Transport driven by velocity-space structures in toroidal plasmas

Maxime Lesur

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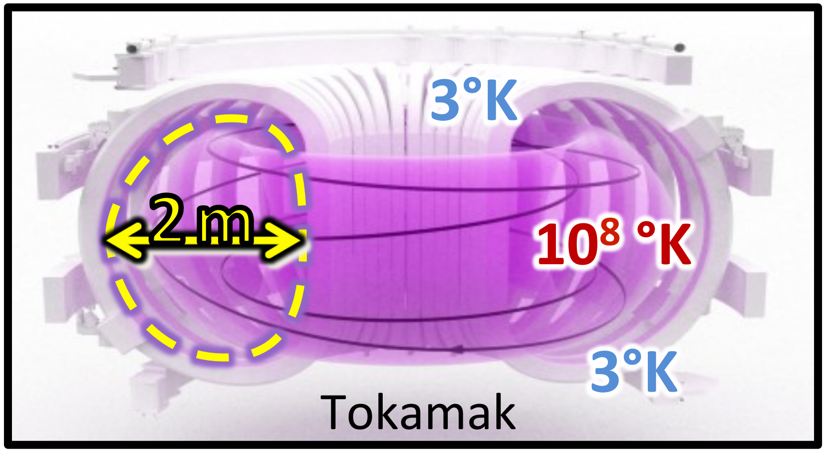
JSPS Kakenhi Project 15K18305

Context: Turbulence in the core of fusion plasmas

Core turbulence

- Microscopic (mm) waves
- Driven by pressure gradients
- In this work: electrostatic
- Transport particles and energy from core to edge via **ExB** drift
  $\Rightarrow$ Degrade confinement
- Collisional mean free path (km) >> λ



Turbulence can be channeled or mitigated

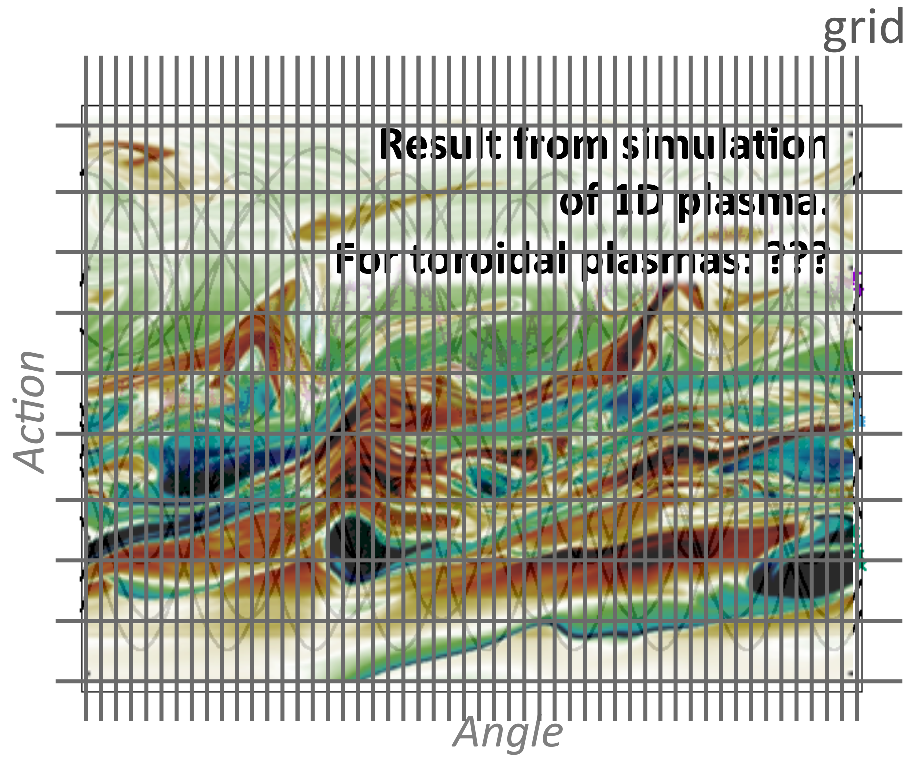
- Turbulence can generate large coherent eddies which degrade confinement further. *Huld '90*
- Turbulence (prey) can feed macroscopic flows (predators), which significantly improve confinement. *Diamond '05*
- Theory could provide selection criteria ? Other methods of control ?

Validity limits of conventional approaches

Analytic theory

- Based on linear theory.
- Example: quasi-linear theory.

  $\Rightarrow$ Random walk
- Caveat: assumes collection of waves with random phases.
- Qualitative discrepancies with measured transport.



Numerical simulations

- Kinetic simulations (gyrokinetics), large efforts on fine scales in real space.
- Caveat: may miss fine scales in velocity (or energy) space.

Experimental measurements

- Direct measurement challenging.

$\Delta v \ll v_{th}$ $\Delta r \sim \rho_{ci} \sim$ mm

$\tau \sim 1$ - $10\ \mu s$

$\Rightarrow$ Microscopic velocity-space structures?

$\Rightarrow$ Macroscopic impacts?

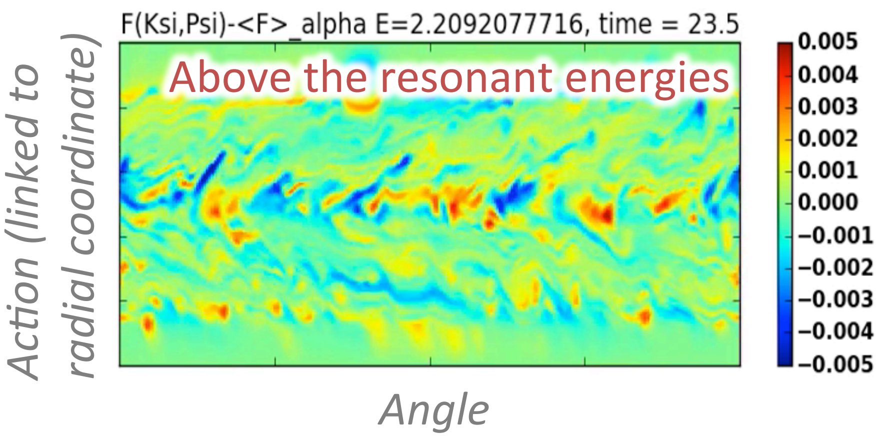
Conclusion: microscopic phase-space structures *can* be responsible for MOST of the transport!

Analytic theories

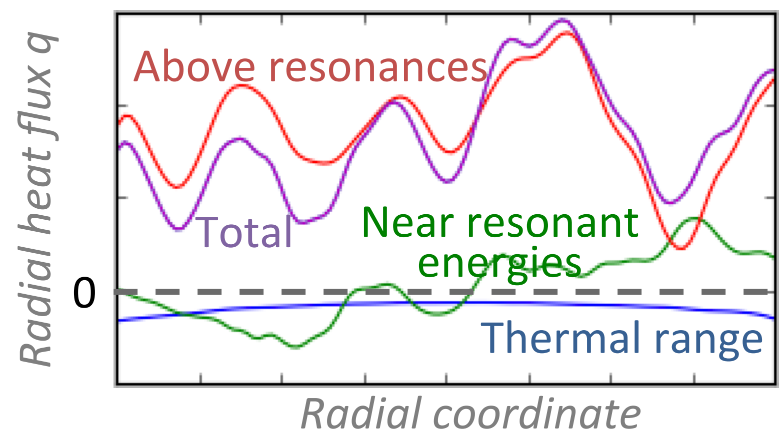
- Either robust structures with long life-time, or microscopic "**granulation**" of the phase-space.
- Essential macroscopic impacts:
  - Turbulence, transport *Dupree '72*
  - Zonal flows *Biglari '88*
  - Corrugation, torque *Kosuga '13, '14, '16, '17*
  - Collisionless friction

Numerical simulations

- Self-organisation of structures



- Heat transport due to structures



⇒ **Essential impacts of small-scale phase-space structures in fusion plasmas!**

But…

… many caveats.

Reduced (toy) model

- Prototype for low-frequency turbulence

  $\Rightarrow$ Coupling with higher-frequency turbulence is neglected

  $\Rightarrow$ Quasi-periodic orbits with faster time-scale are averaged out
- Heavily simplified geometry (but retain toroidal effects and inhomogeneity of equilibrium plasma)

Decaying turbulence

- Initial perturbations, no stirring. Only the transient behavior is analyzed.

Weak turbulence

- Small equilibrium gradients
- Small system size

No collision operator

$\Rightarrow$ Future work: remove these caveats one by one

Approach: reduced model for trapped particles (bananas)

<u>Kinetics:</u> **6D Phase-space** (3D + 3V)

$\omega \ll \omega_{gyro}$

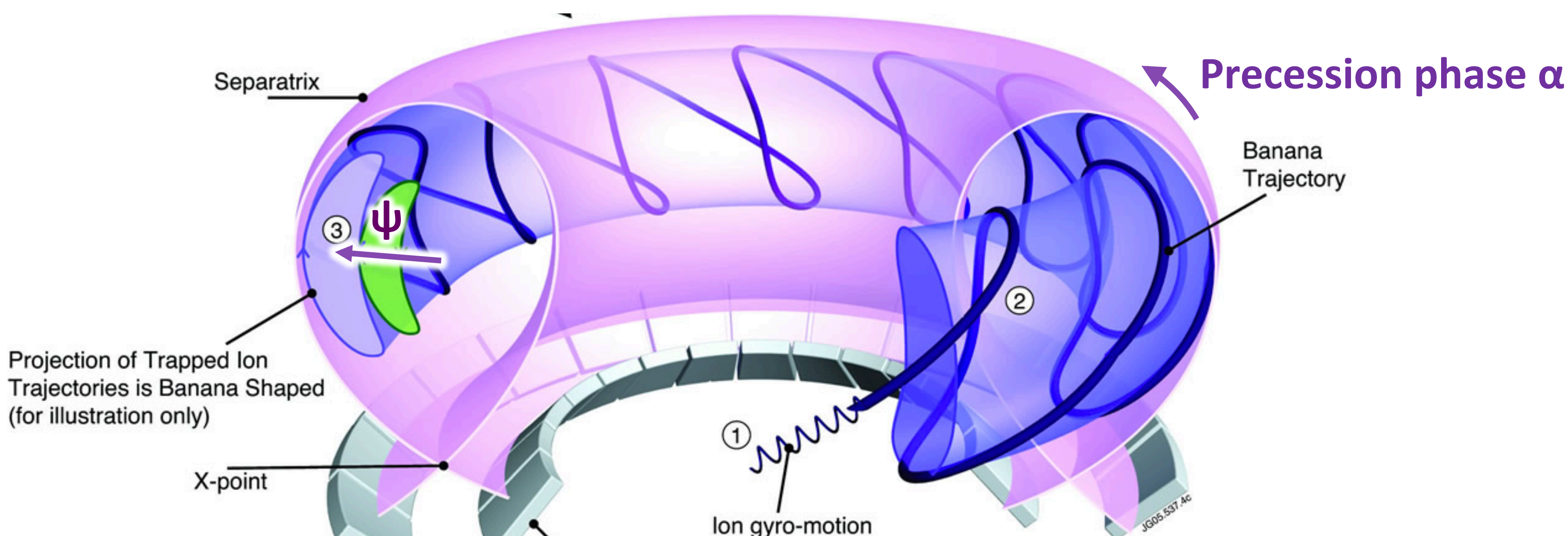
<u>Gyrokinetics:</u> **4D Phase-space + 1 parameter**

