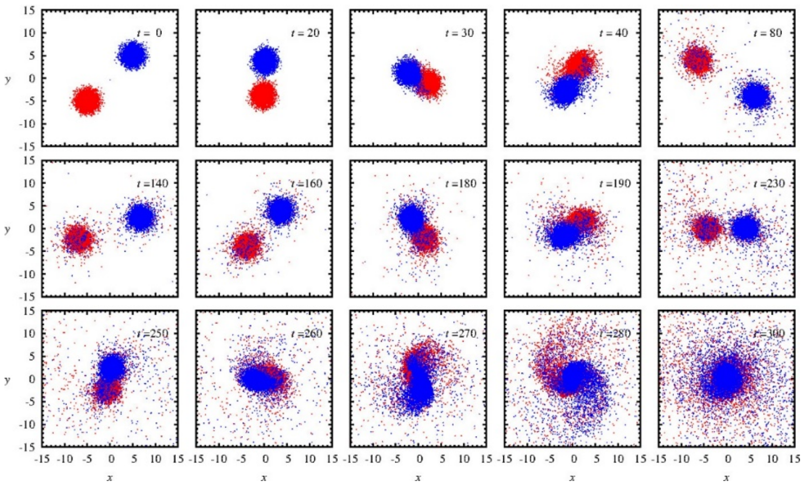
Phase-Space Evolution of Merging Collisionless Stellar Systems with a Self-Consistent Field Method



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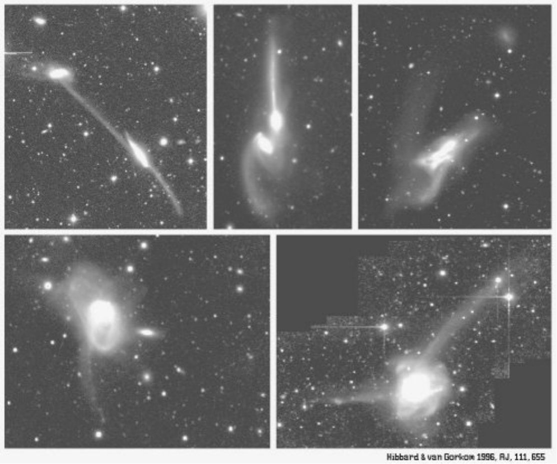
Mean-Field Approach to Merging Systems

- Merging
  - Fundamental process of structure formation in the CDM universe
    - galaxy-galaxy mergers ⇨ collisionless systems
- Galaxy Simulation
  - *N* –Body Approach
    - *N* –gravitating bodies ⇨ $\boldsymbol{F}_i = \sum_{\substack{j=1 \\ i \neq i}}^{N} \frac{G m_i m_j (\boldsymbol{r}_j - \boldsymbol{r}_i)}{|\boldsymbol{r}_j - \boldsymbol{r}_i|^3}$
    - Gravitational softening ⇨ $\boldsymbol{F}_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{G m_i m_j (\boldsymbol{r}_j - \boldsymbol{r}_i)}{(|\boldsymbol{r}_j - \boldsymbol{r}_i|^2 + \varepsilon^2)^{3/2}}$
    - Two-body relaxation ⇨ $t_{\rm relax} \approx \frac{0.1 N}{\ln N} t_{\rm cross}$
  - Mean-Field Approach
    - Force field ⇨ Collisionless nature
      ⇨ One *N* -body problem is reduced to *N* one-body problems
    - No softening ⇨ Pure Newtonian force



Hibbard & *van Gorkom*, 1996, AJ, **111**, 655

1. Merging Simulation

Self-Consistent Field (SCF) Method

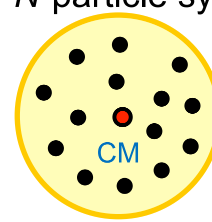
References:
Clutton-Brock, 1972, Ap&SS, **16**, 101
Clutton-Brock, 1973, Ap&SS, **23**, 55
Hernquist & *Ostriker*, 1992, ApJ, **386**, 375

- Bi-orthogonal basis set $(\rho_{nlm}(\boldsymbol{r}),\ \Phi_{nlm}(\boldsymbol{r}))$
  - Poisson's equation : $\nabla^2 \Phi_{nlm}(\boldsymbol{r}) = 4\pi G \rho_{nlm}(\boldsymbol{r})$
  - Bi-orthogonality : $\int \rho_{nlm}(\boldsymbol{r}) [\Phi_{n'l'm'}(\boldsymbol{r})]^* \, d\boldsymbol{r} = \delta_{nn'} \delta_{ll'} \delta_{mm'}$

- Expansions in a set of basis functions

$$\begin{cases} \rho(\boldsymbol{r}) = \sum_{nlm} A_{nlm}\, \rho_{nlm}(\boldsymbol{r}) \\ \Phi(\boldsymbol{r}) = \sum_{nlm} A_{nlm}\, \Phi_{nlm}(\boldsymbol{r}) \end{cases} \qquad \left[\begin{array}{c} \nabla^2 \Phi_{nlm}(\boldsymbol{r}) = 4\pi G \rho_{nlm}(\boldsymbol{r}) \\ \Downarrow \\ \nabla^2 \Phi(\boldsymbol{r}) = 4\pi G \rho(\boldsymbol{r}) \end{array} \right]$$

N-particle system



particle distribution m_k
⇩
operate $\Phi_{nlm}(\boldsymbol{r}_k)$
⇩
calculate A_{nlm}
⇩
obtain $\begin{cases} \Phi(\boldsymbol{r}) & \textit{potentail field} \\ \boldsymbol{a}(\boldsymbol{r}) & \textit{force field} \end{cases}$

  - Expansion coefficients for a particle system : $A_{nlm} = \sum_{k=1}^{N} m_k\, \Phi_{nlm}(\boldsymbol{r}_k)$

