Simulating the Collisionless Dynamics of Dark Matter

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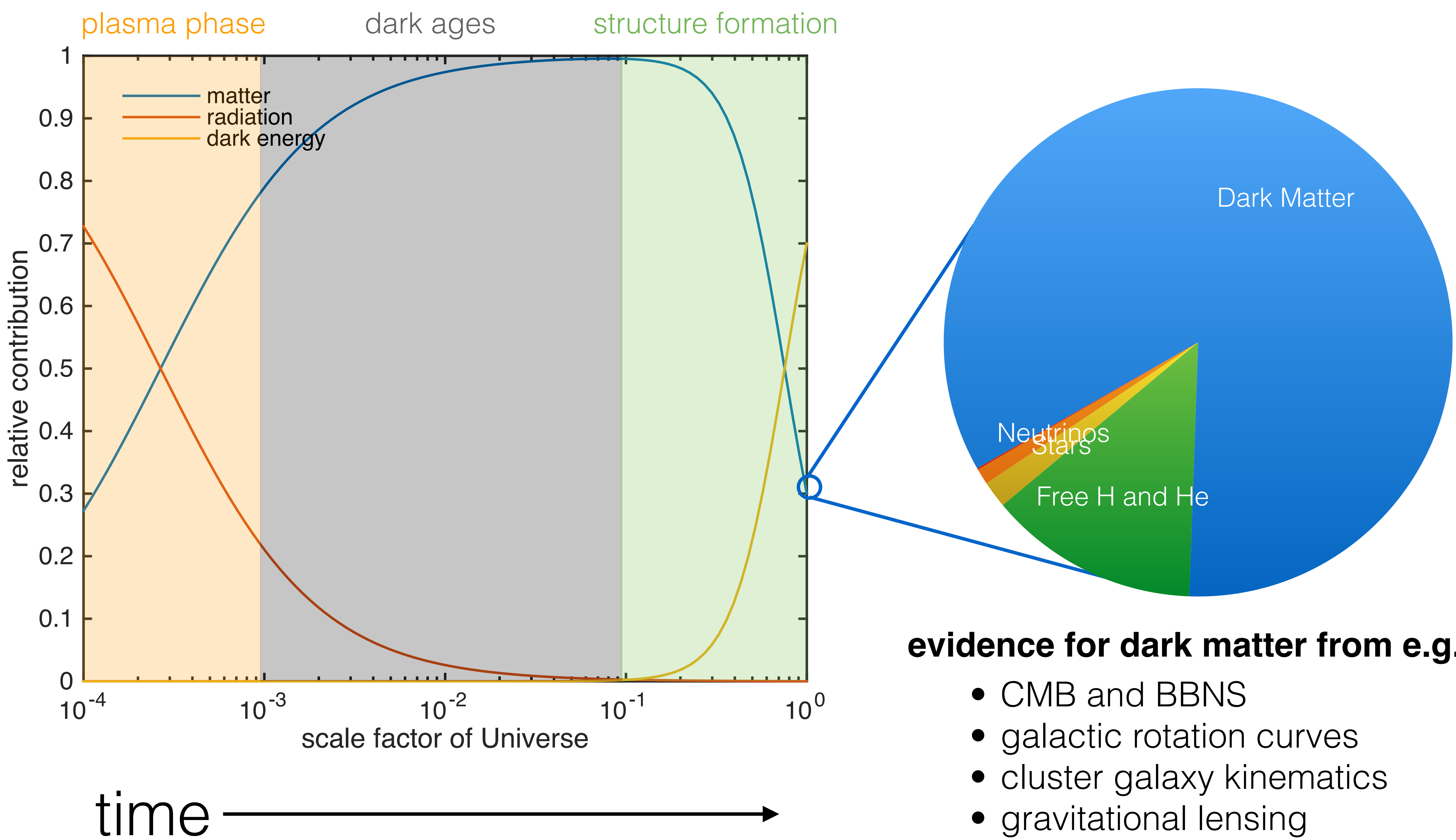
what is dark matter?

Dark matter -
the dominant gravitating component in the Universe



evidence for dark matter from e.g.

- CMB and BBNS
- galactic rotation curves
- cluster galaxy kinematics
- gravitational lensing
- galaxy clustering
- …

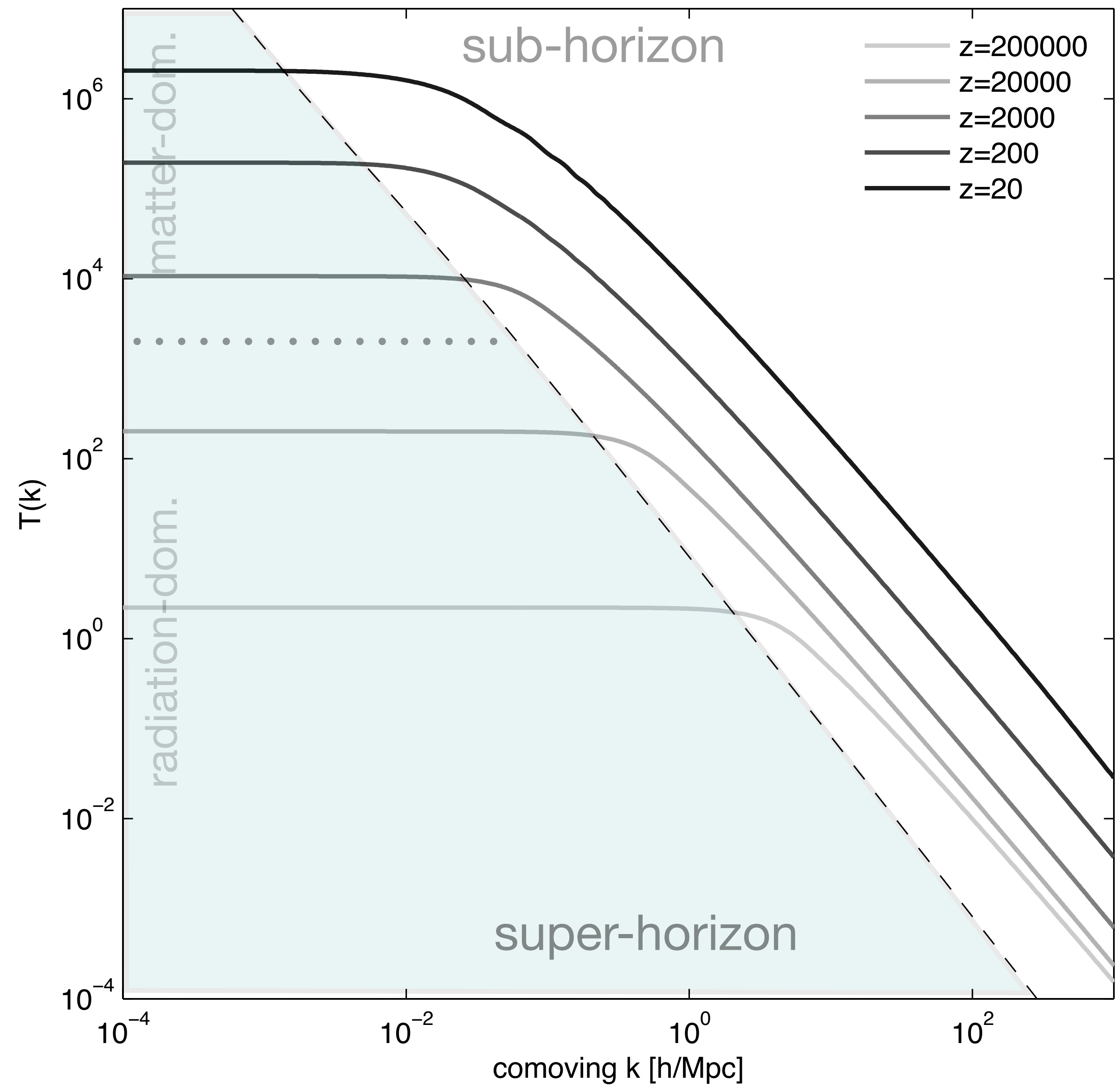
Linear evolution in early Universe

Inflation leads to near scale-invariant primordial density spectrum

$$P_{\mathrm{prim}}(k) = \left\langle \delta\bar{\delta} \right\rangle \propto k^{n_s} \qquad n_s \lesssim 1$$

Gets processed by growth on sub- and super-horizon scales (GR):

$$P_{\mathrm{late}}(k) \propto T^2(k)\, P_{\mathrm{prim}}(k)$$

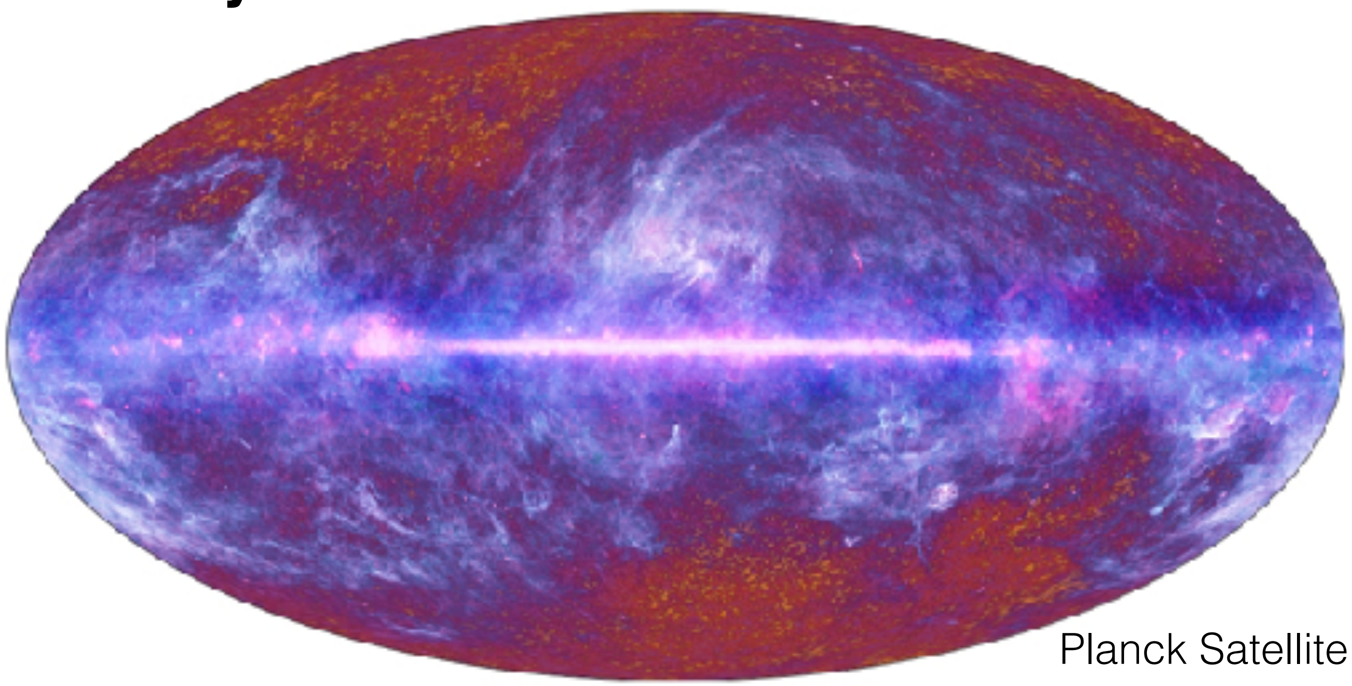


Multi-species fluid of CDM+baryon+photon+neutrino
→**linear** Boltzmann solver
(e.g. Ma & Bertschinger 1995)

Predict CMB and initial conditions

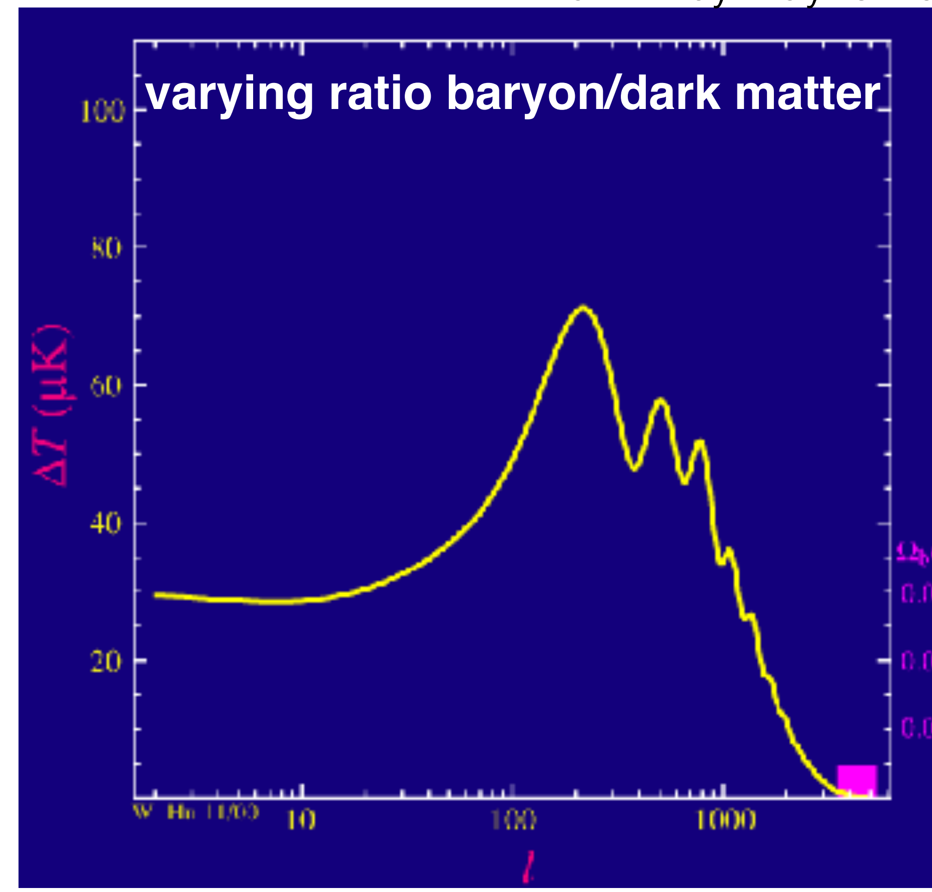
Evidence for Dark Matter from CMB

Full sky in Microwaves

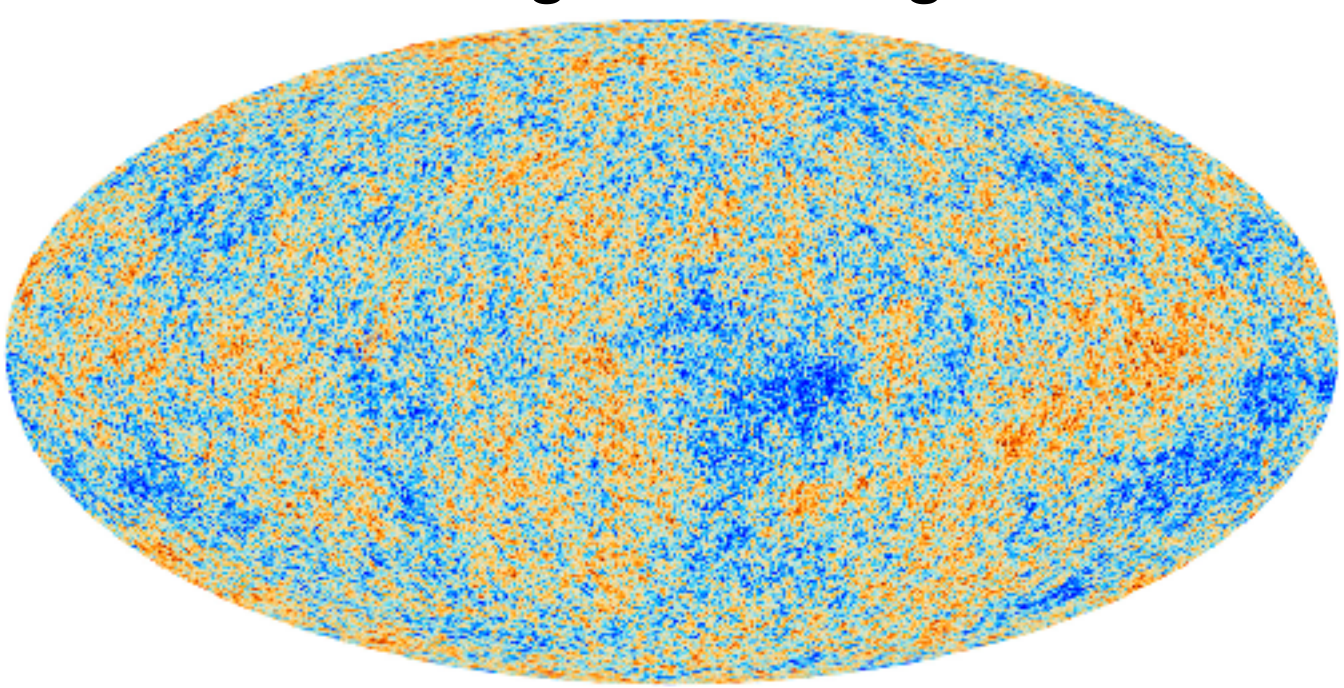


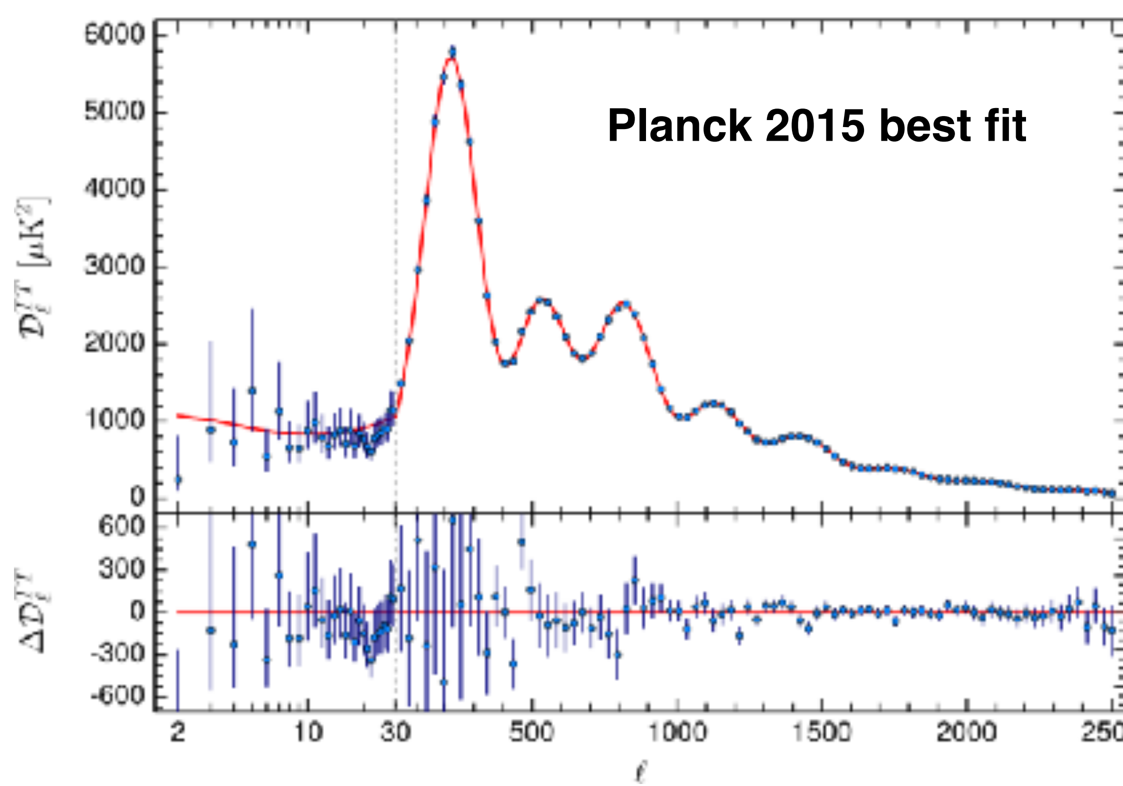
Planck Satellite

anim. by Wayne Hu



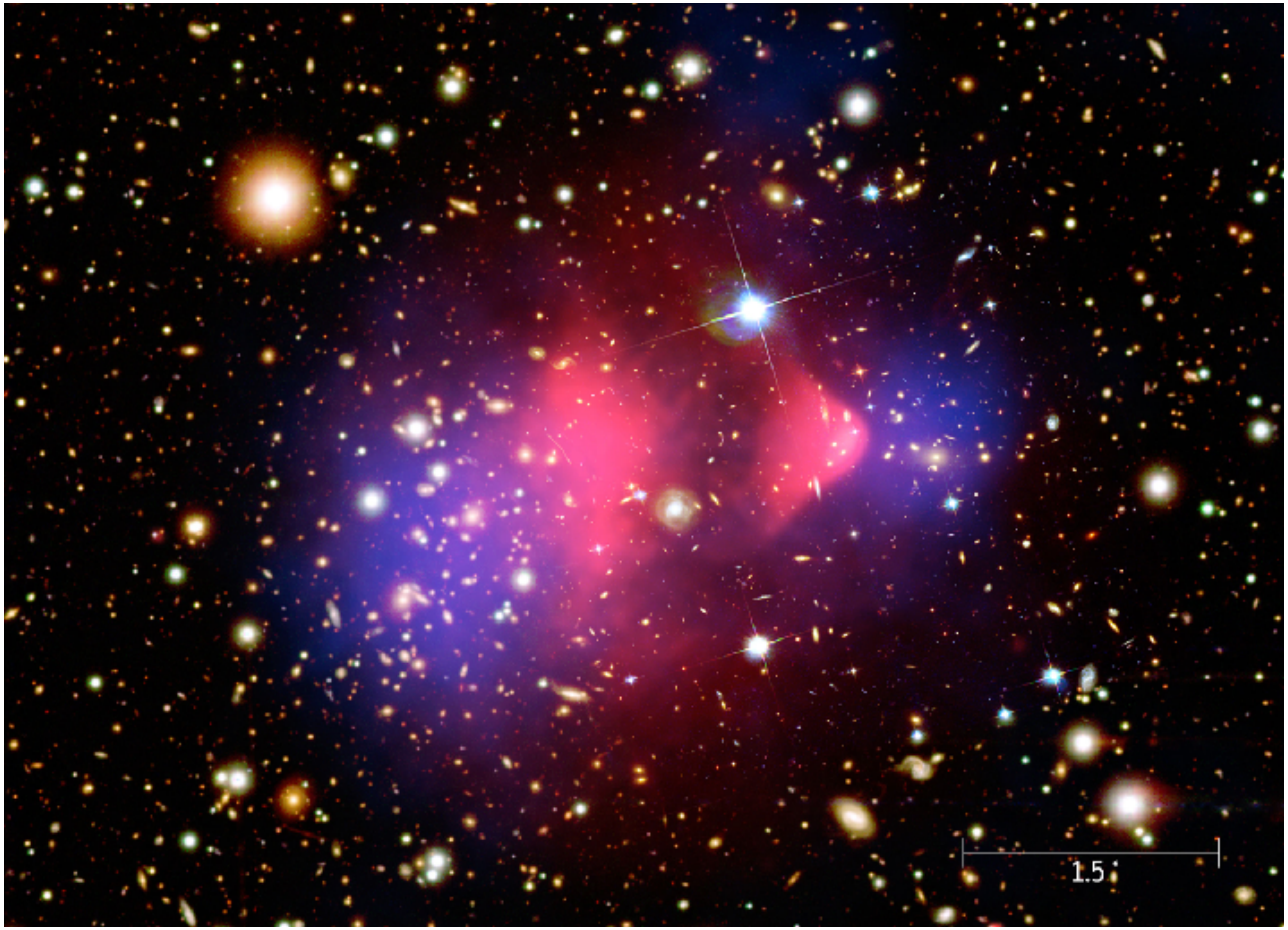
After removal of galactic foregrounds





What do we know about the properties of dark matter?

