DE LA RECHERCHE À L'INDUSTRIE

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THEORETICAL RESULTS FOR THE VLASOV-GAUSS SYSTEM PERTAINING TO NONLINEAR WAVE PROPAGATION IN A PLASMA

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SUMMARY

☞ Theoretical self-consistent resolution of the Vlasov-Gauss system from the linear to the strongly nonlinear regime

For a nearly monochromatic, propagating, plasma wave

$$E(x,t) = E_0(x,t)S[\varphi(x,t)]$$

$$S(\varphi + 2\pi) = S(\varphi) \qquad k \equiv \nabla\varphi,\ \omega \equiv -\partial_t\varphi$$

$$|\nabla(k,\omega)| \ll k(k,\omega),\ |\partial_t(k,\omega)| \ll \omega(k,\omega)$$

IF $|\nabla E_0| \ll kE_0,\ |\partial_t E_0| \ll \omega E_0$

Vlasov → charge density : $\rho \equiv \rho(E_0, k, \omega, \partial_{x,t}E_0, \partial_{x,t}k, \partial_{x,t}\omega, \dots)$

Stop at 1st order : geometrical optics approximation

Gauss law →

$$\{\partial_t\mathcal{N}(E_0,k,\omega) + \partial_x(v_g\mathcal{N}(E_0,k,\omega))\}_{\mathcal{A}_s} + \nu\mathcal{N}(E_0,k,\omega) = 0$$

Or more complicated Envelope equation

Gauss law →

$$\omega \equiv \omega\{E_0, k\}$$

Dispersion relation

OVERVIEW

☞ Modeling of the plasma

☞ Sinusoidal wave

☞ Derivation of the charge density (uniform and stationary plasma)

1. Delta-like distribution function & large enough growth rate
   Application to the nonlinear growth and saturation of plasma instabilities
2. Smooth (e.g. Maxwellian) distribution function & small growth rate
   Envelope equation for a driven wave including
   Nonlinear decrease of the Landau damping rate Generalized collisionless dissipation
   Application to “kinetic inflation” of stimulated Raman scattering

☞ Envelope equation in an inhomogeneous and instationary plasma from a nonlocal variational formalism

☞ Nonlocal adiabatic dispersion relation

MODELING OF THE PLASMA

Plasma : electrons + ions with electrostatic interactions

Ions : fluid → $n(x,t)$

Large scale evolution $k\Delta x \gg 1,\ \omega \Delta t \gg 1$ (hydrodynamical equations)

On large scales : $n_e \approx n_i$

Electrons : small scale evolution from

$$m\frac{dv}{dt} = -eE(x,t)$$

Kinetic effects only addressed for electrons

SINUSOIDAL WAVE

$E \equiv E_0 \sin(\varphi) \Rightarrow$ charge density : $\rho \approx \rho_c \cos(\varphi) + \rho_s \sin(\varphi)$

$$\rho_c(\varphi_0) = \pi^{-1} \int_{\varphi_0-\pi}^{\varphi_0+\pi} \rho \cos(\varphi) d\varphi$$

$$= -(ne/\pi) \int_{-\infty}^{+\infty} \int_{\varphi_0-\pi}^{\varphi_0+\pi} f(v,\varphi) \cos(\varphi) dv d\varphi$$

$$\boxed{\rho_c(\varphi) \equiv -2ne\langle\cos\varphi\rangle}$$

$$\rho_s(\varphi_0) = \pi^{-1} \int_{\varphi_0-\pi}^{\varphi_0+\pi} \rho \sin(\varphi) d\varphi$$

$$= -(ne/\pi) \int_{-\infty}^{+\infty} \int_{\varphi_0-\pi}^{\varphi_0+\pi} f(v,\varphi) \sin(\varphi) dv d\varphi$$

$$\boxed{\rho_s(\varphi) \equiv -2ne\langle\sin\varphi\rangle}$$

$$\varepsilon_0 \partial_x E_0 + 2ne\langle\sin(\varphi)\rangle = 0$$

Envelope equation

Gauss law

$$1 + \frac{2ne\langle\cos(\varphi)\rangle}{\varepsilon_0 k E_0} = 0$$

Dispersion relation

SLOWLY-VARYING WAVE

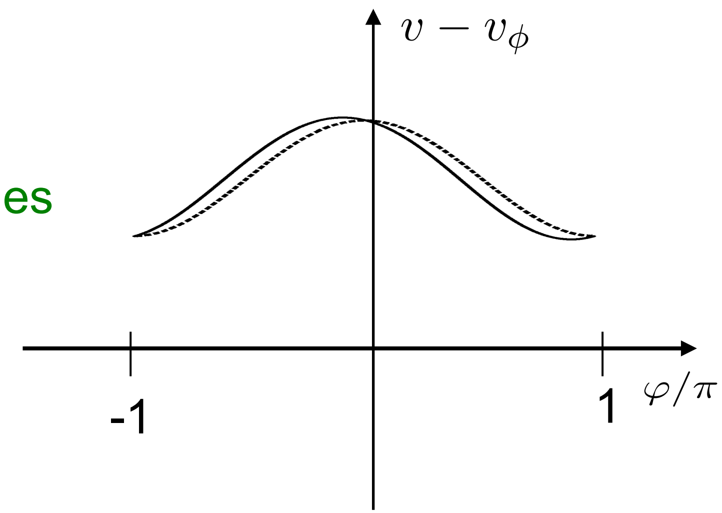
Hamiltonian dynamics

$$H = \frac{(v - v_\phi)^2}{2} - \Phi \cos(\varphi)$$

$v \to v/v_{th}$

$\Phi \equiv eE_0/kT_e$

Slowly-varying wave :
Orbits close to frozen ones
≈ symmetric % v-axis



$$\langle \cos(\varphi) \rangle \approx \langle \cos(\varphi) \rangle_{adiabatic}$$

Whatever the wave amplitude

$$\langle \sin(\varphi) \rangle_{adiabatic} = 0$$

Needs to go beyond the adiabatic approximation to derive $\langle \sin(\varphi) \rangle$

$\langle \sin(\varphi) \rangle$

$\omega_B \equiv \sqrt{eE_0 k/m}$ $\qquad \gamma \equiv E_0^{-1} dE_0/dt$

$\langle \sin(\varphi) \rangle$

small amplitude →

$\omega_B/\gamma \ll 1 \Leftrightarrow \int \omega_B dt \ll 1$

Small amplitude perturbation analysis $(\varepsilon = \omega_B/\gamma)$

Like in KAM or Nekhoroshev theorem

$$\langle \sin(\varphi) \rangle \approx \langle \sin(\varphi) \rangle_{per} \equiv O(\varepsilon) + O(\varepsilon^3) + \dots$$

large amplitude →

$\gamma/\omega_B \ll 1 \Leftrightarrow \int \omega_B dt \gg 1$

Perturbation in the slowness of the dynamics

Like in neo-adiabatic theory $\varepsilon \equiv \gamma/\omega_B$

$$\langle \sin(\varphi) \rangle \approx \langle \sin(\varphi) \rangle_{neoadia}$$

Match when $\int \omega_B dt \approx 5$

$$\langle \sin(\varphi) \rangle \approx \left[1 - Y\left(\int \omega_B/5dt\right)\right] \langle \sin(\varphi) \rangle_{per} + Y\left(\int \omega_B/5dt\right) \sin(\varphi) \rangle_{neoadia}$$

$Y \approx Heaviside$

ENVELOPE EQUATION IN A UNIFORM AND STATIONAY PLASMA

PERTURBATIVE ESTIMATE OF $\langle \sin(\varphi) \rangle$

$$H = \frac{(v - v_\phi)^2}{2} - \Phi\cos(\varphi)$$

For simplicity, $v_\phi \equiv v_\phi(t) \quad \Phi \equiv \Phi(t)$

$$(\varphi, v) \to (\varphi', v') \begin{cases} v' = v - \partial_\varphi F(\varphi, v') \\ \varphi' = \varphi + \partial_{v'} F(\varphi, v') \end{cases}$$

$$\begin{aligned} H' &= H + \partial_t F \\ &= \frac{(v' - v_\phi)^2}{2} + (v' - v_\phi)\partial_\varphi F + \partial_t F - \Phi\cos(\varphi) + \frac{v_\phi^2 + (\partial_t F)^2}{2} \end{aligned}$$

IF $(v' - v_\phi)\partial_\varphi F + \partial_t F + \frac{(\partial_\varphi F)^2}{2} = \Phi\cos(\varphi)$ **(1)**

$$v' = Const., \quad f(v', t) = f(v', t = 0)$$

No exact solution to (1) → expansion

$$F \equiv \sum_n F_n$$

$$F_n = O(\Phi^n)$$

PERTURBATIVE ESTIMATE OF $\langle \sin(\varphi) \rangle$

First order :

$$(v' - v_\phi)\partial_\varphi F_1 + \partial_t F_1 = \Phi \cos(\varphi)$$

$$F_1 = \omega_B^2 \frac{\gamma \cos(\varphi) + [k(v' - v_\phi)] \sin(\varphi)}{\gamma^2 + [k(v' - v_\phi)]^2} + O(d_t\gamma, d_t^2\gamma, \dots) + O(d_t v_\phi + d_t^2 v_\phi + \dots)$$

$$\langle \sin(\varphi) \rangle_{per} = \int f_0(v') \sum_n s_{2n+1} dv' \equiv \langle \sin(\varphi) \rangle_{per}(\omega_B, \gamma)$$

$$s_1 = \frac{\omega_B^2 \gamma [k(v' - v_\phi)]}{(\gamma^2 + [k(v' - v_\phi)]^2)^2}$$

$$s_3 = -\frac{3\omega_B^6}{8} \frac{(43\gamma^4 - 26\gamma^2[k(v' - v_\phi)]^2 - 5[k(v' - v_\phi)]^4)[k(v' - v_\phi)]\gamma}{((9\gamma^2 + [k(v' - v_\phi)]^2)^2(\gamma^2 + [k(v' - v_\phi)]^2)^4)}$$

$$v' \approx v_\phi, \; \frac{s_3}{s_1} = -\frac{\omega_B^4}{216\gamma^4} \to \varepsilon = \left(\frac{\omega_B}{\gamma}\right)^2$$

NEO-ADIABATIC APPROXIMATION

Hamiltonian dynamics

$$\frac{d}{dt}\oint v d\varphi = 0$$

IF $\varphi(t+T) = \varphi(t) + 2\pi \qquad \Phi(t+T) \approx \Phi(T),\ v_\phi(t+T) \approx v_\phi(t)$

Orbit ≈ closed frozen orbit

$$I \equiv \frac{1}{2\pi}\oint_H v d\varphi \approx Const.$$

When there is no separatrix crossing

$$(\Delta I)_{sep} = \mu v_\phi \qquad \mu \in \{-2, -1, 1, 2\}$$

Only valid once $\gamma/\omega_B \ll 1$

DERIVATION OF $\langle \sin(\varphi) \rangle$ (1)

$$t = 0 \begin{cases} f(v,\varphi) = \delta(v - v_0) \\ \Phi \approx 0 \Rightarrow f(I,\theta) = \delta(I - I_0) \end{cases}$$

IF $\gamma/k|v_0 - v_\phi| > 1$

$\langle \sin(\varphi) \rangle \approx \langle \sin(\varphi) \rangle_{per}(\omega_B, \gamma)$ **until** $t = t_{\max}$ when $\langle \sin(\varphi) \rangle_{per} = S_{\max}$

$$f(I, \theta, t_{\max}) = \delta(I - I^*) f_\theta(\theta)$$

$$f(I, \theta, t > t_{\max}) = \delta(I - I^*) f_\theta(\theta - \Omega^* t)$$

$$\langle \sin(\varphi) \rangle \approx \langle \sin(\varphi) \rangle_{neoadia}(\omega_B, t)$$

$$\equiv S_{\max} \frac{K(M)^2}{K(m)^2} \sqrt{\frac{q(m)}{q(M)}} \frac{1 - q(M)}{1 - q(m)} \cos\left[\int_{t_M}^{t} \pi\omega_B / 2K(m) dt'\right],$$

