Simplex in Cell techniques for Vlasov–Poisson and Vlasov–Maxwell modeling

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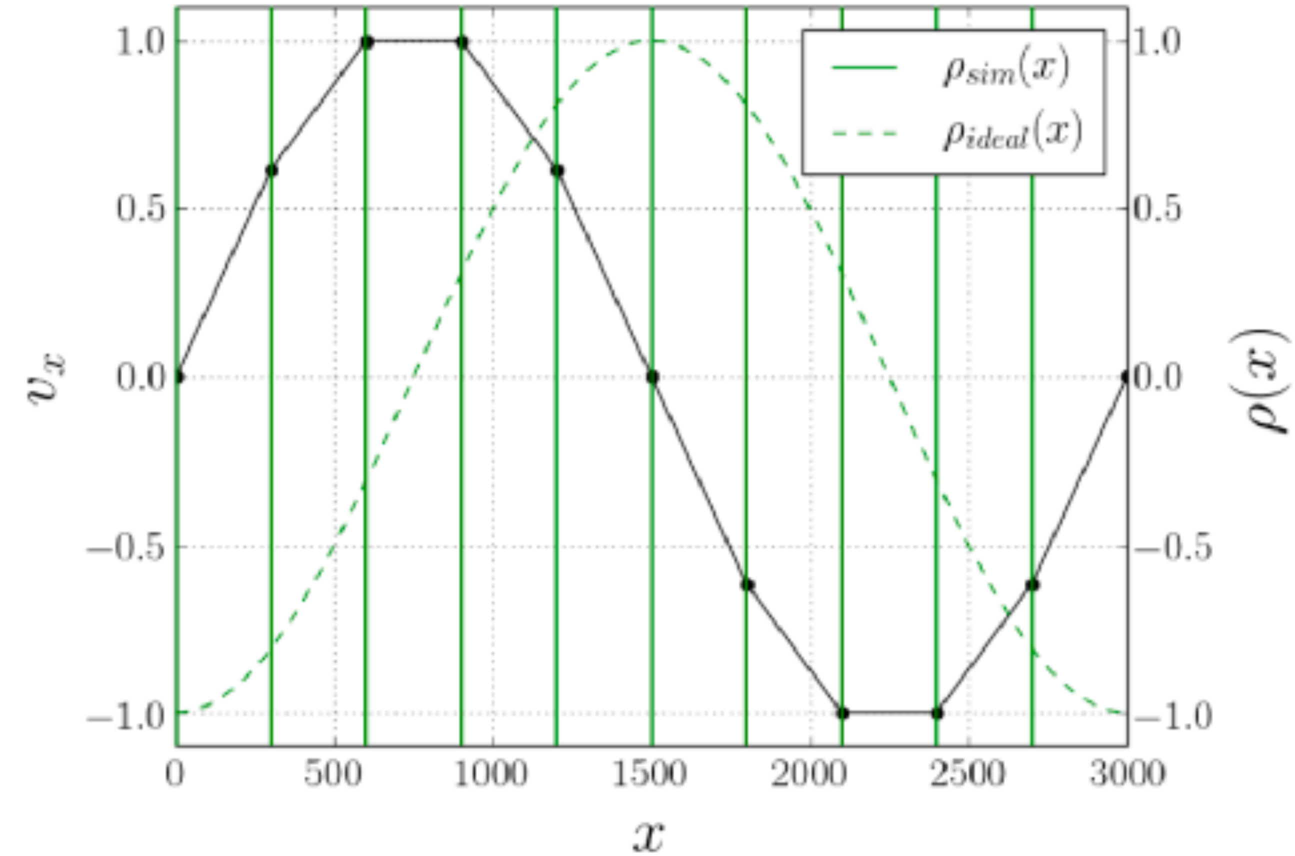
- The Phase Space Sheet for collision-less fluids
- 1D Vlasov Poisson
- 2D/3D Weibel Instability example
- Outlook

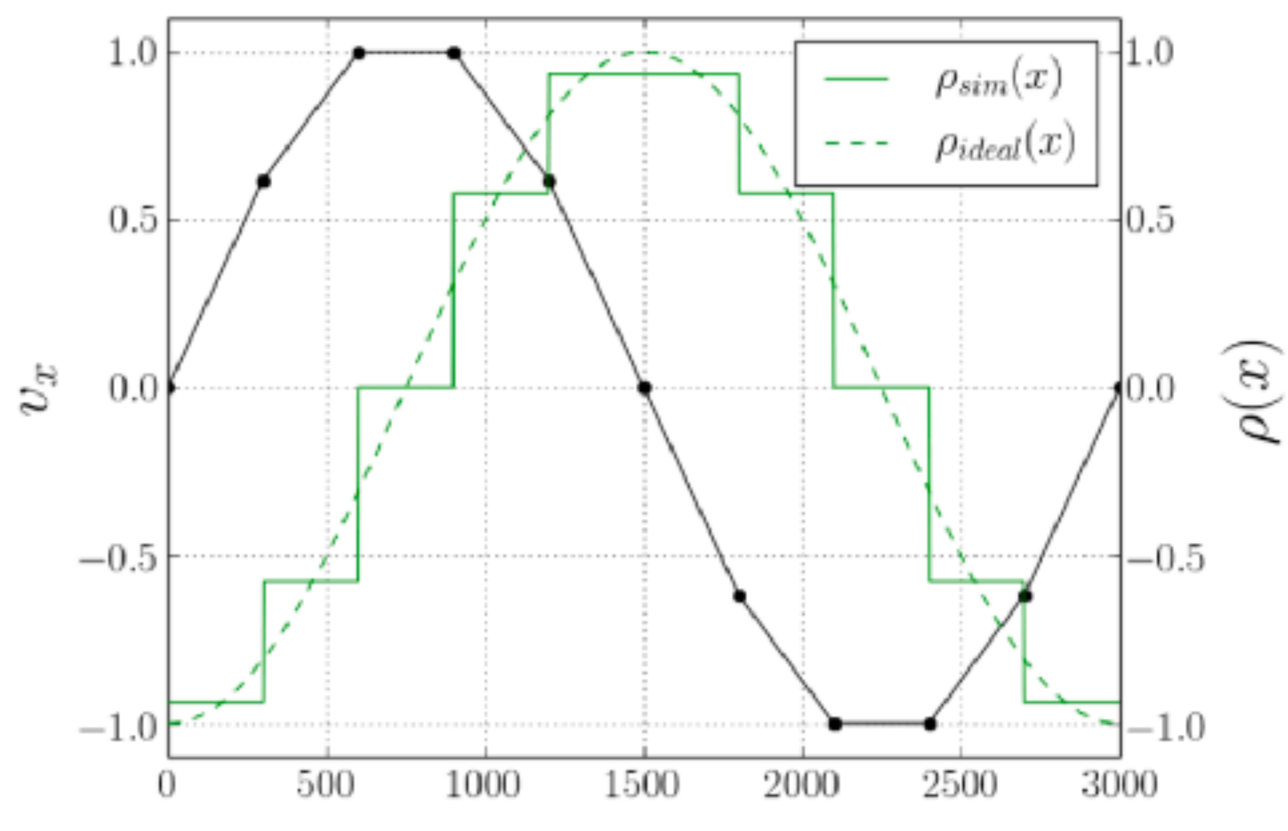
- mostly in collaboration with **Sam Totorica**, Devon Powell, Arka Banerjee, Oliver Hahn, Raul Angulo, Ralf Kähler

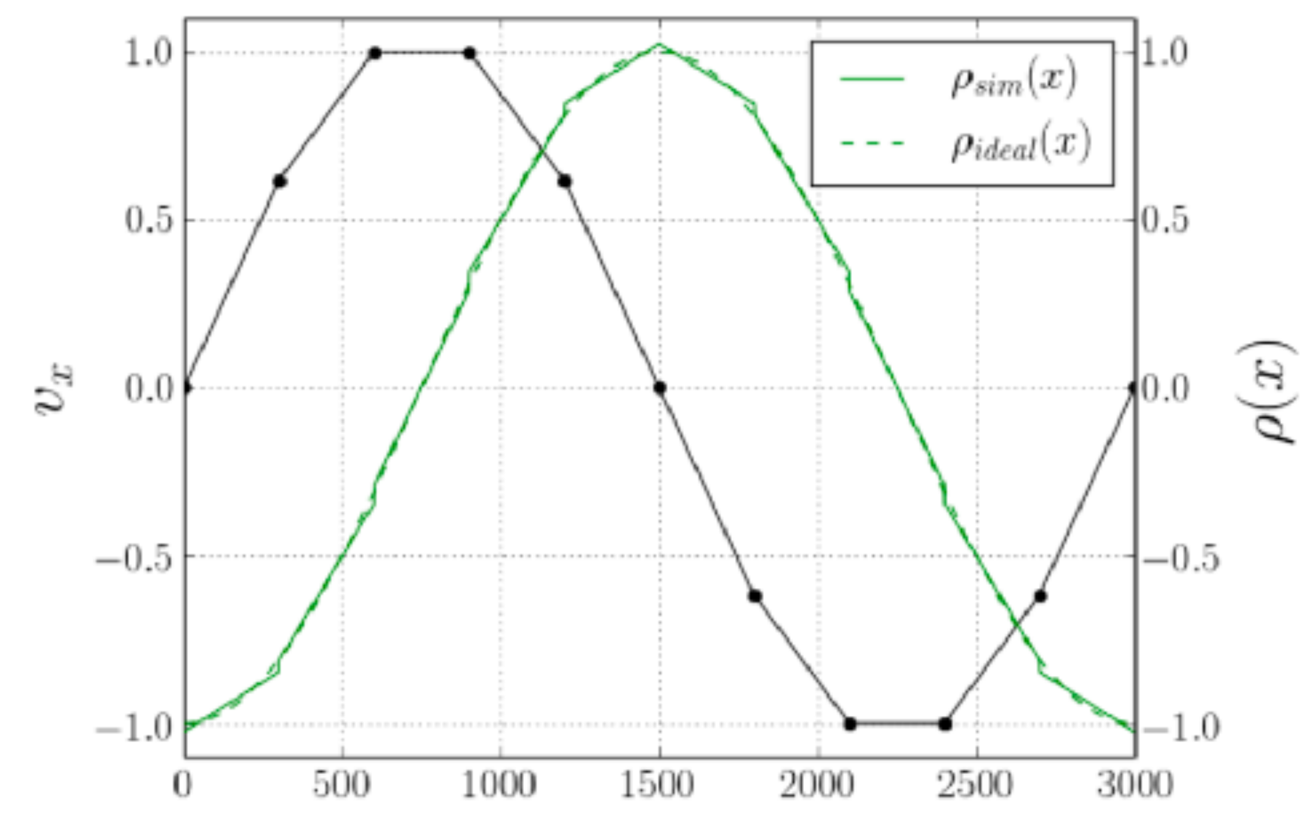
Discuss two papers

- Abel, Hahn, & Kaehler 2012 MNRAS “Tracing the Dark Matter Sheet in Phase Space”
- **SIC method for 1D Vlasov Poisson**
  Kates-Harbeck, Totorica, Zrake & Abel 2016 J.Comp. Phys.
- **Preprint on applications of the phase space sheet method to analyze 2 and 3D Weibel instability Vlasov-Maxwell simulations**
  Totorica, Fiuza, Abel in preparation
- Brief mention of Powell and Abel 2016, J. Comp. Phys.



(a) PIC

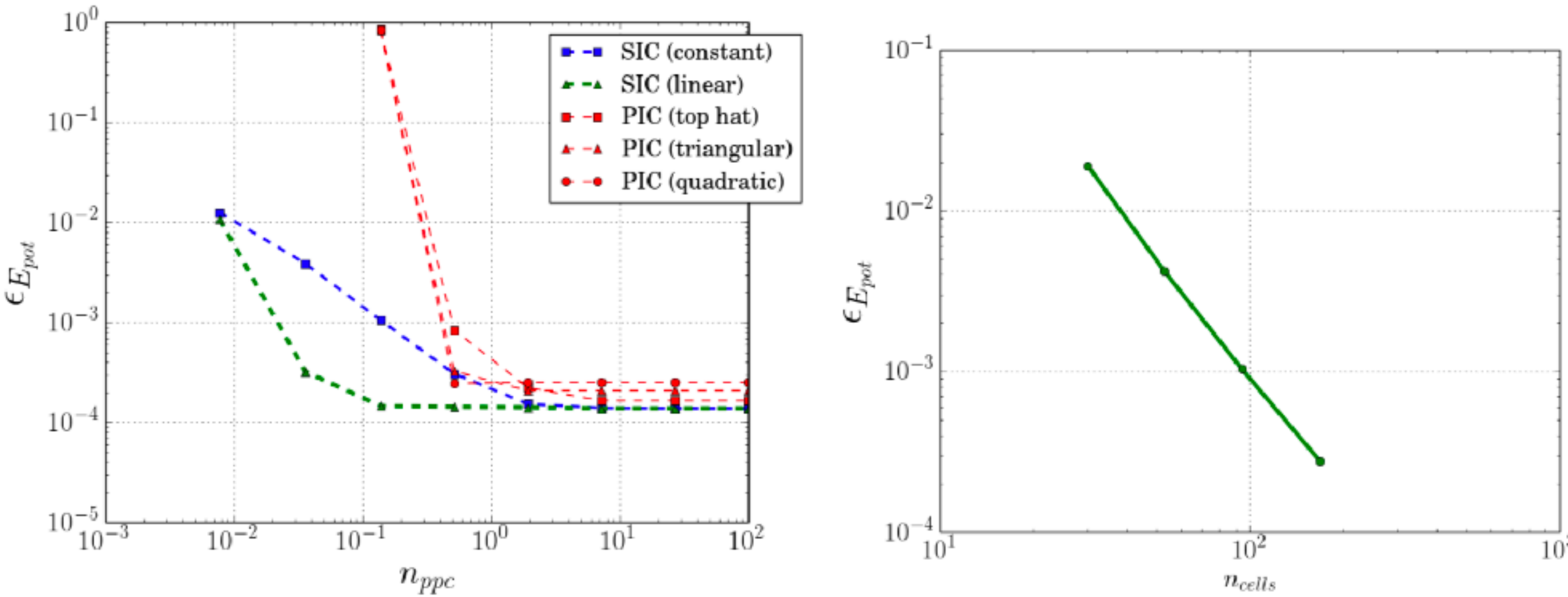




Kates-Harbeck, Totorica, Zrake & Abel 2016 J.Comp. Phys.