- **not baryonic matter (e.g. BBNS)**
- **collisionless (or nearly):**

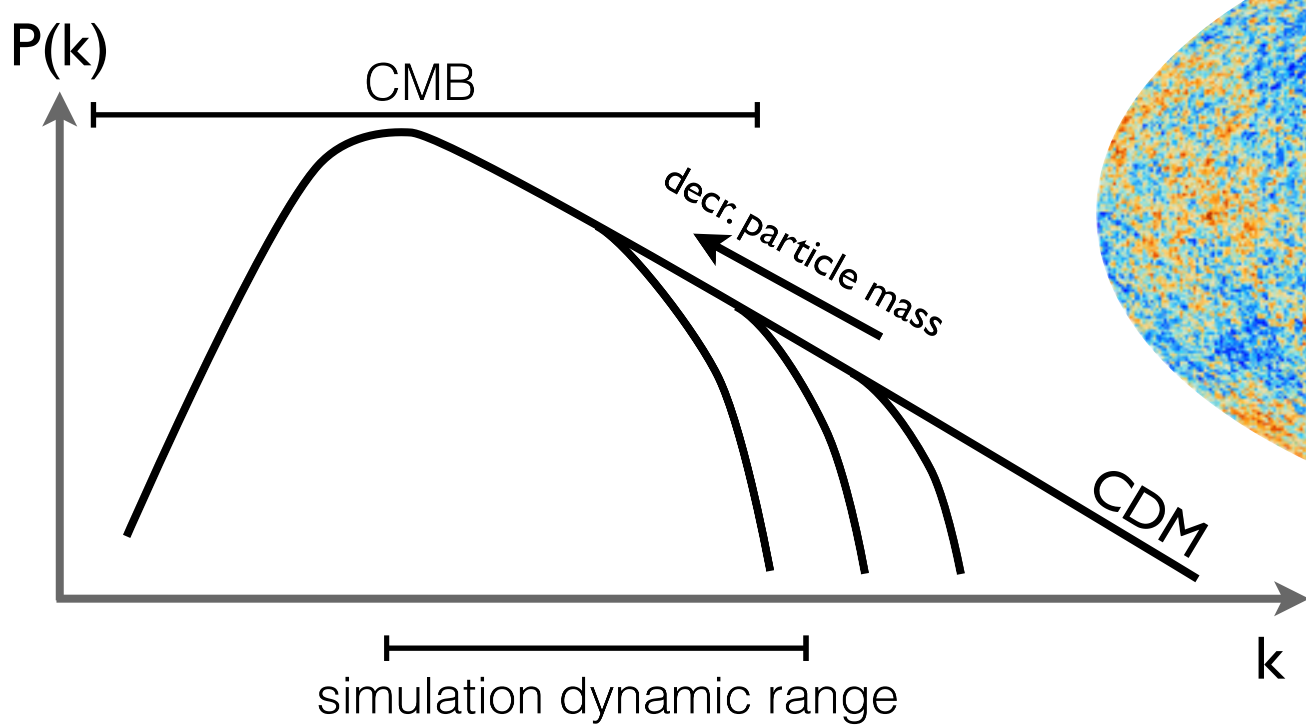


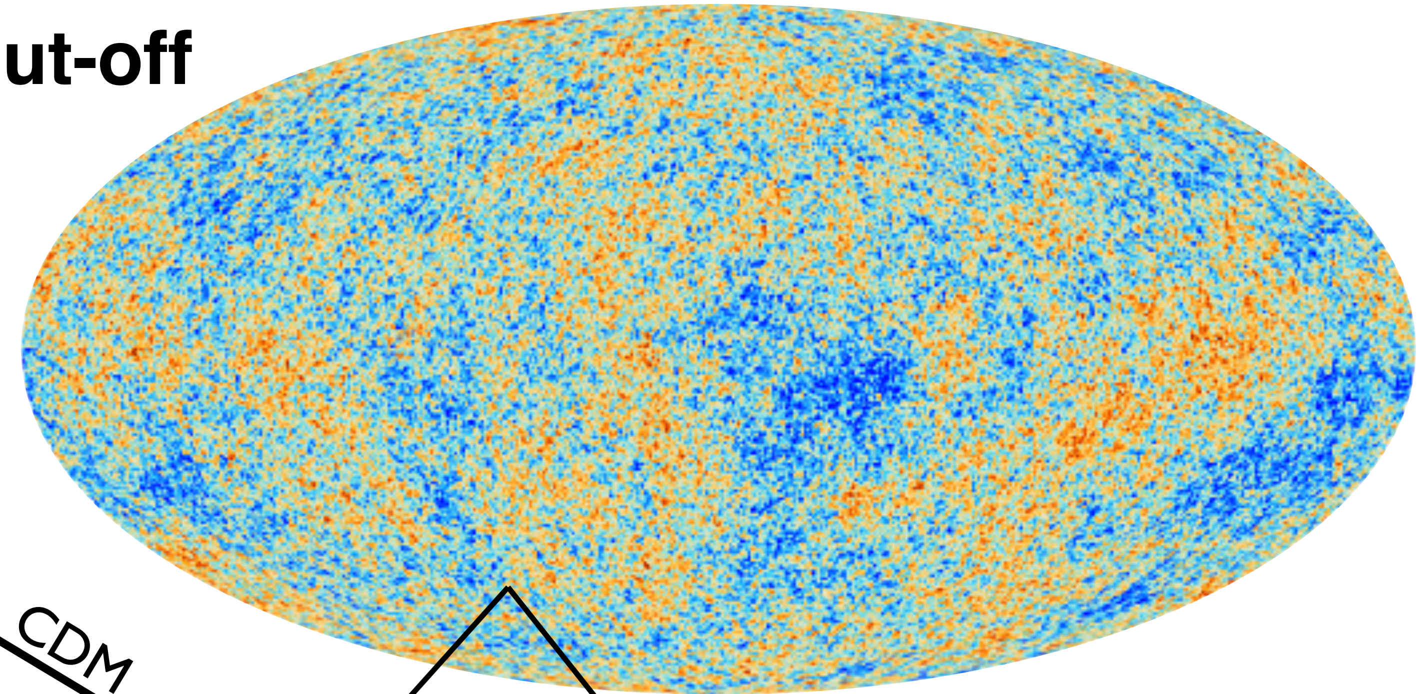
Bullet cluster - 1E 0657−558

NASA/CXC/M. Weiss

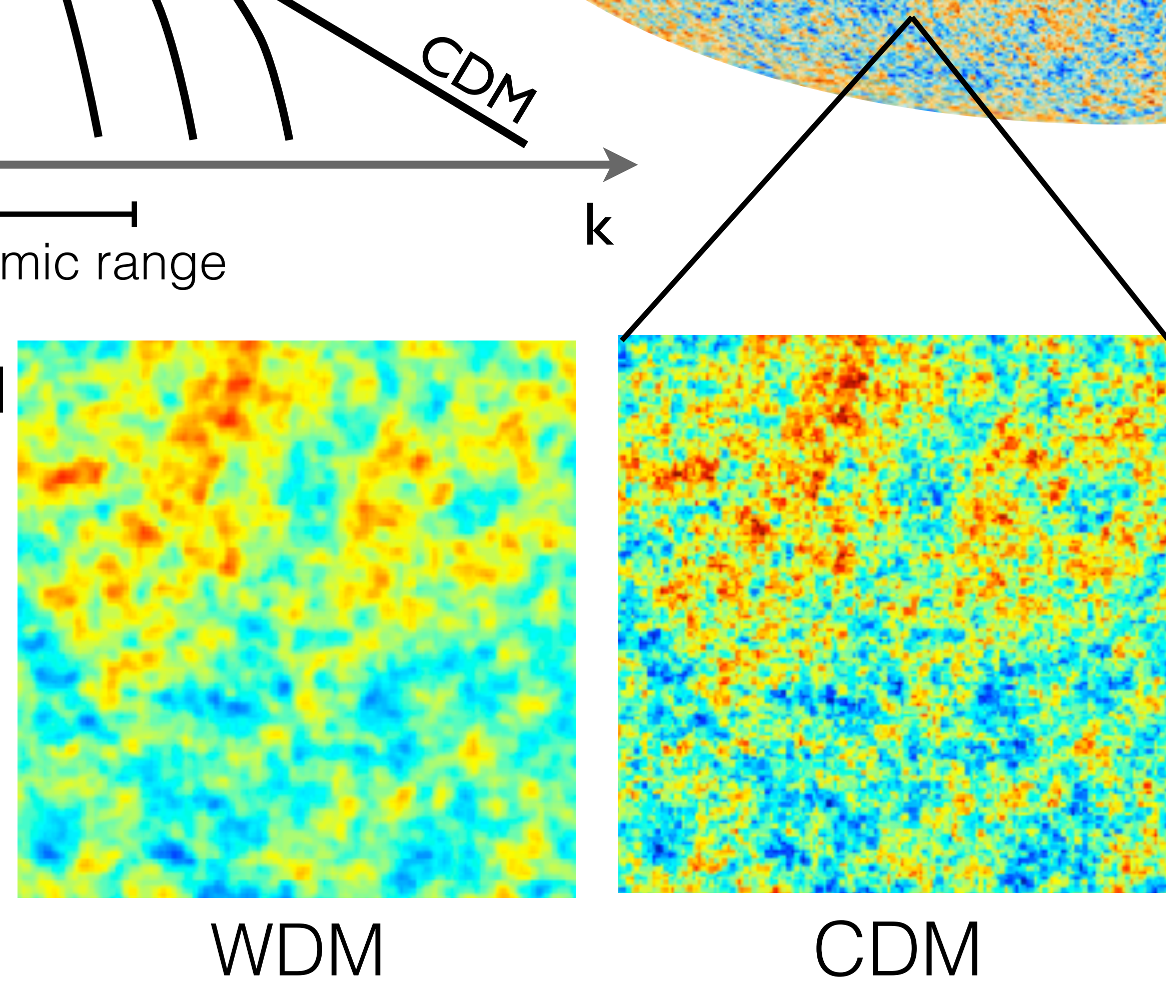
Kinetic temperature of DM — CDM, WDM, HDM, …

- **is encoded in the power spectrum (free streaming)**
- **but Jeans scale much below cut-off**





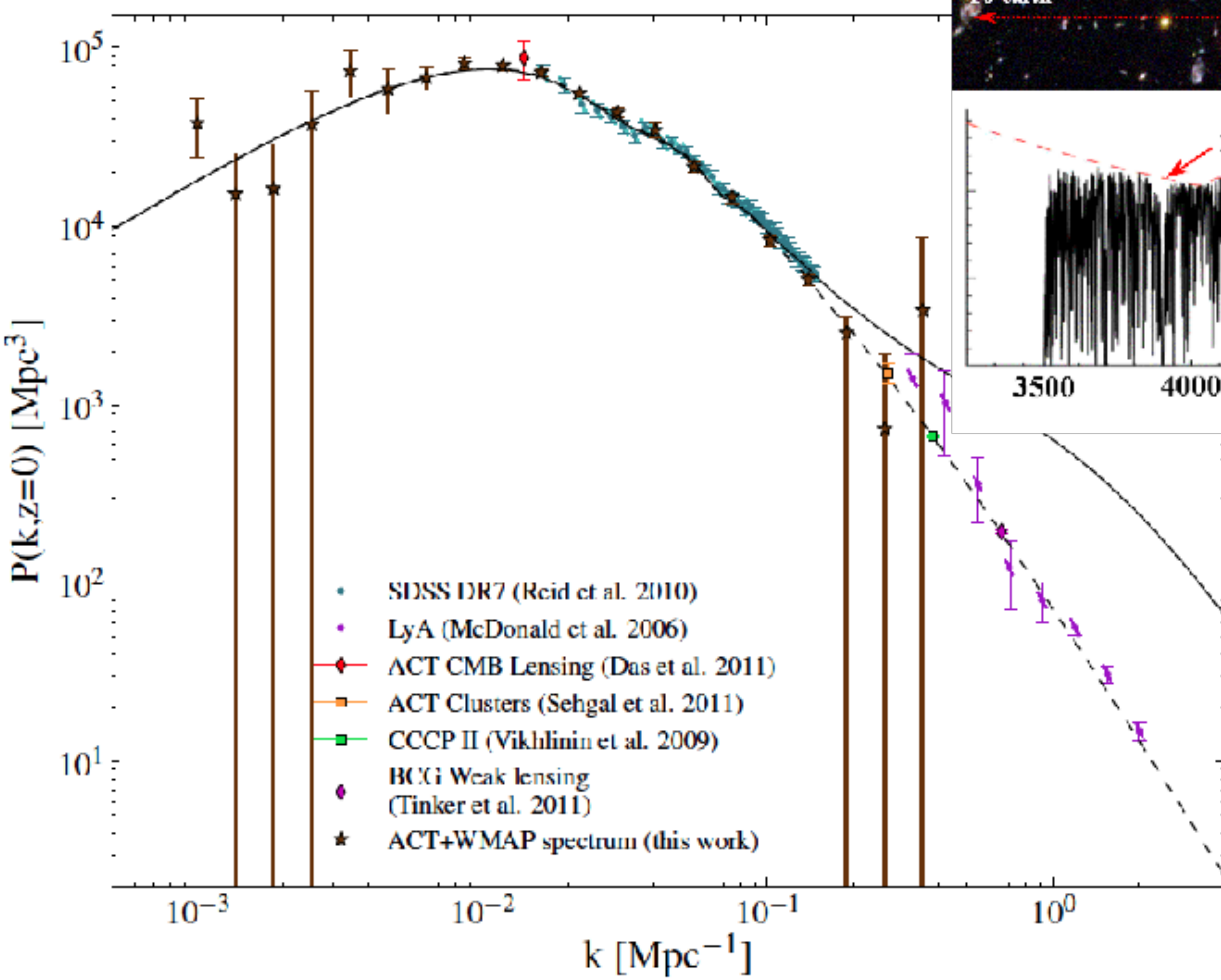
Underlying DM density:

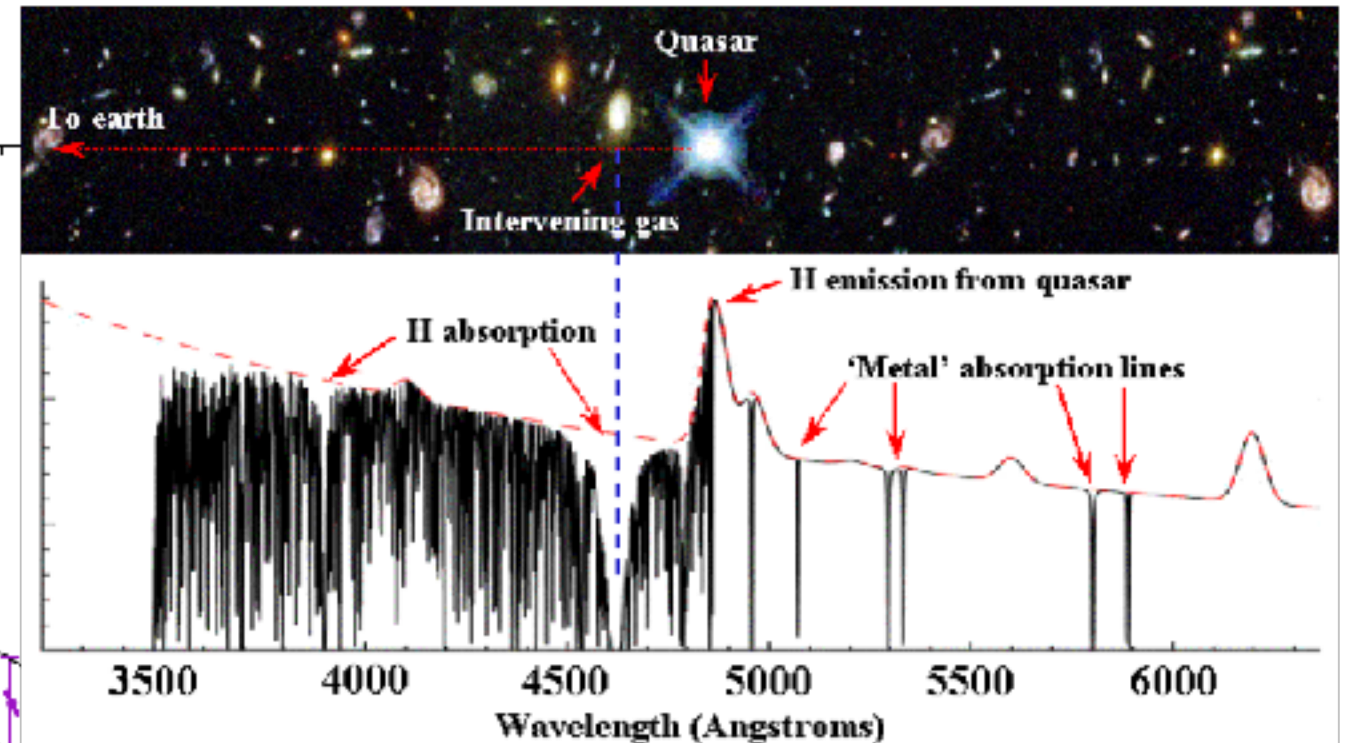


What do we know about the properties of dark matter?

- **kinetic temperature must be cold(ish)**

Hlozek et al. 2012

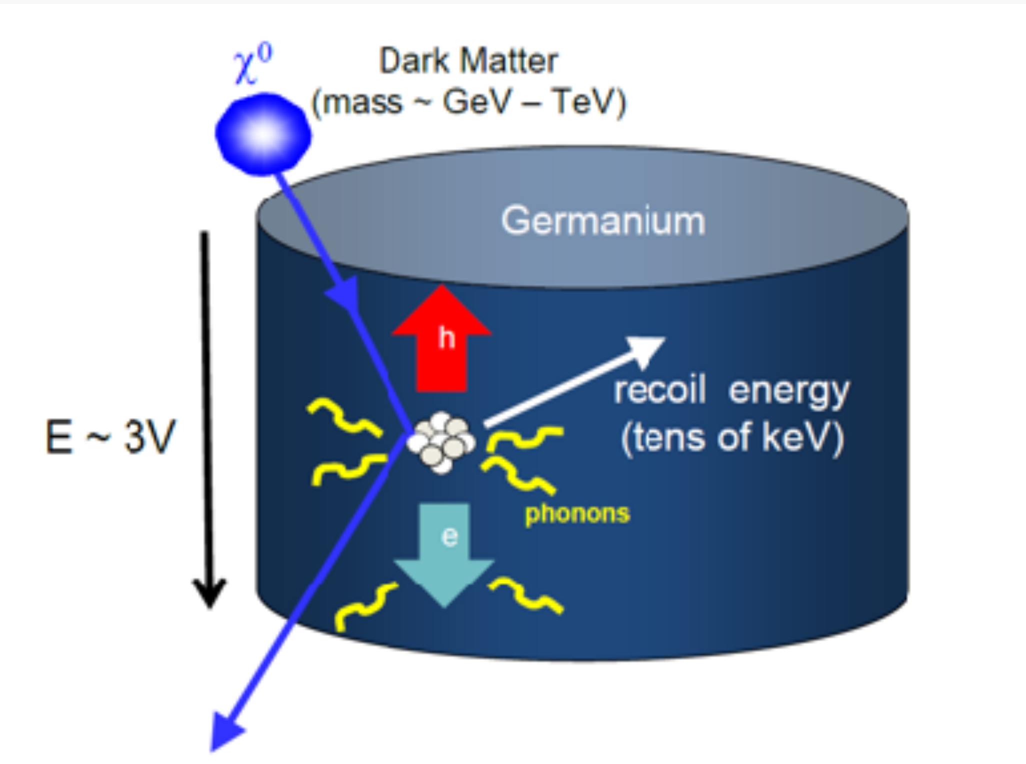




- best constraints from Ly-α forest (e.g. Viel et al. 2013) >~ 5keV
- constrains WDM, FDM

Searches for Dark Matter: Direct/Indirect Detection

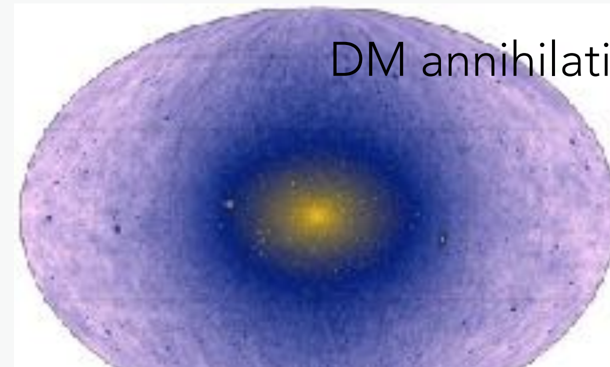
Search for interaction with normal matter



sensitive to n_{DM} and vel.

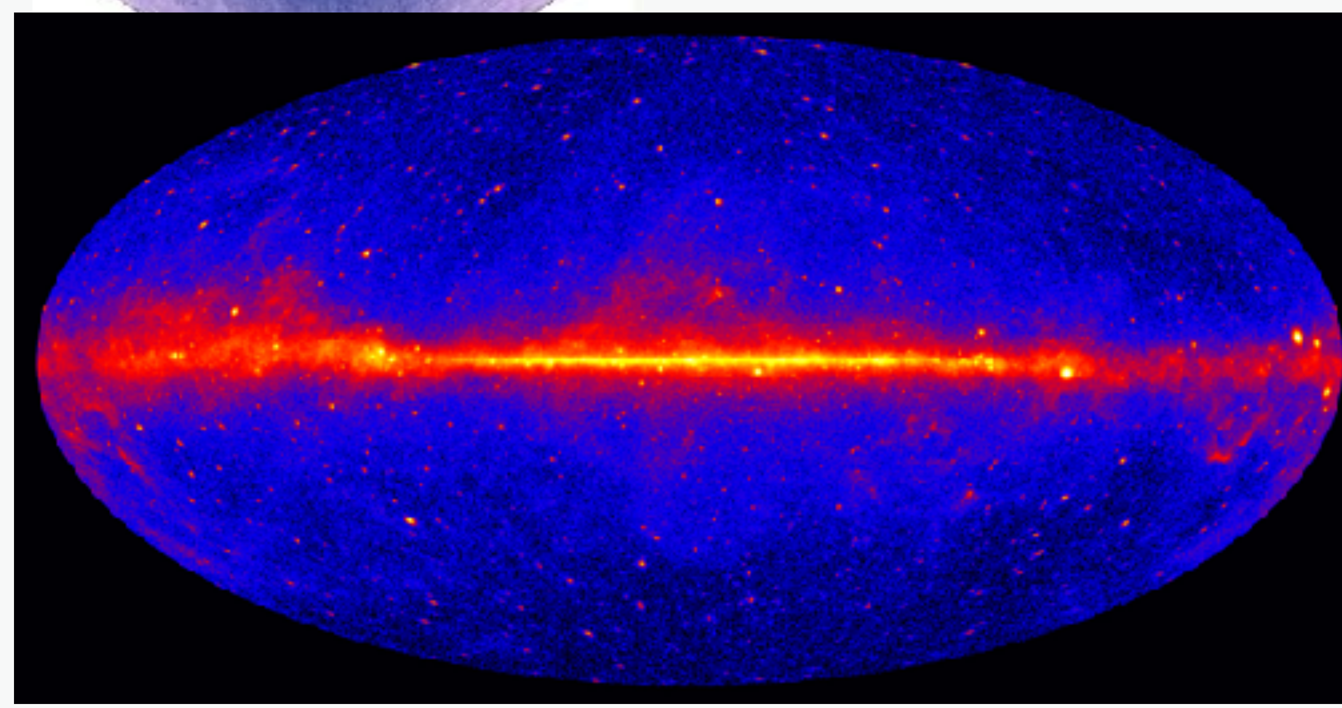
Synthesis / Search for particles beyond standard model

Search for gamma-ray excess due to annihilation



DM annihilation luminosity (e.g. M. Kuhlen et al.)

