2017.10.30 – 2017.11.3
"Collisionless Boltzmann (Vlasov) equation and modeling of self-gravitating systems and plasmas"
CIRM, Luminy, Marseille, France

5D full-*f* gyrokinetic simulation for ion turbulence and transport barrier in tokamak plasmas

Contents

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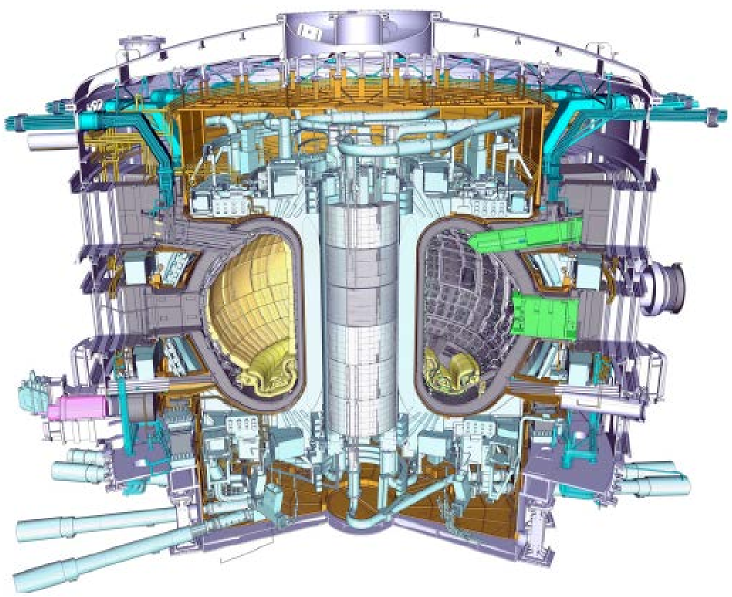
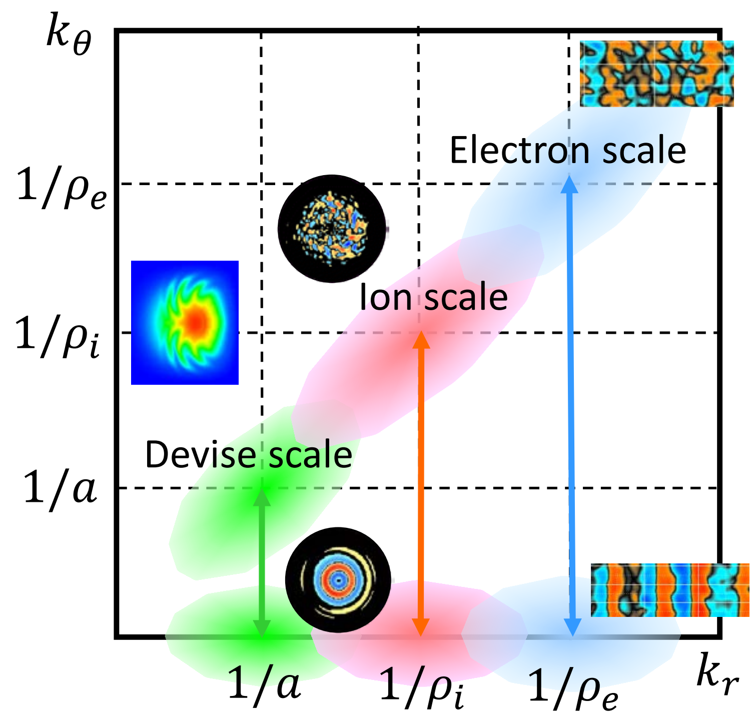
Multi-Physics/Multi-Scales in Fusion Plasmas

- ✓ Fusion burning plasma is a hierarchical complex system dominated by multi-physics and multi-scales.
- ✓ It is a grand challenge to elucidate such a system numerically.

Model of the Tokamak in construction [ITER]





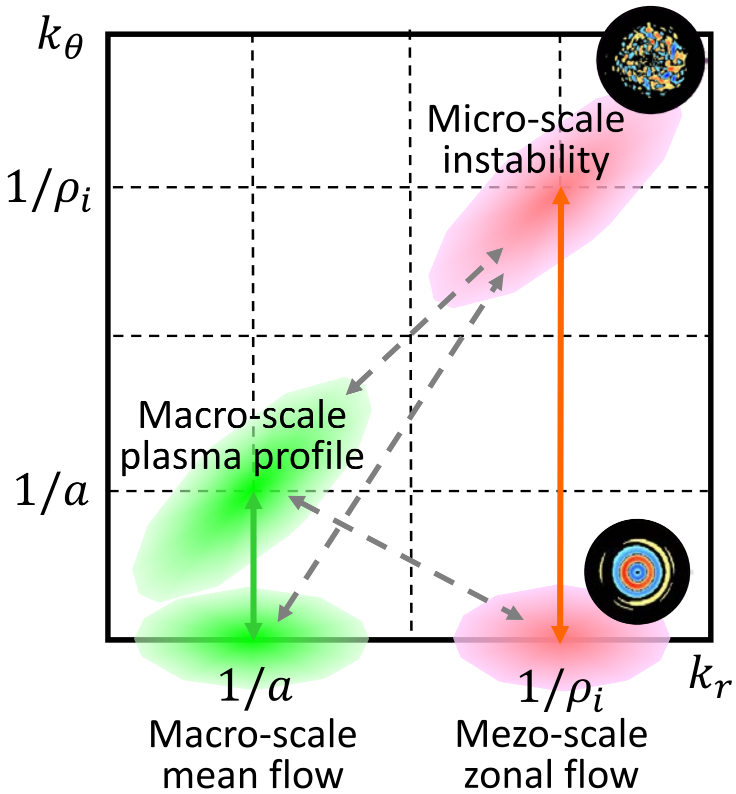
Three fundamental physics

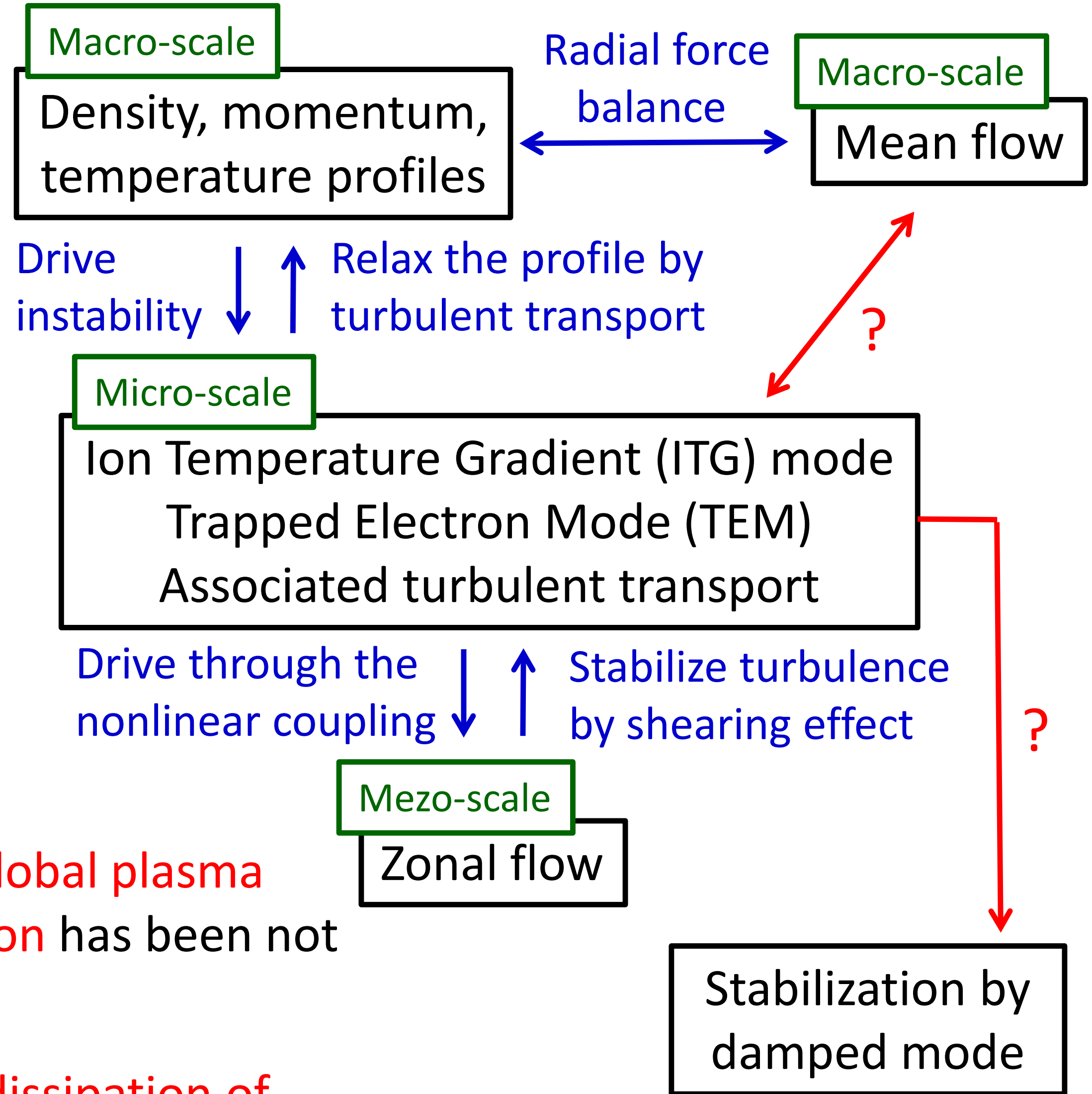
(1) Transport process(classical, neoclassical, turbulent)
(2) MHD dynamics with high energetic particles
(3) Plasma-Wall interaction

Three fundamental scales

(1) Device scale (Plasma profile, MHD)
(2) Ion scale
(3) Electron scale

Cross-Scale Coupling among Profile and Fluctuations





- ✓ Cross-scale coupling among global plasma profile and ion-scale fluctuation has been not understood yet.
- ✓ Role of stable modes for the dissipation of turbulence is also the other concern.

Outline of This Presentation

1. Introduction

2. 5D full-*f* gyrokinetic code *GKNET*
   - Concept of gyrokinetics
   - Full-*f* gyrokinetic model
   - Numerical algorithm of *GKNET*
   - Parallelization & Visualization of *GKNET*

3. 5D full-*f* gyrokinetic simulation for turbulence and transport barrier
   - Non-local ballooning theory
   - Internal transport barrier formation by momentum injection

4. Role of stable modes in gyrokinetic plasmas
   - Theoretical Landau damping
   - Eigenvalue analysis in discretized velocity space
   - Stability analysis in coupled system

5. Summary

2. 5D full-*f* gyrokinetic code *GKNET*

- Concept of gyrokinetics
- Full-*f* gyrokinetic model
- Numerical algorithm of *GKNET*
- Parallelization & Visualization of *GKNET*

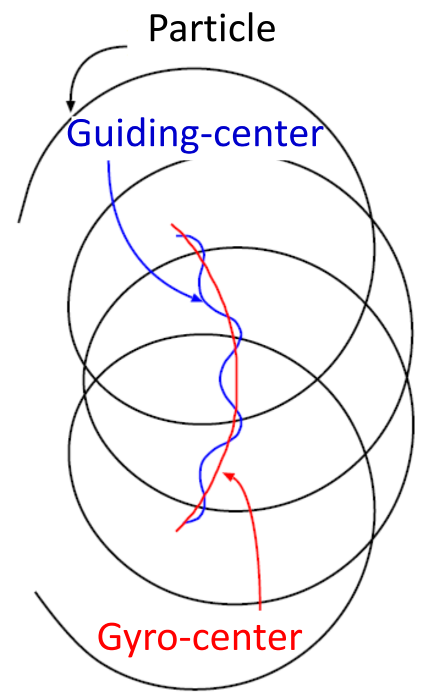
Concept of Gyrokinetics - 1

✓ Gyro-radius is the minimum scale of turbulence.

- ion gyro-radius～5[mm]

✓ Fast-scale gyro-motion can be assumed to be adiabatic.

- fast-scale gyro motion～1[GHz]
- slow-scale drift motion～100[kHz]



Lagrangian of single particle

↓

Single particle one-form ⇒ $$\frac{\partial f}{\partial t} + \frac{d\boldsymbol{x}}{dt}\cdot\frac{\partial f}{\partial \boldsymbol{x}} + \frac{d\boldsymbol{v}}{dt}\cdot\frac{\partial f}{\partial \boldsymbol{v}} = 0 \qquad f(\boldsymbol{x}, \boldsymbol{v})$$

↓ Coordinate transformation

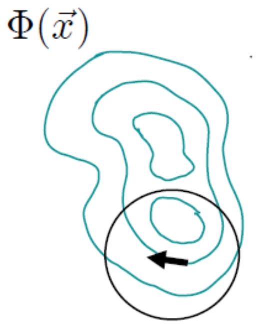
Guiding-center one-Form ⇒ $$\frac{\partial f}{\partial t} + \frac{d\mathbf{R}}{dt}\cdot\frac{\partial f}{\partial \mathbf{R}} + \frac{dv_{||}}{dt}\frac{\partial f}{\partial v_{||}} + \frac{d\theta}{dt}\frac{\partial f}{\partial \theta} + \frac{d\mu}{dt}\frac{\partial f}{\partial \mu} = 0 \qquad f(\mathbf{R}, v_{||}, \theta, \mu)$$

↓ Perturbative Lie transformation

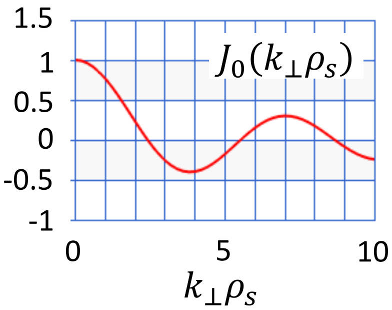
Gyro-center one-Form ⇒ $$\frac{\partial \bar{f}}{\partial t} + \frac{d\bar{\mathbf{R}}}{dt}\cdot\frac{\partial \bar{f}}{\partial \bar{\mathbf{R}}} + \frac{d\bar{v}_{||}}{dt}\frac{\partial \bar{f}}{\partial \bar{v}_{||}} + \frac{d\bar{\theta}}{dt}\underbrace{\frac{\partial \bar{f}}{\partial \bar{\theta}}}_{=0} + \underbrace{\frac{d\bar{\mu}}{dt}}_{=0}\frac{\partial \bar{f}}{\partial \bar{\mu}} = 0 \qquad \bar{f}(\mathbf{R}, v_{||}, \mu)$$

Concept of Gyrokinetics - 2

1. Kinetic effect can be taken into account.
   - MHD/Fluid model: $\omega \gg kv_{||}$ $\longleftrightarrow$ Landau resonance: $\omega \sim kv_{||}$
   - Closed problem of fluid moment



2. Precise treatment of gyro-radius size scale turbulence by FLR effect
   - MHD/Fluid model: $k_\perp \rho_s \ll 1$ $\longleftrightarrow$ Turbulence: $k_\perp \rho_s \sim 1$
   - FLR effect: $< \phi(t, \boldsymbol{R}) >_\alpha = \sum_{\boldsymbol{k}_\perp} \phi(t, \boldsymbol{R}) J_0(k_\perp \rho_s) exp\,(i\boldsymbol{k}_\perp \cdot \boldsymbol{R})$



3. Cost reduction: time step and grid size relaxed from

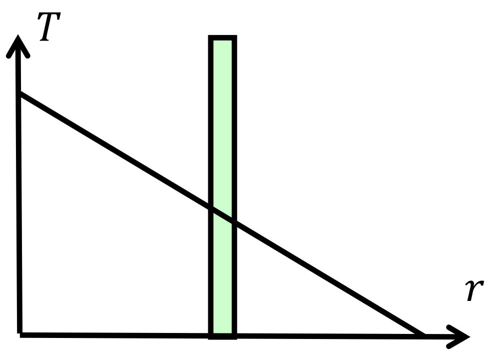
$$6D: \begin{cases} \omega_{ps}\Delta t < 1 \\ \Delta x \ll \lambda_{Ds} \end{cases} \longrightarrow 5D: \begin{cases} \omega_{cs}\Delta t < 1 \\ \Delta x \ll \rho_s \end{cases}$$

