James Binney (University of Oxford)

Modelling our barred Galaxy with angles & actions

Galaxy surveys

- Massive surveys of our Galaxy now underway (LAMOST, APOGEE, Galah,…,Gaia)
    - Extraction of science from these a major focus of astronomy in next decade
- How is it structured?
- How does it function (as a machine)?
- How did it form?

Outline

- Axisymmetric galaxy models
- Quasiperiodic orbits --> f(J) modelling
- Staeckel Fudge (successes, limitations) -->
- Torus Mapping
- Mapped tori near resonance --> p-theory
- Non-axisymmetry via p-theory
  - OLR
  - CR
- f(J) --> local v-space
- Getting J(x,v)
- Conclusions & outlook

Axisymmetric Galaxy models

- MW a cooperative exercise
- Φ generated by stars of many types and zillions of DM particles
- We have to track the DM, & we can do that only in so far as the MW is in statistical equilibrium
- So our 1st job is construction of axisymmetric mean-field models

Orbits

- Orbits in plausible (strongly flattened) Φs are quasiperiodic
- Implies that orbits admit 3 constants of motion
- For many reasons it's wise to choose these to be *action* integrals
  - Actions are (nearly) unique
  - In standard axisymmetric case they are J_r J_z J_ϕ

- Analogy to gyrokinetics?

Distribution functions

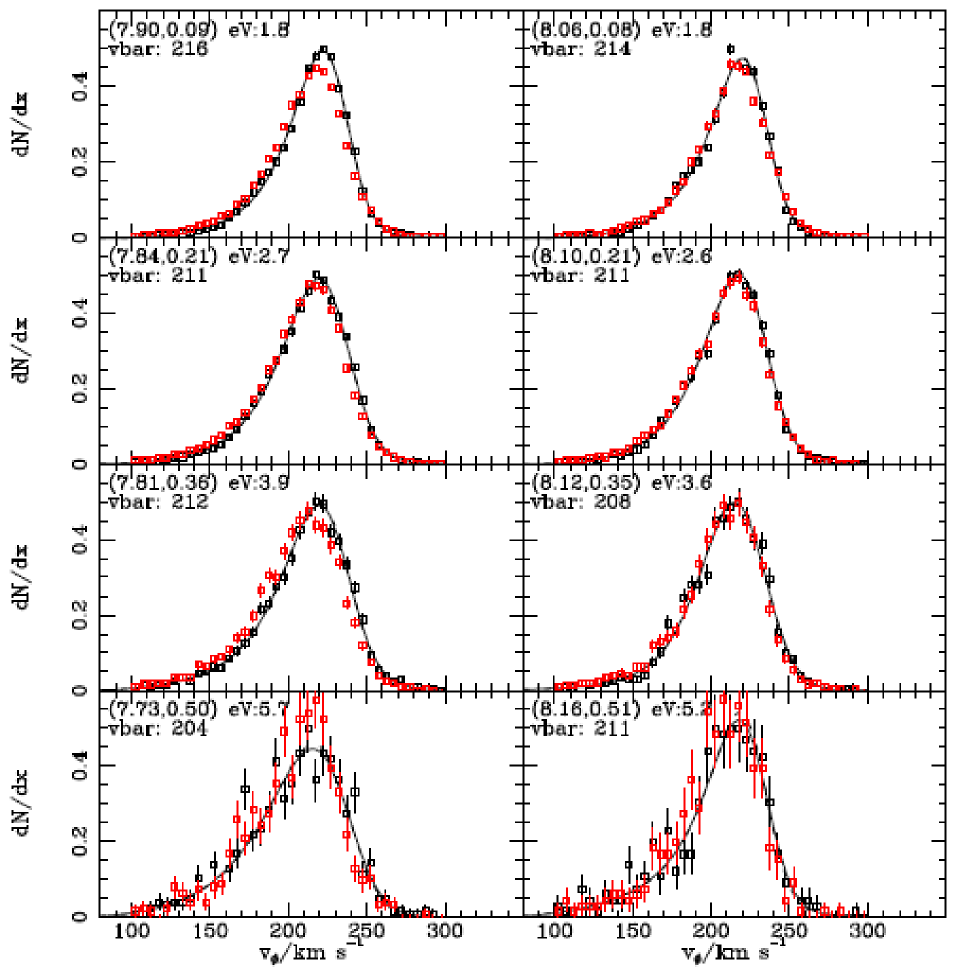
- Each species (G MS stars, WDs, …, DM particles) has a DF f(J)
- Given J(x,v) and the DFs f(J) we can compute ρ(x) and from Poisson find Φ(x)
- The form of J(x,v) depends on Φ so we have to iterate
- But the iterations converge rapidly (B 2014, Piffl Penoyre & B 2015)

How to get actions J(x,v)

- Classically we get J(x,v) by solving the Hamilton-Jacobi eqn
- We need H-J eqn to separate, which requires either spherical symmetry or Φ is of Staeckel form
- B 2012 introduced the Staeckel Fudge which extends J(x,v) to general axisymmetric Φ
- Sanders & B 2016 extended SF to non-rotating triaxial Φs
- SF is a non-rigorous uncontrolled approximation but it *works*
  - Errors in J <~ 5% typically

Predicting kinematics Binney, Burnett + RAVE 2014

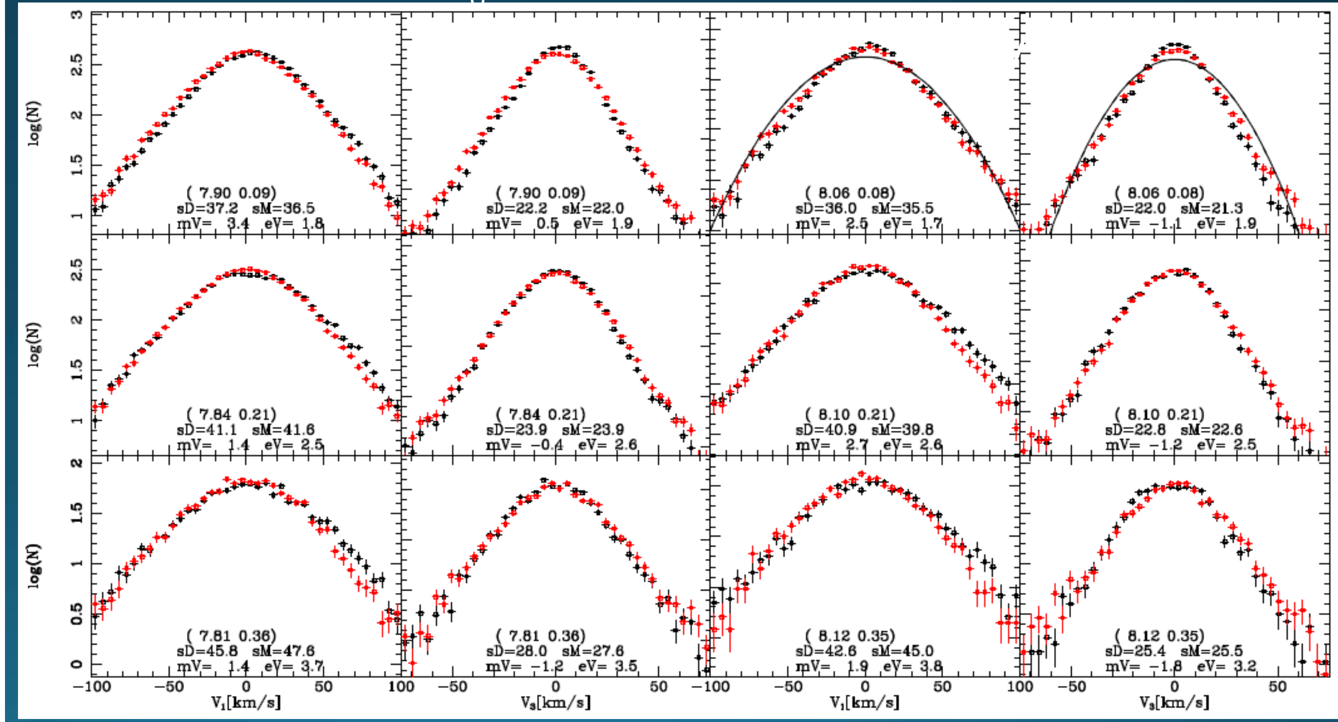
- Binney (2012) fitted disc f(J) to GCS data (s <~ 0.1 kpc)
- Binney + (2014) tested its *predictions* for kinematics of RAVE stars in 8 volumes with s <~ 2 kpc