$\omega \ll \omega_{bounce} \ll \omega_{gyro}$

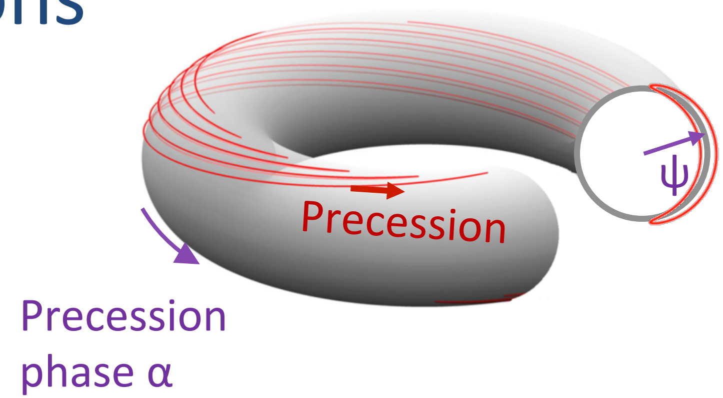
<u>Gyrobounce gyrokinetics:</u> **2D Phase-space + 2 parameters**

Pitch-angle = 0

(α, ψ)
2D Phase-space + 1 parameter (Kinetic energy E)



Model equations



Vlasov equation:

$$\frac{\partial f}{\partial t} + [J_0\phi, f]_{\alpha,\psi} + E\Omega_d \frac{\partial f}{\partial \alpha} = 0$$

Quasi-neutrality:

$$C_1\Big[\phi - \langle\phi\rangle + \mathcal{F}^{-1}(\imath\delta_m\hat{\phi}_m)\Big] - C_2\bar{\Delta}\phi = \frac{2}{\sqrt{\pi}}\int_0^\infty J_0(E) f \sqrt{E}\,\mathrm{d}E \;\; - \; 1$$

$$J_0(E) \approx \left(1 - \frac{m^2\rho_0^2}{4}E\right)\left(1 - \frac{k^2\delta_b^2}{4}E\right)$$

$$\bar{\Delta} \equiv \left(\frac{q_0\rho_0}{a}\right)^2 \frac{\partial^2}{\partial\alpha^2} + \delta_b^2 \frac{\partial^2}{\partial\psi^2}$$

Tagger, Laval, Pellat '77
Biglari, Diamond, Terry '88
Depret '00
Sarazin '05
Darmet '08
Drouot '14

Main limitation: $\omega \sim \omega_{\mathrm{precession}}$ $\Rightarrow$ TIM, TEM

Retains the main ingredients of PS structure formation:

- Free energy and dissipation mechanism
- Wave-particle interactions
- Equilibrium inhomogeneity

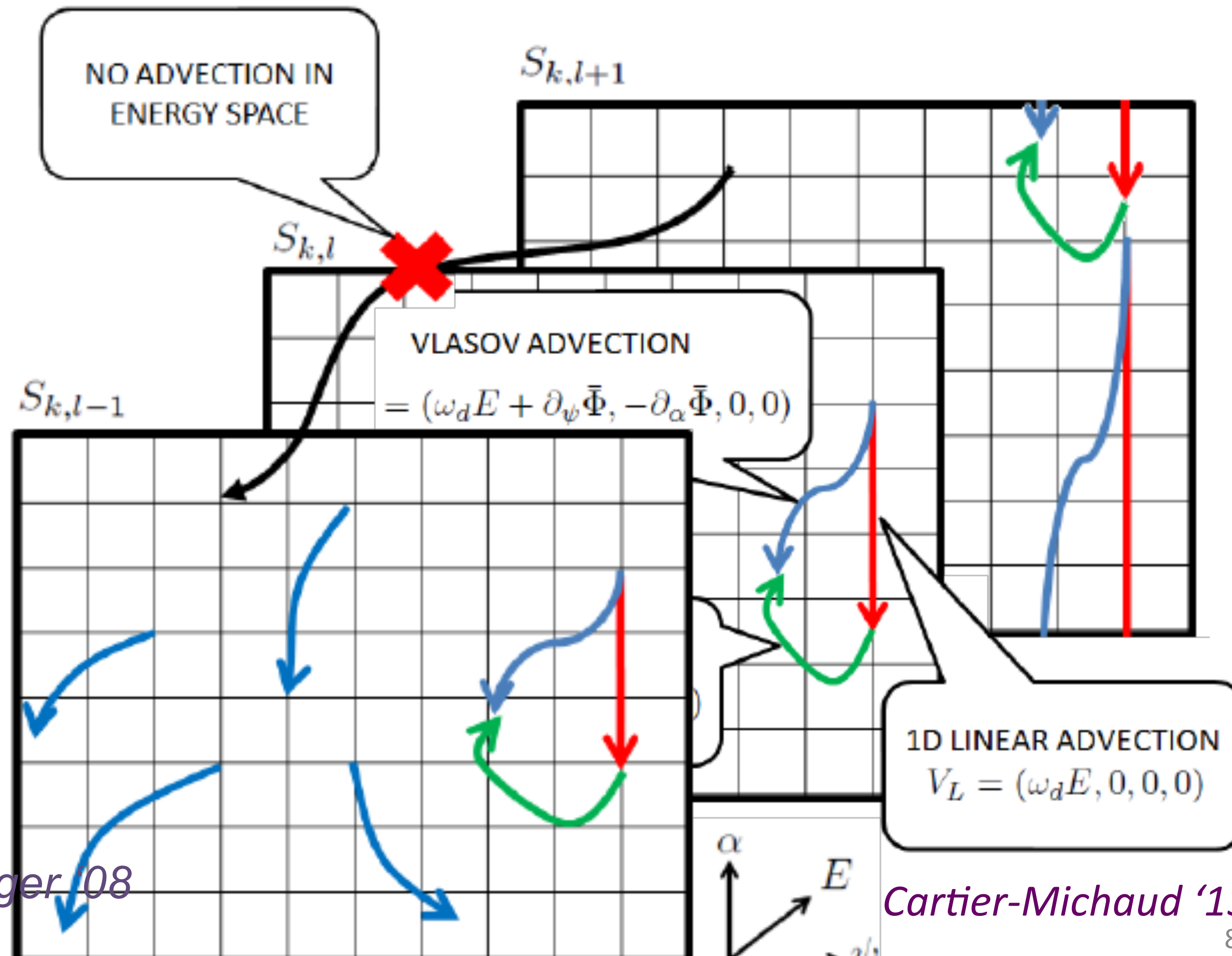
Numerical method (1/2)

$$\partial_t f + \mathcal{V}(X(t),t).\nabla f = 0$$

Vlasov advection: semi-Lagrangian scheme

Sonnendrücker '99

- Based on conservation of f along characteristics X, solutions of



- Step 1: Newton algc
  determine the foot o
  characteristics reacl
  each grid point at t=

- Step 2: bi-cubic splir
  interpolation of f at tl
  of the characteristics

Besse & Mehrenberger '08

Cartier-Michaud '13

Numerical method (2/2)

Charge density $\int_0^{+\infty} dE \;\; \mathcal{J}\bar{f}$

- Pade approximation (tends to over-damp the small scales)

$$\mathcal{J}_{\rho_{c0},\delta_{b0}} f = (1 - \frac{E}{4}\delta_{b0}^2 \partial_\psi^2)^{-1} (1 - \frac{E}{4}\rho_{c0}^2 \partial_\alpha^2)^{-1} f = \bar{f}$$

- Simpson's rule for integration in E

Quasineutrality: $\left\{C_e\left(1 - \delta_{m,0}\varepsilon_e\right) - C_i\left(-m^2 + \bar{\delta}_b^2 \partial_\psi^2\right)\right\} \hat{\Phi}_m(\psi) \;\; = \;\; \hat{\bar{\bar{n}}}_m(\psi)$

- Fourier decomposition in α, 4th order finite-difference in ψ

TERESA, in the jargon of gyrokinetic codes:

Cartier-Michaud '13
Drouot '14

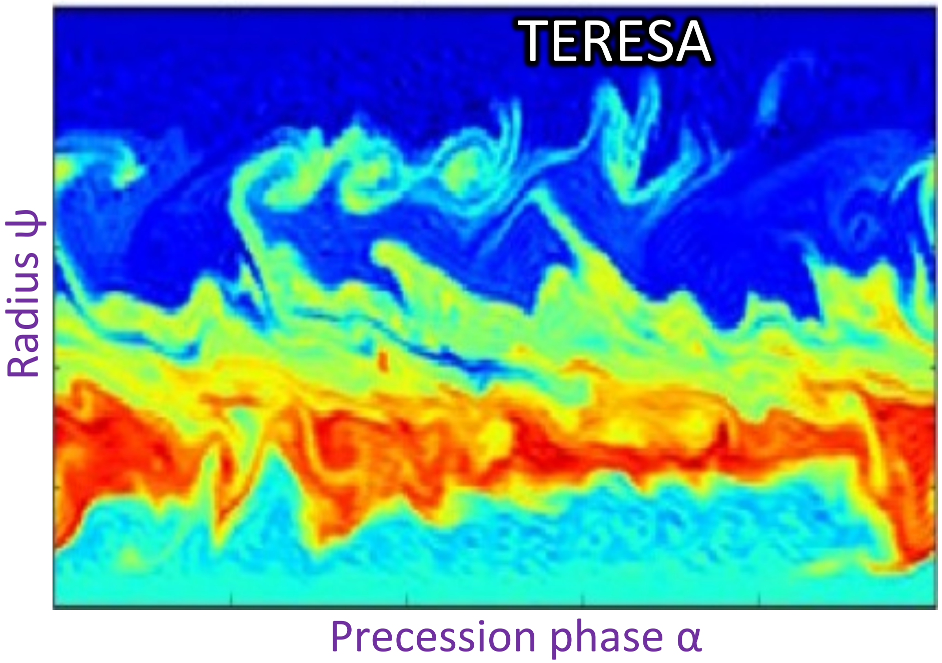
- Global, full-f, fixed-gradient (possibility of flux-driven)
- Collisionless, electrostatic
- Non-adiabatic electrons with a simple model (possibility of kinetic electrons)

$$\frac{\hat{n}_{e,m}}{n_0} = (1 + \imath m\delta)\frac{e\hat{\phi}_m}{T}$$

Kadomtsev '71
Lesur '17

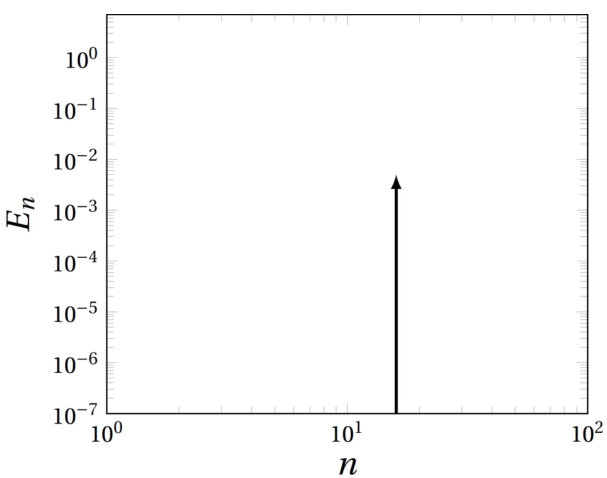
TERESA explores kinetic effects in tokamak turbulence

