Adjacent Vertex Distinguishing Edge Coloring on Powers of Paths

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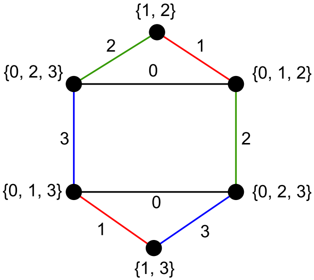
AVD-edge coloring

Set of colors of a vertex

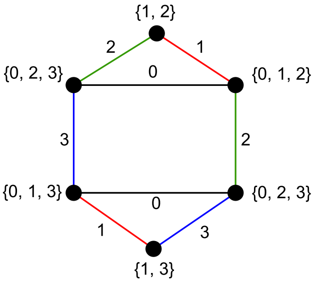
For any vertex v, the **set of colors** of v is composed by the colors of the edges incident to v and, it is denoted by $C(v)$.



AVD-edge coloring

Adjacent vertex distinguishing edge coloring

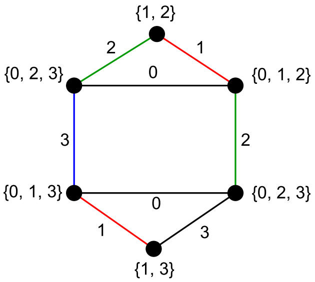
An **adjacent vertex distinguishing edge coloring (AVD-edge coloring)** is a proper edge coloring of G, such that $C(u) \neq C(v)$ for any two adjacent vertices, u and v.



AVD-Edge Coloring Problem

AVD-Edge Coloring Problem

The AVD-Edge Coloring Problem is to determine the least number of colors for which G has an AVD-edge coloring. This number is called **AVD-chromatic index** and it is denoted by $\chi'_a(G)$.



$\chi'_a(G) = 4$

Known results

Theorem (Zhang; Liu; Whang, 2002)

If the degree of any two adjacent vertices of a graph G are different, then $\chi'_a(G) = \Delta(G)$.

Theorem (Zhang; Liu; Whang, 2002)

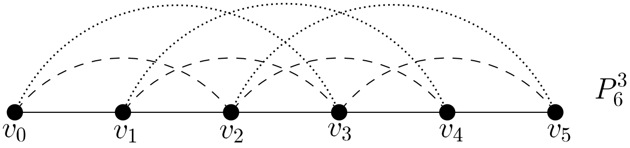
If G is a graph which has two adjacent maximum degree vertices, then $\chi'_a(G) \geqslant \Delta(G) + 1$.

Conjecture (Zhang; Liu; Whang, 2002)

For any connected graph G with at least 3 vertices, $G \ncong C_5$, then $\chi'_a(G) \leqslant \Delta(G) + 2$.

Powers of paths

A power of a path, denoted by P_n^k, is a graph $V(P_n^k) = \{v_0, v_1, \ldots, v_{n-1}\}$ and there is an edge $v_i v_j$ if, and only if, $|j - i| \leqslant k$, $0 \leqslant i, j < n$.



Results

Corolary (OMAI; ALMEIDA; SASAKI, 2017)

Let P_n^k be a power of a path not isomorphic to P_2^1 with exactly q vertices of maximum degree. So,

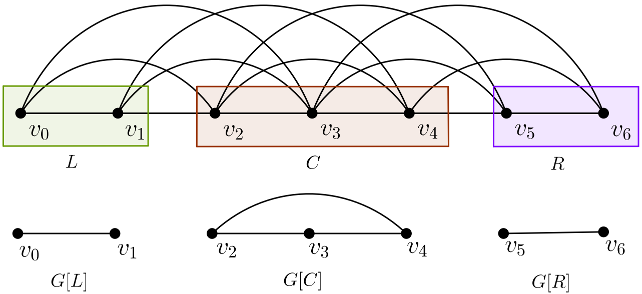
$$\chi'_a(P_n^k) = \begin{cases} \Delta(P_n^k), & \text{if } n = 2k+1, \\ \Delta(P_n^k)+2, & \text{if } k+1 < n < 2k, n \text{ is even, } q > \frac{n}{2} \text{ and } |E(\overline{P_n^k})| < q - \frac{n}{2}, \\ \Delta(P_n^k)+2, & \text{if } n \leqslant k+1 \text{ and } n \text{ is even,} \\ \Delta(P_n^k)+1, & \text{otherwise.} \end{cases}$$

Parcial results

If $n = 2k + 1$ and k is odd, then $\chi'_a(P_n^k) = \Delta(P_n^k)$.