Cool dwarfs

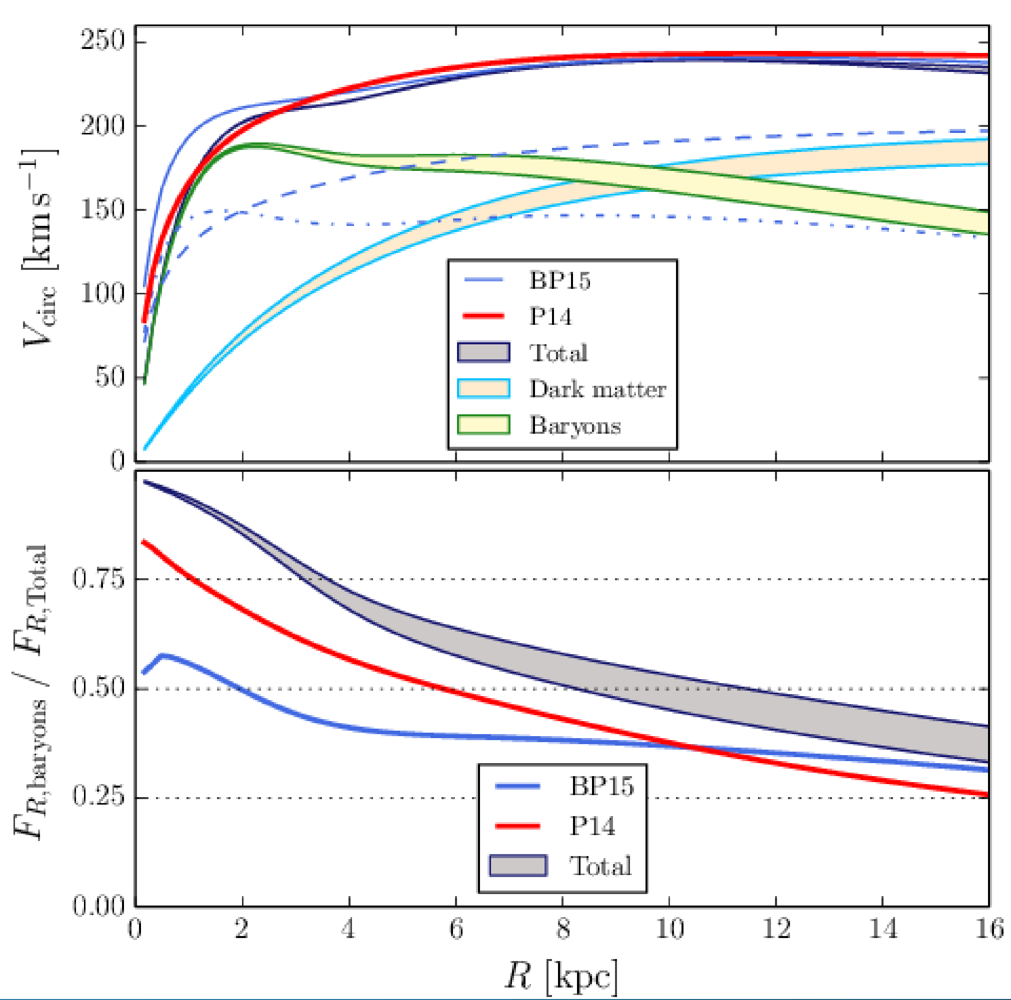
R < R_0

R > R_0



Models with self-consistent Φ

- Show that dark halo of MW has *not* been adiabatically compressed (B & Piffl 2015, Cole & B 2017)



From Fudge to tori

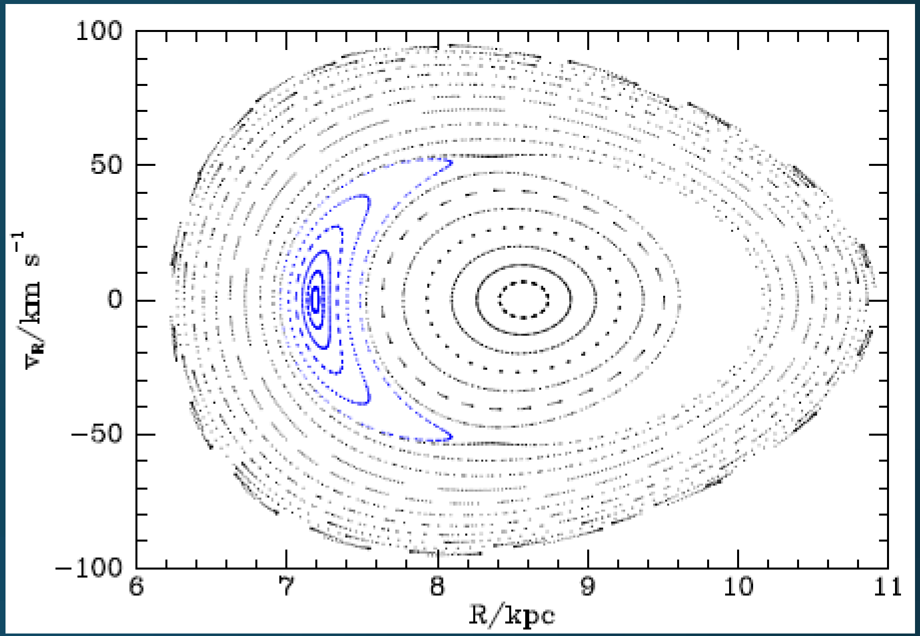
- SF based on separable Φ
- Assumes existence of global AA coordinates
- Actually even in axisymmetric Φ there are islands of “resonantly trapped orbits”
- Also necessary to consider rotating non-axisymmetric Φs
- Torus Mapping provides a way forward (B & McMillan 2016)

