The Continuum Hamiltonian Hopf Bifurcation of Vlasov Theory

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Goal: Explain and prove a Krein-like theorem (G. Hagstrom) for instabilities that emerge from continuous spectra in a large class of Hamiltonian Eulerian matter models including the Vlasov-Poisson (VP) system.

The Continuum Hamiltonian Hopf Bifurcation of Vlasov Theory

and

Beatification: Flattening the Vlasov Poisson Bracket

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Goal: Explain and prove a Krein-like theorem ... .
and obtain weakly nonlinear Hamiltonian theory for Vlasov and other systems by a procedure called beatification.

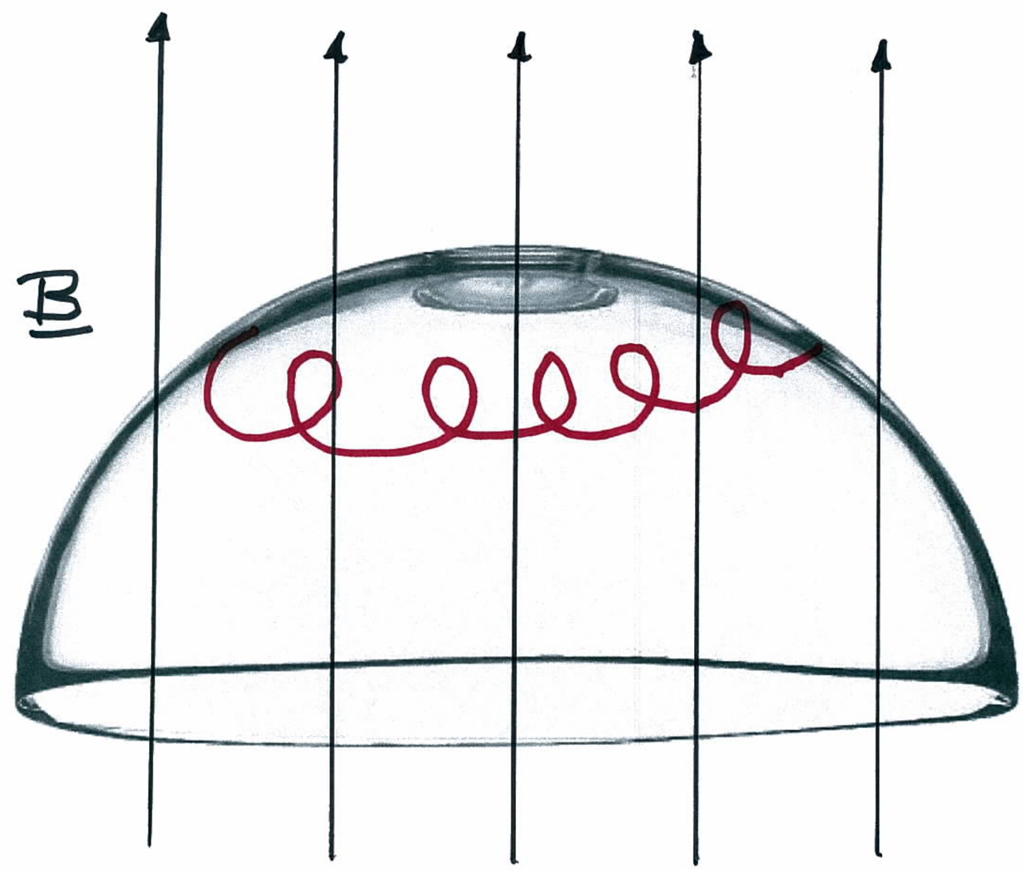
Beatification: Flattening Poisson Brackets

Q: What does this mean?

A: Transfer of nonlinearity from Poisson bracket to Hamiltonian.

What is Hamiltonian Hopf (Krein) bifurcation?

Charged Particle on Slick Mountain



Falls and Rotates $\Rightarrow$ Precession

Charged Particle on Quadratic Mountain

Simple model of FLR stabilization $\rightarrow$ plasma mirror machine.

Lagrangian:

$$L = \frac{m}{2}\left(\dot{x}^2 + \dot{y}^2\right) + \frac{eB}{2}\left(\dot{y}x - \dot{x}y\right) + \frac{K}{2}\left(x^2 + y^2\right)$$

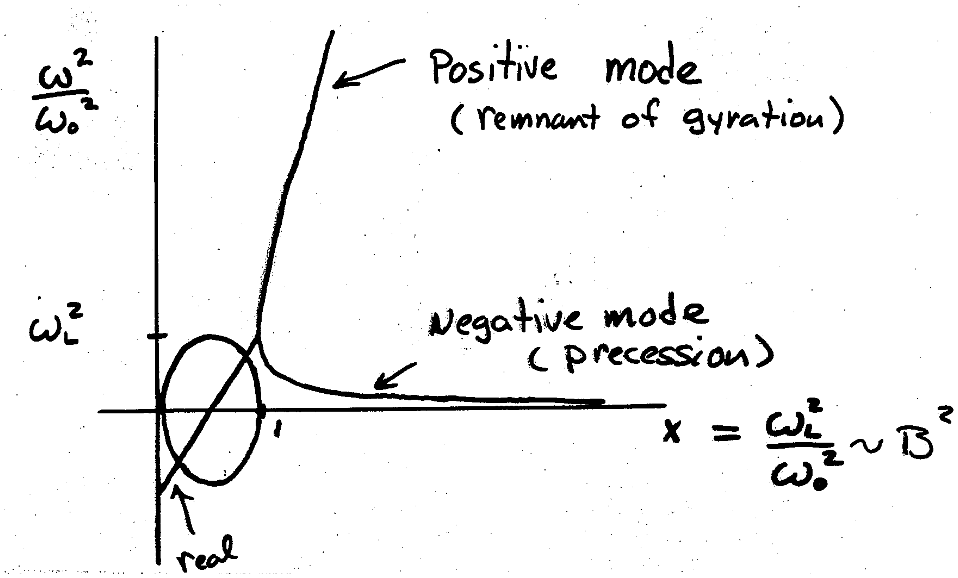
Hamiltonian:

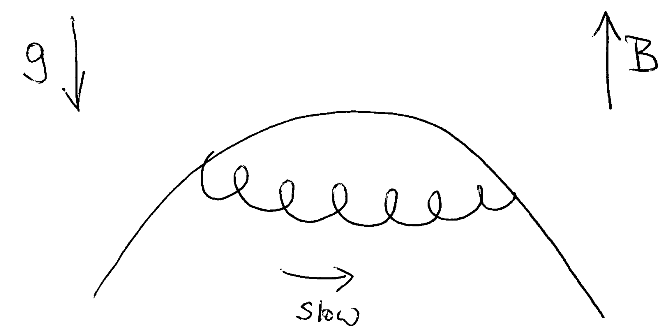
$$H = \frac{m}{2}\left(p_x^2 + p_y^2\right) + \omega_L\left(yp_x - xp_y\right) - \frac{m}{2}\left(\omega_L^2 - \omega_0^2\right)\left(x^2 + y^2\right)$$