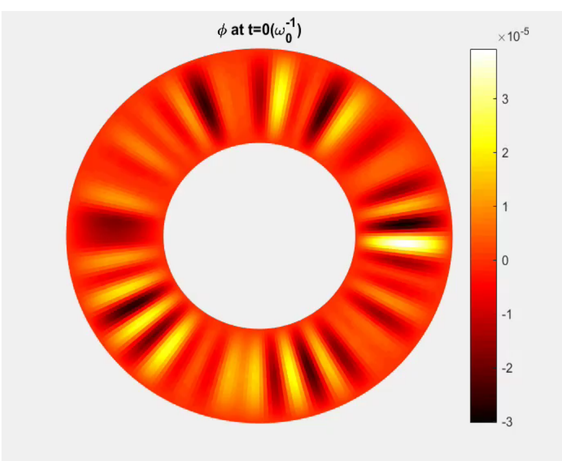
Kinetic version of classic instabilities

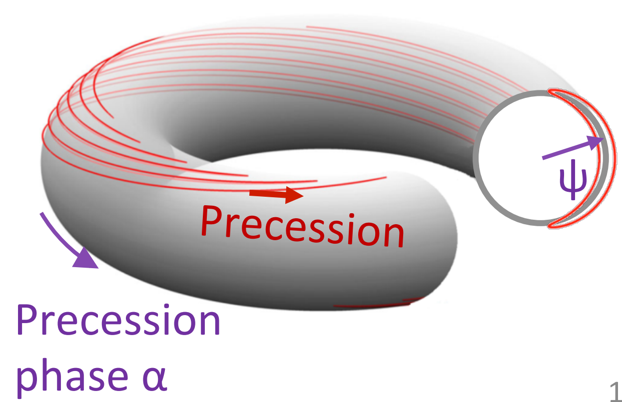


Cartier-Michaud '14

Palermo '15



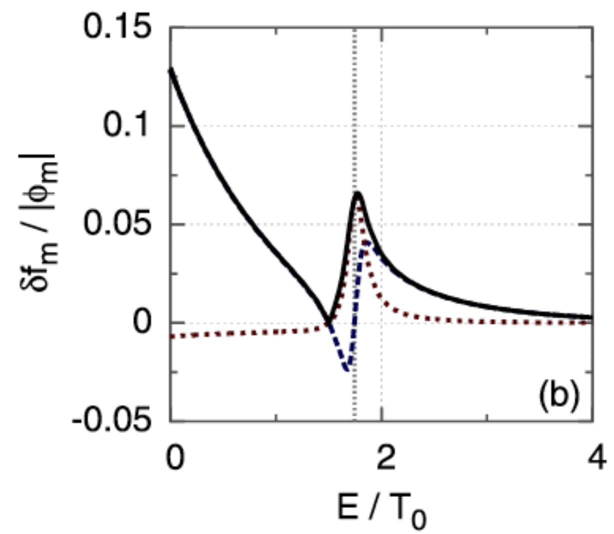


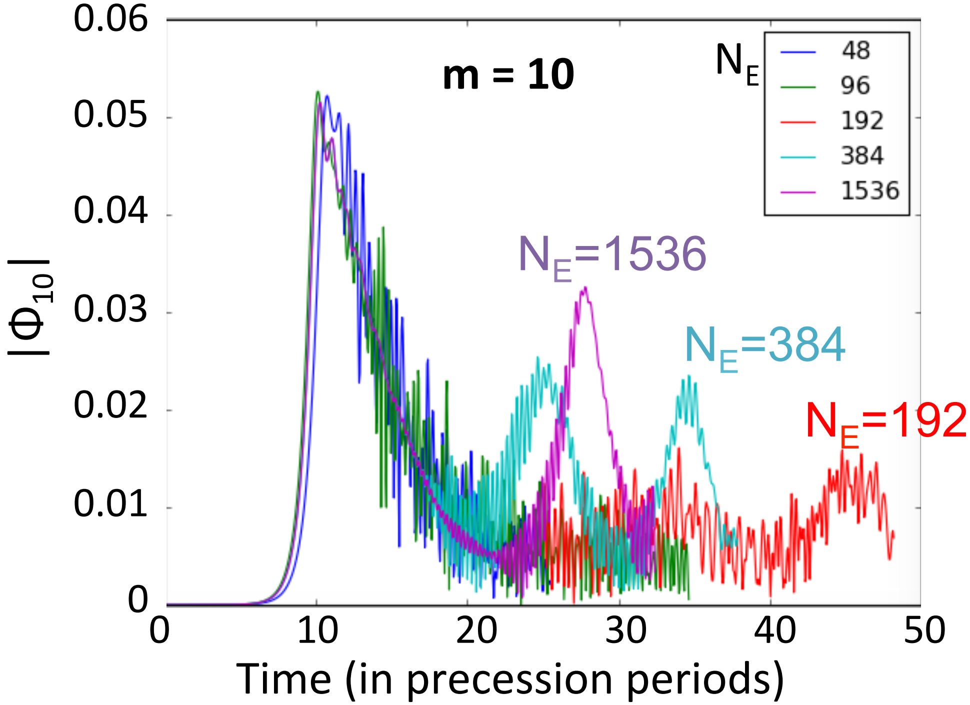


Case study of a single mode (all other modes set to 0)

Resonant wave-particle interactions can create a narrow structure in energy-space

- But linear growth-rate is not so sensitive (0.1% inaccuracy for $N_E \approx 200$).





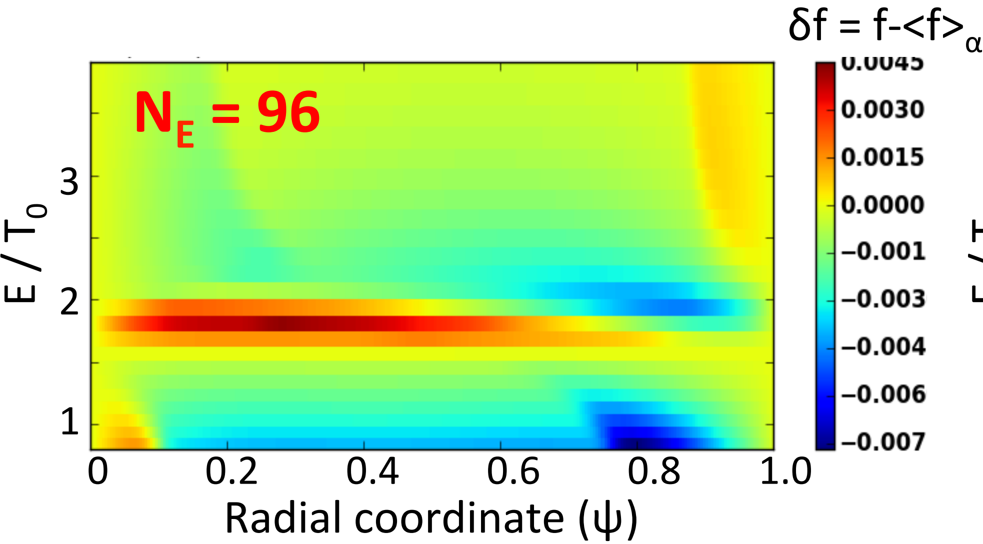
First peak is not so sensitive to N_E

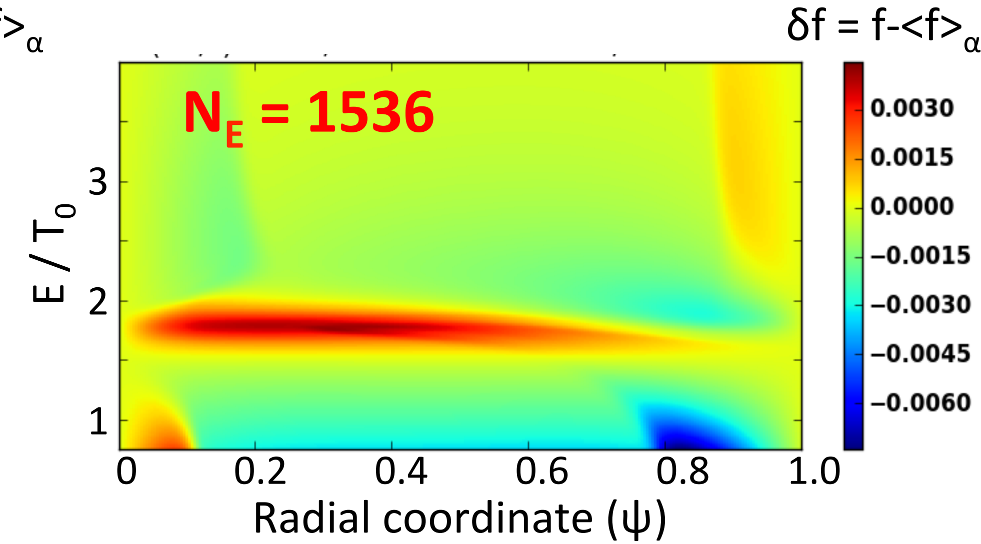
But then, convergence requires large N_E

- Appears to converge for $N_E \approx 30$
- But new peaks for $N_E > 300$

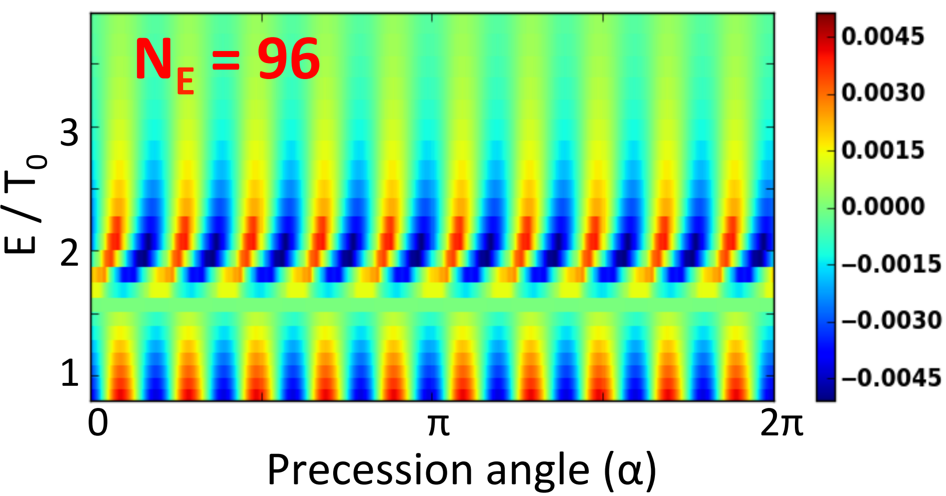
First peak: not-so-fine scales in energy-space

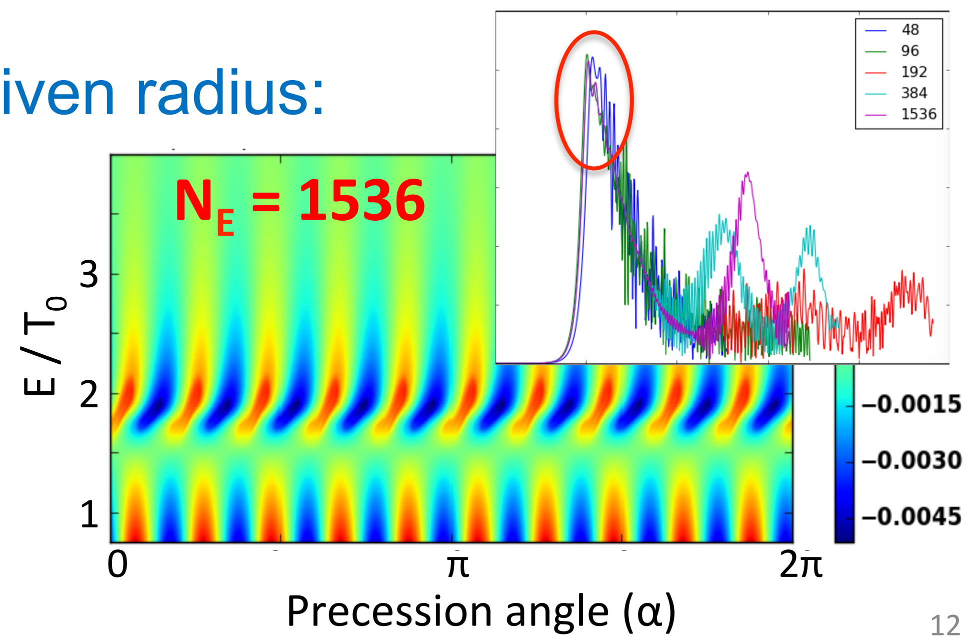
Slice at a given precession angle:





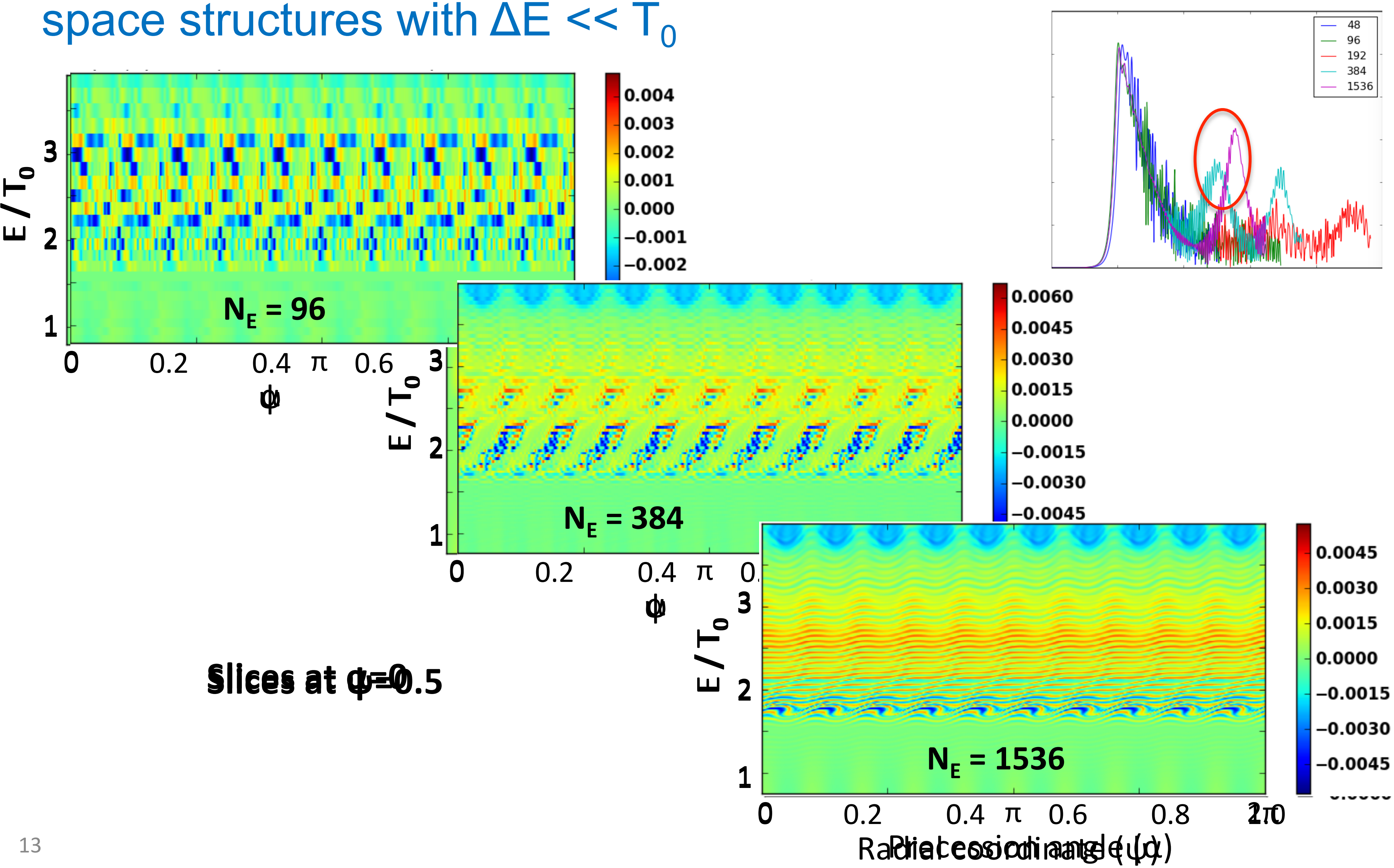
Slice at a given radius:





Second peak: fine-scales in energy-space

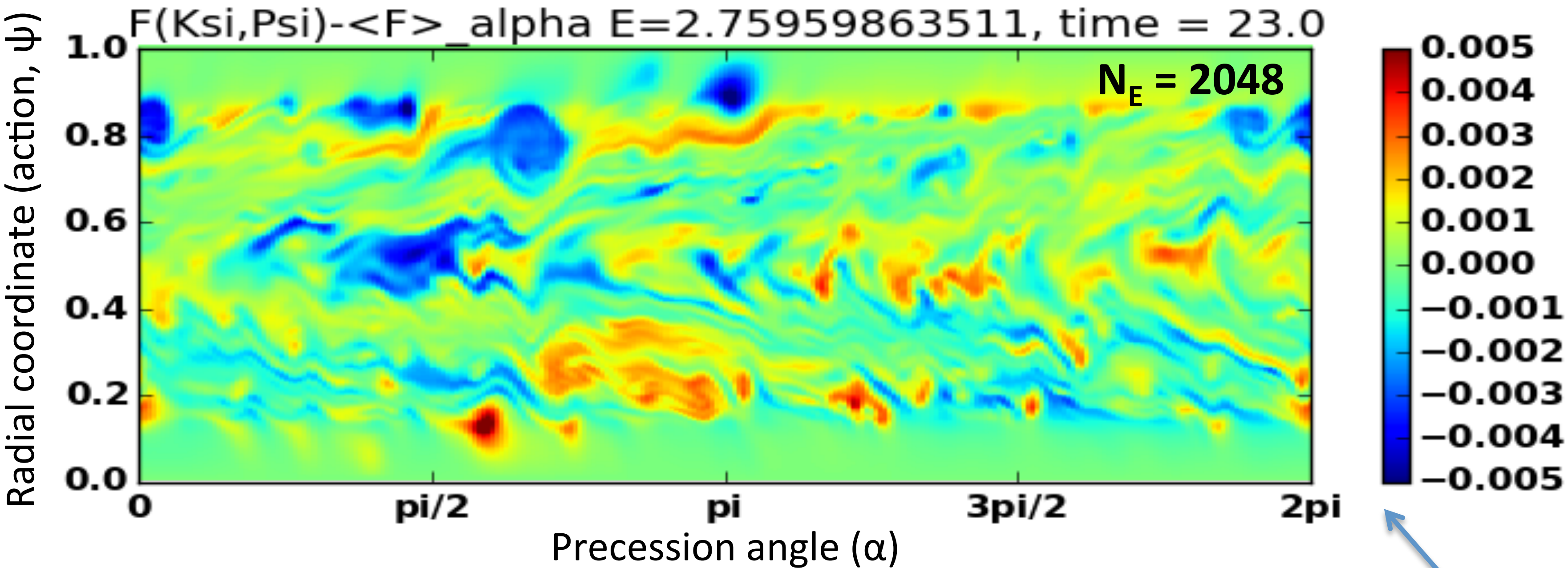
The second peak is associated with the growth of phase-space structures with $\Delta E << T_0$



Slices at ψ=0

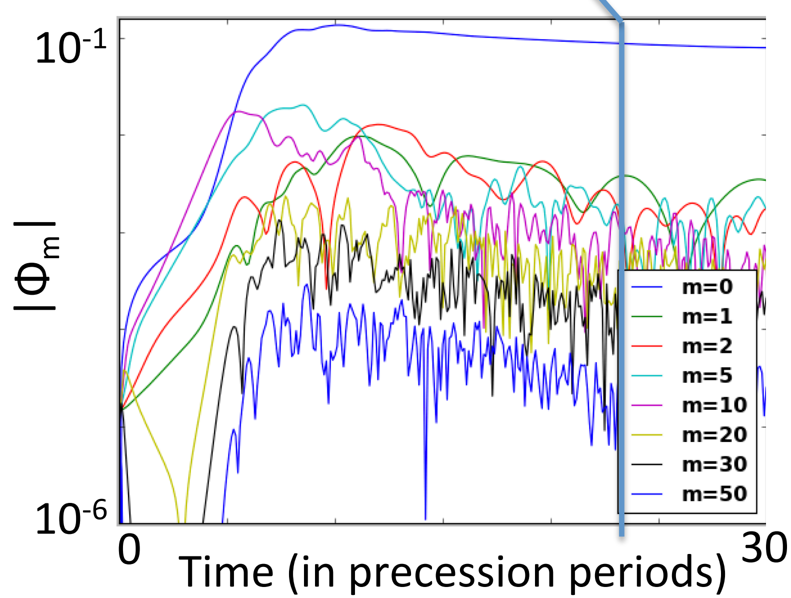
Slices at ψ=0.5

These structures survive turbulence



Properties

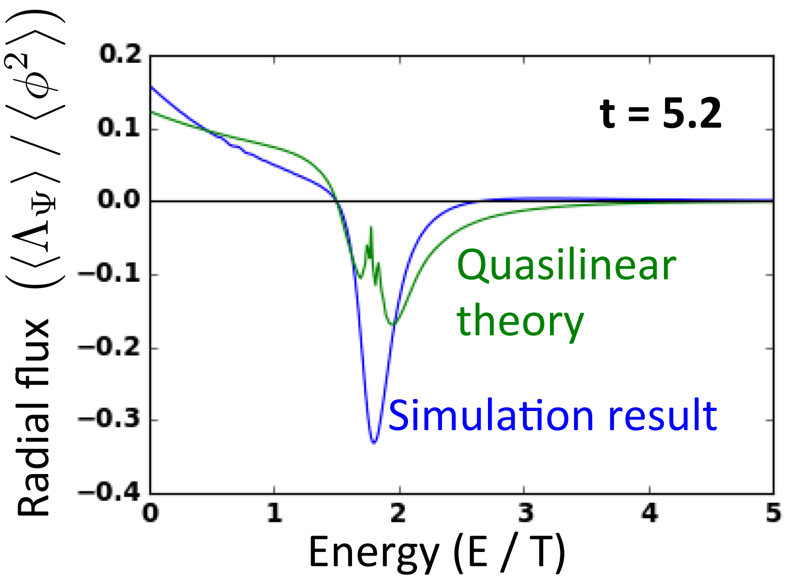
1. Self-organized from seeds born at the resonant energy.
2. Evolved toward higher energies.
3. Localized in radius, with $\Delta r \sim \rho_{ci}$
4. Localized in energy, with $\Delta E \ll T$
5. Lifetime >> drift period