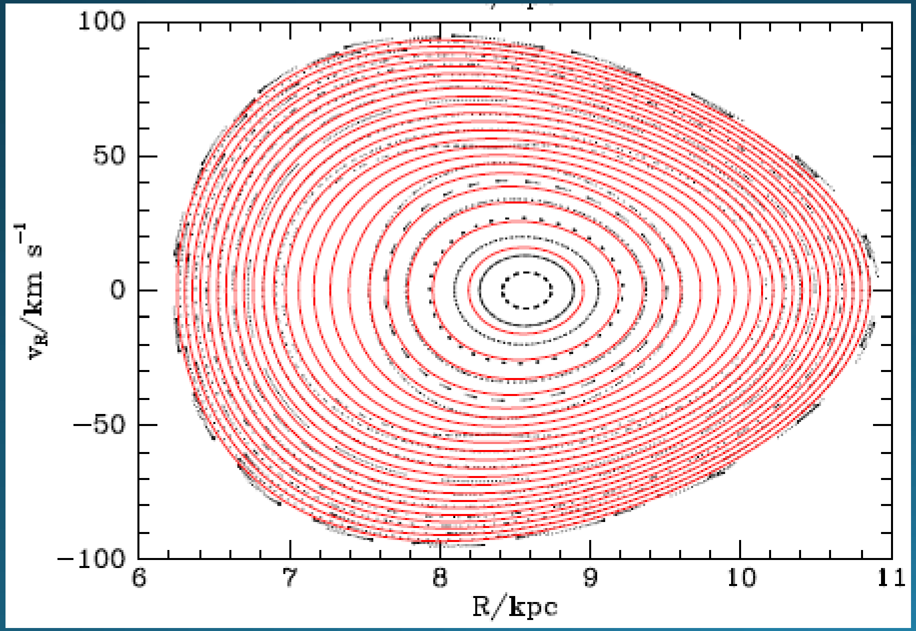
Torus Mapping

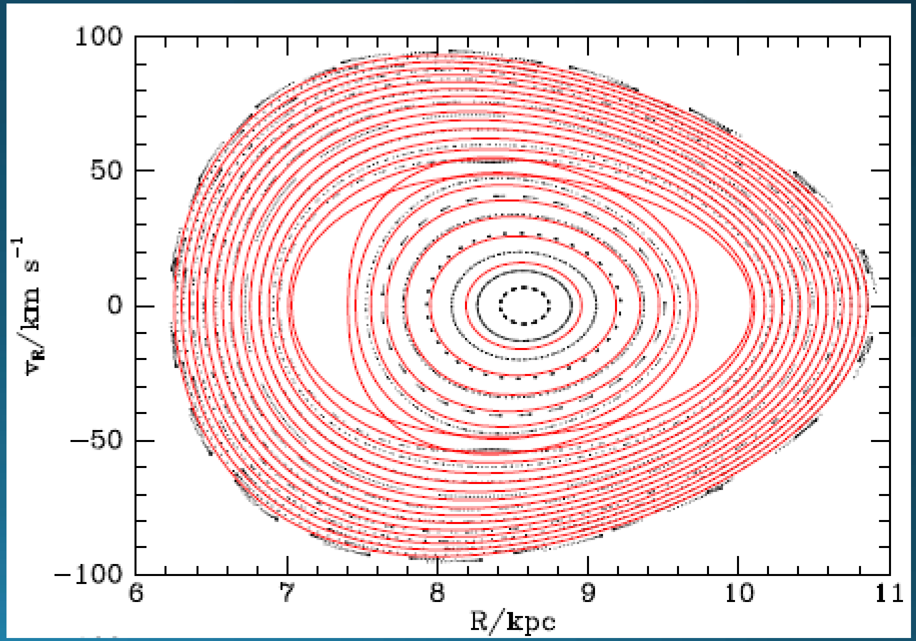
- Numerically construct generating function that maps analytic torus (of harmonic oscillator or isochrones potential) into MW's phase space such that H ~ const on a given torus J
  - Technique started with McGill & B 1990, much developed by Kaasalainen 1994
  - Code released by B & McMillan 2016
  - Extended to resonantly trapped orbits (B 2016, 2017)

Resonance
$\text{Omega}_r = \text{Omega}_z$

- Force-fitting tori of wrong type
- We flop from one minimum to another



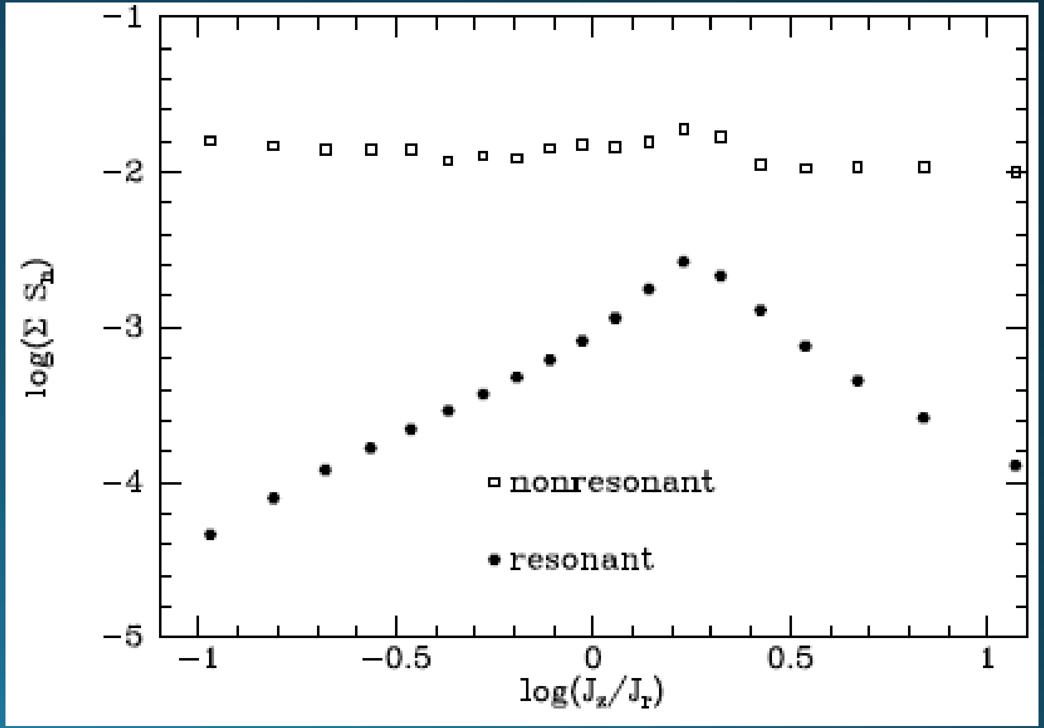


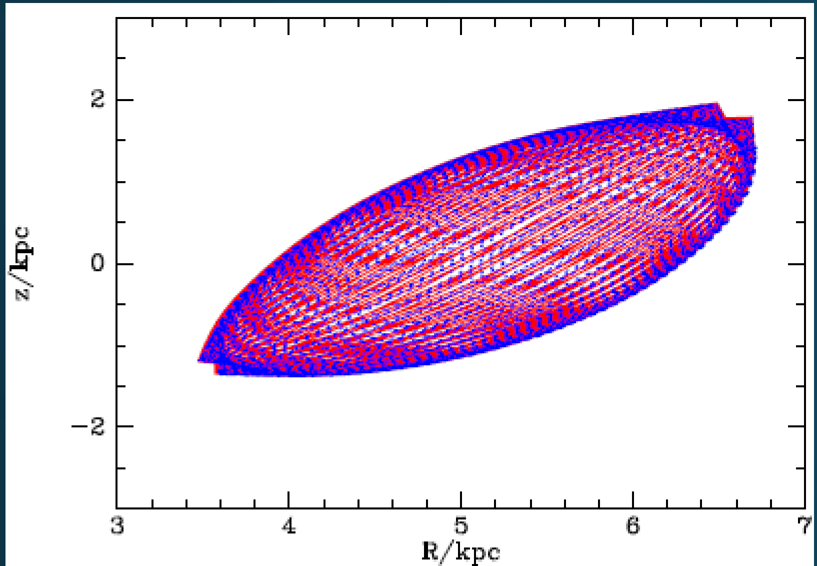


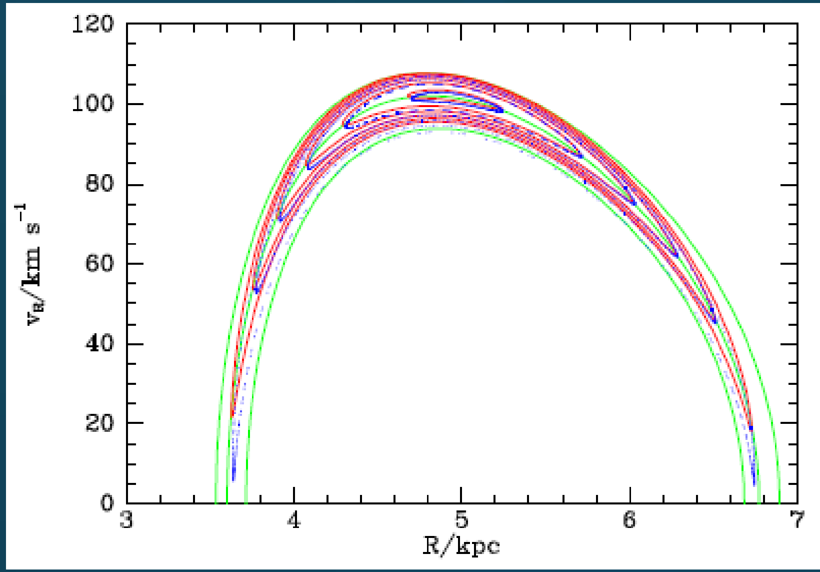
Application of perturbation theory

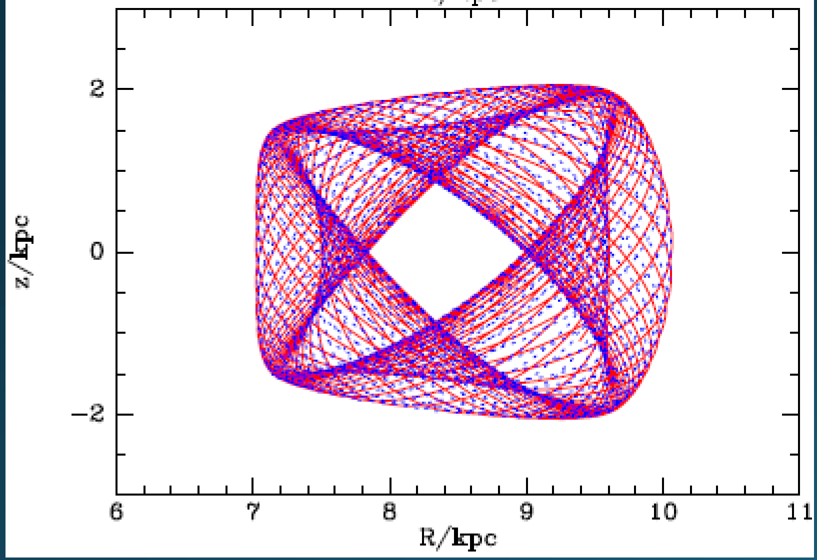
- Use AA cords obtained by interpolating between good tori on either side of trapping zone
- Fourier decompose complete H on these tori
- Drop non-resonant terms
- Solve enhanced pendulum eqn for 1d motion

