k-quasi-transitive digraphs of large diameter

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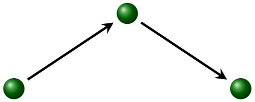
Quasi-transitive digraphs



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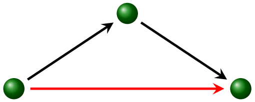

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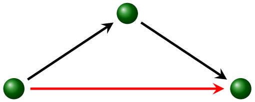

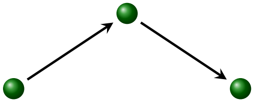

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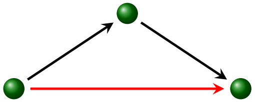

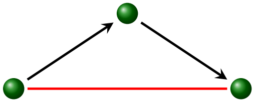

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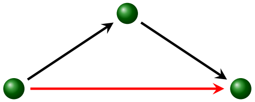

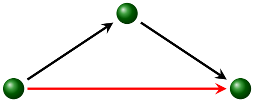

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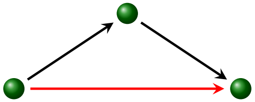

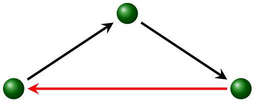

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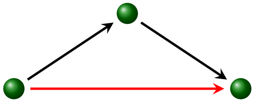

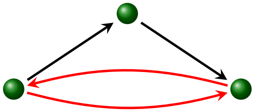

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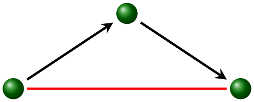

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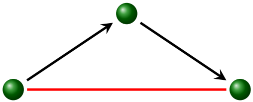

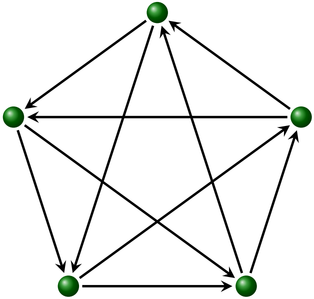

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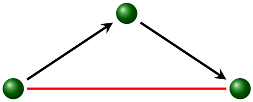

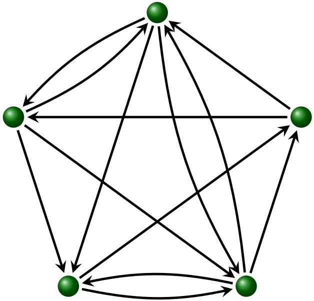

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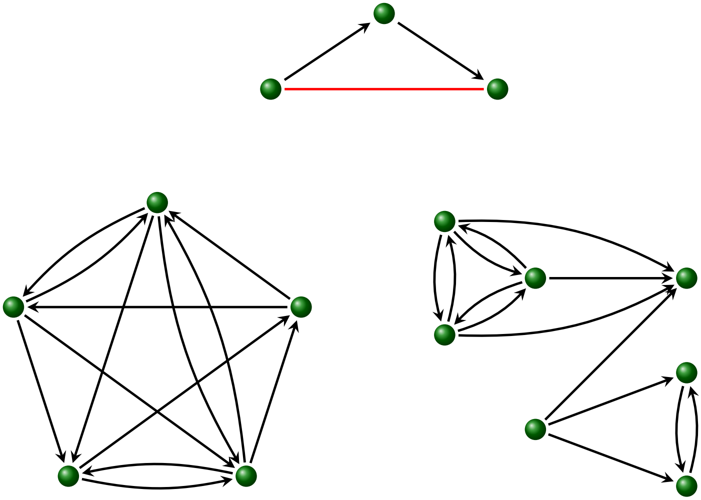
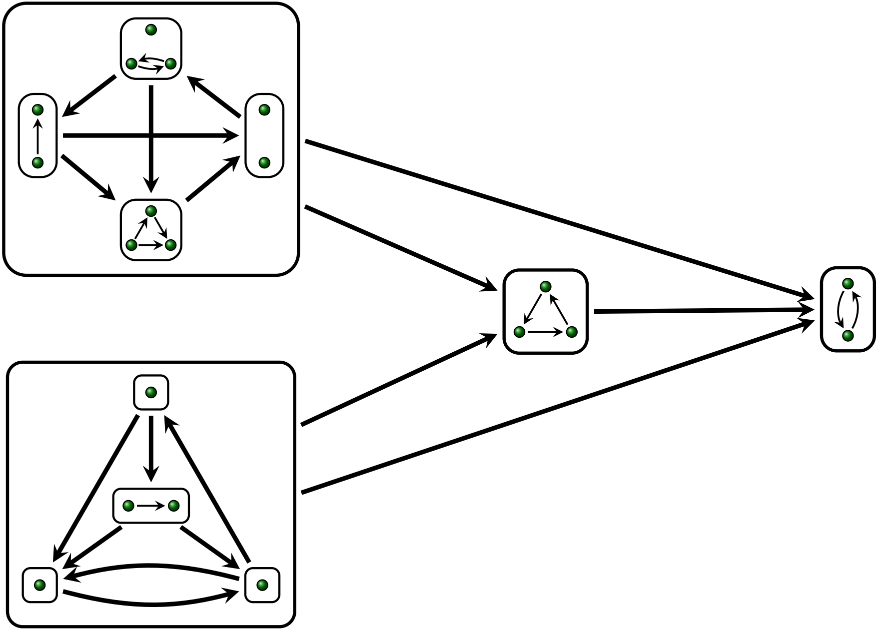


The Canonical Decomposition

Theorem (Bang-Jensen, Huang, 1995)

Let D be a digraph which is quasi-transitive.