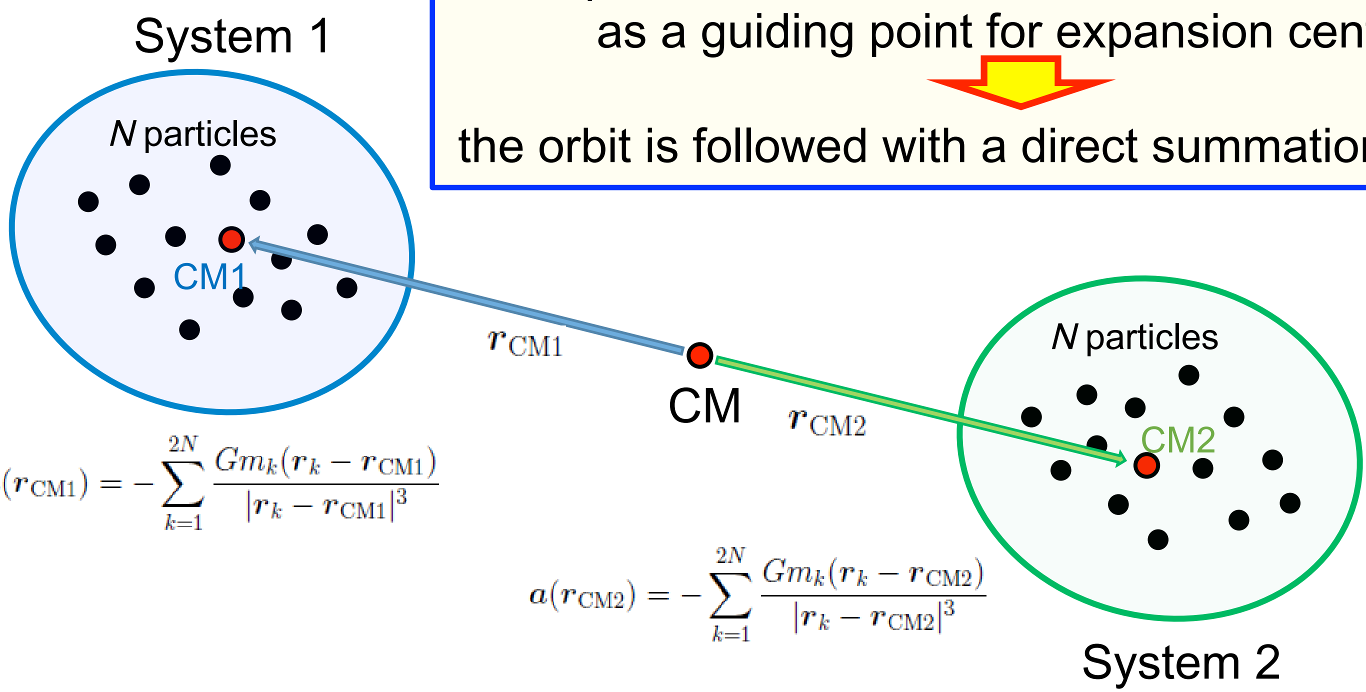
⇨ $\boldsymbol{a}(\boldsymbol{r}) = -\nabla \Phi(\boldsymbol{r}) = -\sum_{nlm} A_{nlm} \nabla \Phi_{nlm}(\boldsymbol{r})$ ⇨ **Gravitational field ⇒ *Field Method***

Expanding ρ and Φ of Merging Systems

Each expansion center is moving in space

test particle at the center of mass of each galaxy
as a guiding point for expansion center

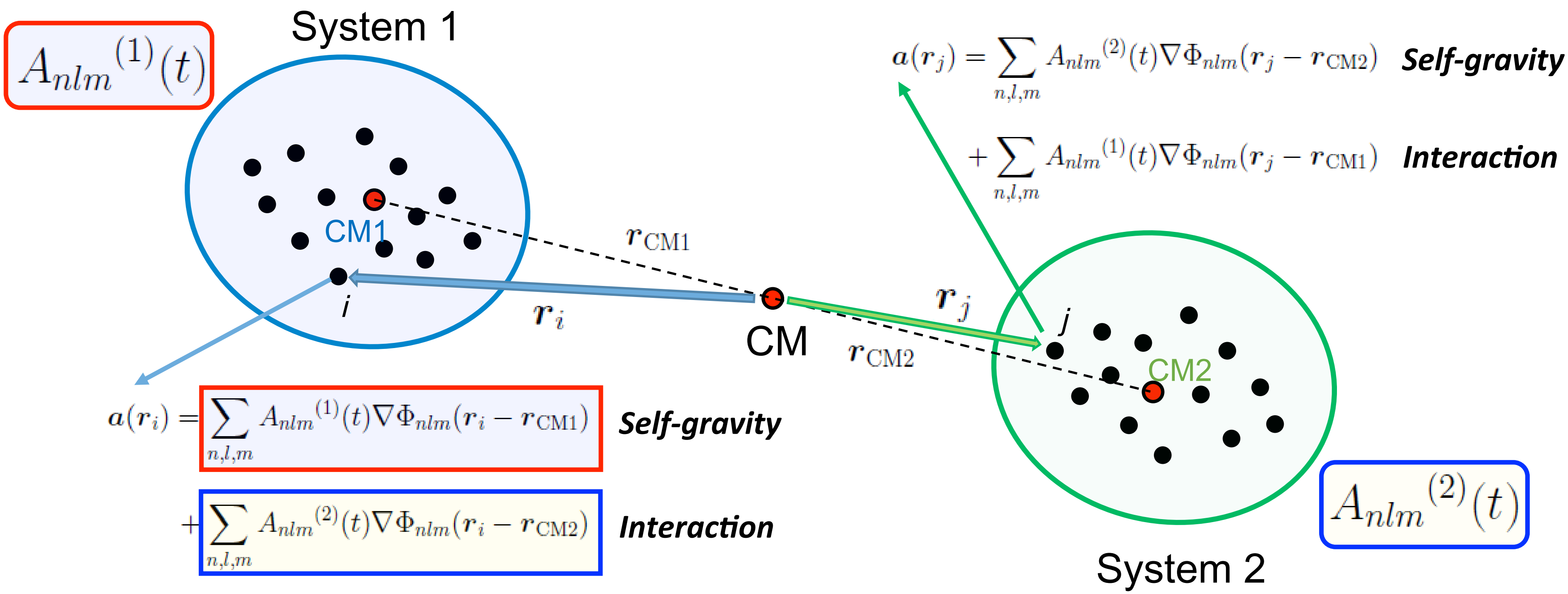
the orbit is followed with a direct summation method



