Quantum gravity = random geometry

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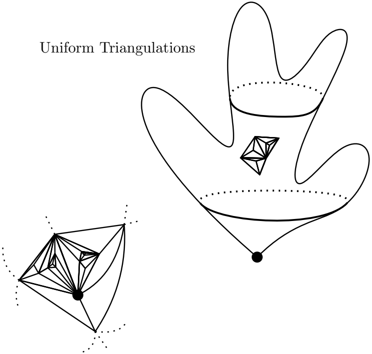
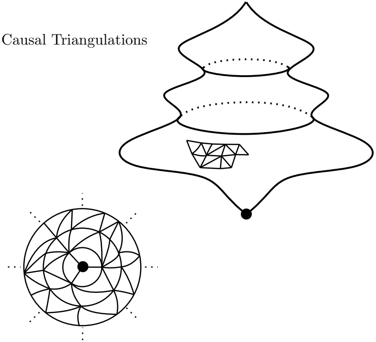
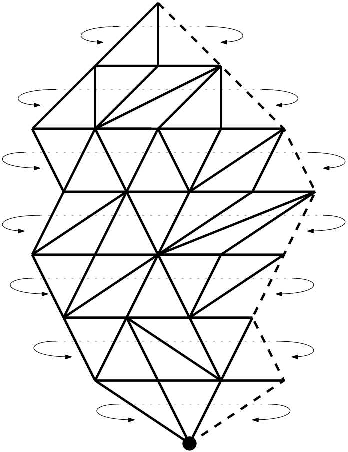
In the late 90's Ambjørn and Loll proposed a Lorentzian model for $1+d$-quantum gravity where the time dimension plays a special role.





Causal triangulations

Definition by picture :



Causal triangulations

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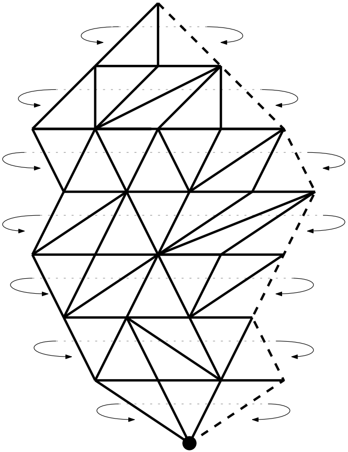
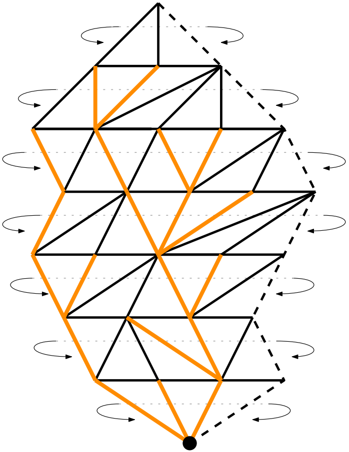
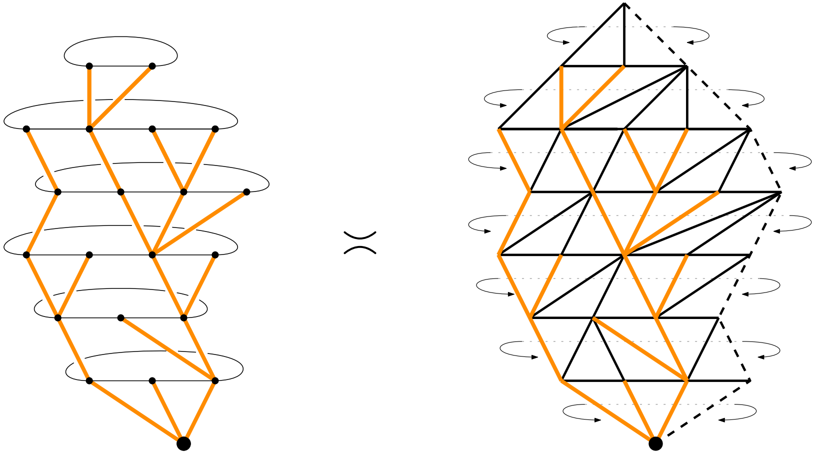




