On the equilibrium states of the distribution function

Darío Núñez

Instituto de Ciencias Nucleares, UNAM
Red Temática de Agujeros negros y Ondas Gravitatorias, CONACyT
Sociedad Mexicana de Física

Collisionless Boltzmann (Vlasov) equation and modeling of self-gravitating systems and plasmas, CIRM, Marseille, France

Main collaborators: Paola Domínguez, Miguel Alcubierre, Olivier Sarbach, Erik Jiménez, Paola Rioseco.

November 2nd, 2017

Initial Considerations

- Dark matter nature remains ...

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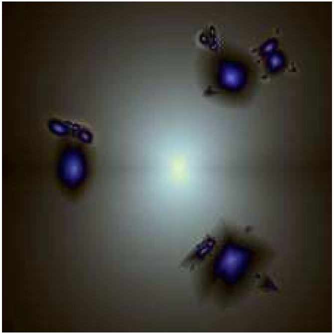
- Obscure!



Legend : Dark matter unknown nature.

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- It has a null or very small cross section with us, barionic matter,
- It has no electric charge $q = 0$,
- It is stable
- It composes 21 % of the matter - energy content of the Universe, and is 90 % of the matter in galaxies

Legend : It forms a kind of halo

Theoretical tools

- In order to understand and determine dark matter properties, and then, some possible observable consequences, one has to study with the best theory available, and with the least assumptions on its Nature.

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- **General relativity!**, where matter is described in general only by its effect on the geometry of the space-time, through the stress-energy tensor, $T_{\mu\nu}$, with three main different types of matter:

$$T_{\mu\nu} = \begin{cases} \rho\, c^2\, h\, \frac{u_\mu\, u_\nu}{c^2} + p\, g_{\mu\nu}, \ \text{fluid} \\ \frac{c^4}{8\,\pi\, G}\, \left(\phi_{,\mu}\,\phi_{,\nu} - \frac{1}{2}\, g_{\mu\nu}\, \left(\phi^{,\alpha}\,\phi_{,\alpha} + 2\, V(|\phi|^2)\right)\right), \ \text{scalar field} \\ \rho_f\, < u_\mu\, u_\nu >, \ \text{Particle collection} \end{cases}$$

Theoretical tools

- In the quest for the determination of which type of matter better describes the dark matter's nature, we perform simple experiments in different astrophysical contexts to test first the viability of the description and second, to look for specific features of the description which could affect the barionic matter and thus, have observable consequences.

Theoretical tools

- In the quest for the determination of which type of matter better describes the dark matter's nature, we perform simple experiments in different astrophysical contexts to test first the viability of the description and second, to look for specific features of the description which could affect the barionic matter and thus, have observable consequences.
- As we will see, in the case of a collection of particles, we look for some peculiar features within each dark matter halo model, which would allow us to discriminate among them.

Types of matter

With this this goal in mind, we can see that

- It is unlikely that the dark matter has a fluid - like nature, almost by definition.

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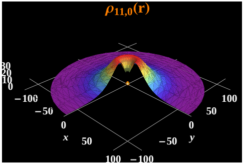
- It is unlikely that the dark matter has a fluid - like nature, almost by definition.
- The scalar field description is interesting and a subject of intense research, by itself, in Astrophysics and in Cosmology, see *e. g.* our series of wigs, **PRD 96**, 024049, (2017), **PRD 89**, 083006 (2014), **PRL 109**, 081102 (2012), **PRD 84**, 083008 (2011); Matos' et al works: *e. g.* **CQG 17**, L75 (2000), **PRD 63**, 063506 (2001), **PRD 96**, 043005 (2017); and also see Schive et al. **Nature Phys. 10**, 496 (2014).