- *If D is not strong, then there exists a transitive oriented graph T with vertices $\{u_1, u_2, \ldots, u_t\}$ and strong quasi-transitive digraphs $H_1, H_2, \ldots, H_t$ such that $D = T[H_1, H_2, \ldots, H_t]$, where H_i is substituted for u_i, $i \in \{1, 2, \ldots, t\}$.*
- *If D is strong, then there exists a strong semicomplete digraph S with vertices $\{v_1, v_2, \ldots, v_s\}$ and quasi-transitive digraphs $Q_1, Q_2, \ldots, Q_s$ such that Q_i is either a vertex or is non-strong and $D = S[Q_1, Q_2, \ldots, Q_s]$, where Q_i is subsituted for v_i, $i \in \{1, 2, \ldots, s\}$.*



Quasi-transitive digraphs

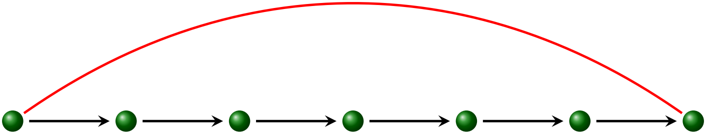
Polynomial time verifiable:

- Hamiltonicity
- Traceability
- Existence of a k-linkage
- Existence of arc-disjoint in- and out-branchings rooted at a given vertex.

True for this class of digraphs:

- Path-partition Conjecture
- Seymour's Second Neighbourhood Conjecture

k-quasi-transitive digraphs



4-quasi-transitive digraphs

k-quasi-transitive digraphs

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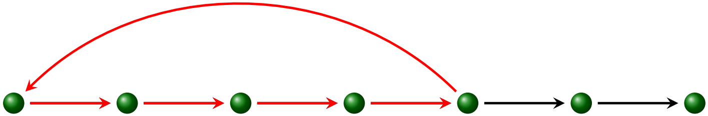

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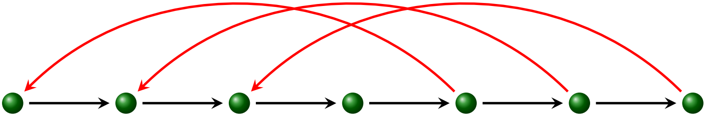

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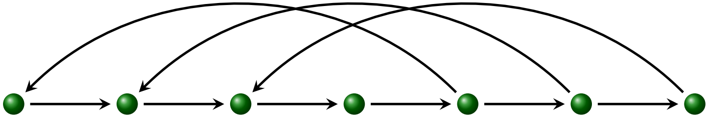

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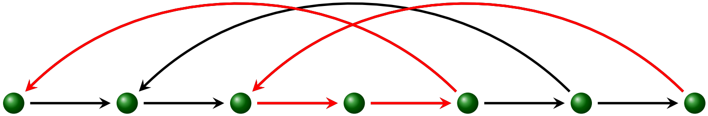

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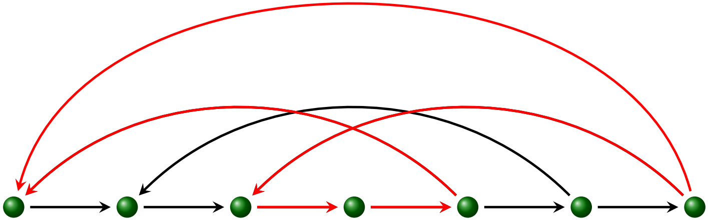

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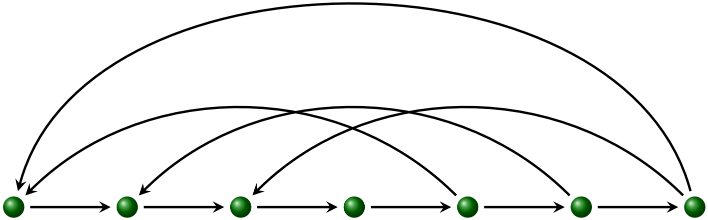

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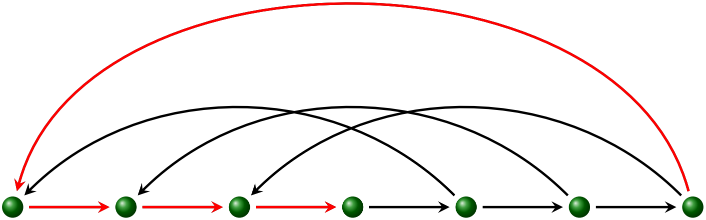