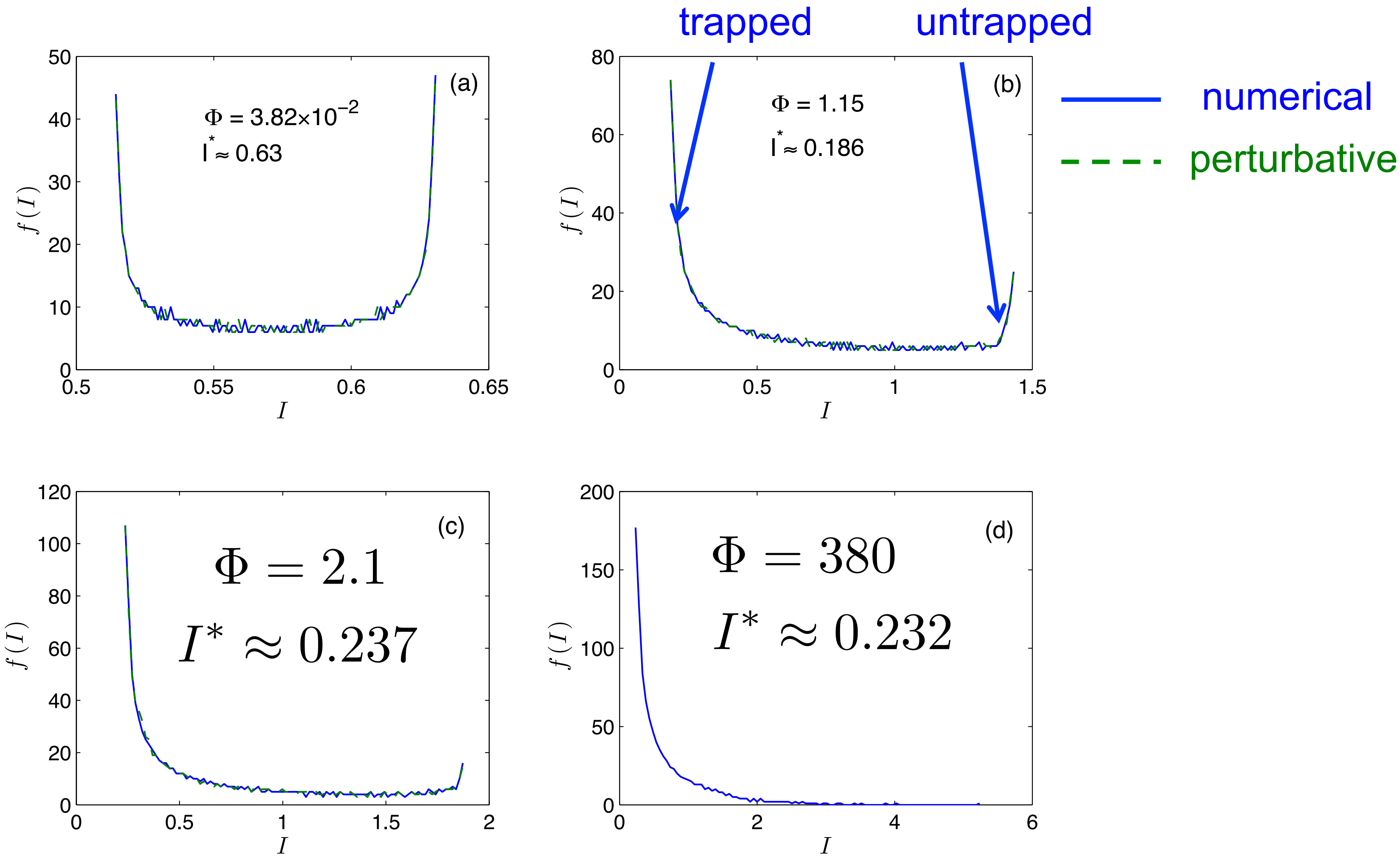
$$q(m) \equiv e^{-\pi K(1-m)/K(m)}$$

$$\frac{4\omega_B}{\pi k}\left[E(m) + (m - 1)K(m)\right] = I^*$$ (adiabatic approximation)

$$m = M, \langle \sin(\varphi) \rangle \equiv S_{\max}$$

$f(I, t*)$

$$I_0 = 1/\sqrt{3}$$



APPLICATION

Uniform plasma and wave amplitude $E \approx E_0(t)\sin(\varphi)$

Charge density : $\rho = \rho_c \cos(\varphi) + \rho_s \sin(\varphi)$

$$= -2ne\left[\langle\cos(\varphi)\rangle\cos(\varphi) + \langle\sin(\varphi)\rangle\sin(\varphi)\right]$$

Gauss law : $kE_0\cos(\varphi) = -2ne\left[\langle\cos(\varphi)\rangle\cos(\varphi) + \langle\sin(\varphi)\rangle\sin(\varphi)\right]$

$\Rightarrow \langle\sin(\varphi)\rangle = 0$

$$\langle\sin(\varphi)\rangle = \int f_0(v_0)\langle\sin(\varphi)\rangle(v_0)dv_0$$

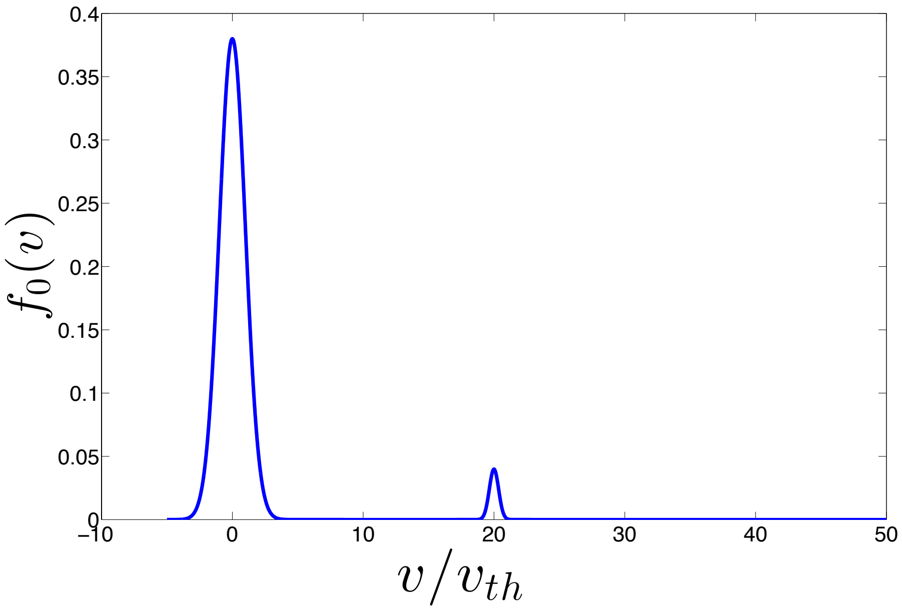
$$\equiv \langle\sin(\varphi)\rangle(\gamma, \omega_B, t)$$

$\langle\sin(\varphi)\rangle(\gamma, \omega_B, t) = 0$ Algebraic equation for the NL growth rate

Very powerful way to derive NL growth and saturation of electrostatic plasma instabilities

May be generalized to $E = E_0(x,t)\sin(\varphi)$

EXAMPLE : COLD BEAM-PLASMA INSTABILITY

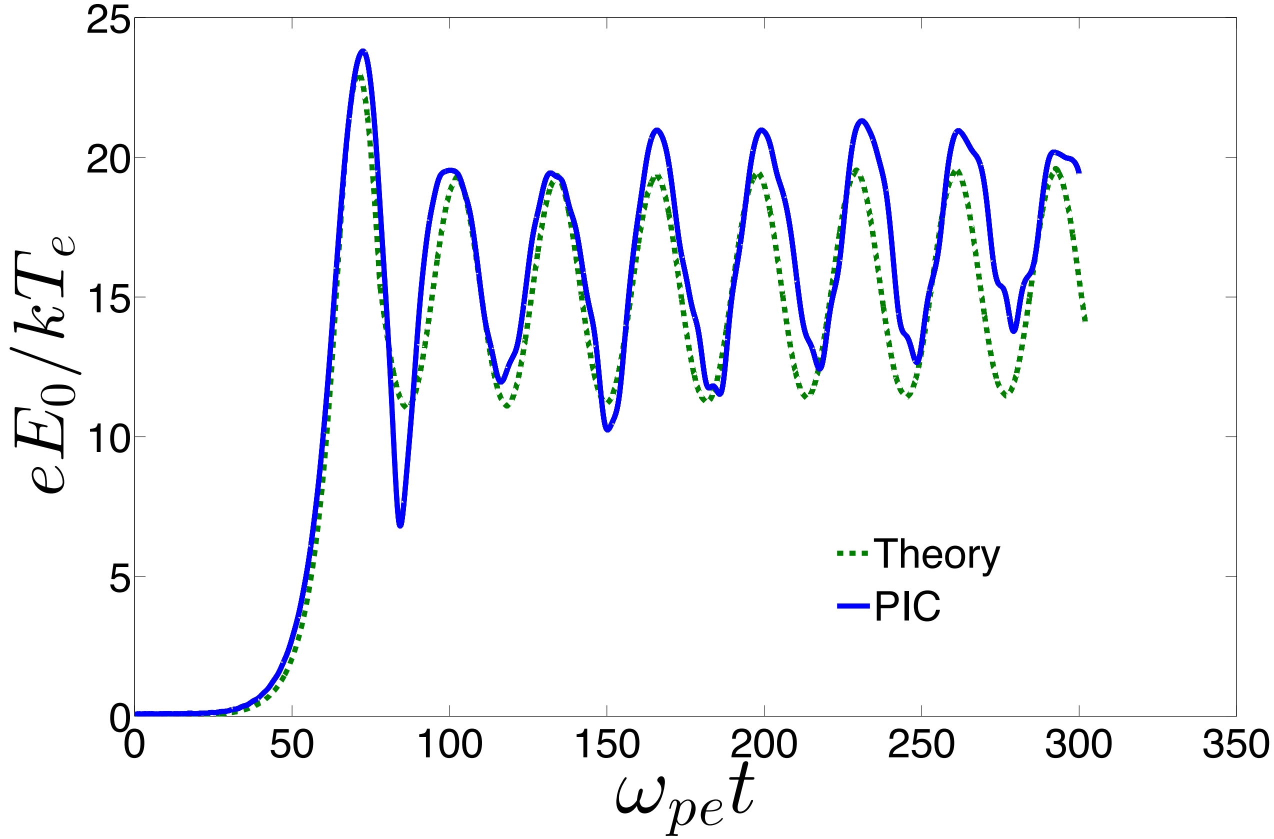


Unstable growth of a nearly monochromatic wave $v_\phi \approx v_{beam}$

$$\gamma_{lin}/k(v_{beam} - v_\phi) > 1$$

Periodic boundary conditions $E \approx E_0(t)\sin(\varphi)$

COLD BEAM - PLASMA INSTABILITY



$T_p = 10eV$

$T_b = 0$

$v_b = 20v_{th}$

$n_b/n_p = 10^{-2}$

N.B. $\gamma_{per} \approx \gamma_{lin}$

Vindicates the approximation $\dot{\gamma} \approx 0$ when performing the perturbative expansion

DERIVATION OF $\langle \sin(\varphi) \rangle$ (2)

IF : $\gamma / k v_T < 0.1 \qquad E \approx E_0(t)\sin(\varphi)$

$v_T = \text{typical range of variation of } f_0(v)$

For an initial Maxwellian it is the thermal velocity : $v_T = v_{th}$

For a growing wave $\langle \sin(\varphi) \rangle \approx \langle \sin(\varphi) \rangle_{per}$ until $\int \omega_B dt \approx 5$

When $\int \omega_B dt > 5$ heuristic approach (proven by variational calculation)

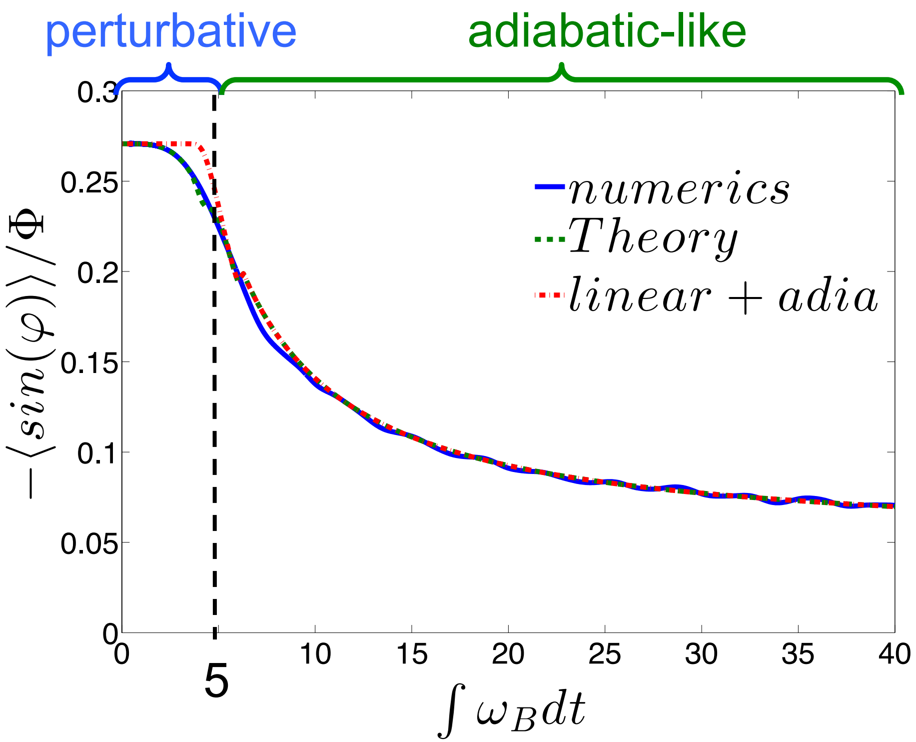
$$\langle e^{i\varphi} \rangle(\omega + i\gamma) \approx \langle e^{i\varphi} \rangle(\omega + i0) + i\gamma\partial_\omega \langle e^{i\varphi} \rangle(\omega + i0)$$