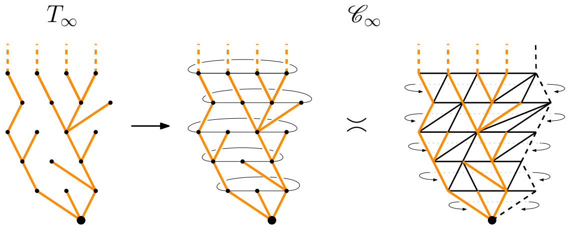
$\mathcal{CT} \simeq$ trees + horizontal connections

Approximate model :



The beast

Take a large uniform (plane) tree T_n (conditioned geometric BGW) and consider $\mathscr{C}_n$ the graph obtained by either considering the associated causal triangulation or simply adding the horizontal connections between successive vertices at height.



When $n \to \infty$ the local limit of T_n is given by T_∞ Kesten's critical geometric Bienaymé–Galton–Watson tree conditioned to survive, and one associates similarly the infinite graph $\mathscr{C}_\infty$.

Simulations

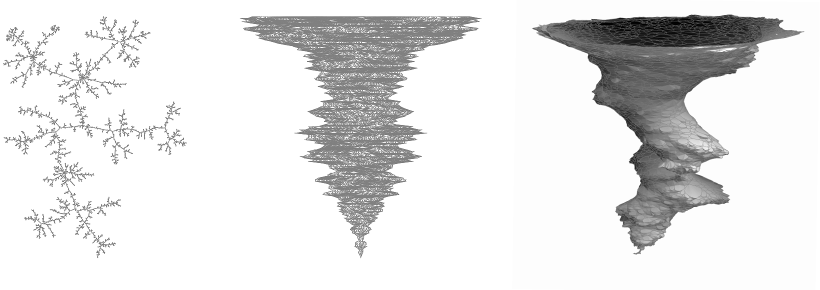


Figure – A large ball around the root in T_∞ in spring-electrical embedding, layered representation and 3D embedding of its associated causal triangulation.

Simulations

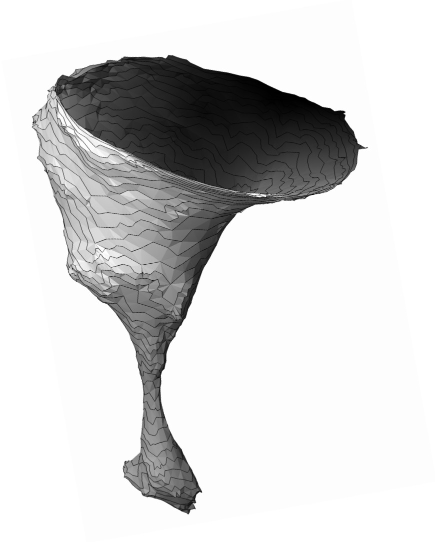


Figure – Contour lines

Simulations

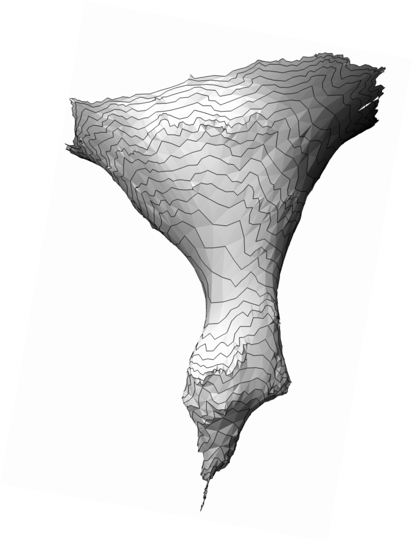
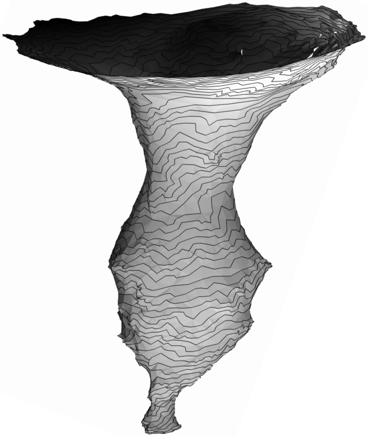