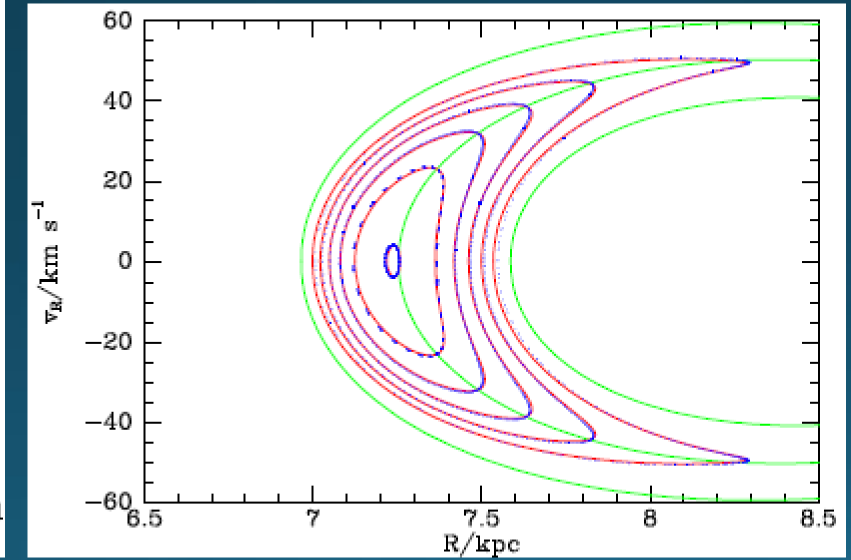
Fourier components of GF

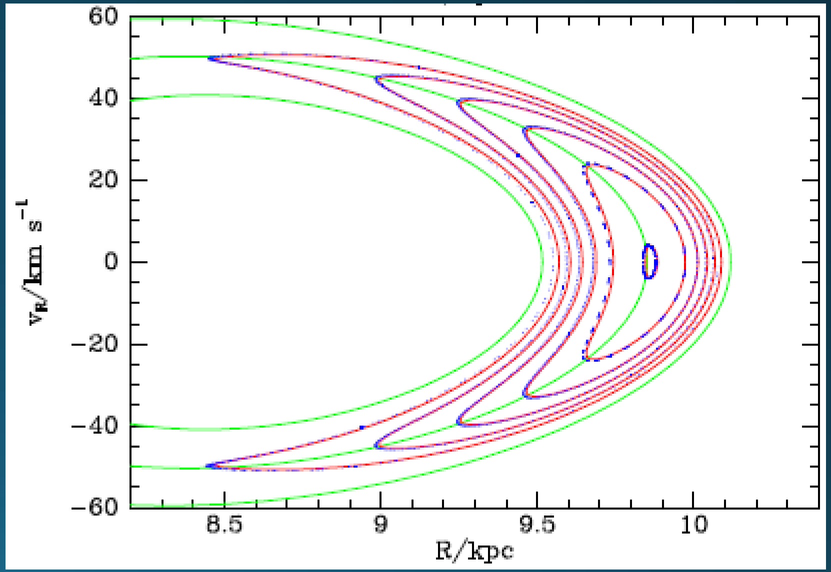



z/kpc
R/kpc
2
0
−2
3
4
5
6
7

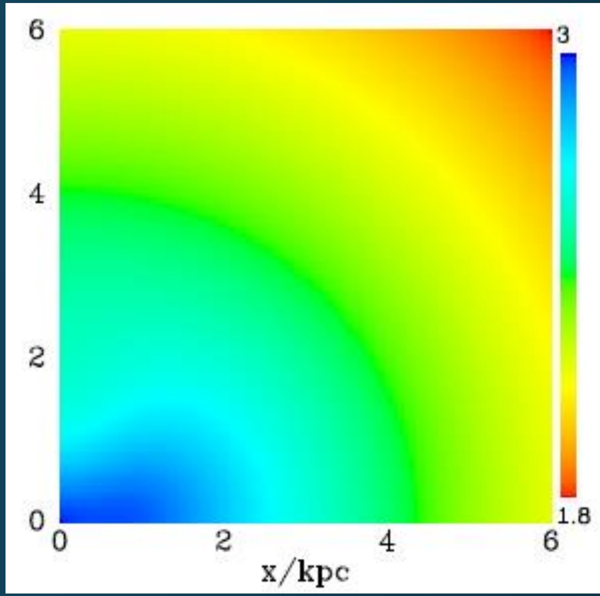

v_R/km s^{-1}
R/kpc
120
100
80
60
40
20
0
3
4
5
6
7


z/kpc
R/kpc
2
0
−2
6
7
8
9
10
11

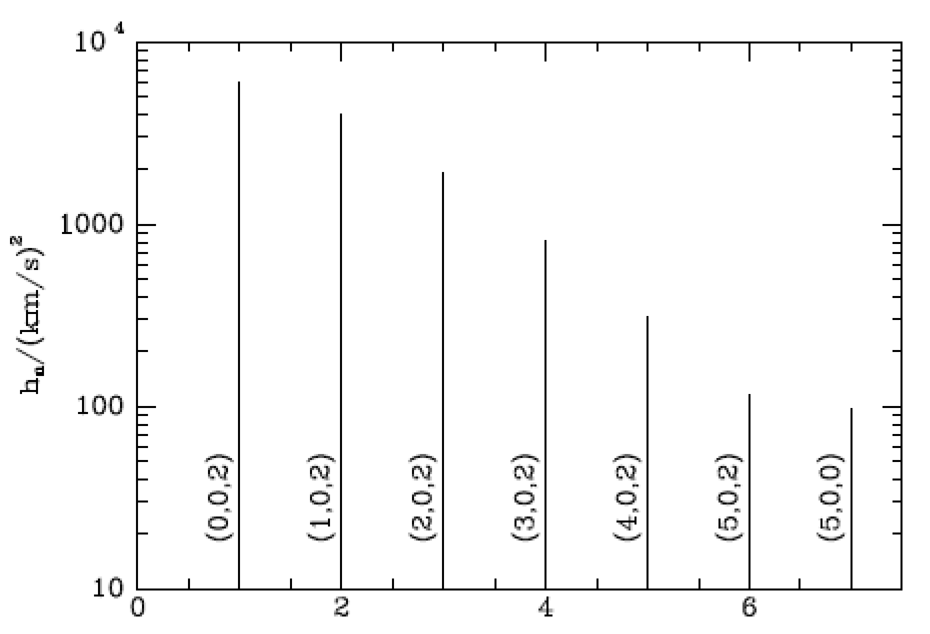

v_R/km s^{-1}
R/kpc
60
40
20
0
−20
−40
−60
6.5
7
7.5
8
8.5


v_R/km s^{-1}
R/kpc
60
40
20
0
−20
−40
−60
8.5
9
9.5
10