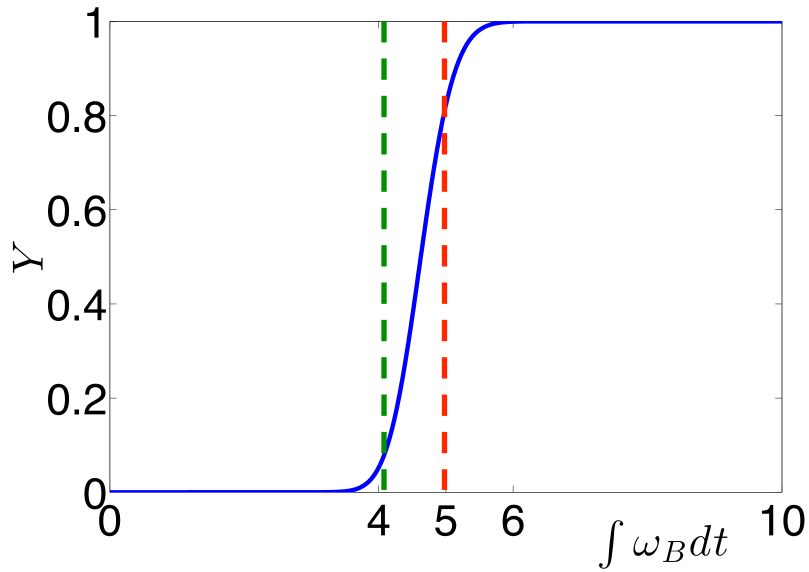
$$\langle \sin(\varphi) \rangle(\omega + i\gamma) \approx \underbrace{\langle \sin(\varphi) \rangle_{adia}}_{=0} + \gamma\partial_\omega \langle \cos(\varphi) \rangle_{adia}$$

$$\boxed{\langle \sin(\varphi) \rangle(\omega + i\gamma) \approx \gamma\partial_\omega \langle \cos(\varphi) \rangle_{adia}}$$

WAVE SLOWLY GROWING IN A MAXWELLIAN PLASMA

Example : $\gamma/kv_{th} = 0.05$ $\quad v_\phi = 3v_{th}$





$$\langle \sin(\varphi)\rangle_{th} = (1-Y)\langle \sin(\varphi)\rangle_{per} + Y\gamma\partial_\omega\langle\cos(\varphi)\rangle_{adia}$$

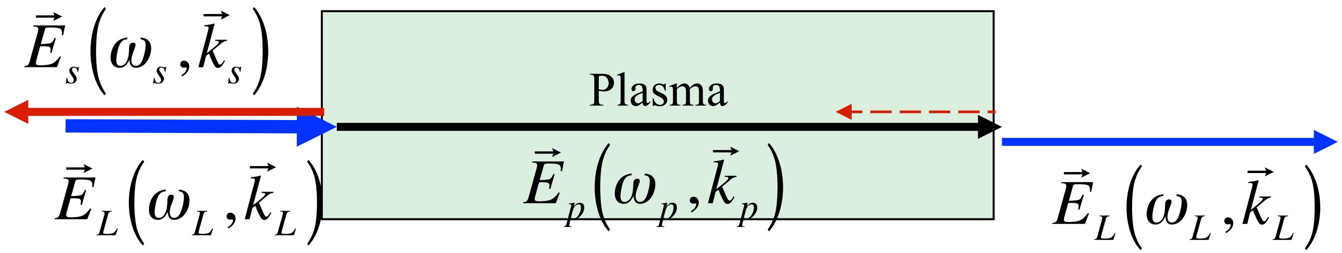
$$\approx (1-Y)\langle \sin(\varphi)\rangle_{lin} + Y\gamma\partial_\omega\langle\cos(\varphi)\rangle_{adia}$$

Gauss law for a driven wave : $$-\frac{2}{(k\lambda_D)^2}\left\langle \frac{\sin(\varphi)}{\Phi} \right\rangle E_0 = E_d$$

$(\lambda_D \equiv v_{th}/\omega_{pe},\ E_d \equiv \text{drive amplitude},\ \Phi \equiv eE_0/kT_e)$

$\left\langle \frac{\sin(\varphi)}{\Phi} \right\rangle \searrow$ when $\Phi \nearrow \Rightarrow$ wave driven more efficiently

Stimulated Raman Scattering (SRS) : kinetic inflation

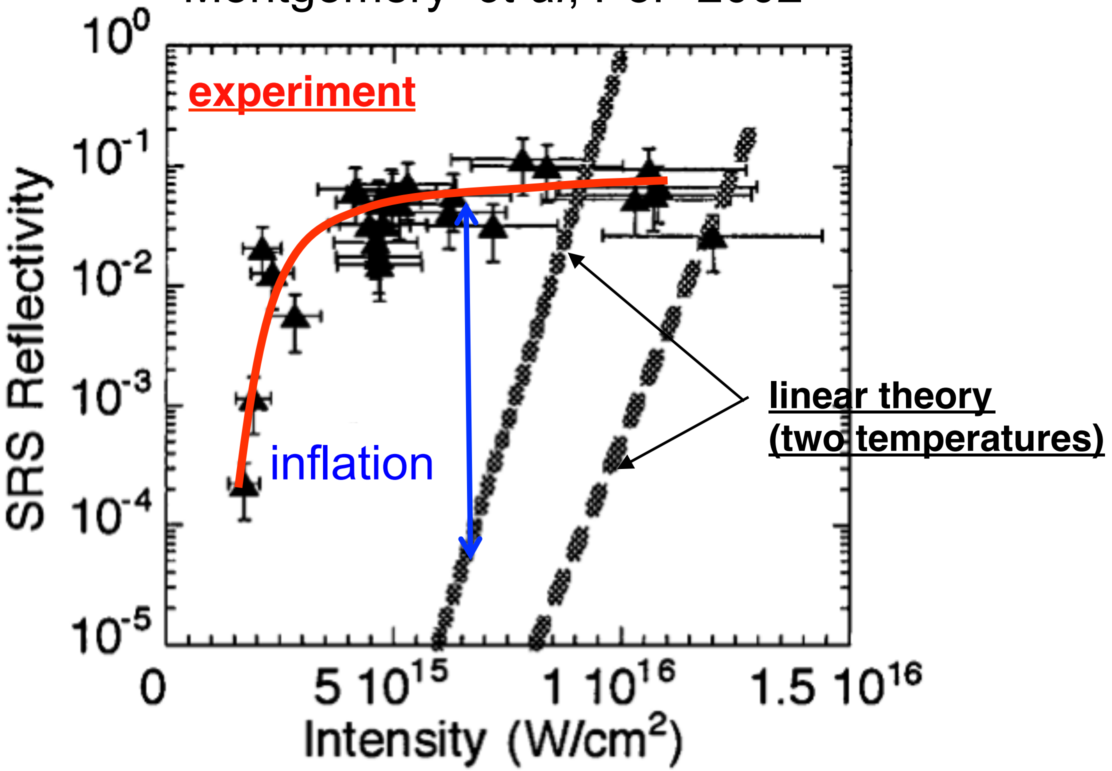


$\omega_L \approx \omega_p + \omega_s$ $\qquad k_L \approx k_p + k_s$

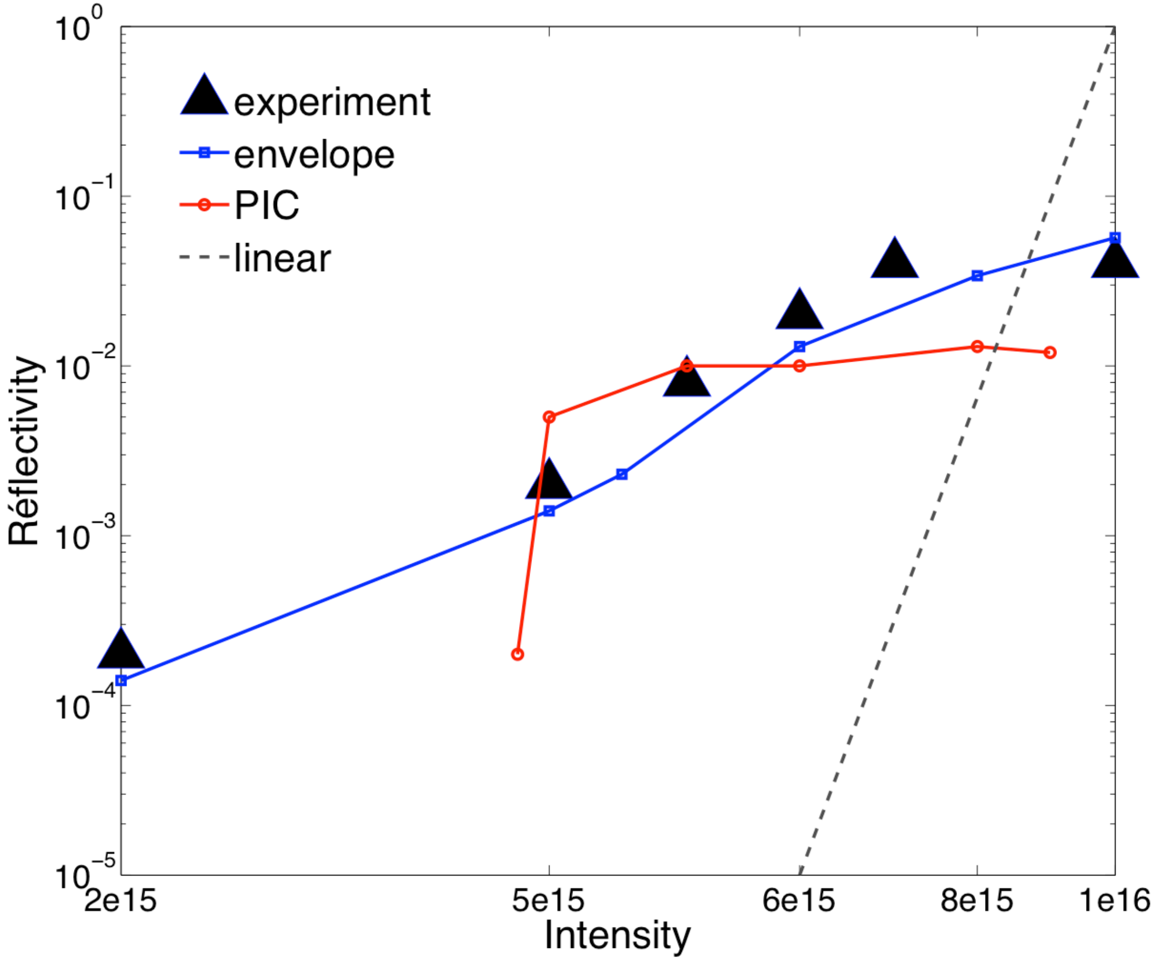
$E_d \propto E_L E_s$

KINETIC INFLATION : EXPERIMENTAL EVIDENCE

Montgomery *et al*, PoP 2002



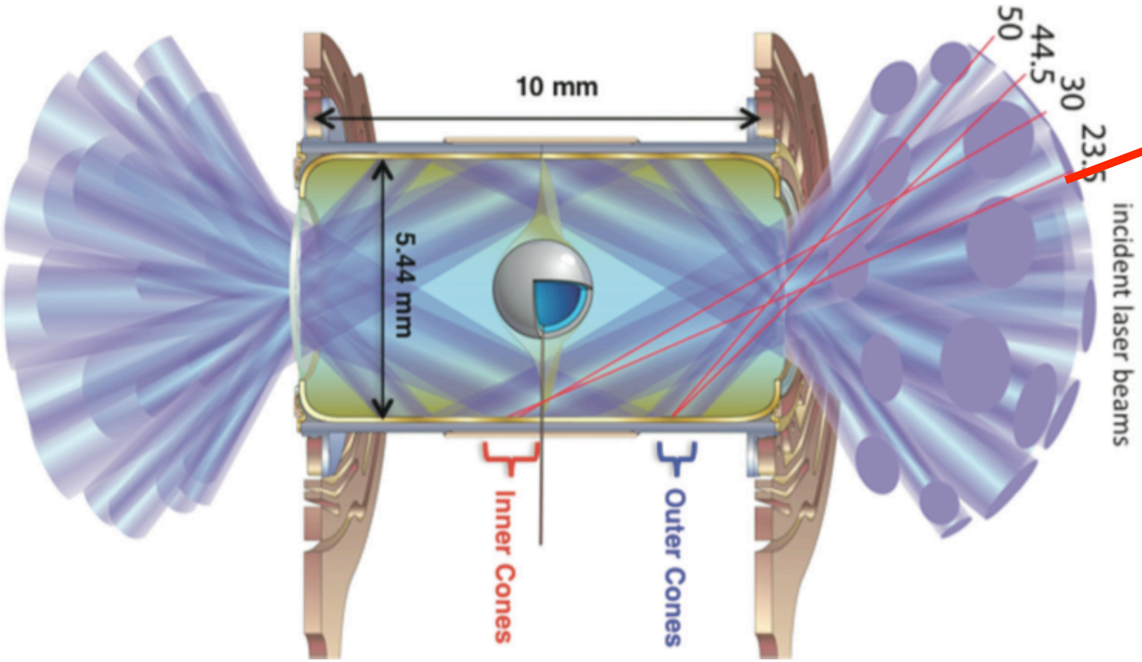
THEORETICAL PREDICTIONS OF SRS REFLECTIVITY



D. Bénisti *et al*,
PoP **19**, 056301 (2012)

Envelope 10^5 times faster than PIC

SRS : THREAT TO INERTIAL CONFINEMENT FUSION



Energy of scattered wave ≈ 50% of incident energy

National Ignition Campaign, Livermore 2009-2010
50% SRS reflectivity in the inner cone
Loss of energy and of symmetry

1D ENVELOPE EQUATION

$E = E_0(x,t)\sin(\varphi)$

Generalization of the heuristic argument to 1D when $\int \omega_B dt > 5$

(proven by variational formalism)

$$\langle \sin(\varphi)\rangle(\omega + i\gamma, k - i\kappa) \approx \gamma\partial_\omega \langle \cos(\varphi)\rangle_{adia} - \kappa\partial_k \langle \cos(\varphi)\rangle_{adia}$$

$$\gamma \equiv E_0^{-1}\partial_t E_0 \;;\; \kappa \equiv E_0^{-1}\partial_x E_0$$

Whatever $\int \omega_B dt$

$$\boxed{\langle \sin(\varphi)\rangle \approx (1-Y)\langle \sin(\varphi)\rangle_{lin} + Y\left[\gamma\partial_\omega \langle \cos(\varphi)\rangle_{adia} - \kappa\partial_k \langle \cos(\varphi)\rangle_{adia}\right]}$$

Gauss law for a driven wave : $-\dfrac{2}{(k\lambda_D)^2}\left\langle \dfrac{\sin(\varphi)}{\Phi}\right\rangle E_0 = E_d$

Writes….