Figure – Contour lines

Simulations



Simulations

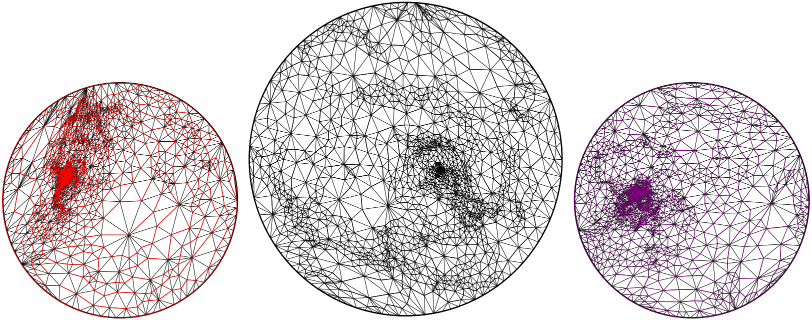


Figure – Guess !

Simulations

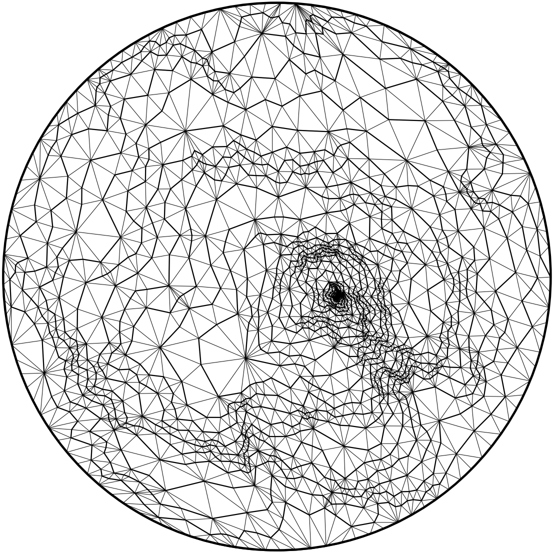


Figure – Tutte embedding

Lower bound on the width : blocks

Definition

A block $\mathcal{G}_r$ of height r is

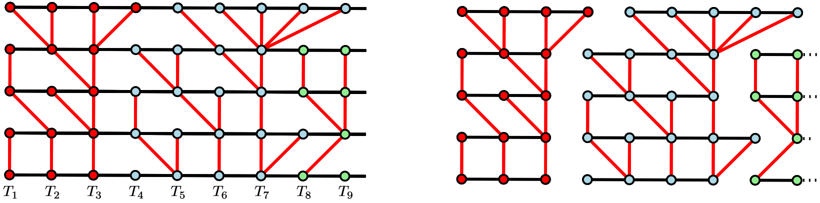


Figure – The layer of height 4 in the random graph obtained from an iid sequence of GW trees. This layer can be decomposed into blocks of height 4 and we represented the first three blocks in this sequence.