(b) SIC: piecewise constant segments

(c) SIC: piecewise linear segments



(a) Error Convergence as a function of n_{ppc} ($n_{cells} = 389$).

(b) Error Convergence as a function of n_{cells} ($n_{ppc} = 100$)

Figure 3: (a) The error in the potential energy evolution $\epsilon_{E_{pot}}$ for the old (PIC) and new (SIC) schemes as a function of particle number per cell. We employ CIC ("top hat"), triangular ("triangular") and "quadratic" (i.e. cubic interpolation) particle shapes for PIC. We compare against SIC with the two cases of piecewise linear and piecewise constant charge densities on the segments. Both PIC and SIC converge to nearly the same "base-line error", set by when the error due to the finite grid spacing becomes dominant. However, SIC more rapidly reaches this baseline, especially with linear segments, which offer higher order convergence. Higher order particle shapes for PIC offer slightly lower errors than standard CIC ("top hat") but converge to a higher baseline error due to their larger size. In (b), we show the common baseline error reached by PIC and SIC as a function of the number of cells. The error was measured as the mean L_1 difference over one plasma period between $\tilde{E}_{\rm pot}(t)$ as obtained from the simulation and the analytical solution $E_{\rm pot}(t) = \sin^2(\frac{2\pi t}{T_p})E_{tot}$, where T_p is the analytical value for the plasma period.

Kates-Harbeck, Totorica, Zrake & Abel 2016 J.Comp. Phys.

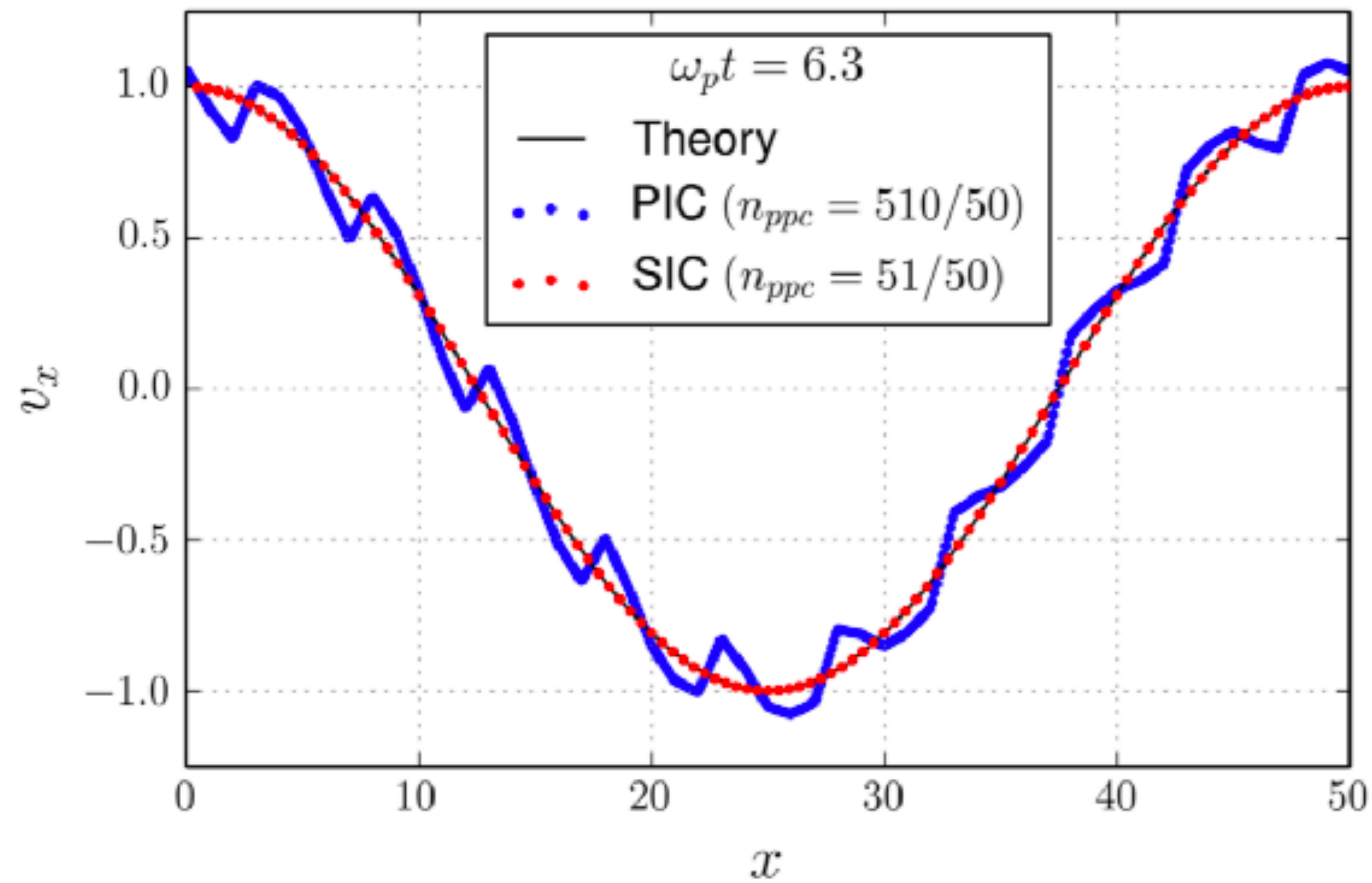
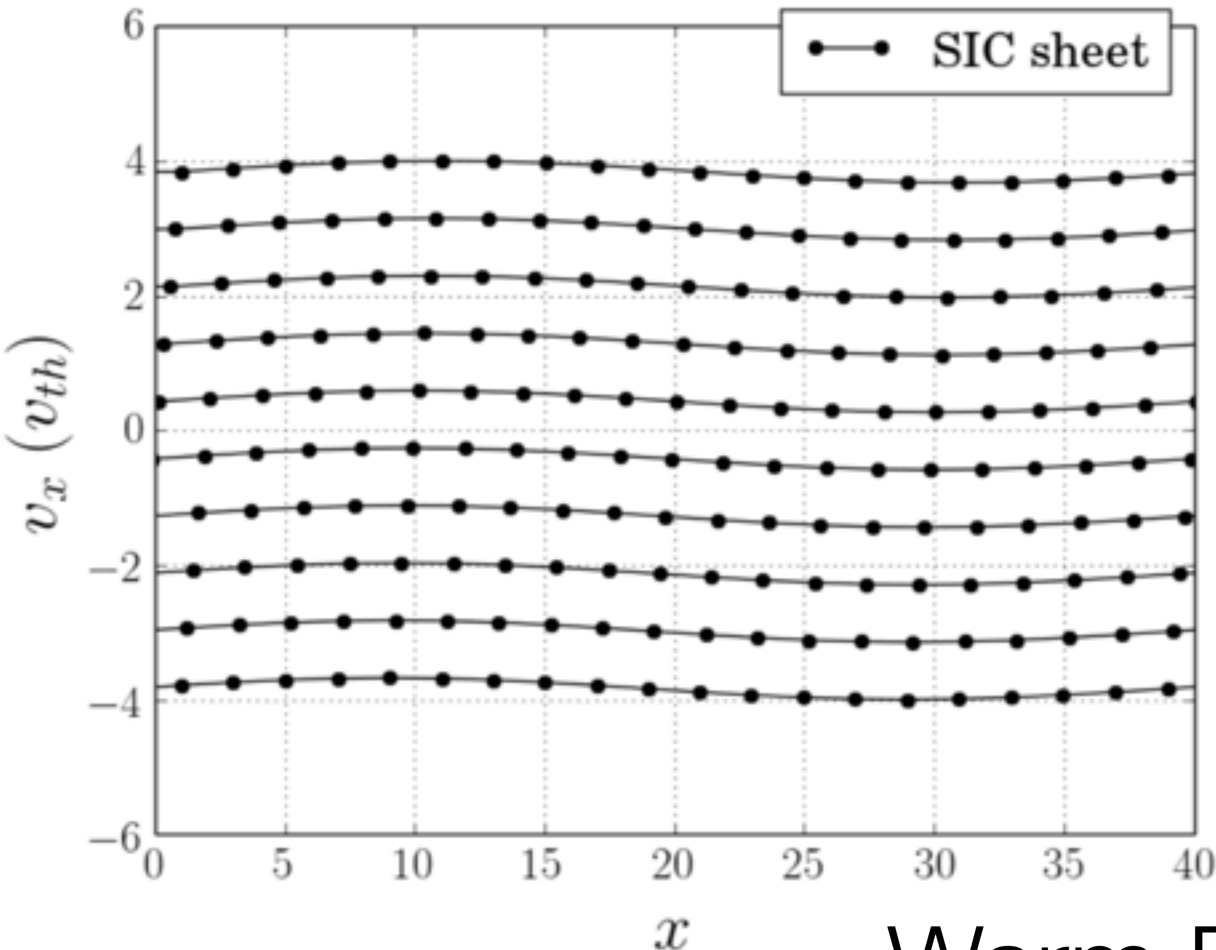
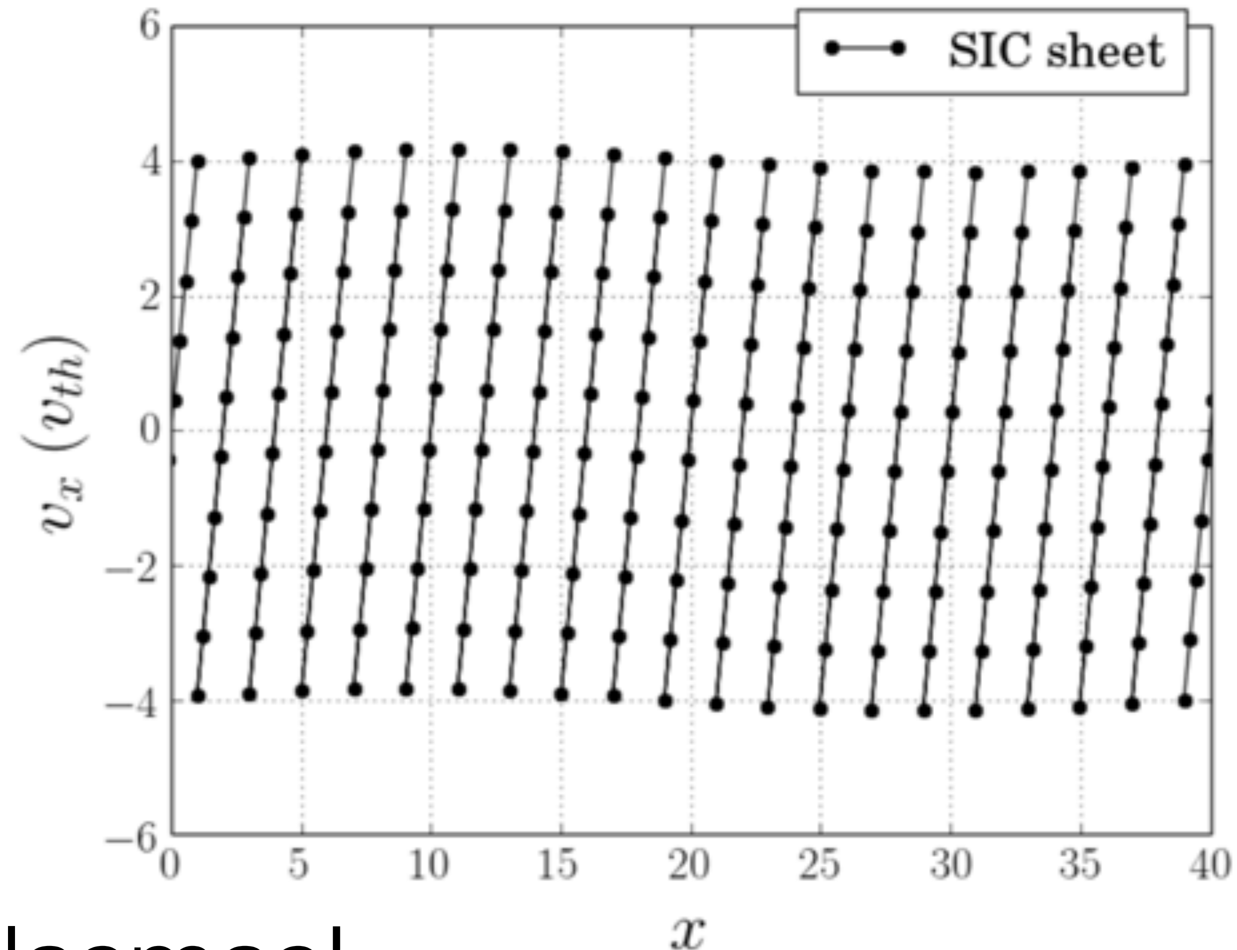


Figure 4: Comparison of phase-space for PIC with linear particles ("top hat" shape) and SIC with constant segments after one plasma oscillation period. The velocity scale is in units of the initial velocity perturbation amplitude v_1. The grid induced noise seen when using PIC is absent in the simulation performed with SIC, even with 1/10 the number of particles.