1. Partite the set of vertices in $L = \{v_0, v_1, \ldots, v_{\lfloor \frac{k}{2} \rfloor}\}$;
$C = \{v_{\lfloor \frac{k}{2} \rfloor + 1}, v_{\lfloor \frac{k}{2} \rfloor + 2}, \ldots, v_{\lfloor \frac{k}{2} \rfloor + k}\}$; and $R = \{v_{\lfloor \frac{k}{2} \rfloor + k + 1}, \ldots, v_{n-1}\}$.

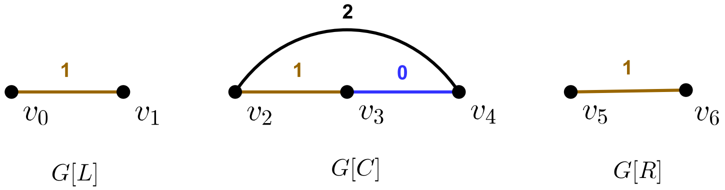
$k = 3$
$L = \{v_0, v_1\}$
$C = \{v_2, v_3, v_4\}$
$R = \{v_5, v_6\}$



Parcial results

If $n = 2k + 1$ and k is odd, then $\chi'_a(P_n^k) = \Delta(P_n^k)$.

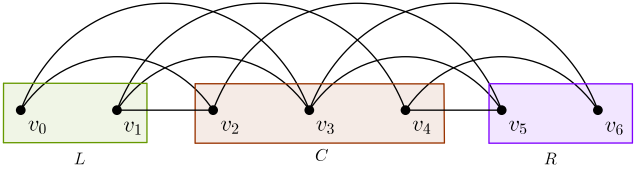
2. $G[L]$, $G[C]$ and $G[R]$ are subgraphs of K_k.
3. $G[L]$, $G[C]$ and $G[R]$ are colored with k colors.



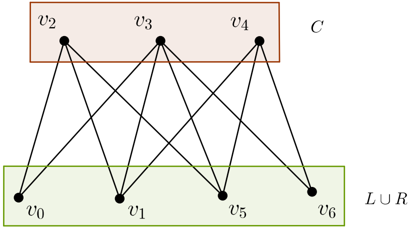
Parcial Results

If $n = 2k + 1$ and k is odd, then $\chi'_a(P_n^k) = \Delta(P_n^k)$.

4. $B = [C, L \cup R, E]$ is a bipartite graph.



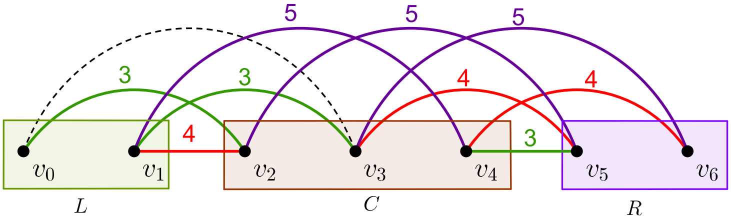
B



Parcial Results

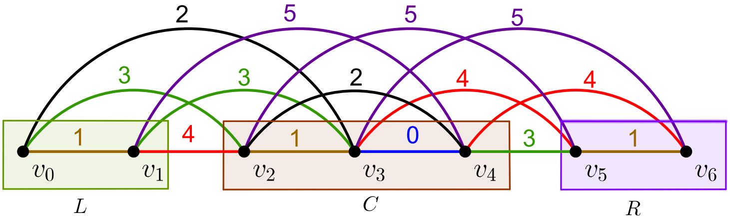
If $n = 2k + 1$ and k is odd, then $\chi'_a(P_n^k) = \Delta(P_n^k)$.

5. $B \setminus \{v_0 v_k\}$ is colored with k new colors.