If $\varnothing(\mathcal{G}_r)$ is the left-right width of the block we define

$$f(r) = \sup\left\{k \geqslant 0 : \mathbb{P}(\varnothing(\mathcal{G}_r) \geqslant k) \geqslant 1/2\right\}.$$

Lower bound on the width : renormalisation

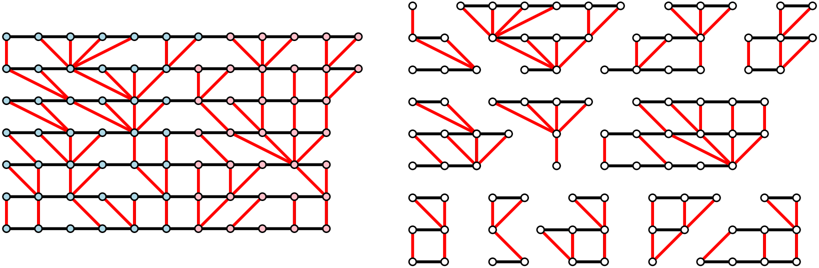


Figure – Decomposing a big block into smaller blocks

Renormalisation

Proposition

For some constant $c > 0$ we have

$$f(r) \geqslant c \cdot \Big(m \wedge (r/m) f(m) \Big).$$

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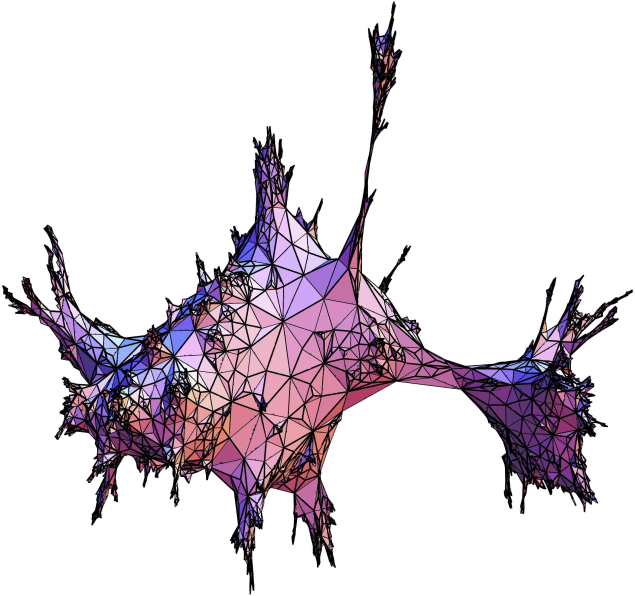
$$f(r) \geqslant c \cdot \Big(m \wedge (r/m) f(m) \Big).$$

$$\Rightarrow f(r) \geqslant r^{1-o(1)}.$$



Krikun's skeleton

Krikun (2005) found a way to encode bi-pointed triangulations by a "reverse decorated tree".