Local/Global Gyrokinetics

δf local approach

$$\cancel{\partial_t f_{eq}} - \cancel{[H, f_{eq}]} = \cancel{C(f_{eq})} + \cancel{S}$$

$$\partial_t \delta f - \underbrace{[H, \delta f]}_{\text{linear}} - \underbrace{[\delta H, f_{eq}]}_{\text{driving}} - \underbrace{[\delta H, \delta f]}_{\text{nonlinear}} = C(\delta f)$$



Fixed *Gradient*

$R/L_T \neq 0$

$T = const$

☺ Very powerful tool to estimate turbulent transport process

☺ Computationally efficient
-> multi-species, EM turbulence

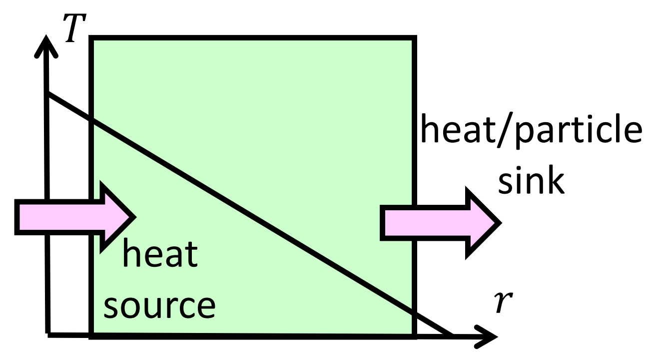
GENE(GER), GYRO(US), GKV(JPN),…

full-f global approach

$$\partial_t f_{eq} - [H, f_{eq}] = C(f_{eq}) + S$$

self-consistently determined Mean E_r

$$\partial_t \delta f - \underbrace{[H, \delta f]}_{\text{linear}} - \underbrace{[\delta H, f_{eq}]}_{\text{driving}} - \underbrace{[\delta H, \delta f]}_{\text{nonlinear}} = C(\delta f)$$



Fixed *Flux*

$R/L_T \neq 0$

$T \neq const$

☺ Global profile shear effect can be taken into account (e.g. ω_D shear)

☺ Profile formation coupled with mean E_r
-> transport barrier formation

GYSELA(FRA), GT5D(JPN), GKNET(JPN), …

Full-*f* Gyrokinetic Code *GKNET* -Vlasov Solver-

GK Vlasov equation for ion

$$\frac{\partial f}{\partial t} + \frac{d\mathbf{R}}{dt} \cdot \frac{\partial f}{\partial \mathbf{R}} + \frac{dv_\parallel}{dt}\frac{\partial f}{\partial v_\parallel} = C_{coll}$$

$$\frac{d\mathbf{R}}{dt} \equiv \{\mathbf{R}, H\} = v_\parallel \mathbf{b}(\mathbf{R}) + \frac{c}{eB_\parallel^*(\mathbf{R}, v_\parallel)}\mathbf{b}(\mathbf{R}) \times \left[e\nabla\langle\phi(\mathbf{R})\rangle_\alpha + m_i v_\parallel^2 \mathbf{b}(\mathbf{R}) \cdot \nabla\mathbf{b}(\mathbf{R}) + \mu\nabla B(\mathbf{R})\right]$$

$$\frac{dv_\parallel}{dt} \equiv \{v_\parallel, H\} = -\frac{\mathbf{B}_\parallel^*(\mathbf{R}, v_\parallel)}{m_i B_\parallel^*(\mathbf{R}, v_\parallel)} \cdot \left[e\nabla\langle\phi(\mathbf{R})\rangle_\alpha + \mu\nabla B(\mathbf{R})\right]$$

Main concern for full-*f* gyrokinetic Vlasov solver

✓ Spectral method is difficult to apply due to the non-periodicity of radial direction.

✓ Since full-*f* simulation treats long-time scale simulation up to confinement time, numerical stability is quite important issue.

✓ Explicit symplectic integration is not directly applied because the gyrokinetic Hamiltonian $H = \frac{m_i v_\parallel^2}{2} + \mu B(\boldsymbol{R}) + e\langle\phi(\boldsymbol{R})\rangle_\alpha$ cannot be separated to the components to apply symplectic integration unlike original Vlasov simulation with $H = \frac{m_i v^2}{2} + e\phi(\boldsymbol{R})$.

Full-*f* Gyrokinetic Code *GKNET* -Vlasov Solver-

GK Vlasov equation for ion

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$$\frac{d\mathbf{R}}{dt} \equiv \{\mathbf{R}, H\} = v_\parallel \mathbf{b}(\mathbf{R}) + \frac{c}{eB_\parallel^*(\mathbf{R}, v_\parallel)}\mathbf{b}(\mathbf{R}) \times \left[e\nabla\langle\phi(\mathbf{R})\rangle_\alpha + m_i v_\parallel^2 \mathbf{b}(\mathbf{R})\cdot\nabla\mathbf{b}(\mathbf{R}) + \mu\nabla B(\mathbf{R})\right]$$

$$\frac{dv_\parallel}{dt} \equiv \{v_\parallel, H\} = -\frac{\mathbf{B}_\parallel^*(\mathbf{R}, v_\parallel)}{m_i B_\parallel^*(\mathbf{R}, v_\parallel)}\cdot\left[e\nabla\langle\phi(\mathbf{R})\rangle_\alpha + \mu\nabla B(\mathbf{R})\right]$$

Vlasov solver: 4th-order Morinishi scheme [Idomura, JCP-2007]

- ✓ Non-dissipative FDM, which conserves L1 and L2 norm
- ✓ Easy to extend to higher-order scheme

Ex. 2D case

$$\frac{\partial f}{\partial t} = -v_x\frac{\partial f}{\partial x} - v_y\frac{\partial f}{\partial x} = -\frac{1}{2}\left[v_x\frac{\partial f}{\partial x} + \frac{\partial (v_x f)}{\partial x}\right] - \frac{1}{2}\left[v_y\frac{\partial f}{\partial y} + \frac{\partial (v_y f)}{\partial y}\right] \quad \left(\because \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0\right)$$

⬇ Discretize each derivative by using 4th-order central FDM

$$\left(\frac{\partial f}{\partial t}\right)^n_{i,j} = -\frac{1}{2}\left\{\left[v^n_{x,i,j}\left(\frac{4}{3}\frac{f^n_{i+1,j} - f^n_{i-1,j}}{2\Delta x} - \frac{1}{3}\frac{f^n_{i+2,j} - f^n_{i-2,j}}{4\Delta x}\right)\right] + \left[\frac{4}{3}\frac{v^n_{x,i+1,j}f^n_{i+1,j} - v^n_{x,i-1,j}f^n_{i-1,j}}{2\Delta x} - \frac{1}{3}\frac{v^n_{x,i+2,j}f^n_{i+2,j} - v^n_{x,i-2,j}f^n_{i-2,j}}{4\Delta x}\right]\right\}$$

$$-\frac{1}{2}\left\{\left[v^n_{x,i,j}\left(\frac{4}{3}\frac{f^n_{i,j+1} - f^n_{i,j-1}}{2\Delta y} - \frac{1}{3}\frac{f^n_{i,j+2} - f^n_{i,j-2}}{4\Delta y}\right)\right] + \left[\frac{4}{3}\frac{v^n_{x,i,j+1}f^n_{i,j+1} - v^n_{x,i,j-1}f^n_{i,j-1}}{2\Delta y} - \frac{1}{3}\frac{v^n_{x,i,j+2}f^n_{i,j+2} - v^n_{x,i,j-2}f^n_{i,j-2}}{4\Delta y}\right]\right\}$$

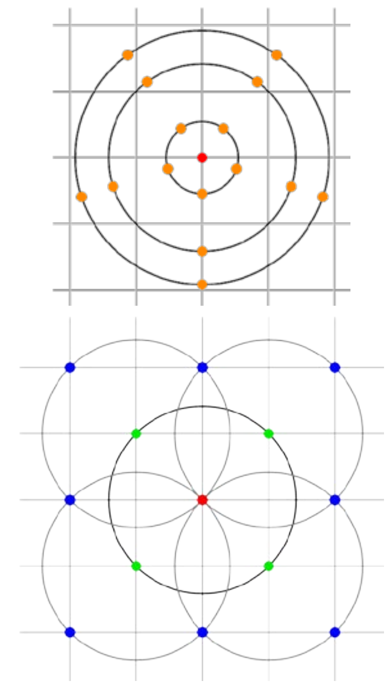
Full-*f* Gyrokinetic Code *GKNET* -Field Solver-

GK quasi-neutrality condition

$$\phi - \ll \phi \gg_\alpha + \frac{1}{T_{e0}(r)}\left(\phi - <\phi>_f\right) = \frac{1}{n_{i0}(r)} \iint <\delta f>_\alpha B_\parallel^* dv_\parallel d\mu$$

<u>*Field solver: Real space field solver*</u> [Kevin, PFR-2015]

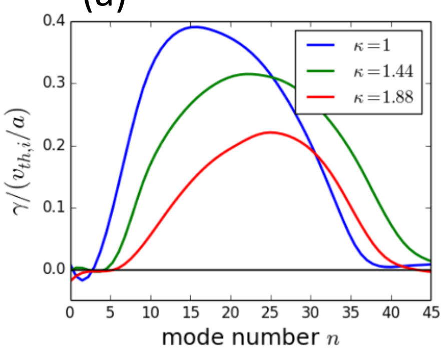
Step 1: Single/double averaging→9th b-spline interpolation+ 18 points averaging
Step 2: Weighted integration for μ
Step 3: Calculation of $<\phi>_\alpha$→ Projection to $R=0$ or diagonalization
Step 4: Calculation of ϕ→Fixed point method [Kevin, CPC-2017]



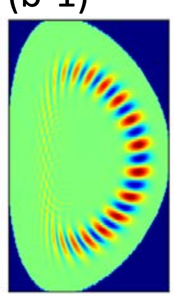
Schematic picture of single/double averaging

✓ Full-order FLR effect (without Tayler/Pade approximation)
✓ Field equation is solved in real space (not k-space)

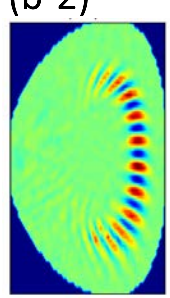
(a)



(b-1)



(b-2)



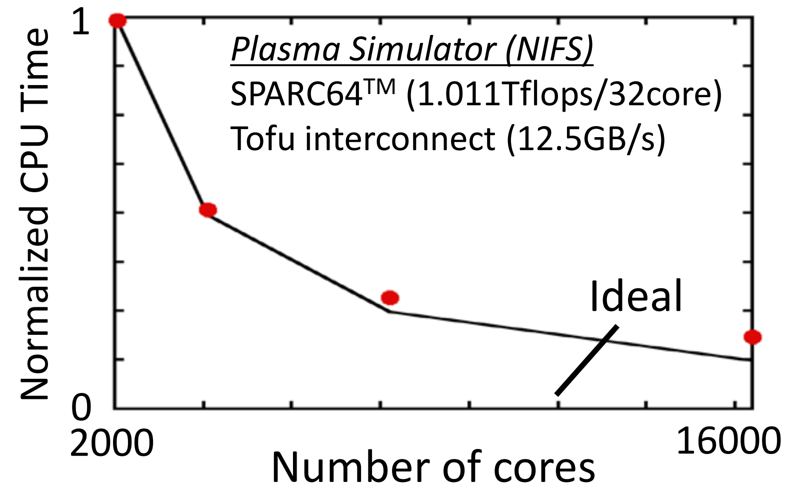
(a) Stabilization effect of elongation on ITG mode and (b) Poloidal mode structure of ITG mode with (1) positive and (2) negative triangularity

Full-*f* Gyrokinetic Code *GKNET* -Parallelization-

Parallelization: MPI+OpenMP
Parallelization rate: 99.9996%

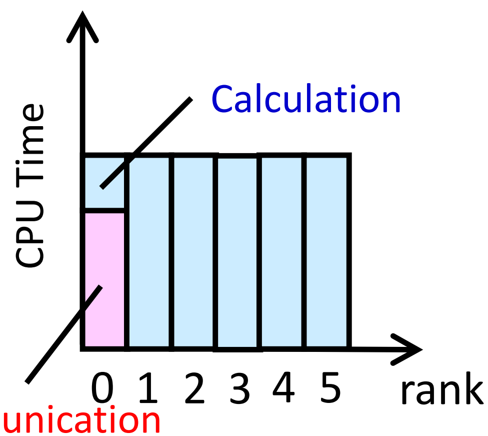
$(N_x, N_y, N_z, N_{v_\parallel}, N_\mu) = (128,128,128,64,16)$
($a/\rho_{ti} = 150$ size plasma)

- Good strong scaling up to 20,000 cores (see right figure)

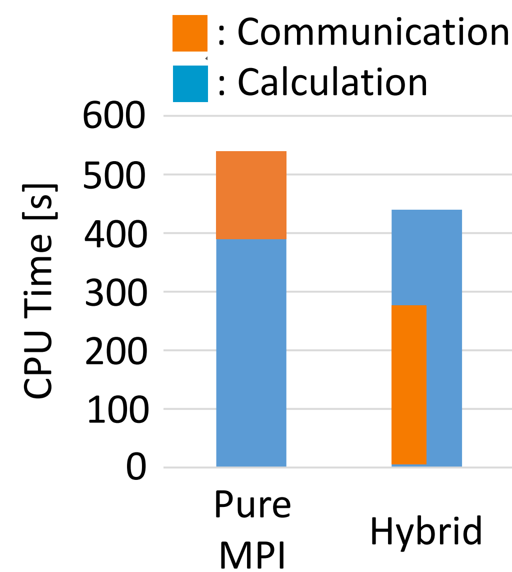


✓ By using communication and computation hiding optimization technique based on hybrid parallelization, we can reduce 21% CPU cost. [Maeyama, PFR-2015]

✓ Memory throughput is the bottle neck in Vlasov solver.