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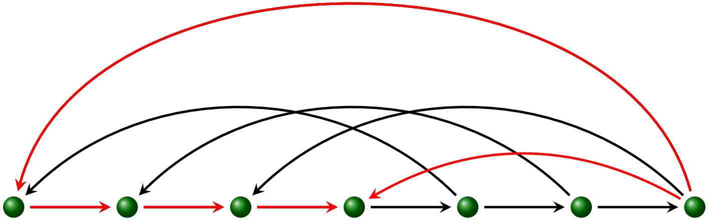

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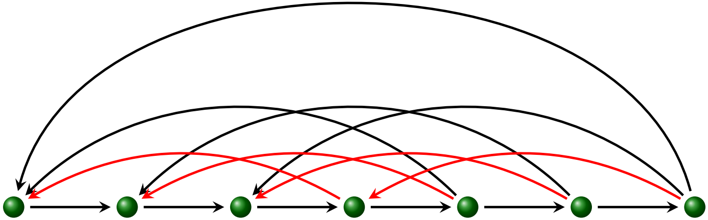

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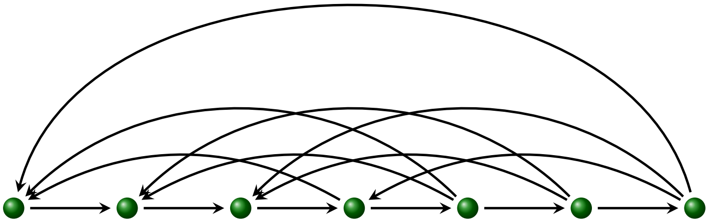

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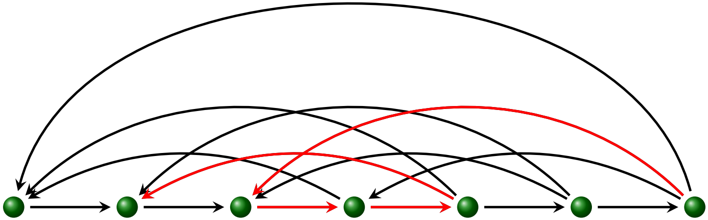

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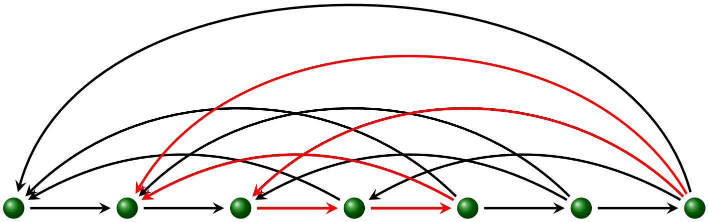

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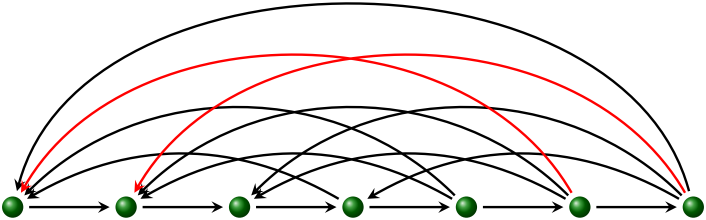

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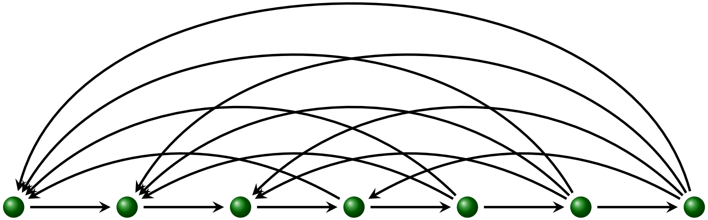

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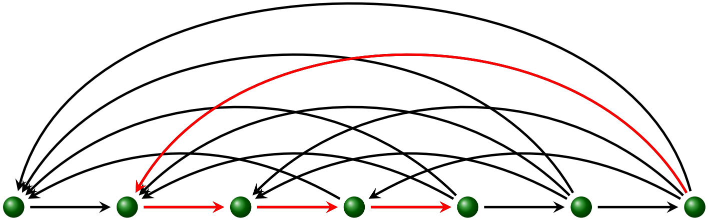

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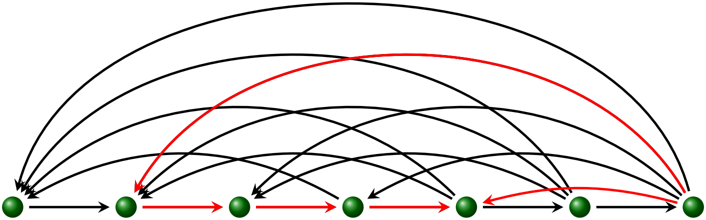