Kates-Harbeck, Totorica, Zrake & Abel 2016 J.Comp. Phys.

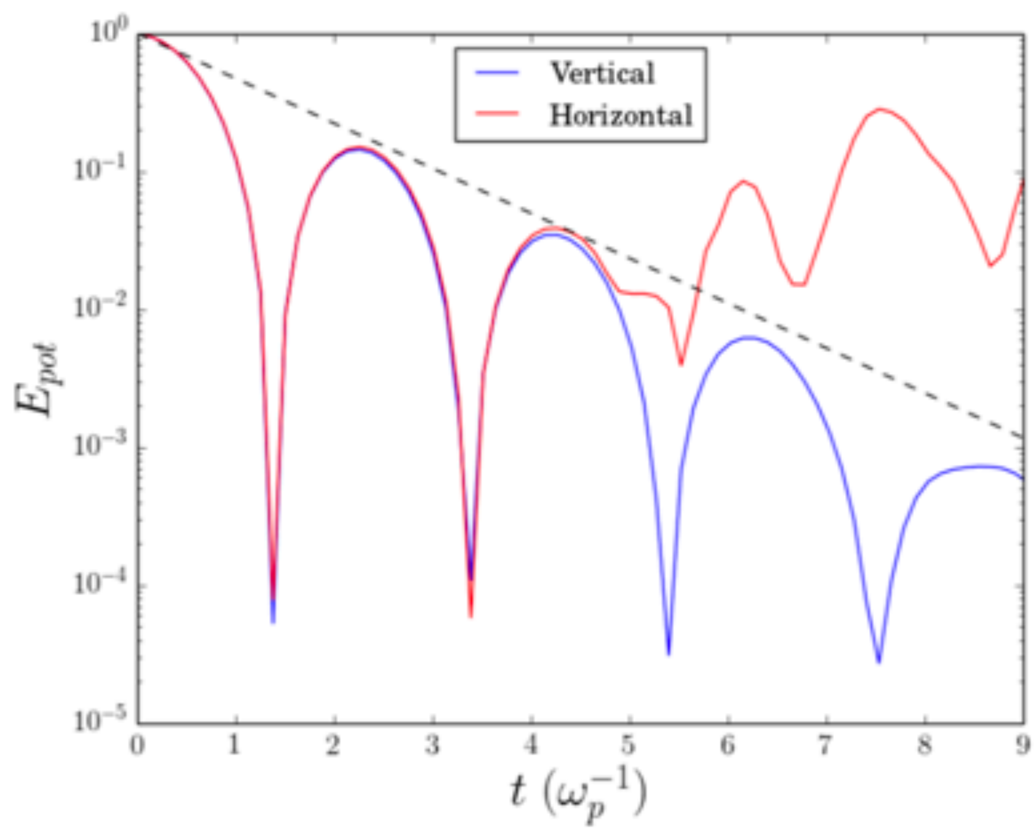


(a) horizontal streams

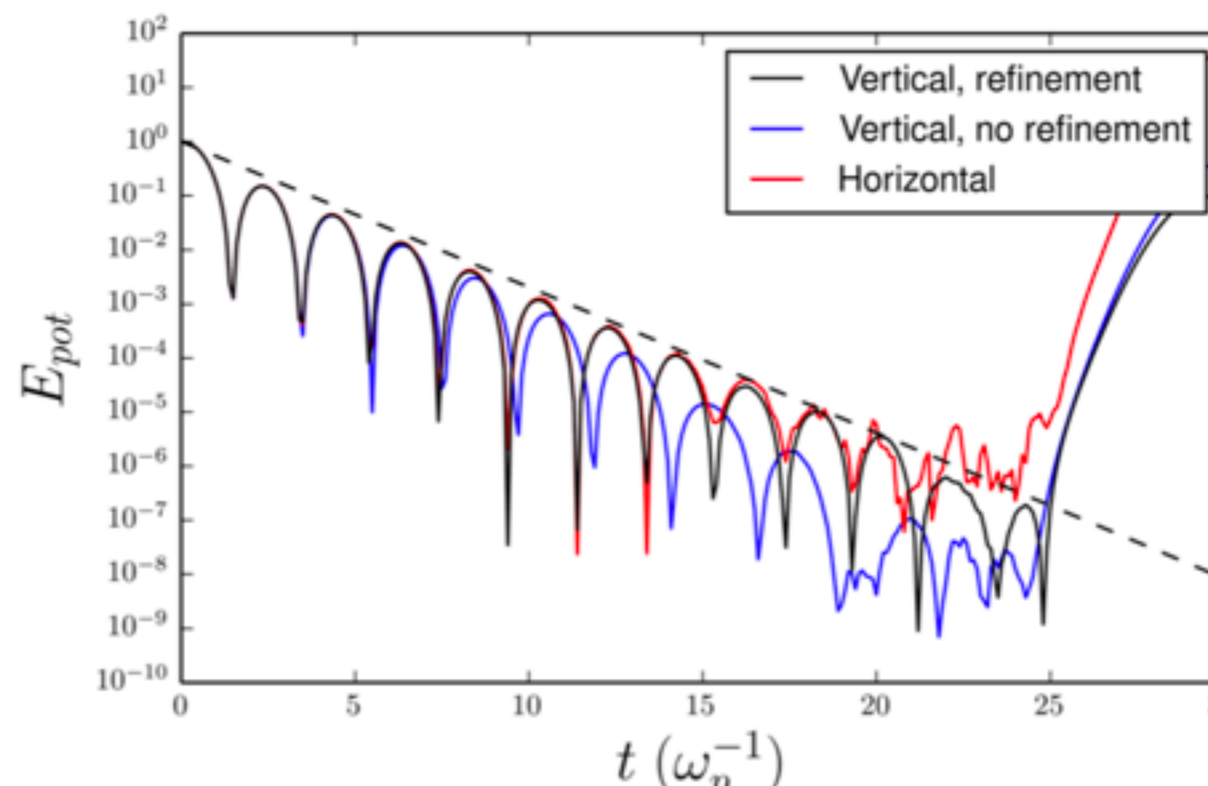


(b) vertical streams

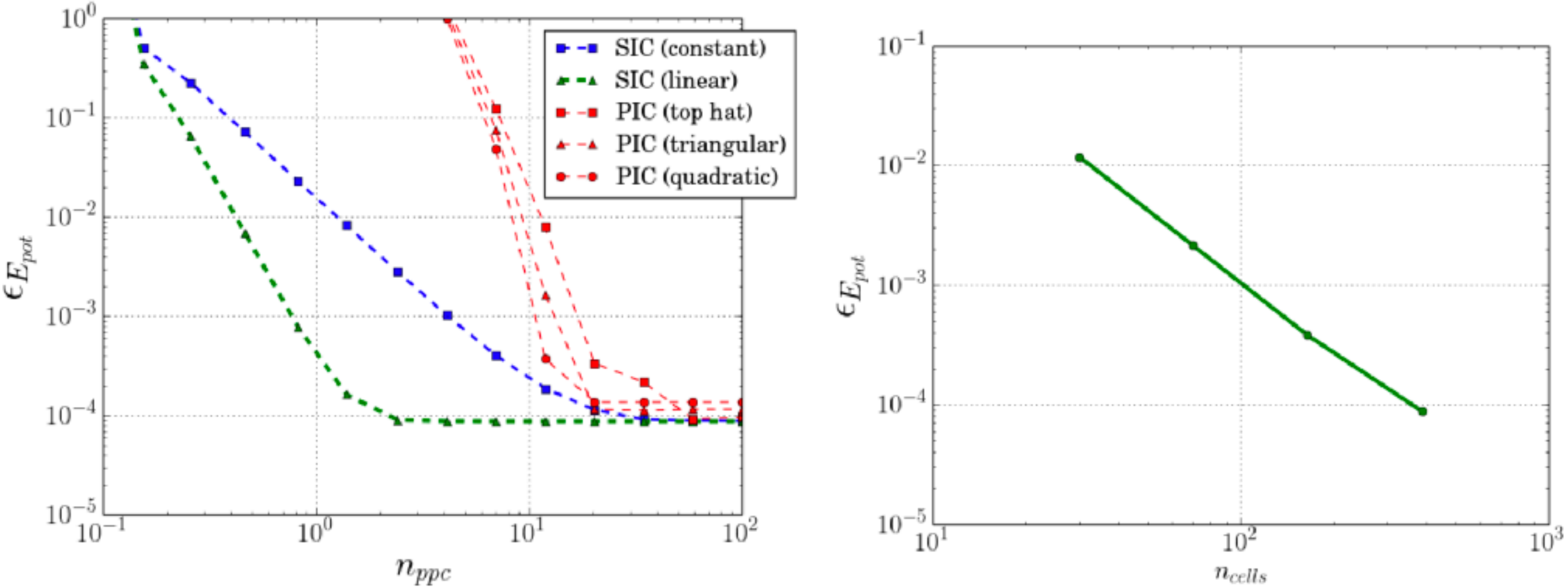
Warm Plasmas!



(a) Nonlinear Damping



(b) Linear Damping



(a) Error Convergence as a function of n_{ppc} ($n_{cells} = 389$)

(b) Error Convergence as a function of n_{cells} ($n_{ppc} = 100$)

Figure 7: (a) Error convergence comparing the SIC method to the standard PIC method in the Landau damping problem. The figure has the same style as figure 3. Note how the new scheme converges faster, requiring far fewer particles than the old scheme for equivalent errors. Note also the increase in convergence order when moving from piecewise constant to piecewise linear segments. A minimum error is reached by all schemes as in figure 3 due to the discretization of the simulation region into cells. As the number of cells is increased, this “baseline error” decreases. The baseline error is plotted in (b) as a function of the number of cells. As the number of cells is increased further, the error again reaches a minimum (around $n_{cells} \sim 750$) due to the finite number of streams and the finite timestep. The convergence was obtained by comparing the evolution of $E_{\rm pot}(t)$ during the simulation to a “gold” run with very high particle number. The error represents the mean L_1 difference between $E_{\rm pot}(t)$ during a given run and $E_{\rm pot}(t)$ for the gold run. In this case, the gold run was performed with the old (PIC) scheme. $M = 20$ streams were used. These results show that in this problem SIC can outperform PIC in accuracy with far fewer resources.