Fermi LAT gamma ray sky



sensitive to n_{DM}^2 ,
so density profiles of haloes matter

Need accurate prediction for
profiles of small haloes and their distribution.

So, what is Dark Matter?

for our macroscopic purposes it suffices to assume that

microscopic → continuum limit

proton = 1GeV, WIMP 100GeV? -> 10^{21}/g

cold (or at most lukewarm) → $v_{thermal} \ll v_{bulk}$

e.g. thermally produced at very early times, cooled since then

negligible cross-section → $\sigma_{DM} \ll \sigma_{em}$ collisionless

weak-scale or even weaker

Compton wavelength << scales → classical

self-gravitating

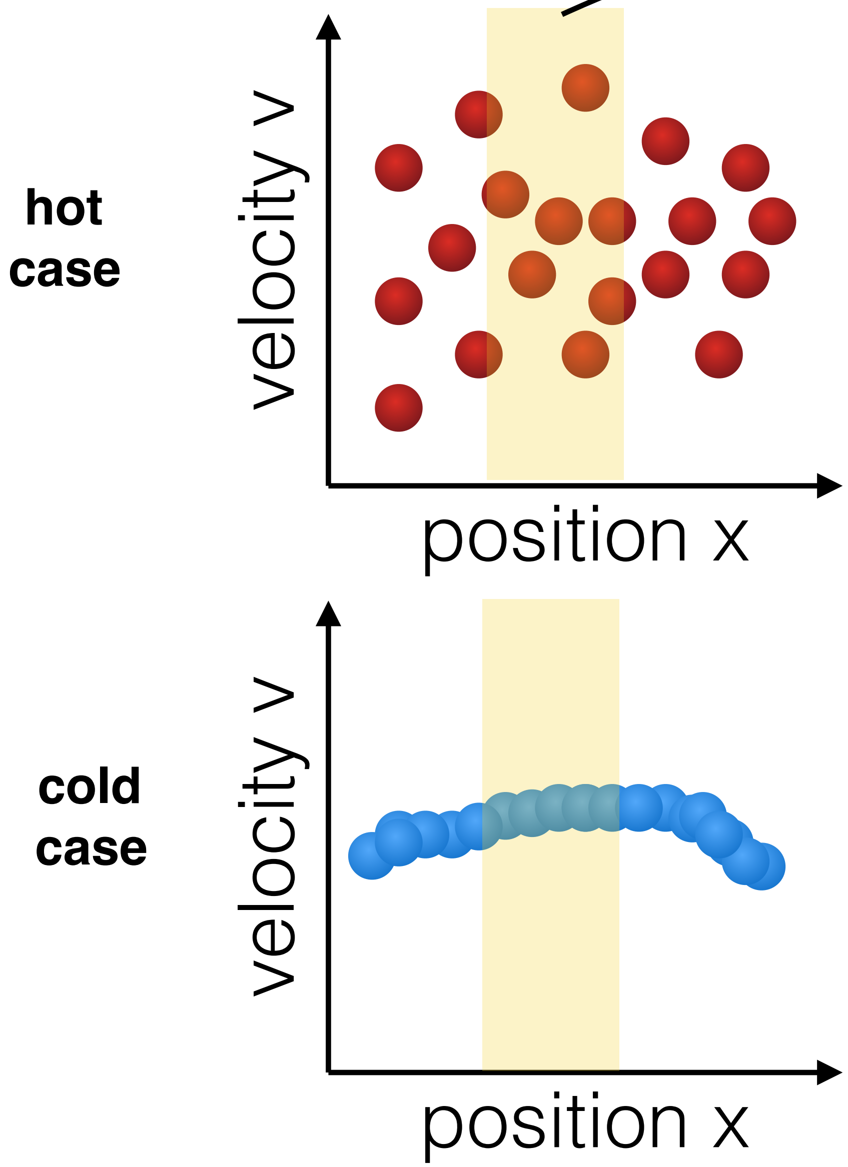
(but in principle any of these can be dropped!)

DM simulations N-body… and beyond…

Kinetic description of dark matter

Density of particles in phase space given by

= distribution function $f(\mathbf{x}, \mathbf{v}, t)$



phase space of n+n dim:

generally,
f is truly 2n-dimensional

in cold limit,
f is only n-dimensional
= **monokinetic**

Kinetic description in terms of Vlasov-Poisson

Density of particles in phase space

= distribution function $f(\mathbf{x}, \mathbf{v}, t)$

Evolution governed by **Boltzmann equation**

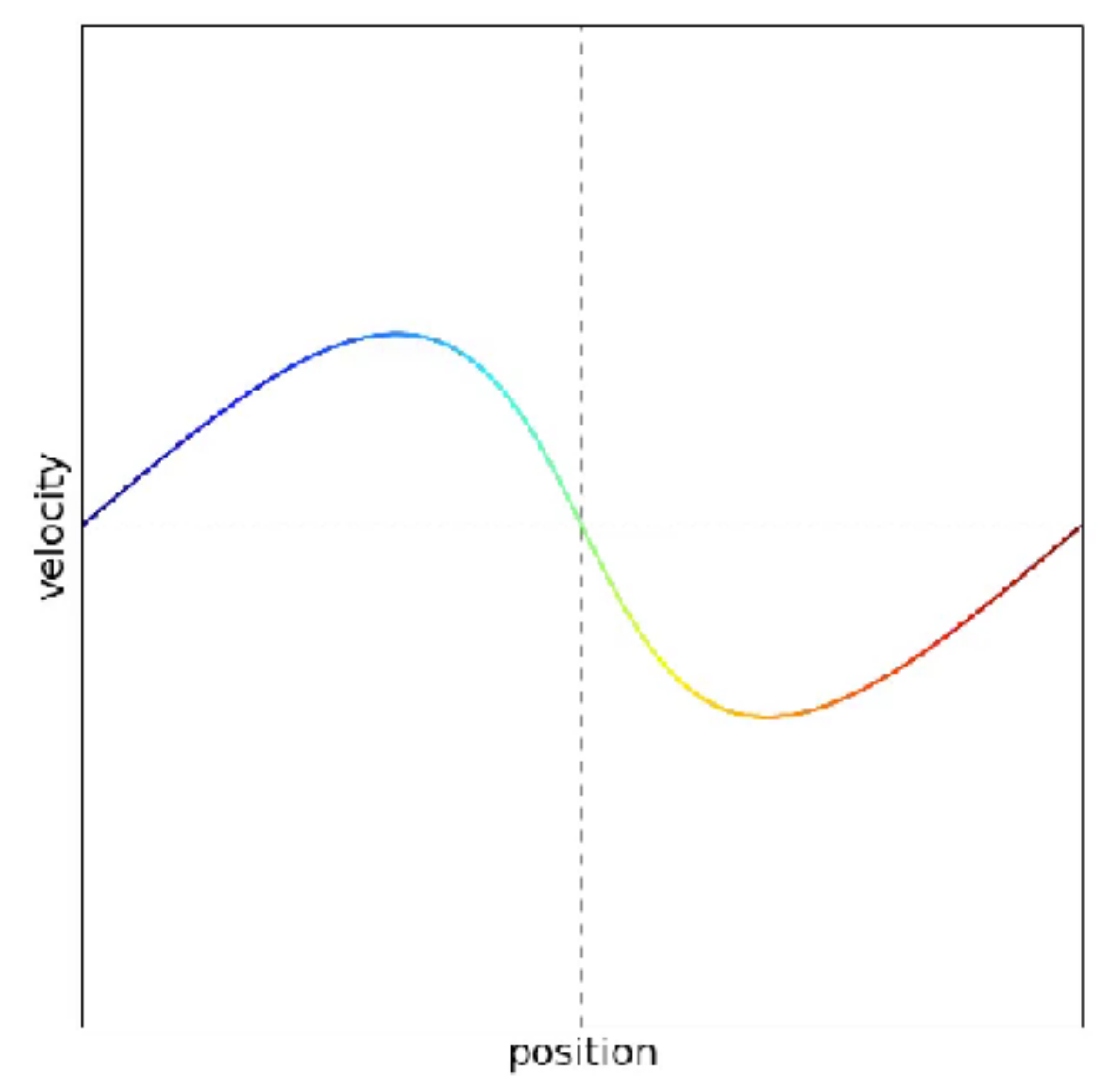
$$\frac{\partial f}{\partial t} + \underbrace{\frac{\mathbf{v}}{a^2} \cdot \boldsymbol{\nabla}_{\mathbf{x}} f}_{\text{velocities advect in configuration space}} - \underbrace{\boldsymbol{\nabla}_{\mathbf{x}} \phi \cdot \boldsymbol{\nabla}_{\mathbf{v}} f}_{\text{grav. forces advect in momentum space}} = \underbrace{C[f]}_{\text{particle-particle interactions } \mathbf{= 0}}$$

+ Poisson equation for grav. potential

$$\boldsymbol{\nabla}_{\mathbf{v}}^2 \phi = \frac{4\pi G}{a^3} \int (f - \bar{\rho})\, \mathrm{d}^n v$$

for C[f]=0:
Vlasov-Poisson

What is special about a cold-collisionless system?



The 1D structure winds up but never tears or mixes! (neighbours stay neighbours!) topologically preserved

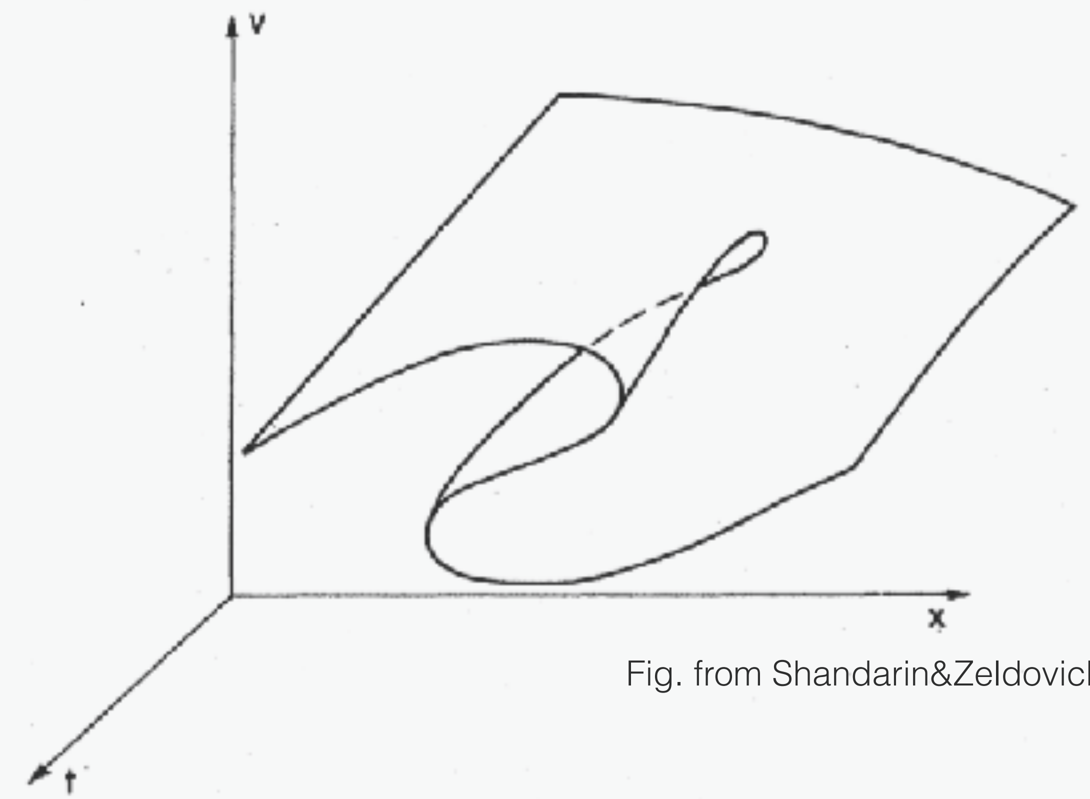


Fig. from Shandarin&Zeldovich 1989

Vanishing collision-term

⇒ not in hydro limit

⇒ velocity can be multi-valued

⇒ cannot stop at low order moments

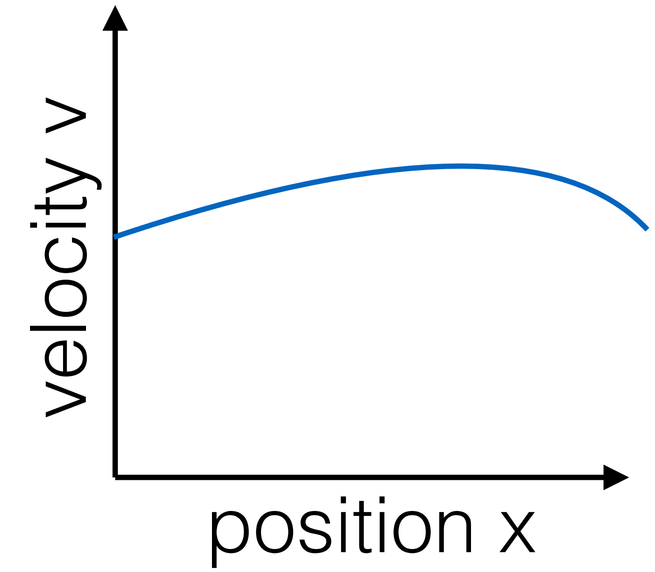
⇒ have to discretize distribution function

⇒ singular caustics emerge (see later)

What about cold fluids in more than 1+1 dimension?

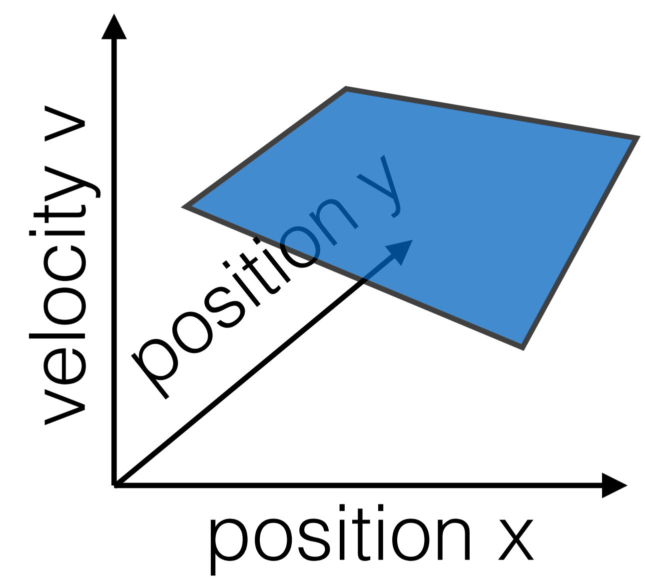
In 1+1D phase space:

f(x,v) is a 1D curve



In 2+2D phase space:

f(x,v) is a 2D surface



In 3+3D phase space:

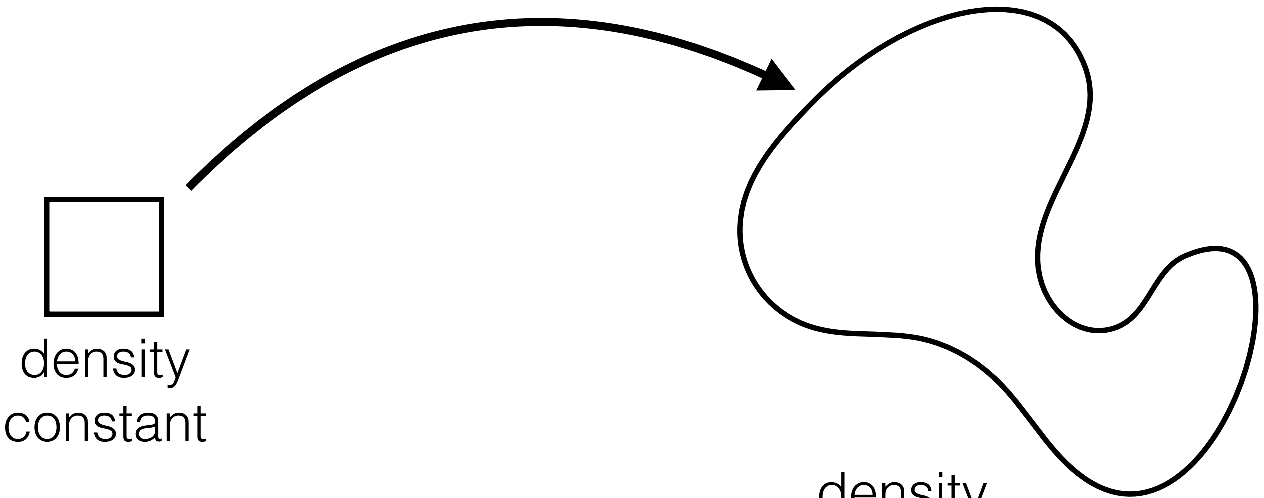
f(x,v) is a 3D hyper-surface

$$\mathbb{Q} \subset \mathbb{R}^3 \to \mathbb{R}^6 \;:\; \mathbf{q} \mapsto (\mathbf{x}_\mathbf{q}(t), \mathbf{v}_\mathbf{q}(t))$$

Lagrangian description

Lagrangian description, evolution of fluid element

$$\mathbb{Q} \subset \mathbb{R}^3 \to \mathbb{R}^6 \;:\; \mathbf{q} \mapsto (\mathbf{x}_\mathbf{q}(t), \mathbf{v}_\mathbf{q}(t))$$



$$\rho = m_{\rm DM} \left| \frac{\partial x_i}{\partial q_j} \right|^{-1}$$

For DM, motion of any point **q** depends only on gravity

$$(\dot{\mathbf{x}}_\mathbf{q}, \dot{\mathbf{v}}_\mathbf{q}) = (\mathbf{v}_\mathbf{q}, -\boldsymbol{\nabla}\phi)$$

unlike hydro, no internal temperature, entropy, pressure

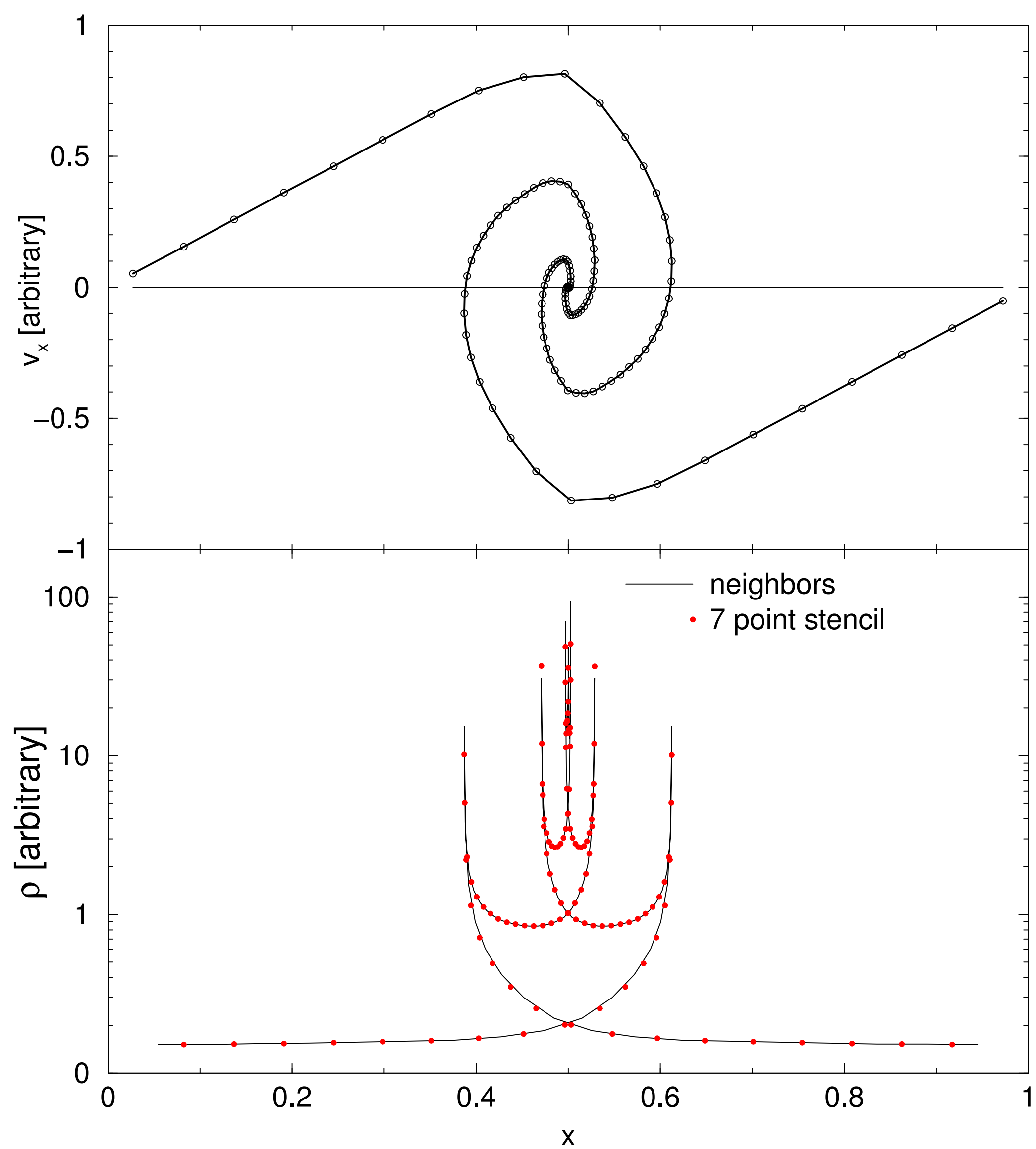
So the quest is to solve Poisson's equation

$$\Delta\phi = 4\pi G \rho$$

So what are those caustics after all??

Density of mapping from Lagrangian to Eulerian space is not guaranteed to be finite!