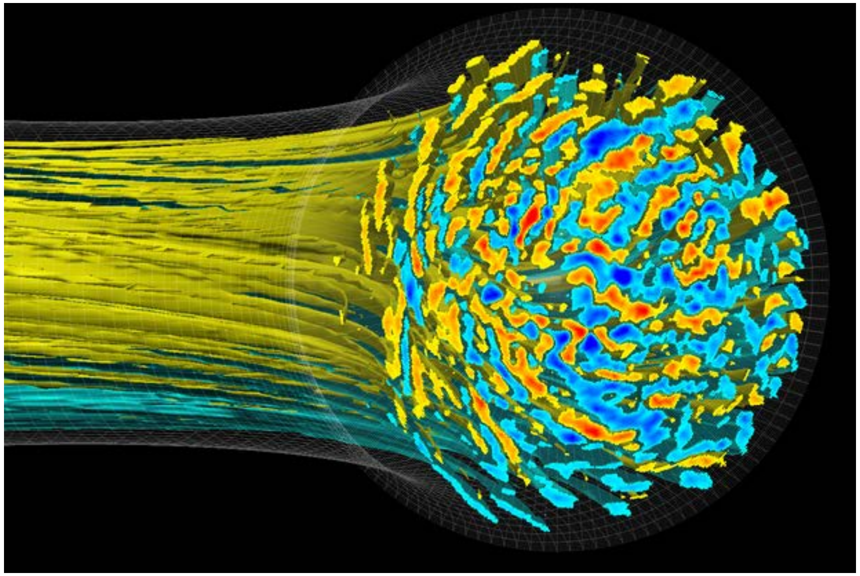


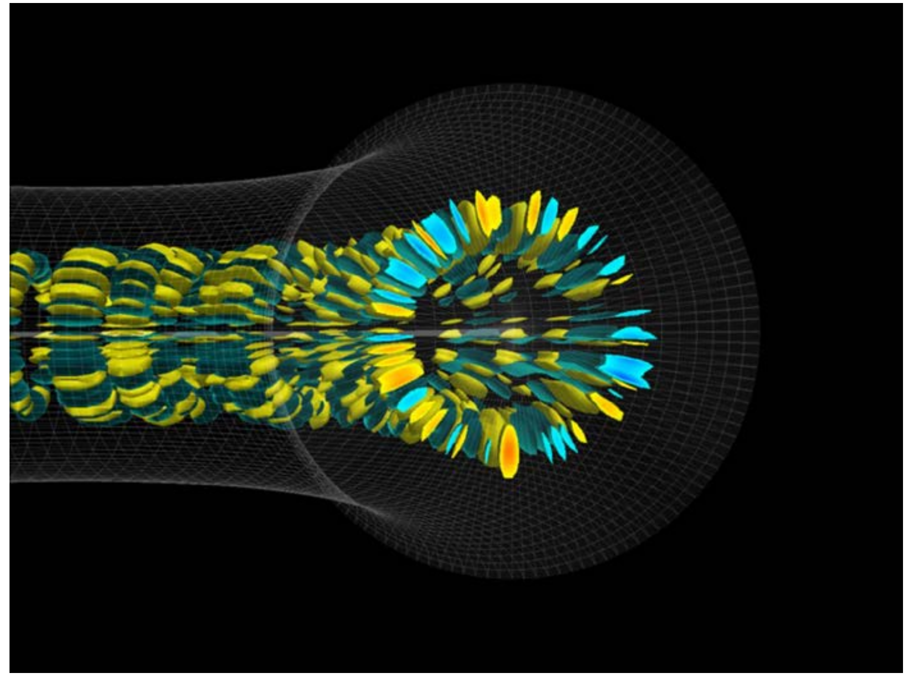

```
!$OMP MASTER
   communication part
!$OMP END MASTER
!$OMP DO SCHEDULE (dynamic)
 DO i = 1, n
   calculation part
 END DO
!$OMP END DO
```

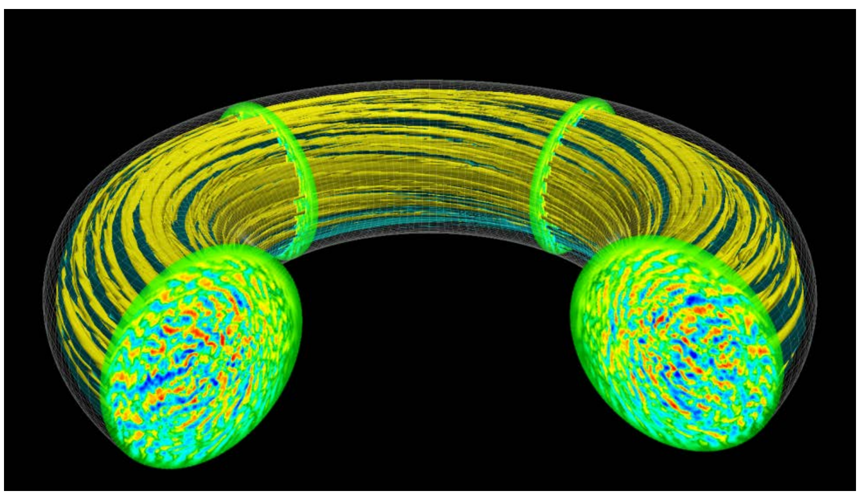


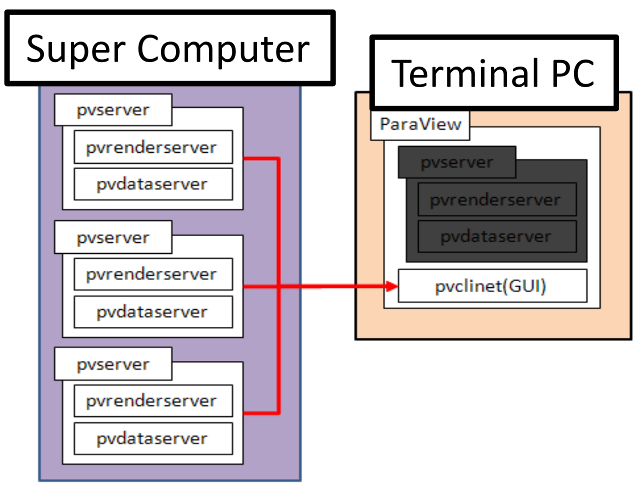
Full-*f* Gyrokinetic Code *GKNET* -Visualization-

Electrostatic potential in flux-driven turbulence simulation



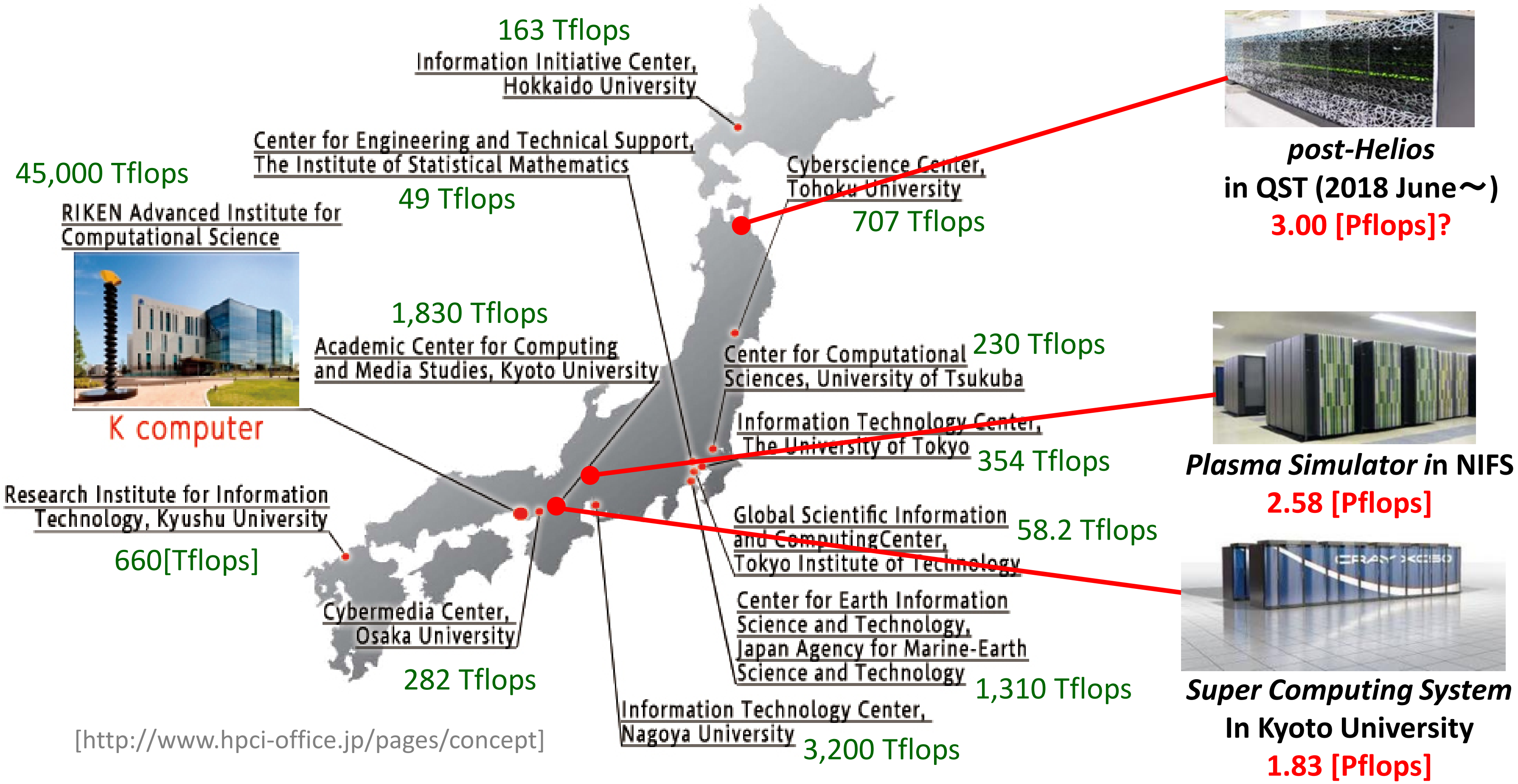






Appendix. HPCI Program in Japan

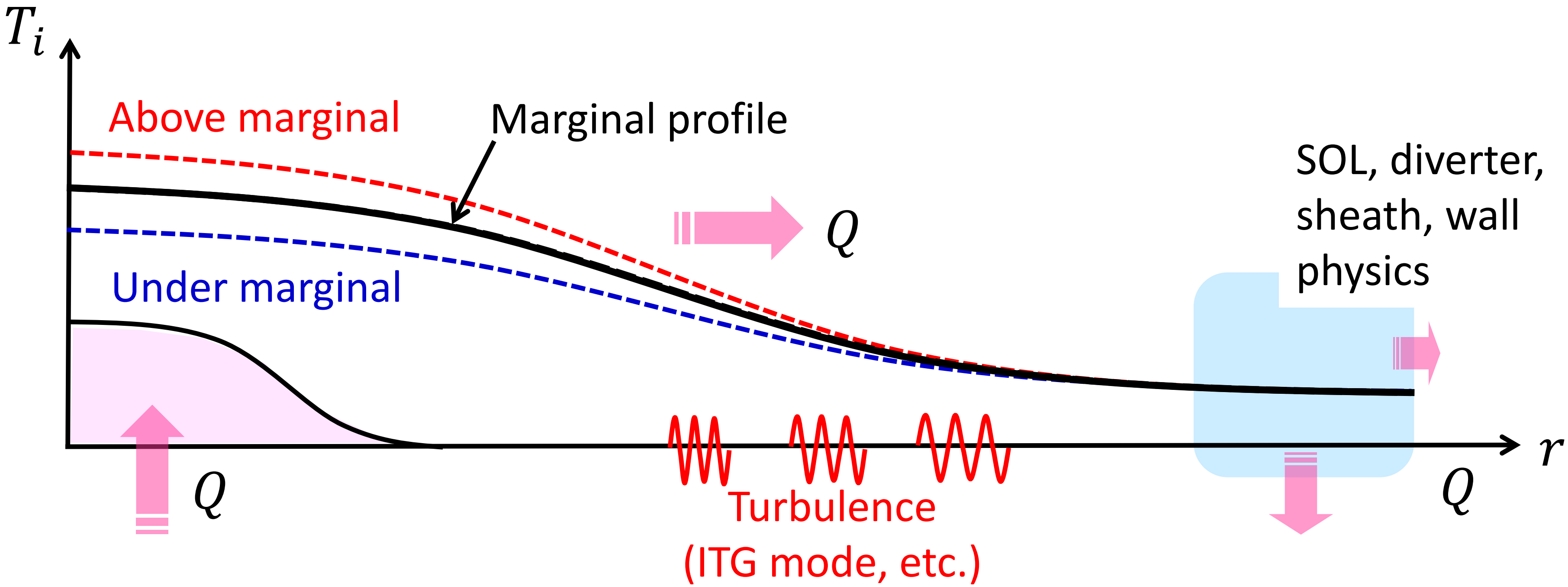
- ✓ High Performance Computing Infrastructure (HPCI) project has been constructed by connecting flagship K computer and other major supercomputers in Japan via high speed networks.
- ✓ Our group utilize Kyoto Univeristy super computer (1.83 [Pflops] under this framework), Plasma Simulator (2.58 [Pflops]) and also post-Helios (3 [Pflops]?) from next June.



3. 5D full-*f* gyrokinetic simulation for turbulence and transport barrier

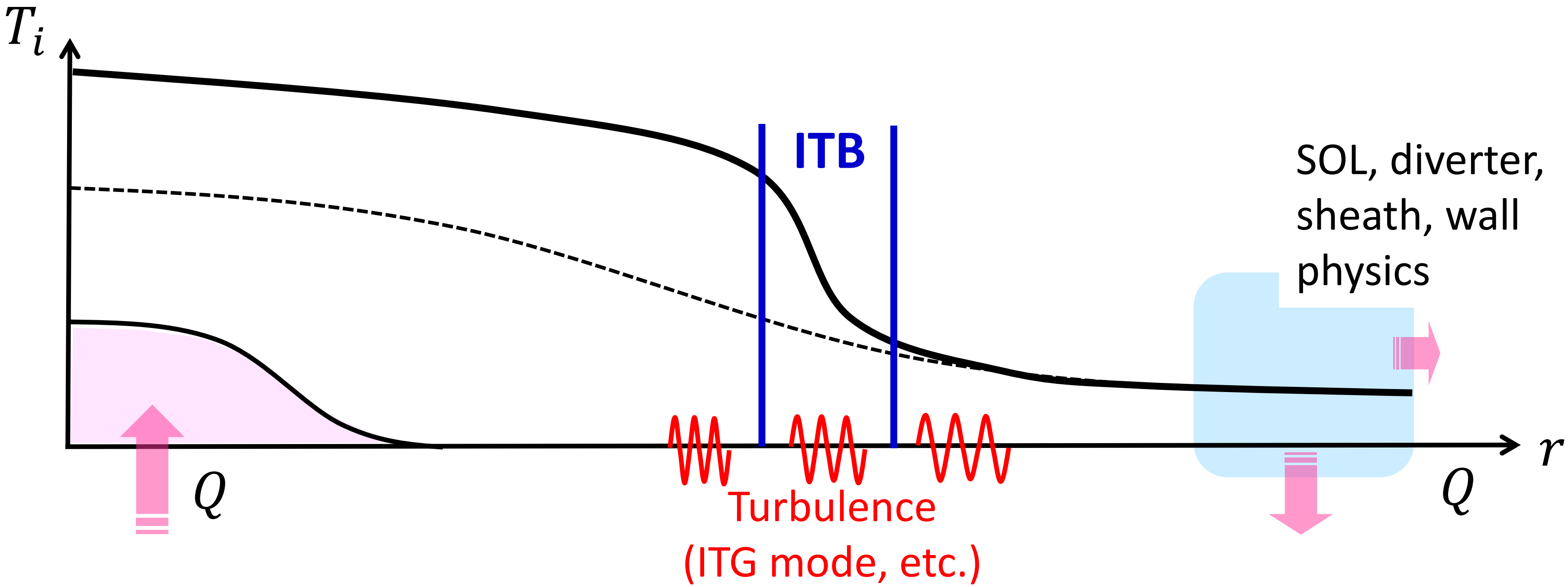
- Background & Motivation
- Non-local ballooning theory
- Internal transport barrier formation by momentum injection

Background: Profile Stiffness in Toroidal Plasmas



- ✓ Temperature gradient is tied to a constant around the critical value to drive micro-scale instability. -> Profile stiffness

Background: Profile Stiffness in Toroidal Plasmas



- ✓ Temperature gradient is tied to a constant around the critical value to drive micro-scale instability. -> Profile stiffness
- ✓ Profile stiffness is locally broken in the presence of Internal Transport Barrier (ITB).
- ✓ Empirical recipe to form clear ion ITBs
  - *Optimized flat-q or reversed-q profiles
  - *Often significant Neutral Beam Injection (NBI)

Purpose of This Work

Purpose of this work

- ✓ Reproducing and understanding ITB formation mechanism in global gyrokinetic ITG turbulence

Radial force balance

$$E_r = -\frac{T_i}{e}\left(\frac{1}{L_n} + \frac{1-k}{L_{T_i}}\right) + \frac{rB}{qR}U_{\parallel}$$

Approaches

1. Non-local ballooning theory
- ✓ Notation of θ_b, Δr and γ
- ✓ Impact of mean flow/toroidal rotation

2. Flux-driven GK ITG simulation
- ✓ Impact of mean flow/toroidal rotation on profile stiffness

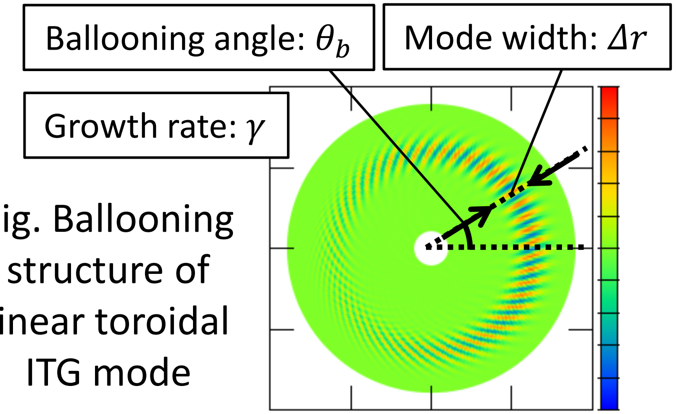


Fig. Ballooning structure of linear toroidal ITG mode

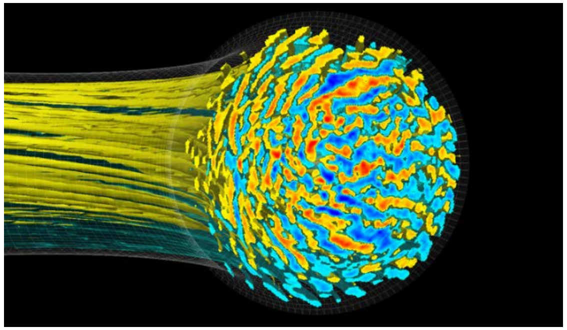


Fig. Typical structure of flux-driven toroidal ITG turbulence calculated by *GKNET*