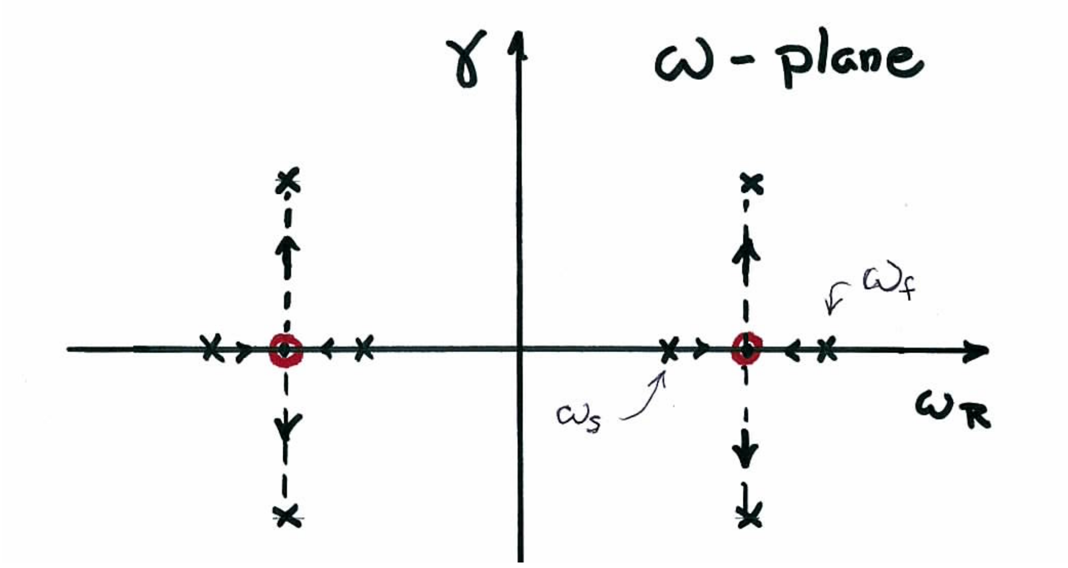
Two frequencies:

$$\omega_L = \frac{eB}{2m} \qquad \text{and} \qquad \omega_0 = \sqrt{\frac{K}{m}}$$

Hamiltonian Hopf Bifurcation - Krein Crash







$$x, y \sim e^{i\omega t} = e^{\lambda t}$$

Stable Normal Form

For large enough B system is stable and $\exists$ a coordinate change, a canonical transformation $(q,p) \to (Q,P)$, to

$$H = \frac{|\omega_f|}{2}\left(P_f^2 + Q_f^2\right) - \frac{|\omega_s|}{2}\left(P_s^2 + Q_s^2\right)$$

Slow mode is a <u>negative energy mode</u> (NEM) – a stable oscillation that lowers the energy relative to the equilibrium state.

NEM Normal Form: Weierstrass (1894), Williamson (1936), ...

Krein: Bifurcation to quartet only possible if modes have opposite signature.

$\rightarrow$ Goal to do analog of this Hamiltonian Hopf for bifurcation with continuous spectrum, viz., the Vlasov equation.

Vlasov-Poisson (VP) System

Phase space density (1 + 1 + 1 field theory):

$$f : \mathbb{T} \times \mathbb{R}^2 \to \mathbb{R}^+ , \qquad f(x,v,t) \geq 0$$

Conservation of phase space density:

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{e}{m}\frac{\partial \phi[x,t;f]}{\partial x}\frac{\partial f}{\partial v} = 0$$

Poisson's equation:

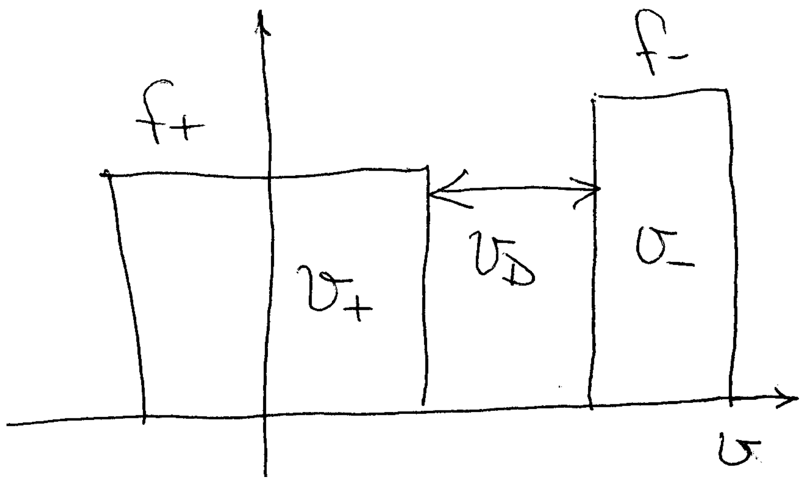
$$\phi_{xx} = 4\pi \left[e \int_{\mathbb{R}} f(x,v,t)\, dv - \rho_B \right]$$

Energy:

$$H = \frac{m}{2}\int_{\mathbb{T}}\int_{\mathbb{R}} v^2 f\, dxdv + \frac{1}{8\pi}\int_{\mathbb{T}} (\phi_x)^2\, dx$$

Fluid Two-Stream

Waterbag distribution function $\rightarrow$ exact closure:



Two-Stream Instability (warm ions & electrons)

$$\frac{\partial v_\alpha}{\partial t} + v_\alpha \frac{\partial v_\alpha}{\partial x} = \frac{e_\alpha}{m_\alpha} E - \frac{1}{\rho_\alpha}\frac{\partial P_\alpha}{\partial x}$$

