

**Tom Abel**

*Simplex in Cell techniques for Vlasov–Poisson and Vlasov–Maxwell modeling*

*Abstract* : We apply a novel phase-space interpolation technique referred to as the simplex-in-cell (SIC)[1] method to analyze two- and three-dimensional electromagnetic particle-in-cell (PIC) simulations. We use calculations of the growth of a Weibel instability in counter streaming mono-kinetic sheets of electrons, calculated with a standard PIC code but varied numbers of simulation particles, from 4 to 65536 particles per cell. I discuss the convergence of physical quantities calculated using SIC compared to standard particle deposits. SIC reduces the noise in the measured physical quantities even with orders of magnitude fewer simulation particles and allows the construction of continuously defined physical quantities through access to the full phase-space distribution function. This allows densities, currents, and even full momentum distribution functions to be measured at individual points without averaging over control volumes[2]. This approach enables new simulation methods that perform extremely well in the one dimensional Vlasov–Poisson system for plasma waves and Landau damping[3]. Given these results we are optimistic that generalizations to higher dimensions and the Vlasov–Maxwell system are possible.

[1] T. Abel, O. Hahn, R. Kaehler, Tracing the dark matter sheet in phase space, Monthly Notices of the Royal Astronomical Society 427 (1) (2012) 6176.

[2] J. Kates-Harbeck, S. Totorica, J. Zrake, T. Abel, Simplex-in-cell technique for collisionless plasma simulations, Journal of Computational Physics 304 (2016) 231251.

[3] Samuel R. Totorica, Frederico Fiuza, Tom Abel, A new method for analyzing and visualizing plasma simulations using a phase-space tessellation, submitted (2017).

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**Robert Blaga, Sergiu Busuioc, Victor E. Ambrus**

*Lattice Boltzmann models for rarefied flows*

*Abstract*: In this talk, we benchmark our finite-difference lattice Boltzmann models against known and new analytic solutions of the Vlasov equation, with a particular focus on the propagation of Cartesian, cylindrical and spherical shock waves for both non-relativistic and ultra-relativistic particle constituents. We discretise the momentum space following the prescription of one-dimensional Gauss quadratures and employ vielbein fields in order to adapt the phase space to the symmetries of the flow. A recent description of our relativistic and non-relativistic models can be found in Refs. [1] and [2], respectively.

[1] Robert Blaga, Victor E. Ambrus, High-order quadrature-based lattice Boltzmann models for the flow of ultrarelativistic rarefied gases, arXiv:1612.01287 [physics.flu-dyn].

[2] Sergiu Busuioc, Victor E. Ambrus, Lattice Boltzmann models based on the vielbein formalism for the simulation of the circular Couette flow, arXiv:1708.05944 [physics.flu-dyn].

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**Håkan Andréasson**

*Approximating gravitational collapse for dust with Vlasov matter*

*Abstract* : In the seminal work by Oppenheimer and Snyder [2] it is shown that a homogeneous ball of dust collapses to a black hole. I will present a result which shows that this gravitational collapse can be approximated arbitrary well by solutions to the Einstein-Vlasov system. Extensions of this result to the inhomogeneous case will also be discussed. In particular, there exist inhomogeneous data for dust [1] which give rise to naked singularities and it is thus important to understand the relation between the dust

solutions and the solutions to the Einstein-Vlasov system in the context of the weak cosmic censorship conjecture. This is a joint work with Gerhard Rein.

[1] D. Christodoulou, Violation of cosmic censorship in the gravitational collapse of a dust cloud, *Commun. Math. Phys.* 93, 171-195 (1984).

[2] J. R. Oppenheimer and H. Snyder, On continued gravitational contraction, *Phys. Rev.* 56, 455-459 (1939).

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### **Mohammad Ansari Fard**

#### *Late time sky as a probe of steps and oscillations in primordial Universe*

*Abstract* : The standard model of cosmology with nearly Gaussian, isotropic, scale invariant and adiabatic initial conditions describes the cosmological observations well. However, the study of any deviation from the mentioned conditions will open up a new horizon to the physics of early universe. In this talk which is based on [1], I will explore the effect of the oscillatory and step-like features in potentials of inflationary models in late time large scale structure observations, both in linear and non-linear regime. In the following I also discuss the importance and difficulty of numerical simulations in future large scale structures experiments.

[1] M. A. Fard and S. Baghram, "Late time sky as a probe of steps and oscillations in primordial Universe," arXiv:1709.05323 [astro-ph.CO].

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### **Claude Bardos**

#### *The Maxwell-Boltzmann approximation for ion kinetic modeling*

*Abstract* : The aim of this contribution, a joint work with F. Golse, T. Nguyen and R. Sentis is to provide a justification of the Maxwell-Boltzmann approximation of electron density from kinetic models. First, under reasonable regularity assumption, we rigorously derive a reduced kinetic model for the dynamics of ions, while electrons satisfy the Maxwell-Boltzmann relation. Second, we prove that equilibria of the electrons distribution are local Maxwellians, and they can be uniquely determined from conserved mass and energy constants. Finally, we prove that the reduced kinetic model for ions is globally well-posed. The constructed weak solutions conserve energy.

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### **Julien Barré**

#### *Bifurcations of Vlasov equation*

*Abstract* : Vlasov-type equations have an infinity of stationary states, and much effort have been spent to analyze their stability, leading to several recent mathematical breakthroughs. I will focus here on the problem: what happens when a stationary state is slightly unstable? While this is a natural and old question, many aspects remain open, and rigorous results are scarce. I will review the case of space homogeneous stationary states, where a bifurcation with some degree of universality is conjectured, and explain the differences with the much less studied inhomogeneous case. This latter situation includes for instance the bifurcation associated to the radial orbit instability. This is a joint work with D. Métivier and Yoshiyuki Yamaguchi [1].

[1] J. Barré, D. Métivier, and Y.Y. Yamaguchi, Trapping scaling for bifurcations in the Vlasov systems, *Physical Review E*, 93(4), 042207 (2016).

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### **Didier Bénisti**

#### *Analytic results for the Vlasov-Gauss system pertaining to nonlinear wave propagation in a plasma*

*Abstract* : This talk addresses the nonlinear propagation of a electron plasma wave (EPW), that may be externally driven or that may result from an instability. The originality of our

approach lies in the derivation of analytical results, directly from the Vlasov-Gauss system. The main step of this derivation is the matching of two different perturbative techniques in order to derive the electron charge density. For small amplitudes, we make use of a canonical perturbation theory similar to that introduced in order to prove KAM theorem [1]. By going to a high enough order, the perturbative results remain accurate up to amplitudes such that neo-adiabatic theory [2] becomes valid.

By inserting the expression thus found for the charge density into Gauss law, we derive an envelope equation for the wave that describes its nonlinear evolution. In particular, when the amplitude only depends on time, we obtain a nonlinear algebraic equation for its growth rate. We solve it in a situation corresponding to the beam-plasma instability, and provide the first analytic derivation of the nonlinear growth and saturation of this instability.

When the plasma is inhomogeneous and non stationary, we resort to a variational formalism in order to derive an envelope equation for the EPW, valid in a three dimensional geometry. This equation includes an explicit theoretical expression for the nonlinear decrease of the Landau damping rate. It is also used to yield a kinetic modeling of stimulated Raman scattering, whose accuracy is tested against experimental results.

[1] A.N. Kolmogorov, Dok. Akad. Nauk. SSSR, 98, 527 (1954).

[2] John R. Cary, D.F. Escande, and J.L. Tennyson, Phys. Rev. A 34, 4256 (1986).

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### **Leandro Beraldo e Silva**

#### *The arrow of time in the collapse of collisionless self-gravitating systems: non-validity of the Vlasov-Poisson equation during violent relaxation*

*Abstract* : The process that brings a self-gravitating system from an initial configuration to a nearly universal quasi-stationary state in the dynamical time-scale is violent relaxation, with a typical particle interacting with the time-changing collective potential. It is traditionally assumed that the transport equation describing violent relaxation is the Vlasov-Poisson equation. In this talk, we show the numerical investigations that we have been performing [1], based on entropy estimation of data from N-body simulations, in order to test the validity of that equation during violent relaxation. We also discuss a possible application of these estimators to constrain the Galactic potential using data provided by the Gaia survey.

[1] Beraldo e Silva, L. and de Siqueira Pedra, W. and Sodré, L. and Perico, E. and Lima, M., The arrow of time in the collapse of collisionless selfgravitating systems: non-validity of the Vlasov-Poisson equation during violent relaxation, accepted by ApJ (with significant changes from the arXiv version in arXiv:1703.07363).

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### **Edmund Bertschinger**

#### *Chaos and the Vlasov Equation*

*Abstract* : Chaos is generally understood to require nonlinearity in the equations of motion. For this reason, the linear Schrödinger equation does not lead to chaos in the usual sense. What about the Vlasov equation? Although the Vlasov-Poisson system is nonlinear, with a fixed potential the Vlasov equation is linear. I discuss the manifestation of chaos in two-dimensional systems described by the linear Vlasov equation. The results raise questions about the relative importance of deterministic chaos and statistical averaging as drivers of relaxation and ergodic behavior.

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### **Nicolas Besse**

#### *A partial review of semi-Lagrangian methods for the Vlasov equation*

*Abstract* : In this talk we give a non exhaustive review of semi-Lagrangian methods for the Vlasov equation, mainly in the context of plasma physics. After a brief history of these

numerical methods, we expose their main steps of development with some detailed examples. Following recent works, we then focus on semi-Lagrangian discontinuous Galerkin methods and their adaptive versions.

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**Nicolas Besse**

*On regularity of weak solutions of the relativistic Vlasov–Maxwell system*

*Abstract* : In this talk we investigate the regularity of weak solutions of the relativistic Vlasov–Maxwell system by using Fourier analysis and the smoothing effect of low velocity particles. This smoothing effect has been used by several authors (see [2] and [3]) for proving existence and uniqueness of  $C^1$ -regular solutions of Vlasov–Maxwell system. This smoothing mechanism has also been used to study the regularity of solutions for a kinetic transport equation coupled with a wave equation (see Bouchut, Golse and Pallard 2004). Under the same assumptions as in the paper [1] we prove a slightly better regularity for the electromagnetic field than the one showed in [1]. Namely, we prove that the electromagnetic field belongs to  $H^s_{loc}(\mathbb{R}^*_+ \times \mathbb{R}^3)$ , with  $s = 6/(14 + \sqrt{142})$ .

[1] F. Bouchut, F. Golse, C. Pallard, Nonresonant smoothing for coupled wave + transport equations and the Vlasov–Maxwell system, *Rev. Mat. Iberoamericana* 20 (2004) 865–892.

[2] R.T. Glassey, W.A. Strauss, Singularity formation in a collisionless plasma could occur only at high velocities, *Arch. Ration. Mech. Anal.* 92 (1986) 59– 90.

[3] S. Klainerman, G. Staffilani, A new approach to study the Vlasov–Maxwell system, *Comm. Pure Appl. Anal.* 1 (2002) 103–125.

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**James Binney**

*Angle-action coordinates for resonantly trapped orbits*

*Abstract* : If a DF is given as a function  $f(\mathbf{J})$  of action integrals  $J_i$  and one knows  $\mathbf{J}(\mathbf{x}, \mathbf{v})$ , a quick iteration yields the self-consistent potential  $\Phi(\mathbf{x})$ . In reality we do not have exact expressions  $\mathbf{J}(\mathbf{x}, \mathbf{v})$ . By torus mapping we can construct an integrable Hamiltonian  $H_i$  that is a close fit to any given realistic  $H_r = v^2/2 + \Phi$ , and for  $H_i$  we then have  $\mathbf{x}(\theta, \mathbf{J})$  and  $\mathbf{v}(\theta, \mathbf{J})$ . I shall discuss the impact of the perturbation  $\delta H = H_r - H_i$  on the orbit structure inherited from  $H_i$ . The impact of  $\delta H$  is significant only when a resonance condition is satisfied. Then new angle-action pairs  $(\mathfrak{A}, \mathfrak{J})$  emerge and one can compute accurate approximations to  $\mathcal{J}(\mathbf{x}, \mathbf{v})$ , etc. These approximations could be used to construct new Vlasov–Poisson equilibria for both axisymmetric and barred galaxies.