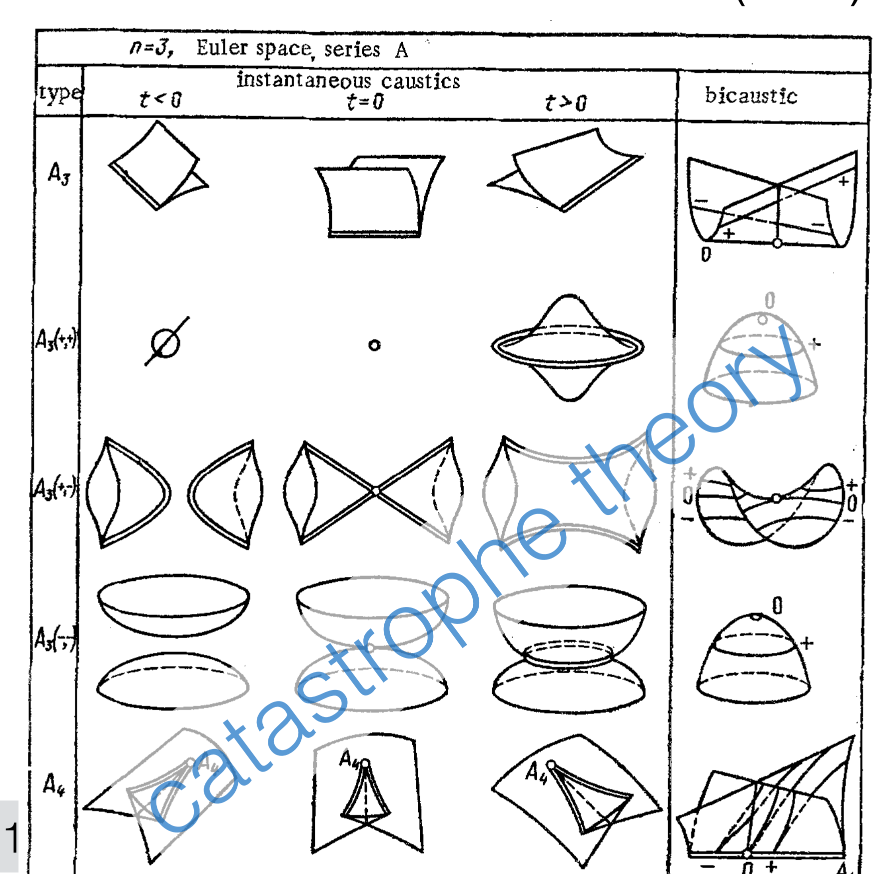
$$\rho = m_{\mathrm{DM}} \left| \frac{\partial x_i}{\partial q_j} \right|^{-1}$$



In 2 and 3d these become entirely non-trivial

See also Hidding et al. 2013

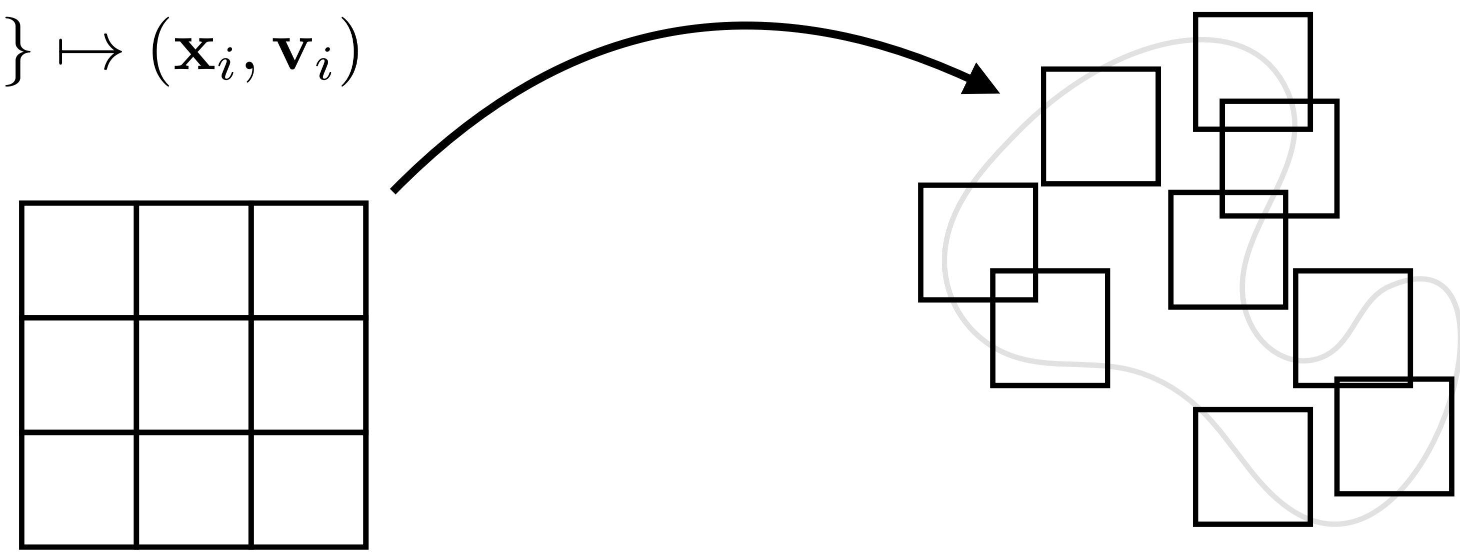
Arnold (1982)



How to solve these systems: the N-body approach…

The N-body approximation:
cover distribution function with N ‘coarse-graining’ particles

$$i \in \{1 \ldots N\} \mapsto (\mathbf{x}_i, \mathbf{v}_i)$$



⇒ EoM are just Hamiltonian N-body eq. (method of characteristics)

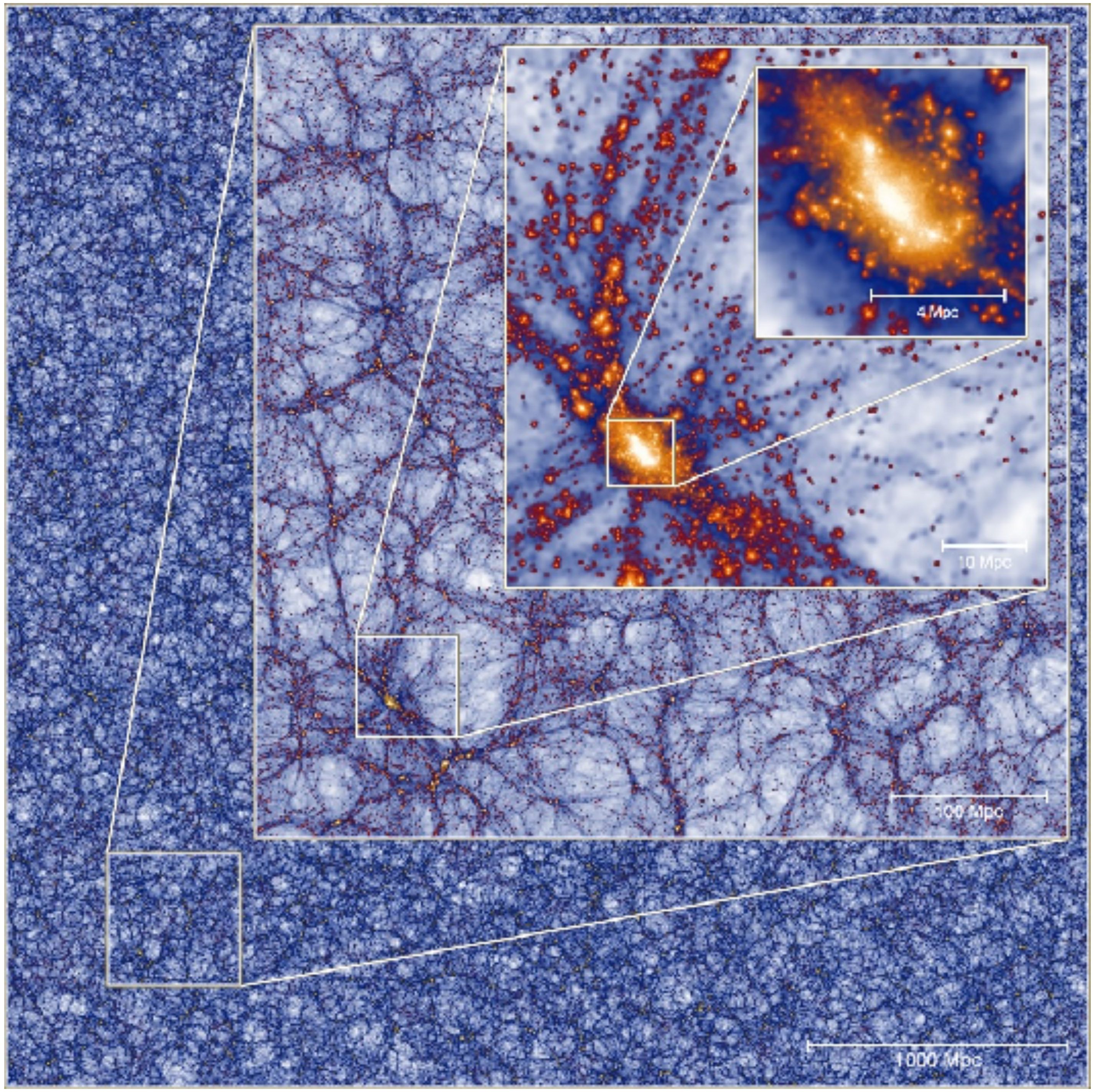
for small N, density field is poorly estimated,

$$\rho = m_p \sum \delta_D(x - x_i) \otimes W$$

continuum structure is given up, but ‘easy’ to solve for forces

hope that as N->very large numbers, approach collisionless continuum, but always ad hoc choice of W

Huge successes! Predicting the LSS of the Universe



Angulo et al. 2012

Input:
Powerspectrum of perturbations
+cosmological model

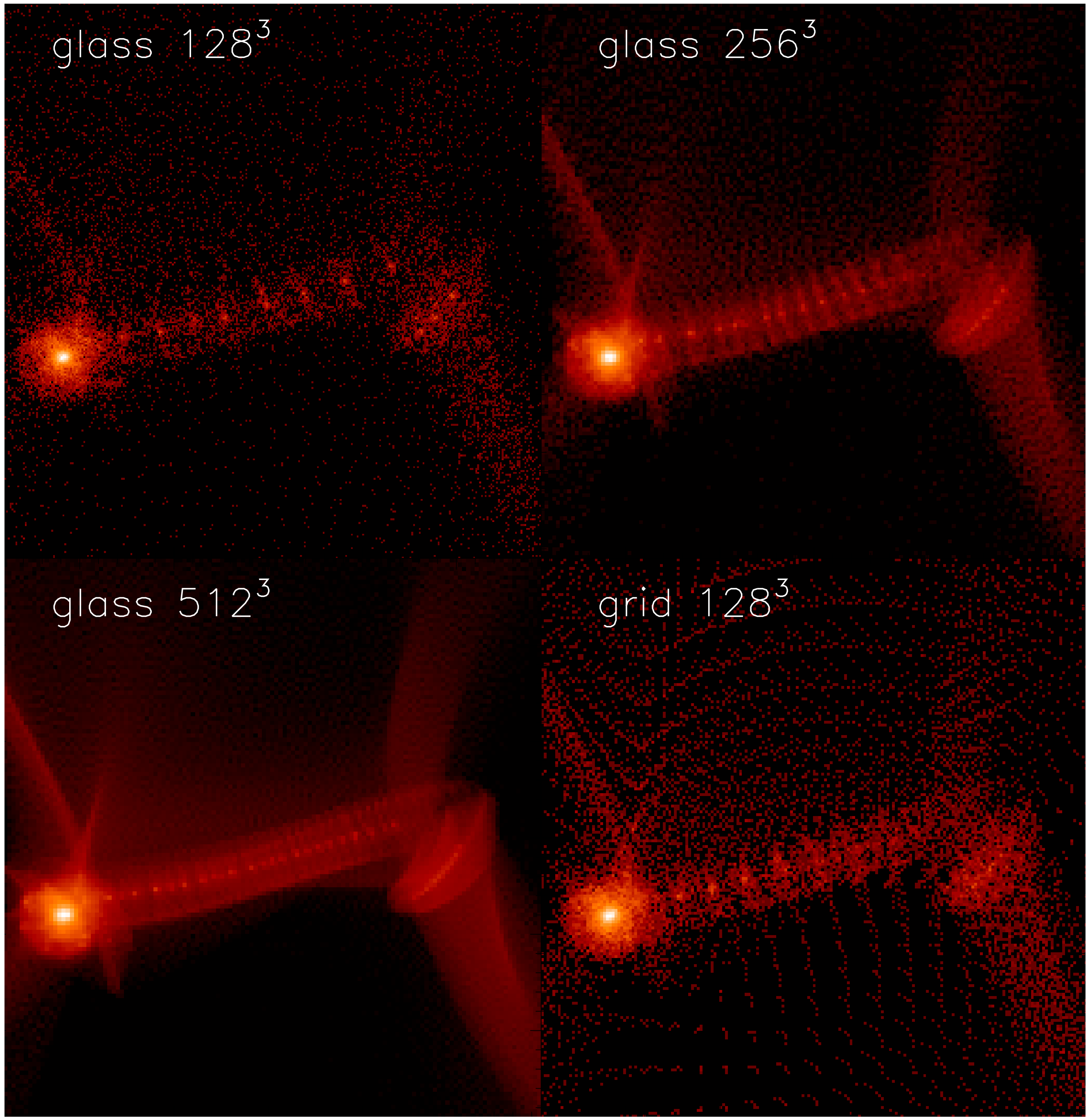
mass functions of clusters
distributions of galaxies
evolution of structure over time
abundance of satellites
density profiles
…

the workhorse
of computational
cosmology

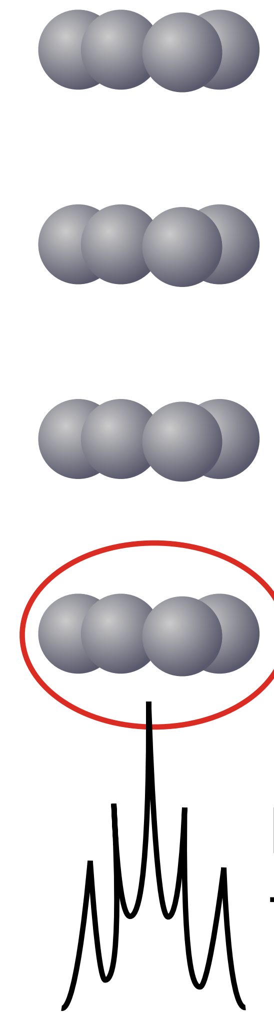
a lot is owed to this method!

“Beads-on-a-string” in WDM simulations

spurious fragmentation is well-known phenomenon in N-body sims with cut-off



Wang&White 2007



large softening needed.
these are no ‘clumps’,
just convergent points!

but want small softening
to get small scale structure!

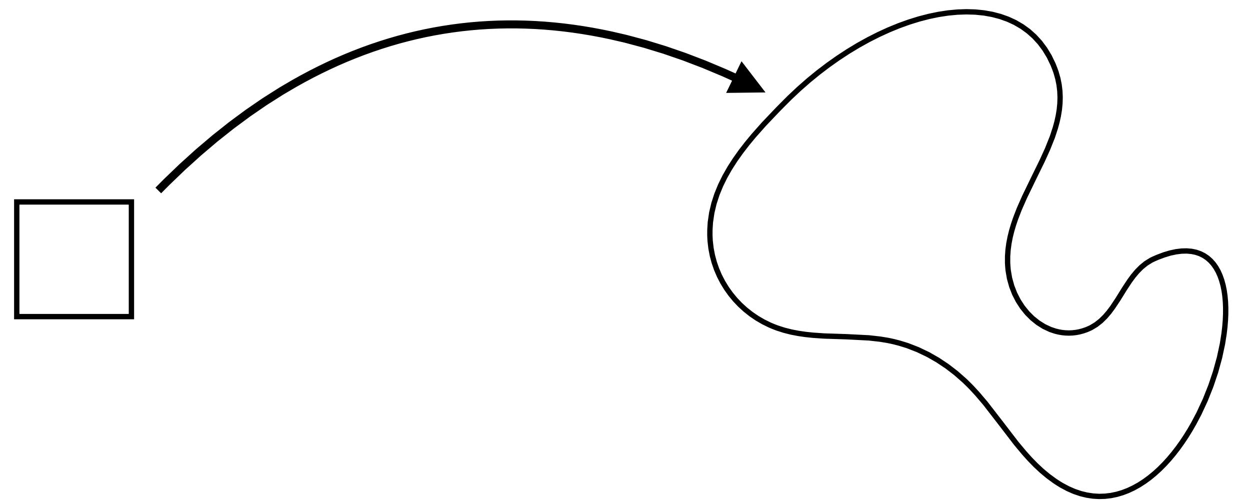
it should not collapse along vertical direction! this info is not local!

Most obvious for non-CDM simulations!
(e.g. Centrella&Melott 1983, Melott&Shandarin 1989, Wang&White 2007)

Try other approach: approximate the continuum…

Lagrangian description, evolution of fluid element

$$\mathbb{Q} \subset \mathbb{R}^3 \to \mathbb{R}^6 \ : \ \mathbf{q} \mapsto (\mathbf{x_q}(t), \mathbf{v_q}(t))$$



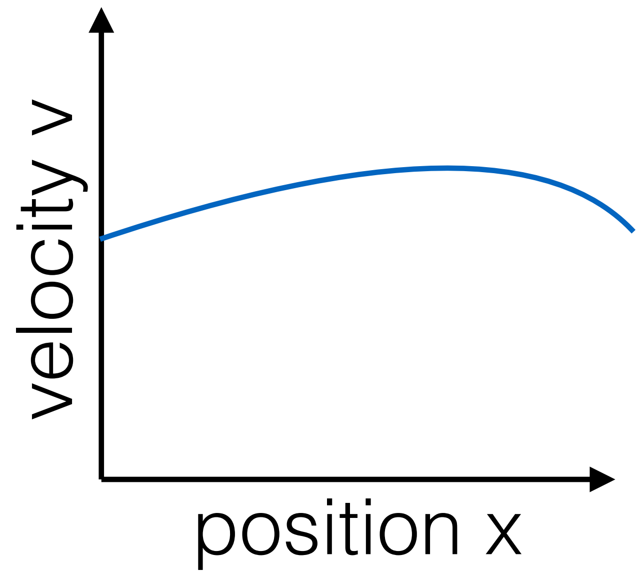
Describe map between Lagrangian and Eulerian space by (infinite dimensional) space of tri-polynomials

$$Q \in P_k = \{\pi(\mathbf{q}) \mid \pi(\mathbf{q}) = \sum_{\alpha,\beta,\gamma=0}^{k} a_{\alpha\beta\gamma}\, q_0^\alpha q_1^\beta q_2^\gamma\}$$

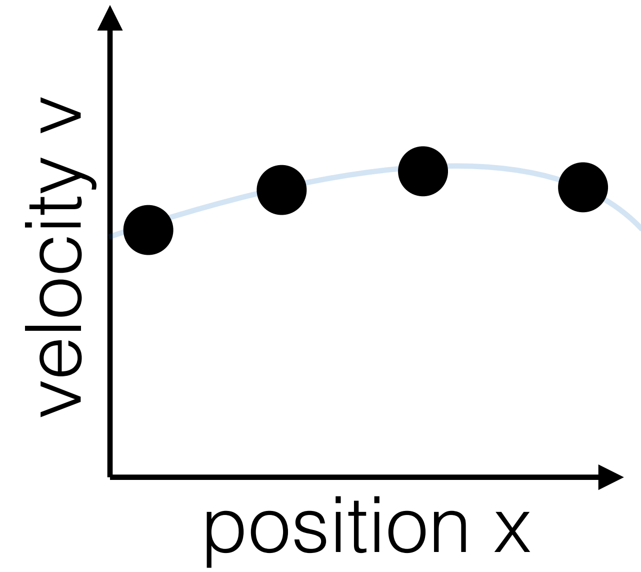
Exact for $k \to \infty$, manifold tracking instead of particles

What does that mean?!?!

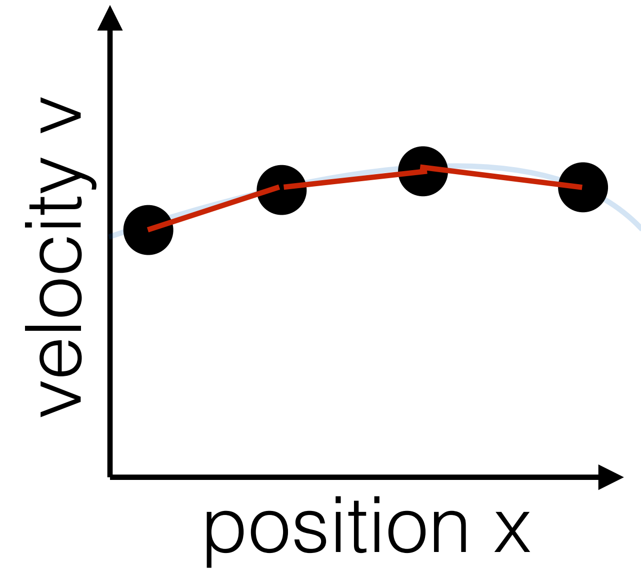
distribution function



N-body just have particles



now: connect particles by interpolating functions



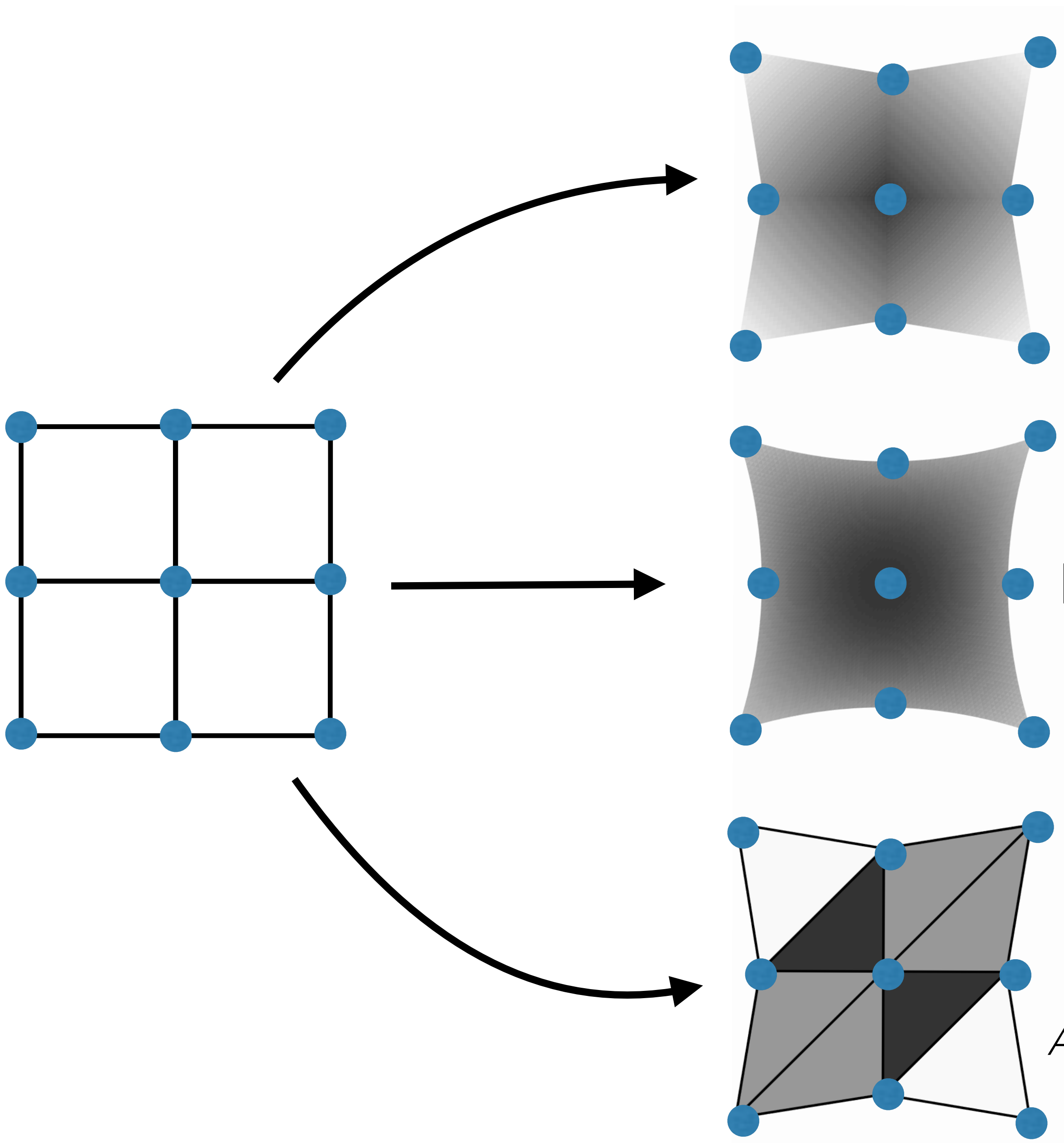
density = 1 / projected length
put mass not at particles, but in-between

But need to split elements, when structure of distribution function becomes complicated -> costly!