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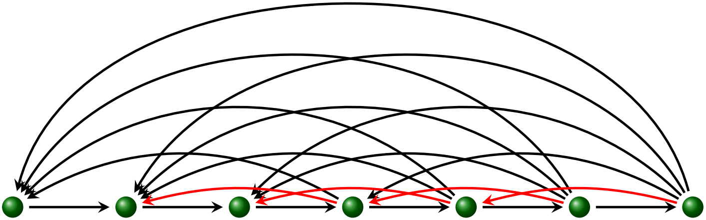

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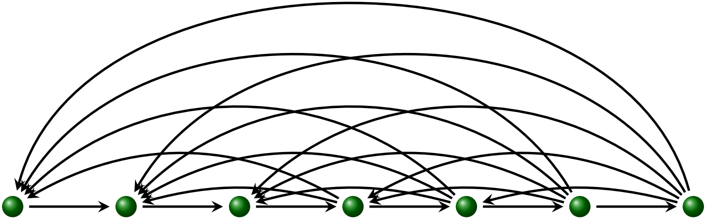

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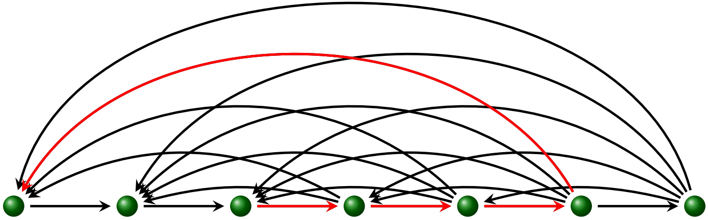

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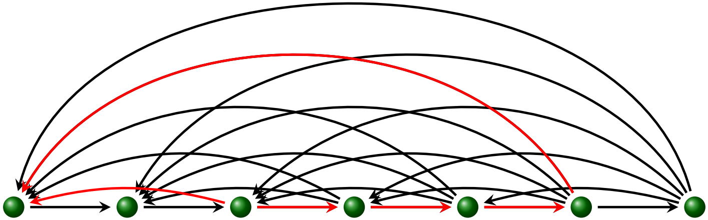

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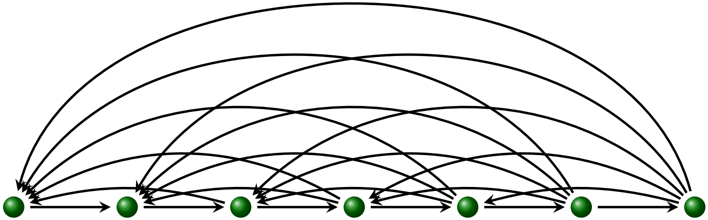

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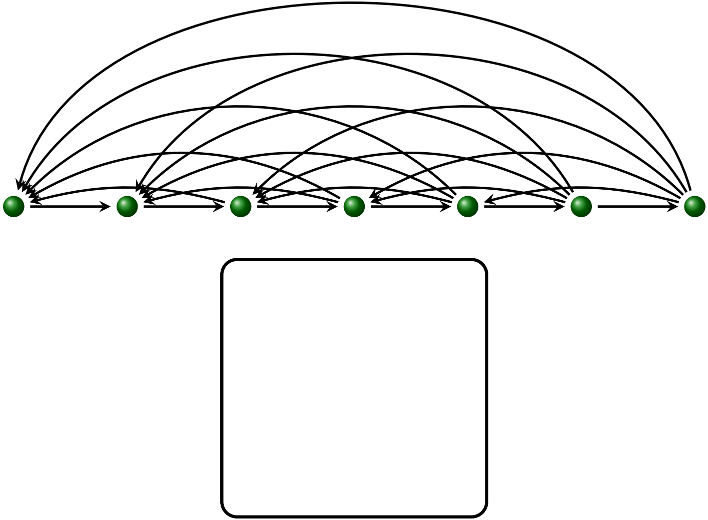

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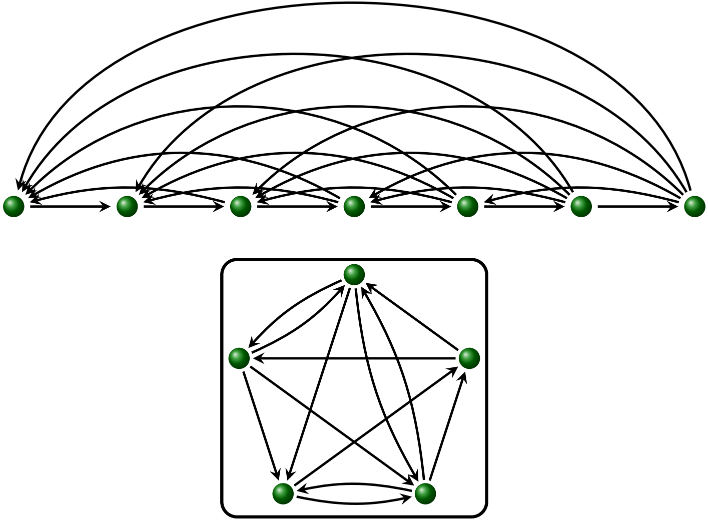
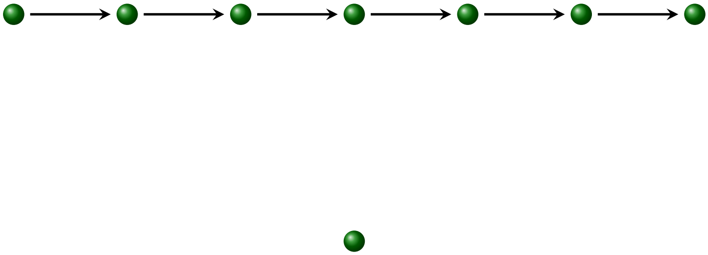