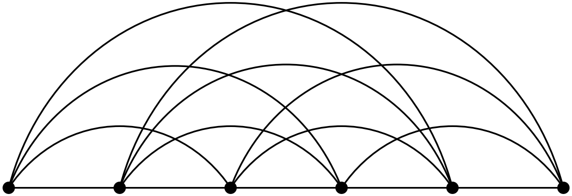
Parcial Results

If $n = 2k + 1$ and k is odd, then $\chi'_a(P_n^k) = \Delta(P_n^k)$.



$$\chi'_a(G) = 2k = 2.3 = 6$$

If $k+1 < n \leqslant 2k$ even and $q > \frac{n}{2}$, so $\chi'_a(P_n^k) = \Delta(P_n^k) + 1$ when $|E(\overline{P_n^k})| \geqslant q - \frac{n}{2}$.

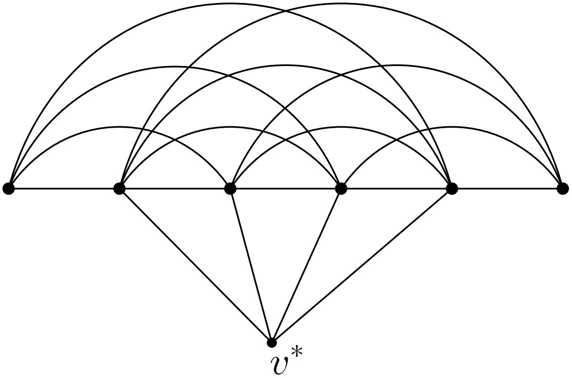


P_6^4

$\Delta(P_6^4) = 4$

Results

1. Build a graph G^* by adding the vertex v^*.

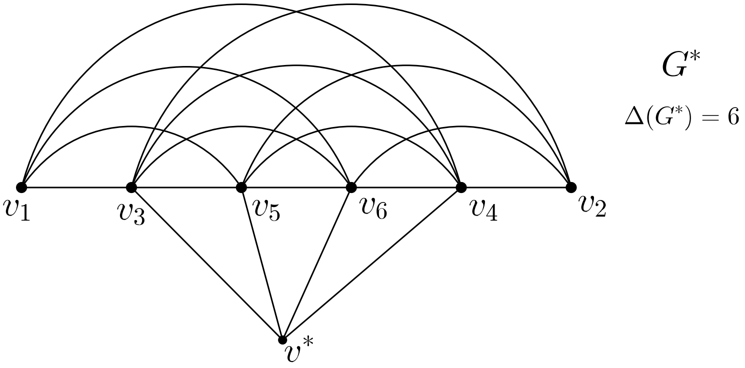


G^*

$\Delta(G^*) = 6$

Results

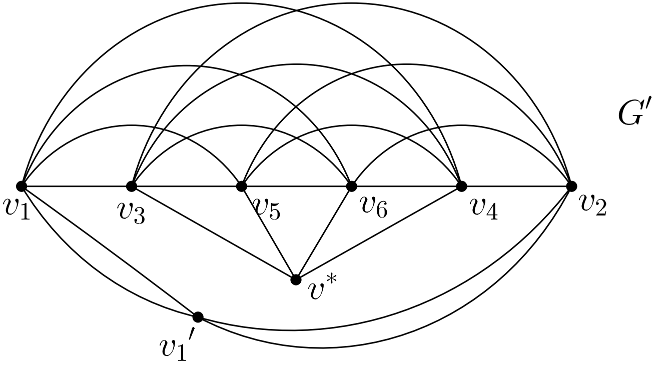
2. Label the vertices of $G^* - v^*$ according to increase order of their degrees.



Let Y be the set of vertices with degree less than $\Delta(G^*)$. So, $Y = \{v_1, v_2\}$ and $y = |Y| = 2$.