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial x}(n_\alpha v_\alpha) = 0 \qquad \frac{\partial E}{\partial x} = 4\pi e(n_i - n_e)$$

equil. n_{0i}, n_{0e}, v_D ← drifting electrons

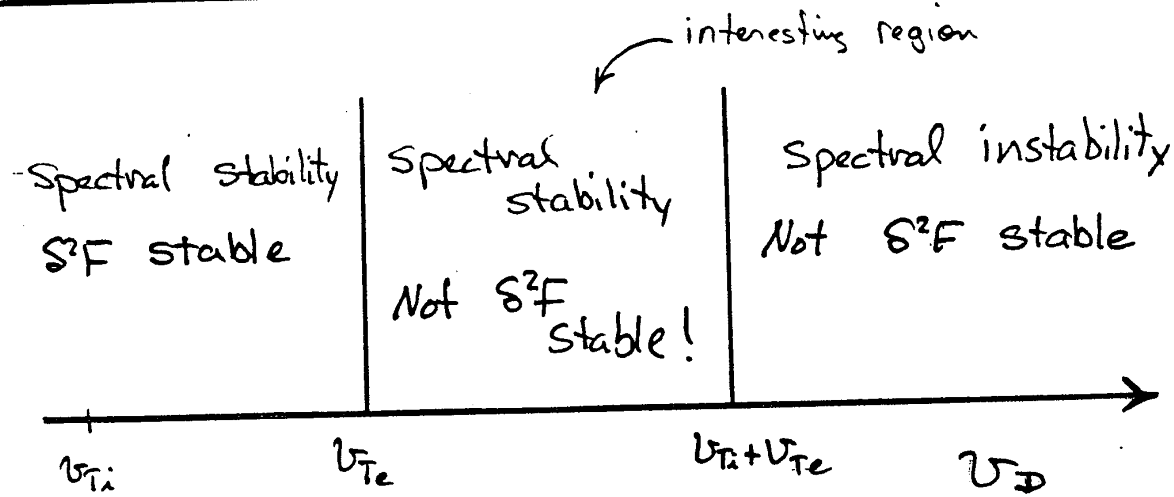
spectral stability condition given via

$$\boxed{0 = 1 - \frac{\omega_{pi}^2}{\omega^2 - k^2 v_{Ti}^2} - \frac{\omega_{pe}^2}{(\omega - k v_0)^2 - k^2 v_{Te}^2} = \varepsilon(k,\omega)}$$

Threshold: $v_D > v_{Ti} + v_{Te}$ ⟹ instab.

$\delta^2 F$:

Threshold: $v_D < v_{Te}$ ⟹ $\delta^2 F$ positive definite



Two-Stream Instability $\leftrightarrow$ Hamiltonian Hopf

Three equivalent definitions of negative energy modes:

- Von Laue 1905:

$$\mathrm{sgn}\left(\omega(k)\frac{\partial\varepsilon(k,\omega(k))}{\partial\omega}\right)$$

- Energy Casmir: $\delta^2 F = \delta^2(H+C) = \sum_k \sigma_k \omega_k (q_k^2 + p_k^2)/2$

- Symplectic signature: H_L on eigenvector or two-form

Krein (1950) – Moser (1958) – Sturrock (1958)
Avoidance crossing etc. Sturrock $\rightarrow$ Cairns ....

Von Laue (wave) energy incorrect for continuous spectrum
pjm and Pfirsch (1992)

Fluid and Plasma Theories – Matter Models

Systems that describe the motion of matter as dynamical systems of the form

$$\frac{\partial \Psi}{\partial t} = \mathcal{O}(\Psi)\,, \qquad \mathcal{O} \text{ nonlinear PDEs, intregrodifferential} \ldots$$

Examples:

- kinetic theories
  - Vlasov equation, drift kinetic equations, gyrokinetics, ...
- multifluid fluid theories
  - 2-Fluid coupled to Maxwell's equations, ...
- magnetofluids
  - MHD, HMHD, IMHD, XMHD, etc.
- hybrids

Common Features:

- Nondissipative part is Hamiltonian. Dissipation should be real!
- Hamiltonian description is noncanonical

Canonical Hamiltonian Form

Hamilton's Equations:

$$\boxed{\dot{p}_i = \{p_i, H\} = -\frac{\partial H}{\partial q^i}\,, \qquad \dot{q}^i = \{q^i, H\} = \frac{\partial H}{\partial p_i}\,,}$$

Natural Hamiltonians:

$$H(p,q) = p^2/2 + V(q) = K + V$$

Poisson Bracket:

$$\{A, B\} = \frac{\partial A}{\partial q^i}\frac{\partial B}{\partial p_i} - \frac{\partial B}{\partial q^i}\frac{\partial A}{\partial p_i}\,, \qquad i = 1, 2, \ldots, N$$

Phase Space Coordinates: $\quad z = (q, p)$

$$\dot{z}^i = J_c^{ij}\frac{\partial H}{\partial z^j} = \{z^i, H\} \qquad (J_c^{ij}) = \begin{pmatrix} 0_N & I_N \\ -I_N & 0_N \end{pmatrix}$$

symplectic 2-form = (cosymplectic form)$^{-1}$: $\quad \omega_{ij}^c J_c^{jk} = \delta_i^k$

Noncanonical Hamiltonian Form $J_c \rightarrow J(z)$

Noncanonical Coordinates:

$$\dot{z}^i = J^{ij}\frac{\partial H}{\partial z^j} = \{z^i, H\}\,, \qquad\qquad \{A,B\} = \frac{\partial A}{\partial z^i}J^{ij}(z)\frac{\partial B}{\partial z^j}$$

Poisson Bracket Properties:

antisymmetry $\longrightarrow$ $\{A,B\} = -\{B,A\}\,,$

Jacobi identity $\longrightarrow$ $\{A,\{B,C\}\} + \{B,\{C,A\}\} + \{C,\{A,B\}\} = 0$

G. Darboux: $det J \neq 0 \Longrightarrow J \rightarrow J_c$ Canonical Coordinates

Sophus Lie: $det J = 0 \Longrightarrow$ Canonical Coordinates plus Casimirs

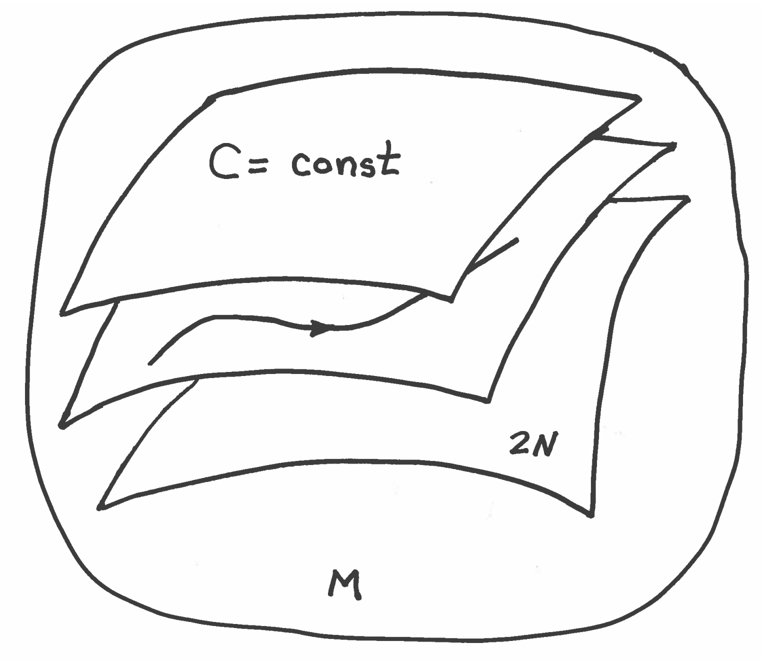
Eulerian Media: $J^{ij} = c_k^{ij} z^k$ $\longleftarrow$ Lie – Poisson Brackets