Hahn&Angulo 2016
Sousbie&Colombi 2016

How would such local maps look like?

Finite order maps q->(x,v):



k=1
bi/tri-linear
Hahn&Angulo 2016

k=2
bi/tri-quadratic
Hahn&Angulo 2016

affine map
tetrahedral
Abel, Hahn & Kaehler 2012
Schandarin et al. 2012

Sousbie&Colombi 2015
(also ***quadratic*** *tetrahedra)*

they define density everywhere, even caustics (for k>=2)

$$\rho = m_{\mathrm{DM}} \left| \frac{\partial x_i}{\partial q_j} \right|^{-1}$$

unlike N-body, which involves ad-hoc kernel

$$\rho = m_p \sum \delta_D(x - x_i) \otimes W$$

split them into smaller ones if they bend too much

With refinement, it is possible to track very complicated orbits

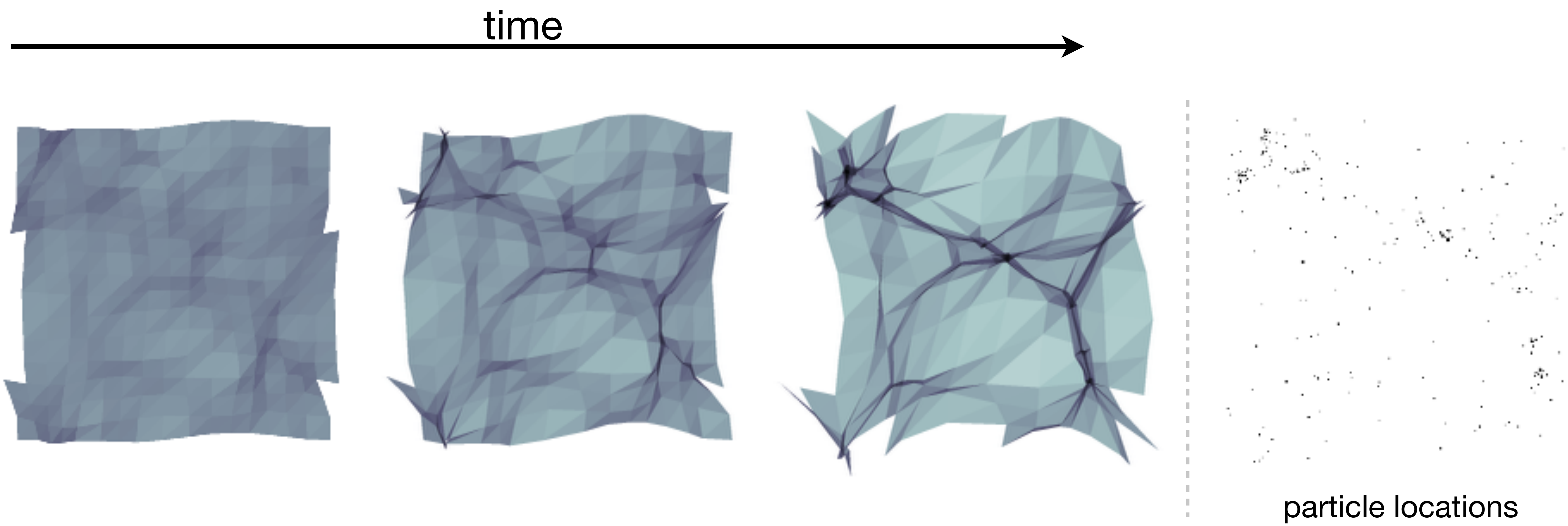
orbit of square in chaotic potential…

movie by T. Sousbie, using ColDICE code (Sousbie&Colombi 2016)

see Thierry's talk right after!

So what do we gain for structure formation?

The real space density, velocity field, etc., at any given point can then be determined from **all** elements that contain that point (see also Shandarin et al. 2012).



Structure formation is like high-dimensional origami: folding a n-dimensional sheet in 2n-dimensional space

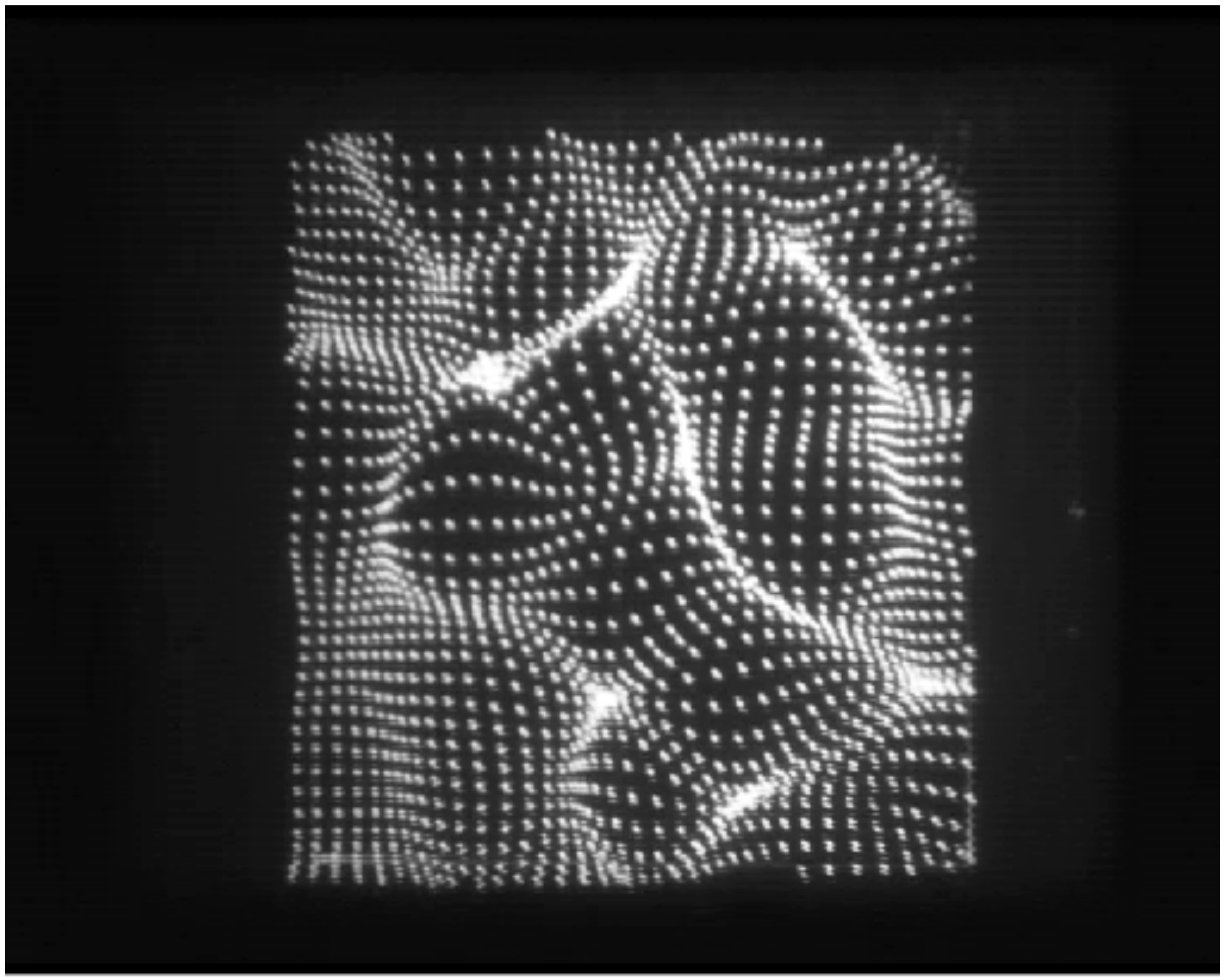
(See also Neyrinck 2014, for the connection to mathematical origami).

each fold is a caustic

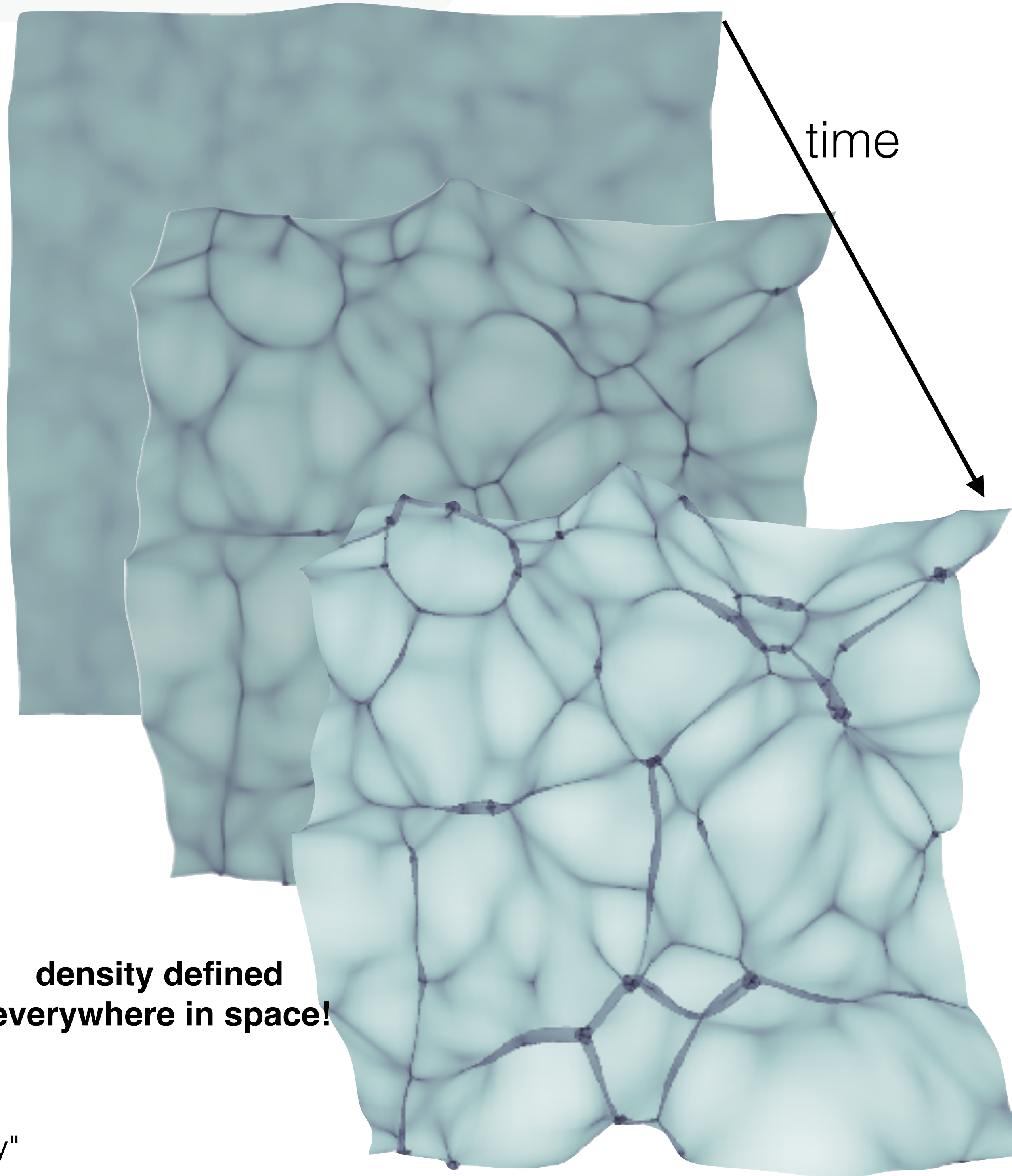
Structure formation as sheet folding….

Formation of structure from catastrophes… Zeldovich pancakes…

$$\rho = m_{\rm DM} \left| \frac{\partial x_i}{\partial q_j} \right|^{-1} \simeq m_{\rm DM} \prod \left(1 + \mathrm{eig}\left\{ \frac{\partial v_i}{\partial q_j} \right\} \right)^{-1}$$



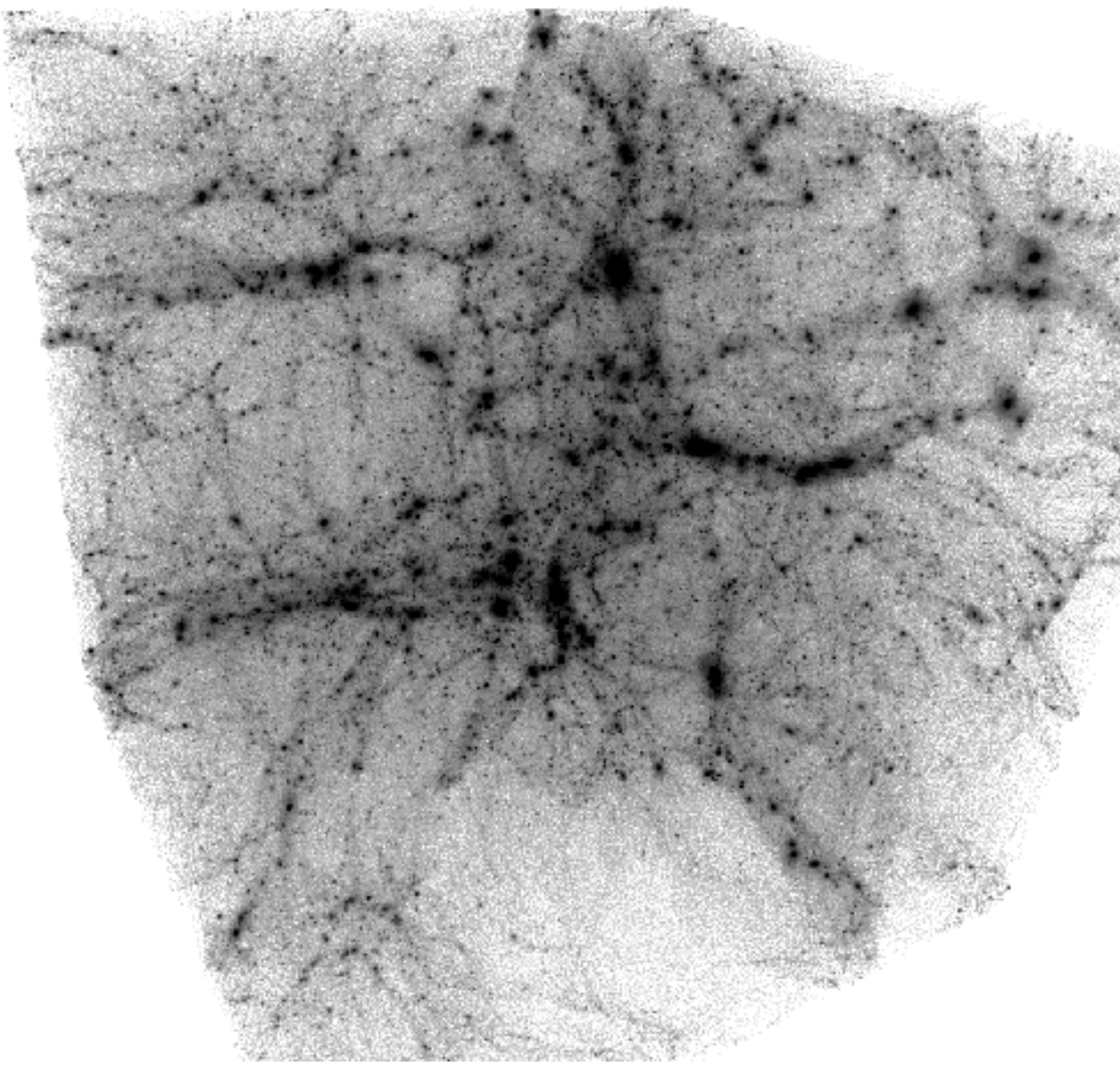
A. G. Doroshkevich, E. V. Kotok, S. F. Shandarin **1977**:
B. "Evolution of the Density Field according to the Theory of Gravitational Instability"
Can be found on webpage of Jaan Einasto



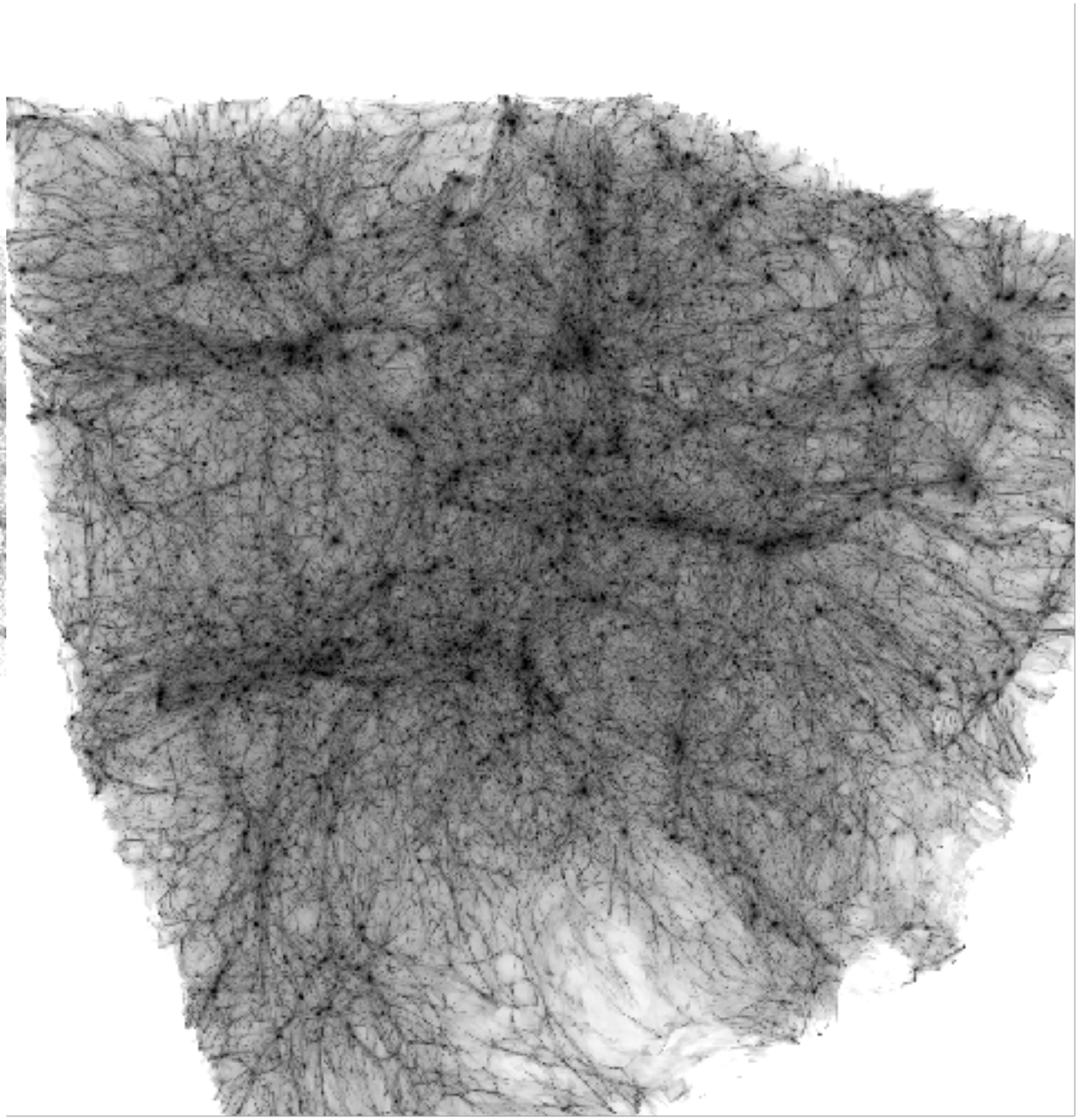
density defined everywhere in space!

today's version… (run on the fly)

So what do we gain in 3+3D?

