nonequilibrium states with temperature inversion

in long-range interacting systems

Lapo Casetti

Dipartimento di Fisica e Astronomia, Università di Firenze, Italy
INFN, sezione di Firenze, Italy
INAF-Osservatorio di Arcetri, Italy

collisionless Boltzmann (Vlasov) equation
and modeling of self-gravitating systems and plasmas

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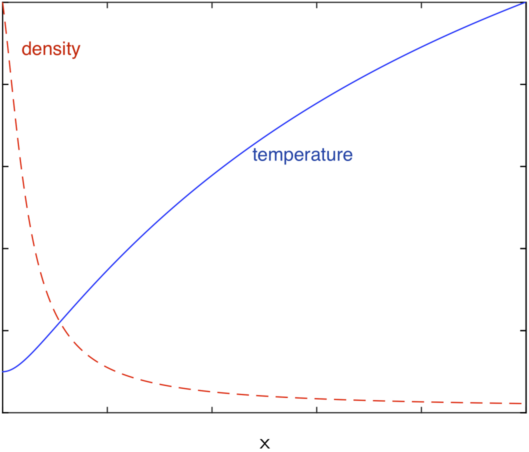
joint work with Pierfrancesco Di Cintio, Shamik Gupta, and Tarcísio N. Teles

LC & Gupta European Physical Journal B **87**, 91 (2014)
Teles, Gupta, Di Cintio & LC Physical Review E **92**, 020101(R) (2015)
Teles, Gupta, Di Cintio & LC Physical Review E **93**, 066102 (2016)
Gupta & LC New Journal of Physics **18**, 103051 (2016)
Di Cintio, Gupta & LC arXiv:1706.01955, MNRAS (submitted, 2017)

introduction & motivation

- temperature inversion
  - nonequilibrium states with anticorrelation between density & temperature
    the sparser the hotter, the denser the colder
  - observed at astrophysical scales
    solar corona, filaments in molecular clouds, (some) cD galaxies, hot gas in galaxy clusters...
- "universal" phenomenon?
  - from a simple idea...
    velocity filtration
  - ...to a toy model...
    the Hamiltonian Mean Field (HMF) model
  - ...and beyond toy models...
    from astrophysical to atomic scales:
    filaments in molecular clouds (2d self-gravity) and atoms in a cavity (mean-field dynamics)
  - ...to a physical picture
    minimal ingredients: long-range interactions & inhomogeneous states
    basic physical mechanism: interplay between spatial inhomogeneity & wave-particle interaction

temperature inversion



temperature $\propto$ locally averaged kinetic energy $\propto$ squared velocity dispersion

temperature inversion

⟶ many examples in astrophysical systems

- the "classic" example: the solar corona
  temperature rises from 10^3 to 10^6 K while density drops by eight orders of magnitude
- filaments in molecular clouds
  clear measurements for dust, only indirect evidence for gas yet [Palmeirim *et al.* 2013; Toci & Galli 2015]
- dense cores in molecular clouds
  velocity dispersion measurements [Goodman *et al.* 1998; Padoan *et al.* 2001; Pineda *et al.* 2010]
- cool cores in galaxy clusters gas
  [Wise *et al.* 2004]
- plasma torus around Jupiter's moon Io
  [Meyer-Vernet *et al.* 1993, 1995; Saur *et al.* 2004]
- some cD galaxies
  velocity dispersion profile of NGC 3311 in Hydra [Loubser *et al.* 2008]

⟶ very different systems and energy/length scales...treated as unrelated phenomena

⟶ some basic physics in common?

velocity filtration

1990s: J. D. Scudder to explain coronal heating [J. D. Scudder, ApJ 1992 & 1994]

...without great success in the solar physics community...

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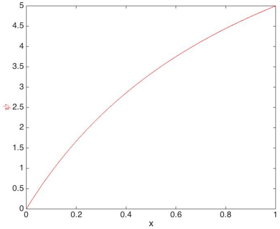
Scudder model

- noninteracting particles in an external field, e.g. gravity
  one dimension: x height above ground
- stationary boundary condition at $x = 0$

collisionless Boltzmann equation for $f(x, p, t)$

$$\frac{\partial f}{\partial t} + p\frac{\partial f}{\partial x} - \frac{d\psi}{dx}\frac{\partial f}{\partial p} = 0$$

...just single-particle energy conservation in this case...



