Non-equilibrium hydrodynamic pressure tensors from kinetic perspectives [Dragos I. Palade]

1. A kinetic model:

$$\partial_t f + \mathbf{v}\nabla f = \mathcal{K}[\varphi] \otimes_{\mathbf{v}} f; \qquad \mathcal{K}[\varphi] = \pm \frac{1}{H} \int d\mathbf{s} e^{-\mathbf{isv}/H} \varphi(\mathbf{r} \pm \mathbf{s}/2)$$

2. Supports a hydrodynamic model:

$$\partial_t n + \nabla \mathbf{j} = 0 \qquad \partial_t \mathbf{j} + \nabla \cdot \left(\frac{\mathbf{j} \otimes \mathbf{j}}{n} + \hat{\Pi}\right) + n\nabla\varphi = 0$$

3. Hydrodynamic linear response function ($\tilde{\chi}^{hydr}$) should be $= \tilde{\chi}^{kin}$
4. Presume functional relation $\Pi = \delta P[n]/\delta n$, where

$$P[n] = \int d\mathbf{r} d\mathbf{r}' dt dt' n^{\alpha}(\mathbf{r}, t) \sigma(\mathbf{r}, \mathbf{r}', t, t') n^{\alpha}(\mathbf{r}', t')$$

5. Enforcing $\tilde{\chi}^{hydr} \approx \tilde{\chi}^{kin}$, we find:

$$\tilde{\sigma}(k, \omega) = \sigma_0(k) + \frac{[\sigma_\infty(k) - \sigma_0(k)]\omega^2}{[\sigma_\infty(k) - \sigma_0(k)]/\sigma_1(k) + \omega^2}$$

$$\hat{\Pi}[n] = \frac{8}{3} n^{4/3}(\mathbf{r}, t)(\hat{\mathcal{D}} + \int d\mathbf{r}' \sigma_\infty(\mathbf{r} - \mathbf{r}') n^{4/3}(\mathbf{r}', t)$$

$$(\partial_{tt} - (\sigma_\infty - \sigma_0)/\sigma_1 \otimes)\hat{\mathcal{D}} = (\sigma_\infty - \sigma_0)^2/\sigma_1 \otimes n^{4/3}$$

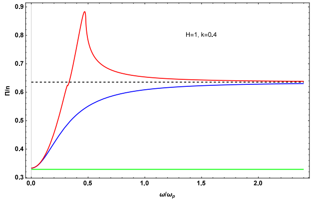

Figure: 1. $\tilde{\Pi}/\tilde{n}$ at fixed $k = 0.4$ versus frequency ω. Red (exact, kinetic), blue (present work), Green (TFvW)

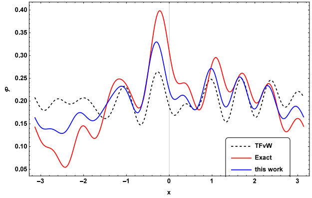

Figure: 2. $H = 1$, $t = 20$; Pressure tensor $\hat{\Pi}_{xx}(x)$ profile obtained in a numerical simulation during the dynamics. The present functional (blue) is considerably closer to the exact (kinetic, red) result, than the existing TFvW approach (black, dashed)

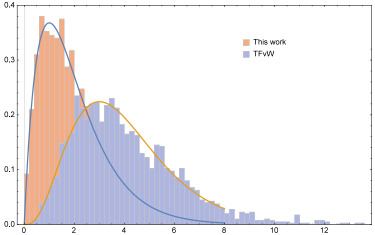

Figure: 3. Histogram of overall error (arb. units) from the TFvW (blue) approximation and the present functional (orange). The error from TFvW peaks considerably at a larger value and has a larger width.