Theorem (Wang, Zhang, 2016)

Let k be an even positive integer. If D is a k-quasi-transitive strong digraph with diameter at least $k + 2$, then $V(D)$ can be partitioned into (V_1, V_2) such that $D[V_1]$ is hamiltonian and, $D[V_1]$, $D[V_2]$ are semicomplete.

Moreover ...

k-quasi-transitive digraphs



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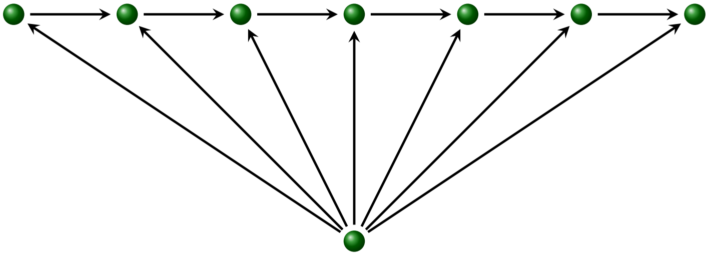

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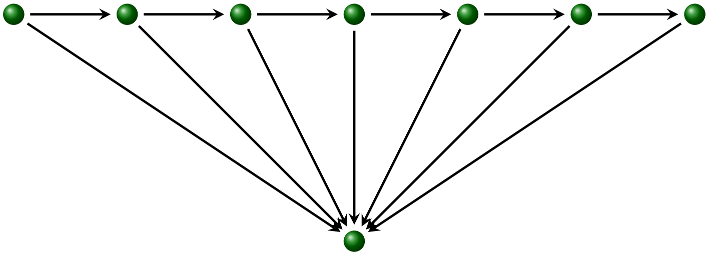

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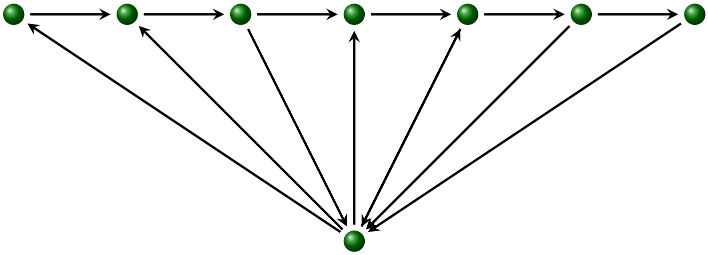

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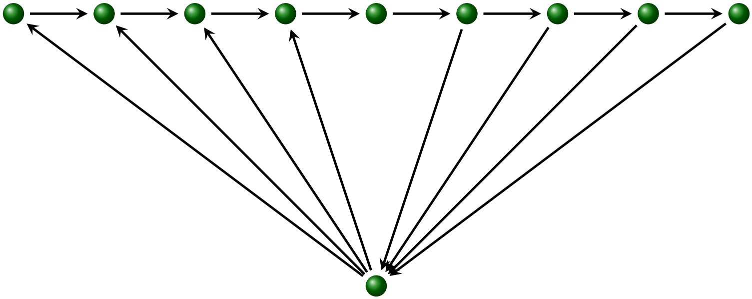
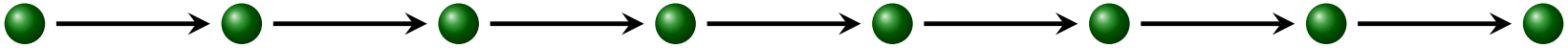


Corollary

Every 4-transitive strong digraph with diameter at least 6 is semicomplete.

k is odd



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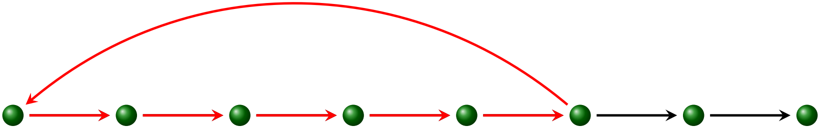

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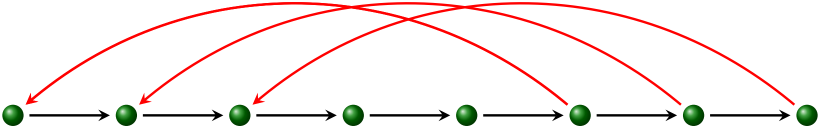

