DYNAMICS ON RANDOM GRAPHS AND RANDOM MAPS

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Abstracts

Louigi Addario-Berry

Two (three?) branching anecdotes

Abstract: I'll describe the ideas behind two or three recent and/or future results about random trees.

1) Relating the height, width and volume of Galton-Watson trees. Based on the paper "Most trees are short and fat".

2) Comparing arbitrary trees with binary trees. A small observation which I think should be useful for solving a set of problems posed by Svante Janson in his paper "Simply generated trees, conditioned Galton?Watson trees, random allocations and condensation".

3) The minimum spanning tree of random 3-regular graphs. In joint work in preparation with Sanchayan Sen, we show that the scaling limit is the same as for the MST of the complete graph. This involves some technical effort, and hinges on a cheap trick that will never work for, e.g., random 4-regular graphs.

Marie Albenque

Local convergence of Ising-weighted triangulations

Abstract: Angel and Schramm proved in 2003, that uniform planar triangulations converge for the local topology. The limit law, known as UIPT (for Uniform Infinite Planar Triangulation) has been much studied since and is now a well understood object.

In this talk, I'll prove a result analogue to Angel and Schramm's but for triangulations that are not uniform anymore but are weighted by a Ising model. The main difficulty is to extend to this framework the combinatorial results known for "regular" triangulations. Inspired by the work of Bernardi and Bousquet-Mélou, we proved that the generating series of triangulations with a fixed boundary condition are algebraic. I will give the main ideas and techniques that appear in this proof.

I'll finish by some open questions.The limiting object is indeed so far ill-understood and could serve as a model to confirm (or not !) for instance the famous Watabiki's prediction.

This is a joint work with Laurent Ménard and Gilles Schaeffer.

Omer Angel

Bootstrap percolation on Erdos-Renyi graphs

Abstract: We consider bootstrap percolation on the Erdos-Renyi graph: given an initial infected set, a vertex becomes infected if it has at least r infected neighbours. The graph is susceptible if there exists an initial set of size r that infects the whole graph. We identify the critical threshold for susceptibility. We also analyse Bollobas's related graph-bootstrap percolation model.
Joint with Brett Kolesnik.

Anna Ben-Hamou

Mixing times of random walks on random graphs

Abstract: In this talk, we will be interested in random walks in random graphs with prescribed degrees. We will present some results establishing the cutoff phenomenon for simple and non-backtracking random walks: for both walks, with high probability, the distance to equilibrium remains very close to one up to the mixing time and abruptly falls from one to zero in a much smaller time scale called the cutoff window. An interpretation of mixing times in terms of the entropies of the walks on a Galton-Watson tree which approximates the graph locally then allows us to compare these two mixing times and to show that, on random graphs, the non-backtracking random walk mixes faster than the simple one.

Those results were obtained in collaboration with Justin Salez (Paris Diderot) on the one hand, Eyal Lubetzky (NYU) and Yuval Peres (Microsoft Reserach) on the other hand.

Timothy Budd

Nesting of loops versus winding of walks

Abstract: Random planar maps with a boundary coupled to an $O(n)$ loop model are expected, upon uniformization, to scale towards Liouville Quantum Gravity (LQG) in the unit disk together with an independent Conformal Loop Ensemble (CLE). This expectation was corroborated by work of Borot, Bouttier & Duplantier in which they found that the asymptotic nesting statistics of loops on the planar map side was related through a Knizhnik-Polyakov-Zamolodchikov (KPZ) relation to the equivalent for CLE obtained by Miller, Watson & Wilson. In this talk I will show that the nesting on the planar map side is closely related, even at the combinatorial level, to another random process in the plane: the winding of two-dimensional random walks around the origin. Inspection of the scaling limit of the latter in terms of Brownian motion sheds some light on the appearance of the KPZ relation.

Van Hao Can

Annealed Ising model on random regular graphs

Abstract: In a recent paper, Giardina, Giberti, Hofstad, Prioriello have proved a law of large number and a central limit theorem with respect to the annealed measure for the magnetization of the Ising model on some random graphs including the random regular graph of degree 2. In this talk, we present a new proof of their results, which applies to all random regular graphs. In addition, we also study the critical behavior of this model.