2D Weibel instability

- Test problem:
  - 2 cold, counter-propagating electron species
  - Weibel seeded in the magnetic field
  - 7 simulations scanning over particles-per cell
    - PPC = 1, 4, 16, 64, 256, 1024, 4096, 65536
- Compare physical quantities calculated using standard method and new method
  - Deposit triangles using PSI (Devon Powell)

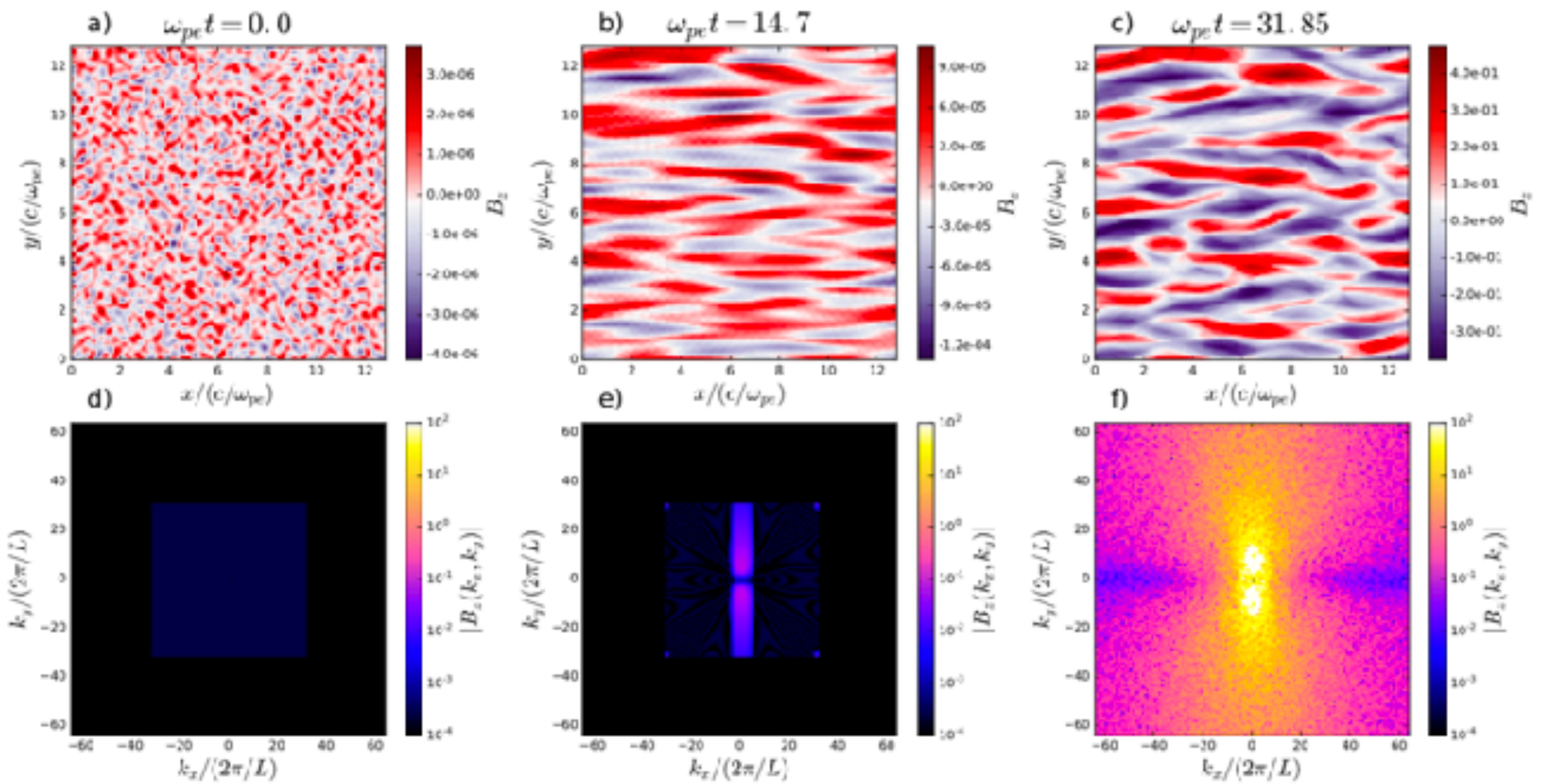
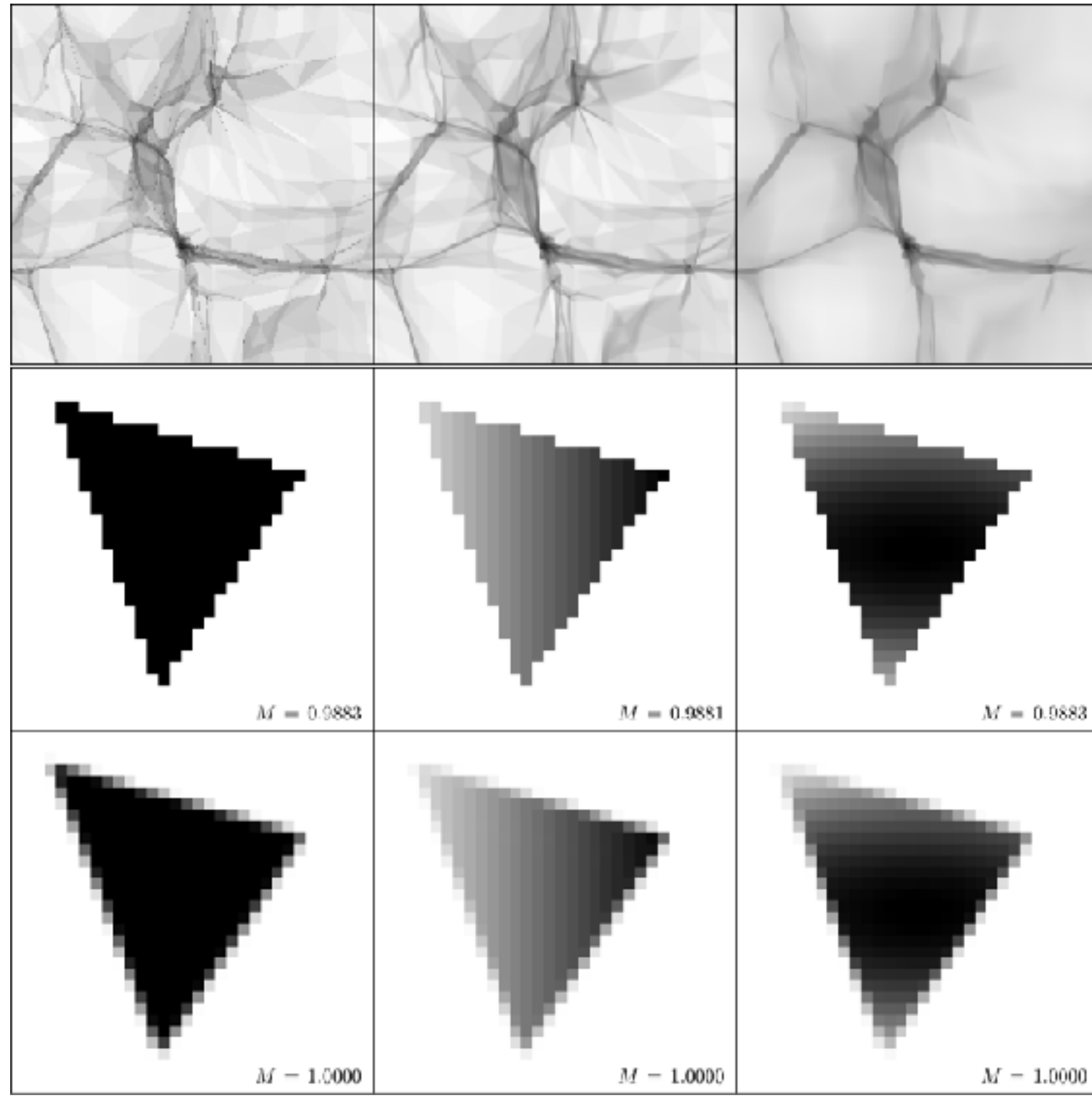


Figure 5: Out-of-plane magnetic field B_z at (a) the beginning of the simulation, (b) during the linear growth of
and (c) at the time of Weibel saturation. (d)-(f) The amplitudes of the discrete Fourier transforms of the fields sh

Totorica, Fiuza, Abel 2017 J.Comp. Phys. in prep

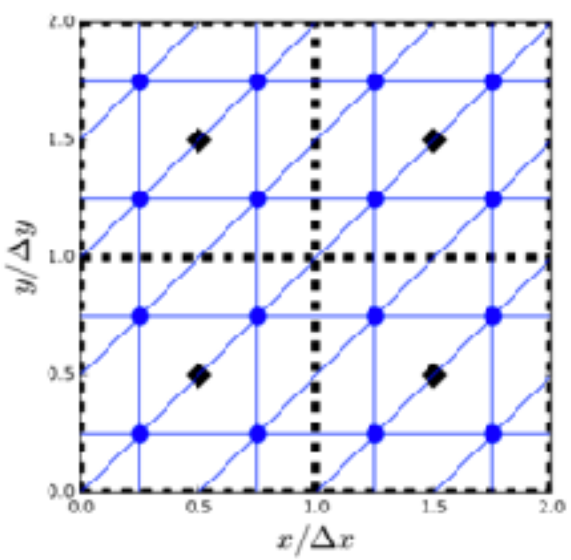
EXACT OVERLAP INTEGRALS. USEFUL BLACK BOX?

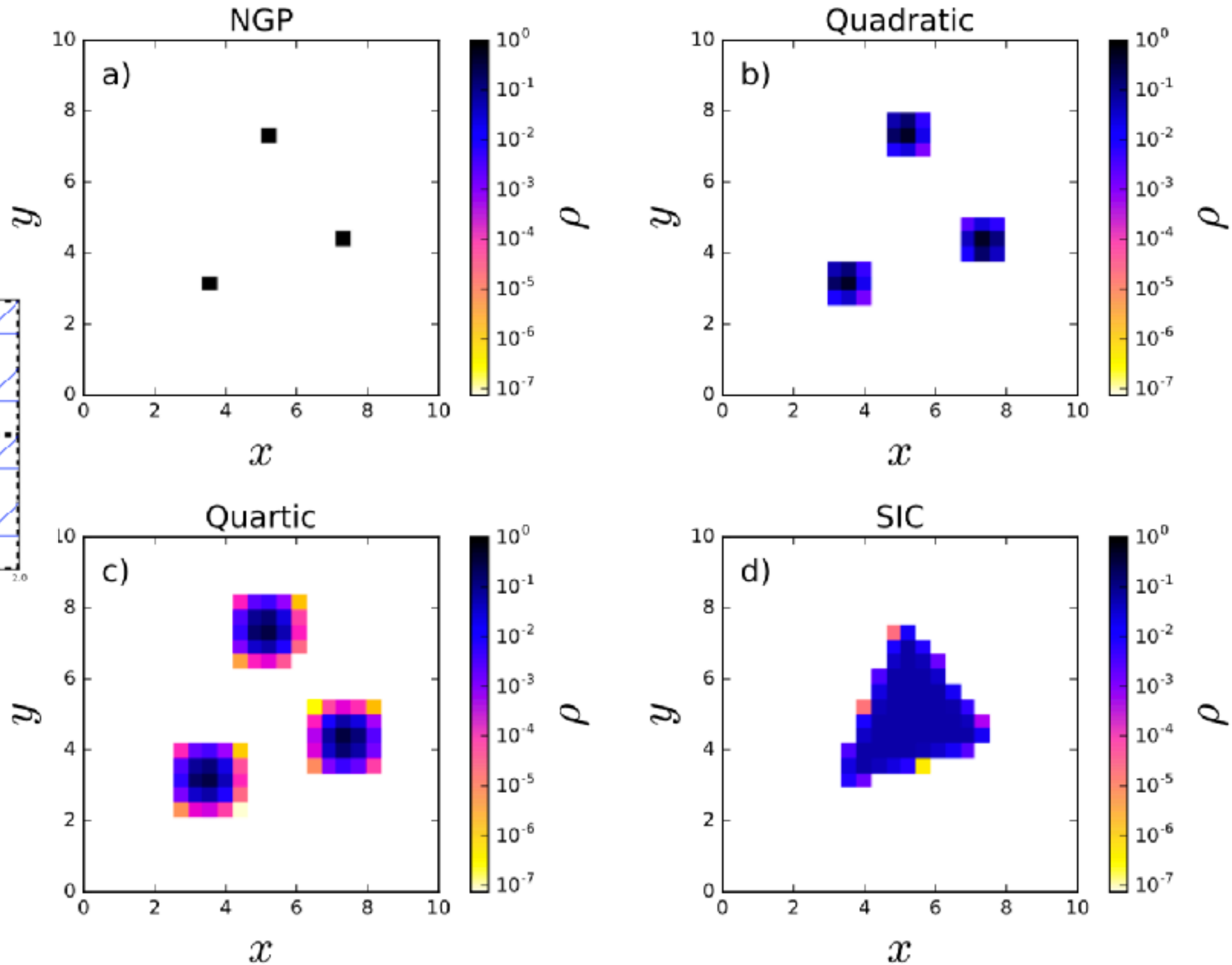
- To write new cosmology and plasma codes exploiting cold three dimensional manifolds in 6D phase space one needs to learn how to robustly calculate overlap integrals of Polyhedra.
- This leads to exact and robust **remeshing** techniques which likely much more generally useful.
- Developed in N-dimensions with N-th order polynomial functions
- Using these ideas to do beam tracing for radiation transport.
- Enables waterbags in higher dimensions.
- New hydro methods? Already being used in ALE codes at Sandia