Legend : Scalar field density distribution around a Black Hole

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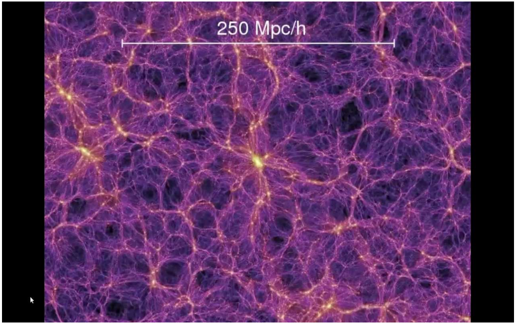
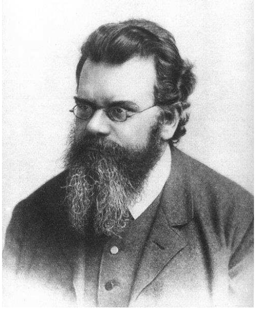
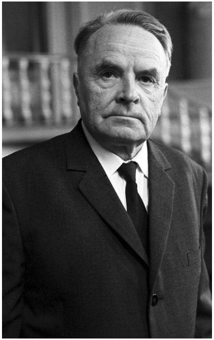
- N-particle simulations *e. g.* the NFW ones, are within this line of description and have astonishing results particularly in Large Scale Structure of the Universe; today $N \sim 10^{12}$ which still makes particles the size of the Sun.



Legend : Large scale structure of the Universe, Millennium simulation

Kinetic theory. Definitions, notes

Describing the dark matter as a collection of non interactive particles, seems sensible (although there is no candidate for such particle within Particle Physics) and it is described by the collision-less Boltzmann (Vlasov) equation

Legend : Ludwig Eduard Boltzmann, 1844-1906, Anatoli Aleksándrovich Vlásov, 1908-1975, Lenin Award 1970.

Kinetic theory. Definitions

Following Paola Domínguez *et al*, **GRG 49**, 9-123 (2017), a test field described by the distribution function $f = f(x^\mu, p_i)$, in spherical static spaces, characterized by the metric tensor, $g_{\mu\,\nu}$, (the gravitational potential $\Phi(r)$ in Newtonian approximation) and study the numerical evolution of a given initial distribution in such space, is a simple model which can give some idea of a density distribution of this specific type of description which in principle could be tested by the corresponding motion of barionic matter:

$$
\begin{aligned}
& & \frac{d\,f}{d\tau} = 0, \\
& \Longrightarrow & \dot{x}^\mu \, \frac{\partial f}{\partial x^\mu} + \dot{p}_i \, \frac{\partial f}{\partial p_i} = 0, \\
& \Longrightarrow & g^{\mu\,0}\, p_\mu \, \frac{\partial\, f}{\partial\, x^0} + g^{\mu\,i}\, p_\mu \, \frac{\partial\, f}{\partial\, x^i} - \frac{g^{\mu\,\nu}{}_{,i}}{2\,m}\, p_\mu\, p_\nu \, \frac{\partial\, f}{\partial\, p_i} = 0, \\
\text{Newtonian} & \Longrightarrow & \frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \cdot \vec{\nabla}\, f - m\, \vec{\nabla}\, \Phi \cdot \vec{\nabla}_p\, f = 0.
\end{aligned}
$$

Vlasov

- For halos, it is accurate to use the Newtonian description.

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- In spherical symmetry, $\frac{p_\theta}{m\,r^2}\,\frac{\partial f}{\partial \theta} = \dot{p}_\theta\,\frac{\partial f}{\partial p_\theta}$, and defining the **pattern quantities** for the system, the distance, R_0 and for the mass, M_0, we can define a characteristic velocity $q^2 = \frac{G\,M_0}{R_0}$, with G the gravitational constant, and thus we define all the needed functions, which allows us to write down the **dimensionless Vlasov equation**:

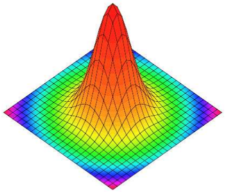
$$\frac{\partial\,\bar{f}}{\partial\,\mathcal{T}} = -\mathcal{P}_r\,\frac{\partial\,\bar{f}}{\partial d} + \left(\frac{\partial\,\bar{\Phi}(d)}{\partial\,d} - \frac{\mathcal{L}^2}{d^3}\right)\,\frac{\partial\,\bar{f}}{\partial\,\mathcal{P}_r},$$

where $t = T_0\,\mathcal{T} = \sqrt{\frac{{R_0}^3}{G\,M_0}}\,\mathcal{T}$, we are considering a single spices of particles with mass m, and $f = f_0\,\bar{f}$, with $\bar{f}$ a unit-less distribution function.