Non-axisymmetric tori from p-theory

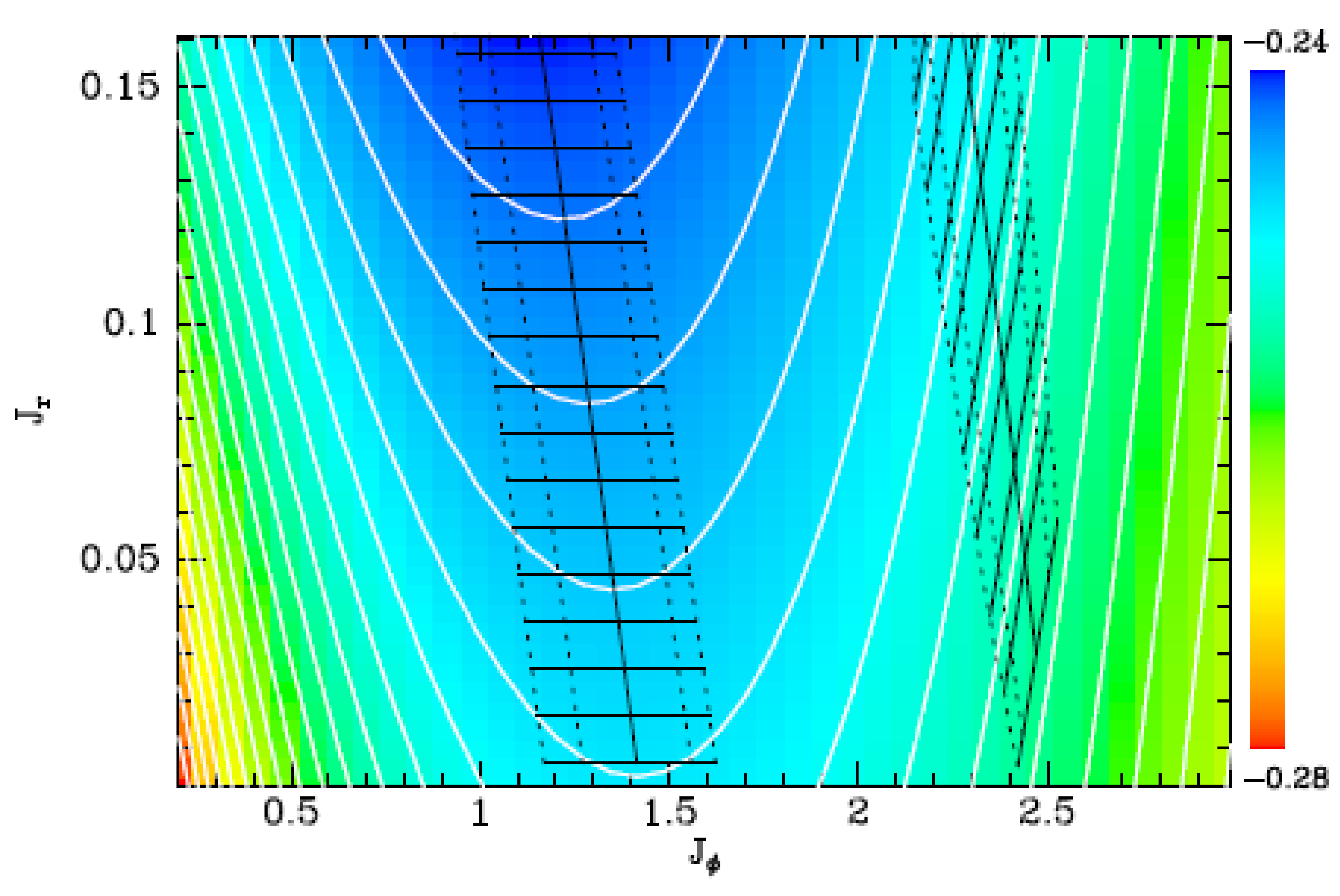


- Superpose an analytic but realistic bar on detailed MW Φ
- Split $H(\theta,\mathbf{J}) = \{\overline{H}(\mathbf{J}) - \omega_{\rm p} J_\phi\} + H_1(\theta,\mathbf{J})$
- Fourier analyse H_1
- Apply enhanced pendulum eqn to resonant terms
- Add in effects of non-resonant terms



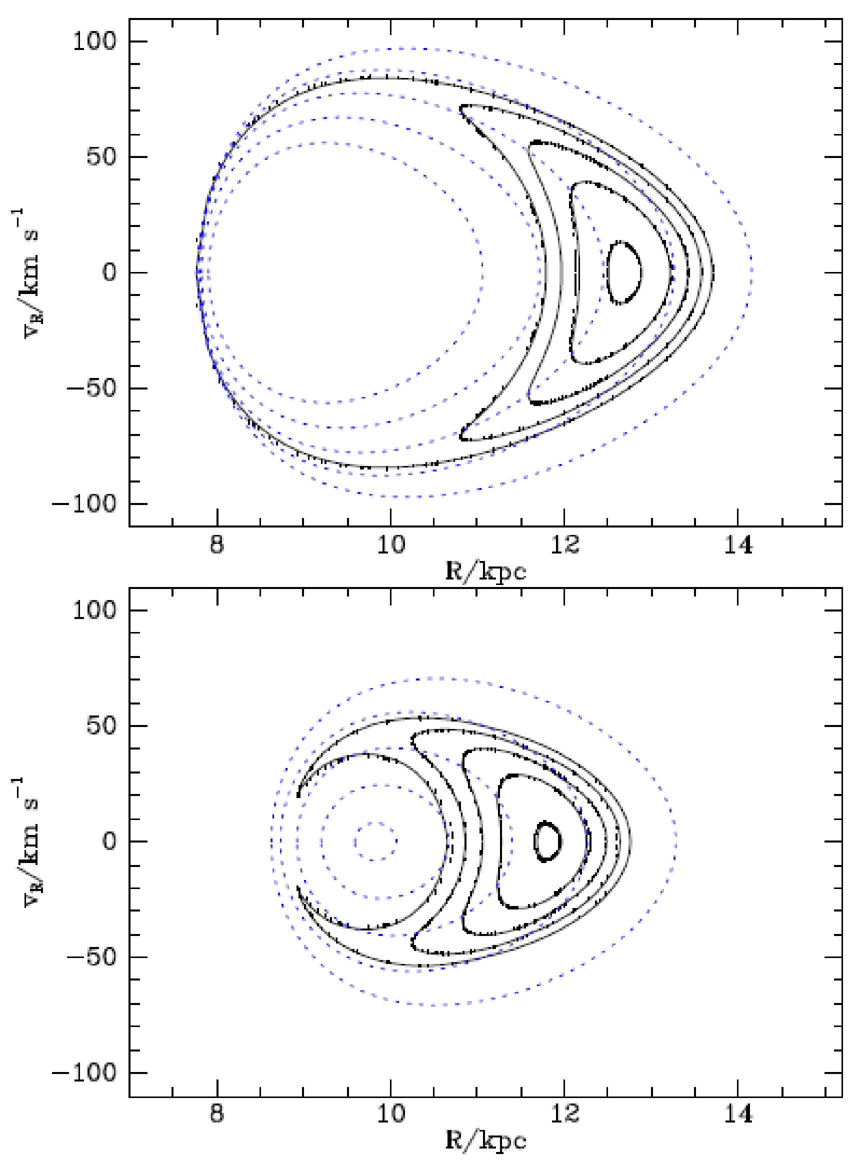
Trapping at CR & OLR takes space!