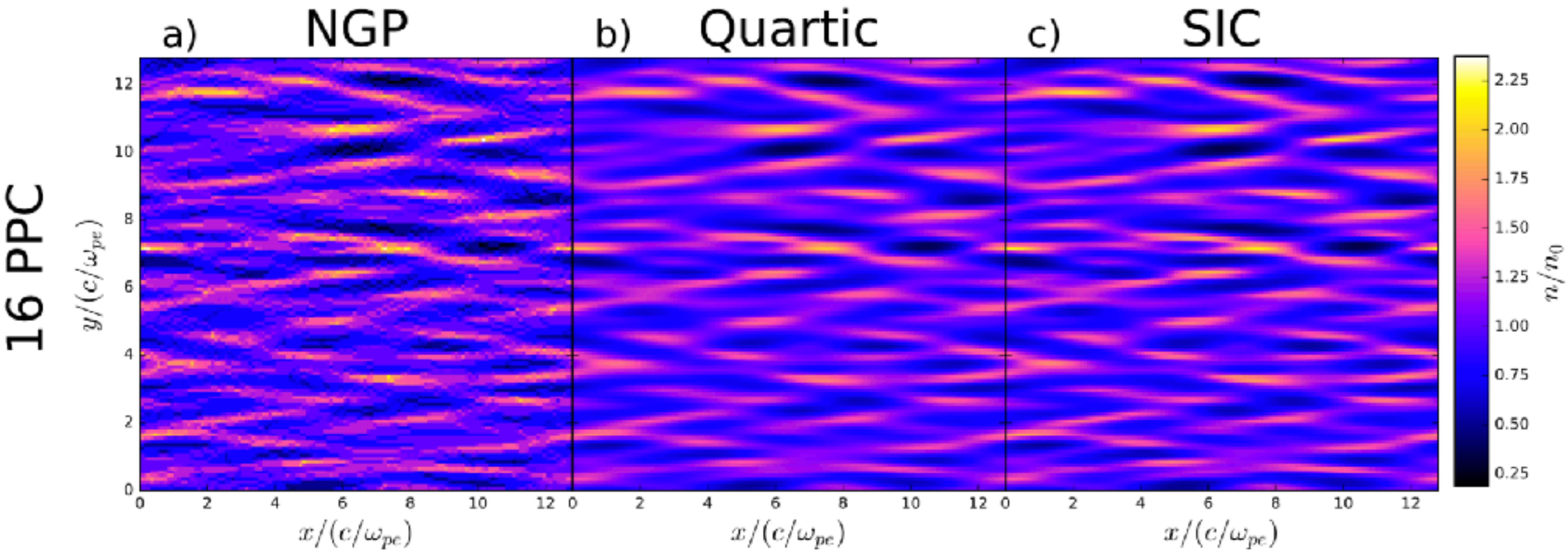
Powell *& Abel 2016. J. Comp. Phys.*

2D SIMPLEX: TRIANGLE



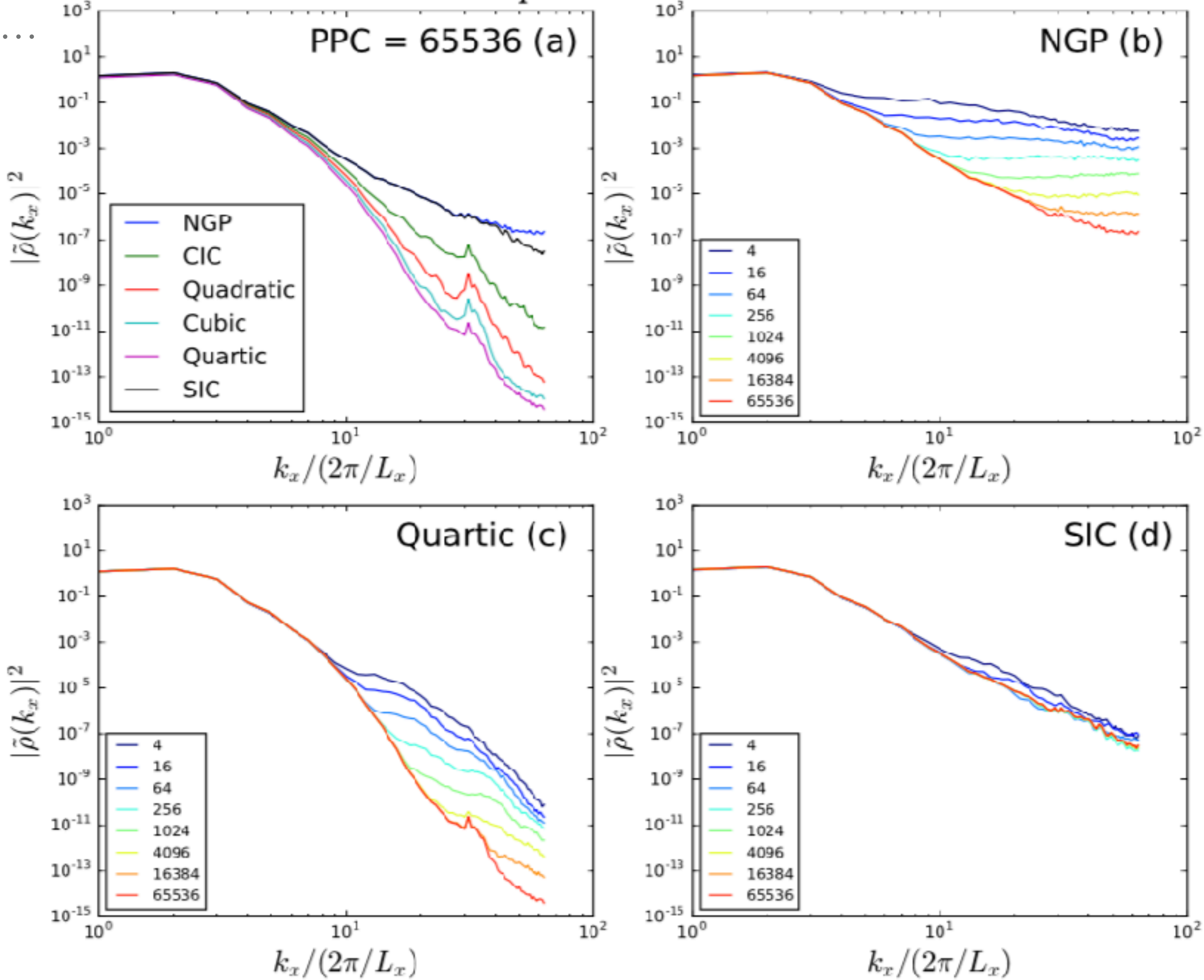


CHARGE DENSITY

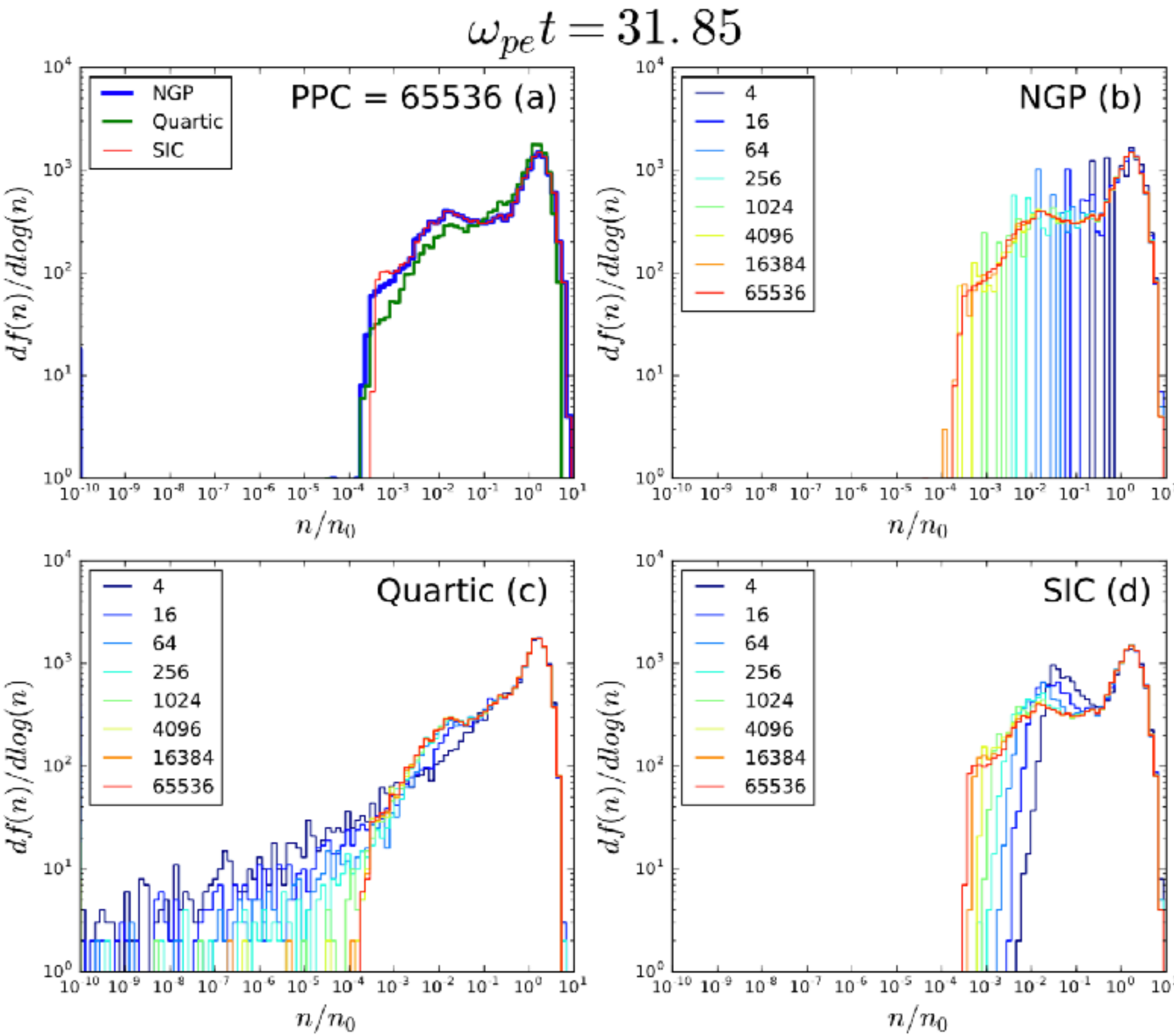


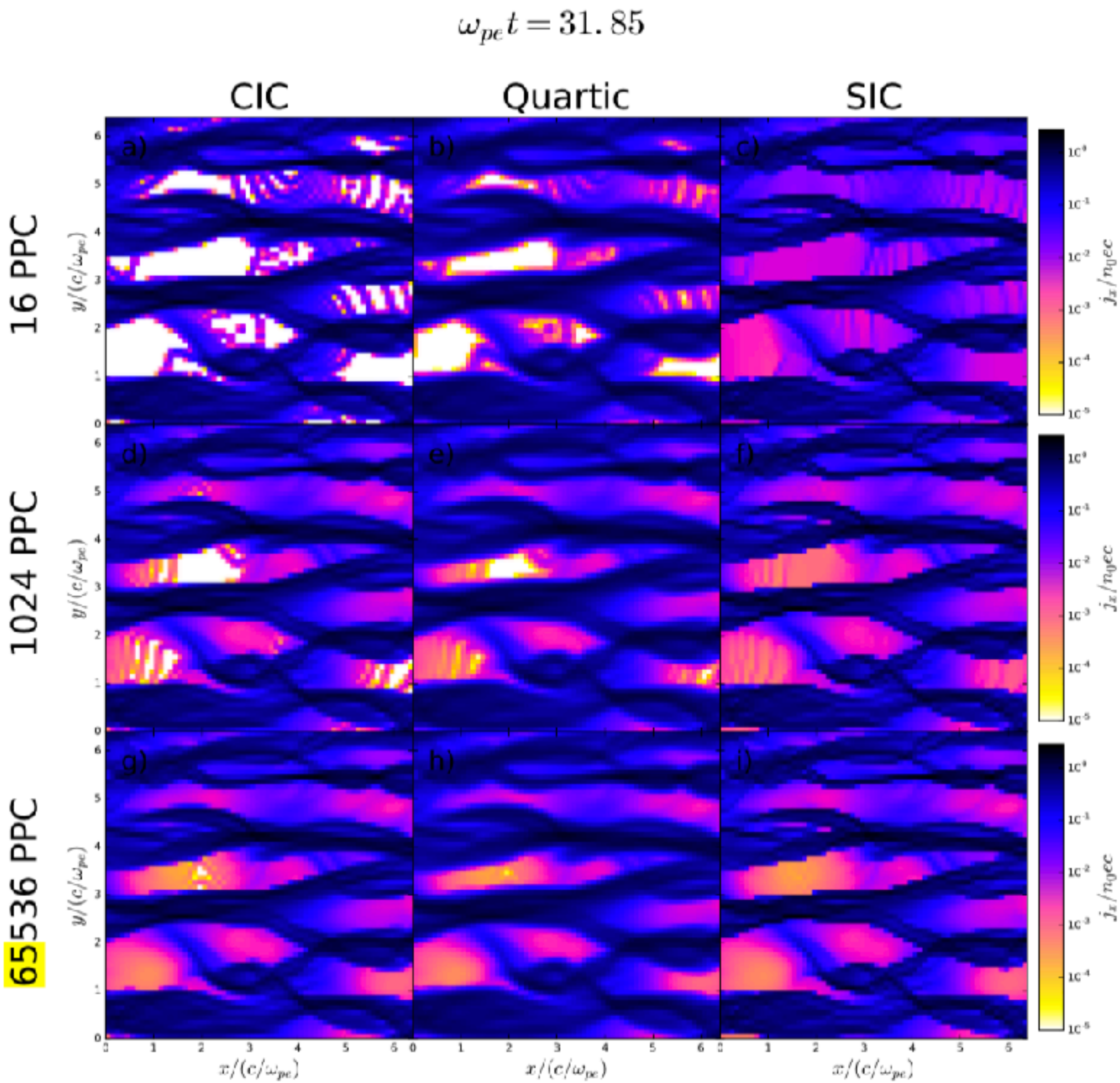
1D LONGITUDINAL POWERSPECTRA

$$\omega_{pe} t = 26.95$$



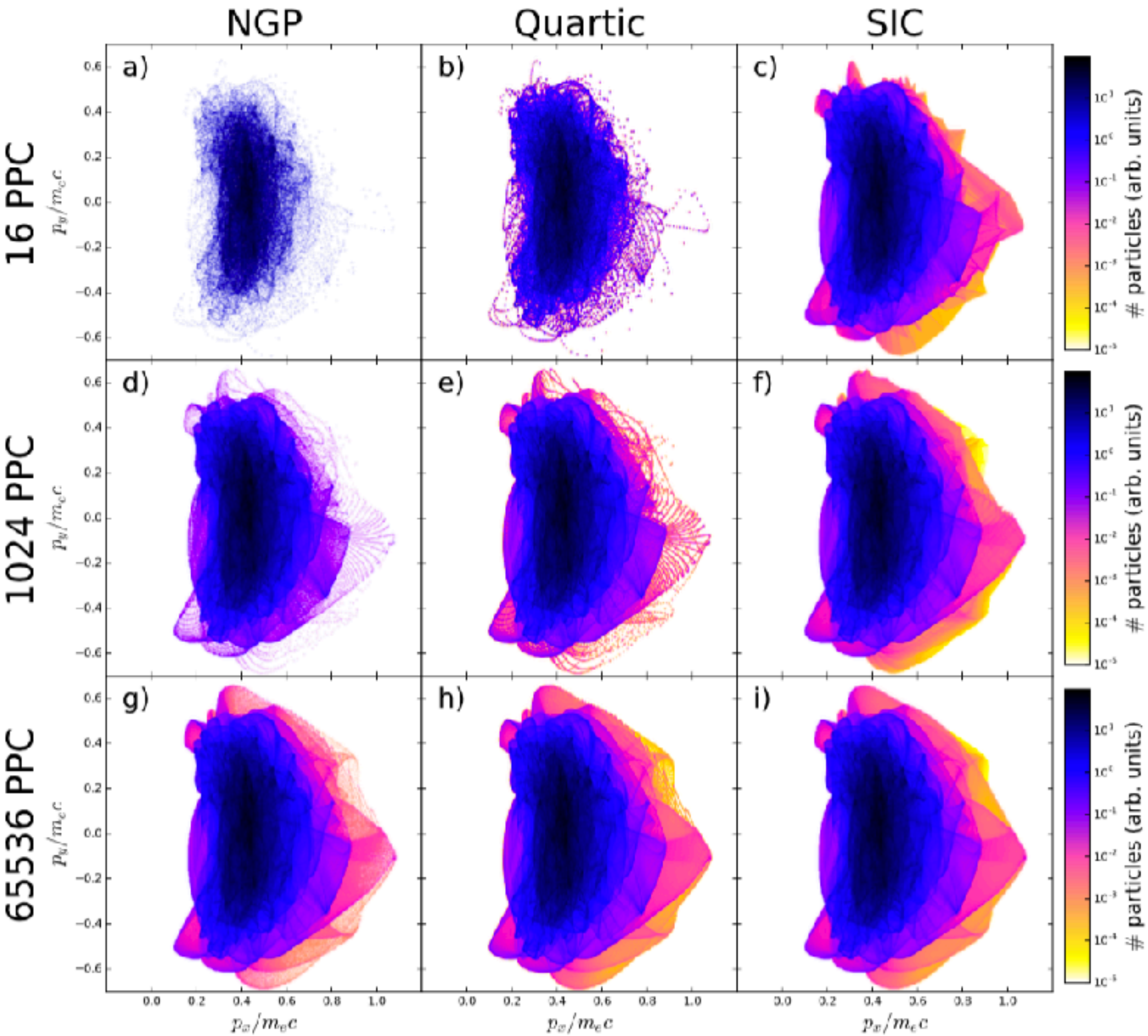
DENSITY PDF