Poisson Manifold $\mathcal{P}$ Cartoon

Degeneracy in $J \Rightarrow$ Casimirs:

$$\{f, C\} = 0 \quad \forall\ f : \mathcal{P} \to \mathbb{R}$$

Lie-Darboux Foliation by Casimir (symplectic) leaves:



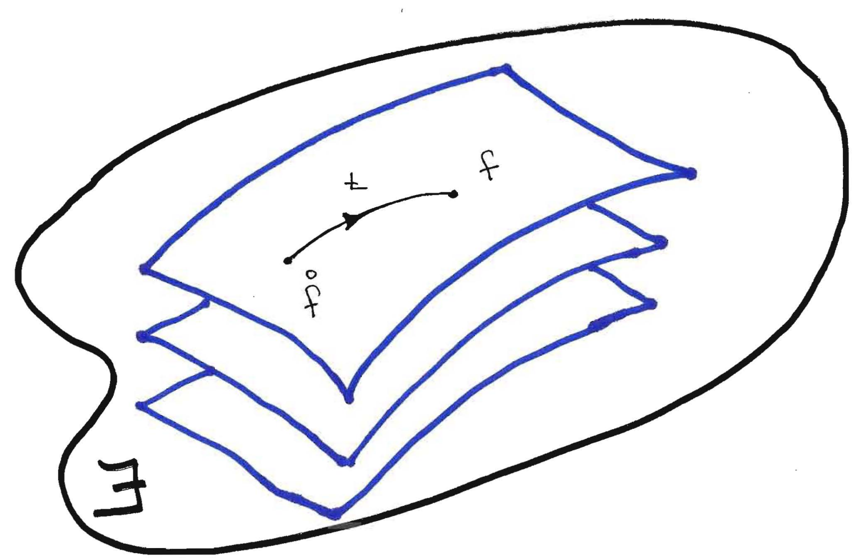
For infinite dof Vlasov leaves are symplectic rearrangements.

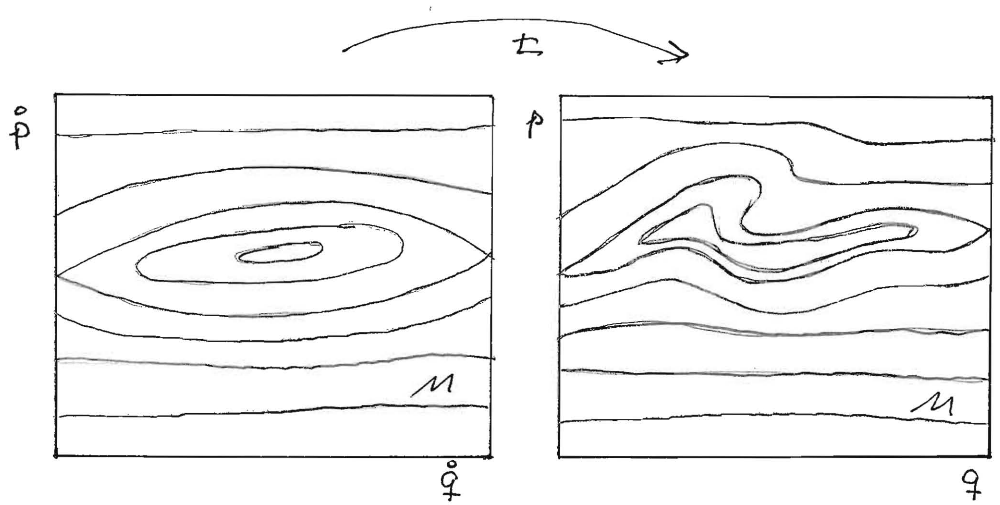
VP Cartoon– Symplectic Rearrangement

$f(x, v, t) = \mathring{f} \circ \mathring{z}$

$f \sim g$ if $f = g \circ z$

with z symplectomorphism





$p = mv$

μ volume measure

$$f(x, v, t) = \mathring{f}(\mathring{x}(x, v, t), \mathring{v}(x, v, t))$$

Infinite-Dimensional Hamiltonian Structure – Field theory

Finite dimensions to infinite dimensions:

Fréchet Derivative $\rightarrow$ Variational Derivative:

$$\delta F = \frac{d}{d\epsilon} F[q + \epsilon \delta q]\Big|_{\epsilon=0} = DF \cdot \delta q \qquad \rightarrow \qquad \frac{\delta F}{\delta q} \equiv F_q$$

Canonical Poisson Bracket:

$$\{F, G\} = \int_D d^3a \left(\frac{\delta F}{\delta q^i} \frac{\delta G}{\delta \pi_i} - \frac{\delta G}{\delta q^i} \frac{\delta F}{\delta \pi_i} \right)$$

EOM:

$$\dot{q} = \{q, H\} \qquad \dot{\pi} = \{\pi, H\}$$

Infinite-Dimensional Plasmas Systems
Noncanonical Hamiltonian Field Theory

Field Variables: $\psi(\mu, t)$ e.g. $\mu = x$, $\mu = (x, v)$, . . .

Poisson Bracket:

$$\{A, B\} = \int d\mu \, \frac{\delta A}{\delta \psi} \, \mathcal{J}(\psi) \, \frac{\delta A}{\delta \psi}$$

Lie-Poisson Bracket:

$$\{A, B\} = \left\langle \psi, \left[\frac{\delta A}{\delta \psi}, \frac{\delta A}{\delta \psi} \right] \right\rangle$$

Cosymplectic Operator:

$$\mathcal{J} \,\cdot\, \sim [\psi, \,\cdot\,]$$

Form for Eulerian theories: ideal fluids, Vlasov, Liouville eq, BBGKY, gyrokinetic theory, MHD, tokamak reduced fluid models, RMHD, H-M, 4-field model, ITG . . . .