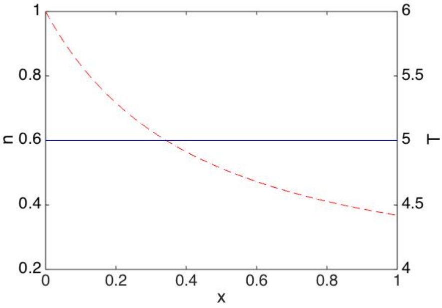
velocity filtration

only particles with kinetic energy $k(0) \geq \psi(x)$ reach x where $k(x) = k(0) - \psi(x)$

velocity filtration thermal boundary condition

thermal boundary condition (Maxwellian) from now on $k_B = 1$

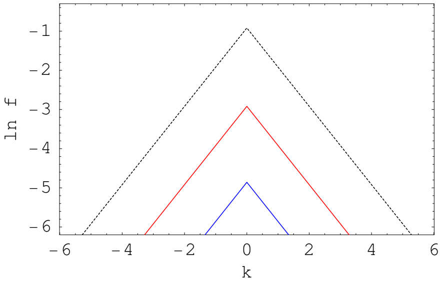
$$f_0{}^M(p) = \frac{n_0}{(2\pi T_0)^{1/2}} \exp\left(-\frac{p^2}{2T_0}\right) \implies f(x,p) = \exp\left[-\frac{\psi(x)}{T_0}\right] f_0{}^M(p) \implies T(x) \equiv T_0$$



only with thermal boundary condition $f_0{}^M$

velocity filtration how does it work?

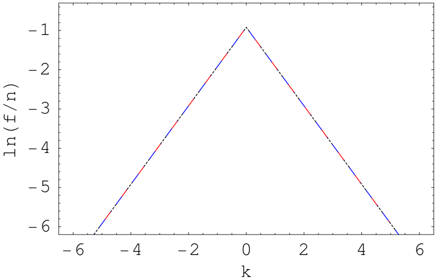
plot ln f as a function of (signed) kinetic energy k



$\cdots\ x = 0$ — $x = 0.25$ — $x = 0.65$

velocity filtration how does it work?

rescale f with n



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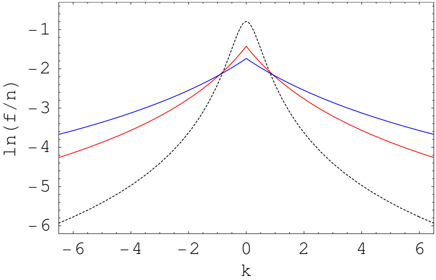
velocity filtration suprathermal boundary condition

suprathermal f_0, i.e., with tails fatter than a Maxwellian

$$f_0(p) = \frac{\sqrt{2}}{\pi\,(1+p^4)}$$



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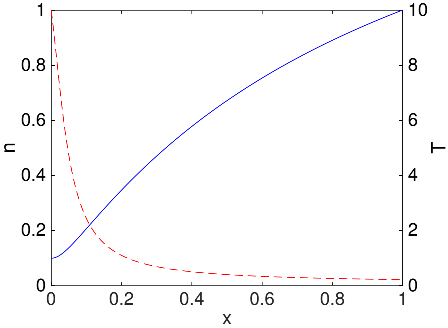

temperature inversion

velocity filtration summary

pros

- simple and general mechanism for temperature inversion
- needs no active energy injection in sparser regions of the system
- makes no use of specific ingredients (magnetic fields, turbulence,...)