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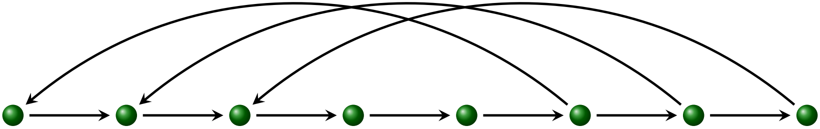

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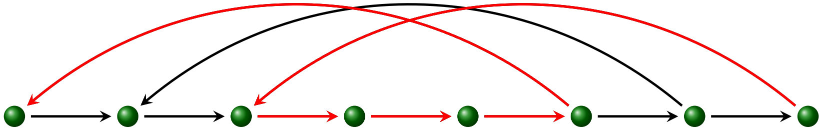

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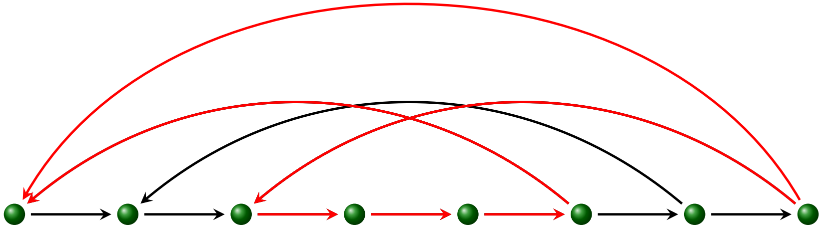

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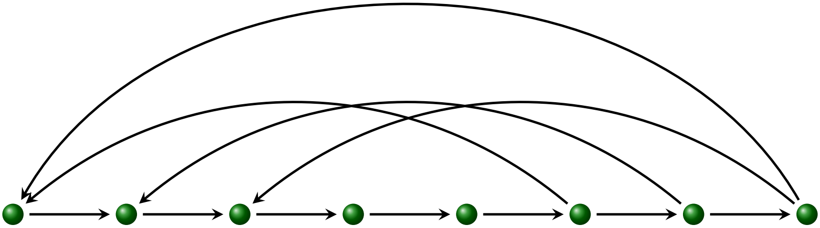

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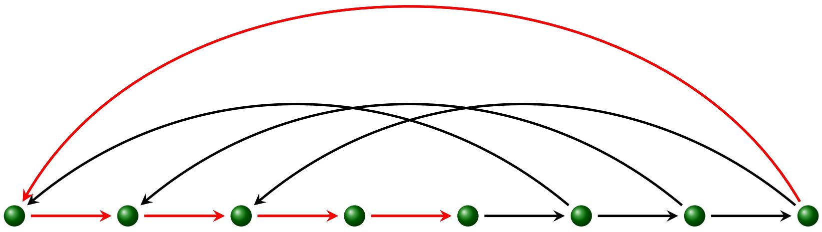

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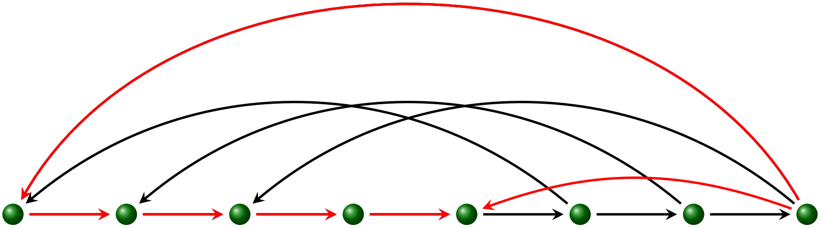

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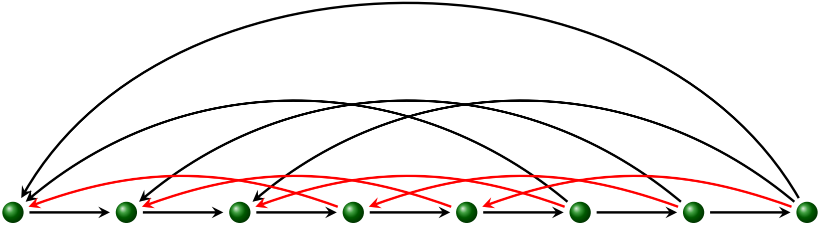

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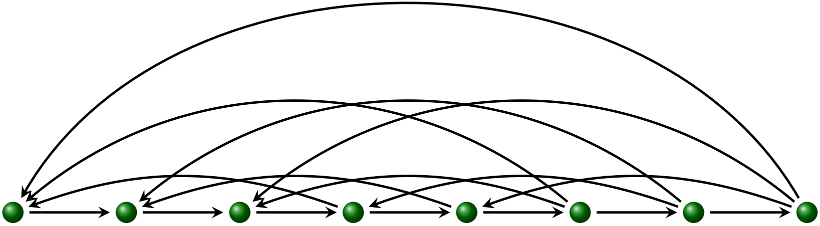