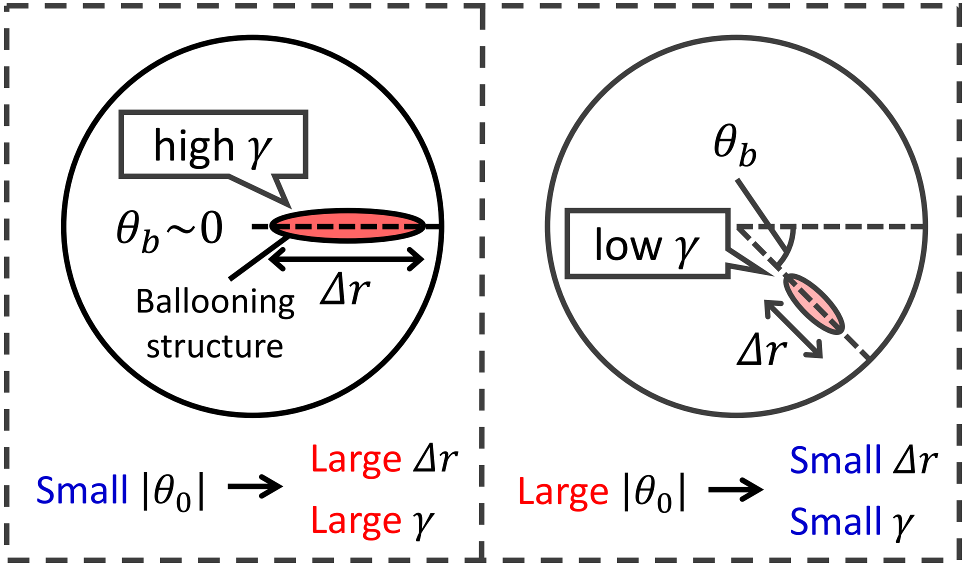
Non-Local Ballooning Theory

[Kishimoto, PPCF-1998]

$$\theta_b = \mp \left| \frac{\partial_r(\omega_r + \omega_f)}{2k_\theta \gamma_0 \hat{s}} \right|^{1/3}$$

$$\Delta r = \left| \frac{\sin\theta_b}{k_\theta^2 \hat{s}^2 \theta_b^3} \right|^{1/2}$$

$$\gamma = \gamma_0 \cos\theta_b$$



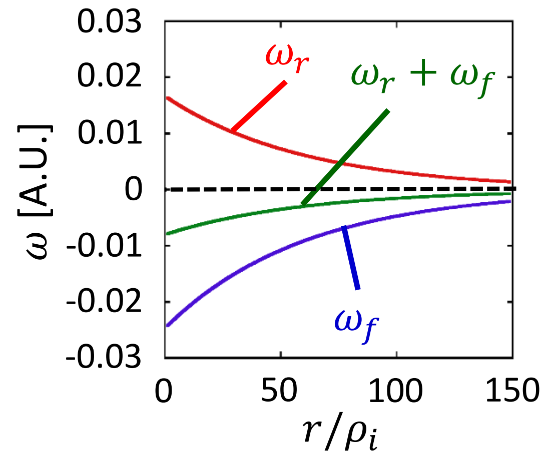
Radial force balance

$$E_r = \frac{rB}{qR} U_\parallel - \frac{T_i}{e}\left(\frac{1}{L_{n_i}} + \frac{1-k}{L_{T_i}}\right) \quad \left(\begin{array}{l} n_i = n_{i0}\exp(-r/L_{n_i}) \\ T_i = T_{i0}\exp(-r/L_{T_i}) \end{array} \right)$$

Eigenfrequency + Doppler shift frequency

$$\omega_r + \omega_f \sim \frac{k_\theta}{eB}\left[\left(\frac{2}{R_0} - \frac{1}{L_{n_i}} - \frac{1-k}{L_{T_i}}\right) T_i - \frac{erB}{qR} U_\parallel \right]$$

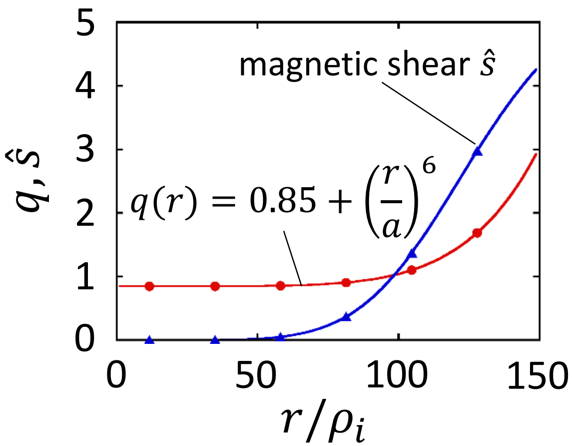
ω_r part ω_f part-1 Toroidal rotation (ω_f part-2)

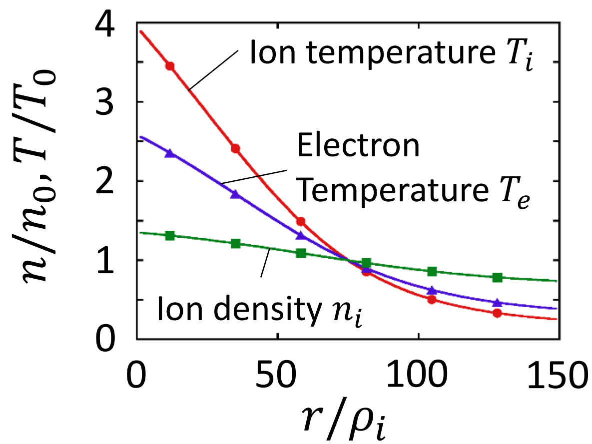


- ✓ Cancellation by mean flow
- ✓ Impact of toroidal rotation

Flux-Driven ITG Simulation with Momentum Injection

Simulation condition



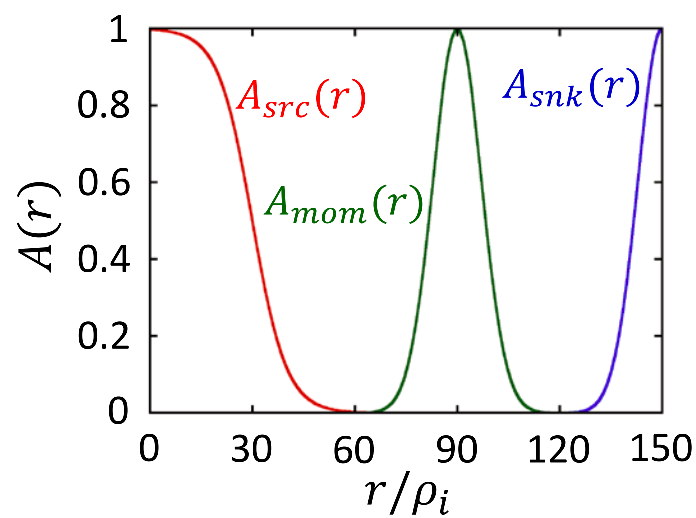


Parameter	Value
a_0/ρ_i	150
a_0/R_0	0.36
$(R_0/L_{n_i})_{r=a_0/2}$	2.22
$(R_0/L_{T_i})_{r=a_0/2}$	10.0
$(R_0/L_{T_e})_{r=a_0/2}$	6.92
ν_*	0.28
$P_{\rm in}$	4 [MW]
$T_{\rm in}$	5.64 [N・m]

Momentum source operator

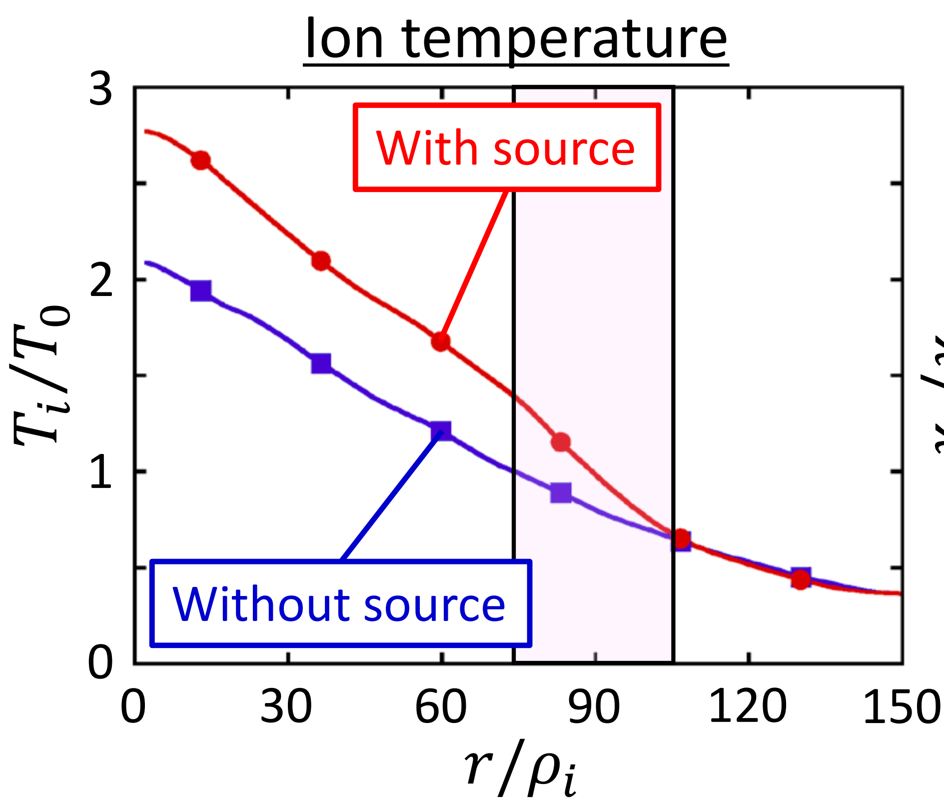
$$S_M = \tau_M^{-1} A(r)[f_{LM}(n_0, 0.5v_{ti}, T_0) - f_{LM}(n_0, 0, T_0)]$$

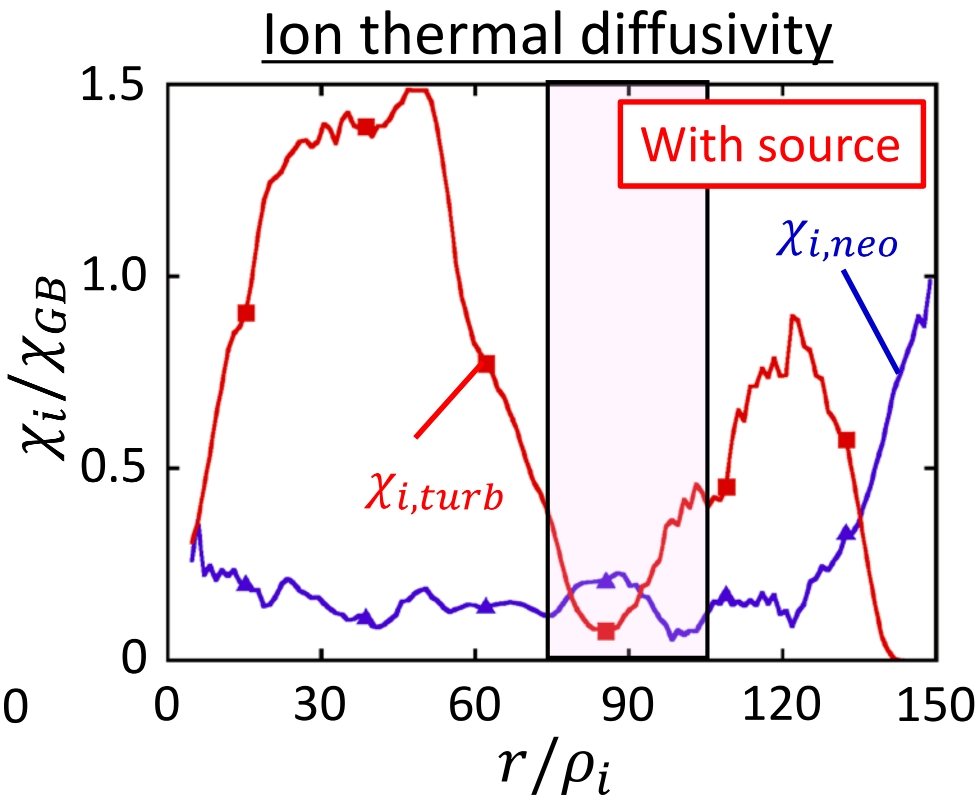
$$f_{LM}(n, U_{||}, T) = \frac{n}{\sqrt{2\pi T^3/m_i^3}} exp\left[-\frac{0.5(v_\parallel - U_\parallel)^2 + \mu B}{T/m_i}\right]$$



✓ We compare two cases;
- (A) without momentum source
- (B) with momentum source at $r = 90\rho_i$