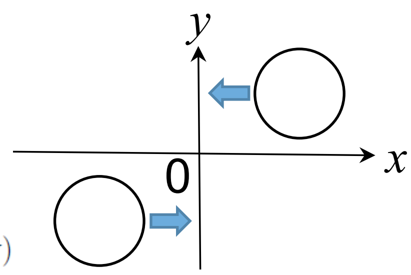
$$a(r_{\mathrm{CM1}}) = -\sum_{k=1}^{2N} \frac{Gm_k(r_k - r_{\mathrm{CM1}})}{|r_k - r_{\mathrm{CM1}}|^3}$$

$$a(r_{\mathrm{CM2}}) = -\sum_{k=1}^{2N} \frac{Gm_k(r_k - r_{\mathrm{CM2}})}{|r_k - r_{\mathrm{CM2}}|^3}$$

Motions of Particles in Each Galaxy



Merging Simulations



- Two identical King models: $f(\mathcal{E}) = \begin{cases} \dfrac{\rho_1}{(2\pi\sigma^2)^{3/2}}(e^{\mathcal{E}/\sigma^2} - 1) & (\mathcal{E} > 0) \\ 0 & (\mathcal{E} \le 0) \end{cases}$
- Central potential : $W = \Psi(0)/\sigma^2 = 3 \Rightarrow r_t = 4.70\, r_0 \quad (r_0 = \dfrac{3\sigma}{\sqrt{4\pi G \rho_0}},\ \rho_0 : \text{central density})$
- Central velocities : (0.2, 0), (-0.2, 0)
- Central coordinates : (-5, -y), (5, y) with y = 3, 5 ⟹ *impact parameter* : 6, 10
- Orbital plane : x-y plane
- Number of particles : N =10,000,584
- Computational units : $G = M = r_0 = 1$

SCF simulations

- Clutton-Brock's basis set

$$\begin{cases} \rho_{nlm}(\boldsymbol{r}) = K_{nl}\dfrac{M}{4\pi a^3}\dfrac{(r/a)^l}{[1+(r/a)^2]^{l+5/2}}C_n^{(l+1)}(\xi)\sqrt{4\pi}Y_{lm}(\theta,\,\phi) \\ \Phi_{nlm}(\boldsymbol{r}) = -\dfrac{GM}{a}\dfrac{(r/a)^l}{[1+(r/a)^2]^{l+1/2}}C_n^{(l+1)}(\xi)\sqrt{4\pi}Y_{lm}(\theta,\,\phi) \end{cases}$$

  - Scale length of the basis fn. : $a = 1.15$

- number of expansion terms
  - radial direction : $n_{\max} = 16$
  - angular directions : $l_{\max} = m_{\max} = 10$

Tree-code simulations

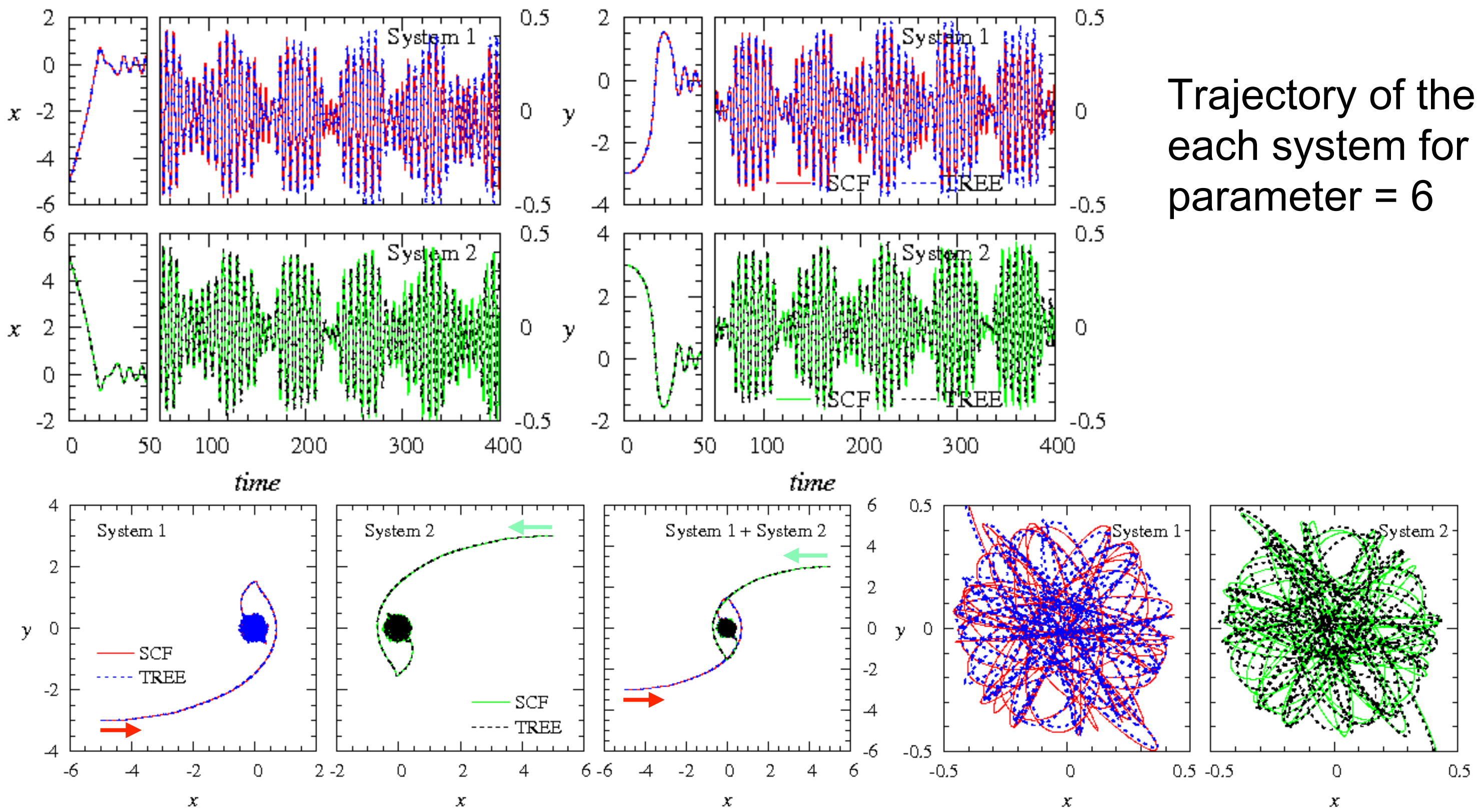
- *Plummer* softening: $\varepsilon = 0.0074$