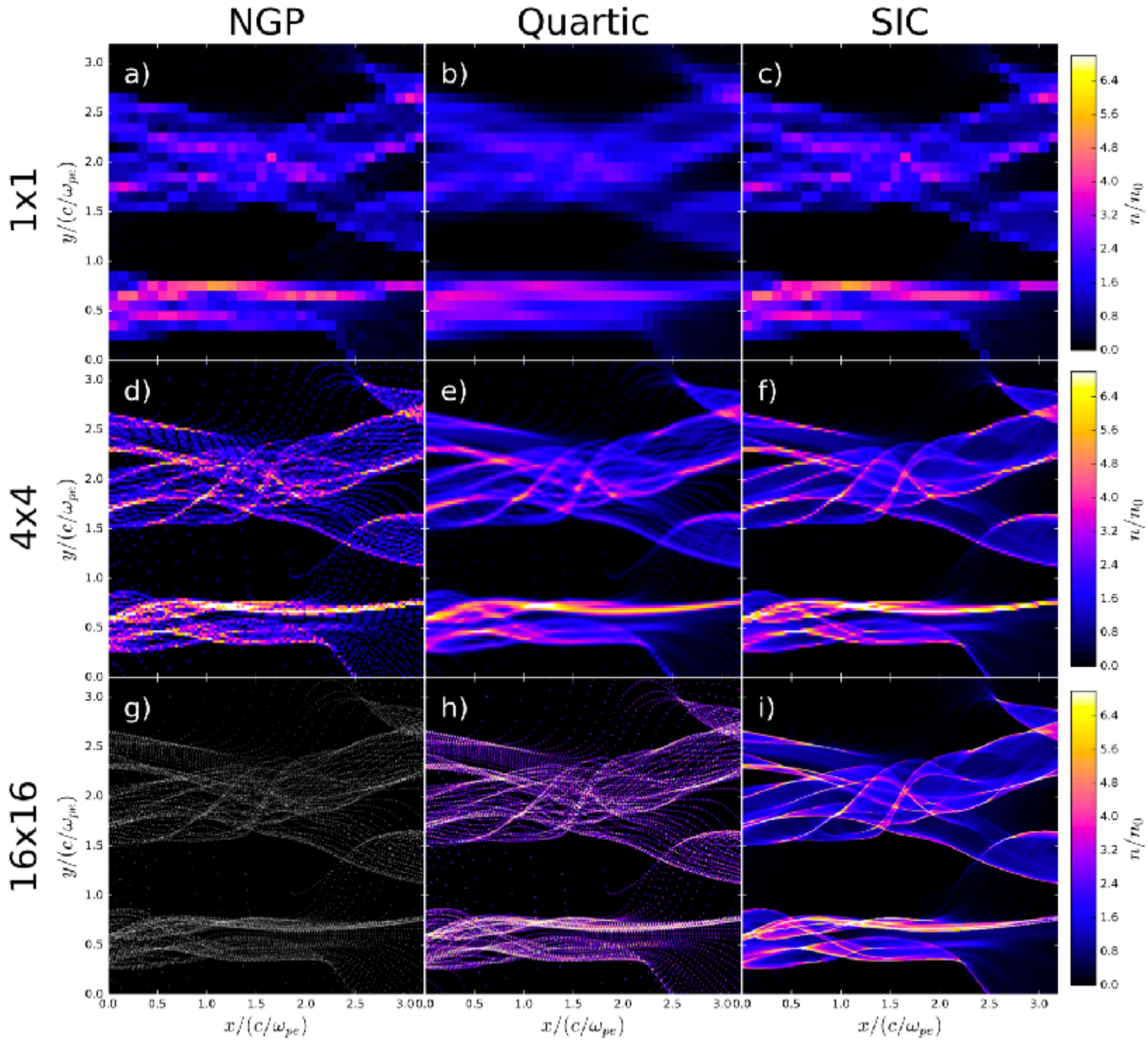

$\omega_{pe} t = 31.85$
PPC = 65536 (a)
NGP
Quartic
SIC
NGP (b)
Quartic (c)
SIC (d)
4
16
64
256
1024
4096
16384
65536
$df(n)/dlog(n)$
n/n_0

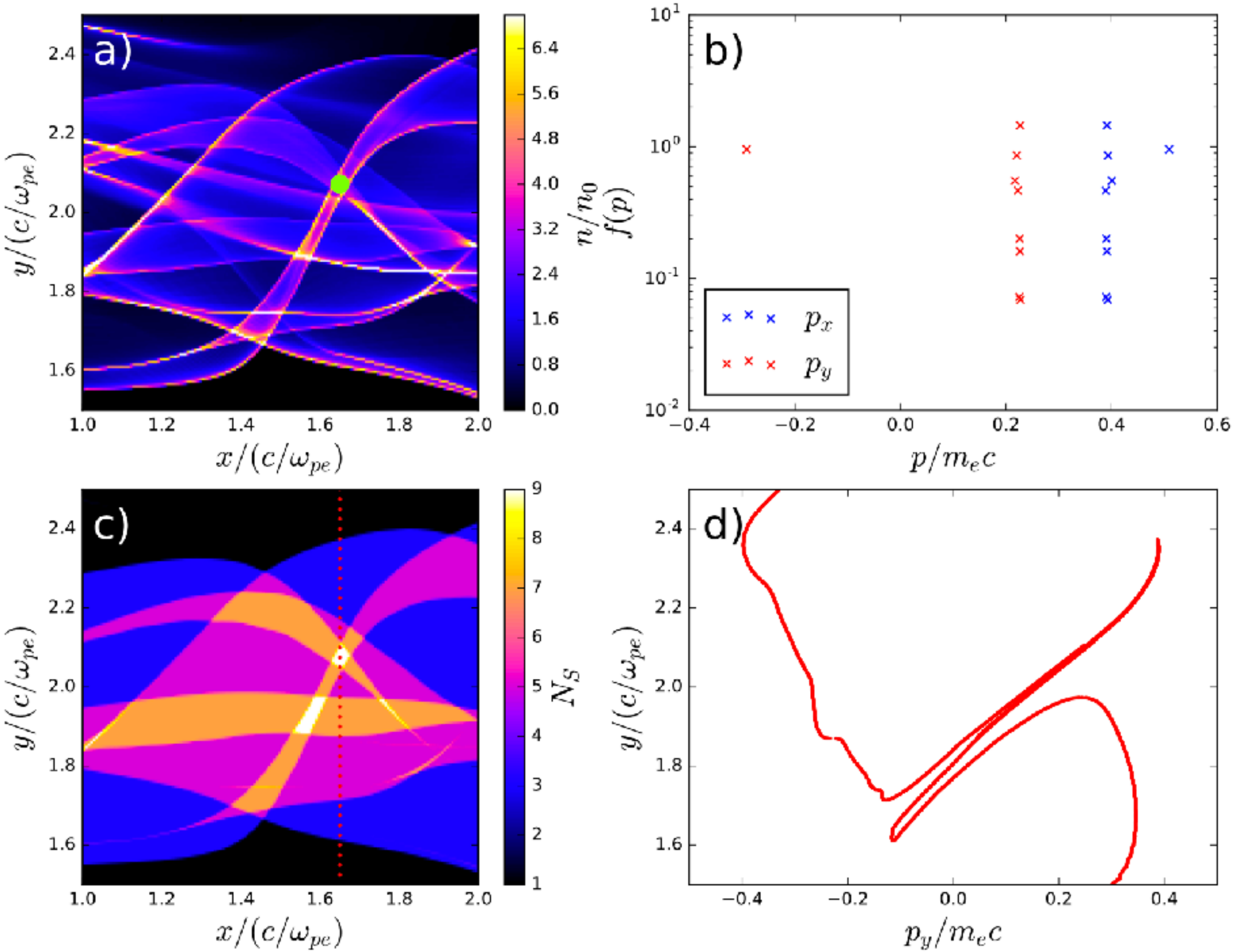


$\omega_{pe} t = 31.85$
CIC
Quartic
SIC
16 PPC
1024 PPC
65536 PPC
a)
b)
c)
d)
e)
f)
g)
h)
i)
$y/(c/\omega_{pe})$
$x/(c/\omega_{pe})$
j_x/n_0ec