1D ENVELOPE EQUATION

$$(1-Y)\partial_\omega \chi_{lin}\left[\partial_t E_0 + v_{g_{lin}}\partial_x E_0 + \nu_L E_0\right] + Y\partial_\omega \chi_{adia}\left[\partial_t E_0 + v_{g_{adia}}\partial_x E_0\right] = E_d$$

$$\chi_{lin} = -\frac{\omega_{pe}^2}{k} P.P.\left(\int \frac{f_0'(v)}{kv-\omega}dv\right).$$

$$v_{g_{lin}} \equiv -\partial_k \chi_{lin}/\partial_\omega \chi_{lin}$$

$$\nu_L = -\frac{\omega_{pe}^2 f_0'(v_\phi)}{k_p^2 \partial_\omega \chi_{lin}}$$: Landau damping rate

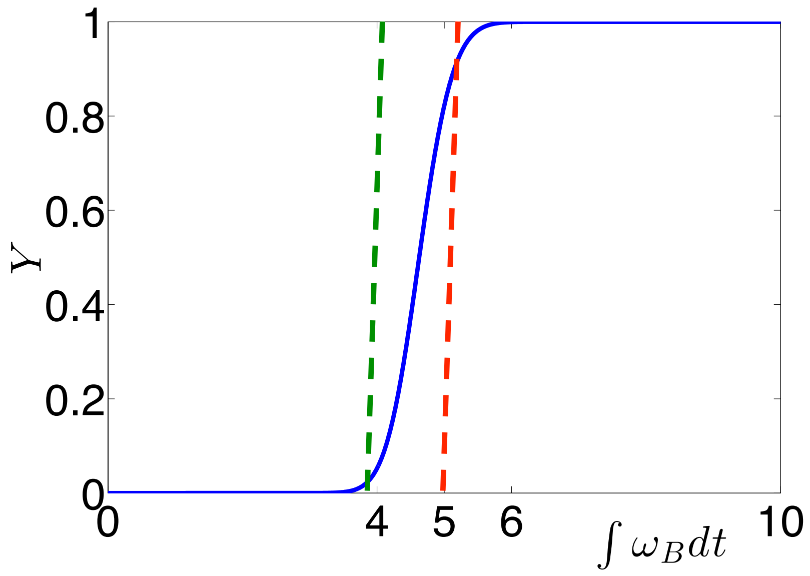
$$\chi_{adia} \equiv \frac{2}{(k\lambda_D)^2 \Phi}\langle \cos(\varphi)\rangle_{adia}$$

$$v_{g_{adia}} \equiv -\partial_k \chi_{adia}/\partial_\omega \chi_{adia}$$

When $E_0 \to 0$

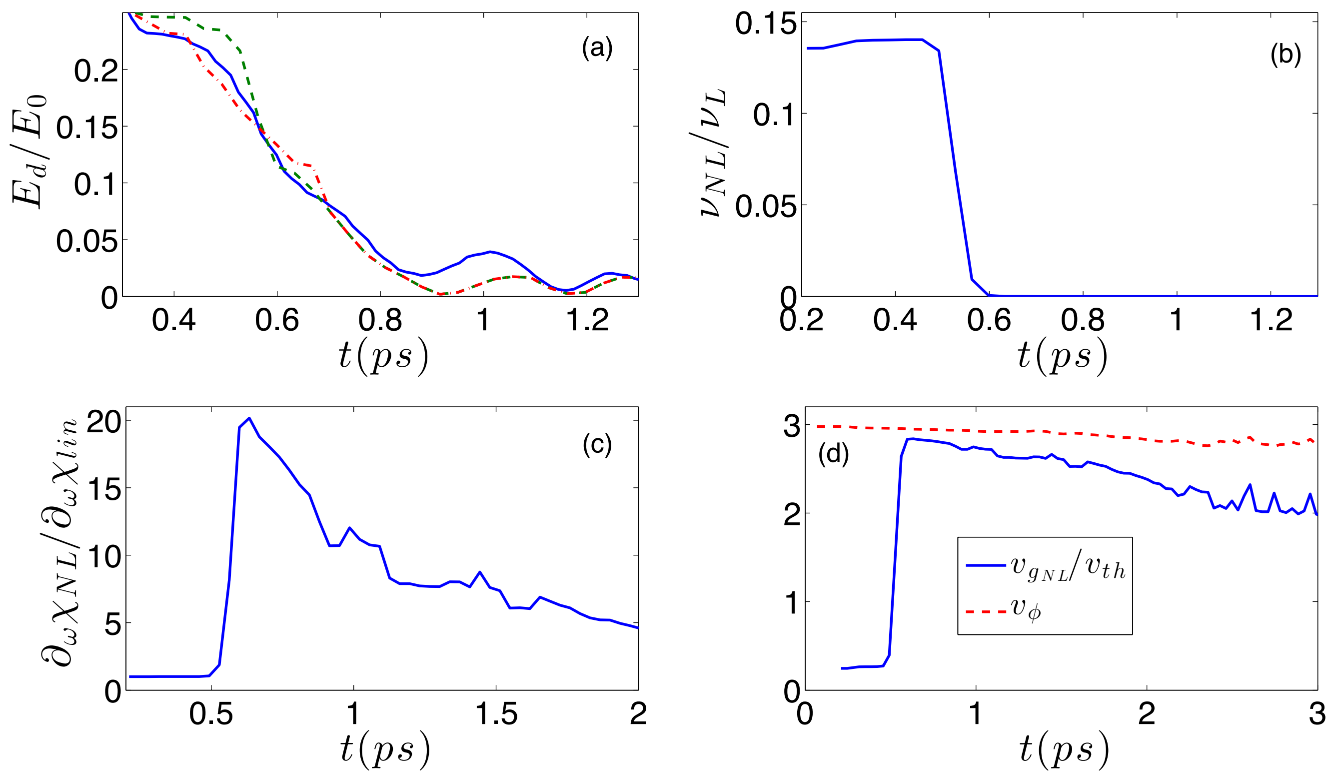
$$\chi_{adia} \longrightarrow \chi_{lin}$$

$$\partial_\omega \chi_{adia} \gg \partial_\omega \chi_{lin}$$



PHASE-LIKE TRANSITION IN 1D

$$\partial_\omega \chi_{NL}\left[\partial_t E_0 + v_{g_{NL}}\partial_x E_0 + \nu_{NL} E_0\right] = E_d$$



Linear wave → wave with trapped electrons

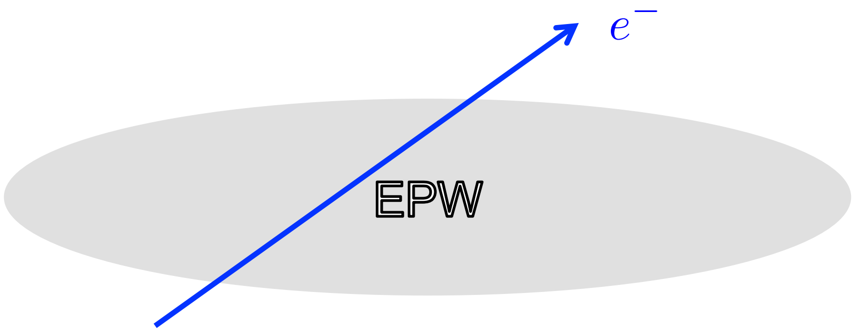
$$E_d/E_0 = \partial_\omega \chi_{NL}\left[\gamma + v_{g_{NL}}\kappa + \nu_{NL}\right]$$

Panel (a) : numerical test of the envelope equation

NONLINEAR LANDAU-LIKE DAMPING RATE

1D : some arbitrariness in the decrease of ν_{NL} from the choice of Y

3D : $\partial_\omega \chi_{NL} \left[\partial_t E_0 + \mathbf{v}_{g_{NL}} . \partial_{\mathbf{x}} E_0 + \nu_{NL} E_0\right] = E_d$



$\int \omega_B dt$ along the electron motion

$$\nu_{NL} \equiv (1 - Y_{3D})\nu_L$$

$$Y_{3D} \equiv \int f(v_\perp) Y \left(\int \omega_B dt \right) dv_\perp$$

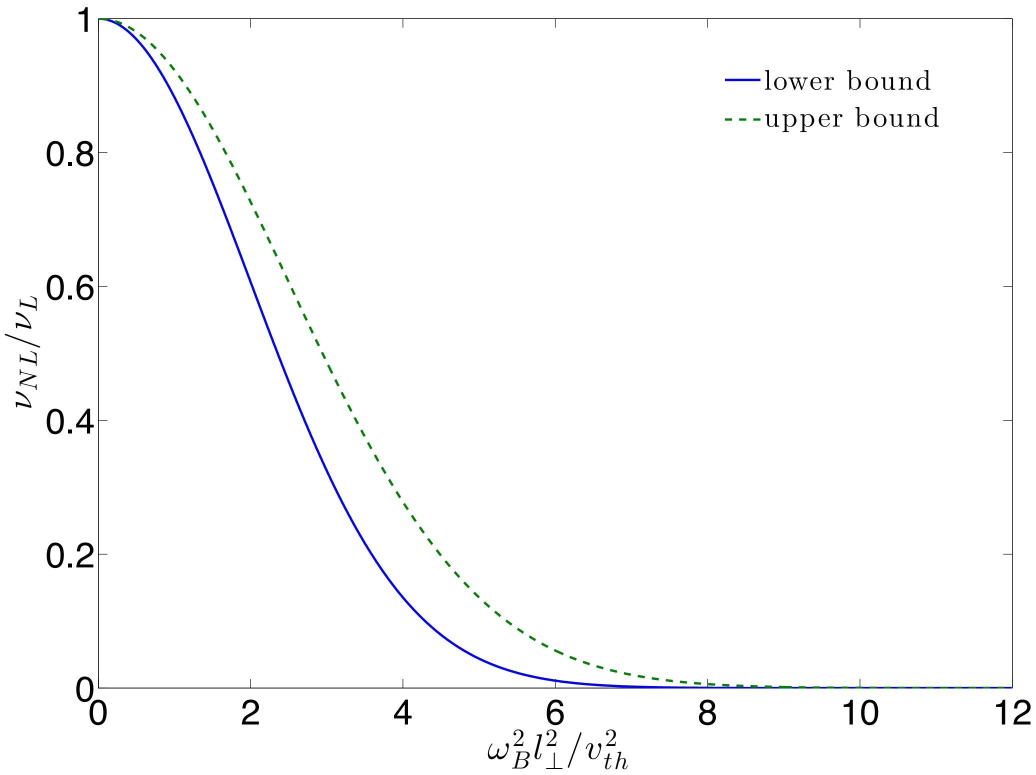
$$\int \omega_B dt < 4 \; ; \; Y \approx 0 \quad Y = \mathcal{H}\left(\int \omega_B dt - 4 \right) \rightarrow \text{upper bound for} Y_{3D}$$

$$\int \omega_B dt > 5 \; ; \; Y \approx 1 \quad Y = \mathcal{H}\left(\int \omega_B dt - 5 \right) \rightarrow \text{upper bound for} Y_{3D}$$

