by directly comparing the effects of the fully nonlinear Landau operator and a linearized version.

$$\frac{\partial f(\boldsymbol{v},t)}{\partial t} = \pi\left(\frac{3}{2}\right)^{3/2}\frac{\partial}{\partial v_i}\int \mathrm{d}^3v'\,U_{ij}(\boldsymbol{u})\left[f(\boldsymbol{v}',t)\frac{\partial f(\boldsymbol{v},t)}{\partial v_j} - f(\boldsymbol{v},t)\frac{\partial f(\boldsymbol{v}',t)}{\partial v'_j}\right],$$

$$\frac{\partial f(\boldsymbol{v},t)}{\partial t} = \pi\left(\frac{3}{2}\right)^{3/2}\frac{\partial}{\partial v_i}\int \mathrm{d}^3v'\,U_{ij}(\boldsymbol{u})\left[f_0(\boldsymbol{v}')\frac{\partial f(\boldsymbol{v},t)}{\partial v_j} - f(\boldsymbol{v},t)\frac{\partial f_0(\boldsymbol{v}')}{\partial v'_j}\right].$$

$$U_{ij}(\mathbf{u}) = \frac{\delta_{ij}u^2 - u_iu_j}{u^3}$$

Coefficients of the Fokker-Planck like structures of the Landau operator have been linearized by introducing the Maxwellian distribution function f_0 associated with f

Nonlinearities in the collisional operator

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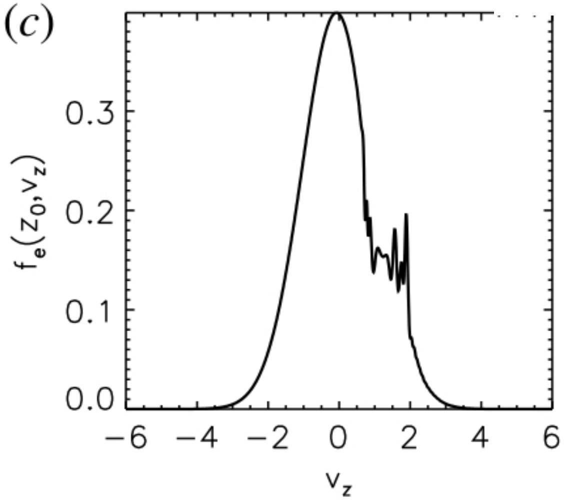
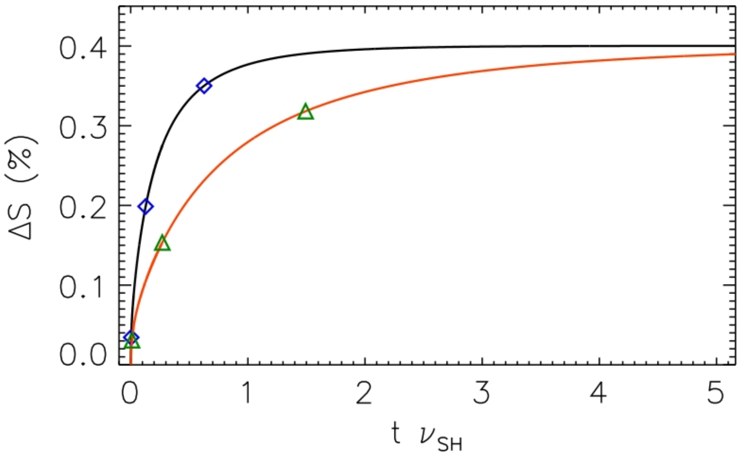
$$\frac{\partial f(\boldsymbol{v},t)}{\partial t} = \pi\left(\frac{3}{2}\right)^{3/2}\frac{\partial}{\partial v_i}\int \mathrm{d}^3v'\,U_{ij}(\boldsymbol{u})\left[f(\boldsymbol{v}',t)\frac{\partial f(\boldsymbol{v},t)}{\partial v_j} - f(\boldsymbol{v},t)\frac{\partial f(\boldsymbol{v}',t)}{\partial v'_j}\right],$$

$$\frac{\partial f(\boldsymbol{v},t)}{\partial t} = \pi\left(\frac{3}{2}\right)^{3/2}\frac{\partial}{\partial v_i}\int \mathrm{d}^3v'\,U_{ij}(\boldsymbol{u})\left[f_0(\boldsymbol{v}')\frac{\partial f(\boldsymbol{v},t)}{\partial v_j} - f(\boldsymbol{v},t)\frac{\partial f_0(\boldsymbol{v}')}{\partial v'_j}\right].$$

$$U_{ij}(\mathbf{u}) = \frac{\delta_{ij}u^2 - u_i u_j}{u^3}$$





NONLINEAR

(i) $\tau_1^{nl} = 3.5\times10^{-3}\,\nu_{SH}^{-1} \rightarrow \Delta S_1^{nl}/\Delta S_{tot} = 13\,\%$,
(ii) $\tau_2^{nl} = 1.3\times10^{-1}\,\nu_{SH}^{-1} \rightarrow \Delta S_2^{nl}/\Delta S_{tot} = 42\,\%$,
(iii) $\tau_3^{nl} = 4.9\times10^{-1}\,\nu_{SH}^{-1} \rightarrow \Delta S_3^{nl}/\Delta S_{tot} = 40\,\%$.