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**Marcel Braukhoff**

*Global existence of a solution of the Vlasov–Dirac–Benney equation with a small linear relaxation time collision operator*

*Abstract* : The global existence of a solution of the Vlasov–Dirac–Benney equation

$$\partial_t f + \square \square_p \varepsilon(p) \cdot \square_x f - \square_x \int_B f(x,p,t) dp \cdot \square_p f = 0, \quad x \in \mathbb{R}^d, \quad p \in B, \quad t > 0$$

is still an open problem. The interaction potential here is significantly more singular than the Coulomb potential such only local existence and ill-posedness results were found so far [HN15, BB13]. In this presentation, we prove the global existence of a solution for small analytic initial data by assuming that the r.h.s. =  $-\gamma f$  for  $\gamma > 0$ . Here, we exploit the techniques of [MV11] by using Gevrey-type norms which vary over time. We will see that these ideas will apply to the far more general setting  $\partial_t f + Lf = Q(f)$ , where  $L$  is a generator

of a  $C_0$ -group with  $\|e^{tL}\| \leq e^{\omega t}$  for  $\omega > 0$  and all  $t \in \mathbb{R}$  and where  $L$  and  $Q$  satisfy additionally further technical properties.

[BB13] Claude Bardos and Nicolas Besse. The Cauchy problem for the Vlasov-Dirac-Benney equation and related issues in fluid mechanics and semiclassical limits. *Kinet. Relat. Models*, 6(4):893–917, 2013. ISSN 1937-5093; 1937-5077/e. doi: 10.3934/krm.2013.6.893.

[HN15] Daniel Han-Kwan and Toan T. Nguyen. Ill-posedness of the hydrostatic Euler and singular Vlasov equations. *Analysis of PDEs (math.AP)*, 2015.

[MV11] Clément Mouhot and Cédric Villani. On Landau damping. *Acta Math.*, 207(1):29–201, 2011. doi: 10.1007/s11511-011-0068-9.

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## Léon Brenig

### *About the convergence of the kinetic equation for gravitational and Coulomb systems*

*Abstract* : Due to its divergence at large impact parameters, the Boltzmann collision integral for 3D gravitational and plasma systems must be replaced by the Balescu-Lenard collision term. However, the later diverges at small impact parameters: it only involves weak interactions while strong collisions between close particles are neglected. More generally, the convergence of the collision integral depends on the spatial dimension and on the exponent  $c$  of the interaction potential  $1/r^c$  [1]. A quantum derivation of the collision integral leads to a convergent collision term [2]. Here, we present a solution to this question in the classical framework [3]. It involves the binary phase-space correlations between close particles localized in a small sphere of radius the Landau length. The correlation integral in the first BBGKY equation is divided into an integral in that sphere and an integral over the rest of the space. The first integral leads to a fractional Laplacian operator in velocity space and the second integral gives the Balescu-Lenard collision term with a natural lower cut-off at the Landau impact distance. For weakly coupled systems and homogeneous initial conditions, the fractional Laplacian contribution leads to a long-tailed velocity distribution for very short times during which the inhomogeneity remains small. Simulations confirmed the existence of such long tails.

[1] P.-H.Chavannis, Kinetic Theory of spatially homogeneous systems with longrange interactions: III. Application to power-law potentials, plasmas, stellar systems, and to the HMF model, *Eur.Phys.J. Plus* (2013) 128: 128.

[2] H.A.Gould and H.E.DeWitt, Convergent Kinetic Equation for a Classical Plasma, *Phys.Rev.* (1967) 155, 68.

[3] L.Brenig, Y.Chaffi and T.M.Rocha Filho, Long velocity tails in plasmas and gravitational systems, arXiv:1605.05981v1 [physics.plasm-ph].

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## Stéphane Brull

### *Angular moment models for plasma physics*

*Abstract* : This talk is devoted to angular moment models constructed from the Landau-Fockker-Plank operator. These models are a compromise between the precision of the kinetic models and the low computational cost of the fluid models. The construction of such models is based on an angular moment extraction on the velocity variable for the kinetic model and the prescription of a closure. In the present talk, entropic closures based on the resolution of an entropy minimisation problem are considered. However, the moments extraction on the nonlinear Landau operator is complicated. Hence one classical approximation consists in considering that the main contribution of the distribution function comes from its isotropic part. But this approached model does not conserve the realizability domain defined as the set of vectors that are the moments of positive distribution functions. Therefore, a new electron-electron collision operator is proposed. In this model, the angular integration leads to a electron-electron collision operator for the

electronic  $M_1$  model which preserves the admissible states. In a last part, the electron transport coefficients are derived for the electronic  $M_1$  model.

- [1] J. Mallet, S. Brull, B. Dubroca, An entropic scheme for an angular moment model for the classical Fokker-Planck equation of electrons, *Comm. Comput. Phys.*, 15 (2), 422-450, (2014)
- [2] J. Mallet, S. Brull, B. Dubroca, General moment system for plasma physics based on minimum entropy principle, *Kin. Rel. Mod.*, 8, (3), 533-558, (2015)
- [3] S. Guisset, S. Brull, E. d’Humières, B. Dubroca, V. Thikhonchuck, Classical transport theory for the collisional electronic  $M_1$  model, *Physica A*, 446, 182-194, (2016)

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## Lapo Casetti

### *Nonequilibrium states with temperature inversion in long-range interacting systems*

*Abstract*: Temperature inversions occur in nature, e.g., in the solar corona and in interstellar molecular clouds: denser parts of the system are colder than sparser ones. We try to understand which are the minimal ingredients and the basic physical mechanism behind such phenomena. In a system with long-range interactions the interplay between collective oscillations and spatial inhomogeneity may drive the system to nonequilibrium stationary states that generically exhibit nonuniform temperature profiles with temperature inversion. Collective oscillations may arise after bringing a thermal state out of equilibrium by a sudden perturbation or by quenching a parameter of the Hamiltonian. We demonstrate our findings by means of numerical simulations of mean-field toy models [1] as well as of semiclassical models of cold atoms in a cavity [2] and of two-dimensional self-gravitating systems, modeling filaments in galactic molecular clouds [3]. In the latter case we observe temperature inversion also in cold collapses, that could be the way these structures form.

- [1] T. N. Teles, S. Gupta, P. Di Cintio, and L. Casetti, Temperature inversion in long-range interacting systems, *Physical Review E* 92, 020101(R) (2015).
- [2] S. Gupta and L. Casetti, Surprises from quenches in long-range-interacting systems: temperature inversion and cooling, *New Journal of Physics* 18, 103051 (2016).
- [3] P. Di Cintio, S. Gupta, and L. Casetti, Dynamical origin of non-thermal states in galactic filaments, arXiv:1706.01955, submitted to *Monthly Notices of the Royal Astronomical Society* (2017).

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## Pierre-Henri Chavanis

### *Kinetic theory of stellar systems*

*Abstract*: The collisional evolution of stellar systems is usually described by the Fokker-Planck equation introduced by Chandrasekhar, or by the Landau equation. These equations rely on a local approximation (as if the system were spatially homogeneous) and neglect collective effects (i.e. the dressing of the stars by their polarization cloud). In this talk, I will present a more general kinetic equation that takes into account spatial inhomogeneity through the use of angle-action variables (inhomogeneous Landau equation) and collective effects through a response matrix (inhomogeneous Lenard-Balescu equation) [1, 2, 3, 4]. These equations have recently been applied to stellar discs, providing an excellent agreement with direct numerical simulations [5]. Previously introduced kinetic equations are recovered in particular limits. I will also make a short historical review of the development of kinetic theories in plasma physics, stellar dynamics, and two-dimensional hydrodynamics.

- [1] J. Heyvaerts A Balescu-Lenard-type kinetic equation for the collisional evolution of stable self-gravitating systems *Mon. Not. Royal Astron. Soc.* 407 (2010) 355 .
- [2] P.H. Chavanis Kinetic theory of long-range interacting systems with angleaction variables and collective effects *Physica A* 391 (2012) 3680.

- [3] P.H. Chavanis Kinetic theory of spatially inhomogeneous stellar systems without collective effects *Astron. Astrophys.* 556 (2013) A93.
- [4] J. Heyvaerts, J.B. Fouvry, P.H. Chavanis, C. Pichon Dressed diffusion and friction coefficients in inhomogeneous multicomponent self-gravitating systems *Mon. Not. Royal Astron. Soc.* 469 (2017) 4193.
- [5] J.B. Fouvry, C. Pichon, J. Magorrian, P.H. Chavanis Secular diffusion in discrete self-gravitating tepid discs II : accounting for swing amplification via the matrix method *Astron. Astrophys.* 584 (2015) 129 .

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**Stéphane Colombi & Christophe Alard*****Solving Vlasov-Poisson equations with a "metric" approach***

*Abstract* : We present a new semi-Lagrangian Vlasov-Poisson solver. It employs metric elements to follow locally the geometry of the flow at second order. Using a percolation algorithm in Lagrangian space to compute the region of influence of each metric element, the algorithm allows one to find quickly and accurately the initial phase-space position of any test particle. Hence, the phase-space distribution function can be reconstructed at any time from initial state by direct application of Liouville theorem. When deformation of the metric is too important, the procedure is repeated again by taking the phase-space distribution function at present time as a new initial condition.

- [1] S. Colombi and C. Alard, A "metric" semi-Lagrangian Vlasov-Poisson solver *Journal of Plasma Physics* 83 (2017), 705830302.

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**Paul Dellar*****Studying the nonlinear suppression of Landau damping in the Vlasov-Poisson system using Hermite mode fluxes in a Fourier-Hermite spectral representation***

*Abstract* : This talk is based on joint work with Joseph Parker [1, 2, 3]. We diagnose the propagation of disturbances towards, or away from, fine velocity space scales in the 1+1D Vlasov-Poisson system using fluxes between Hermite modes in a Fourier-Hermite representation. The linearised V-P system supports both backwards and forwards propagating modes, the latter being responsible for linear Landau damping. For sufficiently large amplitude initial disturbances, nonlinearity excites backwards propagating modes with amplitudes that increase exponentially over time until they balance the forward propagating modes. There is then no net flux towards finer velocity-space scales, so Landau damping is suppressed.

- [1] J. T. Parker and P. J. Dellar, Fourier-Hermite spectral representation for the Vlasov-Poisson system in the weakly collisional limit, *J. Plasma Phys.* 81 (2015) 305810203 [arXiv:1407.1932].
- [2] J. T. Parker, Gyrokinetic simulations of fusion plasmas using a spectral velocity space representation, DPhil thesis, University of Oxford (2015) [arXiv:1603.04727]
- [3] J. T. Parker, E. G. Highcock, A. A. Schekochihin and P. J. Dellar, Suppression of phase mixing in drift-kinetic plasma turbulence, *Phys. Plasmas* 23 070703 (2016) [arXiv:1603.06968]

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**Erwan Deriaz*****Six-dimensional Adaptive Hierarchical Basis for Vlasov Equations***

*Abstract* : We present in [1] an original adaptive scheme using a dynamically refined grid for the simulation of the six-dimensional Vlasov-Poisson equations. The distribution function is represented in a hierarchical basis [2] retaining only the most significant coefficients. This allows considerable savings in terms of computational time and memory usage. The proposed scheme involves Multiresolution Analysis regarding its mathematical formalism and Adaptive Mesh Refinement regarding its computer

implementation. We apply a finite difference method to approximate the Vlasov-Poisson equations although other numerical methods can be considered. A C-implementation resulted in numerical experiments for the d-dimensional Vlasov-Poisson equations in the full 2d-dimensional phase space for  $d = 1, 2$  and  $3$ . The six-dimensional [3] case is compared to a Gadget N-body simulation.