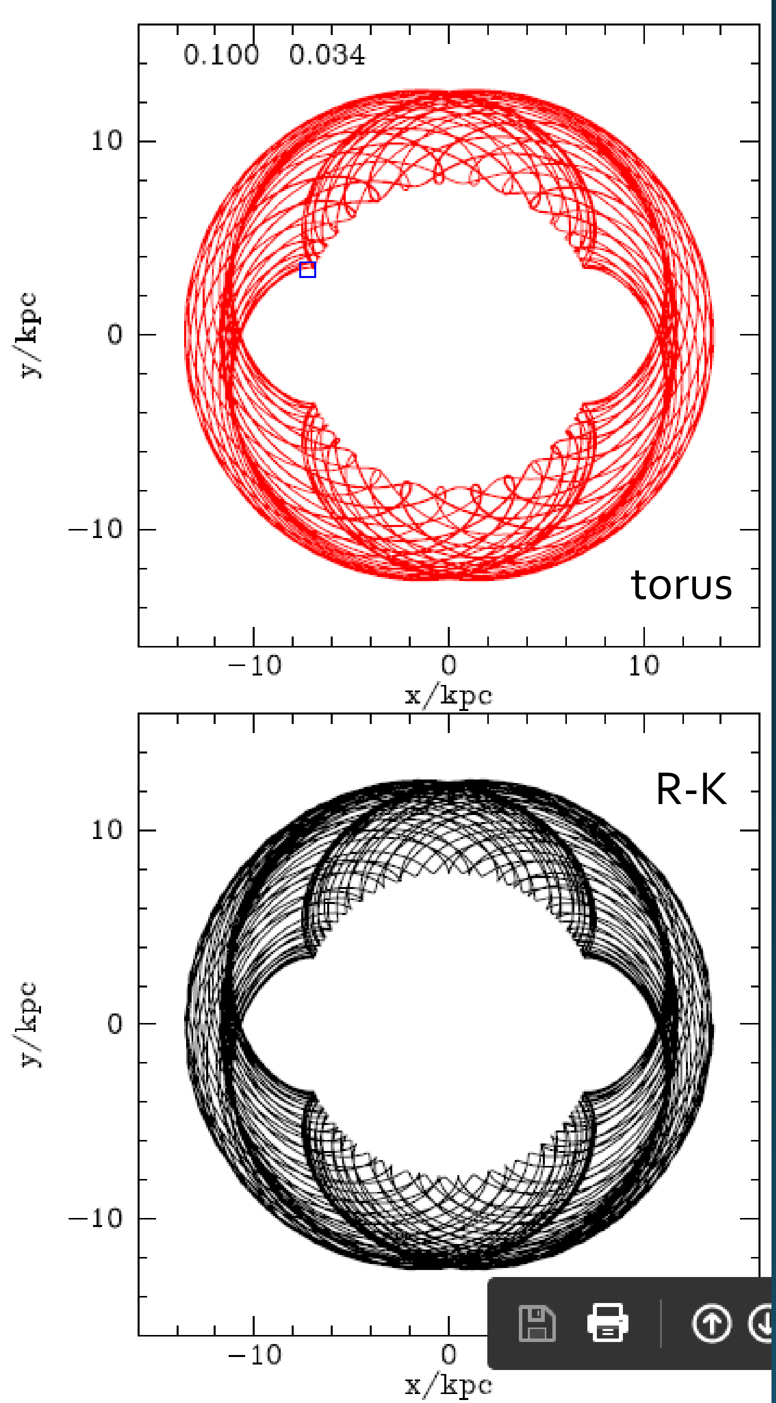
- Slice of J-space



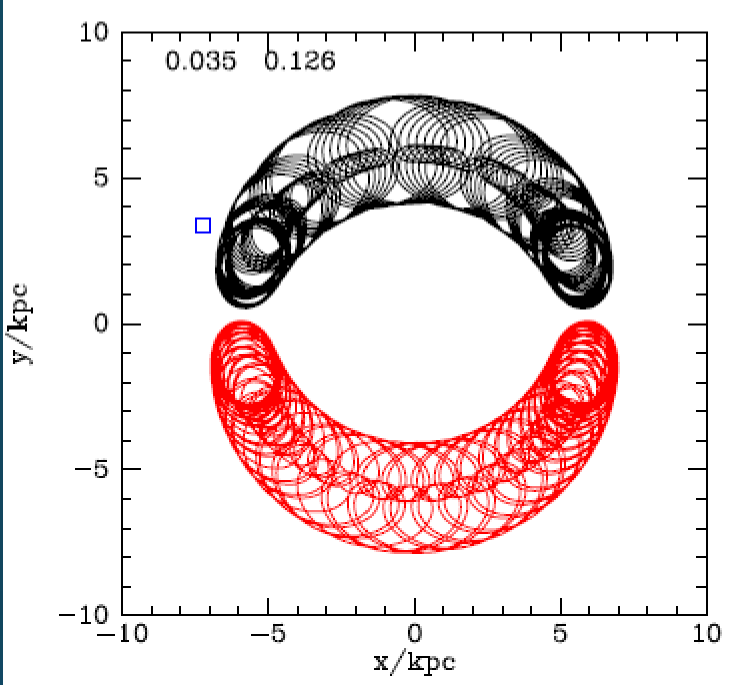
Enhanced pendulum really works

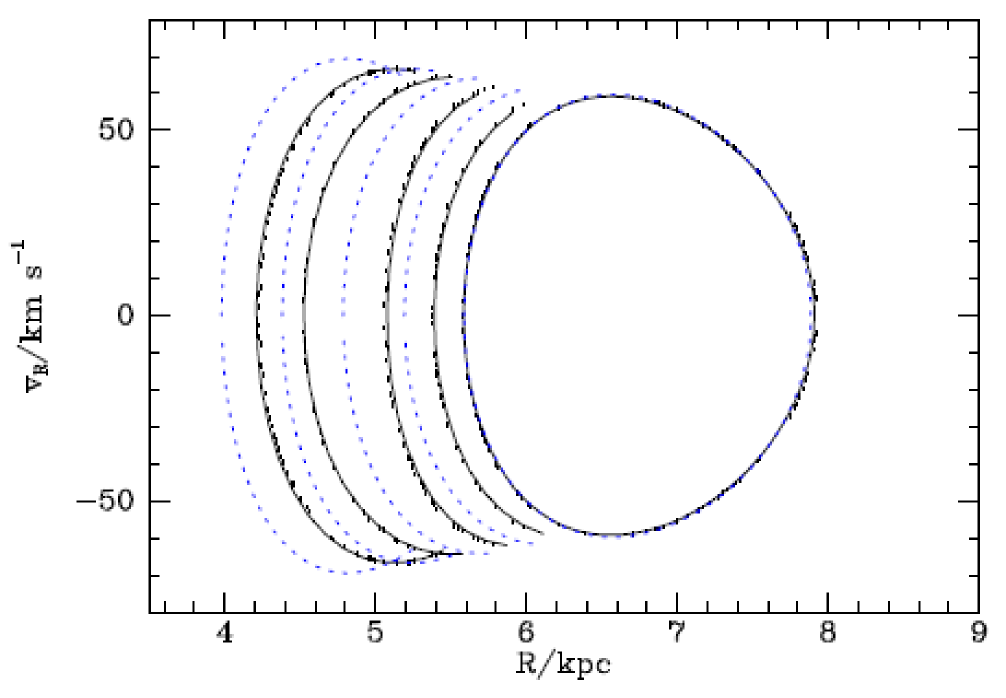
- Orbit at OLR

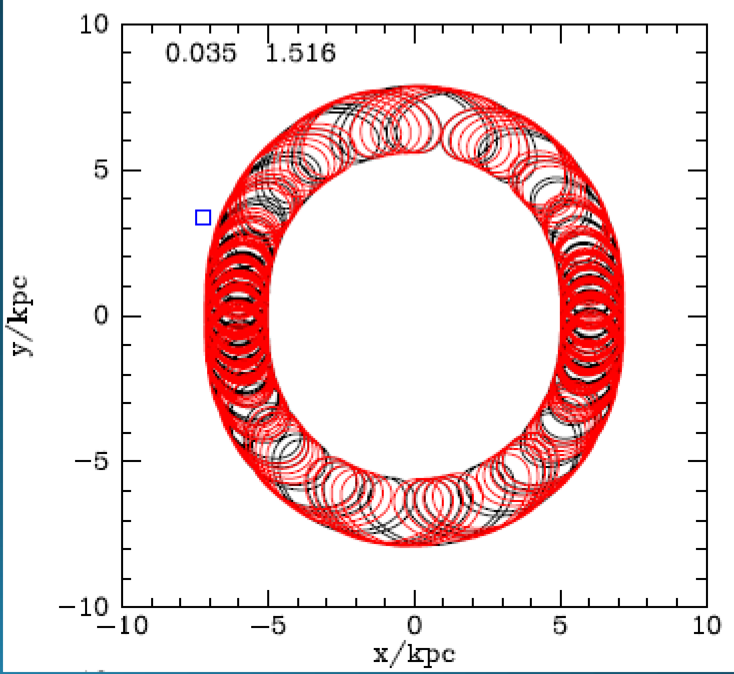




At CR too


0.035 0.126
10
5
0
−5
−10
−10
−5
0
5
10
x/kpc
y/kpc

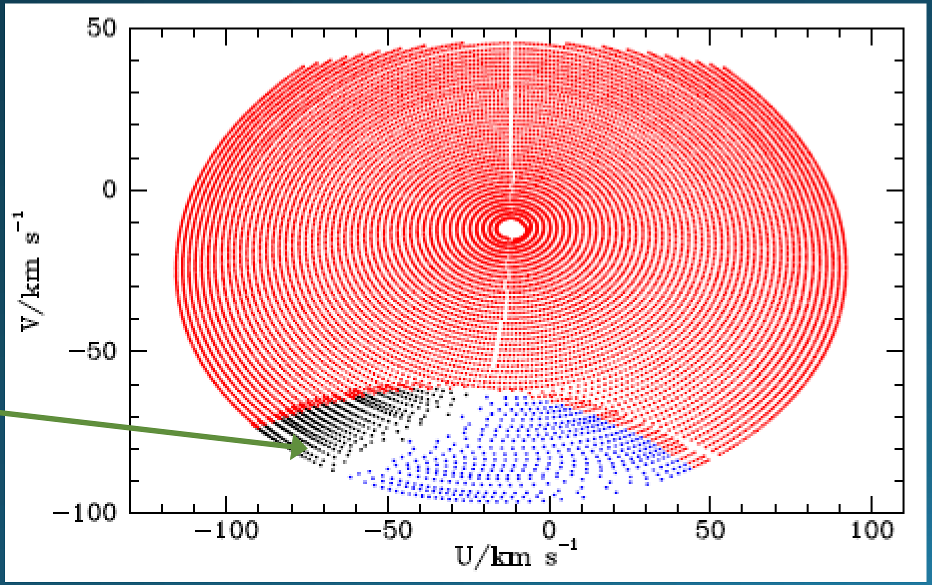