$$\boldsymbol{F}_i = \sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{G m_i m_j (\boldsymbol{r}_j - \boldsymbol{r}_i)}{(|\boldsymbol{r}_j - \boldsymbol{r}_i|^2 + \varepsilon^2)^{3/2}}$$

  as a mean interparticle separation within the half mass radius

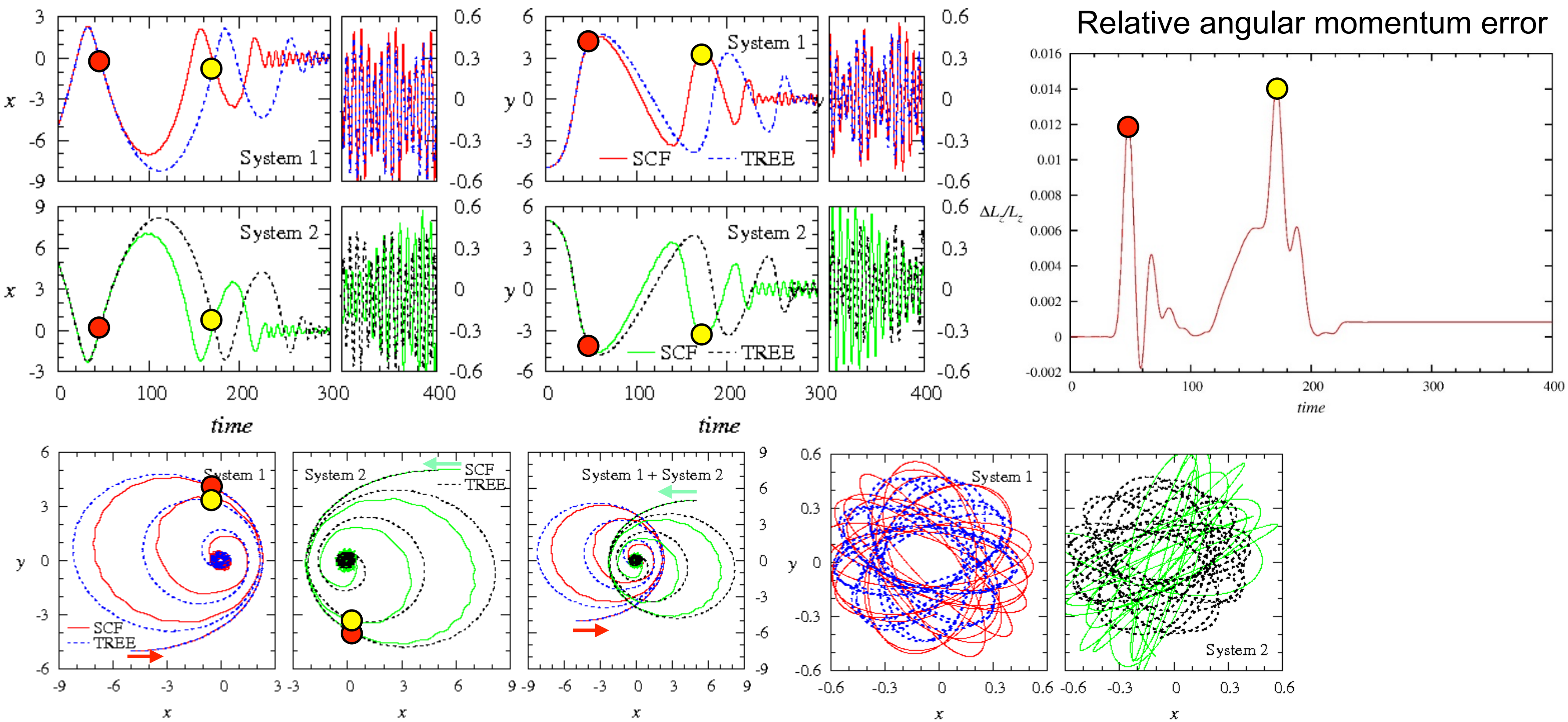
- *Tolerance* parameter: $\theta = 0.5$
- Including up to *quadrupole* terms

Merging Simulation for a Small Impact Parameter



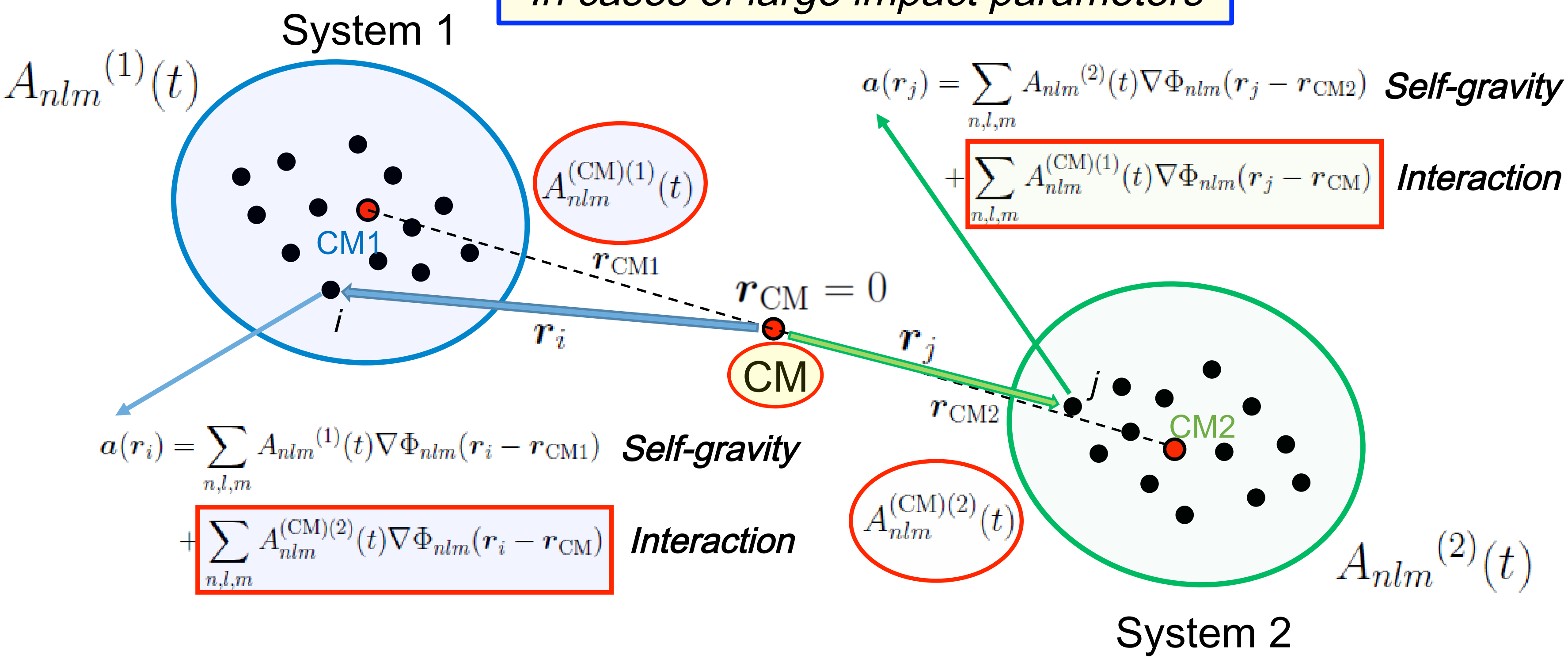
Trajectory of the center of each system for impact parameter = 6