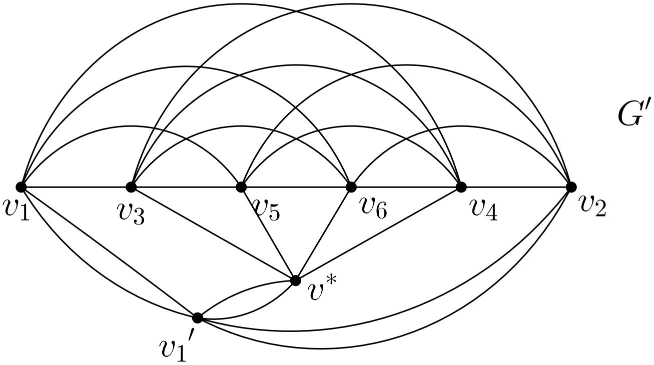
Results

3. Build a multigraph G' by adding a vertex v_i' and connecting it with v_{2i-1} and v_{2i} by $n - d_{G^*}(v_{2i-1})$ edges each one, $1 \leqslant i \leqslant \frac{y}{2}$.



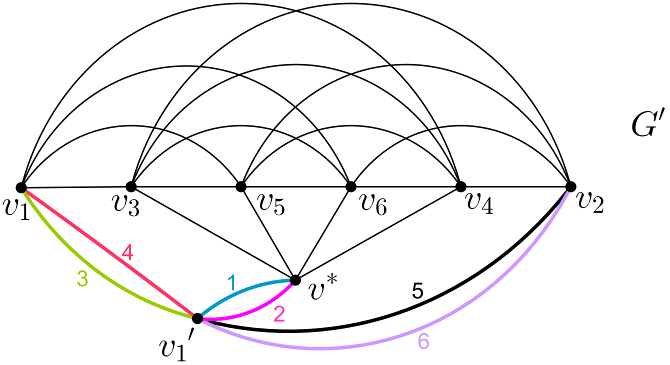
Results

4. Add y edges between v_1' and v^*.



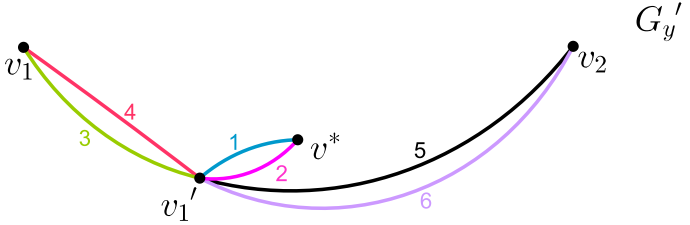
Results

5. Let $V' = \{v'_1, v'_2, \ldots, v'_{\frac{y}{2}}\}$. Make the coloring of the edges incidents to V' using n colors.



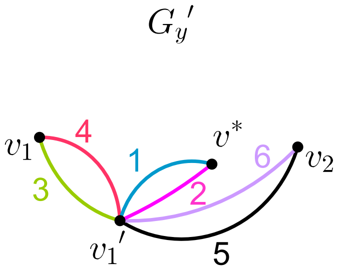
Results

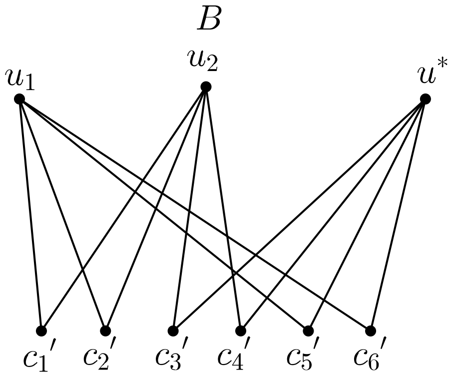
Let $G'_y = G'[Y \cup V' \cup \{v^*\}]$. We extend the coloring of $G'_y = G'_2$ for coloring $G'_{y+1} = G_3$, using the same n colors, until G'_n be colored.



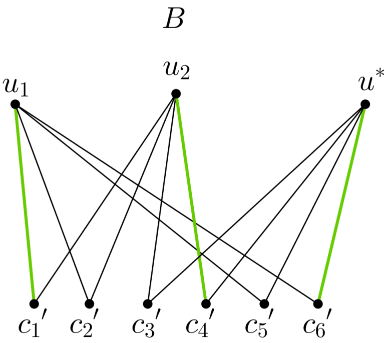
Results

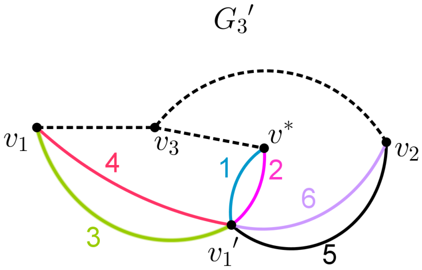
6. Build a bipartite graph B with partition (A_1, A_2), where $A_1 = \{u_1, u_2, \ldots, u_y, u^*\}$ and $A_2 = \{c_1', c_2', \ldots, c_n'\}$.