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cons

- what about interactions?
- needs a non-thermal boundary condition for all times
  a very strong assumption, seems to rule out isolated systems
- who keeps the system in a non-thermal state at the boundary?
  still an *ad hoc* "active" ingredient

a toy model

N unit mass particles, generic long-range interaction V

$$\mathcal{H} = \sum_{i=1}^{N} \frac{p_i^2}{2} + \frac{1}{N} \sum_{i=1}^{N} \sum_{j<i}^{N} V\left(\left|\mathbf{r}_i - \mathbf{r}_j\right|\right)$$

restrict to $d = 1$ and expand V in a Fourier series to the lowest order

$$\mathcal{H} = \sum_{i=1}^{N} \frac{p_i^2}{2} + \frac{J}{N} \sum_{i=1}^{N} \sum_{j<i}^{N} \left[1 - \cos\left(\vartheta_i - \vartheta_j\right)\right]$$

Hamiltonian Mean Field (HMF) model

particles on a ring with all-to-all interactions

XY spins on a complete graph (mean-field interactions)

$J > 0$ attractive/ferromagnetic interactions; $J < 0$ repulsive/antiferro

$J > 0$ equilibrium phase transition breaking the $O(2)$ symmetry at small energy (temperature)

broken symmetry phase: clustered/magnetized

[M. Antoni & S. Ruffo PRE 1995]

HMF dynamics

$t < \tau_{coll} \Longrightarrow$ Vlasov equation for $f(\vartheta, p, t)$

$$\frac{\partial f}{\partial t} + p\frac{\partial f}{\partial \vartheta} - \frac{\partial\left(\langle u\rangle + \psi\right)}{\partial \vartheta}\frac{\partial f}{\partial p} = 0$$

self-consistent interaction

$$\langle u\rangle(\vartheta, t) = \int d\vartheta' \int dp'\, u(\vartheta - \vartheta') f(\vartheta', p', t)$$

+ (possibly) external field ψ

for the HMF model

$$\begin{aligned} u(\vartheta - \vartheta') &= J\left[1 - \cos(\vartheta - \vartheta')\right] \\ \psi(\vartheta) &= -h\cos\vartheta \end{aligned}$$

initial conditions ($t = 0$) $\longrightarrow$ "violent relaxation" ($t = \mathcal{O}(1)$) $\longrightarrow$ QSS (stable stationary Vlasov solution) ($t < \tau_{coll}$) $\longrightarrow$ thermal equilibrium ($t > \tau_{coll}$)

HMF velocity filtration and temperature inversion

- if f is stationary (QSS)
- if the net effect of $\langle u \rangle + \psi$ is attractive (clustered QSS)

Vlasov equation for HMF $\approx$ collisionless Boltzmann equation of the Scudder model

HMF velocity filtration and temperature inversion

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  ...if the velocity distribution is suprathermal!

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  ...if the velocity distribution is suprathermal!

⟶ suprathermal velocity distribution as initial condition of the dynamics

[LC & Gupta EPJB 2014]

...still not very appealing as a general mechanism...it seems you need to prepare such a very particular state

kicking a long-range system away from equilibrium

- prepare a HMF model in a clustered (magnetized) thermal equilibrium at $t = 0$
- evolve until $t = t_0$ then switch on an external field for a short time τ
- look what happens next...

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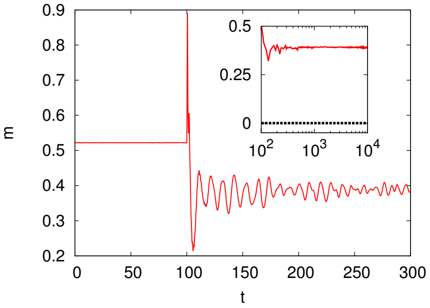

[T. N. Teles, S. Gupta, P. Di Cintio & LC PRE(R) 2015]

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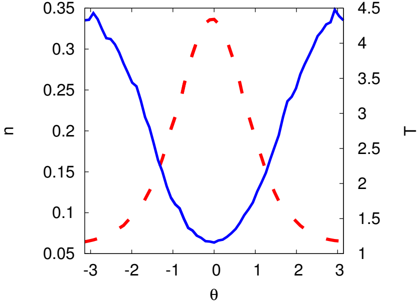

[T. N. Teles, S. Gupta, P. Di Cintio & LC PRE(R) 2015]

quenching a long-range system from equilibrium to nonequilibrium

- prepare a HMF model in a clustered (magnetized) thermal equilibrium at $t = 0$ with external field h
- evolve until $t = t_0$ then quench the external field to another value h'
- look what happens next...