Astrophysics

- As mentioned above, want to test some properties which are specific of the particle collection description, in order to test it as a model for dark matter.

- As mentioned above, want to test some properties which are specific of the particle collection description, in order to test it as a model for dark matter.
- In order to do so, we start with an initial Gaussian distribution for $\bar{f}$ and evolve it in a dark matter halo potential, using our Vlas-Ollin code.

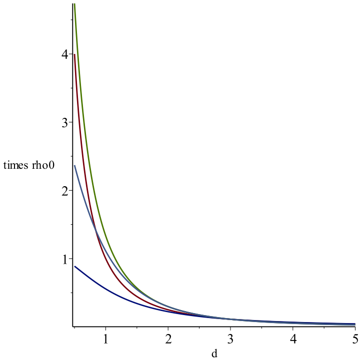


Legend : Initial distribution function representing a shell of inhomogeneity in the halo.

Actually, we considered four halo models: the isothermal, the isothermal truncated, the Burkert one, and the Navarro-Frenk-White one, and that the initial distribution describes a dark matter inhomogeneity in a given halo, the gravitational potential is due to such halo. The corresponding dimensionless density profiles are:

$$\rho_{\rm iso} = \frac{\rho_0}{d^2}, \qquad \rho_{\rm iso-tr} = \frac{10\,\rho_0}{9\,\left(1+d^2\right)},$$

$$\rho_{\rm Burkert} = \frac{40\,\rho_0}{9\,\left(1+d\right)\,\left(1+d^2\right)}, \qquad \rho_{\rm NFW} = \frac{16\,\rho_0}{3\,d\,\left(1+d\right)^2}.$$



Legend : Density profiles for the four halo models considered, with the characteristic speed is related to the characteristic density as $\sigma^2 = \frac{4\,\pi\,G\,\rho_0\,{R_0}^2}{3}$, and we normalized them such that at $d = 3$ the four halos have the same density.

- Now, with the mass defined as $m = 4\,\pi\,\int\,\rho\,r^2\,dr$, spherical, and with the gradient of the gravitational potential as $\phi' = \frac{G\,m}{r^2}$, we obtain that the unit-less potential gradient for each halo, normalized, are

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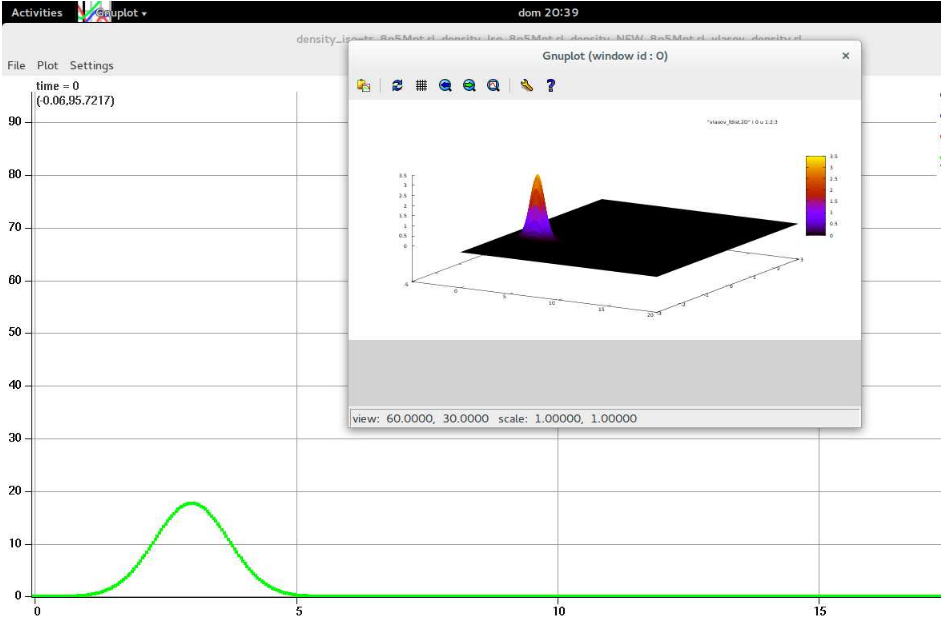
- $$\begin{aligned}
\frac{\partial\,\bar{\Phi}(d)}{\partial\, d}_{\mathrm{iso}} &= \frac{3}{d},\\
\frac{\partial\,\bar{\Phi}(d)}{\partial\, d}_{\mathrm{iso-tr}} &= \frac{10\,\left(1 - \frac{\arctan(d)}{d}\right)}{3\,d},\\
\frac{\partial\,\bar{\Phi}(d)}{\partial\, d}_{\mathrm{Burkert}} &= \frac{10\,\left(\ln\left(\left(1+d^2\right)\,(1+d)^2\right) - 2\,\arctan(d)\right)}{3\,d^2},\\
\frac{\partial\,\bar{\Phi}(d)}{\partial\, d}_{\mathrm{NFW}} &= \frac{16\,\left(\ln(1+d) - \frac{d}{1+d}\right)}{d^3}.
\end{aligned}$$