PHASE SPACE CONVERGENCE

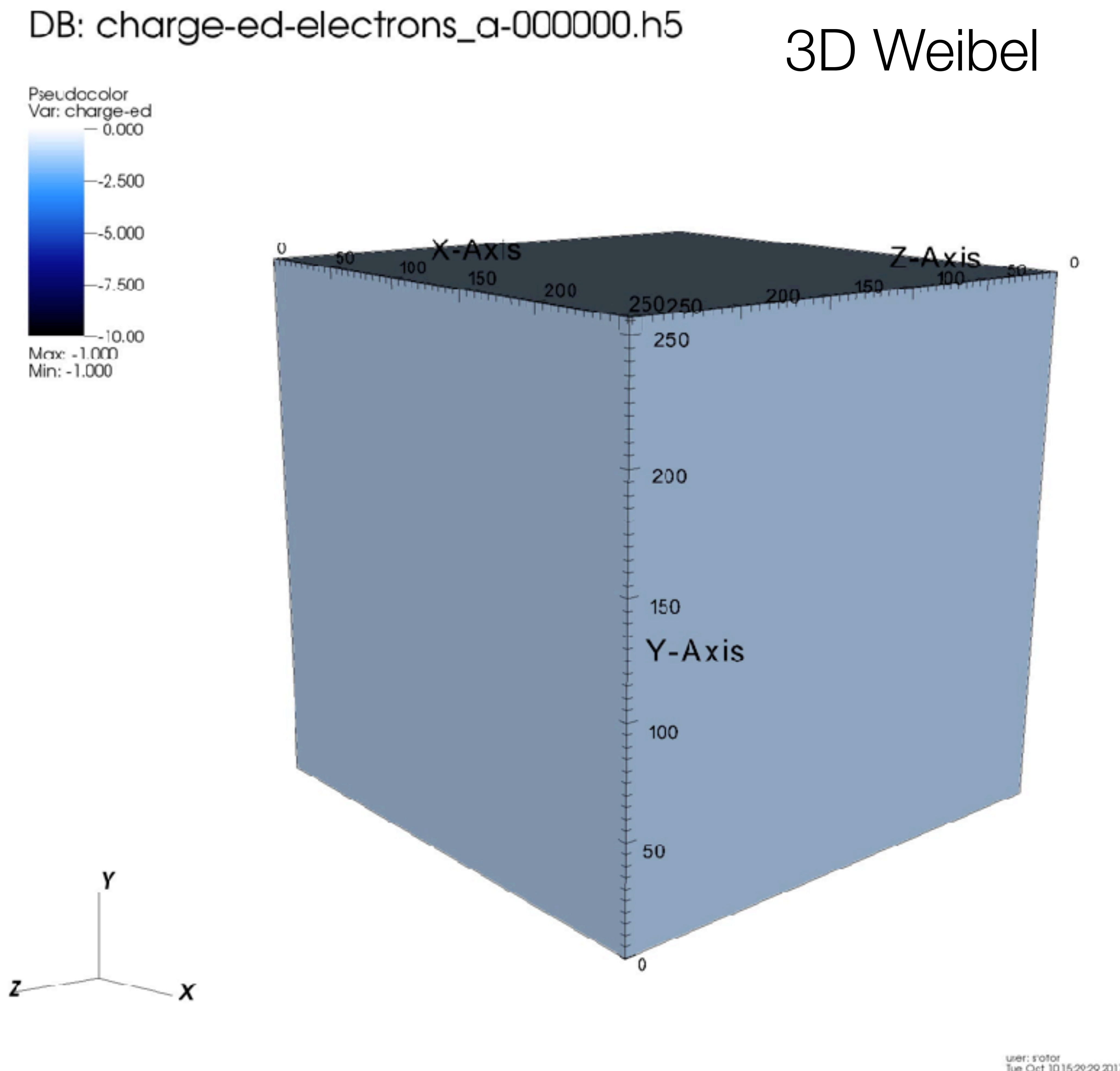



NGP
Quartic
SIC
1x1
4x4
16x16
a)
b)
c)
d)
e)
f)
g)
h)
i)
$y/(c/\omega_{pe})$
$x/(c/\omega_{pe})$
n/n_0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0.8
1.6
2.4
3.2
4.0
4.8
5.6
6.4



a)
b)
c)
d)
$y/(c/\omega_{pe})$
$x/(c/\omega_{pe})$
n/n_0
$f(p)$
p/m_ec
p_x
p_y
N_S
p_y/m_ec

3D Weibel



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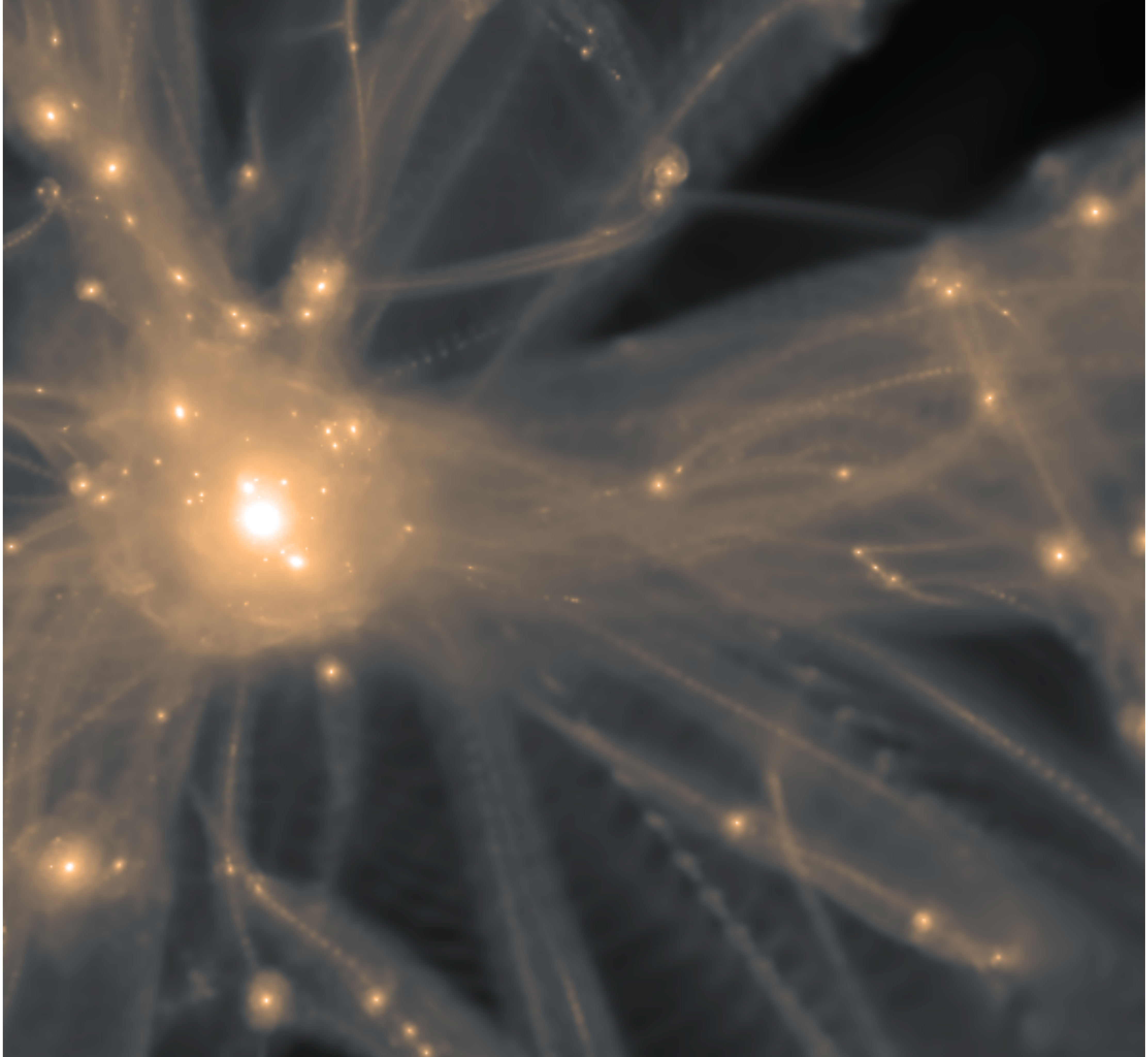
X-Axis
Y-Axis
Z-Axis
0
50
100
150
200
250
Y
Z
X
Tue Oct 10 15:29:29 2017

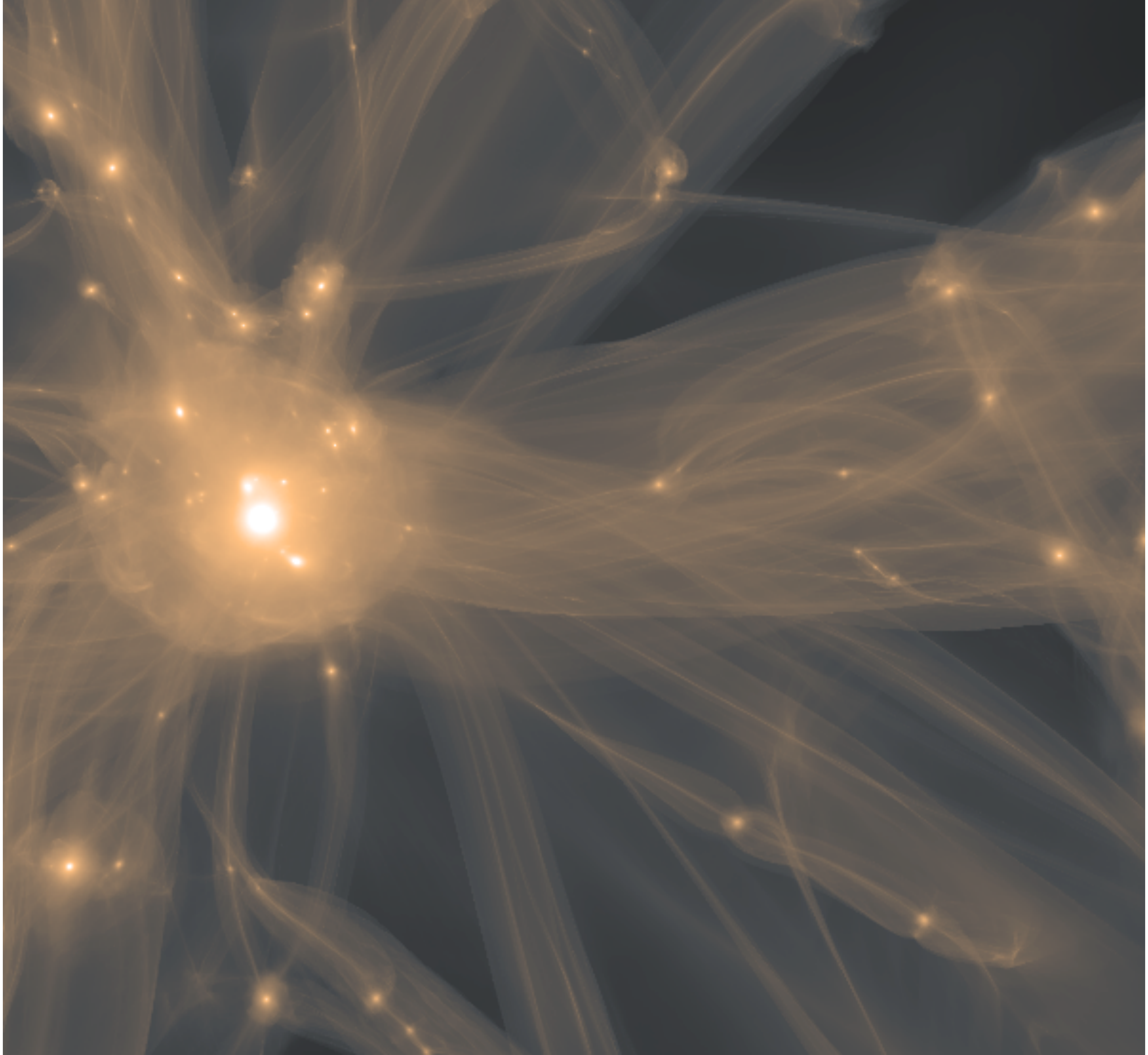
Lagrangian Tessellation: What's it good for? Not complete!