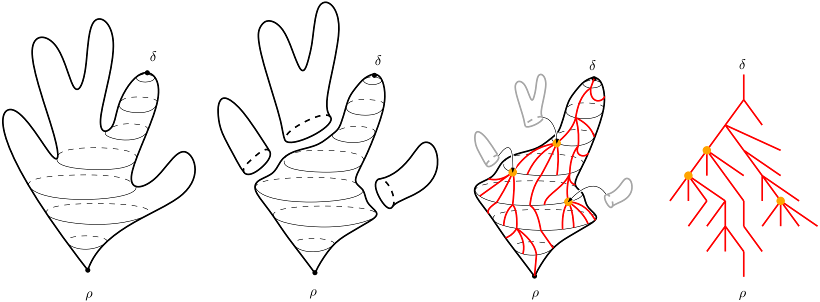
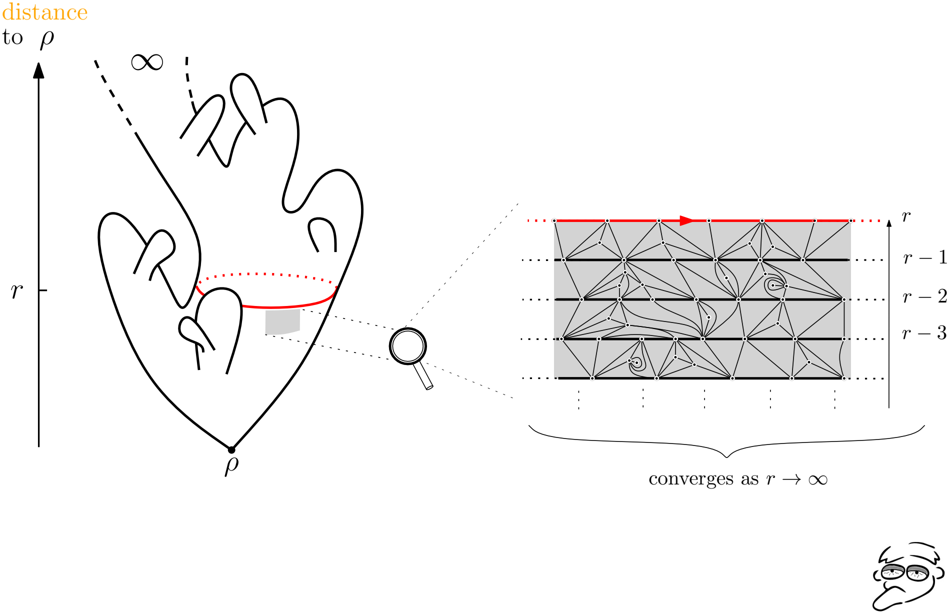
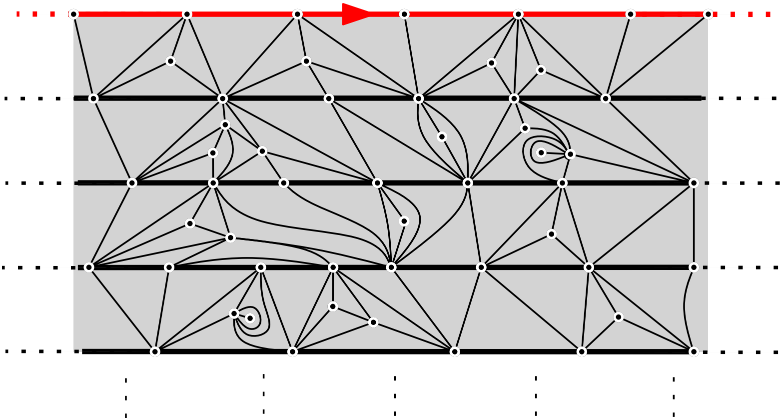