Here we show the initial distribution for the density, and for the distribution function. Due to the spherical symmetry, we are modeling a spherical shell, denser at the central part, and fades towards the rim in both directions. The shell has a total angular momentum, in this case of $\mathcal{L} = 2.5$, and we are showing the data for the four halo models: Isothermal, Isothermal truncated, Burkert and Navarro-Frenk-White. We also show the initial distribution function in the phase space, (r, p_r), for the Isothermal halo. In the next slide, we show the initial distribution function in the phase space for the Isothermal, and initial density distribution for the four halo models.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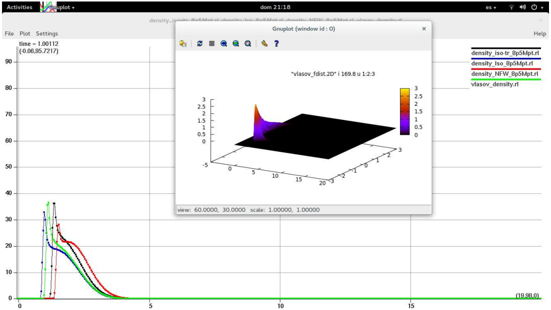
view: 60.0000, 30.0000 scale: 1.00000, 1.00000
90
80
70
60
50
40
30
20
10
0
0
5
10
15

Next, several times for the density and the distribution function evolution. Notice that, as the pattern time is $T0 = R0/q = \sqrt{\frac{{R_0}^3}{G\,M_0}}$, and for a halo with mass of $10^{10}\ M_{\rm Sol}$, ant the pattern distance, R_0 in Kpc, we obtain that the pattern time is $T_0 = 1.48\ \times\ 10^6\ \ \text{years}$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File Plot Settings
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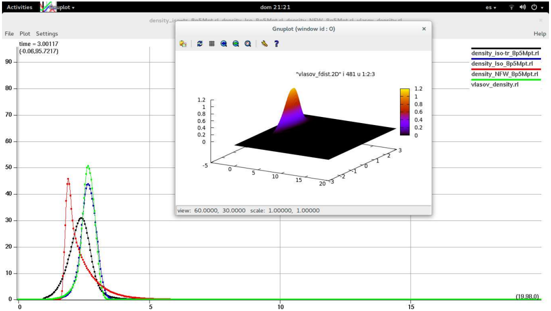
density_Iso_8p5Mpt.rl
density_NFW_8p5Mpt.rl
vlasov_density.rl
(19.98,0)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File Plot Settings
Help
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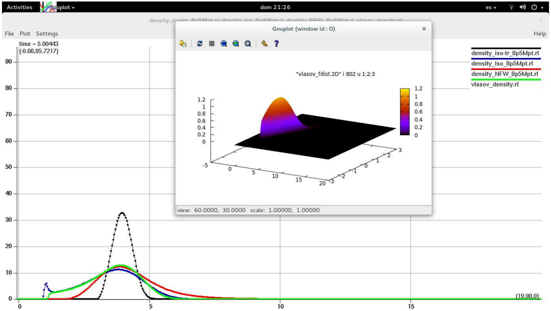
density_Iso_8p5Mpt.rl
density_NFW_8p5Mpt.rl
vlasov_density.rl
(19.98,0)