Impact of Momentum Injection - 1

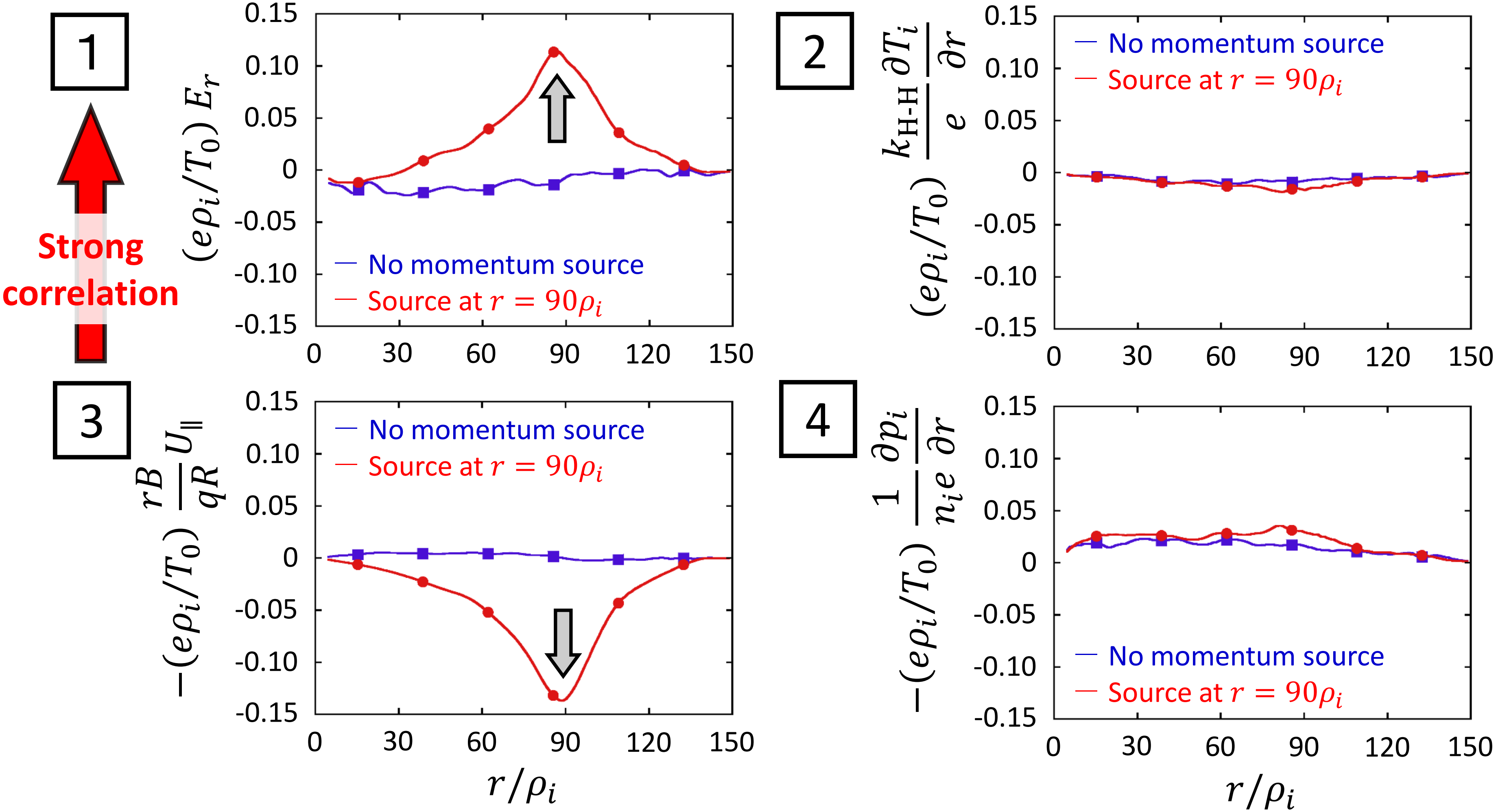




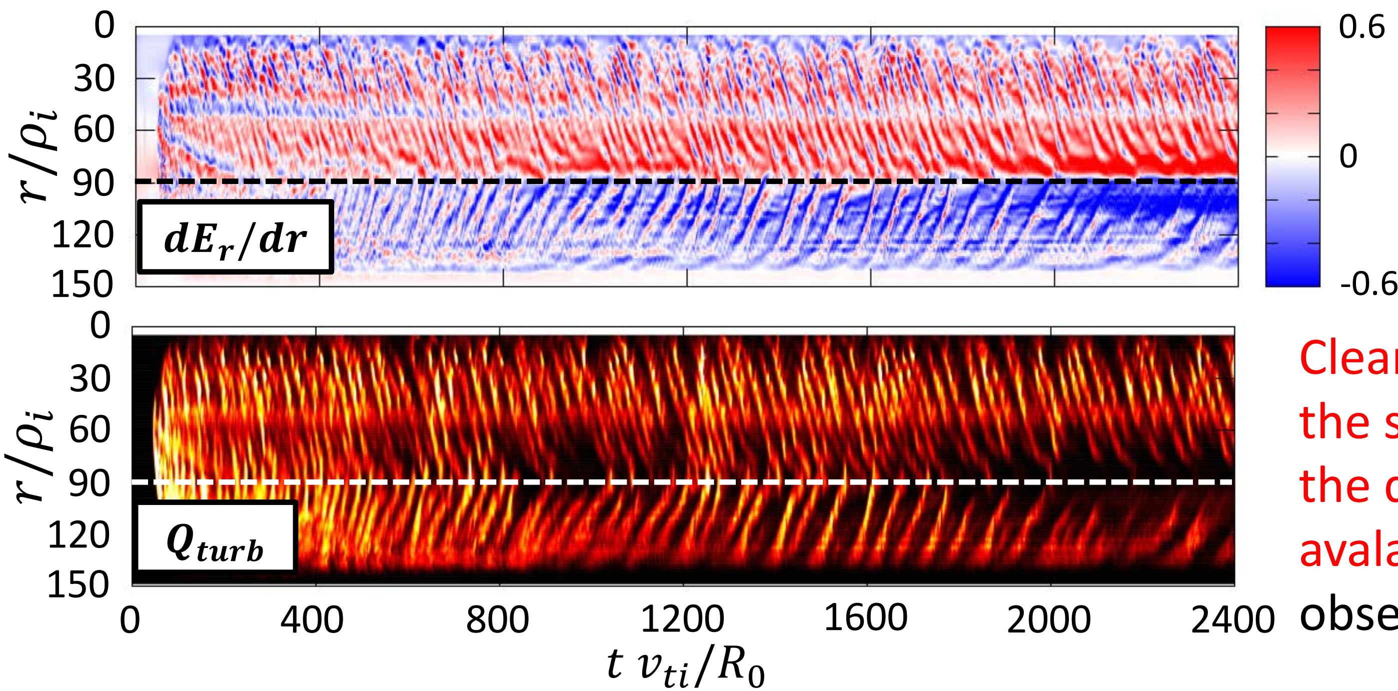
- ✓ Strong impact of momentum source at outer region on temperature build up.
- ✓ Ion turbulent thermal diffusivity decreases to the neoclassical transport level, which is typical tendency of inside of transport barrier. [Imadera, FEC-2016]

Impact of Momentum Injection - 2

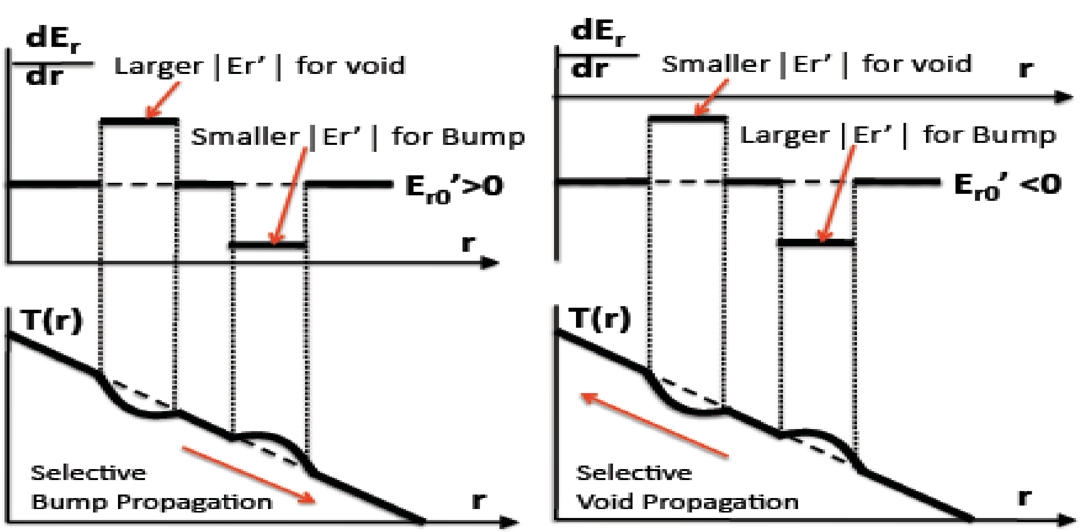
Radial force balance: $\underset{\boxed{1}}{E_r} + \underset{\boxed{2}}{\frac{k}{e}\frac{\partial T_i}{\partial r}} - \underset{\boxed{3}}{\frac{rB}{qR}U_\parallel} - \underset{\boxed{4}}{\frac{1}{n_i e}\frac{\partial p_i}{\partial r}} = 0$



Avalanches in the Presence of Strong Mean Flow



Clear correlation between the sign of E_r shear and the direction of avalanches can be observed.

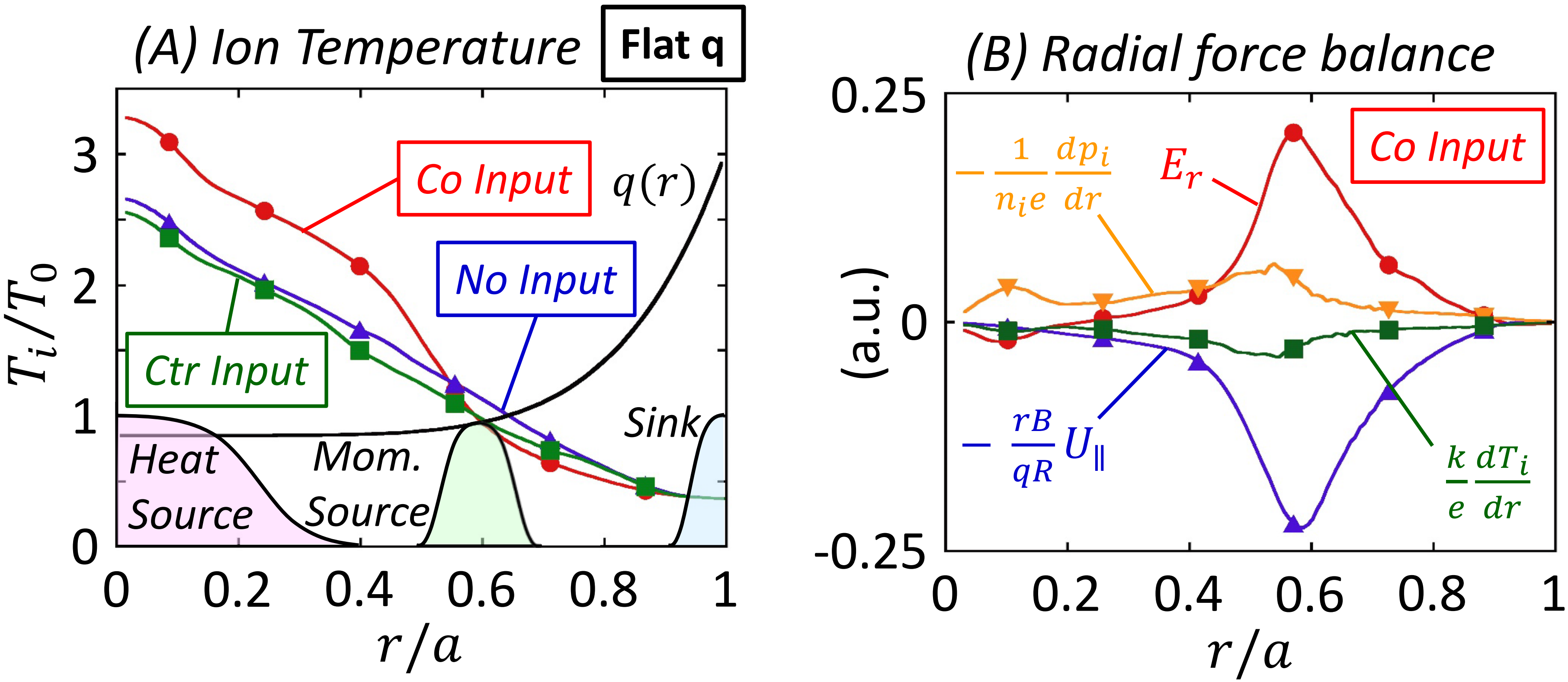


$$E_r = -V_t B_p + V_p B_t + \frac{1}{n_i e}\frac{\partial p_i}{\partial r}$$

$$\frac{dE_r}{dr} \Leftrightarrow \frac{d^2T}{dr^2}$$

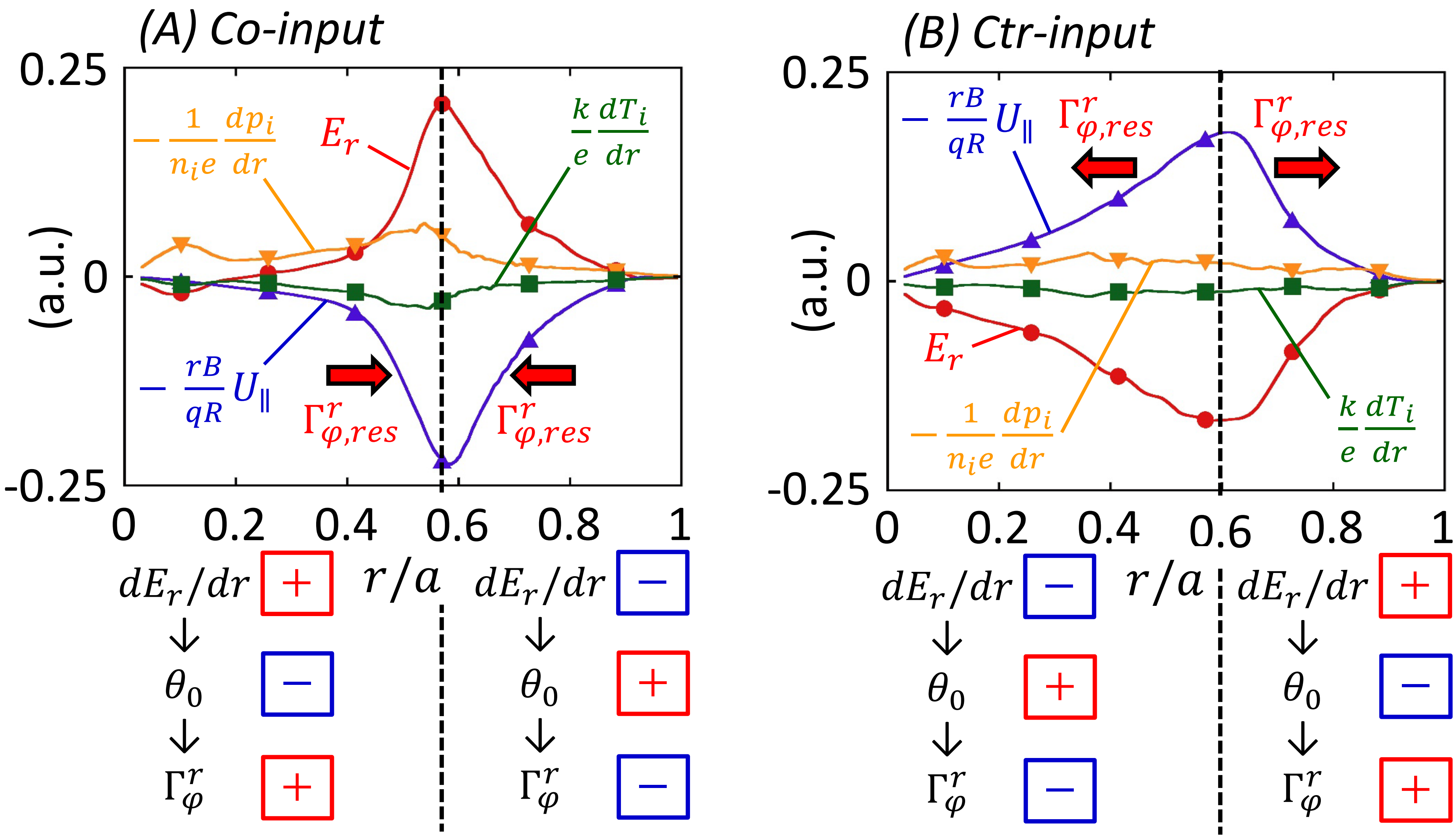
[Idomura, NF-2009]
[Kikuchi, RMP-2012]

Effect of Rotation Direction



- ✓ Only co-current toroidal rotation can benefit the ITB formation in weak magnetic shear plasma.
- ✓ This shows a qualitative agreement with the observations in experiments. [Mantica, PRL-2011] [Yu, NF-2017]

Favorite Trend of Co-input to ITB Formation



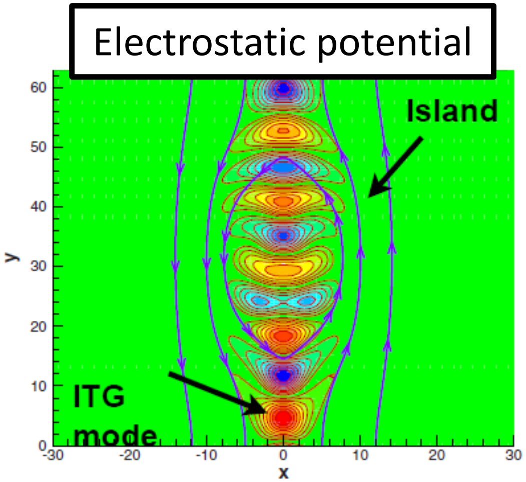
According to momentum transport theory [Camenen, NF-2011], there exists a positive feedback loop between mean E_r shear and momentum transport only in the co-input case.