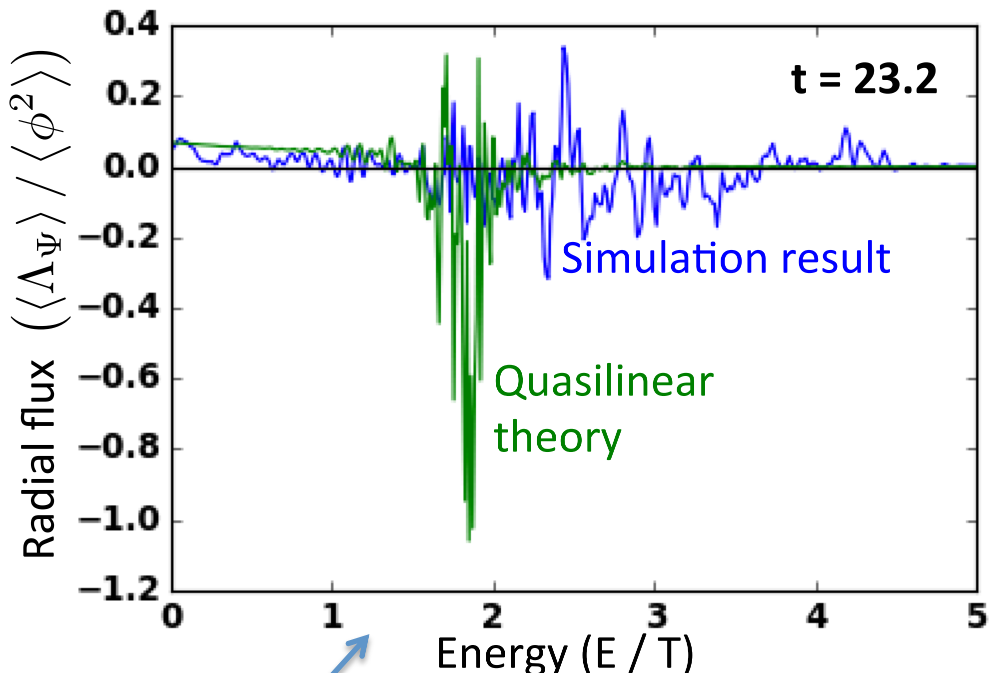


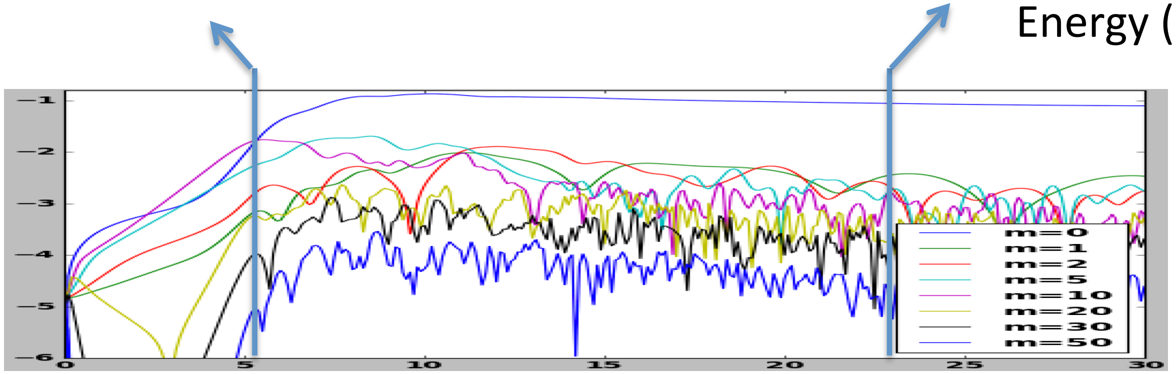
Turbulent case

Small-scale phase-space structures drive radial flux of phase-space density

$$\frac{\partial \langle f \rangle}{\partial t} + \frac{\partial \langle \Lambda_\psi \rangle}{\partial \psi} = 0 \qquad \langle \Lambda_\psi \rangle = \left\langle \dot{\psi}\, \delta f \right\rangle$$







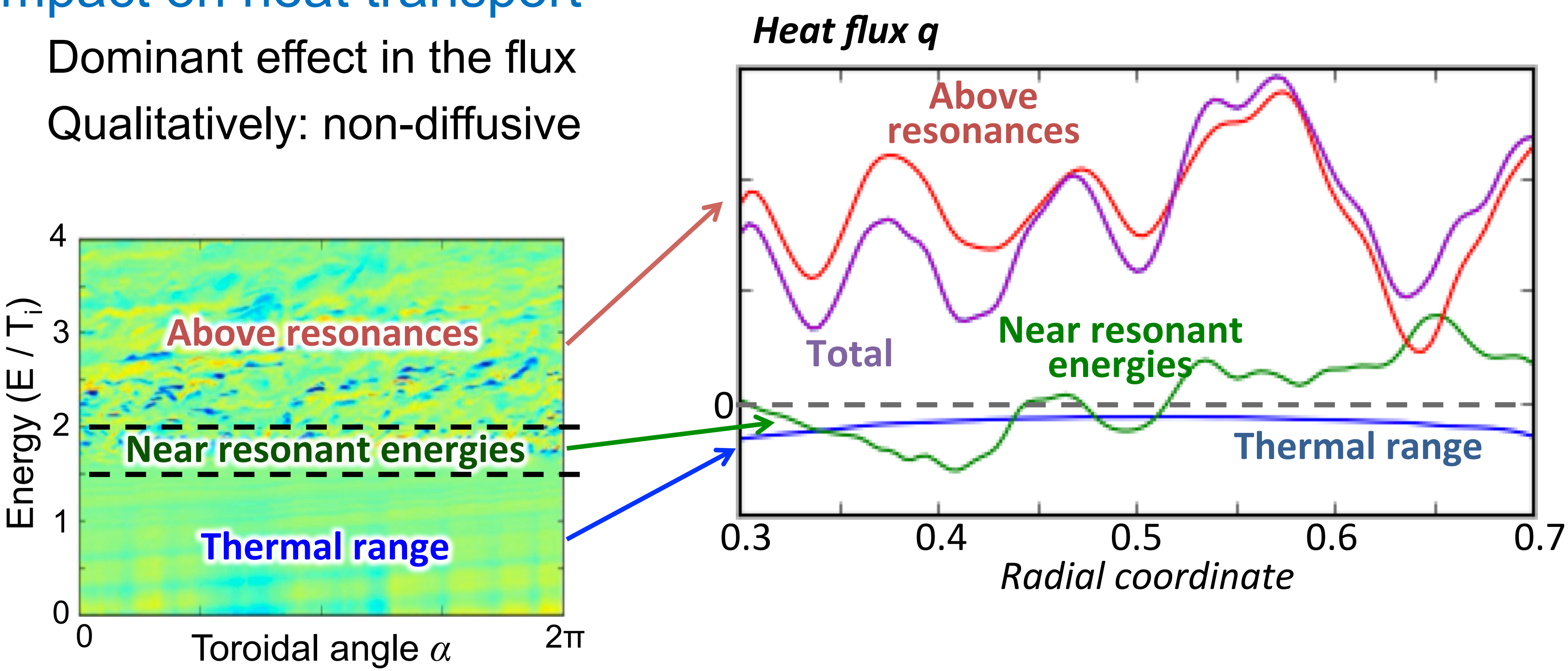
Small-scale phase-space structures drive particle and heat transport

Heat balance

$$\frac{\partial(nT)}{\partial t} + \frac{\partial q}{\partial \psi} = 0 \qquad q \equiv \frac{2}{\sqrt{\pi}} \int \langle \Lambda_\psi \rangle E^{3/2} dE.$$

Impact on heat transport

- Dominant effect in the flux
- Qualitatively: non-diffusive



Summary

TERESA code

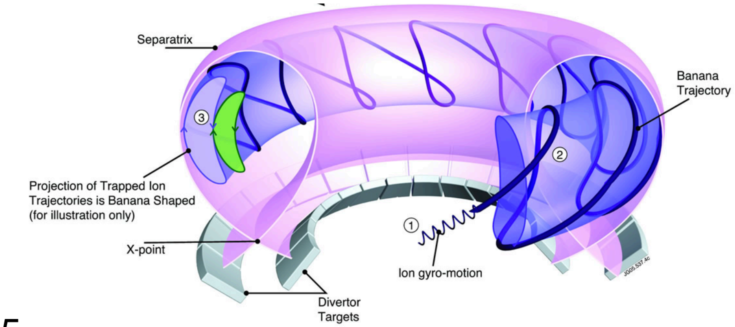
- Focuses on trapped (banana) particles
- Toy-model for low-frequency turbulence
- Retains essential kinetic ingredients

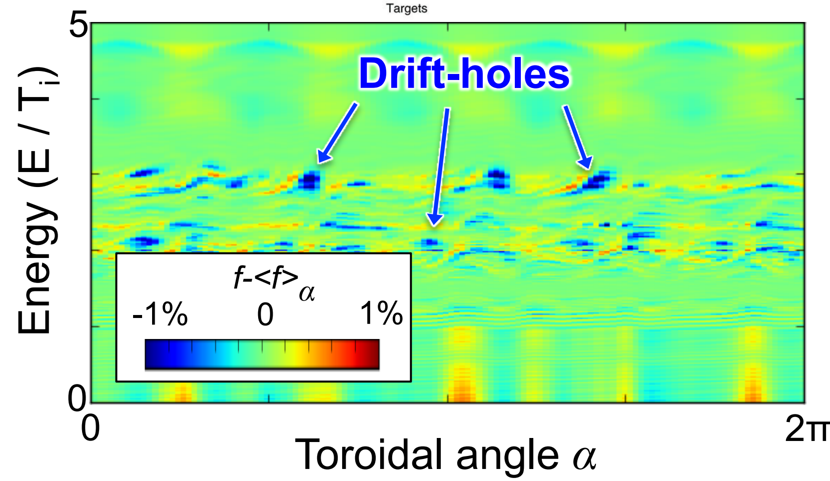
Convergence in N_E is delicate

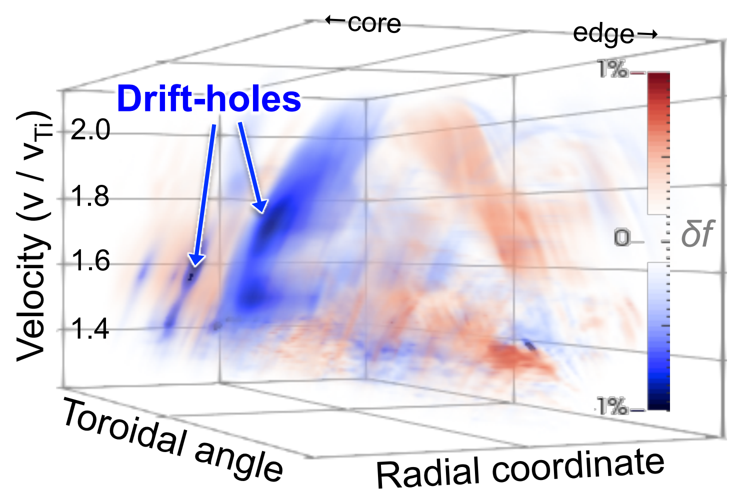
- Small-scale structures localized in energy self-organize near the resonant energy, and travel toward higher energies.
- They drive strong transport, which is missing from simulations with $N_E < 1000$

Perspectives

- Larger systems
- Stronger, stirred turbulence
- Effect of collisions?
- Signatures for experimental observation







Small-scale energy-space structures in GK

<u>Kinetics:</u> **6D Phase-space** (3D + 3V)

$\omega << \omega_{gyro}$

<u>Gyrokinetics:</u> **4D Phase-space + 1 parameter**

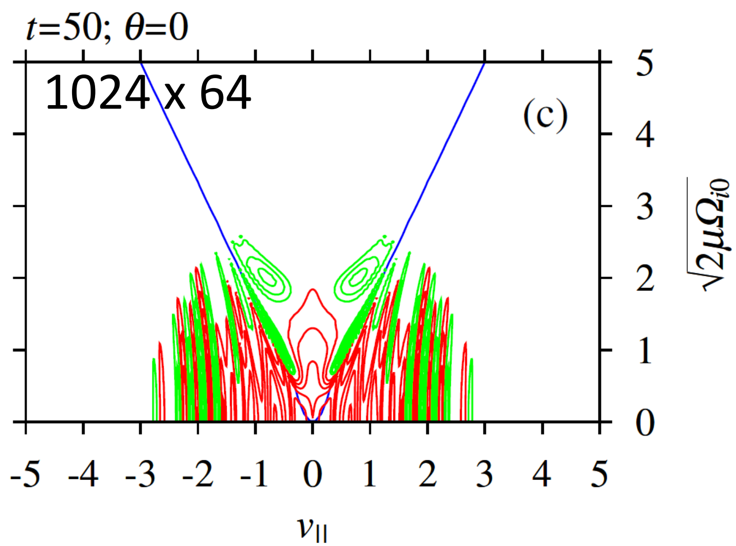
Existence of fine-scale structures in velocity-space

- Not an issue?