Activities
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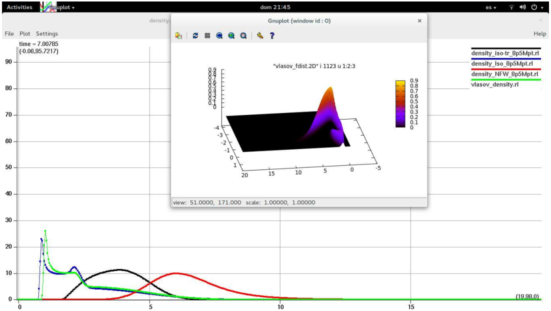
density_Iso_8p5Mpt.rl
density_NFW_8p5Mpt.rl
vlasov_density.rl
(19.98,0)



Activities
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File Plot Settings
Help
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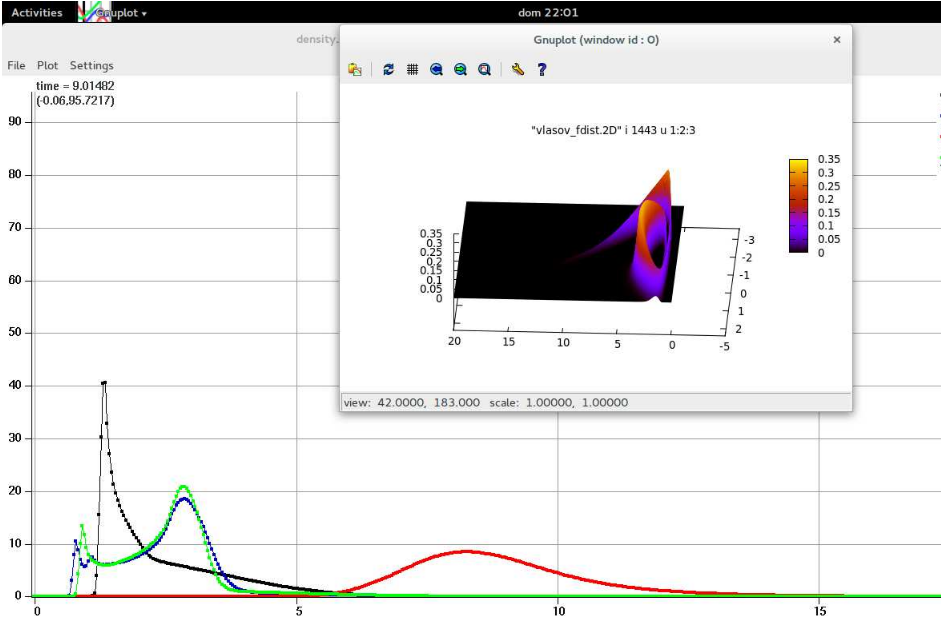
density_Iso_8p5Mpt.rl
density_NFW_8p5Mpt.rl
vlasov_density.rl
(19.98,0)

Next, notice the formation of a camel like distribution for the density mainly for the Isothermal halo and the Burkert on; the density distribution in the NFW halo just moves away. Also notice the spreading of the distribution function into an orbit in the phase space.