[1] E. Deriaz, S. Peirani, hal.archives-ouvertes.fr/hal-01419750 (2017).

[2] G. Deslauriers, S. Dubuc, Constructive Approximation 5(1) 49–68 (1989).

[3] T. Fujiwara, Publ. Astron. Soc. Japan 35, 547–558 (1983).

## Bruno Despres

### *Scattering structure of Vlasov equations around inhomogeneous Boltzmannian states*

*Abstract*: The goal of this research is, by means of a scattering structure for Vlasov-Poisson-Ampère equation, to understand linear Landau damping for a particular family of inhomogeneous Boltzmannian states  $f(x, v, t) = n_0(x) \exp(-v^2/2) + g(x, v, t)$ . The keystone is the derivation of an original integral Lippmann-Schwinger equation. The weak form of the Lippmann-Schwinger equation is: find an electric field  $a \in L^2_0(I)$  such that for all  $b \in L^2_0(I)$

$$\begin{aligned} (a, b) &+ \varepsilon^2 / (2\pi) \sum_{p \neq 0} p^{-2} \text{P.V.} \int_{\mathbb{R}} m^\varepsilon_{a,b,p}(\lambda) (\lambda - i\mu)^{-1} d\lambda \\ &+ \varepsilon^2 / (2\pi) \sum_{p \neq 0} p^{-2} \text{P.V.} \int_{\mathbb{R}} n^\varepsilon_{a,b,p}(\lambda) (\lambda - i\mu)^{-1} d\lambda \\ &= \mathcal{L}(b). \end{aligned}$$

A technical condition shows up: the finite travel time along the characteristics must be strictly monotone with respect to the label of the characteristics.

## Pierfrancesco Di Cintio

### *Dynamics and relaxation of N-body systems with long-range interactions*

*Abstract*: We summarize the main results of our long term exploration of the dynamics of system of particle interacting with  $r^{-\alpha}$  forces. In particular we focus on relaxation phenomena [1, 2], phase space consistency [3] and radial orbit instability [4].

[1] P. Di Cintio and L. Ciotti, Relaxation of spherical systems with long-range interactions: a numerical investigation, IJBC 21 2279 (2011).

[2] P. Di Cintio, L. Ciotti and C. Nipoti, Relaxation of N-body systems with additive  $r^{-\alpha}$  interparticle forces, MNRAS 431 3177 (2013).

[3] P. Di Cintio, L. Ciotti and C. Nipoti, Radially anisotropic systems with  $r^{-\alpha}$  forces: equilibrium states, J. Plas. Phys 81 4904 (2015).

[4] P. Di Cintio, L. Ciotti and C. Nipoti, Radially anisotropic systems with  $r^{-\alpha}$  forces II: radial-orbit instability, MNRAS 468 2222 (2017).

## Claudio Di Troia

### *Non-perturbative guiding center transformation: the gyro-phase is the Kaluza-Klein 5th dimension*

*Abstract*: In the present work we use a non-perturbative approach for describing the particle relativistic motion in a self-consistent e.m. field without approximations. Moreover, the present description is extended to a general relativistic formulation when the presence of a gravitational field is not negligible. It is worth noticing that the solution of an exact Vlasov-Maxwell-Einstein system gives the most complete description of what



concerns the classical field theory approach for studying plasmas. If the (non perturbative) guiding center description of motion [1, 2] is adopted, it is found a similar mechanism to the one proposed by Kaluza and Klein (KK) [3] a century ago. The advantage on using the present description is that now there is no need of looking for a compactification scheme as required in the original KK mechanism. Up to now, all the compactification mechanisms have been shown to give problems, like the inconsistency of the scale of masses with observations. Instead, without compactification and giving a physical meaning to the extra-coordinate, it seems that, finally, the KK mechanism can be accepted as a realistic explanation of the presence of gravitation and electromagnetism treated in a unified manner in classical physics.

[1] C. Di Troia, Phys. Plasmas 22 (2015) 042103.

[2] C. Di Troia, Non-perturbative relativistic guiding center transformation: exact magnetic moment and the gyro-phase proposed as the Kaluza-Klein 5th dimension [plasma-ph/arXiv:1610.00504].

[3] Th. Kaluza, Sitzungsber. Preuss Akad. Wiss. Berlin Math. Phys. K1 (1921) 966; O. Klein, Z. Phys. 37 (1926) 895.

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### **Paola Domínguez Fernández**

#### *Evolution of inhomogeneities on a Dark Matter halo with the Vlasov equation using a TVD scheme.*

*Abstract* : We use a total-variation-diminishing (TVD) scheme for the Vlasov equation in spherical symmetry with a background gravitational potential to determine the evolution of a collection of particles in different models of a galactic halo in order to test its stability. Such collection is assumed to represent a dark matter inhomogeneity which is represented by a distribution function defined in phase-space. Non-trivial stationary states are obtained and determined by the virialization of the system. We describe some features of these stationary states by means of the description of the final distribution function and final density profile obtained. A comparison of our results using different halo models is done, obtaining that the NFW halo model is the most stable of them, showing that an inhomogeneity in this halo model requires a shorter time to virialize. [1]

[1] Domínguez-Fernández, P., Jiménez-Vázquez, E., Alcubierre, Miguel, Montoya, E. and Núñez, D., Description of the evolution of inhomogeneities on a dark matter halo with the Vlasov equation, Gen Relativ Gravit (2017) 49: 123. <https://doi.org/10.1007/s10714-017-2286-8>

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### **Alexander Figotin**

#### *Lagrangian formalism for electron beam interacting with multi-transmission line*

*Abstract* : We construct a Lagrangian field formulation for a system consisting of an electron beam interacting with a slow-wave structure modeled by a possibly non-uniform multi-transmission line (MTL). In the case of a single line we recover the linear model of a traveling wave tube due to J.R. Pierce. Since a properly chosen MTL can approximate a real waveguide structure with any desired accuracy, the proposed model can be used in particular for design optimization. Furthermore, the Lagrangian formulation provides for: (i) a clear identification of the mathematical source of amplification, (ii) exact expressions for the conserved energy and its flux distributions obtained from the Noether theorem. In the case of uniform MTLs we carry out an exhaustive analysis of eigenmodes and find sharp conditions on the parameters of the system to provide for amplifying regimes.

[1] Figotin A. and Reyes G., Multi-transmission-line-beam interactive system, J. Math. Phys., 54, 111901, (2013).

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### **Jean-Baptiste Fouvry**

## *Finite-N effects and secular evolution of self-gravitating systems*

*Abstract* : The dynamics of systems with long-range interactions generically comprise of two phases: first a phase of collisionless relaxation (described by the Vlasov equation), followed by a slower phase of collisional relaxation driven by the fluctuations remaining in the system. When sourced by finite-N effects, this long-term relaxation is captured by the inhomogeneous Balescu-Lenard equation. I will present this formalism, and emphasise in particular how one can account for the particles' intricate individual trajectories (inhomogeneity), and the system's ability to amplify perturbations (self-gravity). I will also present recent applications of this new framework to investigate the long-term orbital restructuring of self-gravitating systems [1, 2].

[1] Fouvry, J.-B., Pichon, C., Magorrian, J., & Chavanis, P.-H. 2015, *A&A*, 584, A129, Secular diffusion in discrete self-gravitating tepid discs. II. Accounting for swing amplification via the matrix method

[2] Fouvry, J.-B., Pichon, C., & Magorrian, J. 2016, *A&A*, 598, A17, The secular evolution of discrete quasi-Keplerian systems. I. Kinetic theory of stellar clusters near black holes

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## **Virginie Grandgirard**

### *Gyrokinetic simulations of magnetic fusion plasmas*

*Abstract* : In magnetic fusion devices, the power gain strongly increases with the energy confinement time. As a matter of fact, the quality of the plasma energy confinement largely determines the size and therefore the cost of a fusion reactor. This confinement time turns out to be mainly governed by the plasma turbulence. Understanding its origin and properties in view of its possible control is one of the critical issues in fusion science. The inhomogeneities in density, temperature, and magnetic field place the plasma naturally out of thermodynamical equilibrium, and tend to excite several micro-instabilities over a wide spectral range. These plasmas exhibit a low collisionality so that conventional fluid models are questionable and kinetic descriptions are more appropriate. In such first-principle descriptions of plasmas, the six dimensional evolution equation for the distribution function – Vlasov or Fokker-Planck equations – is solved for each species, coupled to the self-consistent equations for the electromagnetic fields, namely Maxwell's equations. Fortunately, as far as turbulent fluctuations are concerned, they develop at much lower typical frequencies than the high frequency cyclotron motion. Therefore, this 6D problem can be reduced to a 5D, known as the gyrokinetic model [1]. But even with this dimensional reduction solving 5D gyrokinetic equations for each specie reveals extremely challenging. This presentation will be a brief overview of the different assumptions, different numerical approaches existing now in first-principle gyrokinetic codes which have been developed for this stage [2]. Such codes require state-of-the-art high performance computing. The example of GYSELA code [3] will be used to discuss the actual challenges to prepare future exascale simulations.

[1] A.J. Brizard and T.S. Hahn, Foundations of nonlinear gyrokinetic theory, *Rev. Mod. Phys.* (2007) 2.

[2] V. Grandgirard and Y. Sarazin, Gyrokinetic simulations of magnetic fusion plasmas, *Panoramas et synthèses* (2013) 39-40, pp 91-176.

[3] V. Grandgirard et al., A 5D gyrokinetic full-f global semi-lagrangian code for flux-driven ion turbulence simulations, *Computer Phys. Comm.*, (2016) 207.

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## **Oliver Hahn**

### *Simulating the Collisionless Dynamics of Dark Matter*

*Abstract* : The dynamics of dark matter provides the backbone of studies of cosmic structure formation. Despite our ignorance about the particle physics nature of the elusive dark matter, its microscopic properties leave a distinct imprint on its macroscopic dynamics which can be studied in computer simulations. Such N-body simulations have

driven most of our theoretical knowledge about the distribution of matter in the Universe which in turn reflects properties of the dark matter particle. I will review the theoretical assumptions underlying such simulations and how they are used to study the nature of dark matter through its dynamics. I will particularly focus on recent attempts to model dark matter in the continuum limit. I will discuss how such new methods can be used to overcome known problems of N-body simulations, but also help to gain completely new insights into dark matter dynamics.

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**Anaëlle Halle**

*Phase-space structure of self-gravitating collisionless spherical systems*

*Abstract*: This talk is based on [1]. We study the detailed phase-space structure of collisionless self-gravitating spherical systems with initial power-law density profiles  $\rho(r) \propto r^n$ ,  $n$  ranging from 0 to  $-1.5$ , Gaussian velocity dispersions, and initial virial ratios  $\eta = 0.5$  (“warm”) and  $\eta = 0.1$  (“cool”). We use a Vlasov and a shell code preserving spherical symmetry, and the public N-body treecode Gadget-2. In all the cases, the system experiences a quiescent mixing phase during which it displays, in phase-space, a smooth spiral structure whose properties agree well with predictions from self-similar collapse. At some point, all the simulations display some level of radial instability, particularly in the “cool” case, where some macroscopic resonant modes destroy the spiral, but preserve the coarse-grained structure of the system, particularly the projected density profile  $\rho(r)$ , except for the Gadget-2 simulations with  $n \leq -1$  that are subject to radial orbit instability and thus have a slightly less contrasted central density profile than Vlasov or shells simulations.