Natural Hamiltonian Structure of Matter

Noncanonical Poisson Bracket:

$$\{F,G\} = \int_{\mathcal{Z}} dqdp\, f\left[\frac{\delta F}{\delta f}, \frac{\delta G}{\delta f}\right] = \int_{\mathcal{Z}} dqdp\, F_f \mathcal{J} G_f = \left\langle f, [F_f, G_f]\right\rangle$$

Cosymplectic Operator:

$$\mathcal{J}\ \cdot = \frac{\partial f}{\partial p}\frac{\partial\ \cdot}{\partial q} - \frac{\partial f}{\partial q}\frac{\partial\ \cdot}{\partial p}$$

Vlasov:

$$\frac{\partial f}{\partial t} = \{f, H\} = \mathcal{J}\frac{\delta H}{\delta f} = -[f, \mathcal{E}].$$

Casimir Degeneracy:

$$\{C,F\} = 0 \qquad \forall F \qquad \text{for} \qquad C[f] = \int_{\mathcal{Z}} dqdp\, \mathcal{C}(f)$$

Too many variables and not canonical.
Recall Cartoon – Hamiltonian on leaf.

Linear Vlasov-Poisson System

Expand about Stable Homogeneous Equilibrium:

$$f = f_0(v) + \delta f(x, v, t)$$

Linearized EOM:

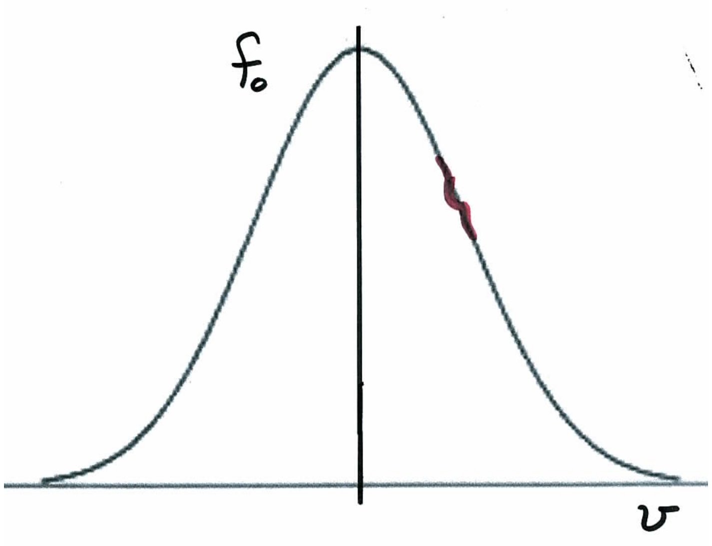
$$\frac{\partial \delta f}{\partial t} + v \frac{\partial \delta f}{\partial x} + \frac{e}{m} \frac{\partial \delta\phi[x, t; \delta f]}{\partial x} \frac{\partial f_0}{\partial v} = 0$$

$$\delta\phi_{xx} = 4\pi e \int_{\mathbb{R}} \delta f(x, v, t)\, dv$$

Linearized Energy (Kruskal-Oberman 1958):

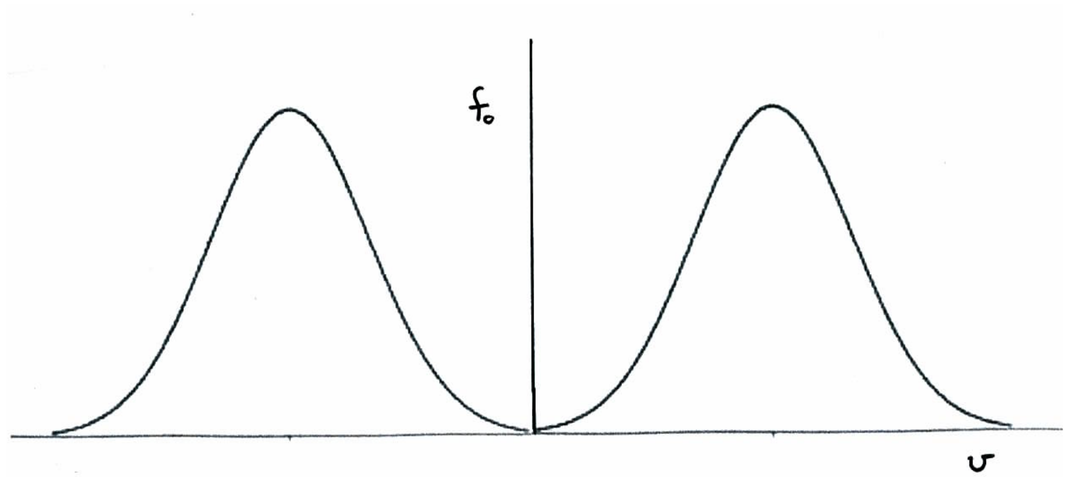
$$H_L = -\frac{m}{2} \int_{\mathbb{T}} \int_{\mathbb{R}} \frac{v\,(\delta f)^2}{f_0'}\, dv dx + \frac{1}{8\pi} \int_{\mathbb{T}} (\delta\phi_x)^2\, dx$$

Sample Homogeneous Equilibria



← Maxwellian

BiMaxwellian →



Linear Hamiltonian Theory

Expand f-dependent Poisson bracket and Hamiltonian $\Rightarrow$

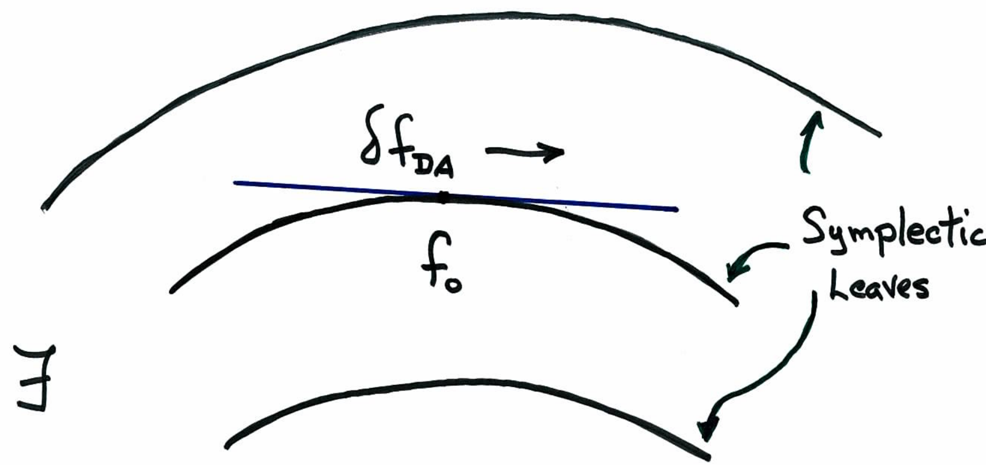
$$\frac{\partial \delta f}{\partial t} = \{\delta f, H_L\}_L \,,$$

where quadratic Hamiltonian H_L is the Kruskal-Oberman energy and linear Poisson bracket is $\{\,,\,\}_L = \{\,,\,\}_{f_0}$.