Activities
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density.
File Plot Settings
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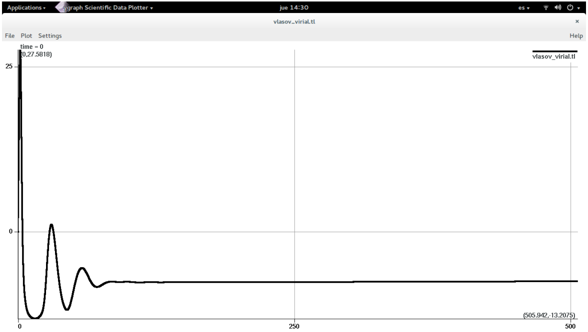
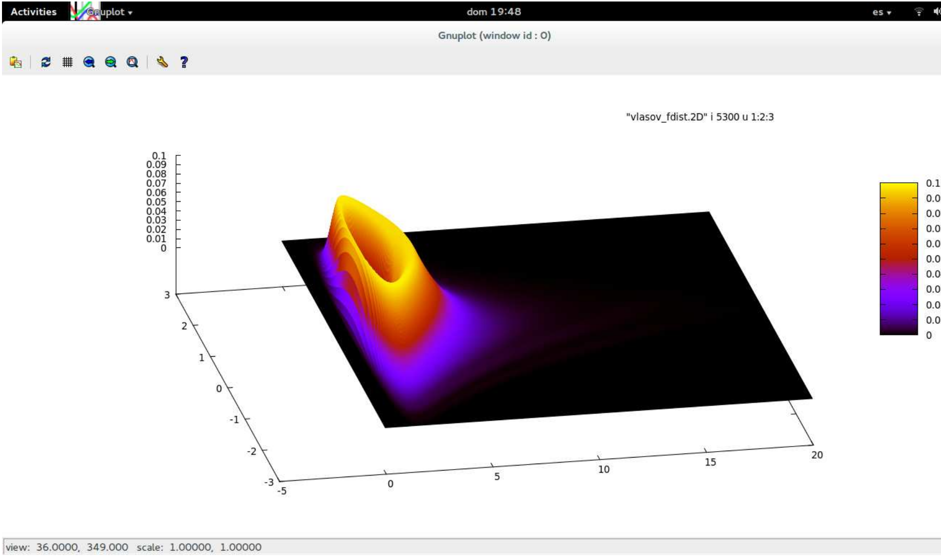
view: 42.0000, 183.000 scale: 1.00000, 1.00000

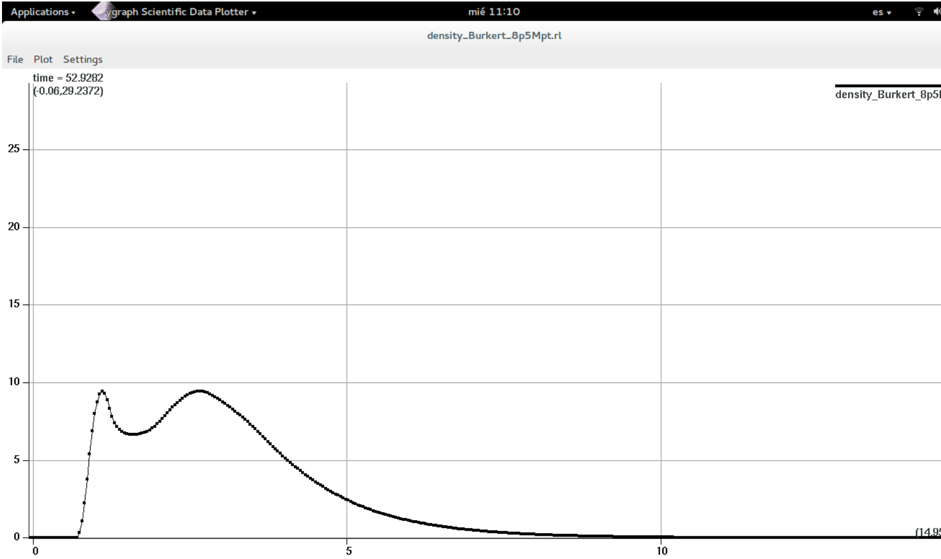
Another form to evaluate that the distribution is reaching the stationary state is by means of the virial, $< x^i \nabla_i \Phi >=< r F >$ which, when reaches a constant, indicates that the system is not changing anymore.



Legend : It is shown the virial $< d\,\bar{\Phi}_{,d} >$, showing how it reaches a constant.



Legend : Final, stationary, shape of the distribution function in the phase space, in a Burkert halo.



Legend : And the corresponding final, stationary, shape of the density function in a Burkert halo.

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- Realistic final distributions should be analyzed searching for observable consequences.