[1] A. Halle, S. Colombi, S. Peirani, Phase-space structure analysis of self-gravitating collisionless spherical systems, arXiv:1701.01384

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**Shunsuke Hozumi**

*Phase-Space Evolution of Merging Collisionless Stellar Systems Reproduced with a Self-Consistent Field Method*

*Abstract*: Phase-space evolution is presented for merging processes of two spherical collisionless stellar systems using a self-consistent field (SCF) method [1] for solving Poisson’s equation. By applying Liouville’s theorem to merging simulations with the help of an SCF method which can provide the force field at each time step, phase space is reproduced by tracing necessary orbits back to  $t = 0$  at which the distribution function is given analytically [2]. In this talk, I first show that merging simulations using an SCF method are in excellent agreement with those using a tree code. Next, I present the phase-space evolution of head-on colliding systems reproduced from an SCF simulation.

[1] L. Hernquist and J. P. Ostriker, A self-consistent field method for galactic dynamics, *ApJ*, 386 (1992) 375

[2] S. Hozumi, A phase-space approach to collisionless stellar systems using a particle method, *ApJ*, 487 (1997) 617

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**Kenji Imadera**

*5D full-f gyrokinetic simulation for ion turbulence and transport barrier in tokamak plasmas*

*Abstract*: Gyrokinetic simulation is considered to be an essential tool to study turbulent transport driven by micro-scale instabilities in tokamak plasmas. It is roughly categorized into two approaches; delta-f local and full-f global approaches. In full-f approach, both turbulent transport and profile evolutions are solved self-consistently under the power balance between external heat source/sink. In this talk, we address (A) numerical technique to treat such full-f gyrokinetic Vlasov-Poisson equations [1] and (B)

characteristics of global ion-scale turbulence and transport barrier [2]. We also discuss (C) the role of stable modes in collisionless or weakly collisional plasmas [3].

[1] Kevin Obrejan, Kenji Imadera, Jiquan Li and Yasuaki Kishimoto, Development of a new zonal flow equation solver by diagonalisation and its application in non-circular cross-section tokamak plasmas, *Comput. Phys. Commun.* 216 (2017) 8.

[2] Kenji Imadera, Jiquan Li and Yasuaki Kishimoto, ITB formation in gyrokinetic flux-driven ITG turbulence, *Proc. of 26th Int. Conf. on Fusion Energy* (2016) TH/P3-3.

[3] Paul P. Hilscher, Kenji Imadera, Jiquan Li and Yasuaki Kishimoto, The effect of weak collisionality on damped modes and its contribution to linear mode coupling in gyrokinetic simulation, *Phys. Plasmas* 20 (2013) 082127.

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## **Pierre-Emmanuel Jabin**

### *Quantitative estimates of propagation of chaos for stochastic systems with $W^{-1,\infty}$ kernels*

*Abstract* : Following the previous ideas introduced in [1], we derive, in this new work [2], quantitative estimates proving the propagation of chaos for large stochastic systems of interacting particles. We obtain explicit bounds on the relative entropy between the joint law of the particles and the tensorized law at the limit. We have to develop for this new laws of large numbers at the exponential scale. But our result only requires very weak regularity on the interaction kernel in the negative Sobolev space  $W^{-1,\infty}$ , thus including the Biot-Savart law and the point vortices dynamics for the 2d incompressible Navier-Stokes.

[1] P.-E. Jabin, Z. Wang, Mean Field limit and Propagation of Chaos for Vlasov Systems with Bounded Forces. *J. Funct. Anal.* 271 (2016), no. 12, 3588–3627.

[2] P.E. Jabin, Z. Wang, Quantitative estimates of propagation of chaos for stochastic systems with  $W^{-1,\infty}$  kernels. Submitted *Inventiones Mathematicae*.

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## **Mehdi Jenab, Felix Spanier**

### *Vlasov simulation study of mutual collisions of ion-acoustic solitons: transition between kinetic and fluid regimes*

*Abstract* : Three different types of mutual collisions of ion-acoustic (IA) solitons, namely head-on, overtaking and scattering, have been studied based on a fully kinetic simulation approach, i.e. temporal evolutions of plasma constituents are followed by the Vlasov equations. The study is based on Vlasov-Hybrid simulation (VHS) approach which is recurrence-free[1]. By adopting the chain formation process, self-consistent IA solitons varying in size and velocity are achieved in the simulation box [2], and then rearranged in pairs inside a new simulation to create variety of scenarios of mutual collisions.

Both kinetic and fluid details of collisions are presented. Although on fluid level head-on and overtaking collision appear to exhibit the same pattern, on kinetic level, considering temporal evolution of distribution function in phase space, they display two distinct processes[3]. Furthermore the kinetic effect arising from the trapped population of electrons on the collision process is focused upon. It is discussed that their effect change the course of collision considerably for the small relative velocity of IA solitons, resulting in soliton-soliton scattering.

[1] H. Abbasi, S. M. Hosseini Jenab, and H. Hakimi Pajouh, *Physical Review E* 84, 3 (2011)

[2] S. M. Hosseini Jenab and F. Spanier, *Physics of Plasmas* 23, 10 (2016) [3] S. M. Hosseini Jenab and F. Spanier, *Physics of Plasmas* 24, 3 (2017)

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## **Michael Kopp**

### *Solving the Vlasov equation in 2D with the Schrödinger method*

*Abstract* : This talk presents work in progress, building upon [1], to solve the Vlasov equation via the Schrödinger-Poisson equation, together with a prescription to construct a phase space density from the wave function, the so-called the Schrödinger method (ScM) [2]. For the first time, we implement the ScM in two spatial dimensions, extending the one-dimensional previous studies through an implementation in CUDA. We present a comparison of our code and the Vlasov solver ColDICE [3], finding excellent agreement. We review how the fully fledged phase space dynamics can be encoded in a wave function with its mere 2 spatial degrees of freedom, and how vorticity, and all higher cumulants can be easily decoded.

[1] C. Uhlemann, M. Kopp, and T. Haugg, Schrödinger method as N-body double and UV completion of dust, *Phys. Rev. D*, 90, 023517 (2014) [astroph/1403.5567].

[2] L. M. Widrow and N. Kaiser, Using the Schroedinger Equation to Simulate Collisionless Matter, *Astrophys. J. Letters*, 416, L71 (1993)

[3] T. Sousbie and S. Colombi, ColDICE: a parallel Vlasov-Poisson solver using moving adaptive simplicial tessellation *Journal of Computational Physics*, 321, 644 (2016), [physics.comp-ph/1509.07720].

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## **Michael Kraus**

### *Discontinuous Galerkin Variational Integrators*

*Abstract* : A new framework for variational integrators based on a discontinuous Galerkin approach will be presented. This framework unifies many of the known variational integrators and, at the same time, allows for the construction of completely new families of integrators. The exploration of the possibilities of the new framework has just begun, but it already became apparent that it has the potential to solve various open problems in geometric numerical integration, most notably a natural treatment of degenerate Lagrangians and Dirac constraints like we often find them in plasma physics, for example in the description of reduced kinetic theories like guiding centre dynamics and gyrokinetics.

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## **Maxime Lesur**

### *Transport driven by velocity-space structures in toroidal plasmas*

*Abstract* : In hot plasmas, collisions are so rare that vortex-like fine scale structures develop in the phase-space of the particle distribution: coupling both real space and velocity space. In this work, numerical simulations are performed to uncover the roles of fine-scale velocity-space structures on micro-turbulence and transport in magnetic confinement fusion plasmas. The simulations are based on a reduced model [1, 2], which isolates one type of low-dimensional turbulence, as a fundamental paradigm for more general turbulence. This model is implemented in the TERESA simulation code [3]. Our simulations indicate important roles of fine-scale velocity-space (or energy-space) structures, which can drive most of the radial particle transport in some regime.

[1] M. Tagger, G. Laval, and R. Pellat, *Nucl. Fusion* 17, 109 (1977).

[2] M. Lesur, T. Cartier-Michaud, T. Drouot, P. H. Diamond, Y. Kosuga, T. Réveillé, E. Gravier, X. Garbet, S.-I. Itoh, and K. Itoh, *Phys. Plasmas* 24, 012511 (2017).

[3] T. Cartier-Michaud, P. Ghendrih, Y. Sarazin, G. Dif-Pradalier, T. Drouot, D. Estève, X. Garbet, V. Grandgirard, G. Latu, C. Norscini, and C. Passeron, *J. Phys.: Conf. Series* 561, 012003 (2014).

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## **Giovanni Manfredi**

### *Cosmology in one dimension: Vlasov dynamics*

*Abstract* : Cosmological simulations should in principle adopt a four-dimensional (4D) space-time to model the evolution of large regions of the universe. However, as billions of

particles are required to represent the matter distribution, significant approximations must be made, which leaves the delicate issue of fractal structure formation largely unresolved. In this contribution, we shall address this question within the framework of a class of idealized 1D models.

Most existing results rely on N-body simulations, which solve the Newtonian equations of motion of a large number of interacting particles. Such simulations generally show the formation of a hierarchical structure [1]. The density power spectrum displays a scale-free range, suggesting a fractal distribution of the particles. The analyses are robust for high-density regions, but give contrasted results for the low-density regions.

To gain a better insight into the distribution of mass in the low-density regions, we resort to Vlasov-Poisson simulations. In Vlasov codes, the entire phase space is covered with a uniform mesh, so that regions of high and low density are sampled with equal precision and the level of numerical noise remains low.

Here we perform one-dimensional Vlasov-Poisson simulations of a long-standing cosmological problem, namely the fractal properties of an expanding Einstein-de Sitter universe in Newtonian gravity. The N-body results are confirmed for high-density regions and extended to regions of low matter density, where the N-body approach usually fails [2].

[1] B. N. Miller, J.-L. Rouet, *J. Stat. Mech.* P12028 (2010); *Phys. Rev. E* 82, 066203 (2010).

[2] G. Manfredi, J.-L. Rouet, B. N. Miller, Y. Shiozawa, *Phys. Rev. E* 93, 042211 (2016).

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**Norbert J. Mauser**

### *The Quantum Vlasov equation*

*Abstract:* We present the Quantum Vlasov or Wigner equation as a "phase space" presentation of quantum mechanics that is close to the classical Vlasov equation, but where the "distribution function"  $w(x,v,t)$  will in general have also negative values.

We discuss the relation to the classical Vlasov equation in the semiclassical asymptotics of small Planck's constant, for the linear case [2] and for the nonlinear case where we couple the quantum Vlasov equation to the Poisson equation [4, 3, 5] and [1].

Recently, in some sort of "inverse semiclassical limit" the numerical concept of solving Schrödinger-Poisson as an approximation of Vlasov-Poisson attracted attention in cosmology, which opens a link to the "smoothed Schrödinger/Wigner numerics" of Athanassoulis et al. (e.g. [6]).

[1] C. Bardos, N.J. Mauser, *Equations cinétiques : une histoire française*, Gazette SMF (2017) to appear

[2] P. Gérard, P. Markowich, N.J. Mauser, F. Poupaud, Homogenization limits and Wigner transforms, *Comm. Pure Appl. Math.*, 50 (1997) 321–377

[3] P.-L. Lions, T. Paul, Sur les mesures de Wigner, *Rev. Mat. Iberoamericana* 9 (3) (1993) 553–618

[4] P. Markowich, N.J. Mauser, The classical limit of a self-consistent quantum Vlasov equation in 3D, *Math. Mod. Meth. Appl. Sci.* 9 (1993) 109–124

[5] P. Zhang, Y. Zheng, N.J. Mauser, The limit from the Schrödinger-Poisson to the Vlasov-Poisson equations with general data in one dimension, *Comm. Pure Appl. Math.* 55 (2002) 582–632

[6] A. Athanassoulis, N.J. Mauser, T. Paul, Coarse-scale representations and smoothed Wigner transforms, *J. Math. Pures et Appl.* 91 (2009) 296–338

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**Julien Medina**

### *Test particles dynamics in low-frequency tokamak turbulence*

*Abstract* : TERESA is a reduced gyrokinetic code which only takes trapped particles into account and is therefore relevant to study low-frequency turbulence such as Trapped Electron Mode (TEM) or Trapped Ion Mode (TIM) in toroidal magnetized fusion plasma.