Note:

δf <u>not canonical</u>

H_L <u>not diagonal</u>



Landau's Problem

Assume

$$\delta f = \sum_k f_k(v,t)e^{ikx}\,, \qquad \delta\phi = \sum_k \phi_k(t)e^{ikx}$$

Linearized EOM:

$$\frac{\partial f_k}{\partial t} + ikvf_k + ik\phi_k\frac{e}{m}\frac{\partial f_0}{\partial v} = 0\,, \qquad k^2\phi_k = -4\pi e\int_{\mathbb{R}} f_k(v,t)\,dv$$

Three methods:

1. Laplace Transforms (Landau and others 1946)
2. Normal Modes (Van Kampen, Case,... 1955)
3. Coordinate Change $\Longleftrightarrow$ Integral Transform (PJM, Pfirsch, Shadwick, ... 1992)

Canonization & Diagonalization

Fourier Linear Poisson Bracket:

$$\{F, G\}_L = \sum_{k=1}^{\infty} \frac{ik}{m} \int_{\mathbb{R}} f_0' \left(\frac{\delta F}{\delta f_k} \frac{\delta G}{\delta f_{-k}} - \frac{\delta G}{\delta f_k} \frac{\delta F}{\delta f_{-k}} \right) dv$$

Linear Hamiltonian:

$$\begin{aligned} H_L = -\frac{m}{2} \sum_k \int_{\mathbb{R}} \frac{v}{f_0'} \, |f_k|^2 \, dv \;\; &+ \;\; \frac{1}{8\pi} \sum_k k^2 |\phi_k|^2 \\ &= \;\; \sum_{k,k'} \int_{\mathbb{R}} \int_{\mathbb{R}} f_k(v) \, \mathcal{O}_{k,k'}(v|v') \, f_{k'}(v') \, dv dv' \end{aligned}$$

Canonization:

$$q_k(v,t) = f_k(v,t) \, , \qquad p_k(v,t) = \frac{m}{ik f_0'} f_{-k}(v,t) \qquad \Longrightarrow$$

$$\{F, G\}_L = \sum_{k=1}^{\infty} \int_{\mathbb{R}} \left(\frac{\delta F}{\delta q_k} \frac{\delta G}{\delta p_k} - \frac{\delta G}{\delta q_k} \frac{\delta F}{\delta p_k} \right) dv$$

Integral Transform

Definintion:

$$\boxed{f(v) = \mathcal{G}[g](v) \;\; := \varepsilon_R(v)\, g(v) + \varepsilon_I(v)\, H[g](v)\,,}$$

where

$$\varepsilon_I(v) = -\pi \frac{\omega_p^2}{k^2} \frac{\partial f_0(v)}{\partial v}\,, \qquad \varepsilon_R(v) = 1 + H[\varepsilon_I](v)\,,$$

and the Hilbert transform

$$H[g](v) := \frac{1}{\pi} \fint \frac{g(u)}{u-v}\, du\,,$$

with $\fint$ denoting Cauchy principal value of $\int_{\mathbb{R}}$.

Theorem (G1) *$\mathcal{G}\colon L^p(\mathbb{R}) \to L^p(\mathbb{R})$, $1 < p < \infty$, is a bounded linear operator; i.e.*

$$\|\mathcal{G}[g]\|_p \leq B_p \, \|g\|_p \, ,$$

where B_p depends only on p.

Theorem (G2) *If $f_0' \in L^q(\mathbb{R})$, stable, Hölder decay, then $\mathcal{G}[g]$ has a bounded inverse,*

$$\mathcal{G}^{-1}\colon L^p(\mathbb{R}) \to L^p(\mathbb{R}) \, ,$$

for $1/p + 1/q < 1$, given by

$$\begin{aligned} g(u) &= \mathcal{G}^{-1}[f](u) \\ &:= \frac{\varepsilon_R(u)}{|\varepsilon(u)|^2} \, f(u) - \frac{\varepsilon_I(u)}{|\varepsilon(u)|^2} \, H[f](u) \, . \end{aligned}$$

where $|\varepsilon|^2 := \varepsilon_R^2 + \varepsilon_I^2$.

Diagonalization

Mixed Variable Generating Functional:

$$\mathcal{F}[q,P'] = \sum_{k=1}^{\infty} \int_{\mathbb{R}} q_k(v)\, \mathcal{G}[P'_k](v)\, dv$$

Canonical Coordinate Change $(q,p) \longleftrightarrow (Q',P')$:

$$p_k(v) = \frac{\delta \mathcal{F}[q,P']}{\delta q_k(v)} = \mathcal{G}[P_k](v)\,, \qquad Q'_k(u) = \frac{\delta \mathcal{F}[q,P']}{\delta P_k(u)} = \mathcal{G}^{\dagger}[q_k](u)$$

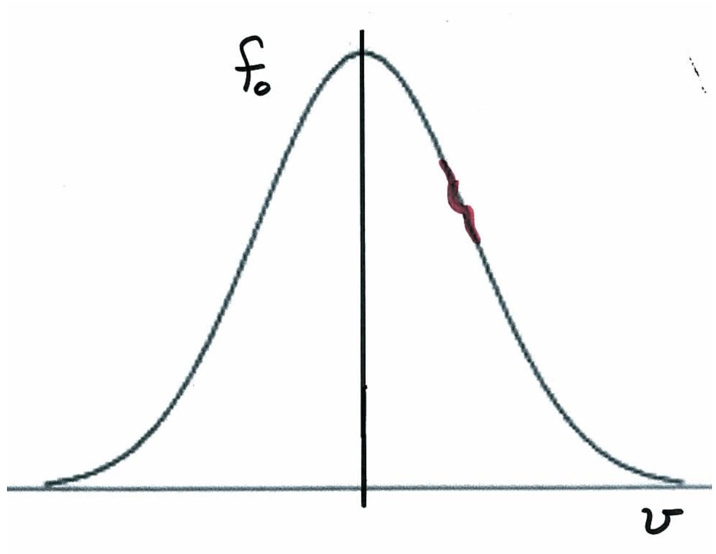
New Hamiltonian:

$$H_L = \tfrac{1}{2} \sum_{k=1}^{\infty} \int_{\mathbb{R}} du\, \sigma_k(u)\omega_k(u)\, \left[Q_k^2(u) + P_k^2(u)\right]$$