Elisabetta Candellero

Percolation and isoperimetric inequalities

Abstract: In this talk we will discuss some relations between percolation on a given graph G and its geometry. There are several interesting questions relating various properties of G such as growth (or dimension) and the process of percolation on it. In particular we will look for conditions under which its critical percolation threshold is non-trivial, that is: $p_c(G)$ is strictly between zero and one. In a very influential paper on this subject, Benjamini and Schramm asked whether it was true that for every graph satisfying $\dim(G) > 1$, one has $p_c(G) < 1$. We will explain this question in detail and present some recent results that have been obtained in this direction.
Based on a joint work with Augusto Teixeira.

Shirshendu Chatterjee

Phase transition for the threshold contact process, an "annealed approximation" of heterogeneous random Boolean networks

Abstract: We consider a model for heterogeneous gene regulatory networks that is an "annealed approximation" of Kauffmann's (1969) original random Boolean networks. In this model, genes are represented by the nodes of a random directed graph G_n on n vertices with specified degree distribution, and the interactions among the genes are approximated by an appropriate threshold contact process (in which a vertex with at least one occupied in-neighbor at time t will be occupied at time $t+1$ with probability q, and vacant otherwise) on G_n. We characterize the order-chaos phase transition curve for the threshold-contact process on G_n segregating the chaotic and ordered random Boolean networks.?

Nicolas Curien

Geometric and spectral properties of random causal triangulations

Abstract: We study the random planar maps obtained from random Galton–Watson plane trees after adding the horizontal connections between successive vertices at each level. This random graphs are closely related to the well-known causal dynamical triangulations introduced by Ambjörn and Loll and are extensively studied in physics. We prove that when the underlying Galton–Watson tree is critical and has finite variance then the horizontal distances are shrunk compared to the vertical distances, although the distance exponent remains the same. This enables us to prove that the spectral dimension of a infinite version of these graphs is almost surely equal to 2.
Based on joint work with Tom Hutchcroft and Asaf Nachmias.

Cecilia Holmgren

Split trees and Galton Watson trees: Two important classes of random trees

Abstract: I will talk about two important classes of random trees, split trees and Galton-Watson trees. Split trees were introduced by Devroye (1998) for unifying many important random trees of logarithmic height. They are interesting not least because of their usefulness as models of sorting algorithms in computer science; for instance can the well-known Quicksort algorithm (introduced by Hoare [1960]) be depicted as the binary search tree. Galton-Watson-trees were introduced already in 1875 to describe under which conditions a (noble) family name would die out or survive forever. The conditioned Galton-Watson trees (also called simply-generated trees) are conditioned on a given total size of the number of vertices and represent important random trees of non-logarithmic height. Examples are ordered (plane) trees, Cayley trees and binary trees. I will give a brief general introduction to the field and the main focus of my talk will then be to discuss some of my own results for these large classes of random trees (e.g., on the total path length, the number of cuttings, the number of inversions, and bootstrap percolation) and some of the methods that I have used (e.g., renewal theory and Aldous Brownian continuum random tree).

Tim Hulshof

Critical percolation on the Hamming graph

Abstract: The Hamming graph $H(d, n)$ is the $(d-1)$-fold Cartesian product of complete graphs K_n. In my talk I will discuss an ongoing project, in which we determine that for critical percolation on $H(d, n)$ with $d = 2, 3, 4$, the vector of ordered cluster sizes, when rescaled in the right way, converges to a vector of proper random variables, which correspond to the excursion times of a reflected Brownian motion with parabolic drift downwards. This is analogous to the result of Aldous on the Erdos-Renyi random graph. The Hamming graph has a non-trivial geometry, and most of our proofs revolve around dealing with the geometry the base graph imposes on the percolation clusters.
Joint work with Lorenzo Federico, Remco van der Hofstad, and Frank den Hollander.