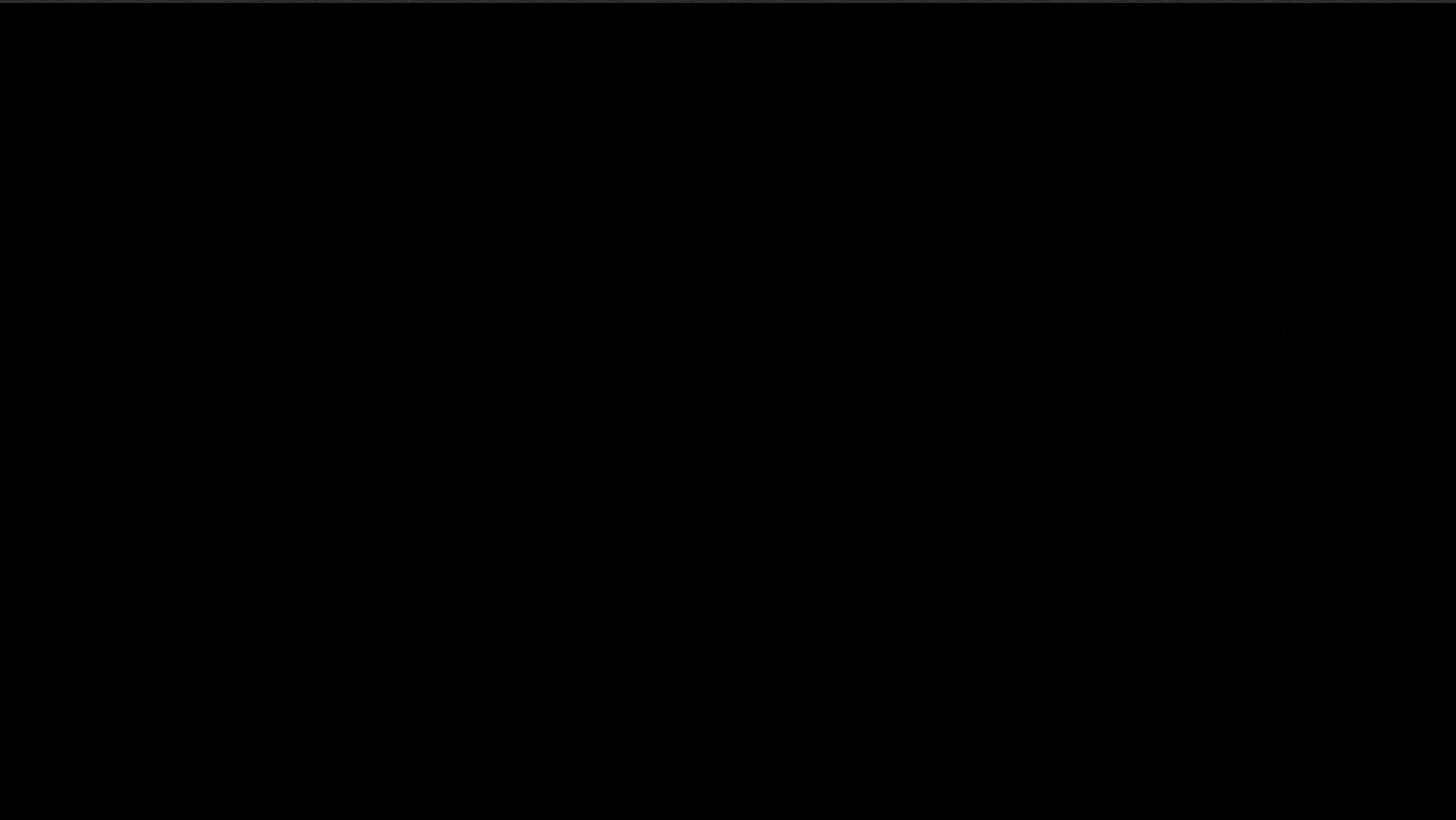


rendering points for particles.



rendering tetrahedral phase space cells.

Same simulation data! (Abel, Hahn, Kaehler 2012)

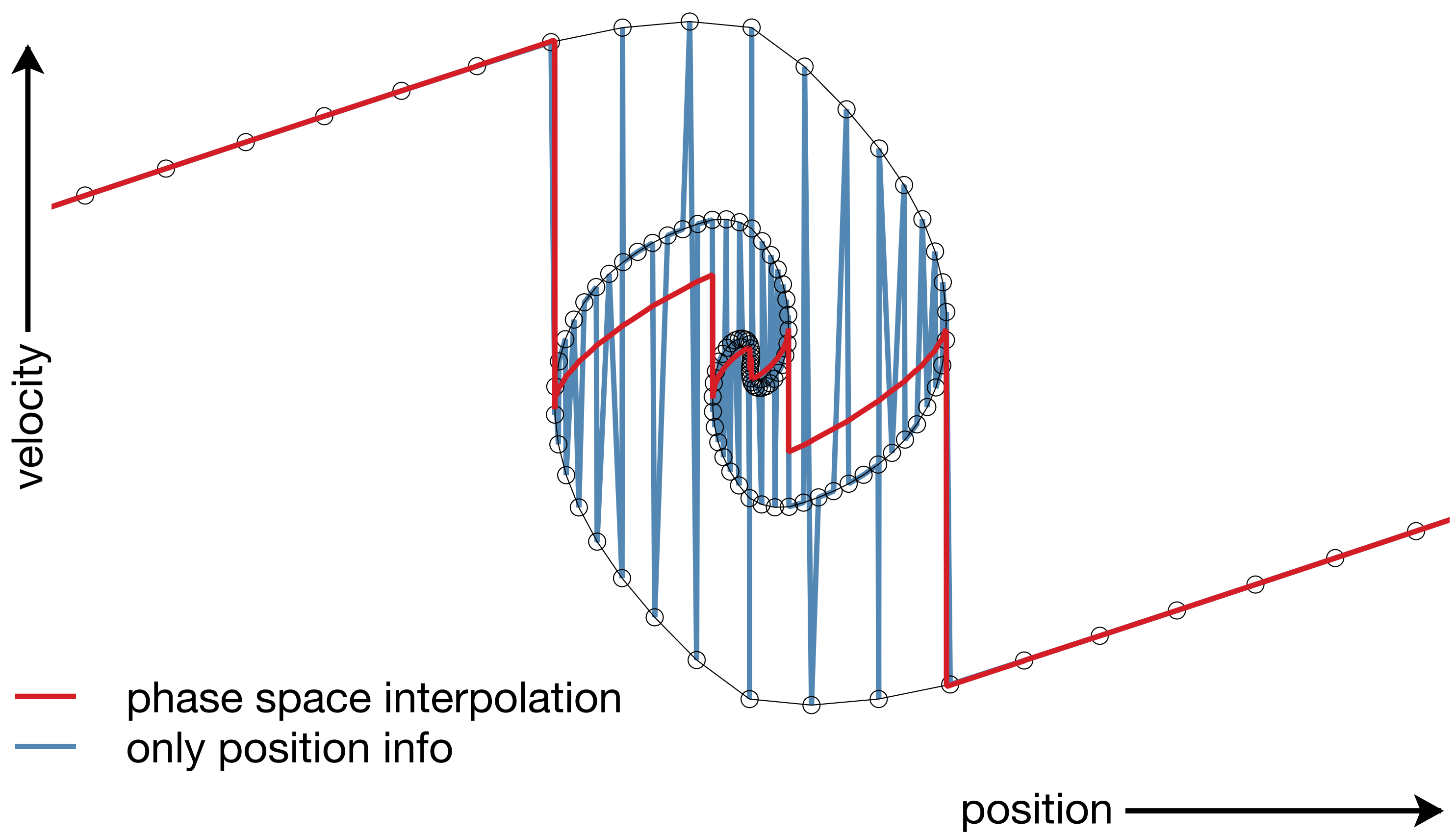


Vis: Kaehler, Emmart & Abel, Sim: Abel & Hahn

the sheet — as an analysis tool

How to measure the mean velocity field?

- Interpolate between neighbouring N-body particles
- "neighbouring" in phase space, not configuration space
- account for averaging over streams (="taking moment")



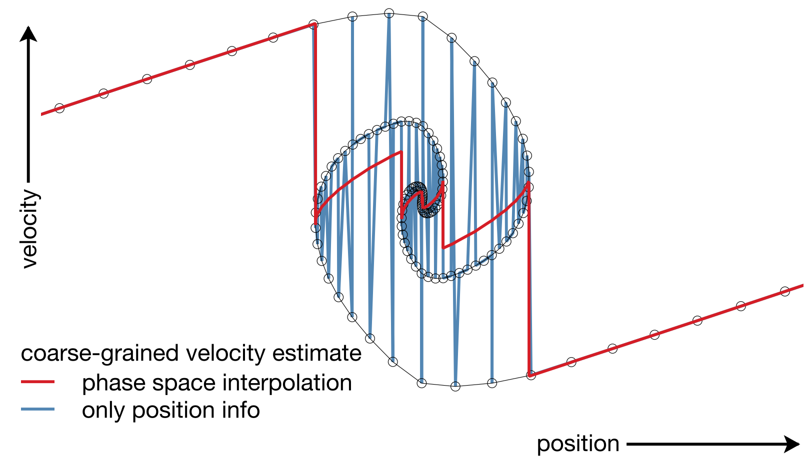
- mean velocity field:

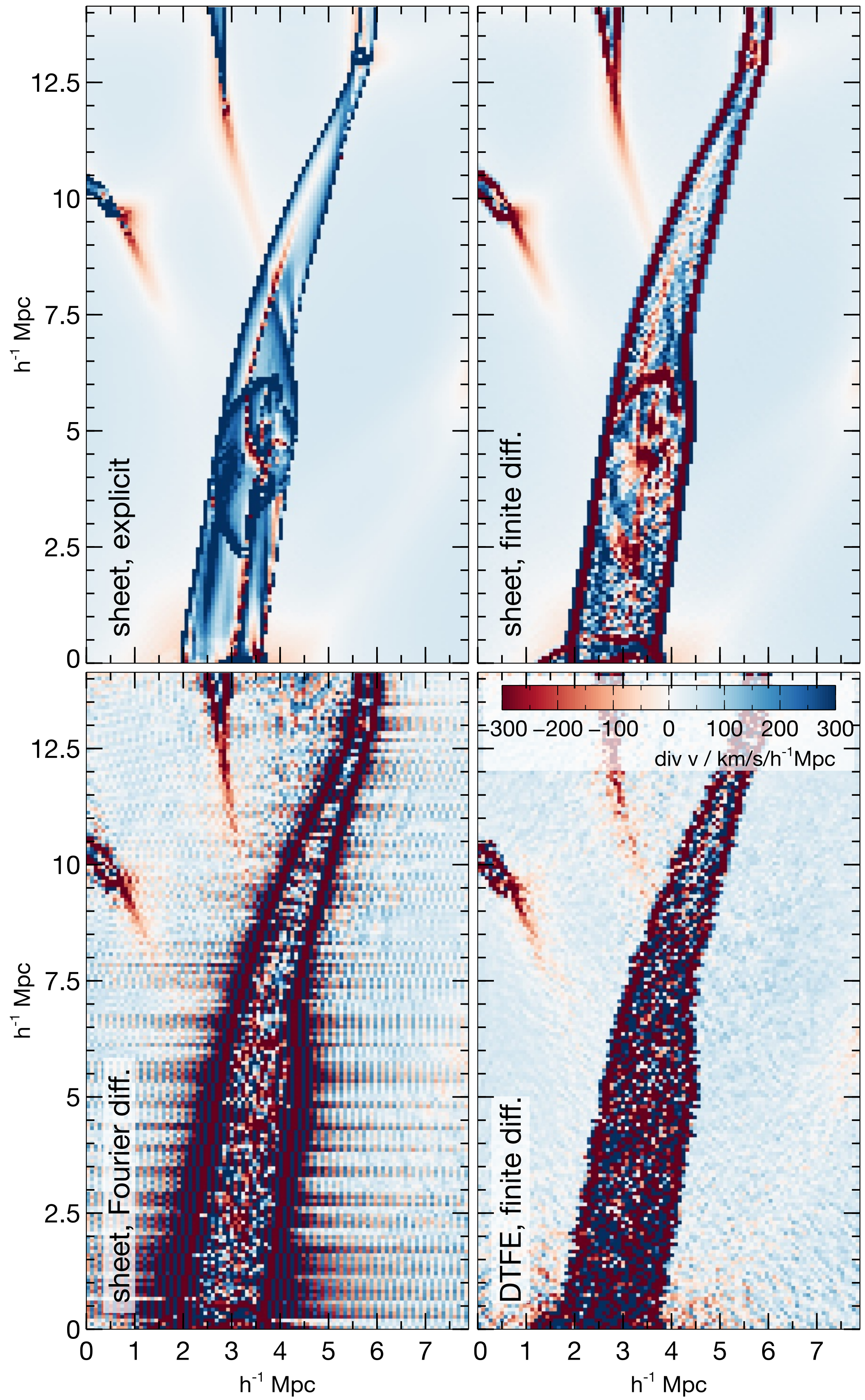
$$\langle \mathbf{v} \rangle \equiv \frac{\int_{\mathbb{R}^3} \mathbf{v}\, f(\mathbf{x},\mathbf{v})\, \mathrm{d}^3 v}{\int_{\mathbb{R}^3} f(\mathbf{x},\mathbf{v})\, \mathrm{d}^3 v} = \frac{\sum_{s\in \mathrm{S}} \mathbf{v}_s(\mathbf{x})\, \rho_s(\mathbf{x})}{\sum_{s\in \mathrm{S}} \rho_s(\mathbf{x})}$$

- result is discontinuous across caustics

New insights from mean field dynamics I

The cosmic mean velocity field





- Discontinuities make ordinary derivatives ill-defined!
- Need to explicitly evaluate coarse-graining+derivative

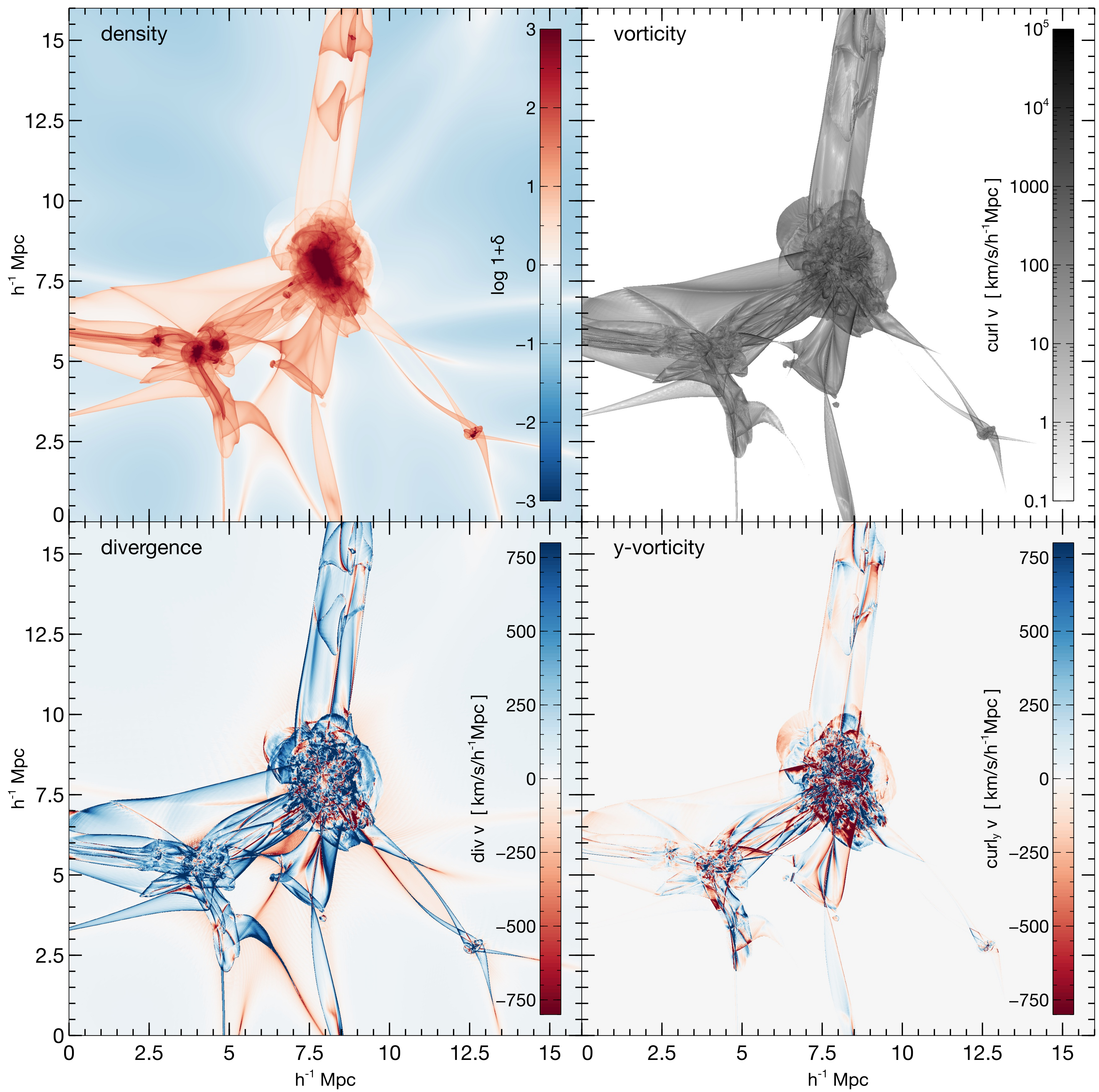
$$\nabla \cdot \langle \mathbf{v} \rangle = \left\langle (\nabla \log \rho) \cdot (\mathbf{v} - \langle \mathbf{v} \rangle) \right\rangle + \langle \nabla \cdot \mathbf{v} \rangle$$

$$\nabla \times \langle \mathbf{v} \rangle = \left\langle (\nabla \log \rho) \times (\mathbf{v} - \langle \mathbf{v} \rangle) \right\rangle + \langle \nabla \times \mathbf{v} \rangle$$

- Vorticity for std. gravity pure multi-stream phenomenon!!

New insights from mean field dynamics II

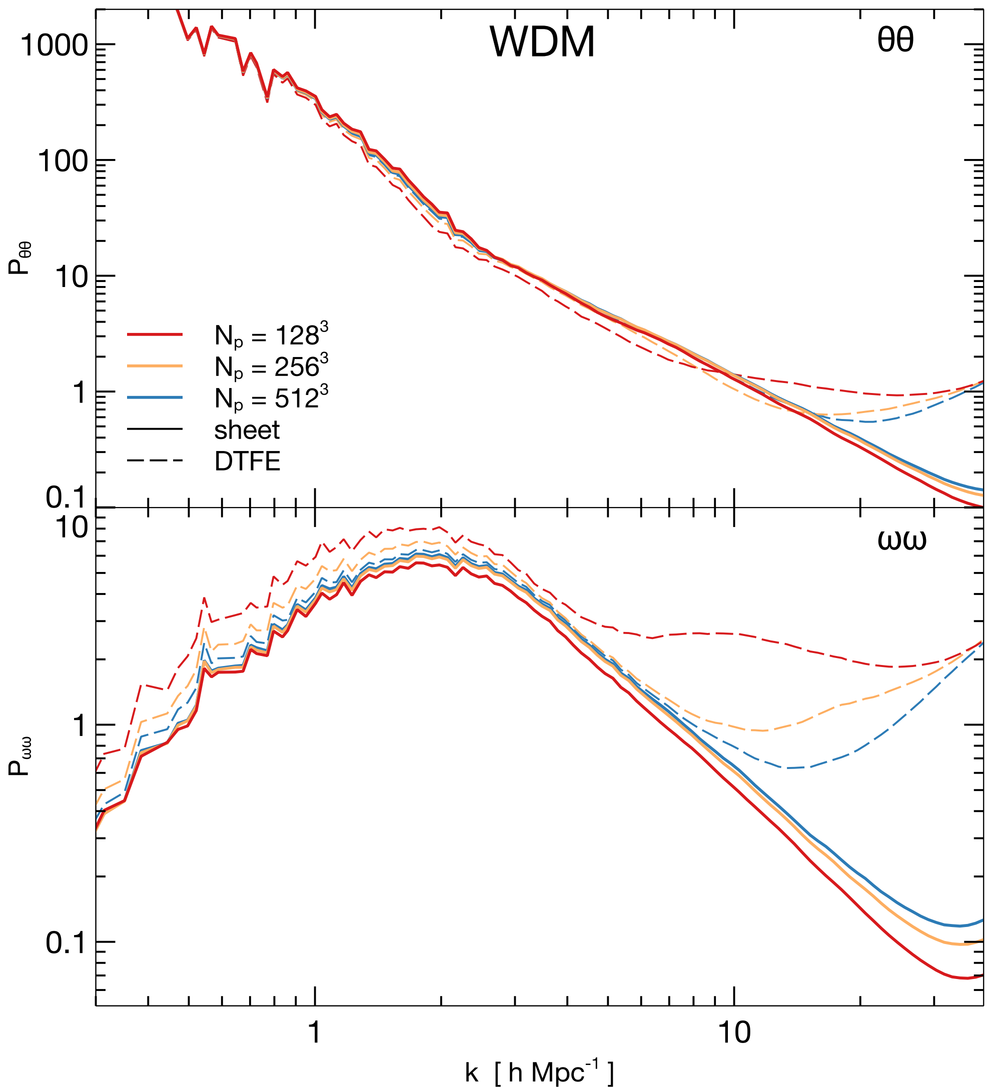
It is possible to investigate moments of the Boltzmann hierarchy a-posteriori