where $\omega_k(u) = |ku|$ and the signature is

$$\boxed{\sigma_k(v) := -\mathrm{sgn}(v f_0'(v))}$$

Sample Homogeneous Equilibria



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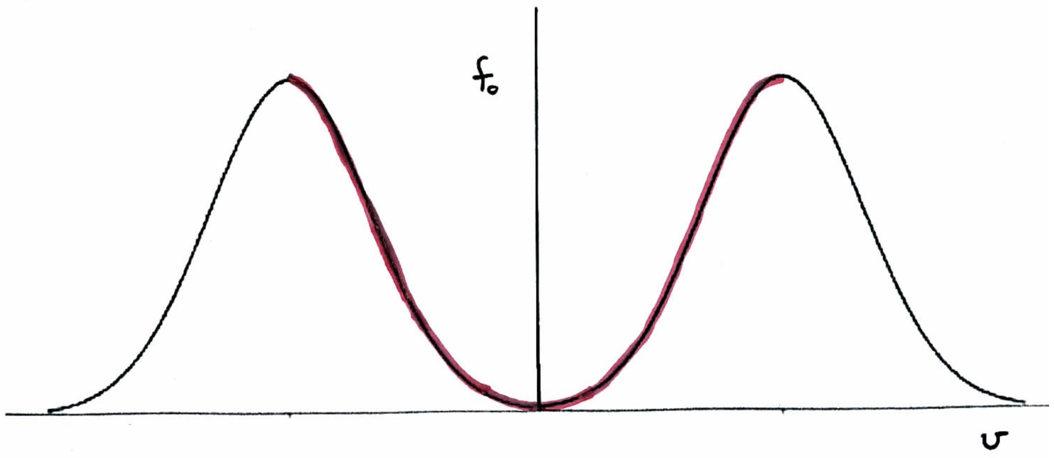


Hamiltonian Spectrum

Hamiltonian Operator:

$$f_{kt} = -ikvf_k + \frac{if_0'}{k}\int_{\mathbb{R}} d\bar{v}\, f_k(\bar{v},t) =: T_k f_k\,,$$

Complete System:

$$f_{kt} = T_k f_k \qquad \text{and} \qquad f_{-kt} = T_{-k} f_{-k}\,, \qquad\qquad k \in \mathbb{R}^+$$

Lemma *If λ is an eigenvalue of the Vlasov equation linearized about the equilibrium $f_0'(v)$, then so are $-\lambda$ and λ^* . Thus if $\lambda = \gamma + i\omega$, then eigenvalues occur in the pairs, $\pm\gamma$ and $\pm i\omega$, for purely real and imaginary cases, respectively, or quartets, $\lambda = \pm\gamma \pm i\omega$, for complex eigenvalues.*

Spectral Stability

Definition The dynamics of a Hamiltonian system linearized around some equilibrium solution, with the phase space of solutions in some Banach space $\mathcal{B}$, is spectrally stable if the spectrum $\sigma(T)$ of the time evolution operator T is purely imaginary.

Theorem *If for some $k \in \mathbb{R}^+$ and $u = \omega/k$ in the upper half plane the plasma dispersion relation,*

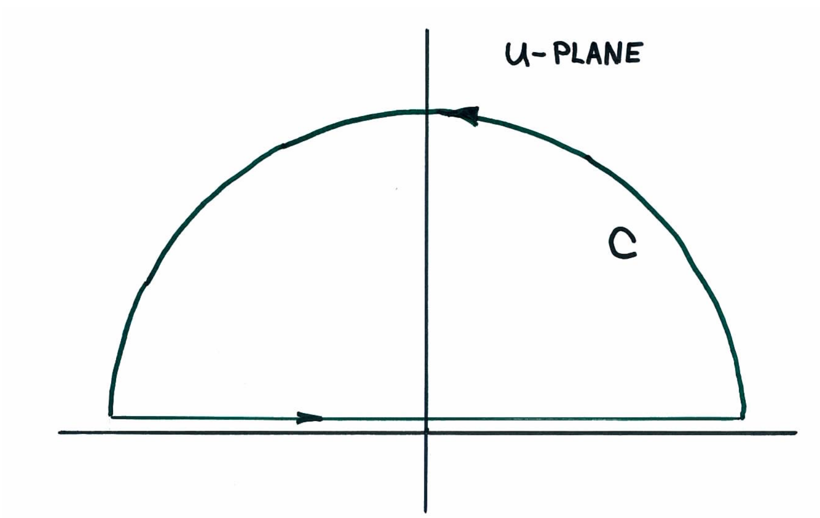
$$\varepsilon(k,u) := 1 - k^{-2} \int_{\mathbb{R}} dv \frac{f_0'}{u-v} = 0\,,$$

then the system with equilibrium f_0 is spectrally unstable. Otherwise it is spectrally stable.

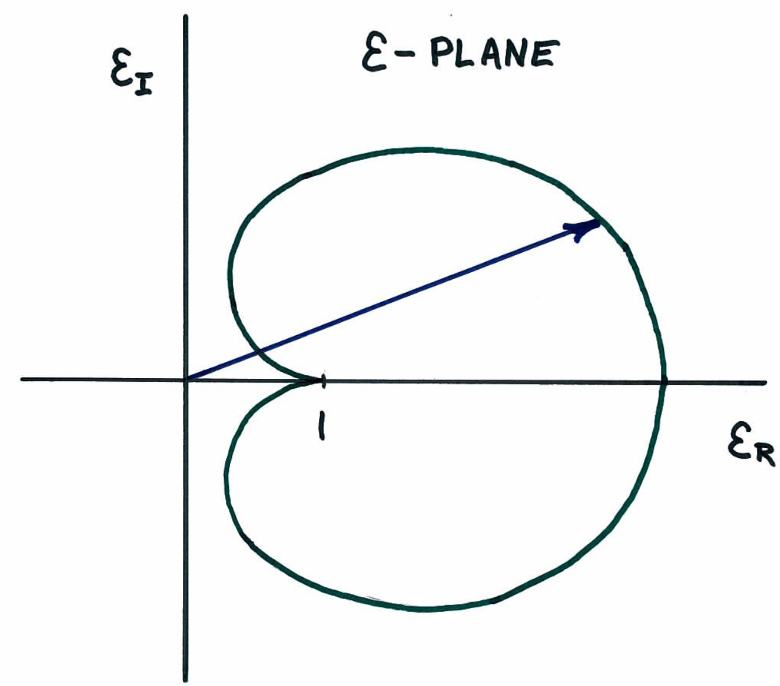
Nyquist Method

$$f_0' \in C^{0,\alpha}(\mathbb{R}) \Rightarrow \varepsilon \in C^{\omega}(uhp).$$

Therefore, Argument Principle $\Rightarrow$ winding # = # zeros of ε



Stable $\rightarrow$



Spectral Theorem

Set $k = 1$ and consider $T\colon f \mapsto ivf - if_0' \int f$ in the space $W^{1,1}(\mathbb{R})$.