In the gyrokinetic framework, the fast cyclotron motion is averaged out. Focusing on kinetic trapped particles (with adiabatic passing particles) allows us to furthermore reduce the model by averaging out the banana bounce motion. The particle dynamics can therefore be described in a 2D phase space (the precession angle  $\alpha$ , and an action  $\psi$  which corresponds to a radial coordinate), parameterized by the energy  $E$  and the pitch-angle  $\kappa$ .

This Vlasov-Poisson model based on action-angle formalism drastically reduces the numerical computation costs while staying physically accurate to study phenomena at the order of the toroidal particle precession timescale. Such phenomena includes macroscopic anomalous transport driven by the plasma turbulence and which degrades the confinement of the fusion plasma.

We study the transport and the diffusion in a realistic turbulent plasma simulation using statistics on a relevant number of test particles.

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### **Michel Mehrenberger**

#### *About recurrence time for a semi Lagrangian discontinuous Galerkin Vlasov solver*

*Abstract* : The recurrence time  $T_R \approx 2\pi/(k\Delta v)$  is known for uniform meshes with one unknown per velocity cell of size  $\Delta v$ . Some investigations have been performed for *several* unknowns per cell [2],[3]. We consider here the semi-Lagrangian discontinuous Galerkin (SLDG) method [1], that uses  $(d + 1)$  Gauss points per cell. At the first recurrence times for the classical simplified equation  $\partial_t f + v\partial_x f = 0$ , a weak part of the energy is restituted and the latter can be cancelled exactly thanks to a spectral quadrature rule.

We then obtain  $T_R \approx 2\pi/(k \lfloor 1+d/2 \rfloor \Delta v)$ . Numerical results of the SLDG method on Landau damping and plasma echo confirm the analysis and a study is done for different points (Gauss-Legendre, Gauss-Lobatto, uniform...).

[1] N. Crouseilles, M. Mehrenberger, F. Vecil, Discontinuous Galerkin semiLagrangian method for Vlasov-Poisson, ESAIM Proc, CEMRACS 2010, October 2011, Vol. 32, p. 211-230.

[2] Y. Cheng, I. M. Gamba, P. J. Morrison, Study of conservation and recurrence of Runge-Kutta discontinuous Galerkin schemes for Vlasov-Poisson systems, Journal of Scientific Computing, 56(2), 319-349 (2013).

[3] E. Madaule, M. Restelli, E. Sonnendrücker, Energy conserving discontinuous Galerkin spectral element method for the Vlasov-Poisson system, Journal of Computational Physics, Volume 279, 15 December 2014, Pages 261288.

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### **Damien F. G. Minenna**

#### *Degree-of-freedom reduction for nonlinear N-body wave-particle interaction*

*Abstract* : To study the resonant wave-particle interaction, and to characterize Langmuir waves in nonlinear regimes (trapping, chaos), we use the finite N-body description [1]. Presently, this description is almost unemployeed because of the tremendous number of degree of freedom involved in plasmas, limiting numerical exploitations. But a recent field decomposition, combined with the self-consistent hamiltonian formalism [2], allows drastic degree-of-freedom reduction while preserving momentum conservation (from symplectic properties) for periodic wave-guides. We propose time domain simulations of vacuum electron tubes (like traveling-wave tubes, klystrons, free electron lasers, or linear accelerators). Tubes are analogues to beam-plasma systems [3] where the medium supporting propagating waves is a slow-wave structure instead of a plasma (the dielectric function is replaced by a wave-guide impedance).

- [1] Y. Elskens, and D. F. Escande, *Microscopic dynamics of plasmas and chaos* (IoP publishing, Bristol) 2003.
- [2] F. André, P. Bernardi, N. M. Ryskin, F. Doveil and Y. Elskens, “Hamiltonian description of self-consistent wave-particle dynamics in a periodic structure”, *Europhys. Lett.*, vol 103 (2013) 28004 (5 pp).
- [3] T. M. O’Neil, J. H. Winfrey and J. H. Malmberg, “Nonlinear interaction of a small cold beam and a plasma”, *Phys. Fluids.*, vol 14 (1971) 1204–1212.
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## **Evelyne Miot**

### *The gyrokinetic limit for the Vlasov-Poisson system with a point charge*

*Abstract :*

We investigate the gyrokinetic limit for the two-dimensional Vlasov-Poisson system in a regime studied by F. Golse and L. Saint-Raymond [1, 3]. First we establish the convergence towards the Euler equation under several assumptions on the energy and on the norms of the initial data. Then we analyze the asymptotics for a Vlasov-Poisson system describing the interaction of a bounded density of particles with a moving point charge, characterized by a Dirac mass in the phase-space.

- [1] F. Golse and L. Saint-Raymond, The Vlasov-Poisson system with strong magnetic field, *J. Math. Pures Appl.* (9) 78 (1999), no. 8, 791–817.
- [2] E. Miot, The gyrokinetic limit for the Vlasov-Poisson system with a point charge, preprint, 2016.
- [3] L. Saint-Raymond, Control of large velocities in the two-dimensional gyrokinetic approximation, *J. Math. Pures Appl.* (9) 81 (2002), no. 4, 379–399.
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## **Philip Mocz**

### *Solving Vlasov-Poisson dynamics on an integer lattice*

*Abstract :* This talk is based on [1], where I revisit the integer lattice (IL) method and use it to numerically solve the Vlasov-Poisson equations. The distribution function lives in a discretized lattice phase-space, and each simulation time-step corresponds to a simple permutation of the lattice sites. Hence, the method is Lagrangian, conservative, and fully time-reversible. IL complements other existing methods, such as N-body/particle mesh and finite volume (FV) direct integration schemes. Being a direct integration scheme like

FV, IL is memory limited (requires  $N^6$  memory scaling for a 3D problem, where  $N$  is the resolution per linear phase-space dimension). However, I describe a new technique that

achieves  $N^4$  scaling. The method offers promise for investigating the full 6D phase-space of collisionless systems of stars and dark matter.

- [1] P. Mocz and S. Succi, Integer lattice dynamics for Vlasov-Poisson, *MNRAS*. 465 (2017) 3, 3154-3162 [arXiv:1611.02757].
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## **Philip J. Morrison**

### *The Continuum Hamiltonian Hopf Bifurcation of Vlasov Theory*

*Abstract :* It is well-known that the Vlasov-Poisson system is a Hamiltonian field theory [1] and, consequently, one expects properties of finite-dimensional Hamiltonian dynamics to appear in the infinite-dimensional setting. One such property is the Hamiltonian Hopf bifurcation where pairs of stable modes collide to form a quartet of eigenvalues with positive and negative real and imaginary parts. By Kreĭn’s theorem [2, 3], this bifurcation to instability is only possible if the colliding eigenvalues have opposite signature, i.e., a necessary condition is that one be a positive energy mode with the other being a negative energy mode. A natural question to ask is what happens to this bifurcation in Vlasov-like



mean field theories, where a continuous spectrum exists. It was shown in [4, 5] how to define signature for the continuous spectrum for the Vlasov equation, opening the possibility of having a Kreĭn-like theorem with the continuous spectrum. In a sequence of papers [5, 6, 7] it was shown how neutral modes of the point spectrum embedded in the continuous spectrum can only bifurcate to instability at certain points where the signature of the continuous spectrum changes sign. Such bifurcations involving opposite signature continuous spectra were termed Continuum Hamiltonian Hopf (CHH) bifurcations. We will describe how  $W^{1,1}(\mathbb{R})$  is the natural Sobolev space for viewing this bifurcation, and several examples will be given where the CHH bifurcation is traced.

- [1] P. J. Morrison, Poisson brackets for fluids and plasmas, AIP Conf. Proc. 88, 13–46 (1982).
- [2] M. G. Kreĭn, A generalization of some investigations on linear differential equations with periodic coefficients. Dokl. Akad. Nauk SSSR 73A, 445–448 (1950).
- [3] J. Moser, New aspects in the theory of stability of Hamiltonian systems, Comm. Pure Appl. Math. 11, 81–114 (1958).
- [4] P. J. Morrison and D. Pfirsch, Dielectric energy versus plasma energy, and Hamiltonian action-angle variables for the Vlasov Equation, Phys. Fluids B 4, 3038–3057 (1992).
- [5] P. J. Morrison, Hamiltonian description of Vlasov dynamics: action-angle variables for the continuous spectrum, Trans. Theory and Stat. Phys. 29, 397–414 (2000).
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- [8] G. I. Hagstrom and P. J. Morrison, Continuum Hamiltonian Hopf bifurcation II, Nonlinear Physical Systems – Spectral Analysis, Stability and Bifurcations, eds. O. Kirillov and D. Pelinovsky (Wiley, 2014).

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**Robert Axel Neiss**

*Generalized symplectization of Vlasov dynamics and application to the Vlasov-Poisson system*

*Abstract* : We study a Hamiltonian structure of the Vlasov-Poisson system, first mentioned by Fröhlich, Knowles, and Schwarz [1]. To begin with, we give a formal guideline to derive a Hamiltonian on a subspace of complexvalued  $L^2$  integrable functions  $\alpha$  on the one particle phase space  $\mathbb{R}^{2d}$ , s.t.  $f = |\alpha|^2$  is a solution of a collisionless Boltzmann equation. The only requirement is a sufficiently regular energy functional on a subspace of distribution functions  $f \in L^1$ . Secondly, we give a full well-posedness theory for the obtained system corresponding to Vlasov-Poisson in  $d \geq 3$  dimensions. Finally, we adapt the classical globality results [2, 3] for  $d = 3$  to the generalized system.

- [1] J. Froehlich, A. Knowles, and S. Schwarz, On the mean-field limit of bosons with Coulomb two-body interaction, Comm. Math. Phys. 288 (2009), pg. 1023–1058.
- [2] P.-L. Lions and B. Perthame, Propagation of moments and regularity for the 3-dimensional Vlasov-Poisson system, Invent. Math. 105 (1991), pg. 415–430.
- [3] K. Pfaffelmoser, Global Classical Solutions of the Vlasov-Poisson System in Three Dimensions for General Initial Data, Journal of Differential Equations 95 (1992), pg. 281–303.

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**Darío Núñez**

*On the equilibrium states of the distribution function*

*Abstract* : We study the Vlasov equation in a gravitational potential with spherical symmetry, high-lighting some of the properties of the distribution function and of the theory. We numerically evolve the equation to determine stationary states in different models of a galactic halo, and discuss on the implications.

[1] P. Domínguez-Fernández, E. Jiménez-Vázquez, M. Alcubierre, E. Montoya, and D. Núñez, Description of the evolution of inhomogeneities on a Dark Matter halo with the Vlasov equation, *Gen. Rel. Grav.* 49, 123, (2017) [grqc/1703.03286].

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**S. Nuritdinov, J. Perez, I. Tadjibaev, A. Muminov**

### *Modelling of self-gravitating systems: Pulsating versions and their instabilities*

*Abstract* : The observed inner structure of self-gravitating systems begins to form in reality against the background of non-linearly non-stationary stages of their evolution. So it would be interesting to find solutions of the non-stationary Boltzmann equation and to study a stability problem of corresponding non-linear models. It is comparatively more interesting new direction of modern astrophysics. Because of the difficulty of direct solution of non-stationary Boltzmann equation we decided to force the equilibrium model to pulsate self-similarly. So we constructed the non-stationary versions of following three equilibrium models with homogeneous volume density: Camms model, Einstein sphere and equilibrium BZK disk model. This report details here are calculations for non-stationary version of Camm model and analysing instabilities on the background of this pulsating model.

