Reducing the Chromatic Number by Vertex or Edge Deletions

Authors:
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- A ***k*-coloring** of G is a mapping $c : V \Rightarrow \{1, 2, \dots k\}$ such that $c(u) \neq c(v)$ for all $uv \in E$.
- The **chromatic number** of G is the smallest integer k such that G admits a k-coloring. It is denoted by $\chi(G)$.

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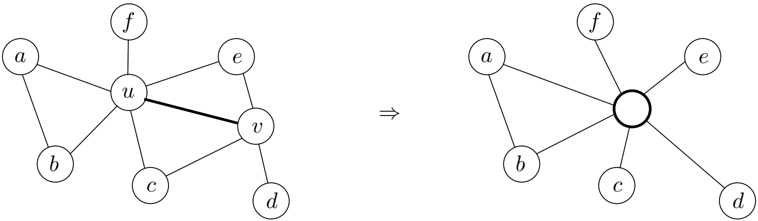
The **contraction** of an edge uv in G removes the vertices u and v from G, and replaces them by a new vertex made adjacent to precisely those vertices that were adjacent to u or v in G.



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- $\Leftarrow$ e is contraction-critical: G' obtained from contracting e has $\chi(G') = \chi(G) - 1$.

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- $\Leftarrow$ e is contraction-critical: G' obtained from contracting e has $\chi(G') = \chi(G) - 1$.Coloring u and v alike, we obtain a coloring of $G-e$ from any coloring of G'.

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□

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I: A graph $G = (V, E)$.
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Theorem

Critical Vertex and *Critical Edge* are both *co-NP-hard* for $(C_5, 4P_1, 2P_1 + P_2, 2P_2)$-free graphs.

Proof

From a restriction of MONOTONE 1-IN-3-SAT which is NP-complete.
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Is there a truth assignment such that each clause is satisfied by exactly one variable ?

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We prove that the problem of deciding whether a graph has a vertex whose deletion reduces $\sigma(G)$ the clique-covering number by 1 is co-NP-hard for $(C_4, C_5, K_4, \overline{2P_1 + P_2})$-free graphs.

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The two problems are equivalent.

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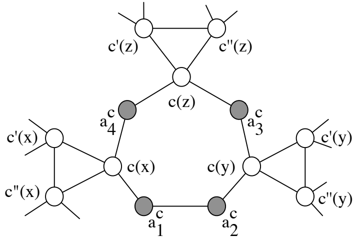
Construction of $G = (V, E)$ from a boolean formula Φ



- Each clause C: a 7-cycle
- Each variable x: a triangle

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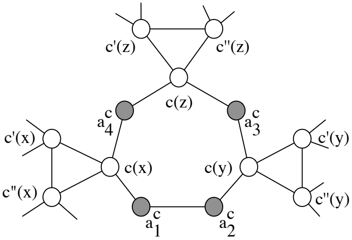

- $|V| = 7n = 7m$
- $(C_4, C_5, K_4, \overline{2P_1 + P_2})$-free
- $\sigma(G) \geq \dfrac{10}{3}n$

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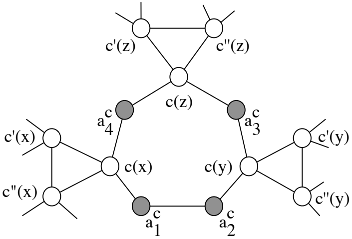

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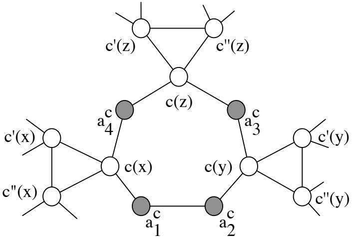

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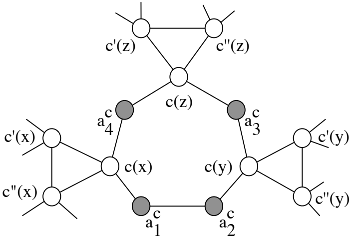

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- If $\sigma(G) > \frac{10}{3}n$, then G has a minimum clique cover $\mathcal{K}$ that contains a clique of size 1.

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Critical Vertex is co-NP-hard for $(C_5, 4P_1, 2P_1 + P_2, 2P_2)$-free graphs.

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consider clique of size two (edges) instead of clique of size one (vertex)

Result

Theorem

Critical Vertex, Critical Edge and Contraction-Critical Edge restricted to H-free graphs are polynomial-time solvable if $H \subseteq_i P_1 + P_3$ or $H \subseteq_i P_4$, and NP-hard or co-NP-hard otherwise.

Proof

Theorem : If a graph $H \subseteq_i P_4$ or $H \subseteq_i P_1 + P_3$, then COLORING is polynomial-time solvable for H-free graphs, otherwise it is NP-complete.
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$d = k = 1$ corresponds to Contraction-Critical Edge

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VERTEX DELETION BLOCKER(χ)
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VERTEX DELETION BLOCKER(χ)
I: $G = (V, E)$ and two integers $d, k \geq 0$
Q: can G be k-vertex-deleted into G' such that $\chi(G') \leq \chi(G) - d$?

$d = k = 1$ corresponds to Critical Vertex

Blocker

Theorem

- If $H \subseteq_i P_4$ CONTRACTION BLOCKER(χ) is polynomial-time solvable for H-free graphs, it is NP-hard otherwise.

Blocker

Theorem

- If $H \subseteq_i P_4$ CONTRACTION BLOCKER(χ) is polynomial-time solvable for H-free graphs, it is NP-hard otherwise.
- If $H \subseteq_i P_1 + P_3$ or P_4 VERTEX DELETION BLOCKER(χ) for H-free graphs is polynomial-time solvable, it is NP-hard or co-NP-hard otherwise.

Merci beaucoup pour votre attention