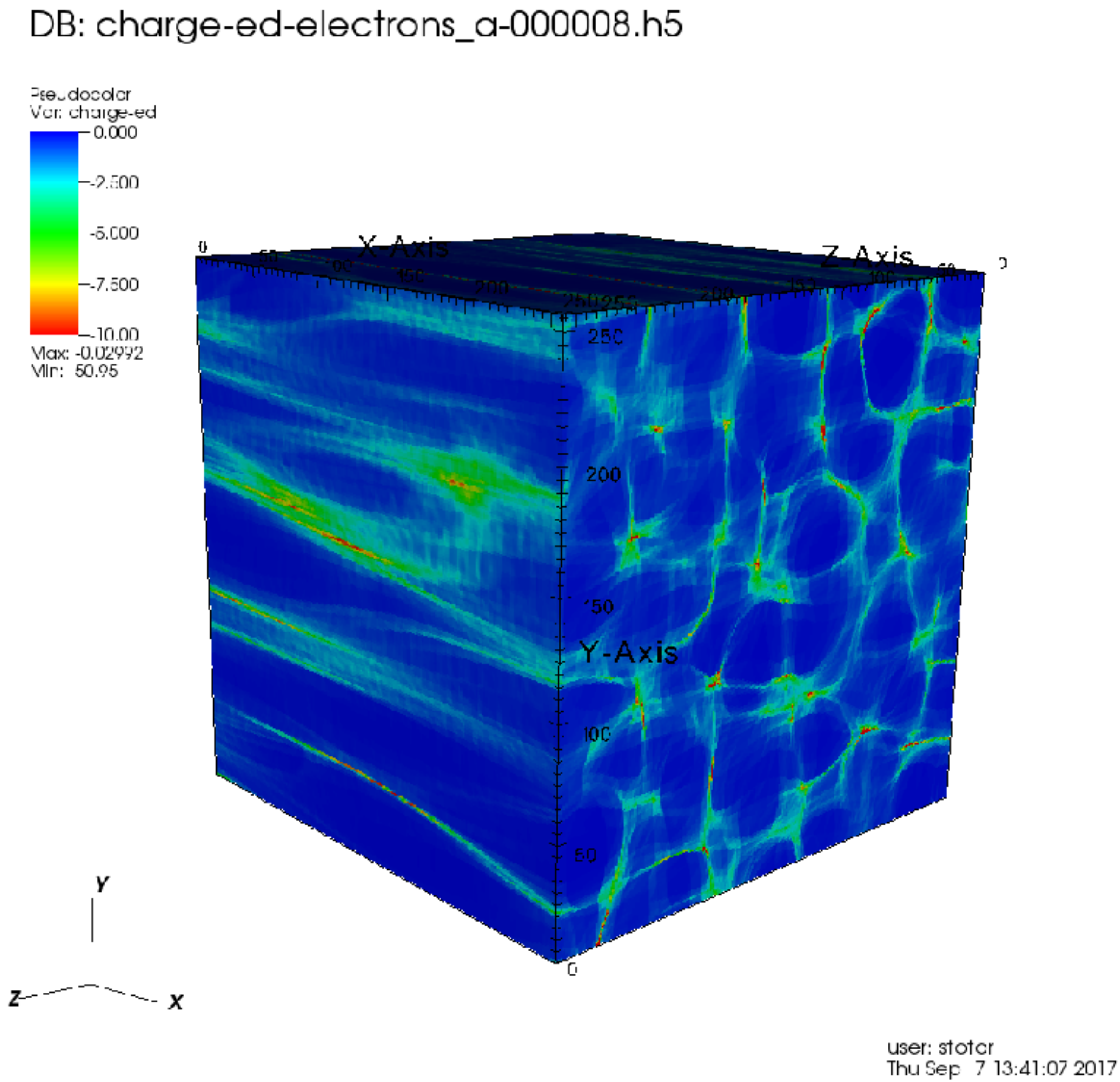
- Analyzing N-body sims, including web classification, velocity dispersion, profiles, resolution study (Abel, Hahn, Kaehler 2012)
- DM visualization (Kaehler, Hahn, Abel 2012)
- Better Numerical Methods (Hahn, Abel & Kaehler 2013, Hahn, Angulo & Abel 2014-, Angulo and Hahn 2016, Sousbie & Colombi 2016)
- Finally reliable WDM mass functions below the cutoff scale (Angulo, Hahn, Abel 2013)
- Gravitational Lensing predictions (Angulo, Chen, Hilbert & Abel 2014
- Cosmic Velocity fields (Hahn, Angulo, Abel 2014)The SIC method for Plasma simulations (Vlasov/ Poisson) (Kates-Harbeck, Totorica, Zrake & Abel 2016, JCompPhys)
- Exact overlap integrals of Polyhedra (Powell & Abel 2015 JCompPhys)
- Void profiles, Wojtak, Powell, Abel 2016 MNRAS
- Stücker, Busch, & White 2017: Median density of the Universe
- East, Wojtak, Abel compare numerical GR and Newtonian cosmology 2017 in prep
- Powell & Abel 2017, Beam Tracing for radiation transport
- Powell, Banerjee, et al: DM annihilation, Neutrino phase streaming
- Totorica, et. al. Weibel instabilities, shocks, particle acceleration in PIC simulations in prep.
- Kopp et al (tomorrow)
- your application here …

SUMMARY

- Methods developed for collision-less particles in cosmology are of clear value to improve on PIC simulations in plasma physics
- Some warm plasmas can be modeled efficiently with a collection of cold sheets
- 1D Vlasov Poisson Simplex in Cell method requires 10-40 times fewer particles for same accuracy in linear and mildly non-linear problems
- 2D analysis of Vlasov-Maxwell Weibel instability simulations suggests that important features are converged with $\sim$3000 fewer particles.
- Very interesting possibility that in 3D some problems may be studied with more than 10,000 times fewer computational resources at the same accuracy and inherently more information.
- We have a working 1D Vlasov-Maxwell SIC code. 2D next.



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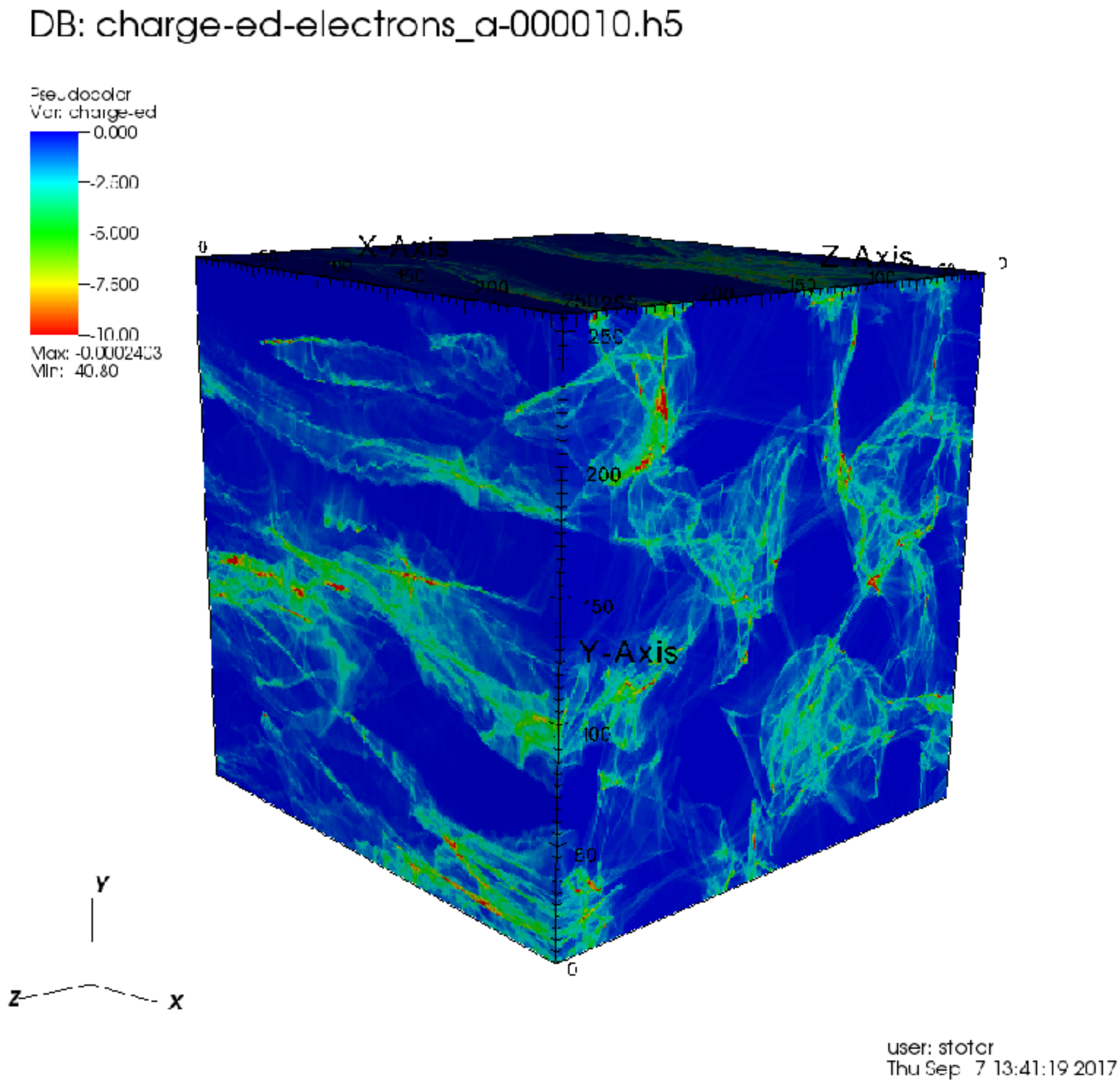

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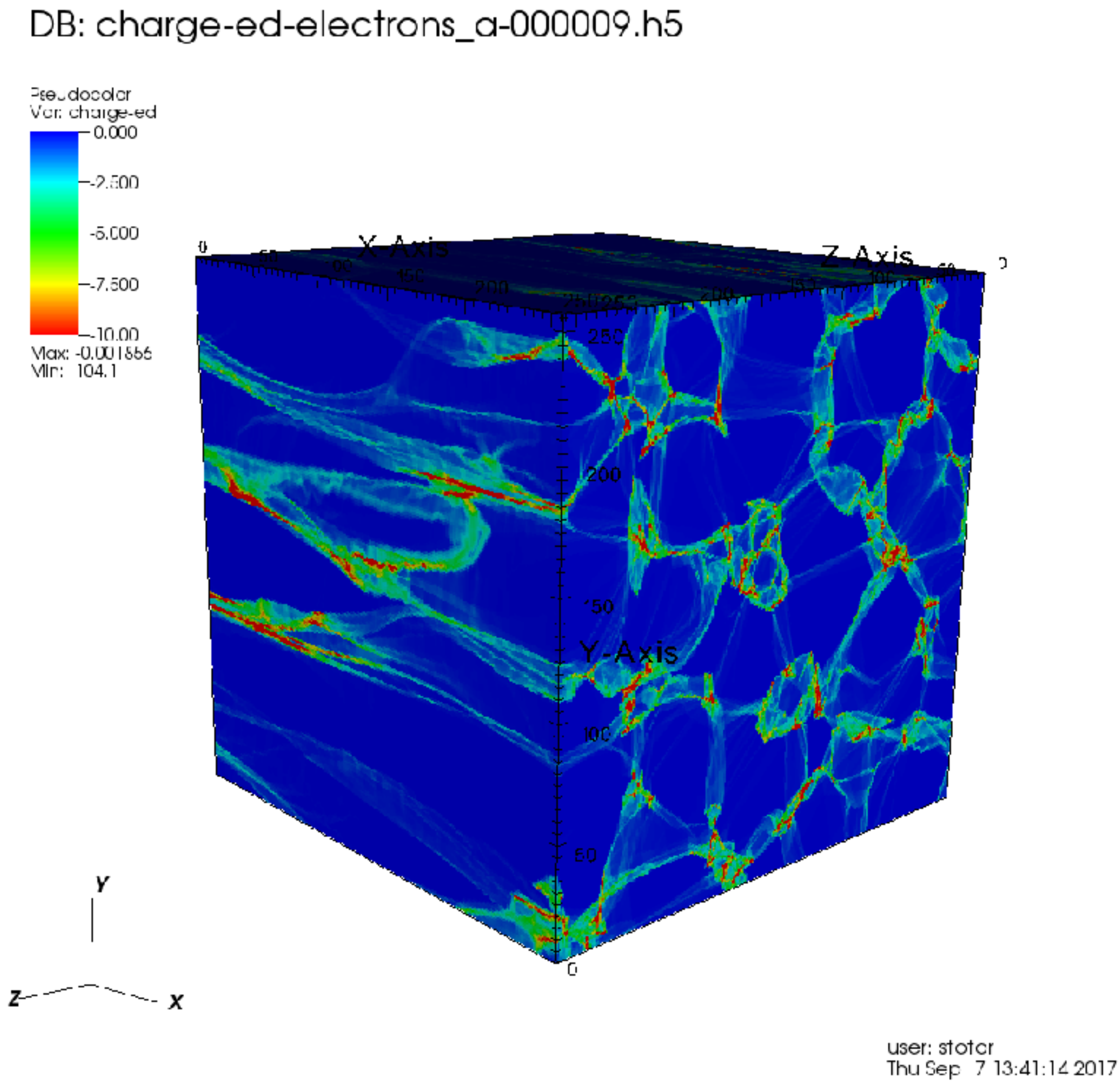
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