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**Go Ogiya**

### *What sets the central density structure of dark matter halos?*

*Abstract* : Dark matter (DM) haloes near the free-streaming cutoff scale of the density perturbations provide important hints to understand the formation and evolution of cosmic structures because they are the progenitors of all larger systems formed later. Recent simulations have found when the free-streaming cutoff is resolved, the central density structure of DM haloes near the scale, formed through monolithic collapse, follows

$r^{-a}$   
the  $r^{-a}$  profile with  $a = 1.5$ , which is steeper than that of the universal Navarro-Frenk-White (NFW) model with  $a = 1$ , and the slope becomes shallower as their masses grow. We study the formation and evolution of the DM haloes using idealized N-body simulations of the collapse of proto-halo patches and of mergers between DM haloes of the  $a = 1.5$  profile. We find (i) the density core formed by the free-streaming motion in the centre of patches leads the rapid collapse and leads DM to free fall motion to form the  $a = 1.5$  profile [1]; and (ii) mergers drive the slope to approach that of the NFW model which is robust to mergers and serves an attractor solution for the density structure of DM haloes [2]. We also discuss the overmerging problem in state-of-the-art simulations of cosmic structure formation and conditions to obtain trustable results [3].

[1] G. Ogiya and O. Hahn, What sets the central structure of dark matter haloes? submitted to Monthly Notices of the Royal Astronomical Society; arXiv:1707.07693

[2] G. Ogiya, D. Nagai and T. Ishiyama, Dynamical evolution of primordial dark matter haloes through mergers Monthly Notices of the Royal Astronomical Society, Volume 461, Issue 3, p.3385-3396; arXiv:1604.02866

[3] F. van den Bosch, G. Ogiya, O. Hahn and A. Burkert, to be submitted

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**D. I. Palade**

### *Non-equilibrium hydrodynamic pressure tensors from kinetic perspectives*

*Abstract* : Hydrodynamic models offer an attractive alternative to kinetic theories such as Vlasov / Wigner-Poisson systems. Unfortunately, they are limited by the unknown

specific form of the pressure tensor. Usually, the latter is approximated by a diagonal tensor as polytropic functional of density (ideal gas). Such an approximation is valid for homogeneous systems at equilibrium, therefore, it is not able to reproduce the propagation of waves even in simple cases, such as the linear regime for a homogeneous electrostatic plasma. In the present work, a modified, non-local functional relation between the pressure tensor and density is proposed, being derived from the analytic solutions of linearized Vlasov/Wigner equation. Our proposal is tested on various non-linear hydrodynamic simulations against their kinetic counterpart for systems with long-range interaction showing an overall improvement of the standard hydrodynamic approach towards the kinetic results.

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**F. Pegoraro, E. D'Avignon and P. J. Morrison**

***Action principle for relativistic magnetohydrodynamics***

*Abstract*: A covariant action principle for ideal relativistic magnetohydrodynamics in terms of natural Eulerian field variables is given based on the article E. D'Avignon, P.J. Morrison, F. Pegoraro Phys. Rev. D 91, 084050 (2015).

This is done by generalizing the covariant Poisson bracket theory of Marsden et al., which uses a noncanonical bracket to effect constrained variations of an action functional. Various implications and extensions of this action principle are also discussed. The approach that will be presented necessitates a handful of new concepts such as a modified enthalpy density, a momentum differing significantly from the standard kinetic momentum, and another "momentum" conjugate to the magnetic field) which may provide new insight into this physical system.

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**Jérôme Perez, Alicia Simon-Petit**

***Why isochrony in self-gravitating systems evolution ?***

*Abstract*: Collisionless Boltzmann dynamics in astrophysics is a challenging problem which is tackled from various approaches. Observations and numerical simulations do not lead to robust established consensus and depend on the considered systems. Theoretical approaches provide results that are too general for equilibria and not stringent enough for stability. Nevertheless, Michel Hénon introduced his isochrone potential describing a classical mean field dynamics in a seminal paper in 1959. After a detailed review of the various approaches mentioned above, we will explain why isochrony seems important in the evolution of self-gravitating systems.

[1] A. Simon-Petit, J. Perez, G. Duval, Isochrony in radial 3D potentials : classification, interpretation and applications, submitted, 2017.

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**Oreste Pezzi, Francesco Valentini and Pierluigi Veltri**

***Collisional Dissipation of Fine Velocity Structures in Weakly Collisional Plasmas***

*Abstract*: Weakly collisional plasmas, such as the solar wind, are usually described by neglecting the effect of collisions. However, the estimation of the plasma collisionality is often based on the restrictive assumption that particle velocity distribution function (VDF) is close to the thermodynamical equilibrium [1], while observations and simulations indicate that marked non-Maxwellian features develop in the three-dimensional VDFs, as a result of the energy cascade towards short, kinetic spatial scales. Since collisional effects are proportional to velocity gradients of the VDF, the collisionless hypothesis may fail locally in velocity space. Here we show that several characteristic times are recovered during the collisional relaxation of fine velocity structures: the entropy growth occurs over several time scales; hence, fine velocity structures are dissipated by collisions in a time much shorter than global non-Maxwellian features, like, for example, temperature anisotropies. This suggests that plasma collisionality can be locally enhanced due to the velocity space deformation of the particle velocity distribution [2, 3].

- [1] L. Spitzer Jr, Physics of Fully Ionized Gases, (Interscience Publishers, New York, NY, 1956).  
[2] O. Pezzi, F. Valentini & P. Veltri, Phys. Rev. Lett. 116, 145001 (2016). [3] O. Pezzi, J. Plasma Phys. 83, 555830301 (2017).
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## **Peter Pickl**

### *Microscopic Derivation of the Vlasov equation*

*Abstract* : The rigorous derivation of the Vlasov equation from Newtonian mechanics of  $N$  Coulomb-interacting particles is still an open problem. In the talk I will present recent results [1, 2], where, similar to [3] an  $N$ -dependent cutoff is used to make the derivation possible. The cutoff is removed as the particle number goes to zero. Our result holds for typical initial conditions, only. This is, however, not a technical assumption: one can in fact prove deviation from the Vlasov equation for special initial conditions for the system we consider.

- [1] N. Boers and P. Pickl, The theory and phenomenology of polarized deep inelastic scattering, J. Stat. Phys. 164, 1–16 (2016).  
[2] D. Lazarovici and P. Pickl: A Mean Field Limit for the Vlasov-Poisson System, Arch. Rat. Mech. Anal. 225, 1201–1231 (2017)  
[3] M. Hauray and P.-E. Jabin:  $N$ -particles Approximation of the Vlasov Equations with Singular Potential, Arch. Rat. Mech. Anal. 183, 489–524 (2017)
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## **Devon Powell**

### *Adaptive Beam Tracing: Radiation transport and synthetic observations of the CDM phase-space sheet.*

*Abstract* : We present “adaptive beam tracing,” a new numerical method for radiation transport and synthetic observations descended from ray-tracing. Beams are defined by polygonal fronts with flux distributed across an extended area. Radiation flux is propagated through each polyhedral mesh cell between upwind and downwind beam fronts. Fluxes are attenuated using volume-averaged opacities in the optically thick case. This is achieved using a geometric algorithm that computes exact overlap integrals for volumes and solid angles [1].

We use beam tracing to create area-sampled synthetic observations of cold dark matter represented by a tetrahedral tessellation of the phasespace sheet [2] . This is done by tracking beams away from a virtual observer through a polyhedral mesh containing source information. The same remapping algorithm [1] is used to integrate source fluxes over the intersection volumes between beam segments and the mesh.

Source fluxes are computed using either the exact integrated density within each beam (in the case of decaying dark matter), or the squared density (in the case of annihilating dark matter). We will describe an updated version of the geometric algorithm and the use of an  $R^*$ -tree to query spatially extended objects in  $\log(n)$  time.

While we focus on the method itself, we will also present our most recent results and discuss prospects for future research.

- [1] Devon Powell and Tom Abel. An exact general remeshing scheme applied to physically conservative voxelization. Journal of Computational Physics, 297:340356, 2015  
[2] Abel, T., Hahn, O., & Kaehler, R. 2012, MNRAS, 427, 61
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## **B. V. Ribeiro, Y. Elskens**

### *Brownian regime of finite- $N$ corrections in the XY hamiltonian mean field model*

*Abstract* : We study the dynamics of the  $N$ -particle system evolving in the XY hamiltonian mean field (HMF) model for a repulsive potential, when no phase transition occurs.

Starting from a homogeneous distribution, particles evolve in a mean field created by the interaction with all others. This interaction does not change the homogeneous state of the system, and particle motion is approximately ballistic with small corrections. For initial particle data approaching a waterbag, it is explicitly proved that corrections to the ballistic velocities are in the form of independent brownian noises over a time scale diverging not slower than  $N^{2/5}$  as  $N \rightarrow \infty$ , which proves the propagation of molecular chaos. Molecular dynamics simulations of the XY-HMF model confirm our analytical findings.

- [1] Antoni M., Ruffo S.: Phys. Rev. E 52 (1995) 2361–2374.
- [2] Ettoumi W., Firpo M-C.: Phys. Rev. E 84 (2011) 030103(R).
- [3] Elskens Y.: J. Stat. Phys. 148 (2012) 591–605.
- [4] B.V. Ribeiro, M.A. Amato, Y. Elskens.: Phys. Scr. 91, 084004 (2016).

**B. V. Ribeiro, Y. Elskens**

*Kinetic limit of wave-particle self-consistent interaction in an open domain*

*Abstract :* We consider a system of  $N$  particles  $\sigma = (x_1, v_1, \dots, x_N, v_N)$  interacting via pair potential  $U$  in a closed domain  $\Omega$ . In this domain, particles also interact with one wave  $Z = A \exp(i\phi)$ . Particles can enter and leave  $\Omega$ , so  $N$  varies in time.

Given initial data  $(Z(0), \sigma(0))$  and a boundary source/sink, the system evolves according to a Hamiltonian to  $(Z(t), \sigma(t))$ . In the kinetic limit  $N \rightarrow \infty$ , this generates a Vlasov-like equation for the distribution function  $f(x, v, t)$  coupled to an envelope equation for  $Z$ , labeled  $Z^\infty$ . The solution  $(Z^\infty, f)$  exists and is unique for initial data with finite energy, under reasonable conditions.

For any finite time  $t$ , given a sequence of initial data such that  $\sigma(0) \rightarrow f(0)$  weakly and  $Z(0) \rightarrow Z(0)$  as  $N \rightarrow \infty$ , the solutions  $(Z(t), \sigma(t))$  to the Hamiltonian dynamics satisfy  $\lim_{N \rightarrow \infty} (Z(t), \sigma(t)) = (Z^\infty, f(x, v, t))$ .

- [1] Y. Elskens and D. Escande, Microscopic dynamics of plasmas and chaos, IoP Publishing, Bristol (2003).
- [2] M-C. Firpo and Y. Elskens, J. Stat. Phys. 93 (1998) 193–209.
- [3] M. K.-H. Kiessling, Commun. Nonlin. Sci. Numer. Simul. 13 (2008) 106–113.
- [4] H. Spohn, Large scale dynamics of interacting particles, Springer, Berlin (1991).

**Tarcísio M. Rocha Filho**

*Vlasov Equation and Violent Relaxation – The Self-Gravitating Ring Model*

*Abstract :* Under suitable conditions, the Vlasov equation correctly describes the dynamics of a system of  $N$  particles with long-range interactions [1]. Nevertheless a zero distance divergence in the inter-particle potential may cause small deviations from the Vlasov dynamics [2]. In this talk we present results from molecular dynamics simulations and direct integration of the Vlasov equation [3] for the one-dimensional self-gravitating ring model. This model has a small softening parameter that regularizes the potential at zero distance. For sufficiently small values of the parameter, deviations of the molecular dynamics results from the solution of the corresponding Vlasov equation are observed. We also briefly discuss a modification of the Vlasov equation considering explicitly the presence of the divergence of the potential for the present model.