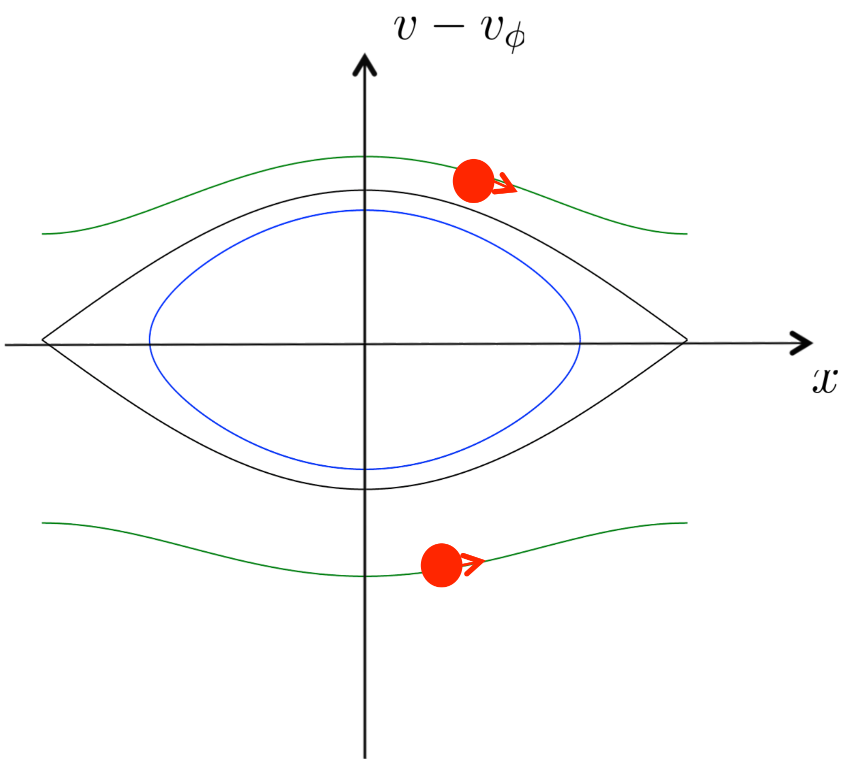
NONLINEAR LANDAU-LIKE DAMPING RATE IN 3D

If $\int \omega_B dt \approx 2\omega_B l_\perp / v_\perp$ $\quad (l_\perp \equiv E_0^{-1}\partial_{x_\perp} E_0)$

$$\exp\left(-\omega_B^2 l_\perp^2 / 8 v_{th}^2\right) < \nu_{NL}/\nu_L < \exp\left(-\omega_B^2 l_\perp^2 / 12.5 v_{th}^2\right)$$



Landau damping :



From perturbation analysis :

$v_0 < v_\phi \rightarrow$ electrons globally accelerated

$v_0 > v_\phi \rightarrow$ electrons globally decelerated

$\langle v \rangle$ gets closer to v_ϕ

Initially Maxwellian plasma :

$$\int f(v) v^2 dv \nearrow$$

⇒ EPW decays

Very local in wave particle interaction, mainly due to electrons such that $|v_0 - v_\phi| < \gamma/k$

⇒ after $\int \omega_B dt > 2\pi, \ \nu_{NL} \approx 0$

COLLISIONLESS DISSIPATION (2)

Dissipation due to adiabatic trapping

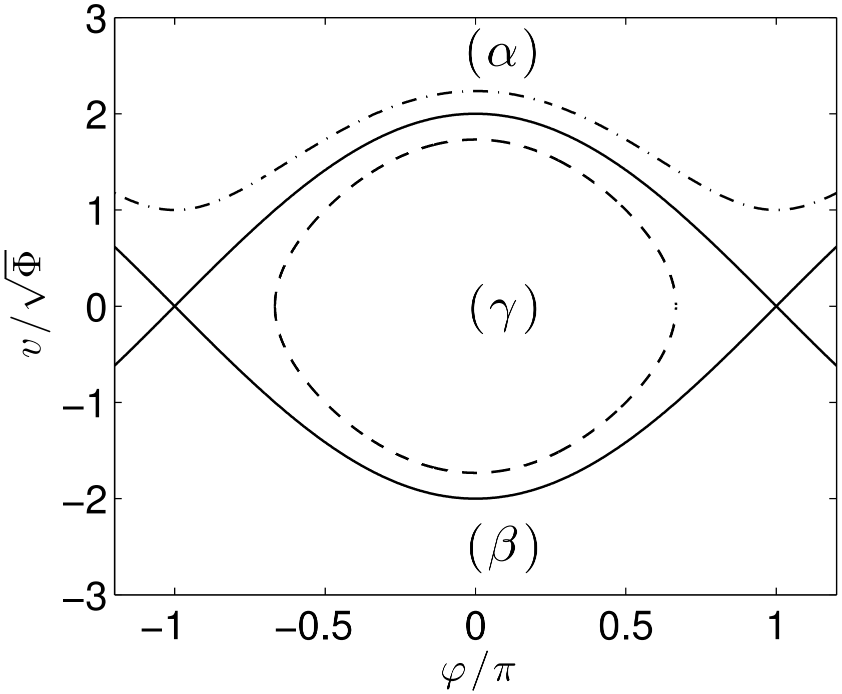
Untrapped electrons $\in (\alpha) \cup (\beta)$

$$I_u \equiv \frac{1}{2\pi} \oint v d\varphi = \frac{1}{2\pi} \int (v - v_\phi) d\varphi + \eta v_\phi$$

$$\eta_\alpha = 1; \eta_\beta = -1$$

Trapped electrons $\in (\gamma)$

$$I_t \equiv \frac{1}{\pi} \oint v d\varphi = \frac{1}{\pi} \int (v - v_\phi) d\varphi$$



Adiabatic approximation : I remains constant within each sub-region

$$(\Delta I)_{sep} = \mu v_\phi \qquad \mu \in \{-2, -1, 1, 2\}$$

Purely geometrical effect

$$f_{adia}(I,t) \neq f_{adia}(I,0)$$

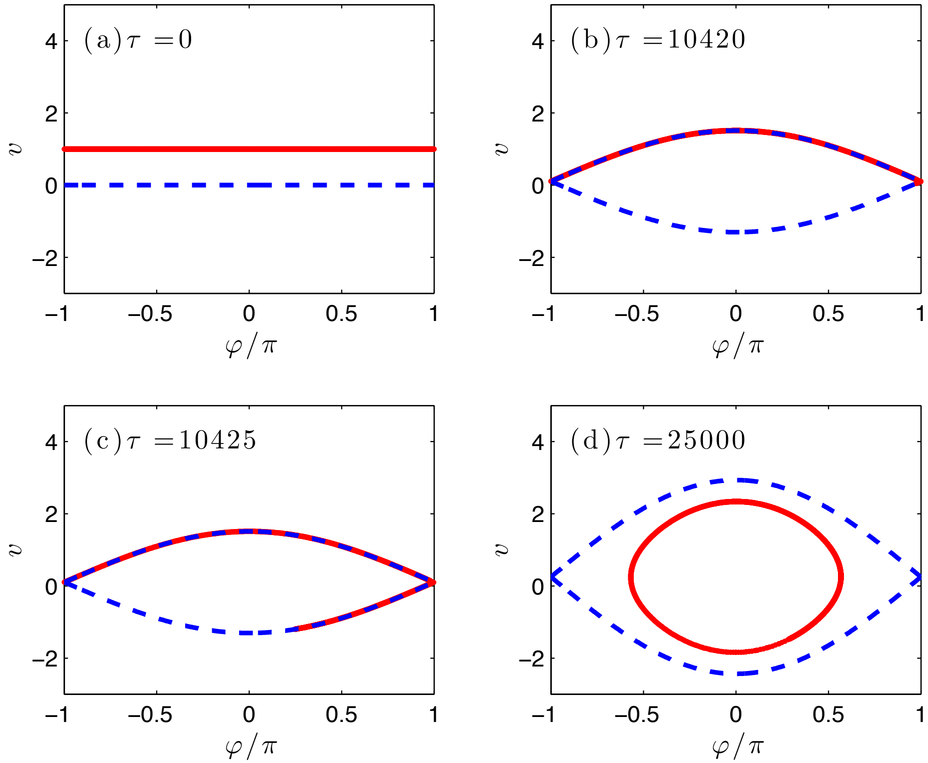
IRREVERSIBLE EVOLUTION OF $f_{adia}(I)$

$$H = \frac{(v - v_\phi)^2}{2v_{th}^2} - \Phi \cos(\varphi)$$

$$\Phi = 1 - \cos(\gamma_1 \tau) \qquad \gamma_1 = 10^{-4}$$
$$v_\phi = \gamma_2 \tau \qquad \gamma_2 = 10^{-5}$$

$$f_0(v) = \delta(v - v_0) \quad v_0 = 1 \Rightarrow f_0(I) = \delta(I - I_0); I_0 = 1$$

Numerically : 32768 initial positions uniforme distributed between $-\pi$ and $+\pi$

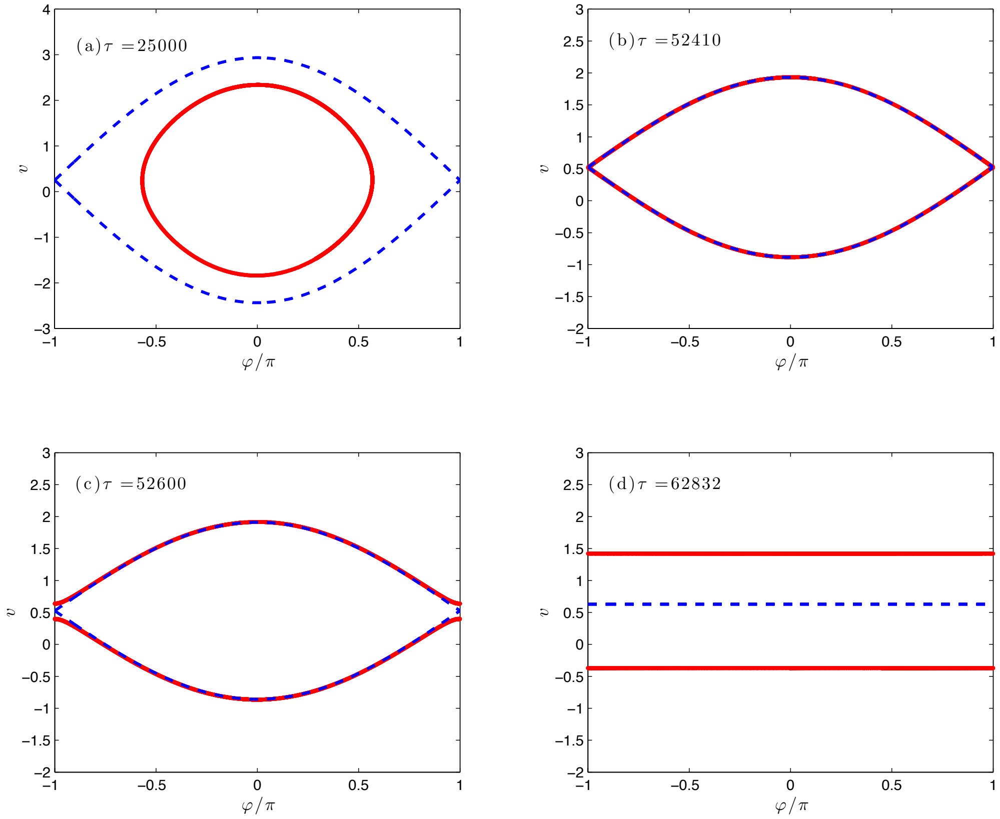


After trapping

$$f(I) = \delta(I - I_0 + v_{\phi_1})$$

The action distribution function explicitly depends on the wave phase velocity when trapping occurs

IRREVERSIBLE EVOLUTION OF $f_{adia}(I)$



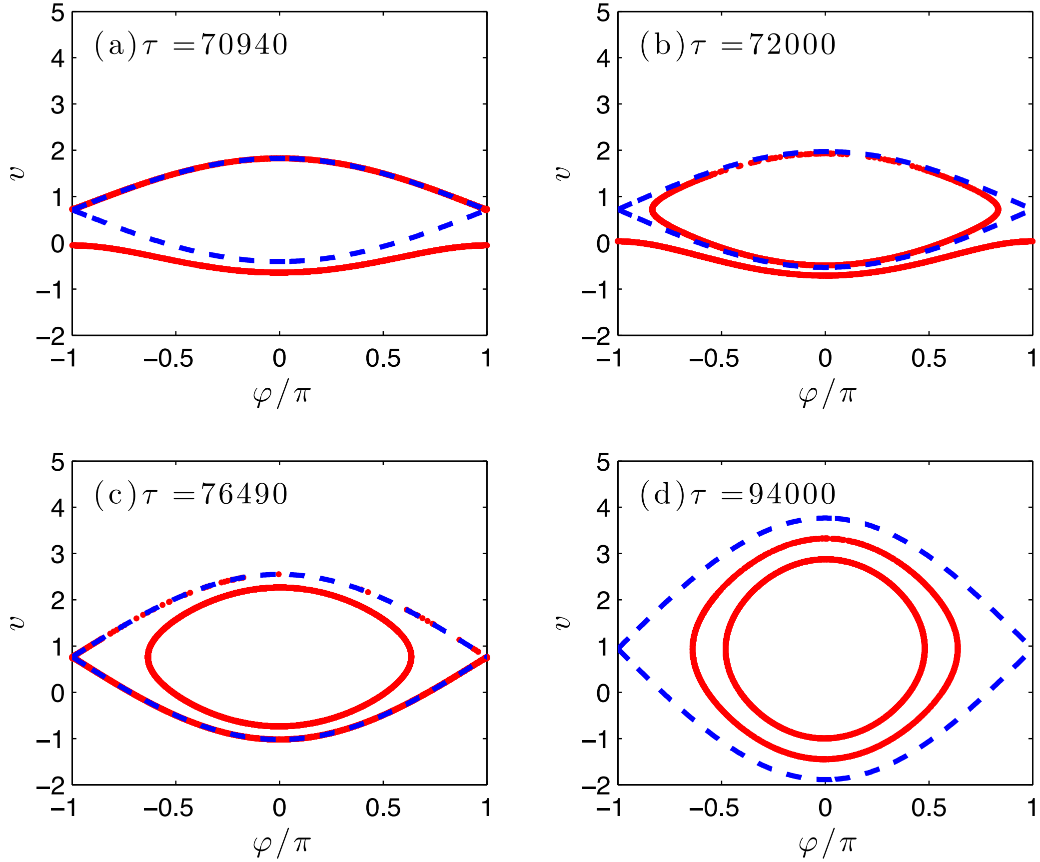
After detrapping

$$f(I) \approx \frac{\delta(I - I_0 + v_{\phi_1} - v_{\phi_2})}{2} + \frac{\delta(I - I_0 + v_{\phi_1} + v_{\phi_2})}{2}$$

$$\delta(I - I_0) \longrightarrow \frac{\delta(I - I_0 + v_{\phi_1} - v_{\phi_2}) + \delta(I - I_0 + v_{\phi_1} + v_{\phi_2})}{2}$$

Irreversible !!