Tom Hutchcroft

Interlacements and the Uniform Spanning Forest

Abstract: The Aldous-Broder algorithm allows one to sample the uniform spanning tree of a finite graph as the set of first-entry edges of a simple random walk. In this talk, I will discuss how this can be extended to infinite transient graphs by replacing the random walk with the random interlacement process. I will then outline how this new sampling algorithm can be used to compute critical exponents for the uniform spanning forest of Z^d.

Julia Komjathy

Weighted distances in scale free random graphs

Abstract: In this talk I will review the recent developments on weighted distances in scale free random graphs as well as highlight key techniques used in the proofs. We consider graph models where the degree distribution follows a power-law such that the empirical variance of the degrees is infinite, such as the configuration model, geometric inhomogeneous random graphs, or scale free percolation. Once the graph is created according to the model definition, we assign i.i.d. positive edge weights to existing edges, and we are interested in the proper scaling and asymptotic distribution of weighted distances.

In the infinite variance degree regime, a dichotomy can be observed in all these graph models: the edge weight distributions form two classes, explosive vs conservative weight distributions. When a distribution falls into the explosive class, typical distances converge in distribution to proper random variables. While, when a distribution falls into the conservative class, distances tend to infinity with the model size, according to a formula that captures the doubly-logarithmic graph distances as well as the precise behaviour of the distribution of edge-weights around the origin. An integrability condition decides into which class a given distribution falls.

This is joint work with Adriaans, Baroni, van der Hofstad, and Lodewijks.

Shen Lin

Harmonic measure for biased random walk on a Galton?Watson tree

Abstract: Consider random walk λ-biased towards the root on a leafless Galton?Watson tree, whose offspring distribution is non-degenerate and has finite mean $m > 1$. In the transient regime $0 < \lambda < m$, the loop-erased trajectory of the biased random walk defines the λ-harmonic ray, whose law is called the λ-harmonic measure on the boundary of the Galton?Watson tree. In this talk, I will present some recent results concerning the Hausdorff dimension of the λ-harmonic measure and the average number of children of the vertices visited by the λ-harmonic ray.

Malwina Luczak

Extinction time for the weaker of two competing stochastic SIS logistic epidemics

Abstract: We consider a simple stochastic model for the spread of a disease caused by two virus strains in a closed homogeneously mixing population of size N. In our model, the spread of each strain is described by the stochastic logistic SIS epidemic process in the absence of the other strain, and we assume that there is perfect cross-immunity between the two virus strains, that is, individuals infected by one strain are temporarily immune to re-infections and infections by the other strain. For the case where one strain has a strictly larger basic reproductive ratio than the other, and the stronger strain on its own is supercritical (that is, its basic reproductive ratio is larger than 1), we derive precise asymptotic results for the distribution of the time when the weaker strain disappears from the population, that is, its extinction time. We further consider what happens when the difference between the two reproductive ratios may tend to 0.
This is joint work with Fabio Lopes.

Edouard Maurel-Segala

From peeling to random walks, an exploration of a critical percolation cluster on the infinite planar map

Abstract: In this joint work with Matthias Gorny and Arvind Singh, using the celebrated peeling process, we exhibit a Markov chain which encodes properties of a critical percolation on the infinite planar triangulation. We will then use classical properties of stable processes to study the tail of the distribution of the Volume and the Perimeter of this cluster. (arxiv:1701.01667)

Gourab Ray

Universality of fluctuation of dimers on Riemann surfaces

Abstract: We investigate the fluctuation of the height 1-form on a Temperleyan graph embedded on a Riemann surface with the goal of proving that the limit does not depend on the graph sequence provided it satisfies certain natural assumptions. We provide a complete proof for the torus and the annulus case, and a conditional theorem for the general case. The proof proceeds through a new result we prove on the universality of cycle rooted spanning forests and the techniques developed for a similar result we had for the simply connected case. This answers a question of Dubedat and Gheissari and also extends a result of Kassel and Kenyon.
Joint work with Nathanael Berestycki and Benoit Laslier.