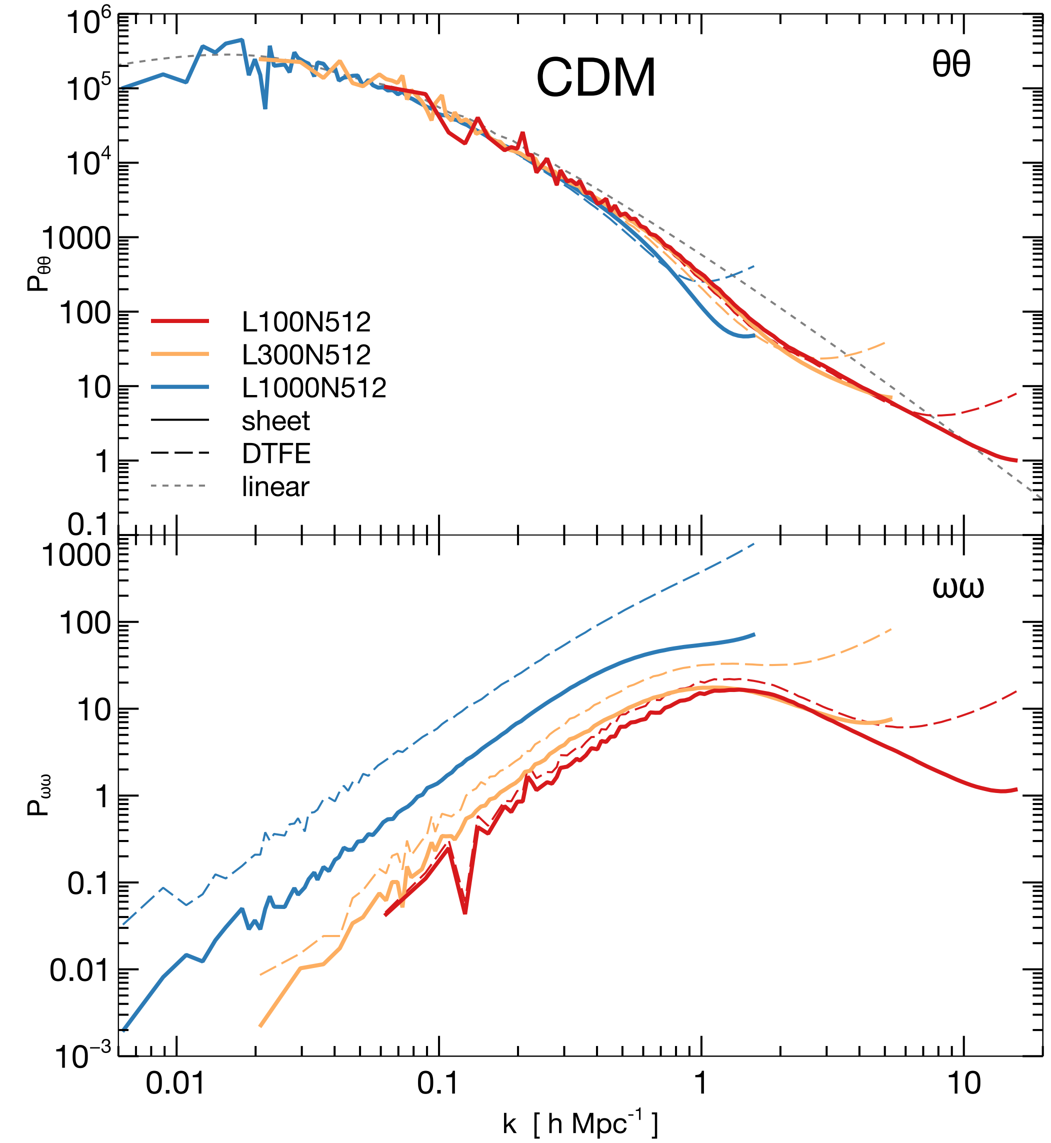


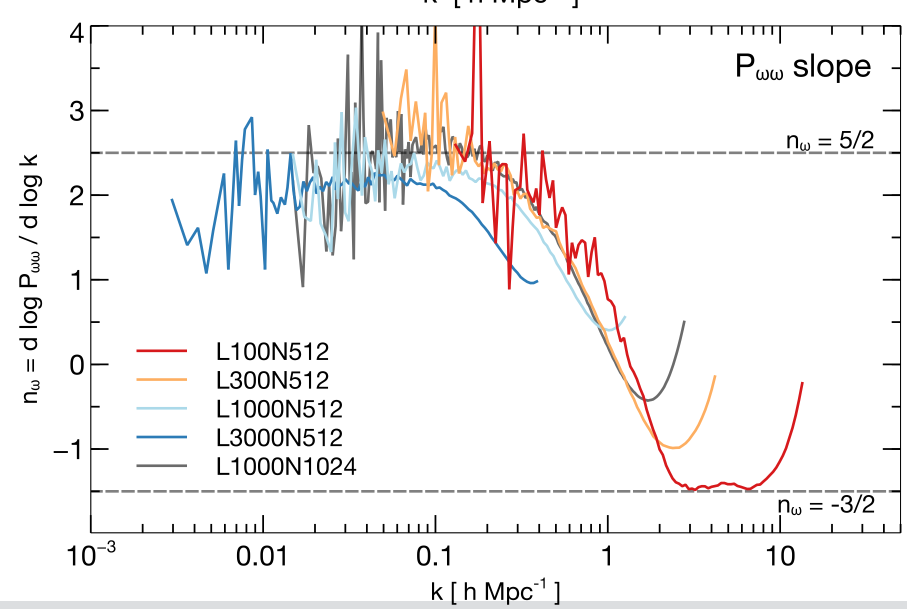
Measurements impossible from N-body

New insights into DM dynamics

Spectral properties of the cosmic velocity field I

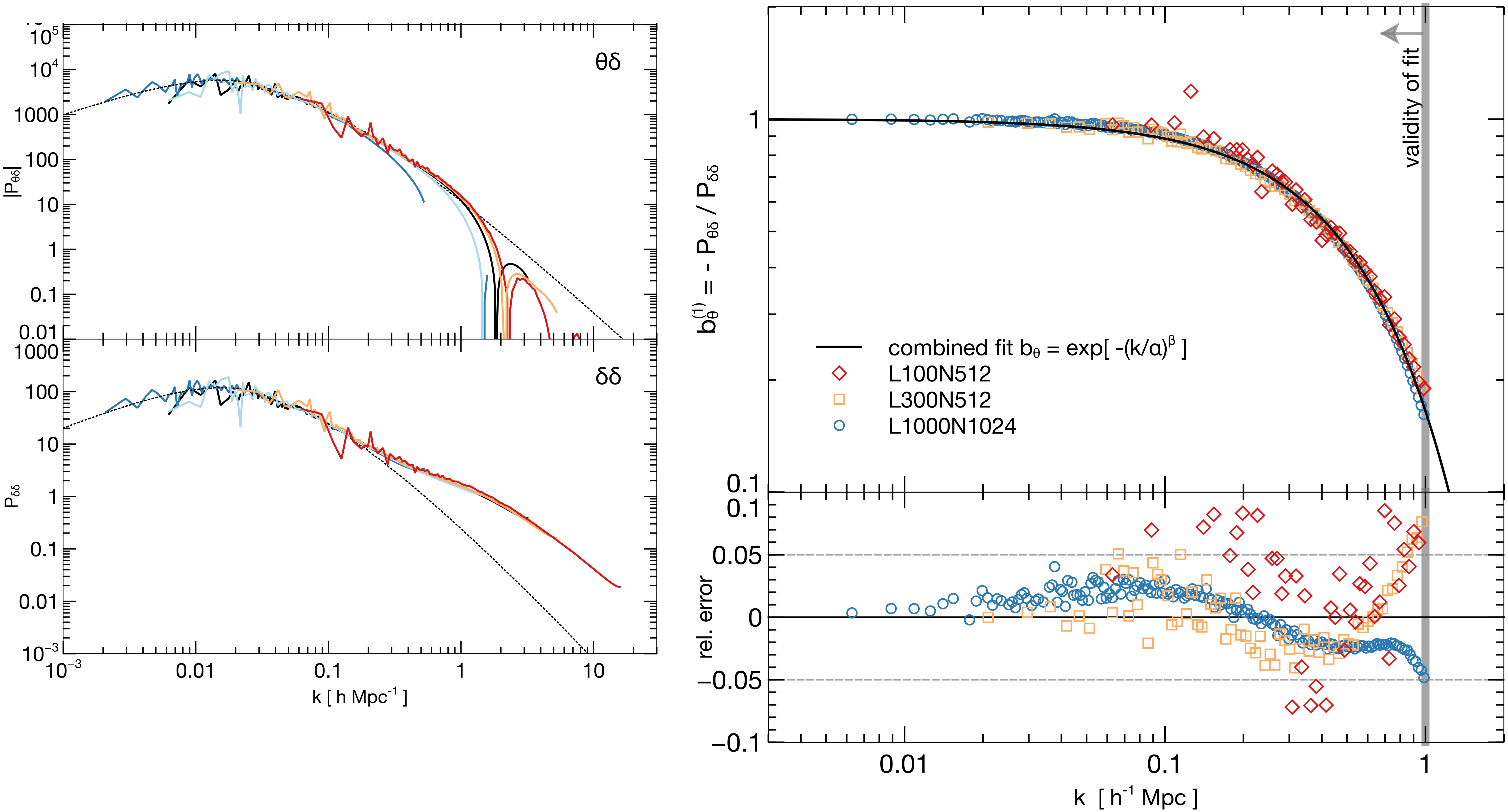






- Faster convergence (for WDM: convergence!)
- Better small scale properties

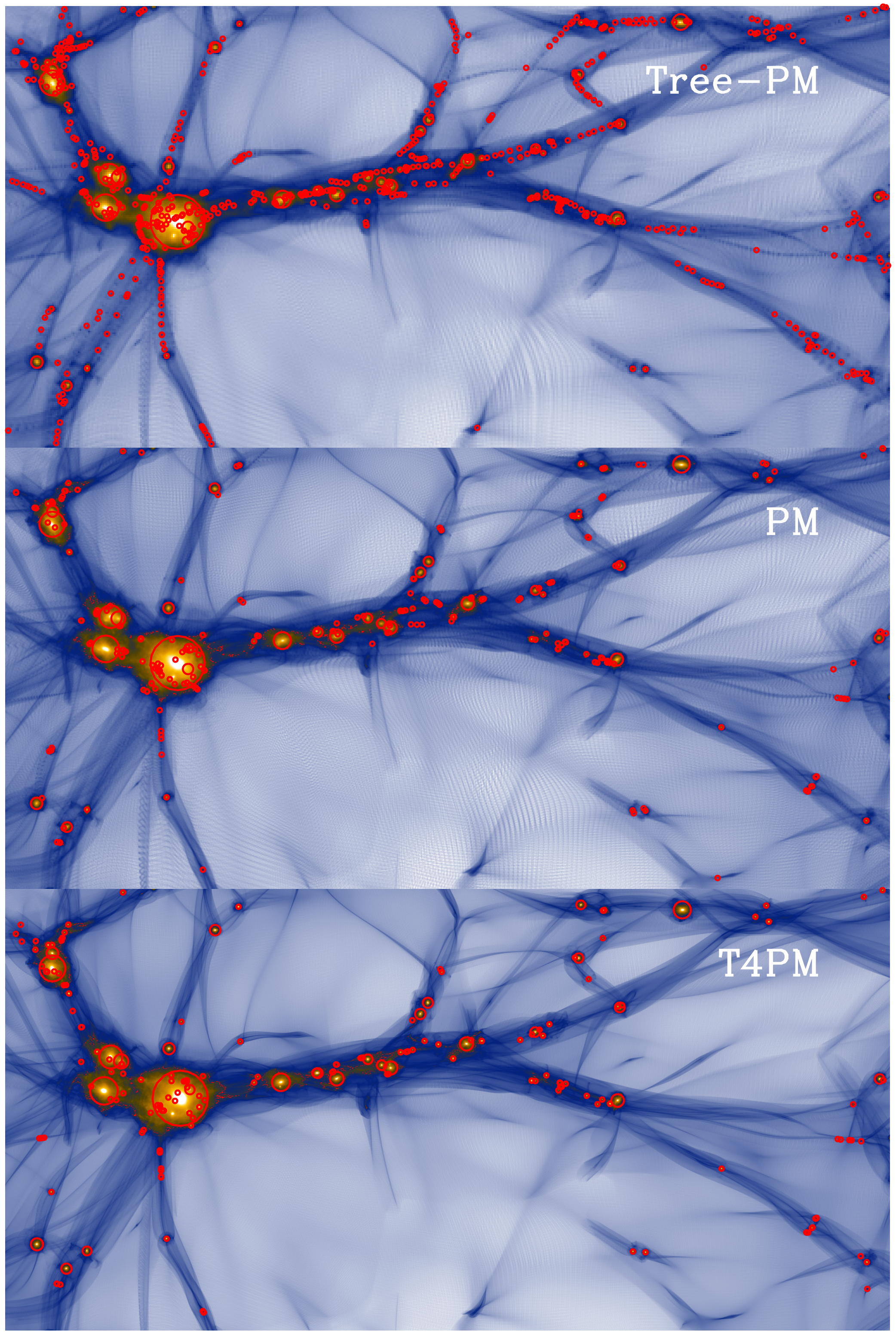
Spectral properties of the cosmic velocity field II



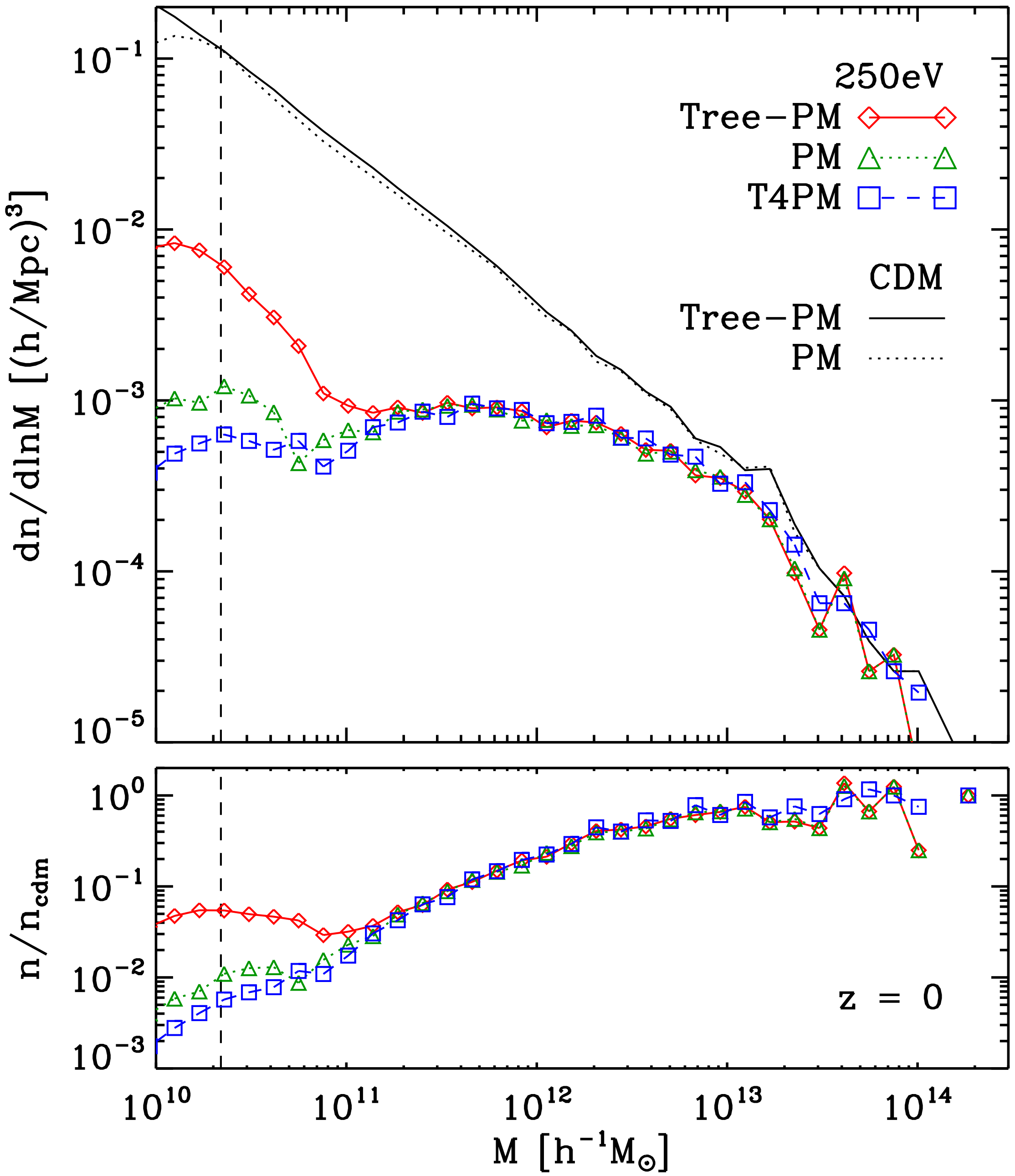
- divergence-density cross spectrum flips sign at k~2
- divergence bias for k<1 well fit by exponential

self-consistent evolution with the sheet

If one uses this approach self-consistently, it cures the fragmentation problem of N-body

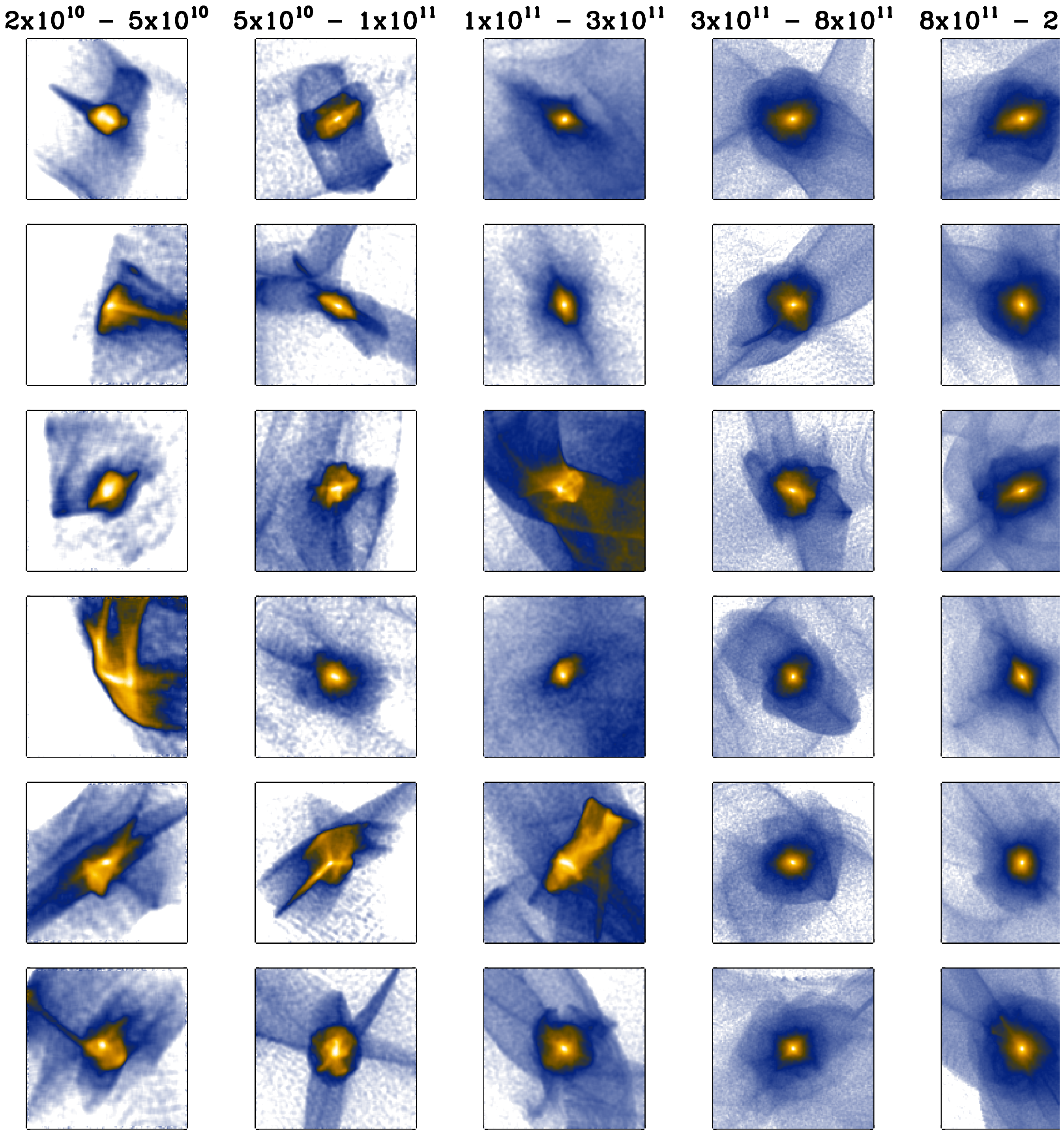


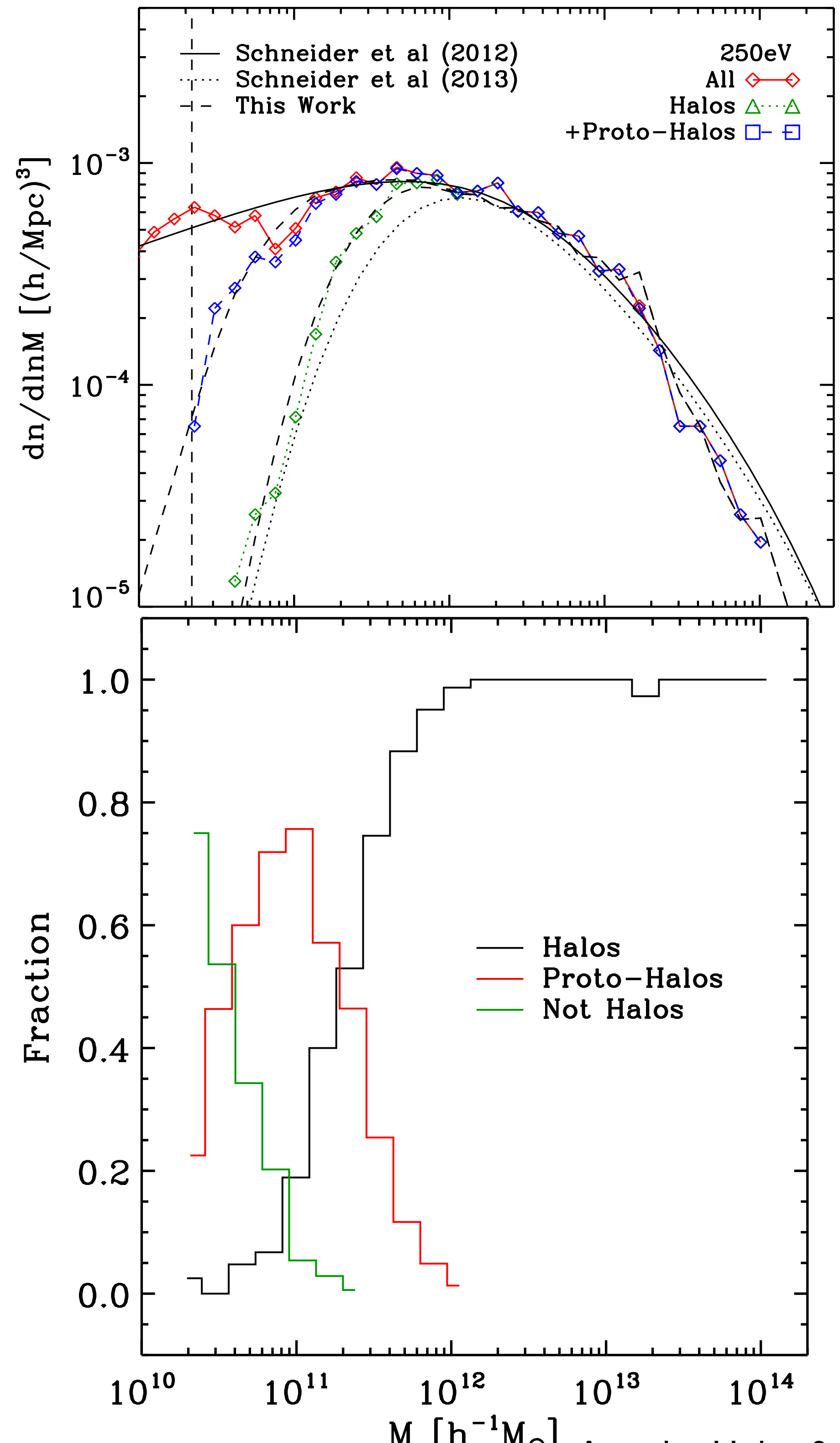
First determination of (true) WDM halo mass function



Angulo, Hahn & Abel 2013

Structure formation in WDM very different than in CDM





Angulo, Hahn & Abel 2013

Mass=evolutionary stage,
no progenitors below some mass

Structure formation in WDM very different than in CDM

collapse from initially smooth field

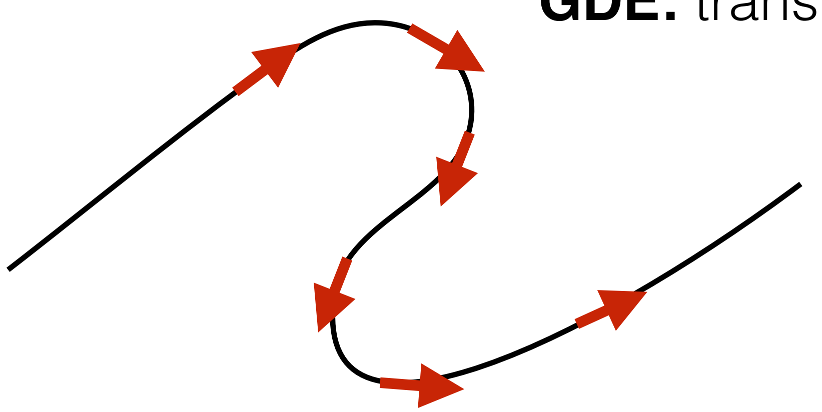
no progenitors below certain mass

caustics everywhere

Density profiles of haloes from smooth ICs?