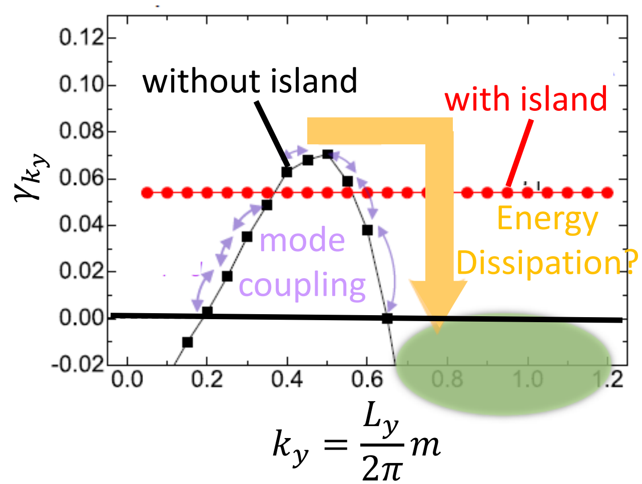
4. Role of stable modes in gyrokinetic plasmas

- Background & Motivation
- Theoretical Landau damping
- Eigenvalue analysis in discretized velocity space
- Stability analysis in coupled system

Background: Role of Stable Modes in Kinetic Plasmas

Multi-scale interaction between magnetic island and ITG mode [Wang, PoP-2009]

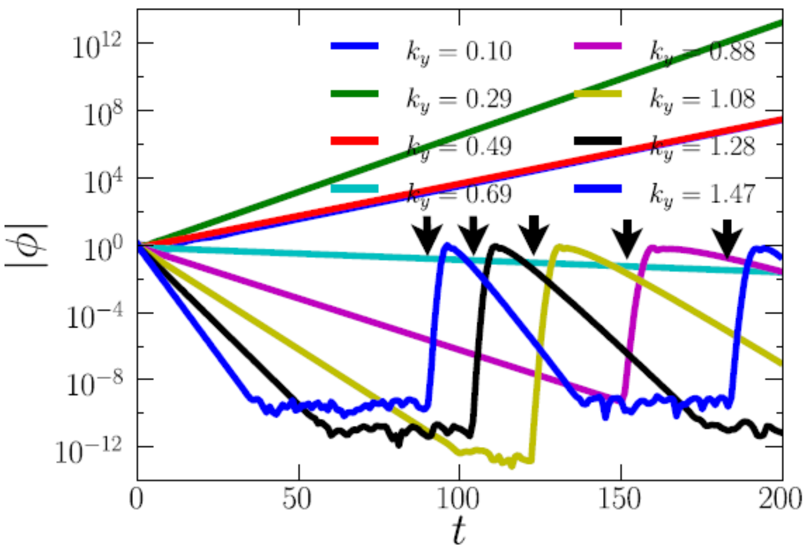




Since magnetic island ($m = 1$) can connect neighbor ITG modes (m_0 and $m_0 \pm 1$), whole the ITG modes have same growth rate.

→Maximum growth rate becomes lower after the coupling, indicating that stable modes work as an energy dissipation.

Local gyrokinetic Vlasov simulation in slab geometry



Damping of stable modes shows the recurrence phenomena.

→Growth rates of such modes are zero in long-time limit.

→Does these modes act as an energy dissipation?

Applied Physical Model

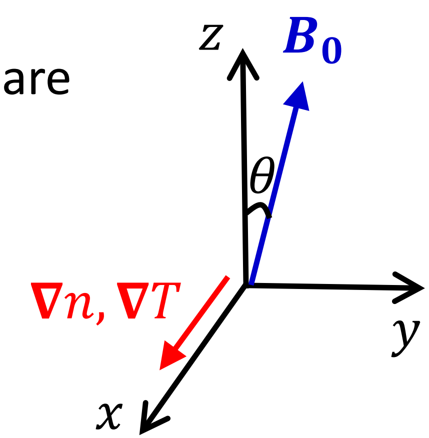
3D($x, y, v_\parallel$) gyrokinetic equation system with LB collision

$$\frac{\partial f_{1,k_\perp}}{\partial t} = -ik_y\left[1+\frac{1}{2}\left(v_\parallel^2 - k_\perp^2 - 1\right)\right]\bar{\phi}_{k_\perp} - i\theta k_y v_\parallel\left(f_{1,k_\perp} + \bar{\phi}_{k_\perp} f_0\right) + C_{LB}$$

$$[2-\Gamma_0(k_\perp^2)]\phi_{k_\perp} = \int_{-\infty}^{\infty} f_{1,k_\perp} e^{-k_\perp^2/2} dv_\parallel \qquad \bar{\phi}_{k_\perp} = \exp(-k_\perp^2/2)\phi_{k_\perp}$$

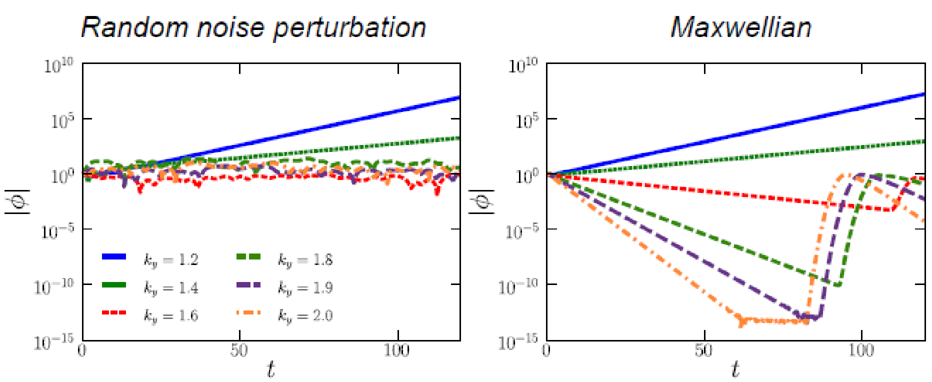
$$C_{LB} = \beta\frac{\partial}{\partial v_\parallel}\left(\frac{\partial f_{1,k_\perp}}{\partial v_\parallel} + v_\parallel f_{1,k_\perp}\right)$$

- ✓ f_0, f_1 are Fourier transformed in x and y directions and equidistantly discretized to $v_\parallel$ direction ($-7 \le v_\parallel \le 7, N_{v_\parallel} = 128$).
- ✓ Shear-less slab geometry and homogeneity along z direction are assumed.
- ✓ Slab ITG mode is traced by setting $\eta_i = 6, \theta = 0.3, k_x = 0$.
- ✓ We set two types of initial perturbation;
  - (A) Random noise
  - (B) $A = \epsilon f_M$

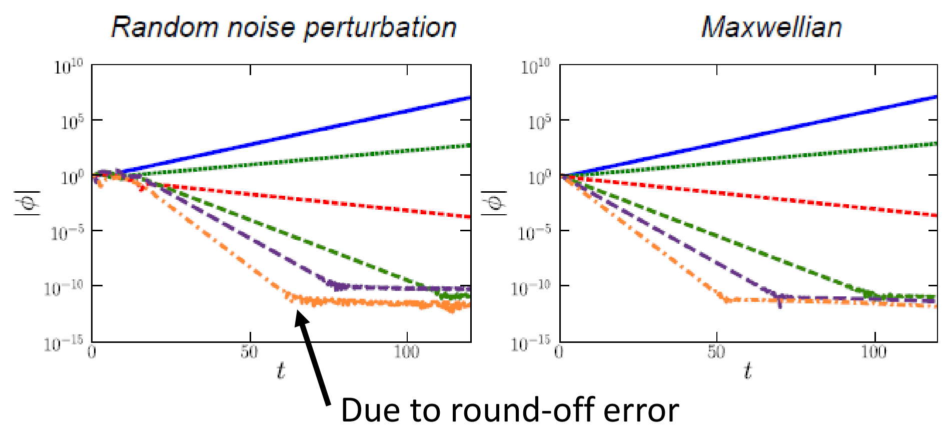


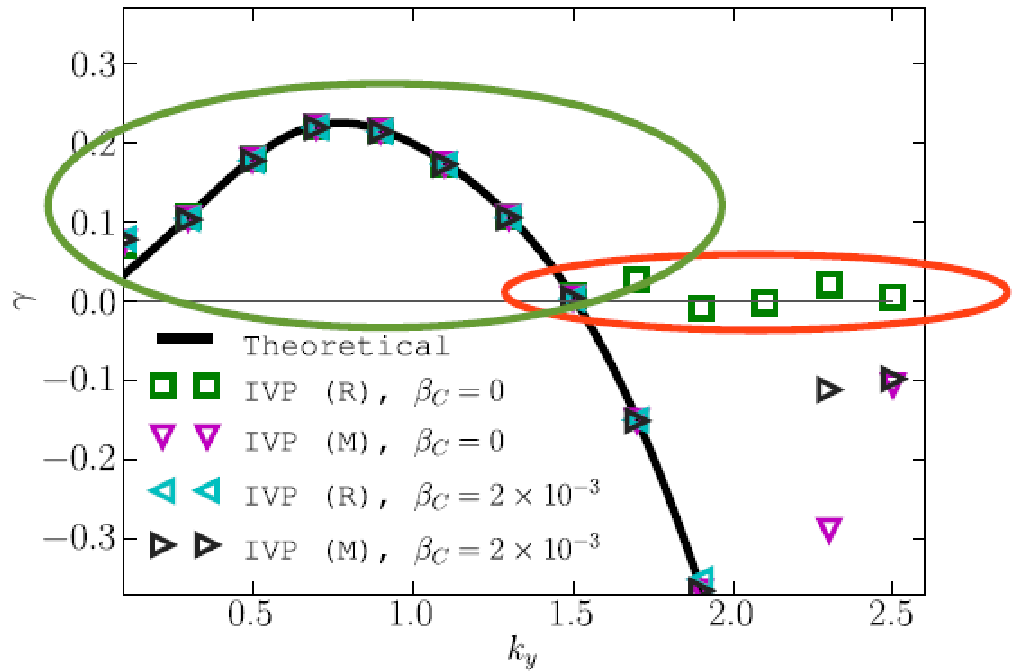
Numerical Solution of Initial Value Problem

Collision-less simulation ($\beta = 0$)



Collisional simulation ($\beta = 5 \times 10^{-3}$)





- ✓ Unstable modes are reproduced well in whole the cases.
- ✓ For stable modes,

$\beta_C = 0$, Random: No damp
$\beta_C = 0$, Maxwellian: Recurrence
$\beta_C \neq 0$, Random: continuous damp
$\beta_C \neq 0$, Maxwellian: Continuous damp

Revisiting Landau Damping (Historical Review)

L. D. Landau, J. Phys. USSR ***10****, 25 (1946)*

He eliminated f and solved initial value problem for electric field E using Laplace transform. Only one eigenmode for E is obtained as the time asymptotic solution.

$$E = -\frac{4\pi e}{k^2 \varepsilon(k,p)} \int_{-\infty}^{\infty} \frac{F_1(k,v,t=0)}{v + ip/k} dv$$

N. G. van Kampen, Physica ***21****, 949 (1955)*

He eliminated E, and obtained the eigenmode for f using Fourier transform.
The linearized Vlasov-Poisson equations have a continuous spectrum of singular eigenmodes, now known as Case van Kampen (CvK) modes;

$$f_1(v) = P\left[\frac{\eta(v)}{v - \omega/k}\right] + \delta(v - \omega/k)\left[1 - P\int_{-\infty}^{\infty} \frac{\eta\,(v')}{v' - \omega/k} dv'\right]$$

A. Lenard and B. Bernstein, Physical Review 1012, 1456 (1958)

They derived dispersion relation with collision and showed that the solution converges to the Landau's solution.

Two Eigenvalue Approaches

(1) Landau-like approach

Eliminating f_1 from the gyrokinetic Vlasov-Poisson system by assuming $\partial_t f_{1,k_\perp} = i\omega f_{1,k_\perp}$,

$$2-\Gamma_0(k_\perp^2)+e^{-k_\perp^2}\left\{1+\frac{\eta_i}{\sqrt{2}\theta}\zeta+\frac{1}{\sqrt{2}\theta}\left[1-\frac{\eta_i}{2}(1+k_\perp^2)+\eta_i\zeta^2+\sqrt{2}\theta\zeta\right]Z(\zeta)\right\}=0,\ \zeta\equiv\omega/k_y$$

(2) CvK-like approach [Paul, PoP-2013] [Paul, PFR-2013]

Eliminating ϕ from the gyrokinetic Vlasov-Poisson system and assuming $\partial_t f_{1,k_\perp} = i\omega f_{1,k_\perp}$,

$$i\omega f_{1,k_\perp} = -ik_y\left[1+\frac{1}{2}\left(v_\parallel^2-k_\perp^2-1\right)\right]\frac{e^{-k_\perp^2}}{2-\Gamma(k_\perp^2)}\int_{-\infty}^{\infty} f_{1,k_\perp}dv_\parallel - i\theta k_y v_\parallel\left(f_{1,k_\perp}+\frac{e^{-k_\perp^2}}{2-\Gamma(k_\perp^2)}\int_{-\infty}^{\infty} f_{1,k_\perp}dv_\parallel\ f_0\right)$$

Discretizing the above equation with equal mesh width $\Delta v_\parallel$,

$$i\omega\vec{f}_{1,k_\perp} = L\vec{f}_{1,k_\perp} \qquad L=\vec{L}_0\cdot I+\left(\vec{L}_1+\vec{D}\right)\otimes P$$

$$P=\begin{pmatrix}\frac{e^{-k_\perp^2}}{2-\Gamma(k_\perp^2)}\Delta v_\parallel & \cdots & \frac{e^{-k_\perp^2}}{2-\Gamma(k_\perp^2)}\Delta v_\parallel \\ \vdots & \ddots & \vdots \\ \frac{e^{-k_\perp^2}}{2-\Gamma(k_\perp^2)}\Delta v_\parallel & \cdots & \frac{e^{-k_\perp^2}}{2-\Gamma(k_\perp^2)}\Delta v_\parallel\end{pmatrix}$$

$$\vec{f}_{1,k_\perp,i} = f_{1,k_\perp}(v_{\parallel,i})$$

$$\vec{L}_{0,i} = -i\theta k_y v_{\parallel,i}$$

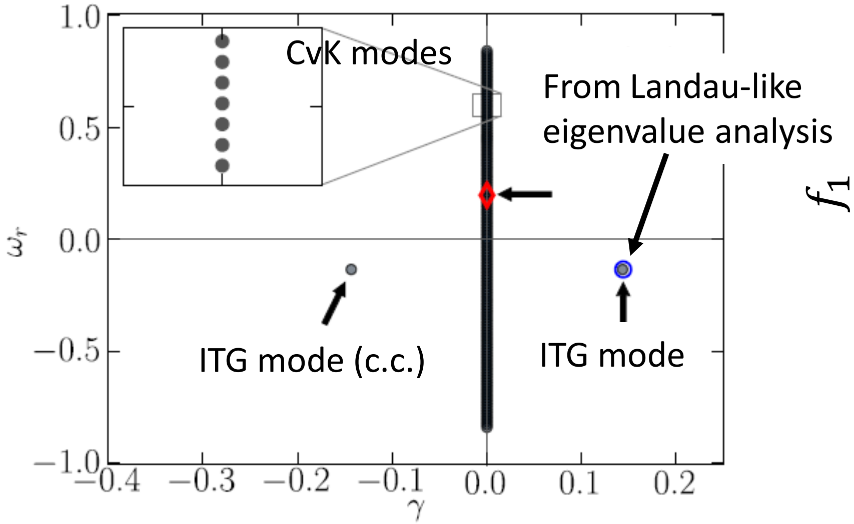
$$\vec{L}_{1,i} = -i\theta k_y v_{\parallel,i} f_0\left(v_{\parallel,i}\right)$$

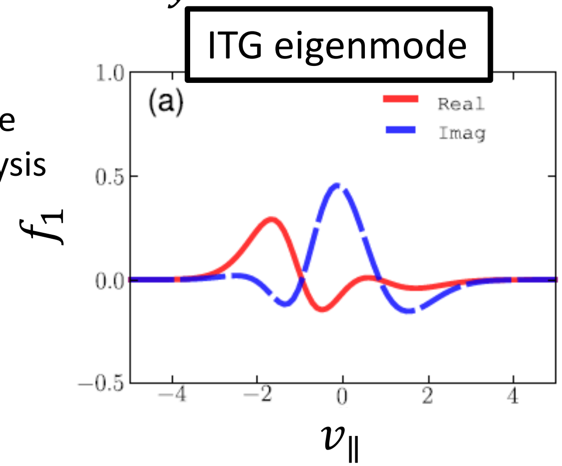
$$\vec{D}_i = -ik_y\left[1+\frac{1}{2}\left(v_{\parallel,i}^2-k_\perp^2-1\right)\right]f_0\left(v_{\parallel,i}\right)$$