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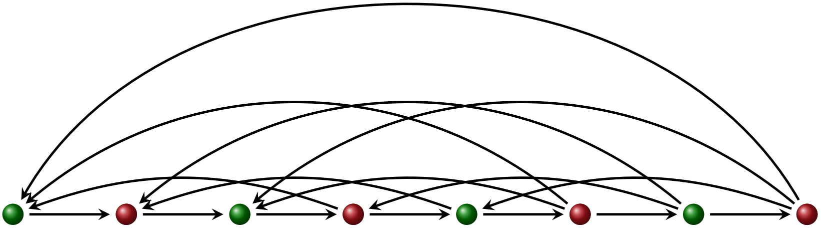

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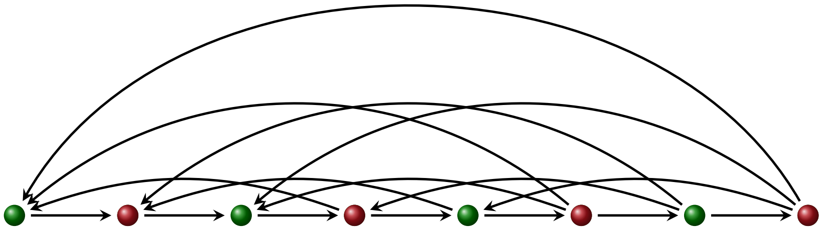

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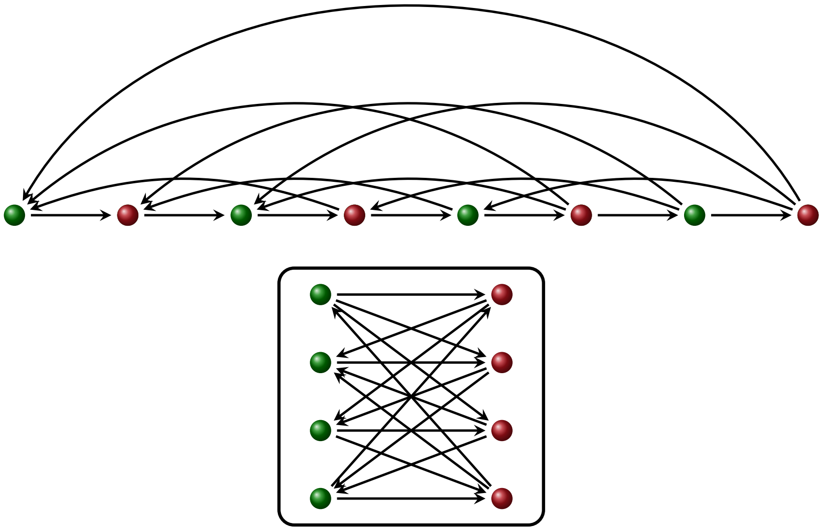

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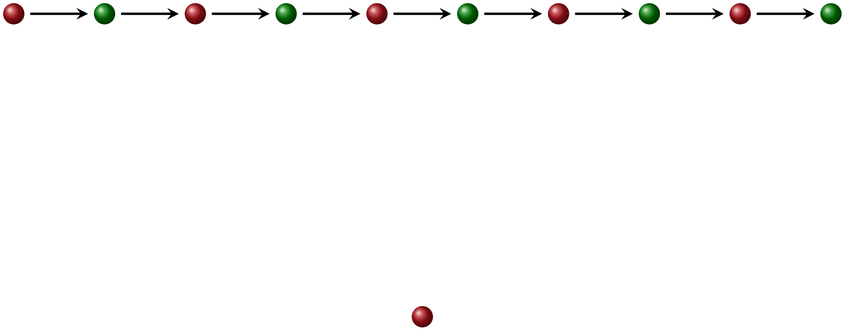

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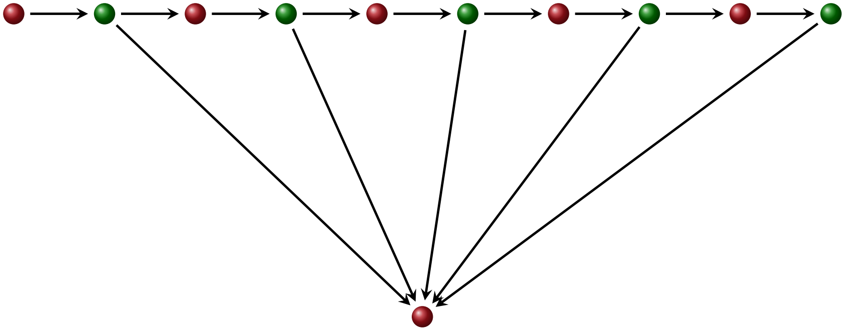

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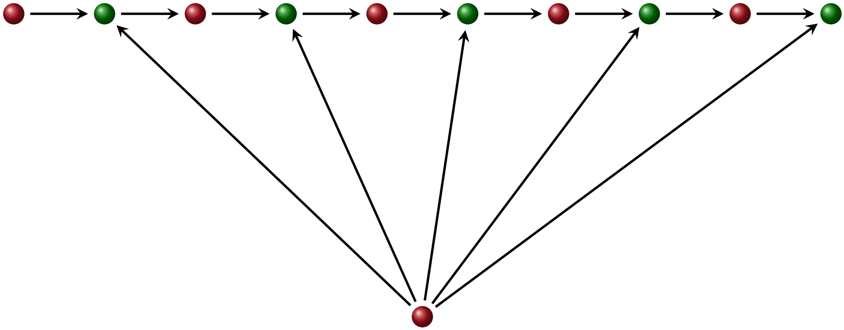