Merging Simulation for a Large Impact Parameter

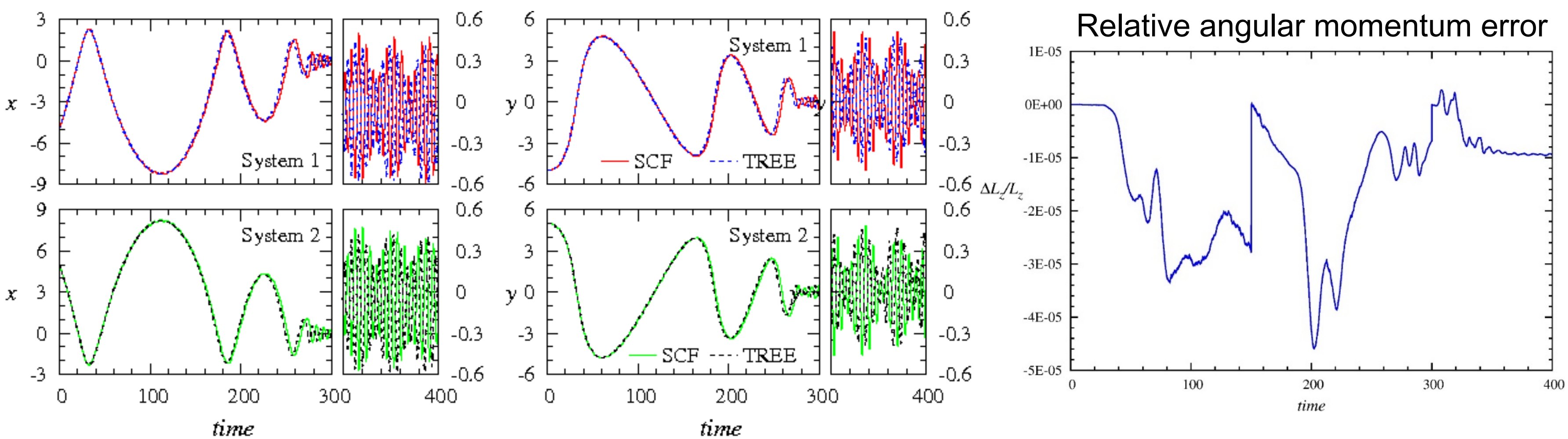


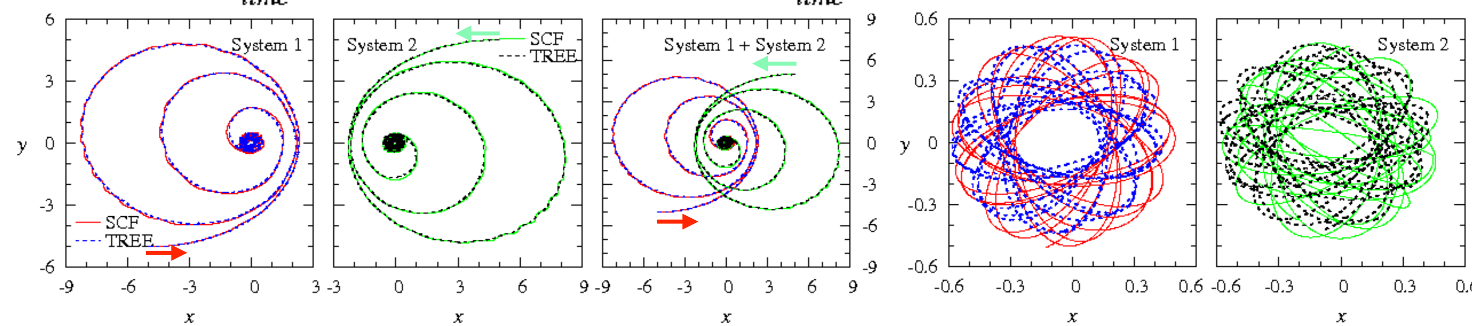
Change of Expansion Center for Interaction Forces