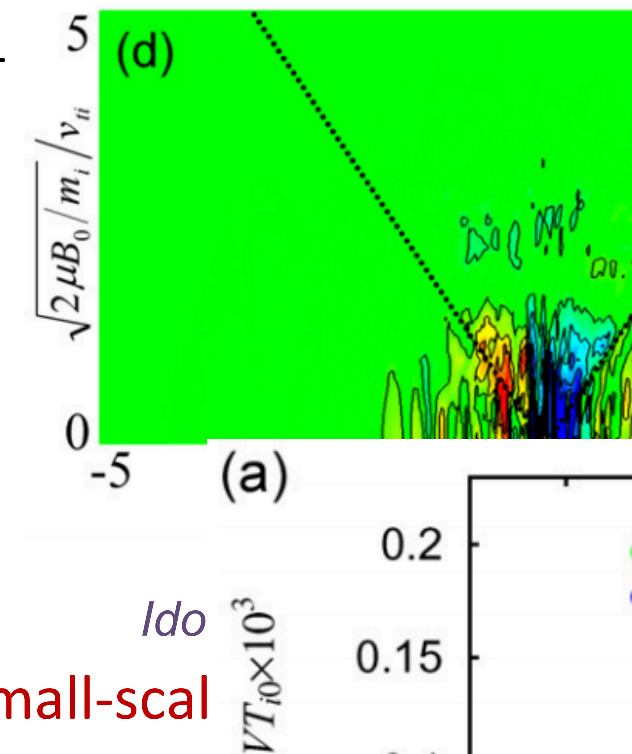
  Entropy and energy flux constant between 4 x 4

- Still an issue!

Watanabe '06



Ido



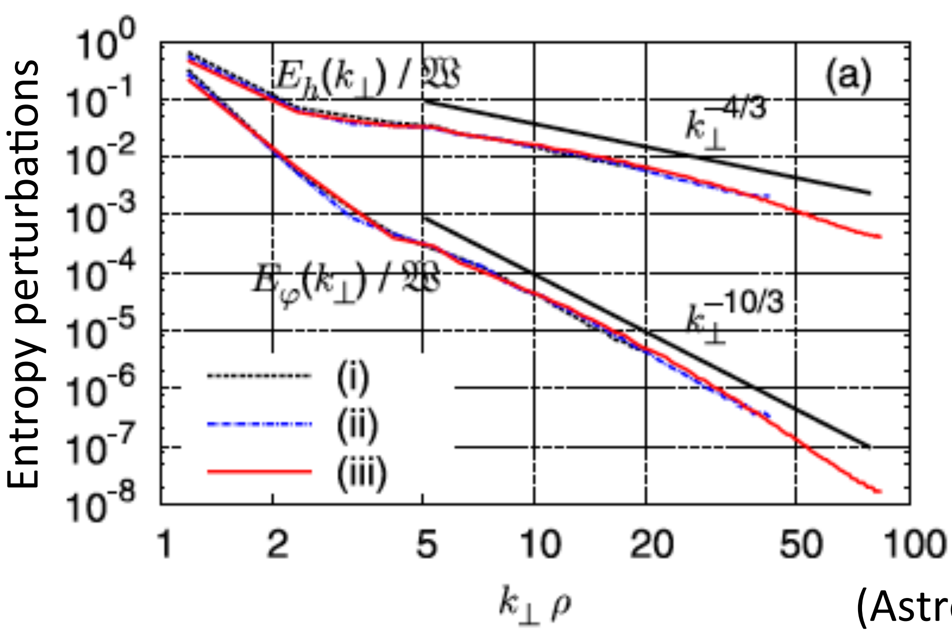
⇒ Existing GK simulations may miss effects of small-scal

Cascade in velocity-space

Turbulent cascade of entropy in phase-space

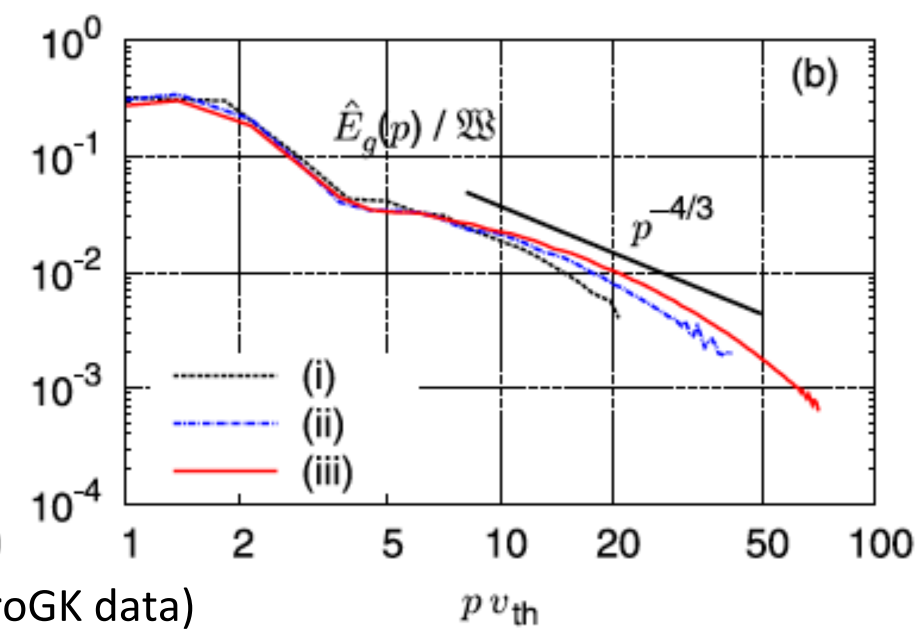
- Driven by nonlinear phase-mixing in strongly turbulent plasmas
- Generation of small-scales in real space and that in velocity space are intertwined.

Schekochihin '08
Tatsuno '09

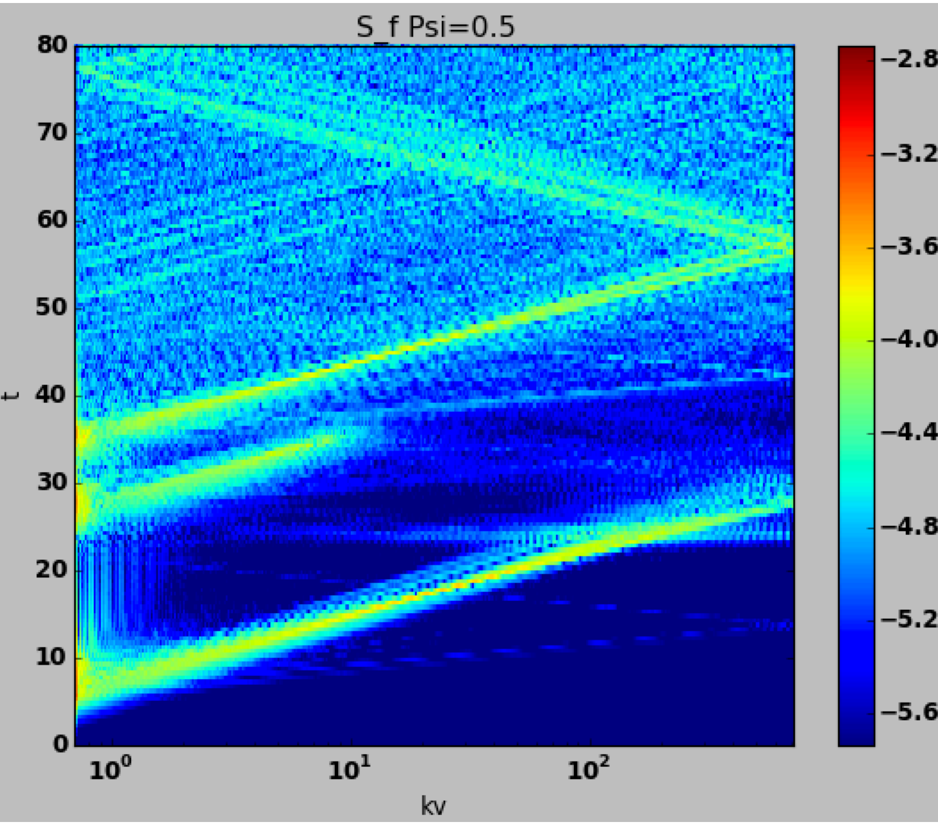


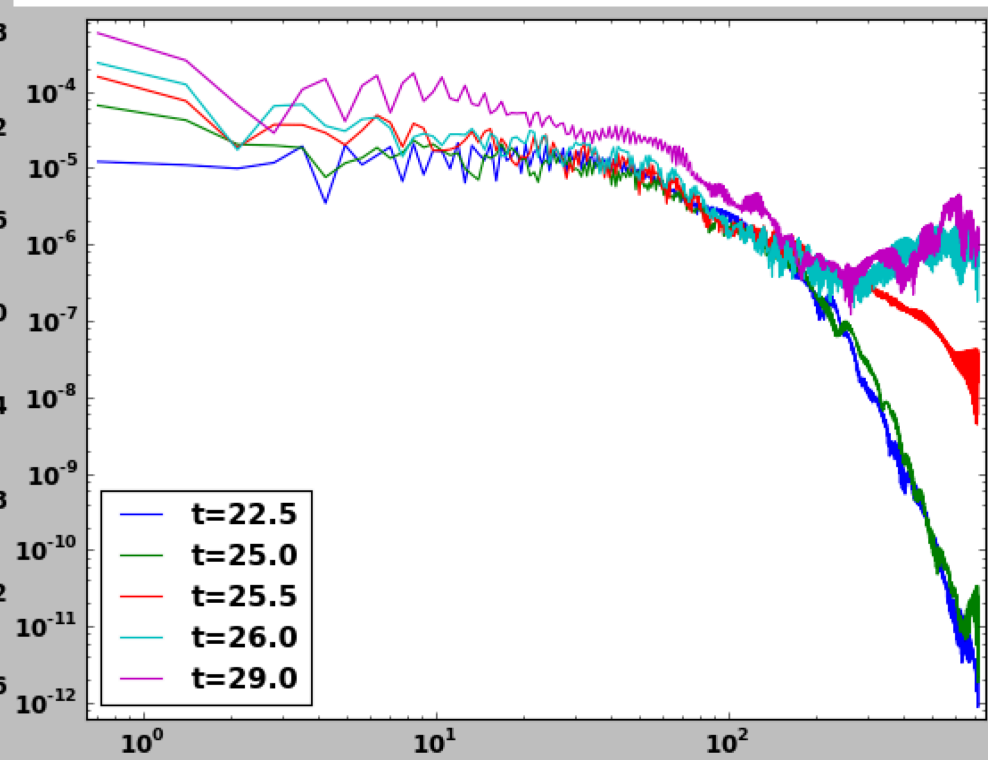
Spectrum in wave number

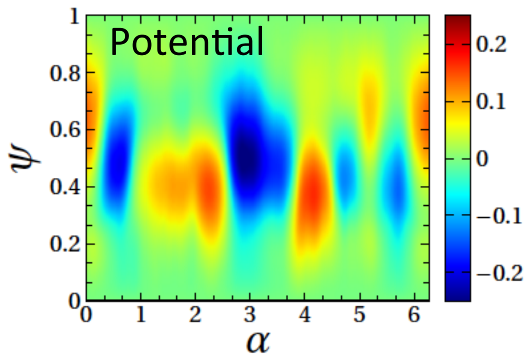
(AstroGK data)