7. Make the coloring of G_3' using the bipartite graph B obtained from G_2'.





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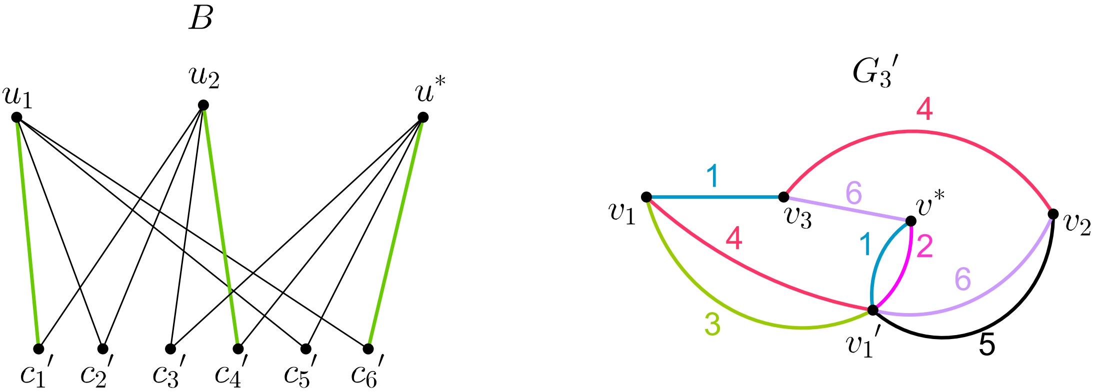
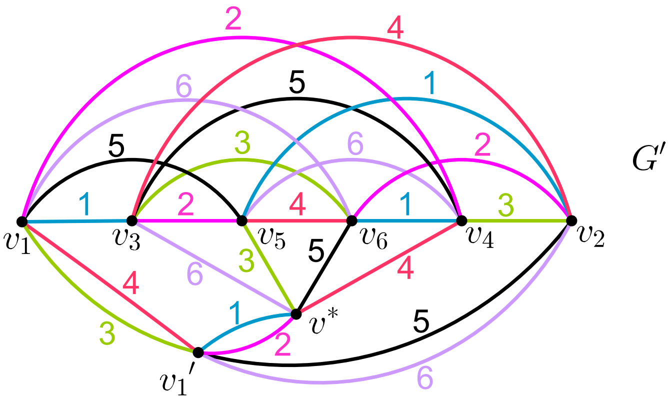


Results

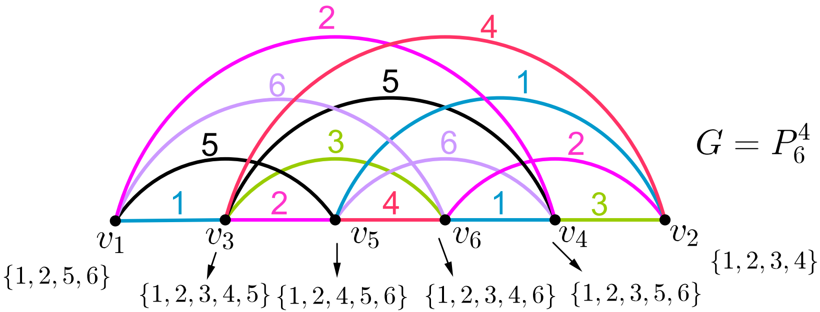
Result of the edge coloring of G_6'.



We only need to remove the vertices v^* and v_i', $1 \leqslant i \leqslant \frac{y}{2}$.

Results

We used n colors, then $\chi'_a(P_n^k) = \Delta(P_n^k) + 1$.



Thank you ;)