50
0
−50
4
5
6
7
8
9
R/kpc
v_R/km s^{-1}

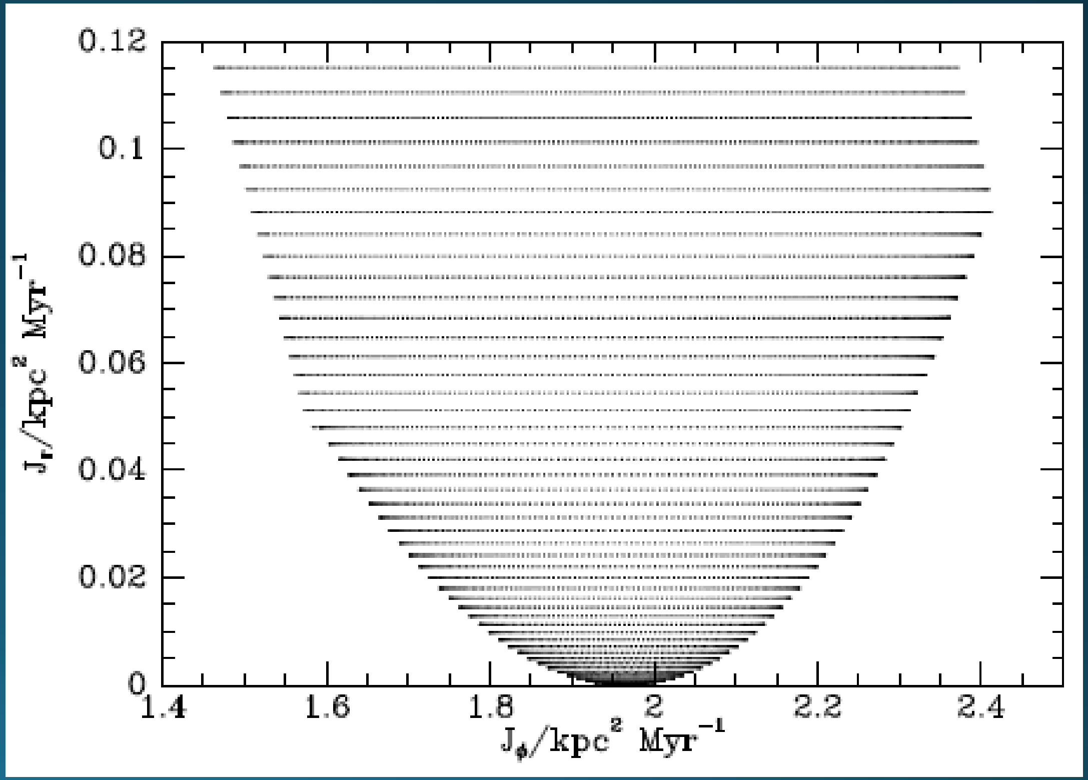

0.035 1.516
10
5
0
−5
−10
−10
−5
0
5
10
x/kpc
y/kpc

Computing observables

- A feature of a torus is ability to find v given x
- Unfortunately v-space samples J-space very non-uniformly
- With care can sample v-space ~uniformly