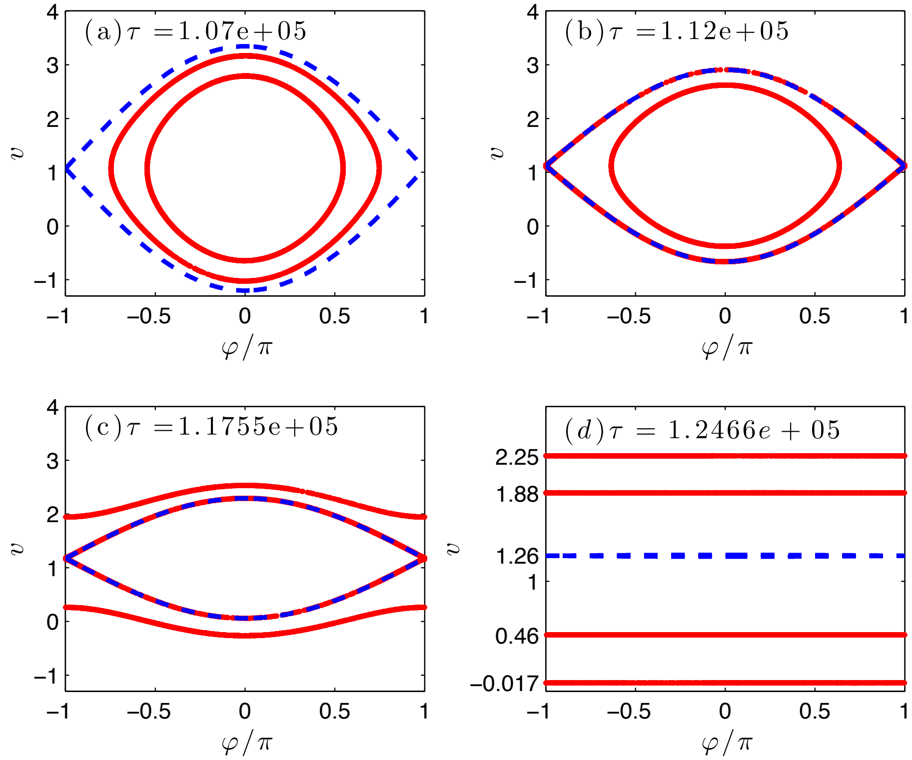
IRREVERSIBLE EVOLUTION OF $f_{adia}(I)$



The distribution function depends on the values assumed by the phase velocity at each trapping and detrapping

$$\delta(I - I_0) \longrightarrow \frac{\delta(I - I_0 + v_{\phi_1} - v_{\phi_2} + v_{\phi_3}) + \delta(I - I_0 + v_{\phi_1} + v_{\phi_2} - v_{\phi_4})}{2}$$

IRREVERSIBLE EVOLUTION OF $f_{adia}(I)$



$\langle v \rangle$ gets closer to v_ϕ

in an irreversible fashion

Initially Maxwellian plasma :

$$\int f(v) v^2 dv \nearrow$$

$\Rightarrow$ Electrostatic energy is dissipated

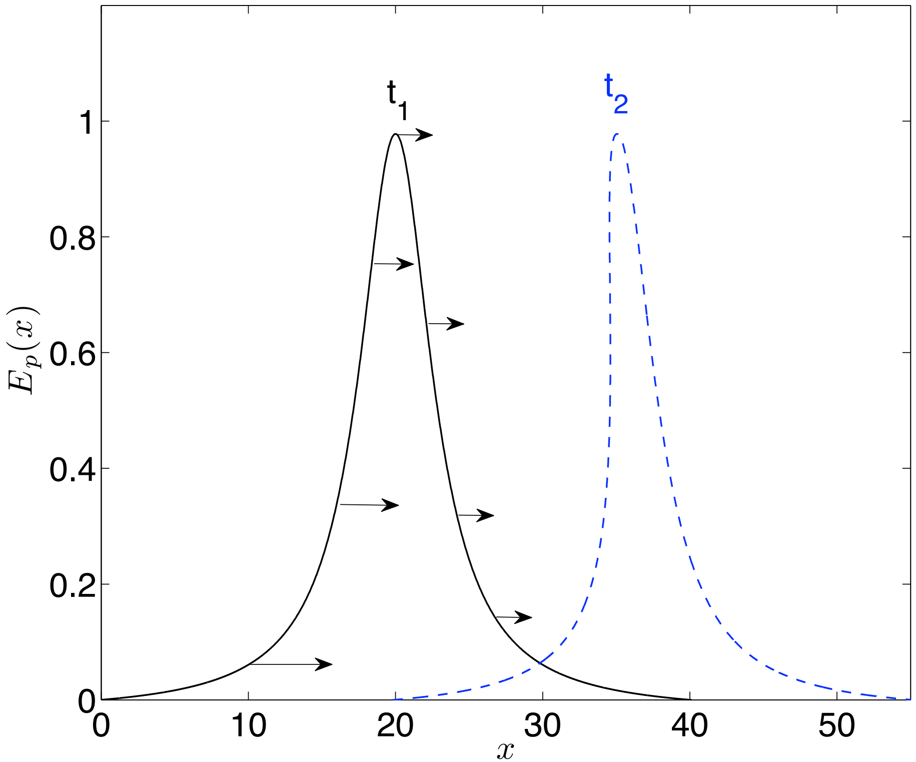
$$\delta(v - v_0) \rightarrow \frac{\delta(v - v_1) + \delta(v - v_2) + \delta(v - v_3) + \delta(v - v_4)}{4}$$

Irreversible !!

MANIFESTATION OF COLLISIONLESS DISSIPATION

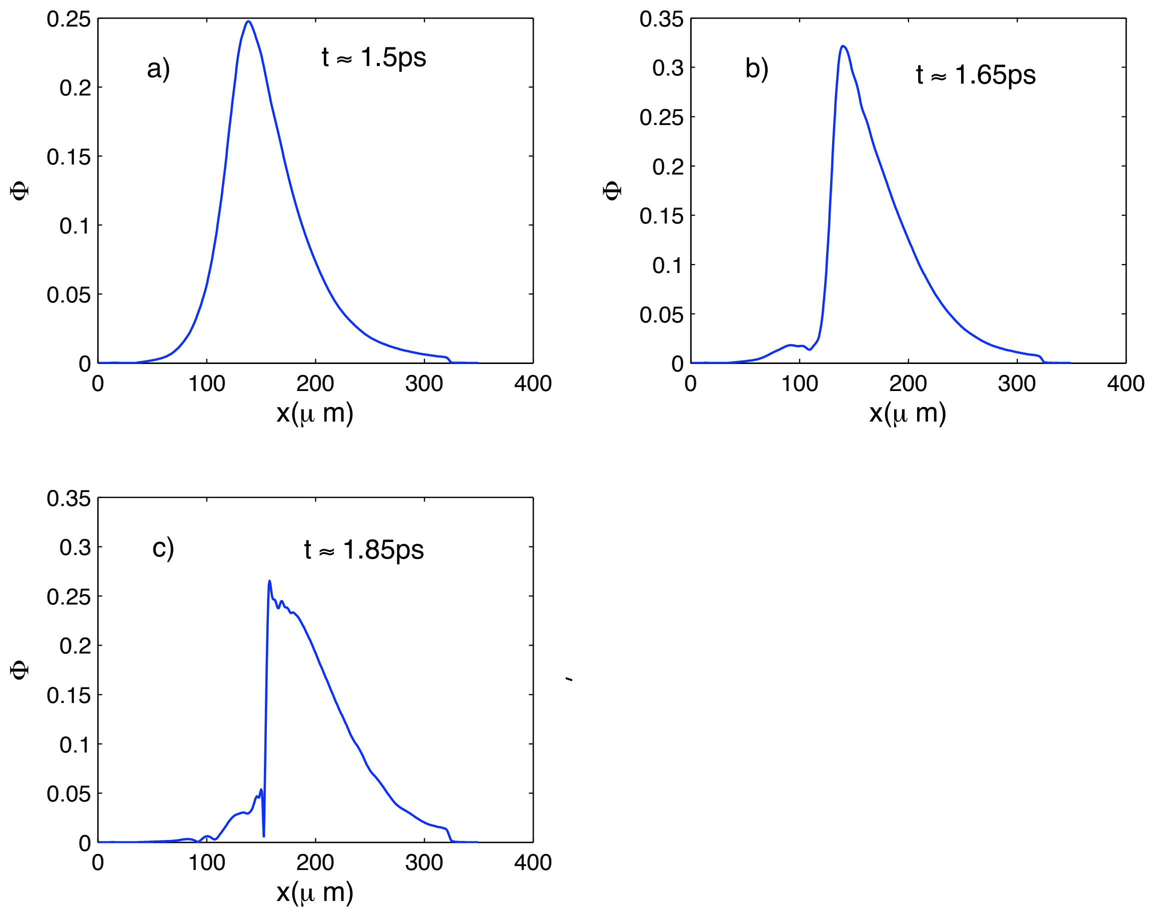
Landau damping : $E_0 \searrow$ along a characteristic

Adiabatic trapping : nonlocal group velocity
$\Rightarrow$shrinking of the wave packet



DISSIPATION DUE TO ADIABATIC TRAPPING

Ex: 1-D Vlasov simulations of SRS



NON-UNIFORM AND NON-STATIONAY PLASMA

VARIATIONAL FORMALISM

Dodin and Fisch, Phys. Plasmas **19**, 012102 (2012) – no separatrix crossing
Bénisti, Phys. Plasmas **23**, 102105 (2016) – with separatrix crossing

Start with general Lagrangian density for wave-particle interaction

$$L(x,t) = \frac{\varepsilon_0 E^2}{2} + \int L_e(x,v,t) f(x,v,t) dv,$$

$$L_e(x,v,t) \equiv \frac{mv^2}{2} - evA(x,t) - \Phi(x,t).$$

$$\boxed{L \approx (1 - Y_{3D}) L_{lin} + Y_{3D} L_{adia} \equiv L(E_0, n, k, \omega, \varphi)}$$

Calculated at 0th-order in the field and plasma variations

Lagrange equation

$$\partial^2_{t\omega} L + \nabla \partial_{\mathbf{k}} L = \partial_\varphi L \ ?$$

yields the first order envelope equation we are looking for

LINEAR NONLOCAL LAGRAGIAN DENSITY

Linear Lagragian density

$$L_{lin} \approx \varepsilon_0(1+\chi_{lin})\frac{|E|^2}{4} - \frac{\varepsilon_0 \pi \omega_{pe}^2 f_0(v_\phi)}{2}\int_0^t \frac{|E|^2(t')}{k}dt',$$

Linear envelope equation $(Y_{3D}=0)$

$$\partial_{t\omega}^2\left(\chi_{lin}E_0^2\right) + \nabla.\partial_{\mathbf{k}}(\chi_{lin}E_0^2) + 2\nu_L\partial_\omega\chi_{lin}E_0^2 = 2E_dE_0$$

$$\nu_L = \frac{-\pi\omega_{pe}^2 f_0'(v_\phi)}{k^2\partial_\omega\chi_{lin}}$$: WKB formula

Advection at $v_{g_{lin}} \equiv -\partial_k\chi_{lin}/\partial_\omega\chi_{lin}$ and damping at $2\nu_L$ of

$\partial_\omega\chi_{lin}E_0^2$: plasmon density

ADIABATIC NONLOCAL LAGRAGIAN DENSITY

Adiabatic Lagragian density

$$L_{adia} \equiv L_u + L_t$$

$$L_u \equiv -\int_{(\alpha)\cup(\beta)} H_u(I, v_\phi, \Phi) f_{adia}(\varphi, I, t) dI$$

$$L_t \equiv -\int_{(\gamma)} H_t I, v_\phi, \Phi) f_{adia}(\varphi, I, t) dI$$

f_{adia} *is not local in* Φ and v_ϕ

$$H_u = \mathcal{E} + mIv_\phi - mv_\phi^2/2$$

$$\frac{e\mathcal{E}}{kT_e} \equiv \frac{(2-\zeta)\Phi}{\zeta} \qquad \frac{I}{v_{th}} = \frac{4}{\pi}\sqrt{\Phi}\frac{K_2(\zeta)}{\sqrt{\zeta}} \pm \frac{v_\phi}{v_{th}}$$

$$H_t = \mathcal{E} - mv_\phi^2/2$$

$$\frac{e\mathcal{E}}{kT_e} \equiv \frac{(2-\zeta)\Phi}{\zeta} \qquad \frac{I}{v_{th}} \equiv \frac{4}{\pi}\sqrt{\Phi}\left[K_2(\zeta^{-1}) + (\zeta^{-1} - 1)K_1(\zeta^{-1})\right]$$