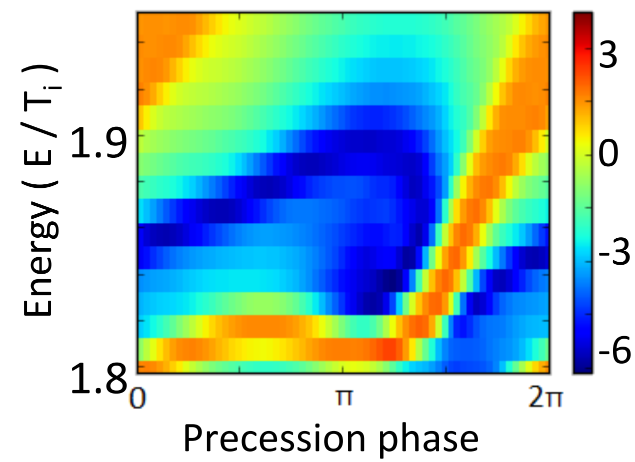
Spectrum in velocity-space
(Hankel spectrum)


S_f Psi=0.5
t
kv
80
70
60
50
40
30
20
10
0
10^0
10^1
10^2
−2.8
−3.2
−3.6
−4.0
−4.4
−4.8
−5.2
−5.6


t=22.5
t=25.0
t=25.5
t=26.0
t=29.0
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9
10^-10
10^-11
10^-12
10^0
10^1
10^2



Trapped ion distribution (perturbation in %)



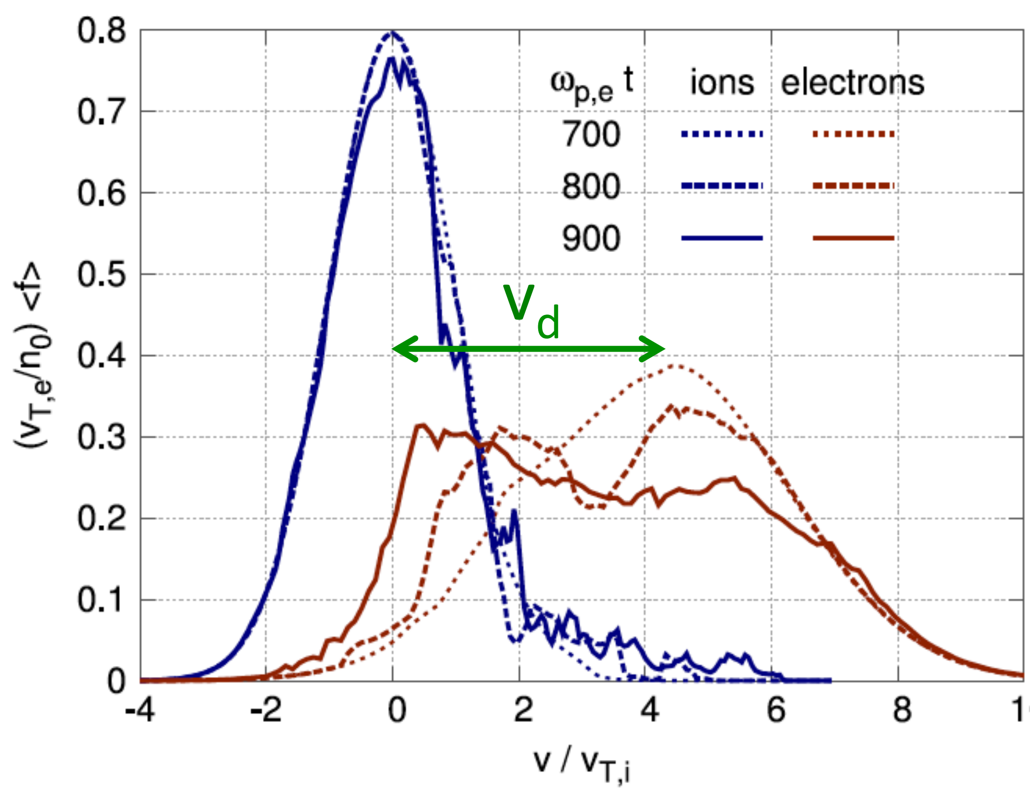
Context 1 : Current-driven ion-acoustic turbulence

1D model for ion-electron plasma with initial velocity drift

- 1D Vlasov for ions and electrons (collisionless)
- Poisson equation

$\Rightarrow$ Current-driven ion-acoustic turbulence

Tonks & Langmuir '29
Revans '33

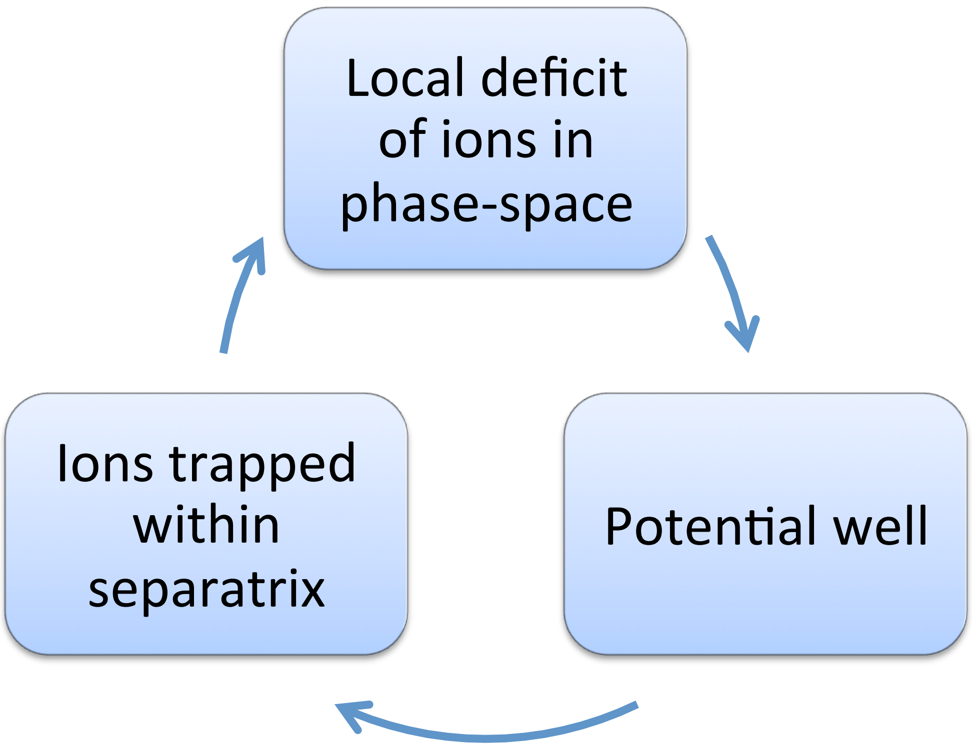


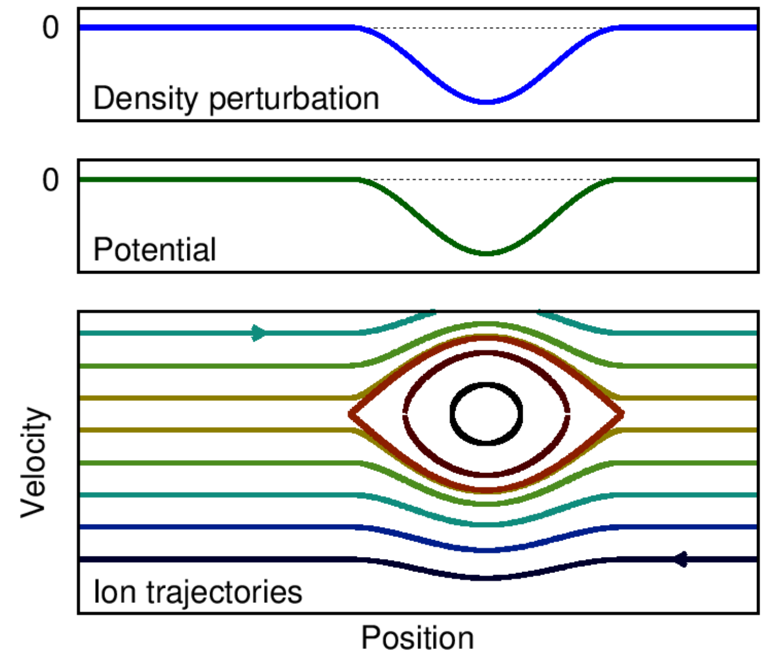
Velocity redistribution

- Anomalous resistivity
- Turbulent "heating"
- Turbulent transport in other contexts

Electrostatic trapping yields phase-space vortex

Self-sustaining structure





$\Rightarrow$ BGK mode

Bernstein & Green & Kruskal '57

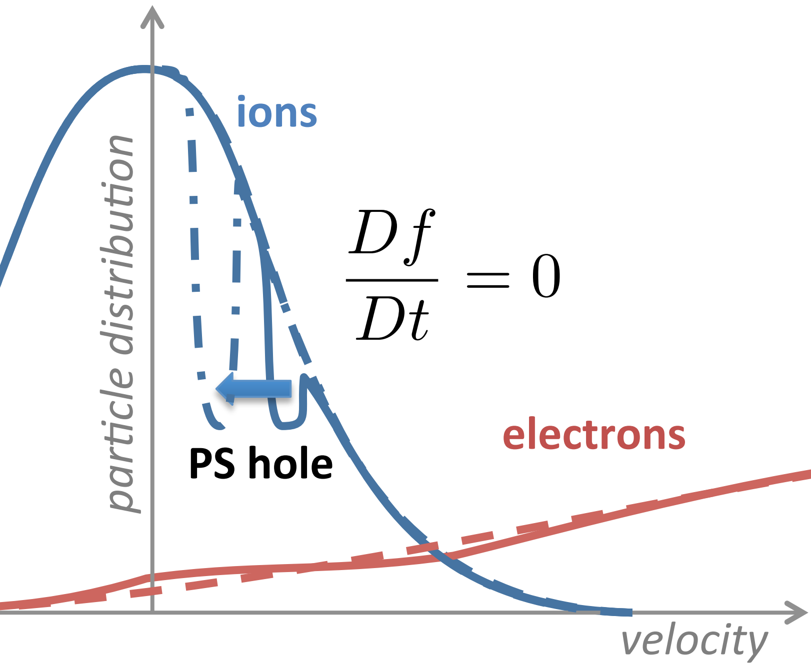
Nonlinear growth of a phase-space vortex