In cases of large impact parameters



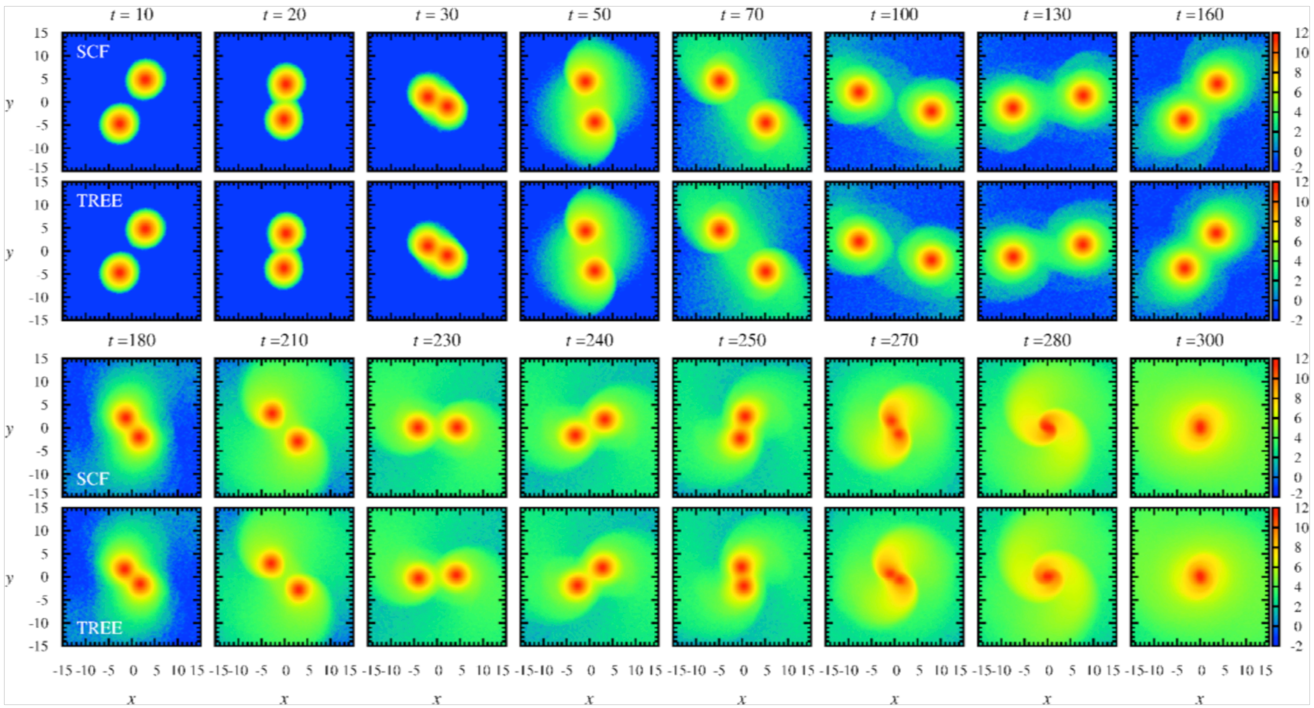
Improved Merging Simulation for a Large Impact Parameter





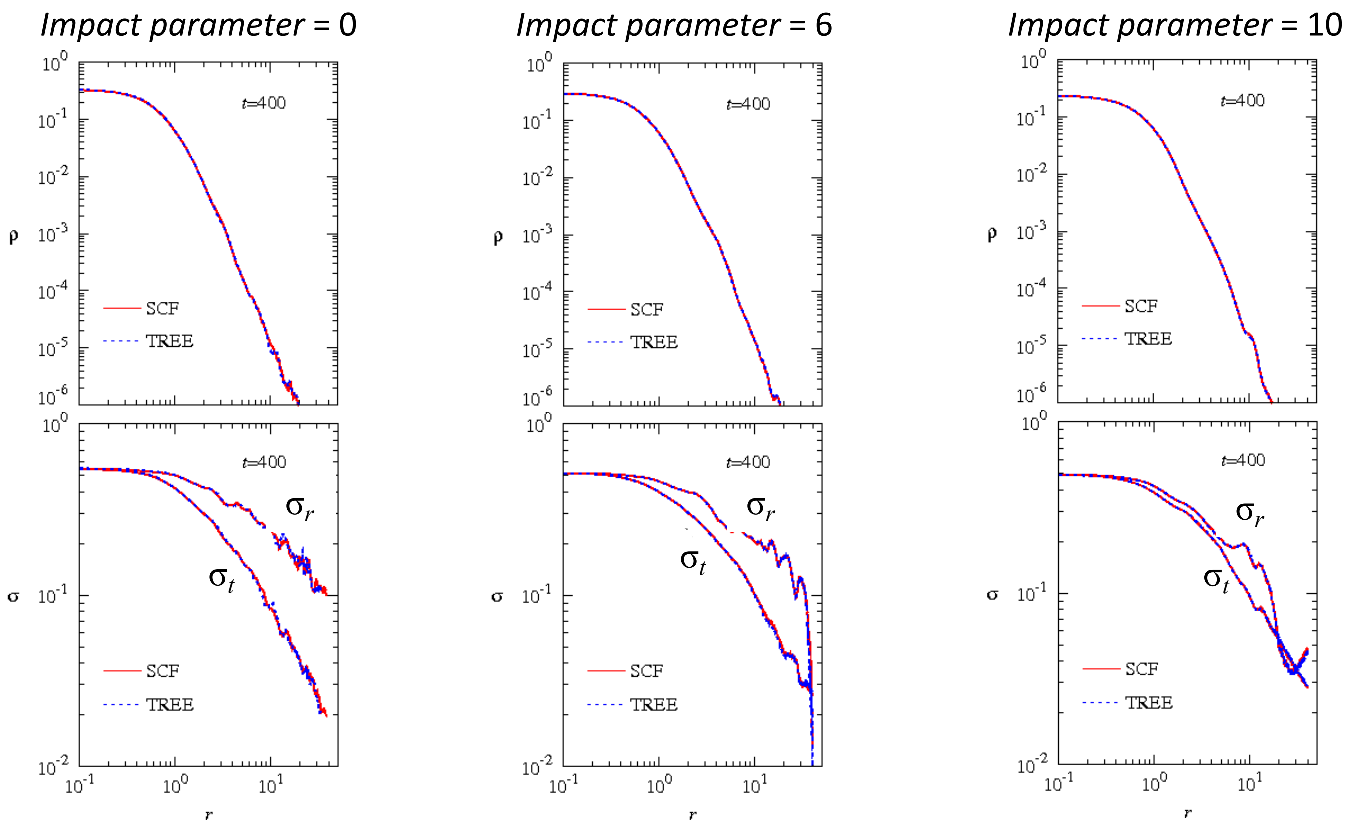
Interactions:
n_{max}=28
l_{max}=m_{max}=28

Time Evolution of Merging Systems for the Large Impact Parameter



- Density evolution of Systems 1 and 2 for impact parameter = 10
- Interaction forces are calculated by expanding the density and potential with respect to the center of mass of the total system with n_{max}=28, and l_{max}=m_{max}=28