K_1 and K_2 Elliptic integrals of 1st and 2nd kind

NONLOCAL ADIABATIC ENVELOPE EQUATION

Adiabatic envelope equation $(Y_{3D}=1)$

$$\varepsilon_0 E_0 E_d/2 - \int_{(\gamma)} H_t \partial_x f_{adia} dI = -[\partial_{t\omega} L_{adia}|_{\mathcal{A}_s} + \nabla \partial_{\mathbf{k}} L_{adia}|_{\mathcal{A}_s}]$$

$|_{\mathcal{A}_s}$: means the derivatives are calculated as though the separatrix did not move i.e. without accounting for the change in the density of trapped and untrapped electrons

LHS : not the advection of $\partial_\omega L_{adia}$

No nonlinear counterpart to the plasmon density

Not true derivatives in the adiabatic wave equation because adiabatic trapping is **dissipative**

For a uniform and stationary plasma

$$\partial_\omega \chi_{adia} \left[\partial_t E_0 + \mathbf{v}_{g_{adia}}.\nabla E_0\right] = E_d$$

$$\mathbf{v}_{g_{adia}} \equiv -\partial_\omega \chi_{adia}/\partial_{\mathbf{k}} \chi_{adia}$$

NONLINEAR DISPERSION RELATION

ADIABATIC DISPERSION RELATION

Gauss : $1 + \frac{2\alpha}{(k\lambda)^2\Phi}\langle\cos(\varphi)\rangle = 0$

Freely propagating wave : $\alpha = 1$

SRS-driven wave : $\alpha > 1$

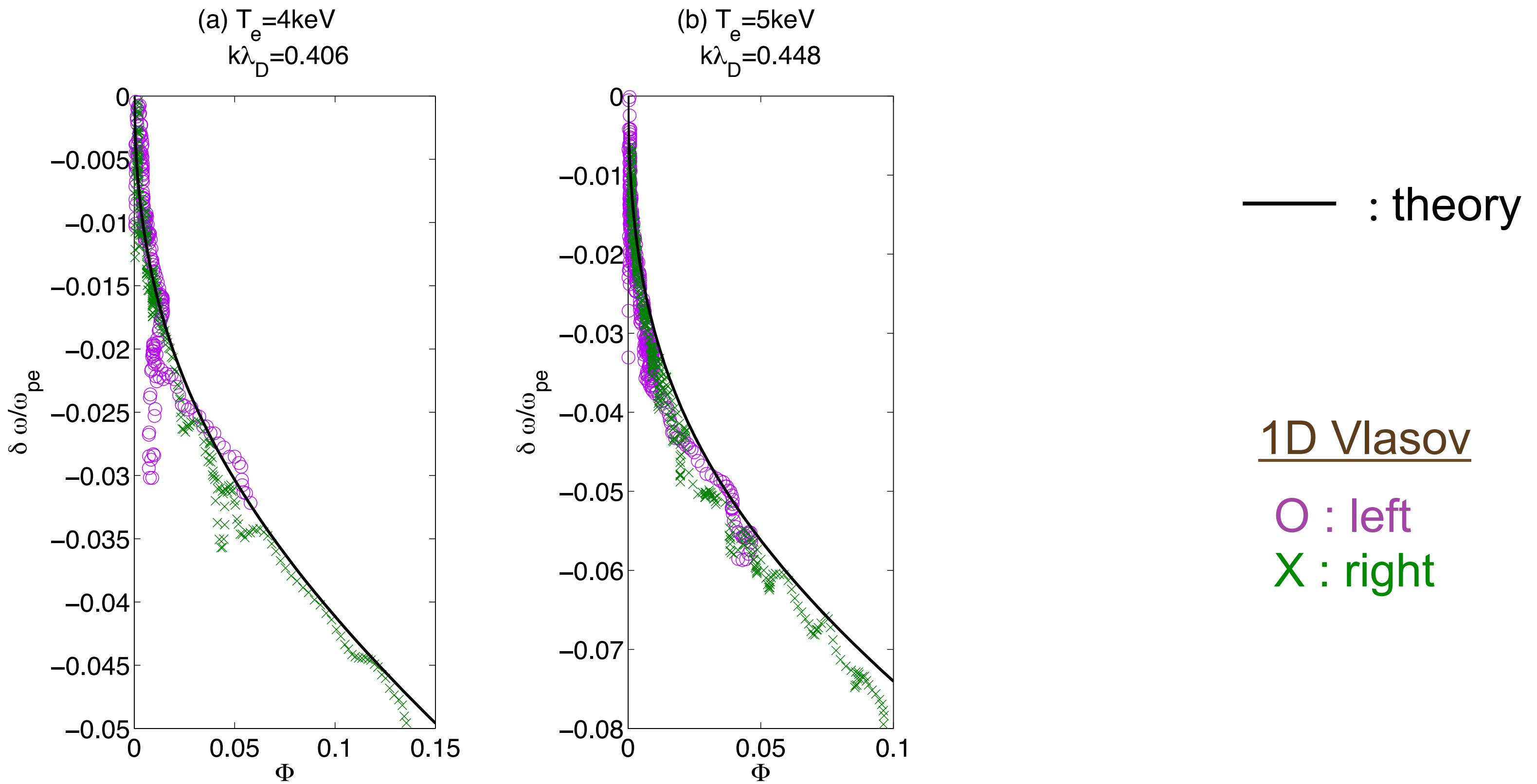
$$\langle\cos(\varphi)\rangle \approx \langle\cos(\varphi)\rangle_{adia}$$
$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{\varphi_0-\pi}^{\varphi_0+\pi} f_{adia}(v,\varphi)\cos(\varphi)dvd\varphi$$

$\mathrm{f_{adia}}(\mathrm{v},\varphi)$ **is not local**

WHEN IS THE DISPERSION RELATION LOCAL ?

Nonlinear frequency shift of an SRS -driven wave in a uniform plasma

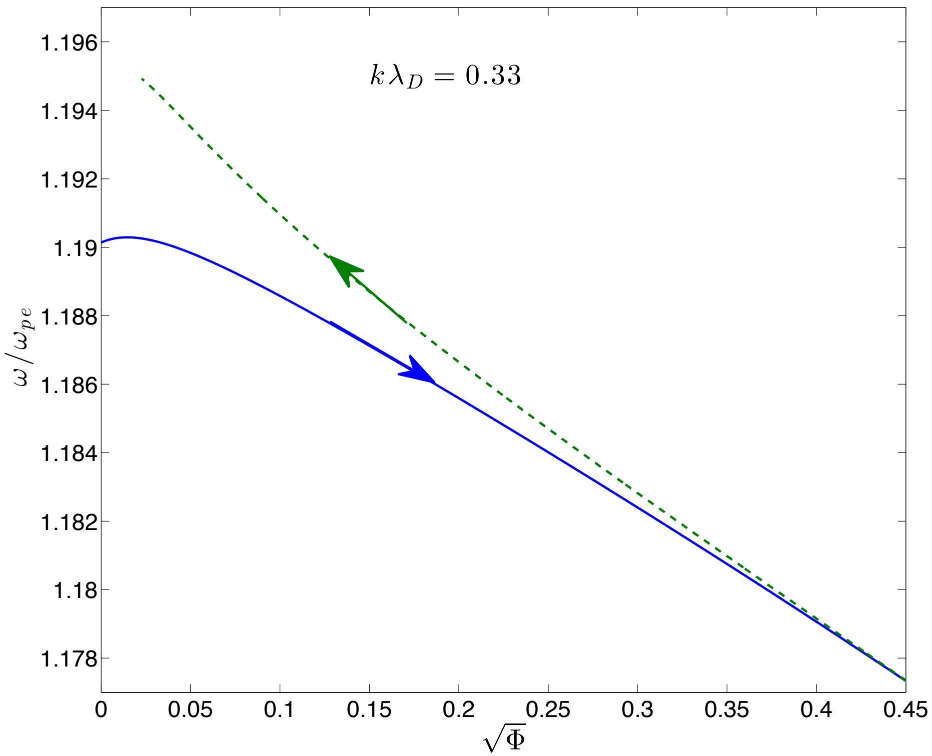
D. Bénisti, D.J. Strozzi, L. Gremillet, PoP **15**, 030701 (2008)



Growing wave in a 1D uniform plasma : $\omega \equiv \omega(k, \Phi)$

WHEN IS THE DISPERSION RELATION NON LOCAL ?

Uniform plasma, $k\lambda_D = 0.33$ $\sqrt{\Phi} = 0 \nearrow \sqrt{\Phi} = 0.45 \searrow \sqrt{\Phi} = 0$



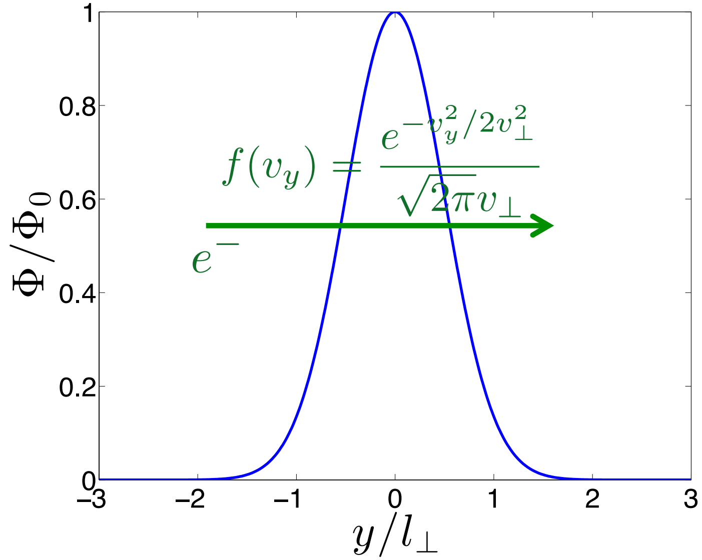
Hysteresis in the wave frequency

D. Bénisti, Phys. Plasmas **24**, 092120 (2017)

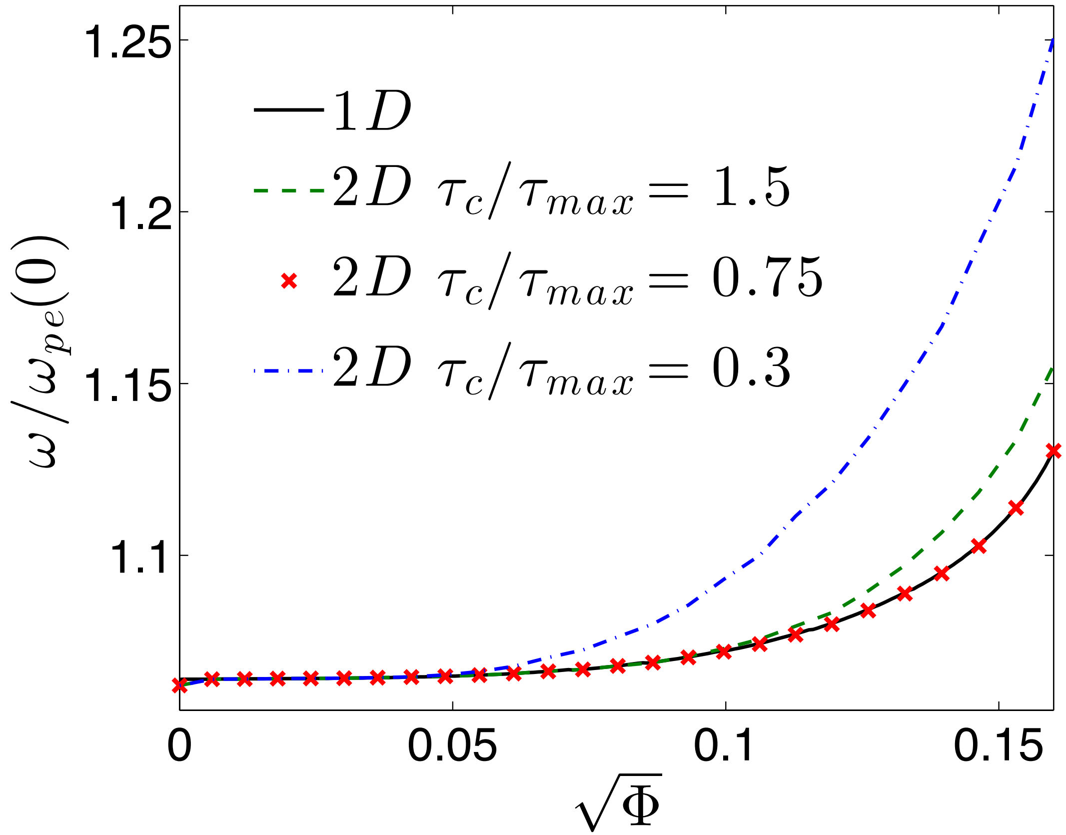
GROWING WAVE IN 2D NON-UNIFORM PLASMA

Gaussian transverse profile crossed by Maxwellian electrons

$$\Phi(y) \equiv \Phi_0 e^{-2y^2/l_\perp^2}$$



$$\tau_c \equiv l_\perp / v_\perp$$



$\tau_c/\tau_{\max} > 1 \sim$ 1D résults

$\tau_c/\tau_{\max} < 1,$ 1D results not recovered due to hysteresis

CONCLUSION

Theoretical results on Vlasov-Gauss obtained by mixing perturbative and “adiabatic” techniques