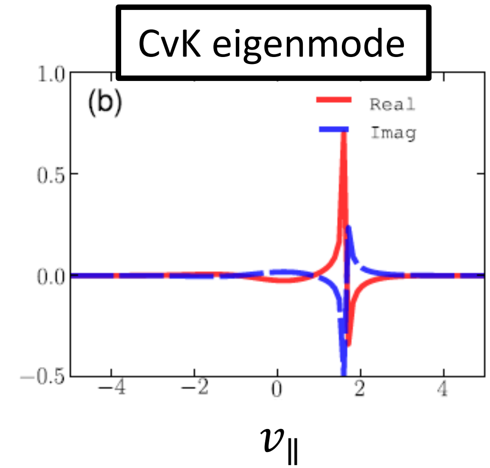
$$v_{\parallel,i} = -v_{\parallel,min}+i*\Delta v_\parallel\ \left(i=1,2,\cdots N_{v_\parallel}\right)$$

Solution of CvK-like Eigenvalue Analysis

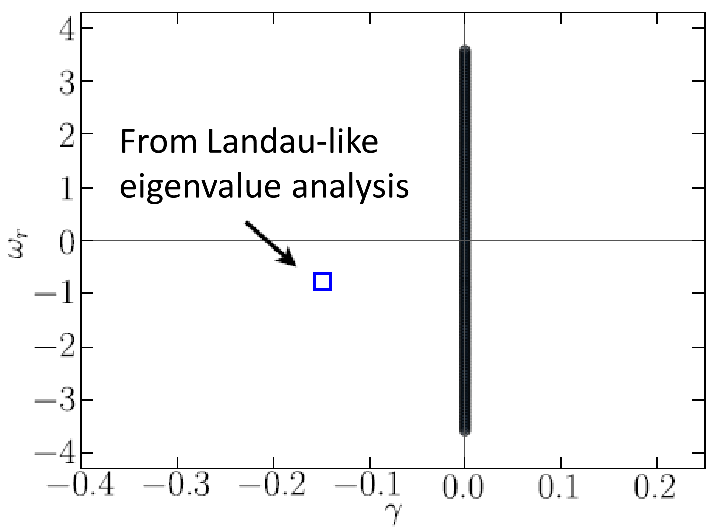
Eigenvalue spectra of unstable ITG mode ($k_y = 0.4$)







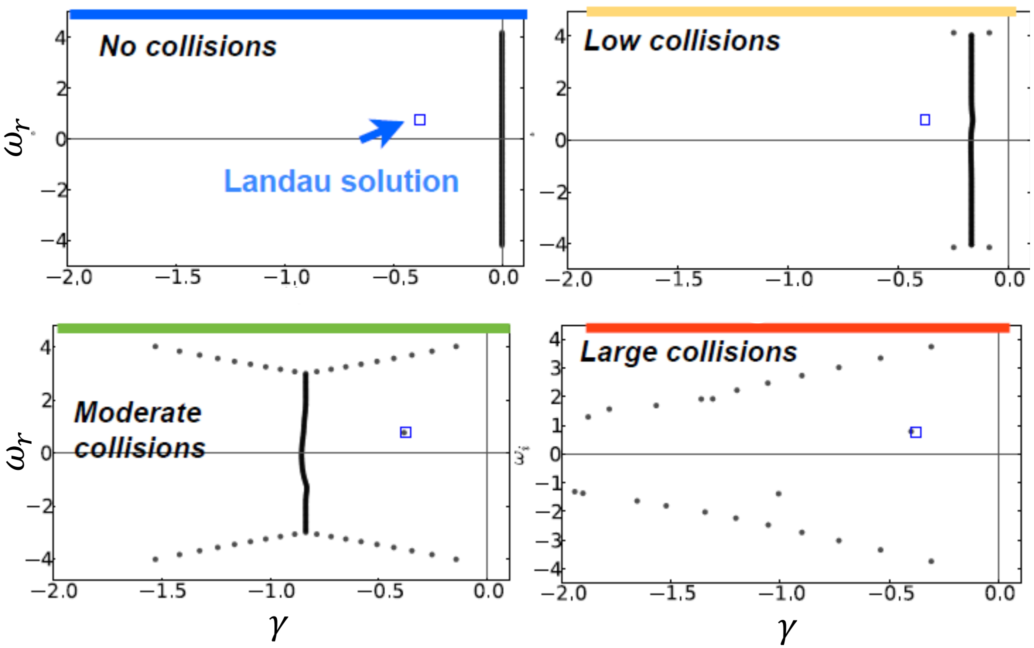
Eigenvalue spectra of unstable ITG mode ($k_y = 1.8$)



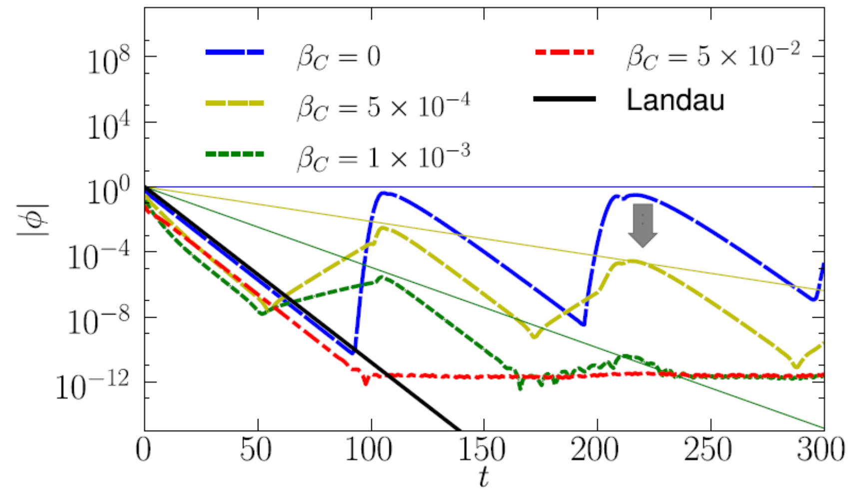
- ✓ Only semi-stable singular CvK eigenmodes appears and Landau mode is not found.
- ✓ The Landau damping with the recurrence originates from the phase-mixing of CvK modes, which is also confirmed by the energy transfer to higher Hermite basis (not shown here).

Transition from CvK Phase Mixing to Landau Eigen-Damping

Eigenvalue spectra



Time evolution of stable mode energy

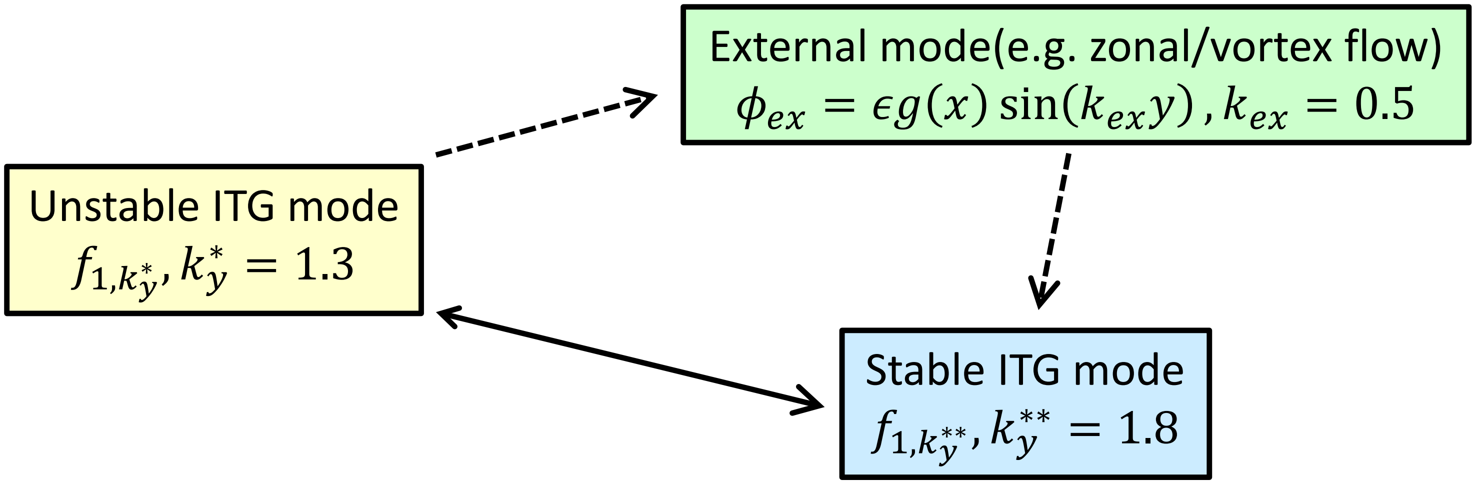


- ✓ Once the damping rate of CvK mode exceeds the Landau damping one, the Landau eigenmode appears.
- ✓ There exists two numerically different Landau damping mechanism; "phase mixing of singular CvK modes" and "Landau eigen-damping".

However, does "phase mixing of singular CvK modes" acts as an energy sink in the coupled system?

Coupling of Unstable and Stable ITG Modes via Vortex Flow

<u>Mode-coupling triad mediated though $E \times B$ nonlinearity</u>



$$-i\omega \begin{pmatrix} f_{1,k_y^*} \\ f_{1,k_y^{**}} \end{pmatrix} = \begin{bmatrix} L_{k_y^*} & C_{k_y^{**}} \\ C_{k_y^*} & L_{k_y^{**}} \end{bmatrix} \begin{pmatrix} f_{1,k_y^*} \\ f_{1,k_y^{**}} \end{pmatrix} \qquad \begin{cases} C_{k_y^*} = 0.5\epsilon k_y^* I \\ C_{k_y^{**}} = 0.5\epsilon k_y^{**} I \end{cases}$$

Coupling components
From linear Poisson bracket

- ✓ As a minimal model, we consider triad coupling with unstable, stable and tertiary “static” mode as a vortex flow.
- ✓ For $\epsilon > 0$, both f_{1,k_y^*} and $f_{1,k_y^{**}}$ have same eigenvalue (global mode).

Eigenvalue Spectra of Mode-Coupled System

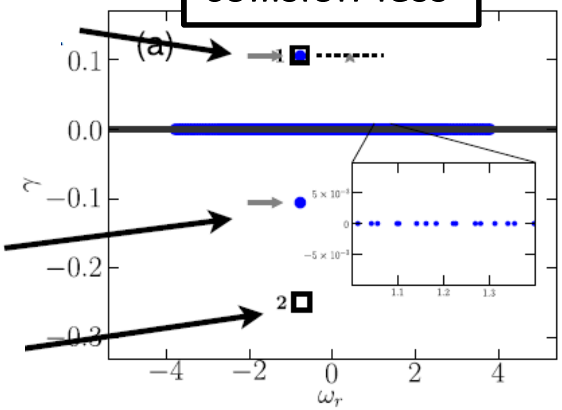
$\epsilon = 0$: Uncoupled system

Unstable ITG mode is captured

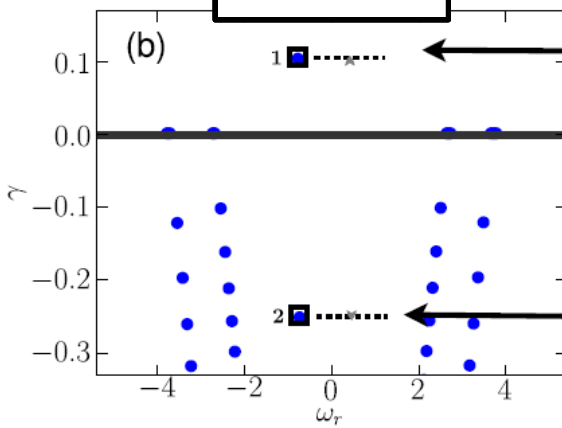
Complex conjugate

Stable eigenmode is not found

collision-less



collisional



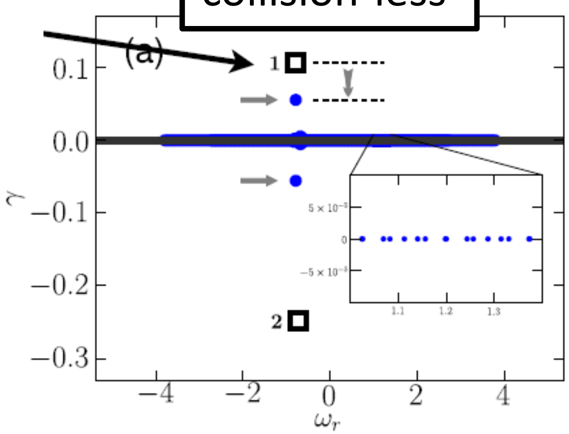
Unstable ITG mode is captured

Stable eigenmode is resolved after CvK collapse

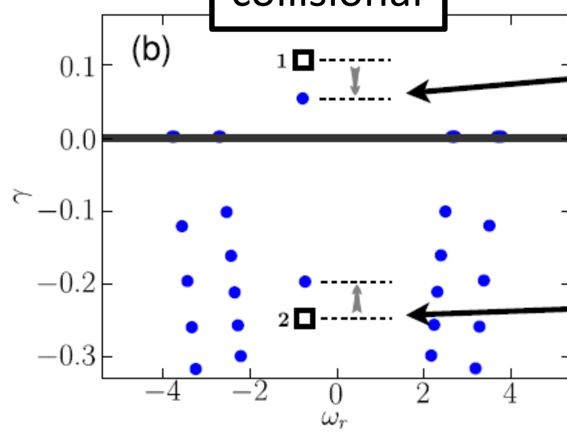
$\epsilon = 0.1$: Coupled system

Unstable global ITG mode is stabilized

collision-less



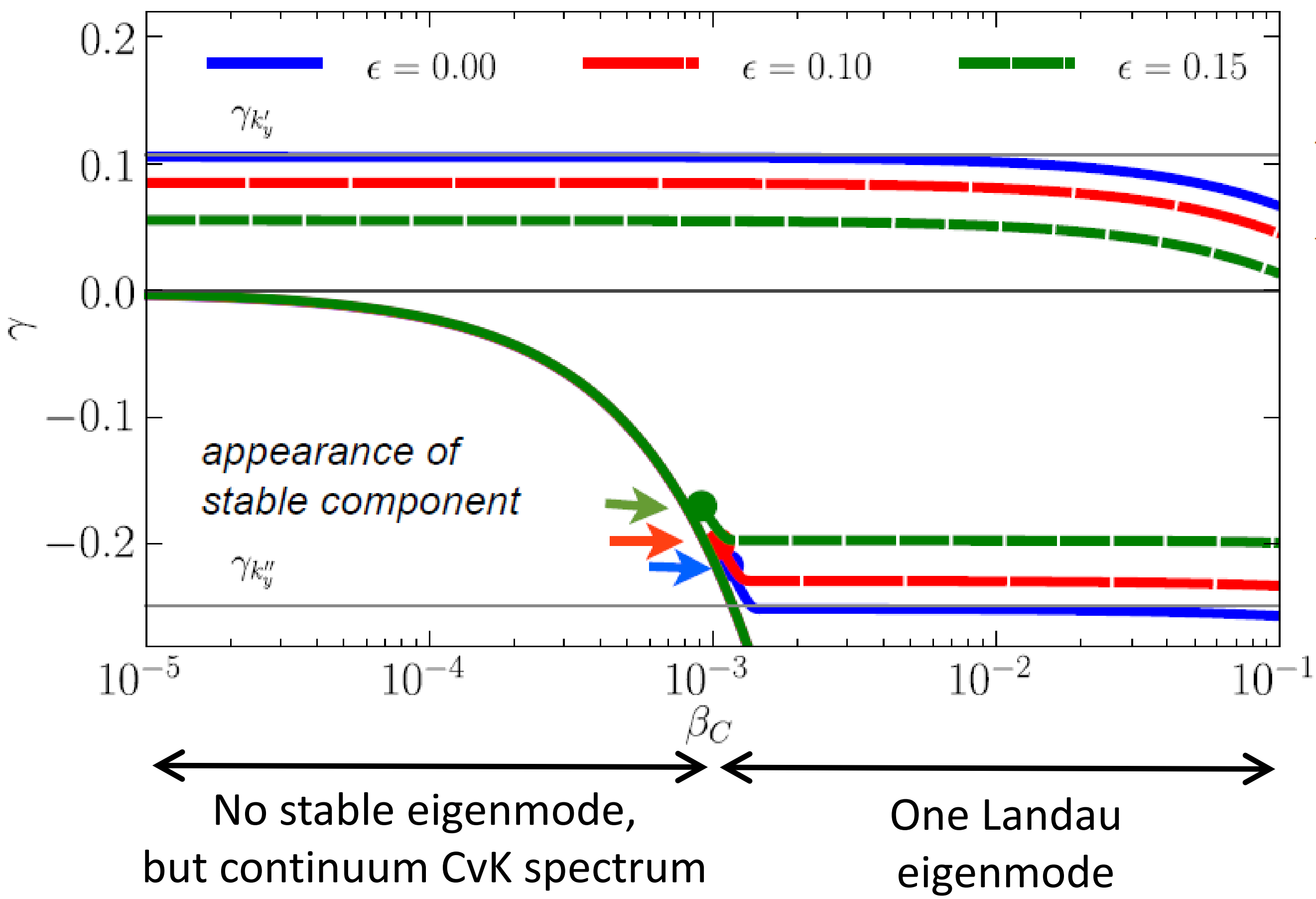
collisional



Unstable global ITG mode is stabilized

Stable global mode is resolved after CvK collapse and is destabilized

Parameter Scan for Collisionality in Mode-Coupled System



- ✓ The linear stabilization effect in the mode-coupled system is independent to whether stable modes arises from phase-mixing or Landau eigen-damping.
- ✓ In the linear coupling models, the dissipation by stable modes correctly works in gyrokinetic Vlasov simulation.