Figure – Krikun's skeleton decomposition in a nutshell

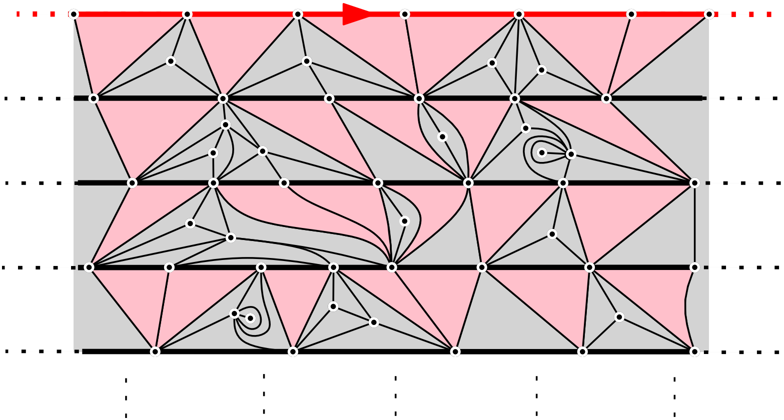
Looking backward



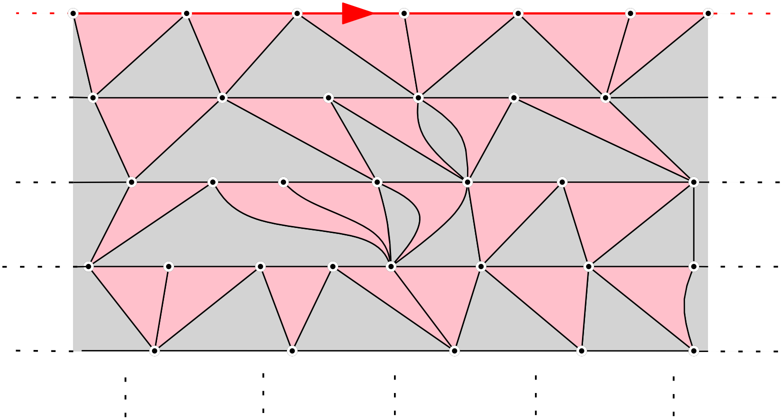
Looking backward



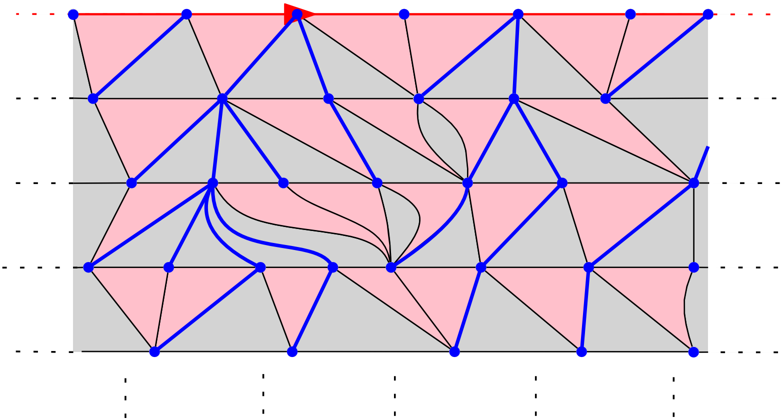
Downward triangles



Skeleton



A hidden causal map



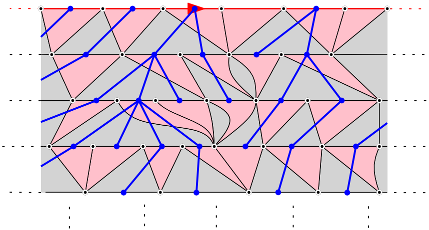
But 3/2-stable

Proposition (Krikun 05, see also C.& Le Gall 16)

The blue trees are (almost) i.i.d. Galton–Watson trees with critical offspring generating function given by

$$\sum_{k=0}^{\infty} \theta(k) z^k = 1 - \left(1 + \frac{1}{\sqrt{1-z}}\right)^{-2},$$

in particular $\theta(k) \sim C k^{-5/2}$ *as* $k \to \infty$. *Conditionally on them the gray holes are filled-in with independent Boltzmann triangulations of the proper perimeters.*



Mutatis Mutandis

What does it change ?

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For some constant $c > 0$ we have

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The girth of causal “stable” triangulations is now linear in the height (C. & Hutchcroft & Nachmias).
Consequences on geometry of uniform random triangulations (work in progress).

Quizz : What is it ? What can we ask ?

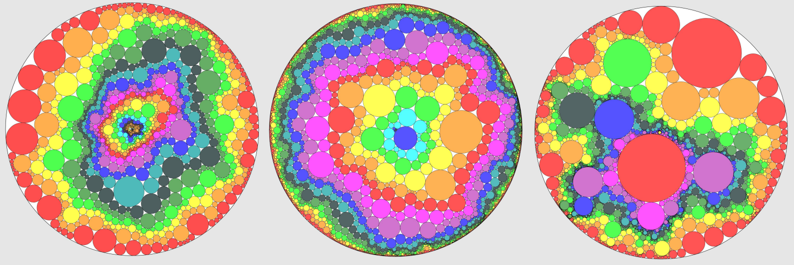

Figure – Simulations by T. Budzinski

Thank you for your attention !