quenching a long-range system from equilibrium to nonequilibrium

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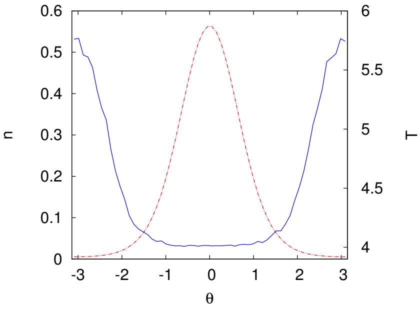
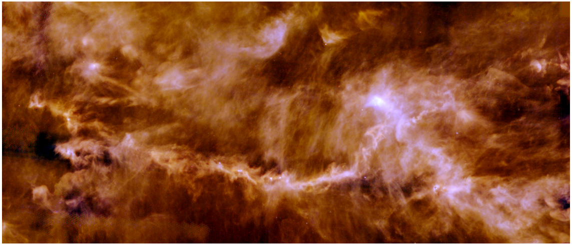


- the same also by quenching J

[S. Gupta & LC NJP 2016]

beyond toy models: from astrophysical scales...

filaments in molecular clouds



[esa/Herschel]

model: cylindrical symmetry $\Longrightarrow$ two-dimensional self-gravitating system (2DSGS)

$$\mathcal{H} = \sum_{i=1}^{N} \frac{|\mathbf{p}_i|^2}{2m} + G m^2 \sum_{i=1}^{N} \sum_{j<i}^{N} \ln |\mathbf{r}_i - \mathbf{r}_j|$$

[Katz & Lynden-Bell 1978; Marcos PRE 2013; Toci & Galli MNRAS 2014]

beyond toy models: from astrophysical scales...

- prepare a 2DSGS model in a thermal equilibrium state at $t = 0$
- evolve until $t = t_0$ then apply a radial perturbation
- look what happens next...

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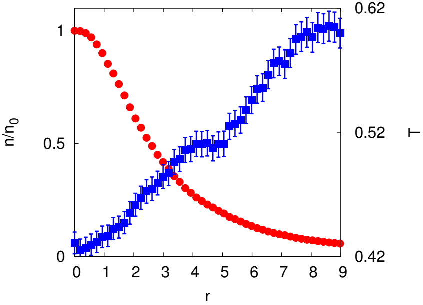
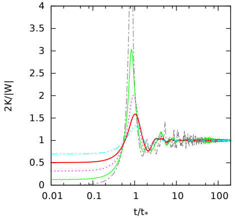


[T. N. Teles, S. Gupta, P. Di Cintio & LC PRE(R) 2015]

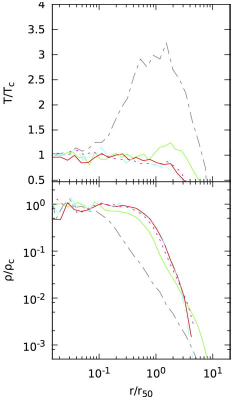
beyond toy models: from astrophysical scales...

– cold collapse

initial condition: Gaussian overdensity with kinetic energy < virialized



⟶

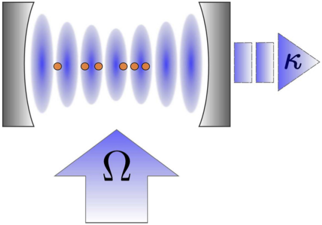


– colder collapses consistent with observed density profiles

[Di Cintio, Gupta & LC arXiv 2017 (MNRAS subm)]

beyond toy models: ...to atomic scales

cold atoms trapped in a 1-d single-mode optical cavity



model: semiclassical + dissipationless limit $\Longrightarrow$ mean-field Hamiltonian dynamics

$$\mathcal{H} = \sum_{i=1}^{N} \frac{p_i^2}{2} + \frac{J}{N}\left[\sum_{j=1}^{N} \cos\left(kx_j\right)\right]^2$$ with $J \propto$ laser intensity

[Schütz & Morigi PRL 2014]

beyond toy models: ...to atomic scales

- take $k = 1$ and prepare the system in an inhomogeneous thermal equilibrium state at $t = 0$
- evolve until $t = t_0$ then quench J by suddenly changing the laser intensity
- look what happens next...