5. Summary

Summary

5D full-*f* gyrokinetic code *GKNET*

- ✓ 5D full-*f* gyrokinetic code *GKNET* was developed based on (1)non-dissipative FDM, (2)real space field solver with full-order FLR effect, (3)hybrid parallelization with communication and computation hiding technique.

5D full-*f* gyrokinetic simulation for turbulence and transport barrier

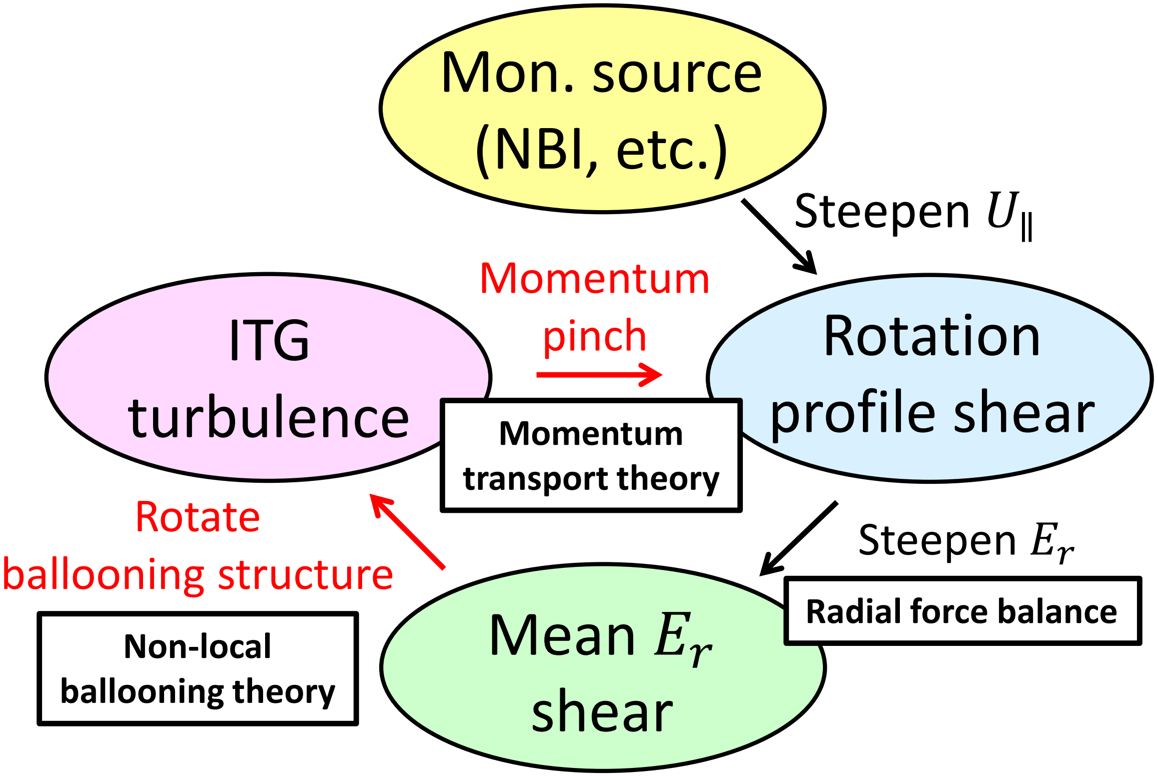
- ✓ A momentum source can change the mean E_r through the radial force balance, leading to ITB formation.
- ✓ The underlying mechanism is identified to originate from a positive feedback loop between the enhanced mean E_r shear and resultant momentum pinch, which can be observed only in co-input case.

Role of stable modes in gyrokinetic plasmas

- ✓ There exists two numerically different Landau damping mechanism; “phase mixing of singular CvK modes” and “Landau eigen-damping”, however, the linear stabilization effect in the mode-coupled system is independent to this type.

Appendix

Feedback Loop between Mean E_r Shear and Momentum Pinch



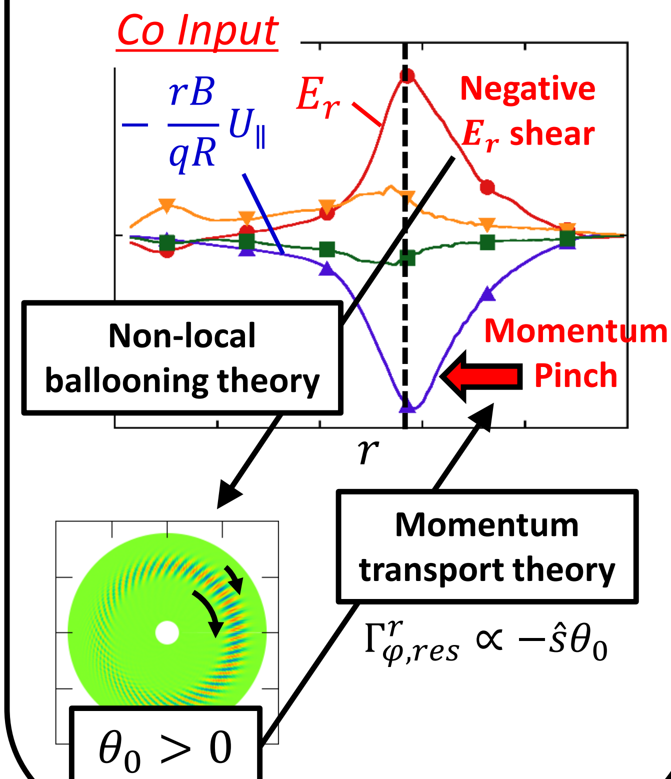
Momentum transport theory

$$R_\varphi \equiv \frac{\Gamma_\varphi^r}{Q_i^r}\frac{v_{ti}}{R_0} = \frac{2}{R/L_T}\frac{\chi_\varphi}{\chi_i}\left[u' + \frac{R_0 V_{co}}{\chi_\varphi}u + \frac{C^*}{\chi_\varphi}\right]$$

$$\frac{C^*}{\chi_\varphi} \sim -\frac{\hat{s}}{2qk_\theta\rho_{ti}}\left[\frac{R_0}{L_n} + 4\right]\theta_0$$

Radial force balance

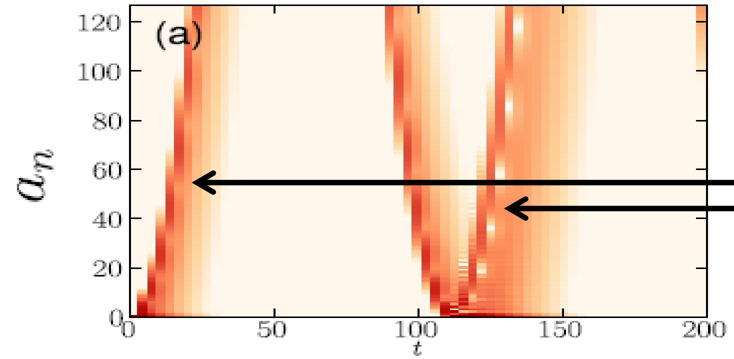
$$E_r + \frac{k}{e}\frac{\partial T_i}{\partial r} - \frac{rB}{qR}U_\parallel - \frac{1}{n_i e}\frac{\partial p_i}{\partial r} = 0$$



[Camenen, NF-2011]

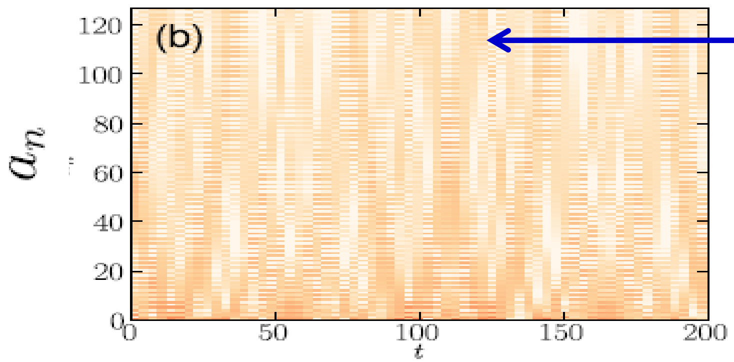
Phase Mixing and Energy Transfer in Velocity Space

0th order Hermite function



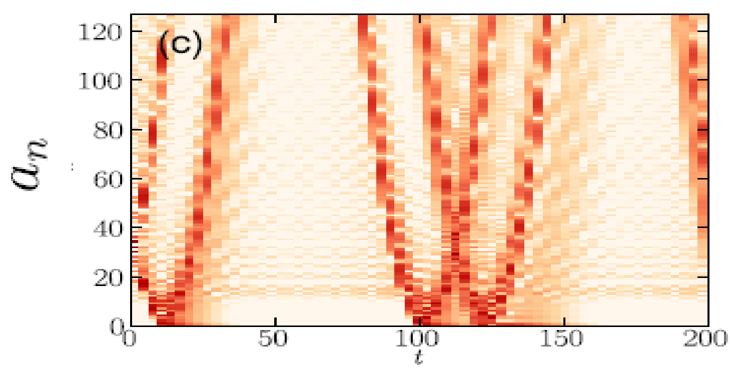
Transfer from a0 to higher order moments

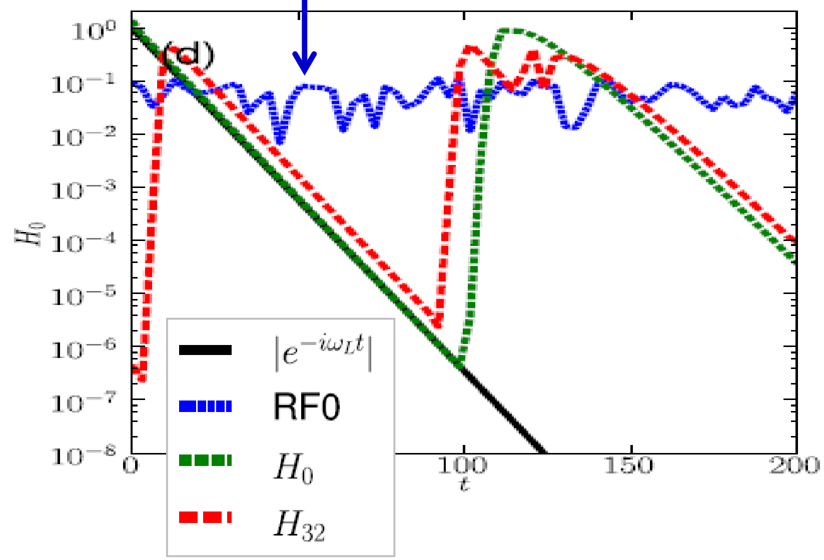
Random noise



All modes are simultaneously excited with no directionality

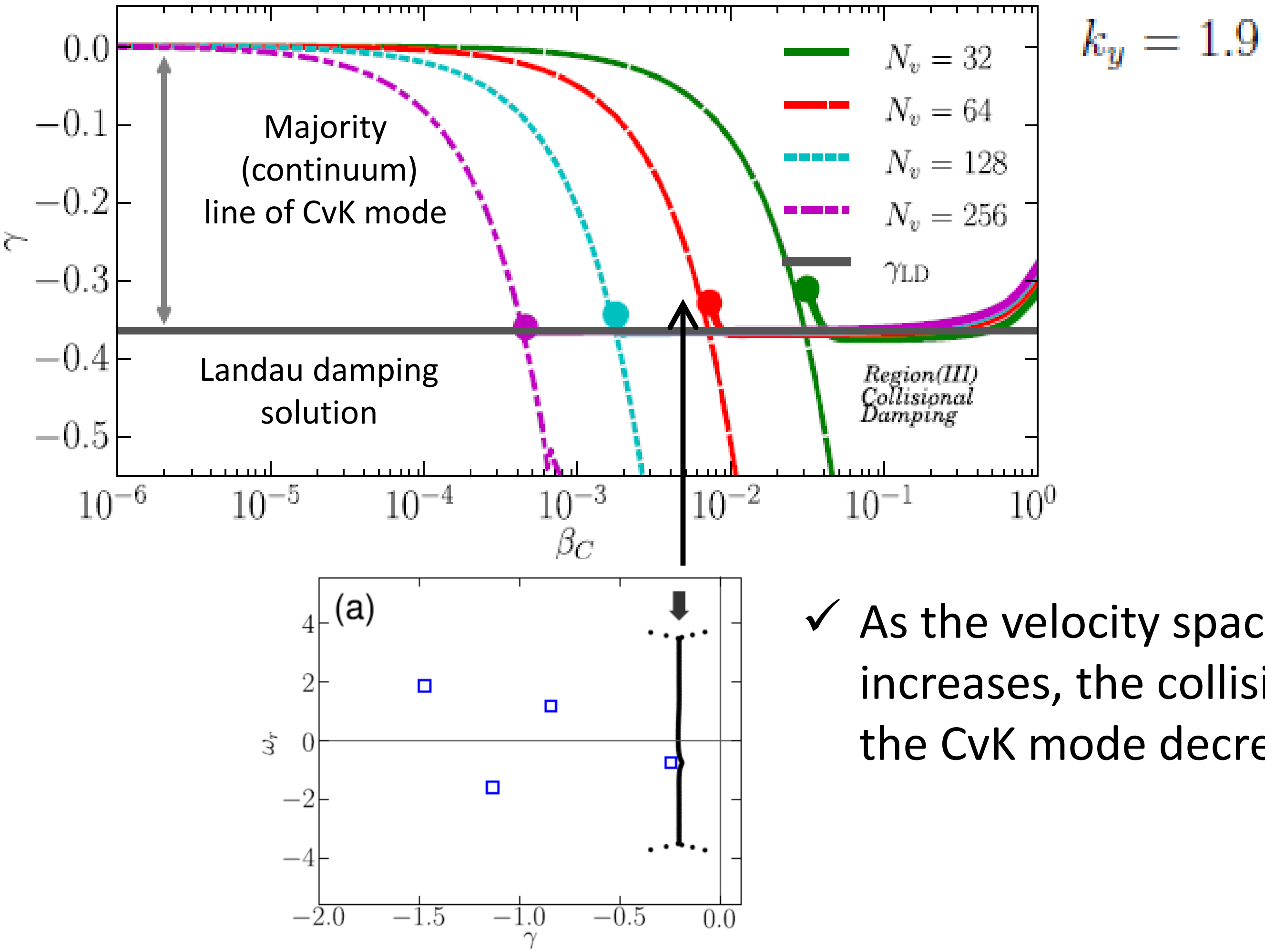
32th order Hermite function





Critical collisionality for getting Landau solution

- Damping rate with respect to collisionality for different velocity space resolutions



✓ As the velocity space resolution increases, the collisionality which damps the CvK mode decreases.