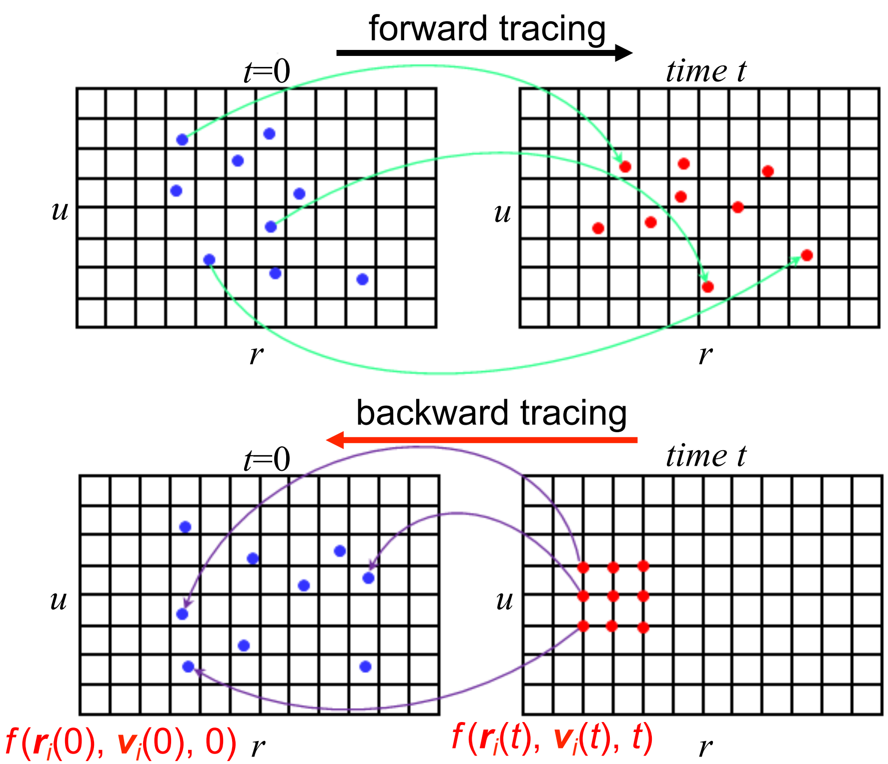
Density and Velocity Dispersion Profiles



2. Construction of Phase Space

Construction of Phase Space with an SCF Method

Hozumi, 1997, ApJ, **487**, 617



- Forward tracing

  Expansion coefficients at each time *t*

  $$A_{nlm}(t) = \sum_{k=1}^{N} m_k \, \Phi_{nlm}(\boldsymbol{r}_k)$$

  are saved.

- Barckward tracing

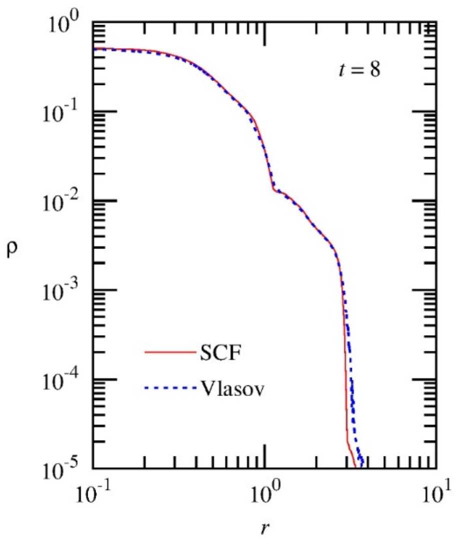
  Using $A_{nlm}(t)$, necessary orbits are traced from time *t* backward to *t* =0, and according to *Liouville's theorem*,

  $$f(\boldsymbol{r}_i(t),\, \boldsymbol{v}_i(t),\, t) = f(\boldsymbol{r}_i(0),\, \boldsymbol{v}_i(0),\, 0)$$

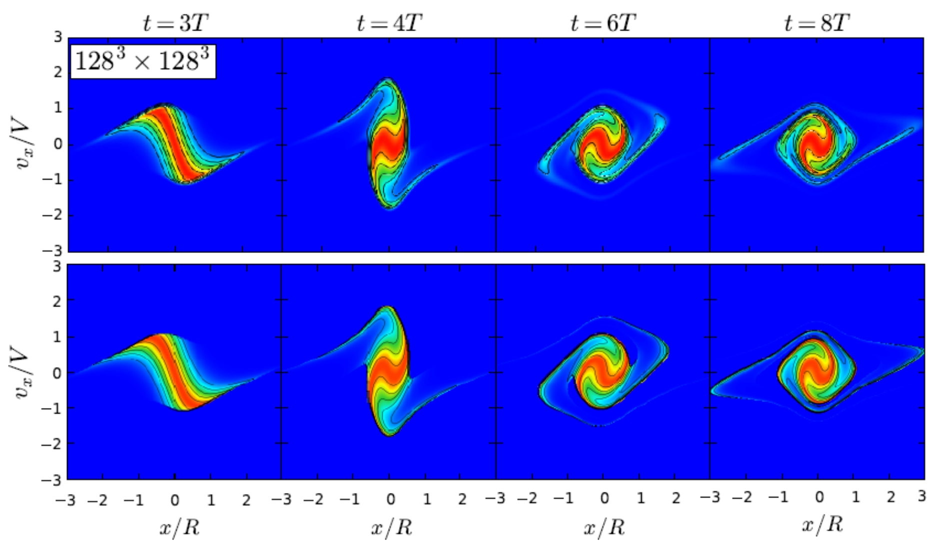
  DF is constructed.

Evolution of the Collapse of a Uniform-Density Sphere in Phase Space – Asymmetric Case