- [1] T. M. Rocha Filho, M. A. Amato, A. E. Santana, A. Figueiredo and J. R. Steiner, Dynamics and physical interpretation of quasistationary states in systems with long-range interactions, *Phys. Rev. E* 89, 032116 (2014).
- [2] L. Brenig, Y. Chaffi and T. M. Rocha Filho, Long velocity tails in plasmas and gravitational systems, arXiv:1605.05981 [physics.plasm-ph].
- [3] T. M. Rocha Filho, Solving the Vlasov equation for one-dimensional models with long range interactions on a GPU, *Comp. Phys. Comm.* 184,34 (2013).
- 

### **Chiara Saffirio**

#### *Lagrangian solutions to the Vlasov-Poisson system with a point charge*

*Abstract* : We consider the Cauchy problem for the repulsive Vlasov-Poisson system in the three dimensional space, where the initial datum is the sum of a diffuse density, assumed to be bounded and integrable, and a point charge. Under some decay assumptions for the diffuse density close to the point charge, under bounds on the total energy, and assuming that the initial total diffuse charge is strictly less than one, we prove existence of global Lagrangian solutions. This is a joint work with G. Crippa and S. Ligabue.

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### **Shohei Saga**

#### *Analysis of the phase-space structure in the pre-collapse perturbation theory*

*Abstract* : Before the breakdown of the single-stream regime of the cold-darkmatter fluid, the system is treated as the Euler-Poisson equation, that is, the fluid approximation, where the perturbative approach remains adaptive. We develop the higher-order perturbation theory in the Lagrangian coordinate for the three-sine wave model [1], which describes the gravitational collapse in the expanding the Universe. We perform three kinds of perturbation schemes for the Euler-Poisson equation up to the much higher order. By comparing perturbative results and Vlasov-Poisson simulation, we discuss the validity of perturbation theory during the precollapse regime.

[1] F. Moutarde, J.-M. Alimi, F.R. Bouchet, R. Pellat, and A. Ramani, *Astrophys. J.* 382 (1991) 377

[2] C. Rampf and U. Frisch, *Mon. Not. Roy. Astron. Soc.* 471 (2017) 671

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### **Sergei Shandarin**

#### *Tracing the Dark Matter Web*

*Abstract* : Dark matter (DM) constitutes almost 85% of all mass able to cluster into gravitationally bound objects. Thus it has played the determining role in the origin and evolution of the structure in the universe often referred to as the Cosmic Web. The dark matter component of the Cosmic Web or simply the Dark Matter Web is considerably easier to understand theoretically than the baryonic component of the web if one assumes that DM interacts only gravitationally. One of the major differences between the DM and baryonic webs consists in the multi stream structure of the DM web. Thus it allows to use three diagnostic fields that do not present in the baryonic web: the number of streams field in Eulerian space, the number of flip flops field in Lagrangian space, and the caustic structure in the both. Although these characteristics have been known for a long time their systematic studies as fields started only a few years ago. I will report new recent results of numerical studies of the three fields mentioned above and also discuss the features of the DM web they have unveiled.

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### **Alicia Simon-Petit, Jérôme Perez**

#### *What is isochrony in 3D static potentials ?*

*Abstract* : Collisionless Boltzmann dynamics is a fruitful problem for which the search of equilibria is usually based on symmetry properties or mass density profiles reconstruction. Michel Hénon focused on stellar orbital properties to introduce isochrone potentials in 1959. Such potentials emerge in spherically symmetric systems formed by a large amount of charges (electric or gravitational) of the same type considered in a mean-field theory. They are defined by the fact that, when it exists, the radial period of any test charge only depends on its energy and not on its angular momentum.

After a characterization and completion of the whole set of isochrone potentials, we will discuss the nature of isochrony which appears to be very consistent in the potential theory. A physical interpretation of isochrony will be proposed. Some symmetries involved in gravitation (Kepler's Laws, Bertrand's theorem) will be eventually revisited.

[1] A. Simon-Petit, J. Perez, G. Duval, Isochrony in radial 3D potentials : classification, interpretation and applications, submitted, 2017.

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## **Fred Skiff**

### *On putting experimental data into the Vlasov-Poisson equations*

*Abstract* : We expand the technique of comparing theory and experiment by computing the terms of the Vlasov-Poisson equations directly from experimental data. This involves techniques for treating a number of systematic effects in measurements as well as trying to address the non-local response of the plasma. Data from laser-induced fluorescence (for ion electrostatic waves [1]) and whistler-mode wave absorption and electromagnetic field probes to measure electron distribution functions and waves [2]. Zero-order quantities, such as the total particle distribution function, first order perturbations, and second order (energy) quantities will be considered.

[1] F. Skiff, G. Bachet, and F. Doveil, Ion dynamics in nonlinear electrostatic structures, Physics of Plasmas Volume 8, p 3931 (2001).

[2] J. W. R. Schroeder, F. Skiff, G. G. Howes, C. A. Kletzing, T. A. Carter and S. Dorfman, Alfvénic Oscillations of the Electron Distribution Function: Linear Theory and Experimental Measurements, AIP Conference Proceedings 1689, 030001 (2015); doi: 10.1063/1.4936466.

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## **Eric Sonnendrücker**

### *Geometric electromagnetic PIC models*

*Abstract* : A hamiltonian framework for the derivation of semi-discrete (continuous in time) Finite Element Particle In Cell approximations of the Vlasov-Maxwell equations was derived in [1]. It is based on a particle (Klimontovitch) discretization of the distribution function and a compatible Finite Element discretization of the grid quantities. The ideas introduced in [1] can be declined in different variants, choosing different discrete spaces for the fields or adding smoothing functions for the particles.

Moreover, starting from such a semi-discretization, which yields a finite dimensional Hamiltonian structure

$$dU / dt = J(U) \square H(U),$$

defined by a Poisson  $J(U)$  matrix and a hamiltonian  $H(U)$ , several classes of different structure preserving time discretization can be derived: hamiltonian splitting methods as in [1], that preserve the Poisson structure, or discrete gradient methods that preserve exactly the hamiltonian. This procedure enables in particular to recover and generalize several wellknown explicit and implicit PIC algorithms.

We are going in this talk to give an overview of the geometric ideas behind this structure and how they can be used to derive fully discrete particle in cell schemes with exact conservation of the Poisson structure, the energy and Gauss' law.

[1] M. Kraus, K. Kormann, P.J. Morrison, E. Sonnendrücker. GEMPIC: Geometric electromagnetic particle-in-cell methods. *Journal of Plasma Physics*, 83(4), (2017).

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### Thierry Sousbie

#### *ColdICE: 6D collisionless phase space hydrodynamics using a lagrangian tessellation*

*Abstract* : In this talk, I will present ColDICE[1, 2], a publicly available parallel numerical solver designed to solve the Vlasov-Poisson equations in the cold case limit. The method is based on the representation of the phase-space sheet as a conforming, self-adaptive simplicial tessellation whose vertices follow the Lagrangian equations of motion. In this presentation, I will mainly focus on describing the underlying algorithm and its practical implementation, as well as showing a few practical examples demonstrating its capabilities.

[1] Sousbie, T., & Colombi, S. 2016, *Journal of Computational Physics*, 321, 644

[2] <https://github.com/thierry-sousbie/dice>

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### Florin Spineanu

#### *Singular states of the 2D Euler fluid and of the density of eigenvalues of random matrices*

*Abstract* : It is known [1] that the resolvent of an ensemble of random matrices verifies the Burgers equation with a viscosity proportional with the inverse of the dimension of the matrix,  $\nu \propto 1/(2N)$ . For the two-dimensional Euler fluid one can formulate the fluctuation regime where coherent structures have not yet emerged, using the random matrix analogue. We examine the possibility that the pre-shock singularity of the viscous Burgers equation corresponds to the pile-up singular accumulation of all elementary Euler vortices (in a discretization of  $N$  vortices) into a single point. Then this corresponds to the condition derived by Chorin [2] for this extreme solution,  $|T| < 1/(8\pi N)$  where  $T$  is the temperature of the statistical ensemble of discrete vortices. This means that the highest (singular) state of order would ideally be obtained if the fluctuations are less than the "viscosity" in the space of density of eigenvalues.

[1] J.P. Blaizot and M. Nowak, Universal shocks in random matrix theory, *Phys Rev E* 82, 051115 (2010).

[2] A.J. Chorin, *Vorticity and Turbulence*, Springer, New York (1994), pg. 80.

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### Jens Stücker

#### *The Complexity of the Dark Matter Sheet*

*Abstract* : Dark matter, as a cold collisionless fluid, effectively occupies a three-dimensional sub-manifold in six-dimensional phase space [1]. In simulations, this "dark matter sheet" can in principle be reconstructed by interpolation techniques to obtain an almost exact density estimate [2]. However, in regions of strong mixing (like dark matter haloes), this is difficult due to the rapid growth of the sheet's complexity [3] and an alternative is required. I will present a hybrid scheme which uses sheet-interpolation techniques where a reconstruction is possible, and N-body-techniques in regions of strong mixing. We anticipate that this will make fragmentation-free warm dark matter simulations with realistic haloes possible.

[1] T. Abel, Tom, O. Hahn and R. Kaehler, Tracing the dark matter sheet in phase space, *Monthly Notices of the Royal Astronomical Society*, Volume 427, Issue 1, pp. 61-76 (2012)

[2] O. Hahn and R. Angulo, An adaptively refined phase-space element method for cosmological simulations and collisionless dynamics, *Monthly Notices of the Royal Astronomical Society*, Volume 455, Issue 1, p.1115-1133 (2016)

[3] M. Vogelsberger and S. White, Streams and caustics: the fine-grained structure of cold dark matter haloes, *Monthly Notices of the Royal Astronomical Society*, Volume 413, Issue 2, pp. 1419-1438 (2011)



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**Yasushi Suto*****Non-sphericity of collisionless gravitating systems in the universe***

*Abstract* : The cosmological structure formation is dictated by gravitational evolution of collisionless dark matter. In particular the nature of dark matter halos is the key to understand the diversity of the hierarchical structure in the universe. We consider the non-sphericity of dark matter halos from a series of cosmological N-body simulations. We find a phenomenological universality in the non-sphericity, and derive its statistical distribution function that can be in principle tested against the future cosmological survey of galaxy clusters.

[1] Y.P. Jing and Y. Suto, Triaxial Modelling of Halo Density profiles with High-Resolution N-body Simulations, *The Astrophysical Journal* 574 (2002) 538 [astro-ph/0202064].

[2] D. Suto, T. Kitayama, K. Osato, S. Sasaki and Y. Suto Confrontation of top-hat spherical collapse against dark halos from cosmological N-body Simulations, *Publication of Astronomical Society of Japan* 68 (2016) 1417 [arXiv:1511.06935].

[3] D. Suto, T. Kitayama, T. Nishimichi, S. Sasaki and Y. Suto Evolution and statistics of non-sphericity of dark matter halos from cosmological N-body Simulations, *Publication of Astronomical Society of Japan* 68 (2016) 97 [arXiv:1608.06494].

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**Atsushi Taruya*****Perturbative description of Vlasov-Poisson system in cosmology: approaching and going beyond shell-crossing***

*Abstract* : Vlasov-Poisson equation in a cosmological background provides a fundamental basis to describe the gravitational dynamics of large-scale matter distribution in the Universe. Starting with cold initial condition, such a system initially follows the single-stream regime, where various analytical techniques have been exploited. Beyond the single-stream regime, however, no adequate treatment has been known. In this talk, based on Ref. [1], we present a perturbative description that can go beyond the single-stream regime in the 1D cosmological setup. Implication of this treatment to the three-dimensional case is also discussed, particularly paying an attention to the accurate description of the shell-crossing.