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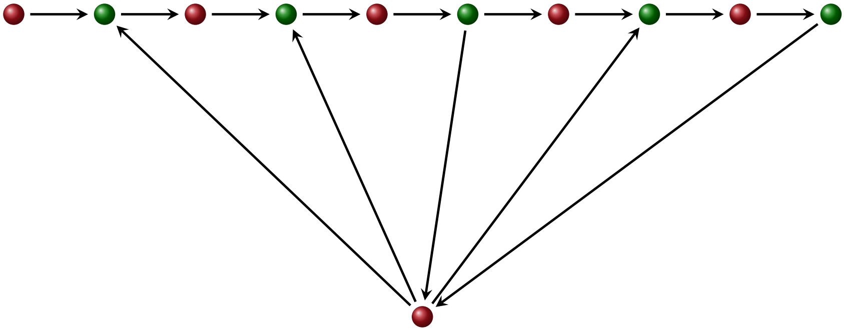

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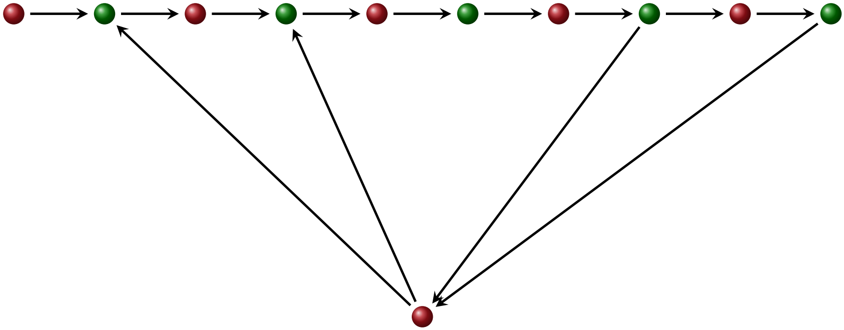

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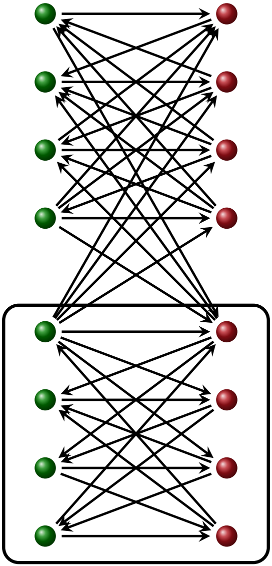

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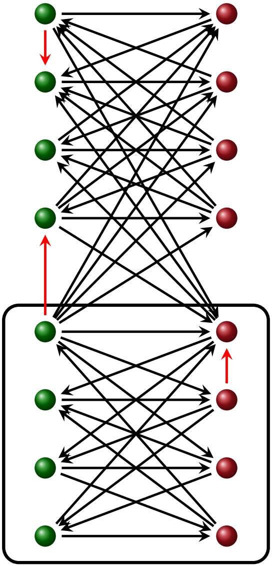

Theorem (Alva-Samos, HC, 2017+)

Let k be an odd integer, $k \geq 3$. If D is a k-quasi-transitive strong digraph with diameter at least $k + 2$, then $V(D)$ can be partitioned into (V_1, V_2) such that $D[V_1]$ is hamiltonian and,

- *$D[V_1]$, $D[V_2]$ are semicomplete bipartite if D is bipartite.*
- *$D[V_1]$, $D[V_2]$ are semicomplete, otherwise.*

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Corollary

If D is a 5-quasi-transitive strong digraph with diameter at least 7, then D is either semicomplete or biparite semicomplete.

Corollary

Let k be an odd integer, $k \geq 3$. If D is a non-bipartite k-quasi-transitive strong digraph with diameter at least $k+2$, then D has a hamiltonian path.

Corollary

If D is a 5-quasi-transitive strong digraph with diameter at least 7, then it can be determined in polynomial time whether D is hamiltonian (traceable). Moreover, if D is non-bipartite, then D is hamiltonian.

Problem (Wang, Zhang, 2016)

Let k be an even integer, $k \geq 4$. Is it true that every strong k-quasi-transitive digraph with diameter at least $k+2$ is hamiltonian?

Problem

Let k be an odd integer, $k \geq 3$. Is there a polynomial algorithm to determine hamiltonicity for k-quasi-transitive digraphs with diameter at least $k+2$?

Problem

Let k be an integer, $k \geq 3$. Is it true that a k-quasi-transitive strong digraph with diameter at least $k+2$ has a hamiltonian cycle if and only if it has a cycle factor?

¡Gracias!