- *3-D Collapse with* $|2T/W| = 1/2$



Spherically averaged density profile

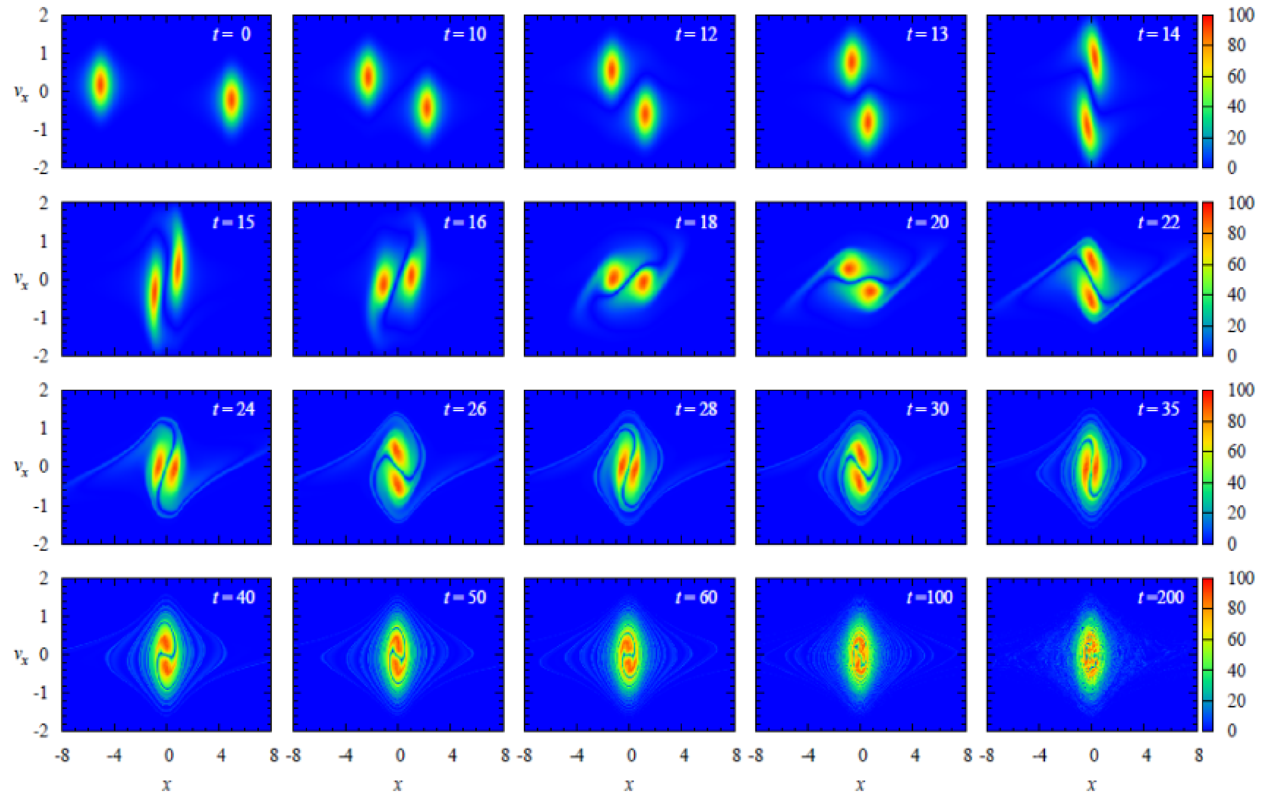


x-v_x plane with $y = z = 0$ and $v_y = v_z = 0$

Vlasov simulation:
$N_r \times N_v = 128^3 \times 128^3$
(*Tanaka et al.*, 2017, ApJ, in press)

SCF simulation:
Clutton-Brock's basis set
N = 10 044 000
n_{max} = 64, l_{max} = m_{max} = 12

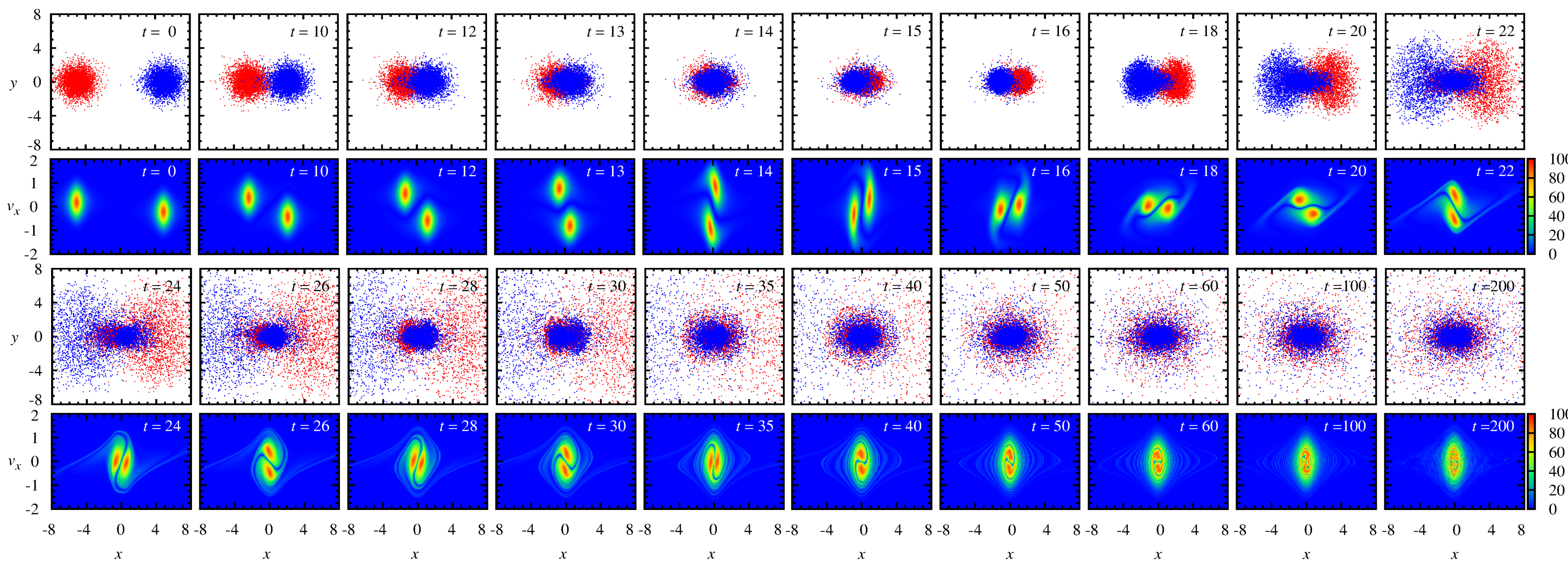
Evolution of Head-On Colliding Systems in Phase Space



x-v_x plane with $y=z=0$ and $v_y=v_z=0$

- $N=10\ 000\ 584$/system
- $n_{\max}=16$, $l_{\max}=m_{\max}=10$

Evolution of Head-On Colling Systems in Configuration and Phase Spaces



Summary

- SCF method can be applied to merging simulations.
- Phase-space evolution in merging processes can be reproduced with SCF method.
- (SCF simulation can avoid the ill-effects of gravitational softening, needed by tree-code simulation, on merging simulations in large impact parameter cases.)
- (SCF simulation is at least twice as fast as tree-code simulation.)