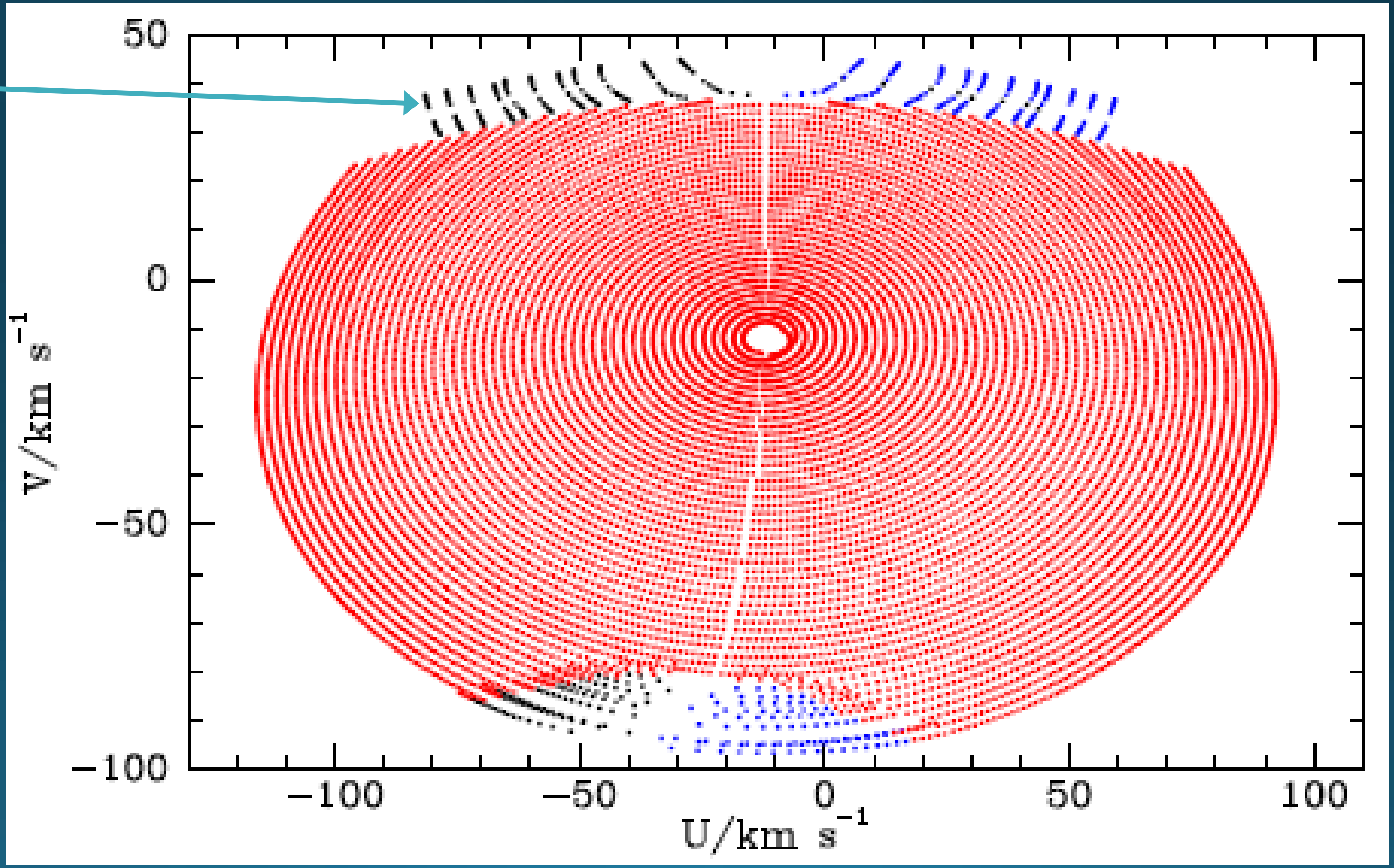
Trapped at CR





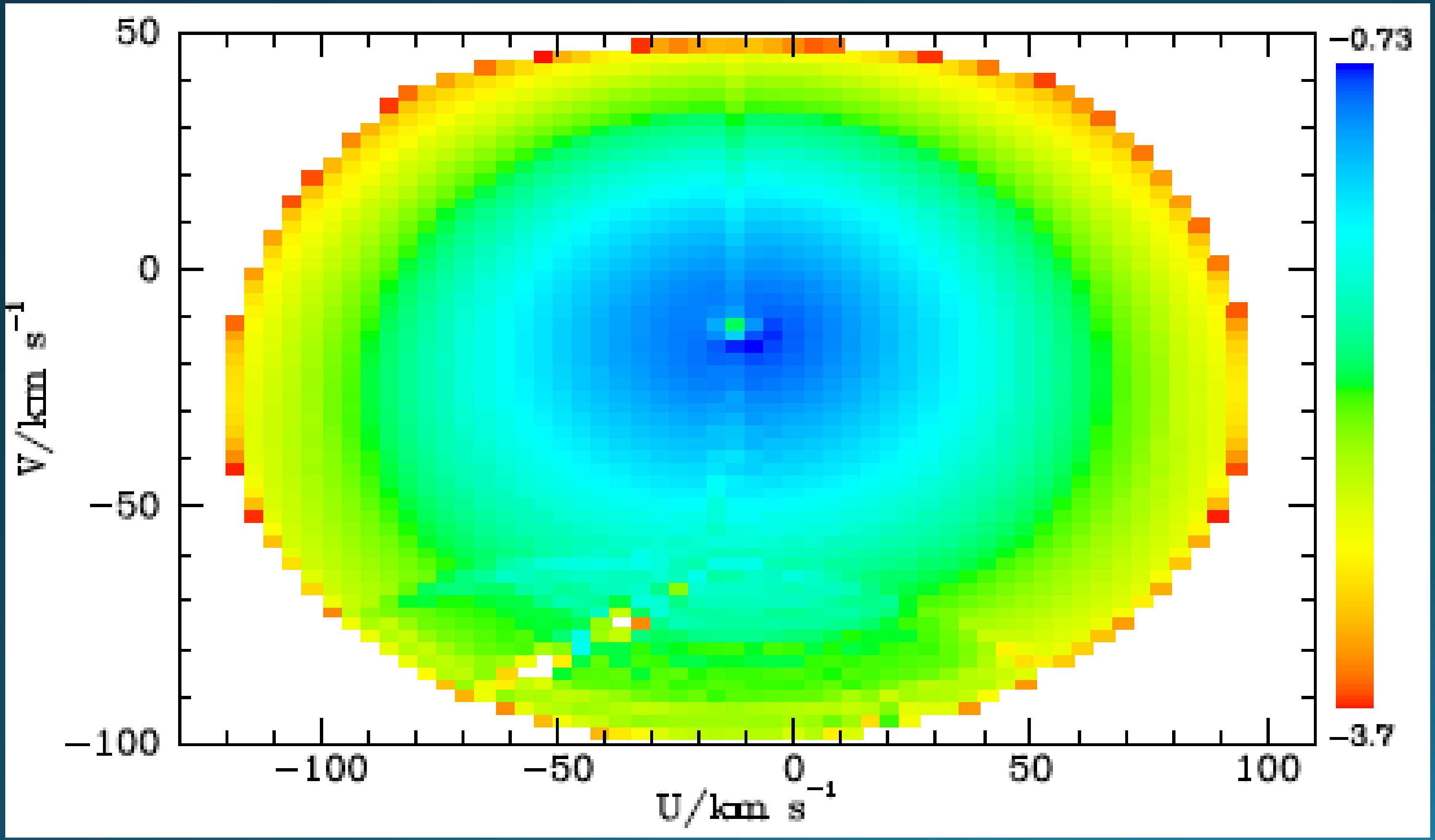
A higher pattern speed

Trapped at OLR



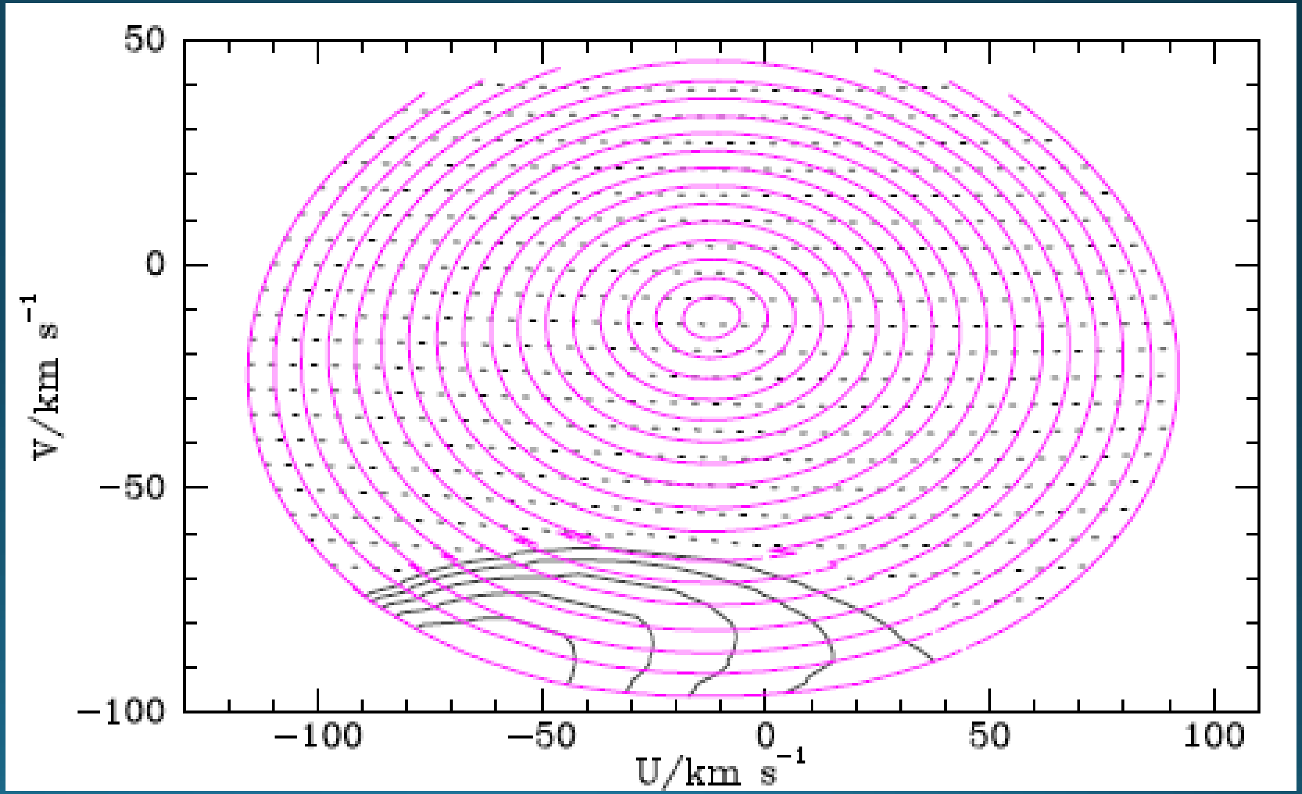
Qualitative modification to orbits may have small impact on observables

- Density of stars in v-space for a particular (realistic) f(J)



Getting J(x,v)

- Alternatively on a grid of x determine J(v) and interpolate to get v(J)
- Magenta const J_r
- Blue dashed const J_ϕ
- Grey const $J_{libration}$



Conclusions & outlook

- f(J) modelling enables
  - Multicomponent modelling
  - In self-consistent potential
- Through the Staeckel Fudge we have had good success in axisymmetric case
- Must move on to steadily rotating non-axisymmetric Φ
- Then SF not available and must resort to orbit-based techniques
- Torus modelling supersedes Schwarzschild modelling
- Basic axisymmetric tori are obtained non-perturbatively
- Enhanced pendulum eqn produces trapped tori with remarkable precision
- Next steps:
  - Build self-consistent axisymmetric model via tori
  - Build self-consistent barred model via tori