beyond toy models: ...to atomic scales

- take $k = 1$ and prepare the system in an inhomogeneous thermal equilibrium state at $t = 0$
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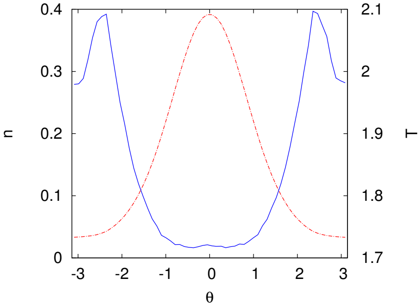


[S. Gupta & LC NJP 2016]

the physical picture

long-range-interacting system in thermal equilibrium

for 2DSGS also cold nonequilibrium initial state

$\Downarrow$

perturbation/quench

or whatever brings the system far from equilibrium with oscillating mean field

$\Downarrow$

inhomogeneous QSS

$\Downarrow$

temperature inversion in the QSS

- generic feature of long-range-interacting systems
  both mean-field & slowly decaying forces, attractive/repulsive with confining external field, 1-d & 2-d (hopefully 3-d too)
- robust w.r.t. changes in the parameters & in the protocol
  temperature inversion is always there!
- what is going on?
  a general mechanism at work...

wave-particle interaction

- the perturbation induces a wave in the system

  after the kick the system gains energy and m oscillates

  a cold collapse starts with virial oscillations

- the wave is damped and the system settles in a QSS

  how can it be? no collisions!

- wave-particle interactions! (Landau damping)

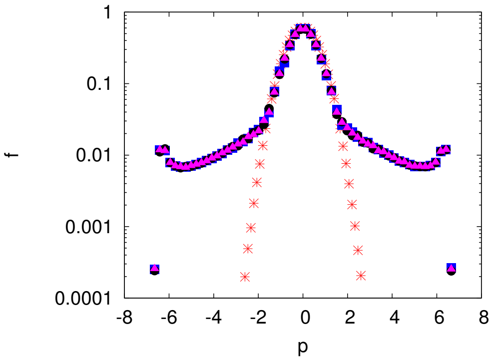
  particles interact with the oscillating mean field

- wave-particle interactions $\approx$ selective in velocity

  interaction $\approx$ locally changes $f(v)$ after the kick

  let's check it...

wave-particle interaction



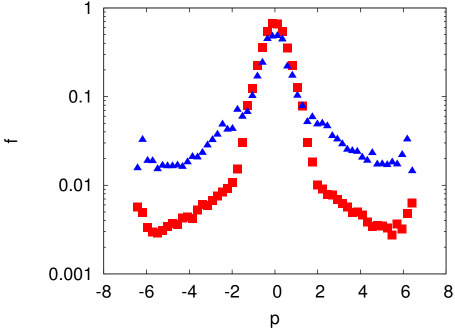
cumulative momentum distribution $f(p)$ as a function of time ($*$ t = 0)

[T. N. Teles, S. Gupta, P. Di Cintio & LC PRE(R) 2015]

wave-particle interaction and velocity filtration

velocity filtration is back!

- $f(v)$ has suprathermal tails in the QSS
- velocity filtration produces temperature inversion



distribution function $f(\vartheta, p)$ ■ $\vartheta = 0$ ▲ $\vartheta = \pi$

[T. N. Teles, S. Gupta, P. Di Cintio & LC PRE(R) 2015]

summary & outlook

summary

- temperature inversion from astrophysical to atomic scales
  astrophysics: examples of a general phenomenon rather than a collection of unrelated phenomena
  may occur in any system with long-range interactions, also at atomic scales: "universality"
- minimal ingredients of temperature inversion
  long-range interactions, clustered steady nonequilibrium state, fat-tailed velocity distributions
- basic and general physical mechanism
  temperature inversion spontaneously appears after the damping of collective oscillations
  interplay between wave-particle interaction and spatial inhomogeneity leading to velocity filtration
- long-living quasi-stationary states obtained disturbing equilibrium
  typically show nonuniform temperature profiles and temperature inversion

summary & outlook

what next?

- 3-d self-gravitating systems
  cD galaxies, Larson's power laws for molecular clouds... (work in progress)
- trapped ions and particle beams
  close to (antiferrro) HMF and 2-d Coulomb systems, respectively (work in progress)
- experiments (hopefully)
  atoms in a cavity, trapped ions, particle beams...
- physical picture $\Longrightarrow$ theory?