Growth mechanism

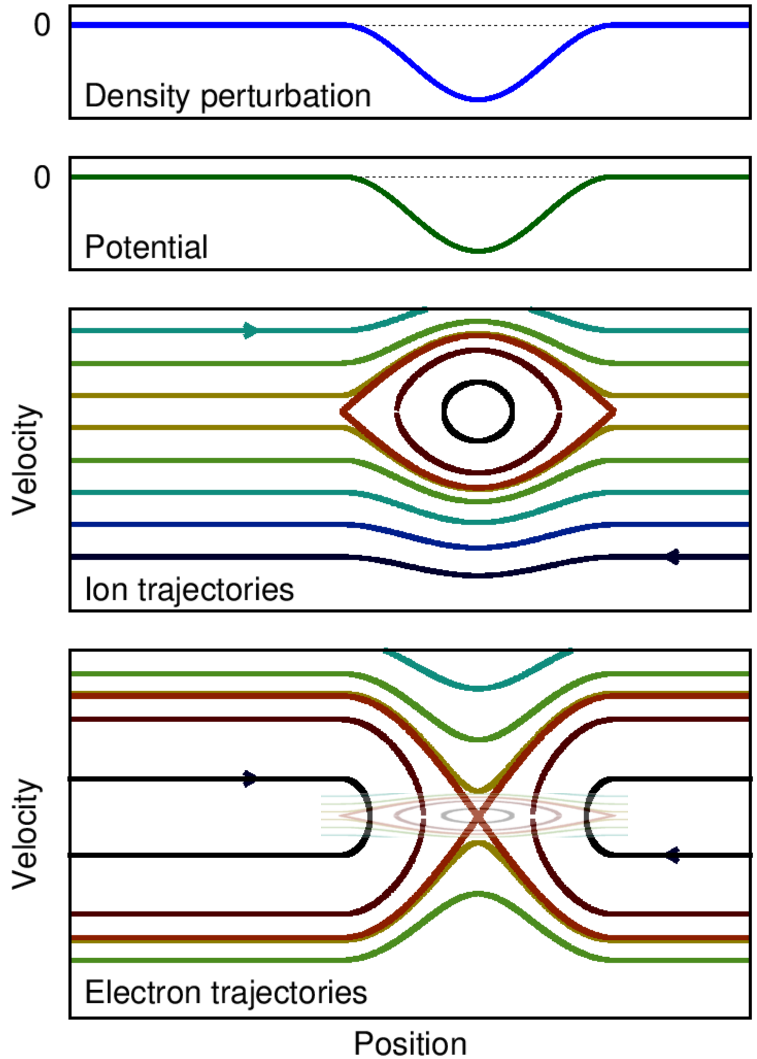
- F-P drag by scattering electrons
- Lagrangian conservation of *f* yields Eulerian growth of *δf*

$$\gamma \sim \sqrt{\varphi}$$

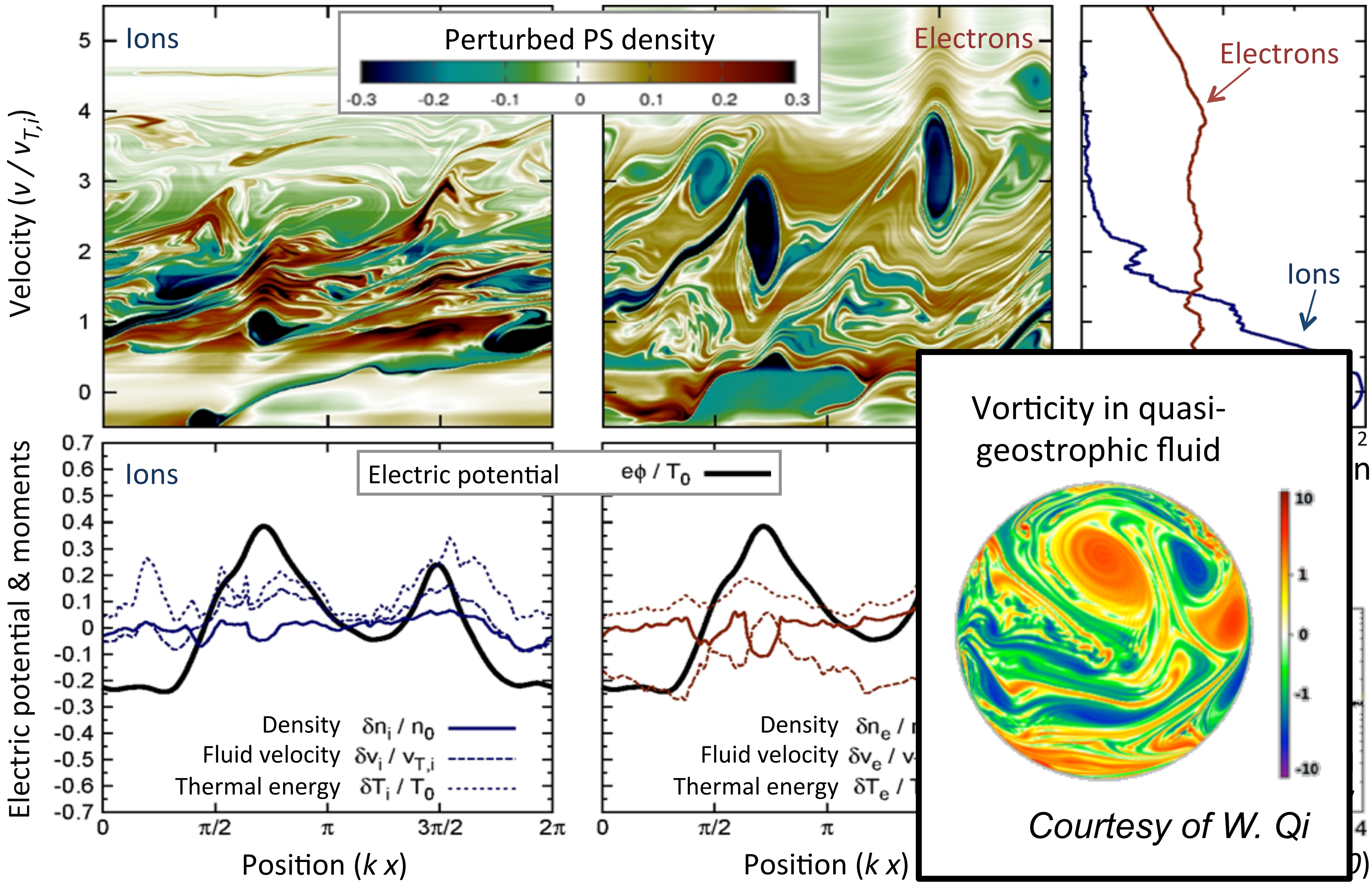
Dupree '83



$$\frac{Df}{Dt} = 0$$

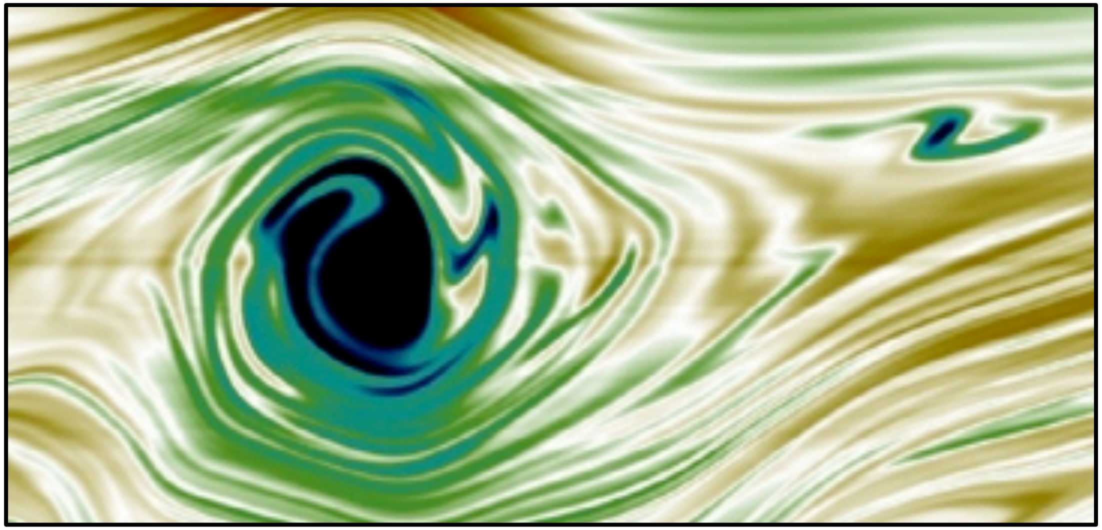


Phase-space dynamics

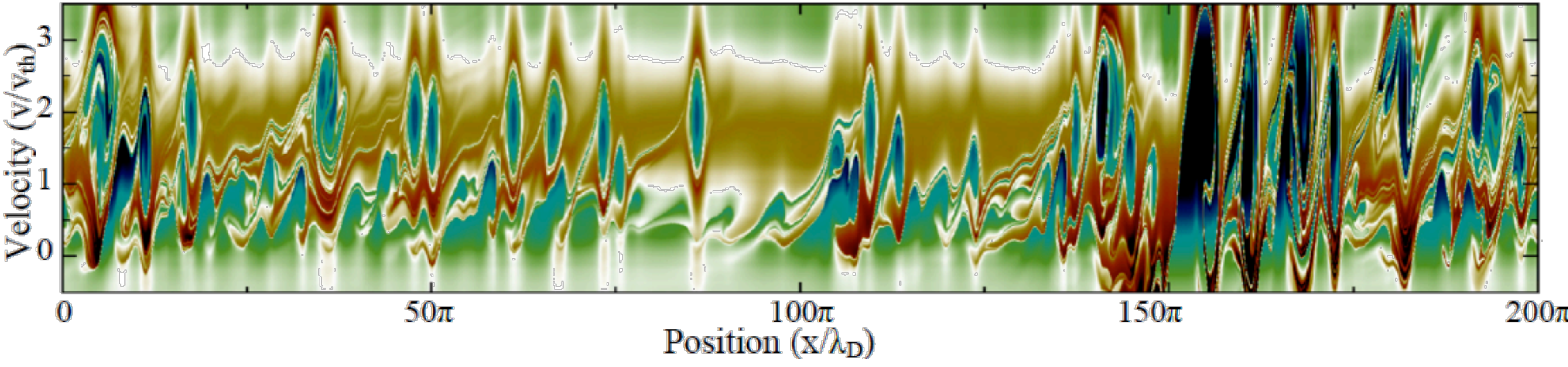


Phase-space turbulence?

Interaction of two structures is deterministic



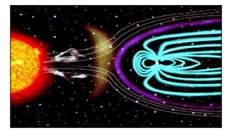
How about the interaction of coexisting structures of various sizes?

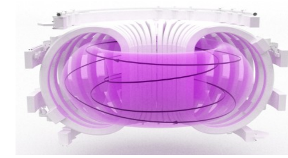


Motivation: essential role of phase-space structures

Vortices in phase-space are observed in experiments

EXPERIMENTS



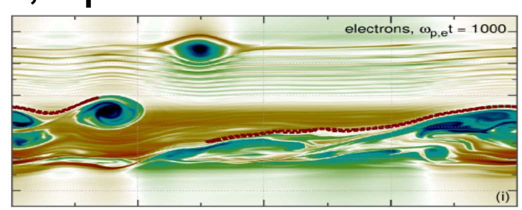


- Space plasmas — *Review: Eliasson & Shukla '06*
- Laboratory linear plasmas — *Saeki '79*
- Magnetic reconnection of toroidal plasmas — *Fox '08*
- Fusion plasmas — *Kusama '99 ; Berk, Breizman & Pekker '96*
- Laser plasmas — *Sarri '10*

Deep implications for instabilities, turbulence, transport, heating

THEORY

- Drive nonlinear instabilities — *Dupree '82*
- Modify the magnitude of saturation, spectrum of turbulence — *Terry '90*
- Qualitative effect on transport — *Biglari '88*
- Interact with large-scale flows — *Kosuga '11*



EXP.

- PS vortex-driven subcritical instability dominates the EM spectrum

Lesur '16 *Ido '17*