Loic Richier

Limits of large loops in the $O(n)$ ***model on random maps***

Abstract: The $O(n)$ loop models consist of random maps endowed with self and mutually avoiding loops drawn on the dual edges of the map, such that each loop receives a non-local weight n. The purpose of this talk is to discuss local and scaling limits of large loops in the rigid $O(n)$ model on random quadrangulations. We will focus on the so-called dense regime, in which stable looptrees appear in the scaling limit. The proof is based on the tight connection with Boltzmann maps established by Borot, Bouttier and Guitter, and also provides information about the geometry of these maps at large scale.

Raphael Rossignol

Scaling limit of dynamical percolation on Erdös-Rényi random graphs

Abstract: Consider a critical Erd?s-Rényi random graph: n is the number of vertices, each one of the $\binom{n}{2}$ possible edges is kept in the graph independently from the others with probability $p(n) = n - 1 + \lambda n - 4/3$, λ being a fixed real number. When n goes to infinity, Addario-Berry, Broutin and Goldschmidt have shown that the collection of connected components, viewed as suitably normalized compact connex metric measure spaces, converges in distribution to a continuous limit made of random real graphs closely linked to the brownian random tree of Aldous. Let us now consider the dynamical percolation on this random graph for finite n. To each pair of vertices is attached a Poisson process of intensity $n - 1/3$, and every time it rings, one resamples the corresponding edge. Under this process, the collection of connected components undergoes coalescence and fragmentation. We shall study the distributional convergence of this process when n goes to infinity, towards a fragmentation-coalescence process on the continuous limit.

Justin Salez

Generic cutoff at the entropic time for sparse exchangeable Markov chains

Abstract: We study convergence to equilibrium for a large class of Markov chains in random environment. The chains are sparse in the sense that in every row of the transition matrix P the mass is essentially concentrated on few entries. Moreover, the entries are exchangeable within each row. This includes various models of random walks on sparse random directed graphs. These models are generally non reversible and the equilibrium distribution is itself unknown. In this general setting we establish the cutoff phenomenon for the total variation distance to equilibrium, with mixing time given by the logarithm of the number of states times the inverse of the average row entropy of P.
Joint work with Charles Bordenave and Pietro Caputo.

Alexandre Stauffer

Multi-particle diffusion limited aggregation

Abstract: We consider a stochastic aggregation model on Z^d. Start with an infinite collection of particles located at the vertices of the lattice, with at most one particle per vertex, and initially distributed according to the product Bernoulli measure with parameter $\mu \in (0, 1)$. In addition, there is an aggregate, which initially consists of only one particle placed at the origin. Non-aggregated particles move as continuous time simple symmetric random walks obeying the exclusion rule, whereas aggregated particles do not move. The aggregate grows indefinitely by attaching particles to its surface whenever a particle attempts to jump onto it. Our main result states that if on Z^d, d at least 2, the initial density of particles μ is large enough, then with positive probability the aggregate grows with positive speed.
This is a joint work with Vladas Sidoravicius.

Daniel Valesin

Spatial Gibbs random graphs

Abstract: Many real-world networks of interest are embedded in physical space. We present a new random graph model aiming to reflect the interplay between the geometries of the graph and of the underlying space. The model favors configurations with small average graph distance between vertices, but adding an edge comes at a cost measured according to the geometry of the ambient physical space. In most cases, we identify the order of magnitude of the average graph distance as a function of the parameters of the model. As the proofs reveal, hierarchical structures naturally emerge from our simple modeling assumptions. Moreover, a critical regime exhibits an infinite number of phase transitions.
Joint work with Jean-Christophe Mourrat (ENS Lyon).

Rob Van den Berg

Frozen and near-critical percolation

Abstract: Motivated by sol-gel transitions, David Aldous (2000) introduced and analysed a fascinating dynamic percolation model on a tree where clusters stop growing ('freeze') as soon as they become infinite.

In this talk I will discuss recent (and ongoing) work, with Demeter Kiss and Pierre Nolin, on processes of similar flavour on planar lattices. We focus on the problem whether or not the giant (i.e. 'frozen') clusters occupy a negligible fraction of space. Accurate results for near-critical percolation play an important role in the solution of this problem.

I will also present a version of the model which can be interpreted as a sensor/communication network.