LINEAR

(i) $\tau_1^{lin} = 1.1\times10^{-2}\,\nu_{SH}^{-1} \rightarrow \Delta S_1^{lin}/\Delta S_{tot} = 11\,\%$,
(ii) $\tau_2^{lin} = 2.7\times10^{-1}\,\nu_{SH}^{-1} \rightarrow \Delta S_2^{lin}/\Delta S_{tot} = 23\,\%$,
(iii) $\tau_3^{lin} = 1.5\,\nu_{SH}^{-1} \rightarrow \Delta S_3^{lin}/\Delta S_{tot} = 63\,\%$.

Characteristic times associated with the linearized operator are systematically bigger compared to the ones recovered for the fully nonlinear operator.

Collisions vs other dynamical "collisionless" processes

$g = 10^{-10}$
$g = \nu_{pp}/\omega_{pp}$

$\tau_{NL} = 10^1\text{-}10^2\ \Omega^{-1}_{cp}$ *Matthaeus et al., APJL 2014*

$\tau_I = 1/\gamma_I = 10^2\text{-}10^3\ \Omega^{-1}_{cp}$

$\tau_{pp} = 10^4\text{-}10^5\ \Omega^{-1}_{cp}$

… obtained within the quasi-Maxwellian approach!

Collisions vs other dynamical “collisionless” processes

$g = 10^{-10}$

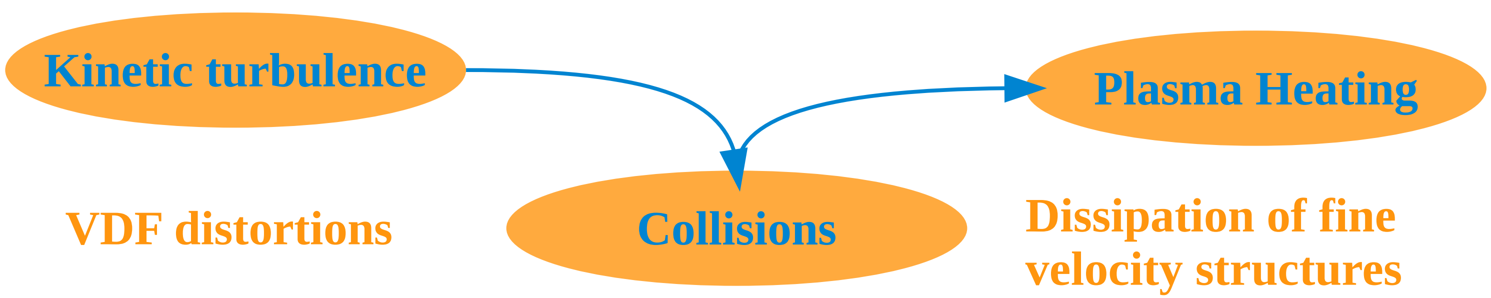
$g = \nu_{pp}/\omega_{pp} \longrightarrow$

$\tau_{NL} = 10^{1}\text{-}10^{2}\,\Omega^{-1}_{cp}$ *Matthaeus et al., APJL 2014*

$\tau_{I} = 1/\gamma_{I} = 10^{2}\text{-}10^{3}\,\Omega^{-1}_{cp}$

$\boldsymbol{\tau_{pp} \ll 10^{4}\text{-}10^{5}\,\Omega^{-1}_{cp}}$

Collisional effects could be enhanced by 1-2 orders of magnitude if small velocity scales are taken into account, hence **collisions could efficiently compete with other mechanisms in the transformation of the distribution function free energy into heat.**



Conclusions

1) Collisions are the most robust physical effect to introduce irreversibility in the system, i.e. to describe dissipation in the general thermodynamic sense.

2)The collisionless assumption may locally fail in weakly collisional plasmas, such as the solar wind, since collisions quickly dissipate fine velocity structures, *which are naturally produced by plasma turbulence at kinetic scales.*

3) Fitting the VDF with bi-Maxwellians may artificially hide the collisionality enhancement due to velocity space gradients. High-resolution VDFs measurements are thus decisive for a deep comprehension of the heating processes.

4) The relaxation towards the equilibrium is qualitatively described with both the nonlinear and linearized Landau operators. However characteristic times gets bigger (i.e. collisions "strength" is smaller) if nonlinearities are neglected.

5) Collision can be efficiently enhanced if fine velocity structures are considered, therefore they may play a role into the plasma heating process.

… and thank for the attention!

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How to model collisions in a weakly collisional plasma?

Plasmas are quite different from neutral gases!

Liouville equation

BBGKY hierarchy

Boltzmann operator

Series expansion

Balescu operator

- NO cut-off for high impact parameter;
- Quasi-equilibrium plasma
- Plasma Dispersion Function is taken into account

Landau operator

- Cut-off for high impact parameter;
- Quasi-equilibrium plasma
- Plasma Dispersion Function is NOT taken into account

- The Boltzmann operator is widely used to model plasma collisions but it is usually introduced by assuming the probability conservation (not from Liouville equation…)
- Both “natural” collisional operators for plasmas involves gradient in velocity space!
- Both “natural” collisional operators for plasmas have been derived under the assumption of quasi-equilibrium plasmas! Turbulence it's “a bit” far from equilibrium! But it is the best we can do…