$W^{1,1}(\mathbb{R})$ is Sobolev space containing closure of functions

$$\|f\|_{1,1} = \|f\|_1 + \|f'\|_1 = \int_{\mathbb{R}} dv(|f| + |f'|)$$

Definition Resolvent of T is $R(T,\lambda) = (T - \lambda I)^{-1}$ and $\lambda \in \sigma(T)$. (i) λ in point spectrum, $\sigma_p(T)$, if $R(T,\lambda)$ not injective. (ii) λ in residual spectrum, $\sigma_r(T)$, if $R(T,\lambda)$ exists but not densely defined. (iii) λ in continuous spectrum, $\sigma_c(T)$, if $R(T,\lambda)$ exists, densely defined but not bounded.

Theorem *Let $\lambda = iu$. (i) $\sigma_p(T)$ consists of all points $iu \in \mathbb{C}$, where $\varepsilon = 1 - k^{-2} \int_{\mathbb{R}} dv\, f_0'/(u - v) = 0$. (ii) $\sigma_c(T)$ consists of all $\lambda = iu$ with $u \in \mathbb{R} \setminus (-i\sigma_p(T) \cap \mathbb{R})$. (iii) $\sigma_r(T)$ contains all the points $\lambda = iu$ in the complement of $\sigma_p(T) \cup \sigma_c(T)$ that satisfy $f_0'(u) = 0$.*

cf. e.g. P. Degond (1986). Similar but different.

Structural Stability

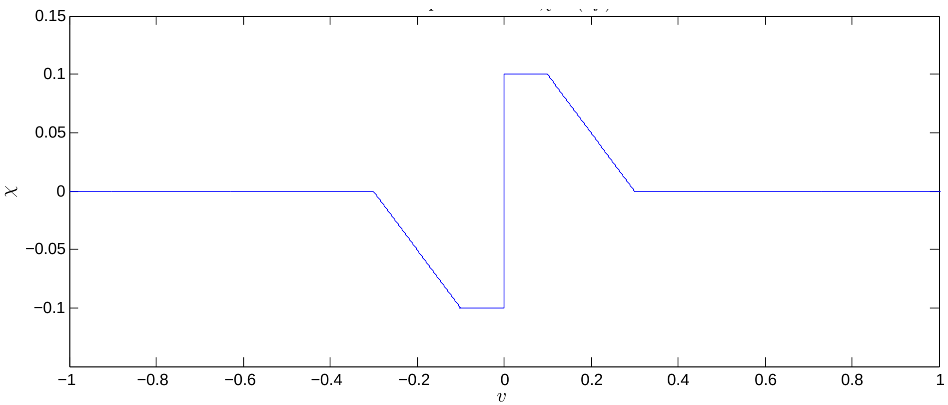
Definition Consider an equilibrium solution of a Hamiltonian system and the corresponding time evolution operator T for the linearized dynamics. Let the phase space for the linearized dynamics be some Banach space $\mathcal{B}$. Suppose that T is spectrally stable. Consider perturbations δT of T and define a norm on the space of such perturbations. Then we say that the equilibrium is <u>structurally stable</u> under this norm if there is some $\delta > 0$ such that for every $\|\delta T\| < \delta$ the operator $T + \delta T$ is spectrally stable. Otherwise the system is <u>structurally unstable</u>.

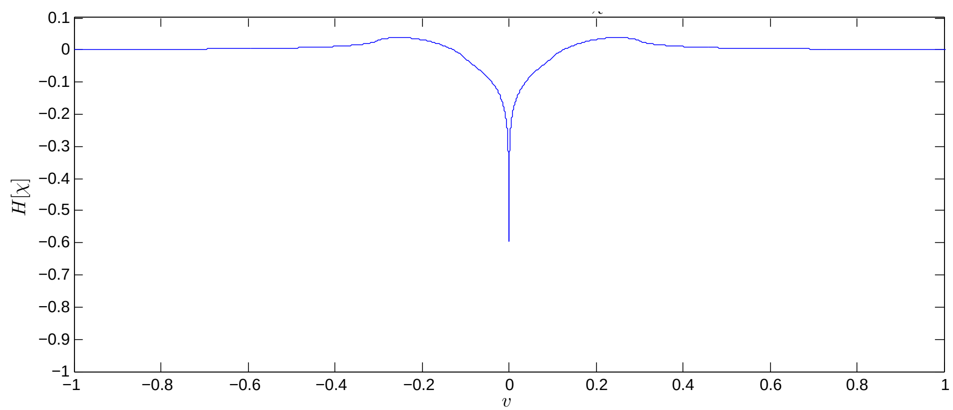
Definition Consider the formulation of the linearized Vlasov-Poisson equation in the Banach space $W^{1,1}(\mathbb{R})$ with a spectrally stable homogeneous equilibrium function f_0. Let $T_{f_0+\delta f_0}$ be the time evolution operator corresponding to the linearized dynamics around the distribution function $f_0 + \delta f_0$. If there exists some ϵ depending only on f_0 such that $T_{f_0+\delta f_0}$ is spectrally stable whenever $\|T_{f_0} - T_{f_0+\delta f_0}\| < \epsilon$, then the equilibrium f_0 is structurally stable under perturbations of f_0.

All f_0 are Structurally Unstable in $W^{1,1}$

True in space where Hilbert transform unbounded, e.g. $W^{1,1}$.
Small perturbation $\Rightarrow$ big jump in Penrose plot.

Theorem *A stable equilibrium distribution is structurally unstable under perturbations of f_0' in the Banach spaces $W^{1,1}$ and $L^1 \cap C_0$.*





Easy to make 'bumps' in f_0 that are small in norm. What to do?

Krein-Like Theorem for VP

Theorem *Let f_0 be a stable equilibrium distribution function for the Vlasov equation. Then f_0 is structurally stable under dynamically accessible perturbations in $W^{1,1}$, if there is only one solution of $f_0'(v) = 0$ (e.g. Maxwellian). If there are multiple solutions, f_0 is structurally unstable and the unstable modes come from the roots of f_0' that satisfy $f_0''(v) < 0$.*

Remark A change in the signature of the continuous spectrum is a necessary and sufficient condition for structural instability. The bifurcations do not occur at all points where the signature changes, however. Only those that represent valleys of the distribution can give birth to unstable modes.

Summary – Conclusions

When a linear system has NEMs:

- Structurally unstable – Krein-Moser via Hamiltonian Hopf

For Hamiltonian pdes with continuous spectrum (CS) like VP:

- Diagonalization by $\mathcal{G}$-transform defines signature for CS
- There is a Krein-like theorem, e.g. valley theorem

Beatification: Flattening Poisson Brackets

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