stay tuned, and see also **Go Ogiya**'s talk on Friday

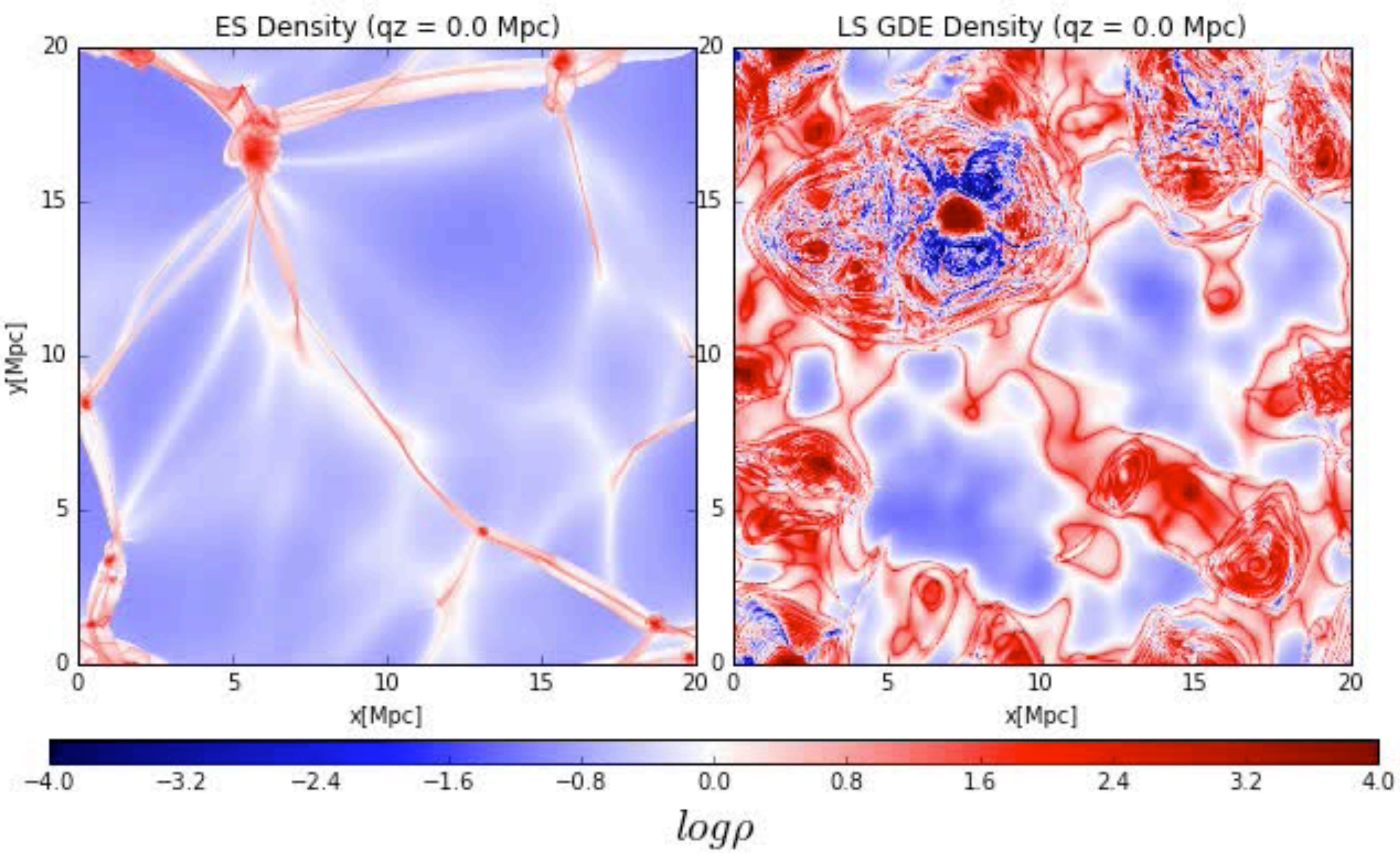
What's next? Validation: The geodesic deviation equation and stream densities



GDE: transport of tangent and normal spaces at particle positions

$$\frac{d}{dt}\begin{pmatrix} D_{xx} & D_{xv} \\ D_{vx} & D_{vv} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ T & 0 \end{pmatrix}\begin{pmatrix} D_{xx} & D_{xv} \\ D_{vx} & D_{vv} \end{pmatrix}$$

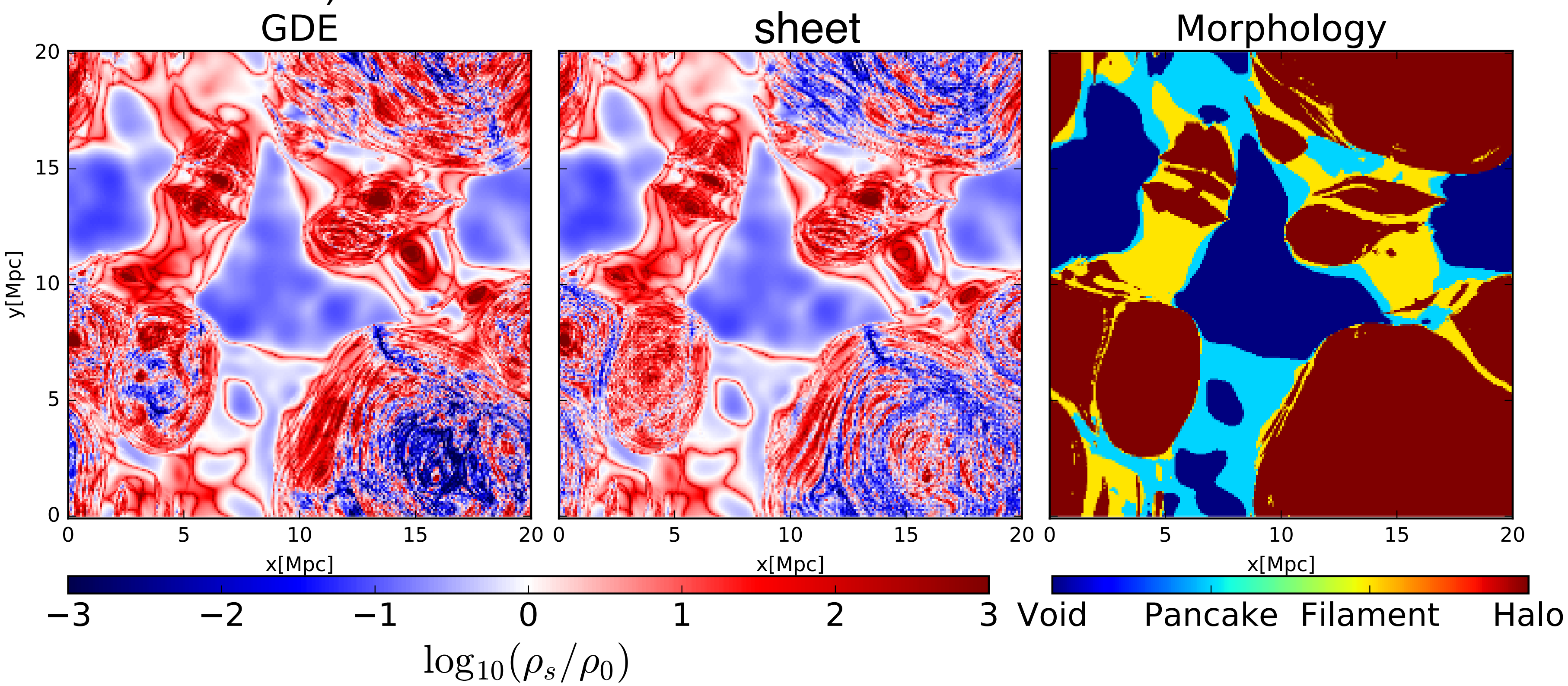
Vogelsberger+(2008)

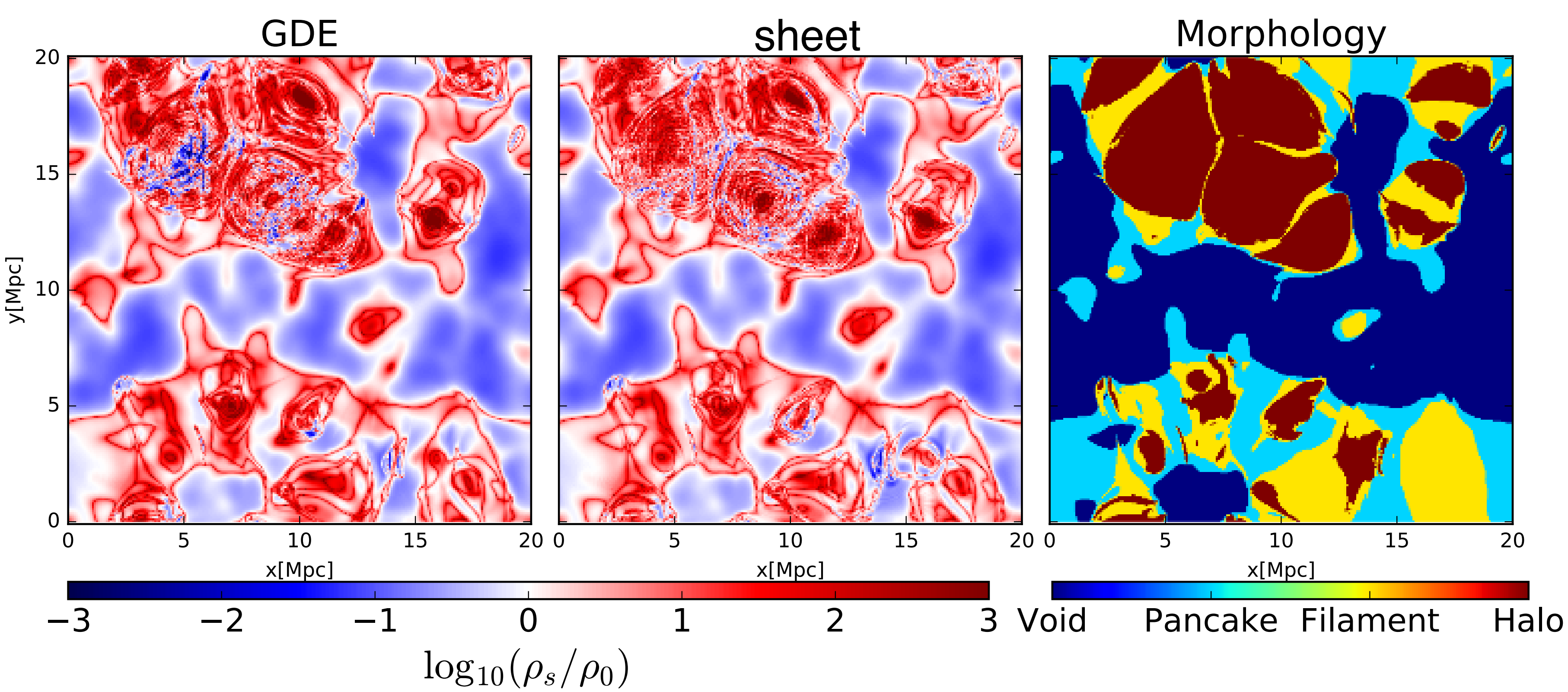


See Jens Stücker's talk on Friday

Comparison GDE - sheet interpolation

(no refinement)





See Jens Stücker's talk on Friday

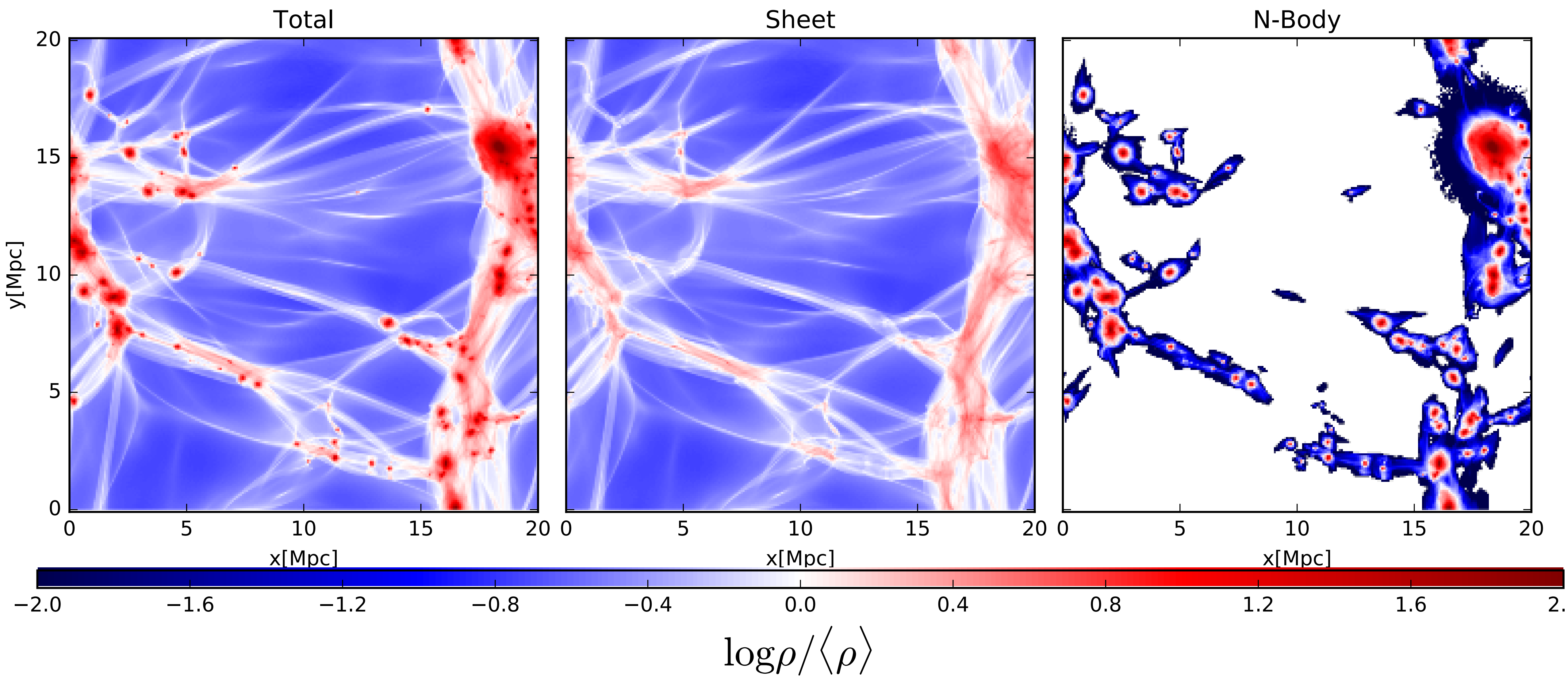
…work in progress…

Stuecker, OH et al. 2017, in prep.

One way forward: Instead of refinement : resort to N-body

Sheet works very well in anisotropic regions, and is expensive when mixing is important

Try best of both worlds: sheet in voids to filaments, N-body when isotropic ergodic, i.e. halos

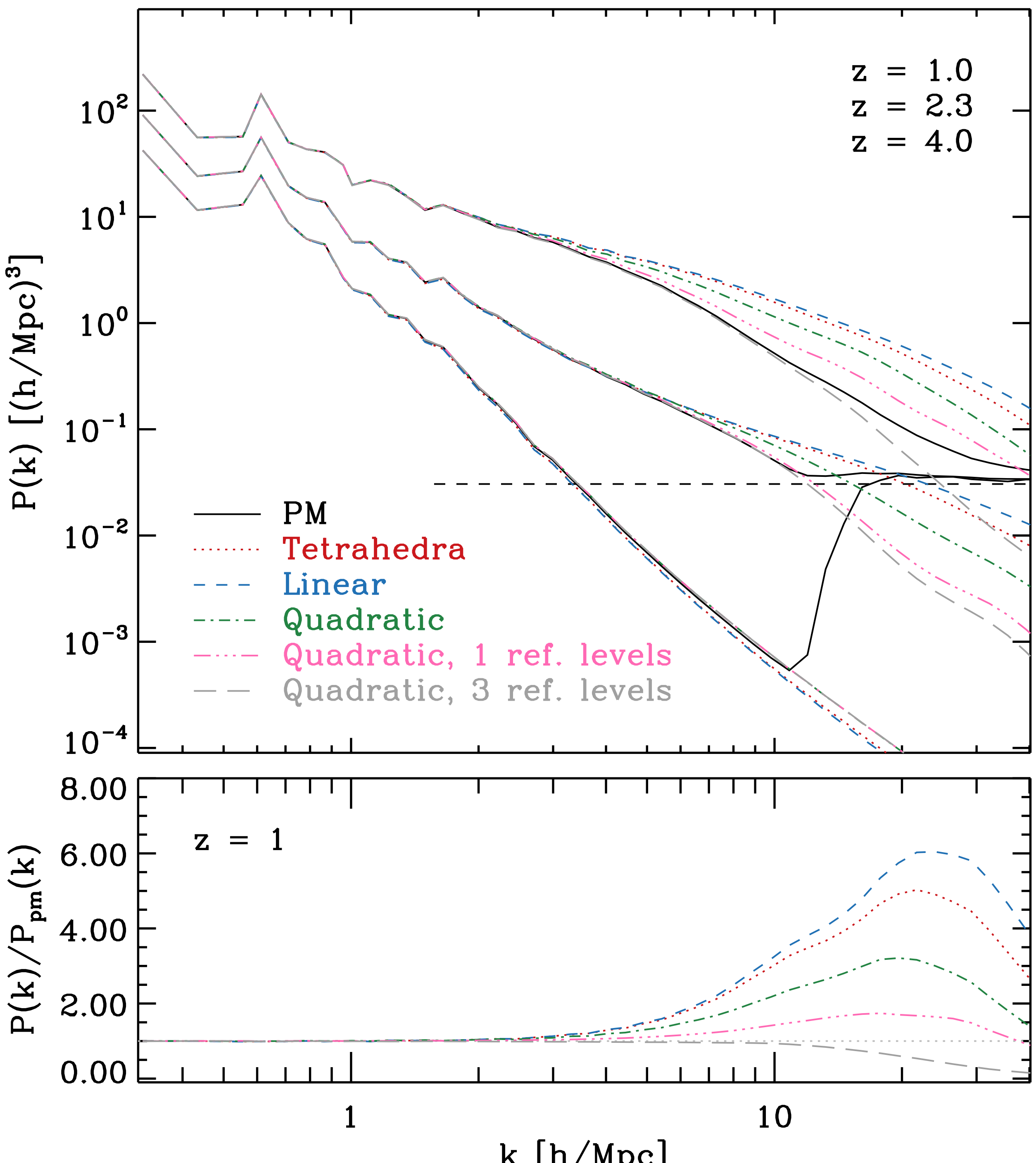


...work in progress...

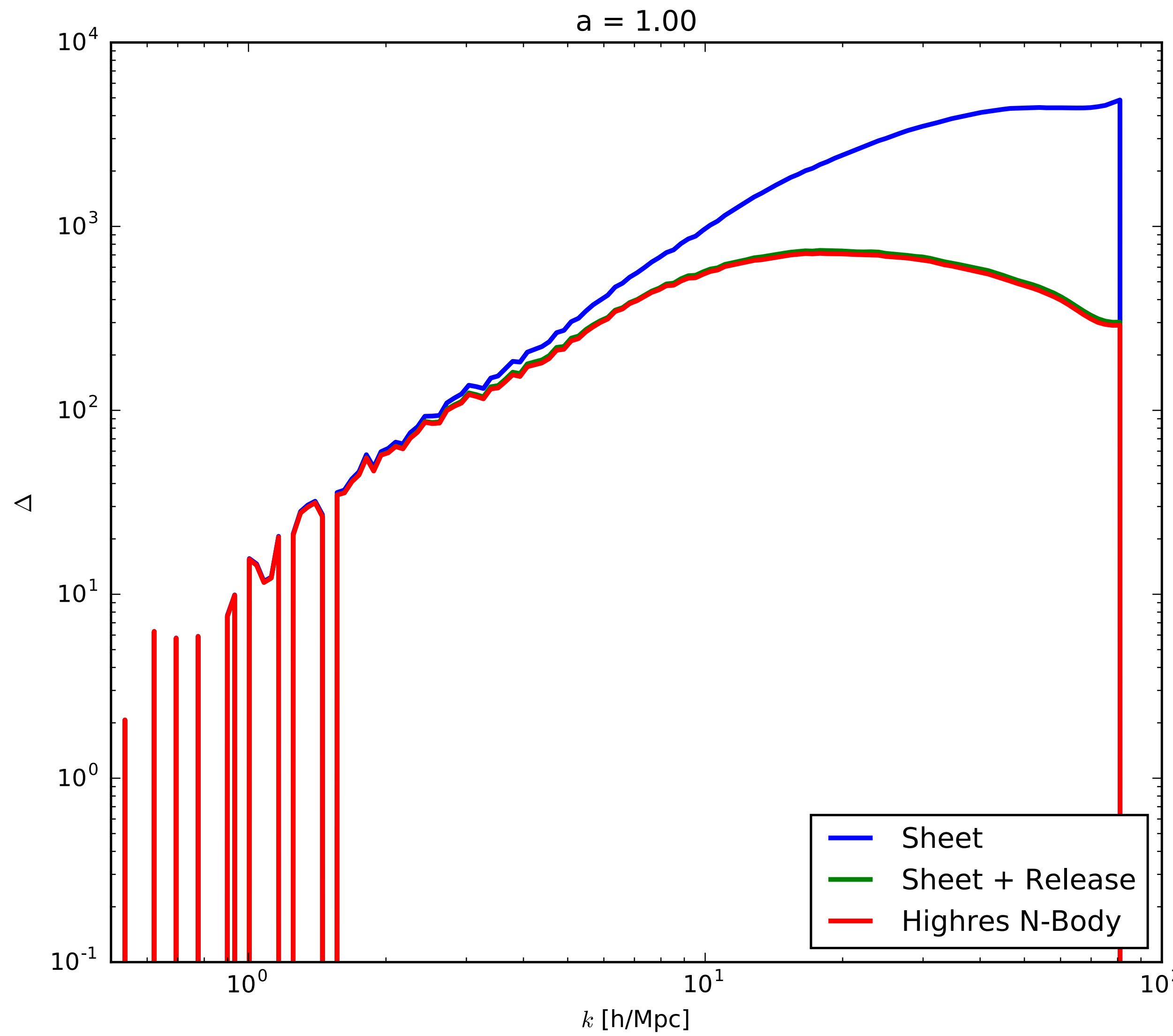
Stuecker, OH et al. 2017, in prep.

Power spectrum convergence

either with refinement and high order



Hahn & Angulo 2016



…work in progress…

Stuecker, OH et al. 2017, in prep.

Summary

Dark matter is collisionless and rather cold -> challenges for modelling

New tessellation methods overcome important limitations of N-body method (virtually noise-free but more costly)

Allow to study wealth of additional properties of collisionless systems

New angle on studying small scale properties of dark matter (improved constraints on particle nature..)

Are halo profiles truly universal? Universality driven by noise and mergers? Role of N-body?