[1] A. Taruya, S. Colombi, Post-collapse perturbation theory in 1D cosmology – beyond shell-crossing, *Mon. Not. R. Astron. Soc.* 470 (2017) 4858 [astro-ph/1701.09088].

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**Emanuele Tassi*****A reduced Landau fluid model for collisionless magnetic reconnection***

*Abstract* : Magnetic reconnection is a modification of the connection between magnetic field lines and infinitesimal plasma volumes, and participates in several processes occurring in astrophysical and laboratory plasmas. In this contribution we present a reduced fluid model for the description of magnetic reconnection driven by electron inertia in a low- $\beta$  plasma, with  $\beta$  the ratio between internal and magnetic pressure. The fluid reduction is obtained by imposing closures accounting for Landau damping. In the non-dissipative limit, the model is shown to possess a noncanonical Hamiltonian structure. Preliminary results of numerical simulations of magnetic reconnection with helical symmetry are also illustrated.

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**Romain Teyssier*****High performance N body simulations for modelling large scale structures in the universe***

*Abstract* : N body simulations are an important tool to compute the non-linear dynamics of the large scale structures in the universe. Large upcoming galaxy surveys will be used in the coming decade to measure with exquisite precision the amount of dark matter, the

mass of the neutrinos and possibly shed light on the nature of dark energy, or a possible modification of general relativity. The required precision, better than 1% up to wave number as large as 10 h/Mpc pushes our existing N body codes to their limit. We need fast gravity solver with low memory footprints. I will report on a recent attempt to model the entire observable universe in the context of the Euclid mission, with 2 trillion dark matter particles simulated in a periodic box of 3000 Mpc (megaparsecs). This unprecedented effort was made possible thanks to the combined use of the Fast Multipole Method efficiently, ported on massively parallel architecture, and of Graphical Processing Units (GPU) to speed up the direct force calculations.

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**Jihad R. Touma**

*Secular Collisionless Dynamics around Massive Central Bodies*

*Abstract* : The orbit-averaged treatment of planetary motion, initiated in its linear form by Laplace and Lagrange, translates cleanly to the nearly-Keplerian motion of self-gravitating clusters of particles (dust, comets, stars) dominated by massive central bodies (planets, stars, supermassive black holes)[1]. Collisionless evolution of such clusters is best captured by an orbit averaged Poisson-Vlasov system over a reduced 4D phase space [2]. In this talk, I will overview the theoretical framework and associated results on equilibria and their stability, then highlight directions for further investigation which are opened up by dedicated numerical experiments of unstable configurations and resulting saturated states. [3].

[1] S. Sridhar and J. Touma, Stellar dynamics around black holes in galactic nuclei, MNRAS (1999).

[2] S. Sridhar and J. Touma, Stellar dynamics around a massive black hole I. Secular collisionless theory

[3] J. Touma, S. Tremaine and M. Kazandjian, Gauss's Algorithm, Softened, MNRAS (2009).

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**Pallavi Trivedi, R. Ganesh**

*Vlasov simulations of Driven Electrostatic Phase Space Vortices in a 1-D electron-ion plasma*

*Abstract* : The paradigm of formation and evolution of electrostatic phase space vortices (PSV) in an unbounded, collisionless plasma is of fundamental interest, both in astrophysical plasmas as well as laboratory plasmas alike. Recently, for both Maxwellian and non-Maxwellian distributions of electrons and stationary background ions, a small amplitude external drive, when chirped, has been shown to couple effectively to the plasma and increase both streaming of "untrapped" and "trapped" particle fraction, eventually leading to large, multi-extrema phase space vortices. [1,2]

In the above studies, the ions have been assumed to be immobile. However, ion motion may significantly change the evolution of high and low frequency motions which in turn may affect the trapping and formation of PSVs. In the present work, using a numerical Vlasov-Poisson solver which treats both electrons and ions kinetically, we bring out several interesting features of driven phase space structures in the dynamics of the background slowly moving ions, in Maxwellian plasma, the details of which will be presented.[3]

[1] Pallavi Trivedi, R. Ganesh, Physics of Plasmas 23, 062112 (2016).

[2] Pallavi Trivedi, R. Ganesh, Physics of Plasmas 24, 032107 (2017).

[3] Pallavi Trivedi, R. Ganesh (Manuscript in preparation.)

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**Natalia Tronko**

*Second order Hamiltonian gyrokinetic models for particle-in-cell codes.*

*Abstract* : Since the full gyrokinetic Vlasov-Maxwell set of equations [1] is currently unsuitable for the direct numerical implementation, the existing codes implement reduced gyrokinetic equations, and each code has its own set of equations. The choice of the reduced gyrokinetic model is most importantly driven by numerical constraints. This makes the interpretation and the verification of the numerical results obtained by different codes extremely challenging. Recently, a serie of articles has been published presenting a systematic framework for a gyrokinetic code verification: numerically [4],[5] and analytically [3],[4]. This talk is based on [2]. An iterative Lie-transform procedure for the derivation of gyrokinetic Vlasov equations suitable for the implementation in PIC codes (like ORB5) will be presented.

[1] A.J. Brizard and T.S. Hahm, Foundation of nonlinear gyrokinetic theory, Rev. Plasma Phys. 79 (2) April, (2007) 861.

[2] N. Tronko, A. Bottino, C. Chandre and E. Sonnendrücker, Hierarchy of second order gyrokinetic Hamiltonian models for particle-in-cell codes, PPCF 59 (2017) 064008.

[3] N. Tronko, A. Bottino and E. Sonnendrücker, Second order gyrokinetic theory for particle-in-cell codes, Phys.Plasmas 23 (2016) 082505.

[4] N. Tronko, A. Bottino, T.Goerler, E. Sonnendrücker D. Told and L.Villard, Verification of Gyrokinetic codes: Theoretical background and applications, Phys.Plasmas 24 (2017) 056115.

[5] T. Goerler, N. Tronko, W. Hornsby et al., Intercode comparison of gyrokinetic global electromagnetic modes, Phys.Plasmas 23 (2016) 072503.

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## **Eugene Vasiliev**

### *Beyond collisionless Boltzmann equation: when relaxation is important*

*Abstract* : Many physical systems are expected to be collisionless on the Hubble timescale. Yet when we simulate a galaxy having  $\sim 10^8 - 10^{11}$  stars with only  $O(10^6)$  N-body particles, the two-body relaxation rate is artificially increased. I present a couple of examples when this may lead to incorrect conclusions about the behaviour of the system: the evolution of binary supermassive black holes [1], and the chaotic diffusion in triaxial elliptical galaxies [2, 3]. I then introduce a different simulation method [4] that has a much lower intrinsic numerical relaxation rate, and demonstrate that it is able to recover the nearly collisionless dynamics in these systems.

[1] E. Vasiliev, F. Antonini, D. Merritt, The final-parsec problem in the collisionless limit, ApJ, 810 (2015), 49.

[2] H. Kandrup, C. Siopis, Chaos and chaotic phase mixing in cuspy triaxial potentials, MNRAS, 345 (2003), 727.

[3] C. Hamilton, E. Vasiliev, Shape evolution of triaxial stellar systems due to chaotic diffusion and two-body relaxation, in prep.

[4] E. Vasiliev, A new Monte Carlo method for dynamical evolution of nonspherical stellar systems, MNRAS, 446 (2015), 3150.

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## **Michel Vittot, Phil Morrison**

### *A Geometrical Version of the Maxwell-Vlasov Hamiltonian Structure*

*Abstract* : We present a geometrization of the Hamiltonian approach of classical electrodynamics, via a (non-canonical) Poisson structure. This relativistic Hamiltonian framework (introduced by Morrison, Marsden, Weinstein) is a field theory where the phase space is a set of differential forms, and is independent of the gauge potentials. This algebraic and geometric description of the Vlasov kinetics is well suited for a perturbation theory, in a strong inhomogeneous magnetic field (expansion in  $1/|B|$ , with all the curvature terms...), like in magnetized plasmas, and in any coordinates, for instance adapted to a Tokamak (toroidal coordinates, or else...) or in a stellar plasma.

[1] P.J. Morrison, M. Vittot, L. de Guillebon: Lifting particle coordinate changes of magnetic moment type to Vlasov-Maxwell Hamiltonian dynamics, *Physics of Plasmas*, 20, 3 (2013).

[2] L. de Guillebon, M. Vittot: Gyro-gauge independent formulation of the guiding-center reduction to arbitrary order in the Larmor radius. *Plasma Physics and Controlled Fusion* 55, 105001 (2013).

[3] A.J. Brizard, P.J. Morrison, J.W. Burby, L. de Guillebon, M. Vittot: Lifting of the Vlasov-Maxwell Bracket by Lie-transform Method, *J. Plasma Phys.* (2016)

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## **Madalina Vlad**

### *Hidden drifts and turbulence evolution*

*Abstract* : Drift type turbulence in magnetically confined plasmas is analyzed in the basic Vlasov-Poisson description. The results presented in [1]-[3] are developed by including plasma rotation. We use the iterated self-consistent method [1], a semi-analytical approach based on connected studies of test particle and test modes on turbulent plasmas. Ion trajectories can have both random and quasi-coherent aspects. We show that a particular effect of the quasi-coherence appears in the presence of plasma rotation. It is represented by the hidden drifts. They are quasi-coherent flows, which have zero average, but can strongly influence the evolution of the turbulence through the generation of zonal flow modes.

[1] M. Vlad and F. Spineanu, Random and quasi-coherent aspects in particle motion and their effects on transport and turbulence evolution, *New J. Phys.* 19 (2017) 025014.

[2] M. Vlad, Ion stochastic trapping and drift turbulence evolution, *Phys. Rev. E* 87 (2013) 053105.

[3] M. Vlad and F. Spineanu, Test particle study of ion transport in drift type turbulence, *Phys. Plasmas* 20 (2013) 122304.

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## **Yoshiyuki Y. Yamaguchi**

### *Relaxation and long-time correlation of finite-size fluctuation in thermal equilibrium*

*Abstract* : We demonstrate that long-range systems provide strange phenomena even in thermal equilibrium by focusing on finite-size fluctuation. Initial states are randomly taken from thermal equilibrium, and temporal evolutions of order parameter are numerically computed. Defining the variance by the time average, we numerically reveal two level relaxation of the variance, which is once trapped at a nonequilibrium level and goes to the thermal equilibrium level [1]. Appearance of  $1/f$  fluctuation of power spectra is also shown, which implies long-time correlation. These phenomena are explained by introducing the concept of pseudo Casimir invariants in finite-size systems.

[1] Y. Y. Yamaguchi, Strange scaling and relaxation of finite-size fluctuation in thermal equilibrium, *Phys. Rev. E* 94 (2016) 012133.

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## **Kohji Yoshikawa**

### *Vlasov-Poisson simulation of collisionless self-gravitating systems and its application to cosmological neutrinos in the large-scale structure of the universe*

*Abstract* : In this talk, we present the Vlasov-Poisson simulation of collisionless self-gravitating systems in the six-dimensional phase space and its application to numerical simulation of cosmological neutrinos in the large-scale structure of the universe. As a key ingredient of Vlasov-Poisson simulation, we introduce spatially high-order advection schemes presented in [2] which preserve the monotonicity and positivity of numerical solutions. We will also briefly present Vlasov-Maxwell simulation of collisionless plasma performed with our new schemes.

[1] Yoshikawa, K., Yoshida, N., & Umemura, M. 2013, *The Astrophysical Journal*, 762, 116

[2] Tanaka, S., Yoshikawa, K., Minoshima, T., & Yoshida, N. 2017, submitted to The Astrophysical Journal, arXiv:1702.08521

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