☞ Nonlinear growth and saturation of an instability
→Application to the cold-beam plasma instability

☞ 3D envelope equation for a slowly-varying EPW in a non-uniform and non-stationaty plasma
→Collisionless dissipation
Simple and accurate expression for NL Landau-like damping rate in 3D
→ Application to SRS

☞ Adiabatic dispersion relation

Growing wave in a 1D uniform plasma : $\omega \equiv \omega(k, E_0)$ local

Otherwise : $\omega \equiv \omega\{k, E_0\}$ non local

In particular, $\omega_{1D}\{k, E_0\} \neq \omega_{2D}\{k, E_0\}$

BACKUP

WHEN IS THE AMPLITUDE SMALL ?

$$H = \frac{(v - v_\phi)^2}{2} - \Phi \cos(\varphi)$$

$$m \equiv \frac{H + \Phi}{2\Phi}$$

$m < 1$ particles are trapped

$m > 1$ particles are untrapped

$$\omega_B \equiv \sqrt{\Phi}$$

$$\gamma \equiv \Phi^{-1} d\Phi/dt$$

For the untrapped small amplitude :

$$|v - v_\phi| \gg \omega_B/k$$

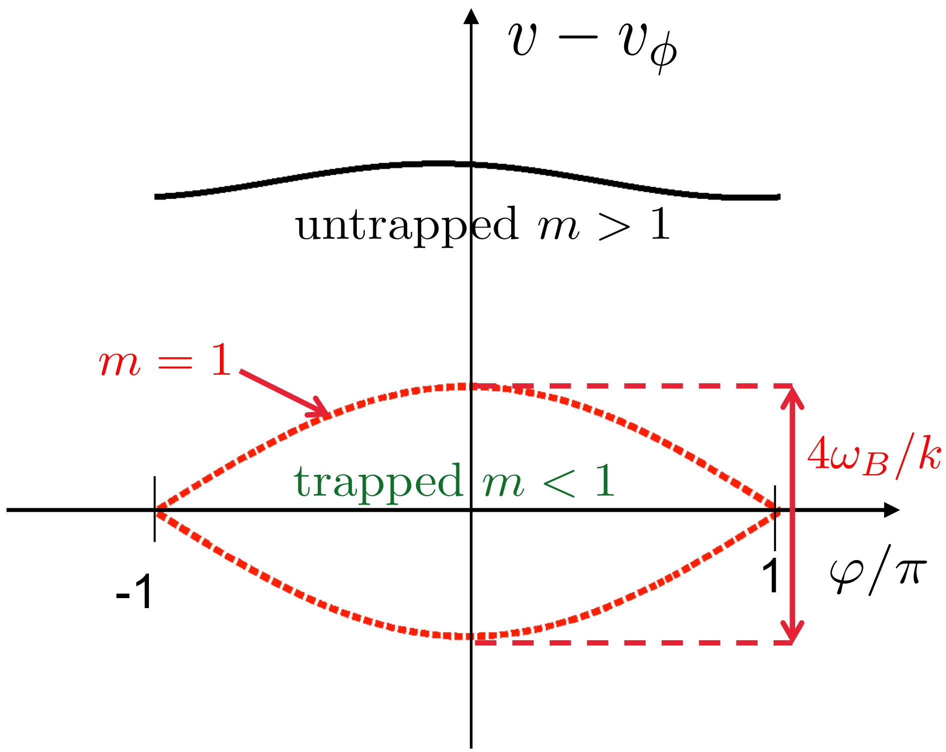
Trapped orbits : period T

$$\int_t^{t+T} \omega_B du \approx 2\pi$$

Small amplitude :

$$\int \omega_B dt \ll 1 \Leftrightarrow \omega_B/\gamma \ll 1$$

for a growing wave



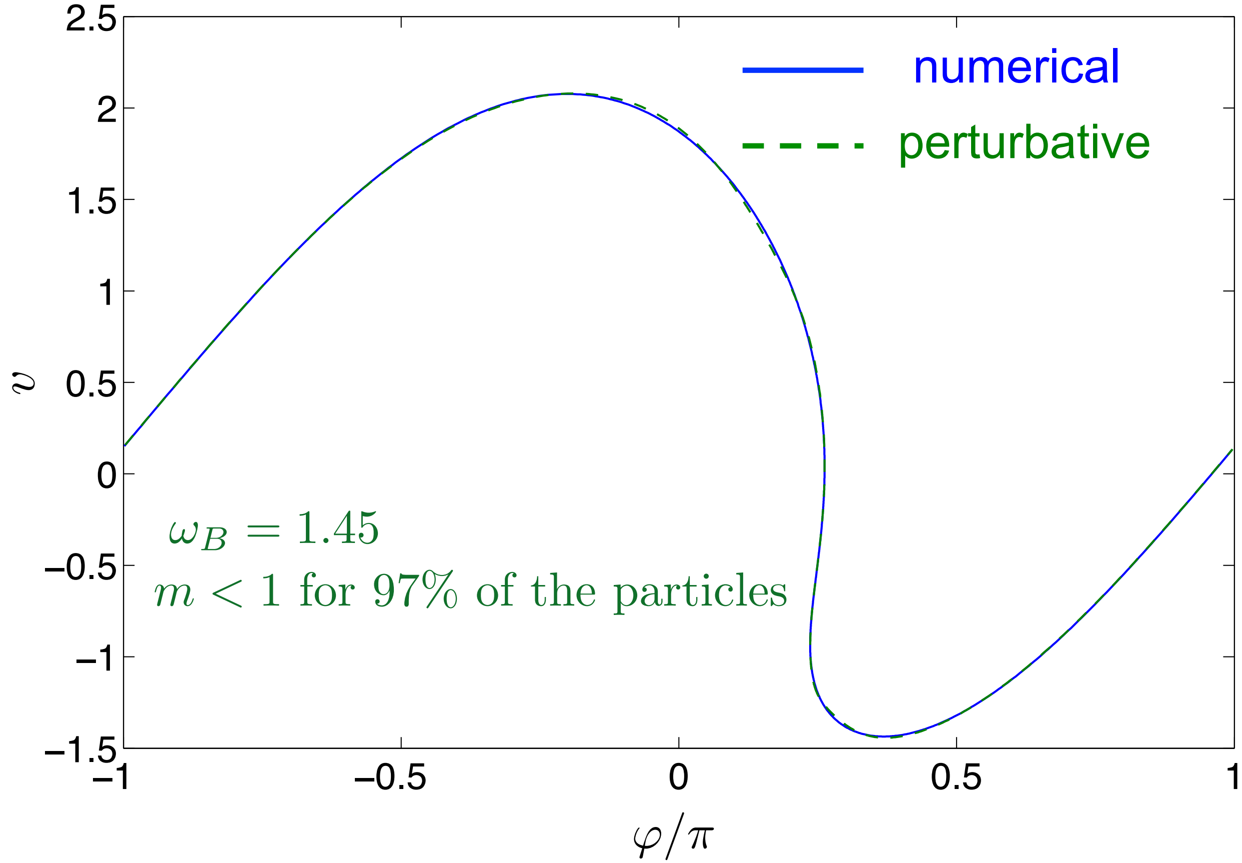
PERTURBATIVE TRAPPED ORBITS

$$H = \frac{(v - v_\phi)^2}{2} - \Phi\cos(\varphi)$$

$v_\phi = 0$

$\Phi = 10^{-8} e^t$

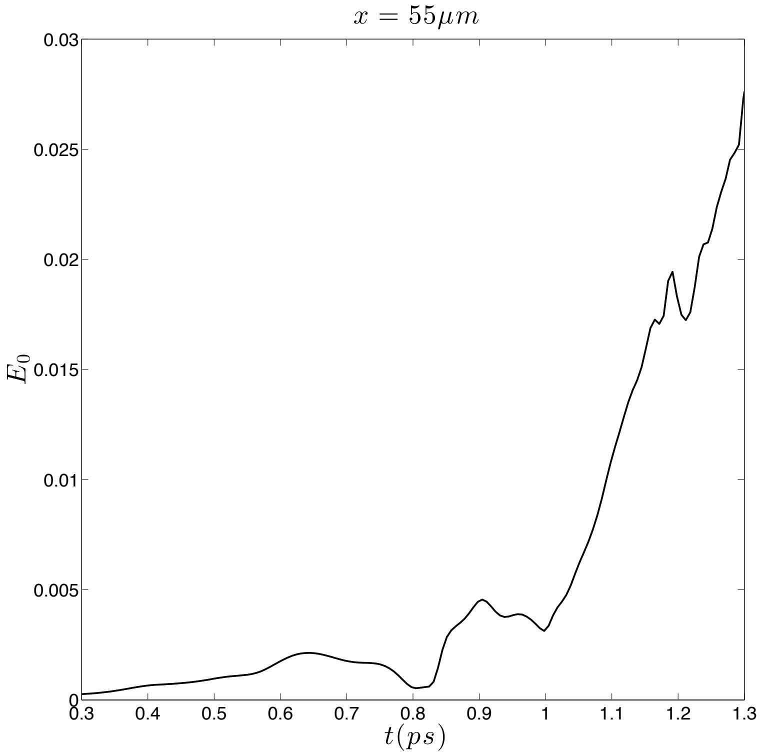
$t = 0, \; v = 1/\sqrt{3}$

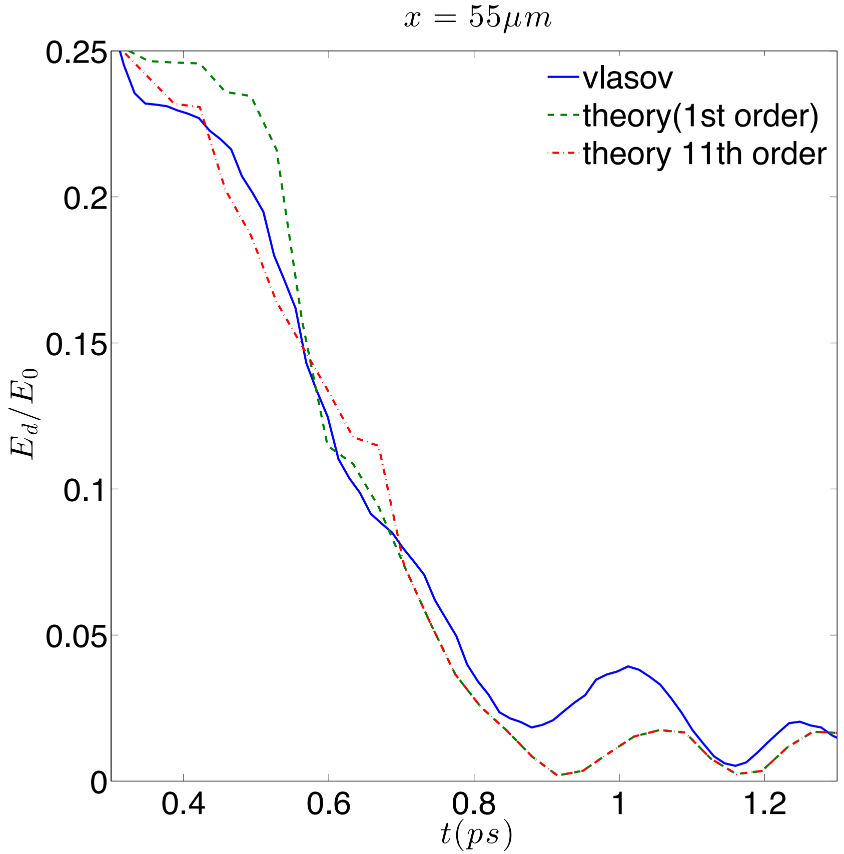


KINETIC INFLATION : THEORY VS NUMERICS

1D Vlasov simulations of SRS

$$L_{\text{plasma}} = 95\mu\text{m},\ T_e = 5keV,\ I_l = 4 \times 10^{15} W/cm^2,\ n_e = 0.1 n_c$$





$$t = 0 \begin{cases} f(v, \varphi) = \delta(v - v_0) \\ \Phi \approx 0 \Rightarrow f(I) = \delta(I - I_0) \end{cases}$$

$$I = I(\varphi_0) \Rightarrow f(I) \propto \frac{1}{\partial_{\varphi_0} I}$$

$I_{per}(\varphi_0)$ → 1 maximum

$I_{per}(\varphi_0)$ → 1 minimum

$$f(I) \approx \frac{\delta(I - I_{\min}) + \delta(I - I_{\max})}{2}$$

When all particles are trapped : $t > t^*$

$$f(I, t > t*) \approx \delta(I - I_{\min})$$

COMPARISON WITH NUMERICAL RESULTS

$$f_0(v) = \delta(v - v_0) \qquad E = E_0 e^{\gamma t} \sin(\varphi)$$

$$\gamma / k(v_0 - v_\phi) > 1$$